Ambiguity and Overconfidence[†]

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Abstract

There are two phenomena in behavioral finance and economics which are seemingly unrelated and have been studied separately; overconfidence and ambiguity aversion. In this paper we are trying to link these two phenomena providing a theoretical foundation supported by evidence from an experimental study. We derive a model, based on the max-min ambiguity framework that links overconfidence to ambiguity aversion. In the experimental study we find that overconfidence is decreasing in ambiguity, as predicted by our model.

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1. Introduction

A common assumption in the neoclassical finance literature, dealing with risk tolerance, is that financial decision makers are able to precisely estimate the probability distribution of returns on assets. This literature includes the concept of ambiguity, or Knightian uncertainty, which assumes that we are not certain about the distribution of returns and that our decisions may be dictated by aversion to ambiguity. At the same time the behavioral finance literature has suggested alternative behavioral traits that may affect our decision making. A key behavioral trait is overconfidence. This begs the question; how are the two, ambiguity and overconfidence related?

The objective of this paper is to investigate the effect of ambiguity on the level of overconfidence in financial decision making.

The theoretical contribution of the paper is in modeling investment behavior under conditions of ambiguity. In reality, financial decisions are made under conditions of ambiguity, so that the expected relative performance of a portfolio is determined not only by its level of risk but also by the degree of ambiguity. We show that ambiguity affects the decision maker's expected performance, thereby affecting his investment decisions. We have conducted experiments that provide results consistent with the predictions of the theoretical model.

Our model assumes that the parameters (mean and variance) of the stochastic process of the decision maker's portfolio, as well as the parameters of the stochastic process of the comparative portfolio (benchmark), are ambiguous. While estimating the odds that his portfolio will beat a benchmark, the decision maker takes into account the ambiguity about the parameters.

The decision maker in our settings obeys the Gilboa and Schmeidler (1989) Max-Min model.[1] He holds two subjective sets of priors: one about his portfolio and the other about the benchmark portfolio. This also means that he estimates the odds conditional on the worst scenario with respect to his beliefs. That is, he acts as if his portfolio follows the worst prior in his subjective set of priors and the benchmark portfolio follows the best possible prior. The model predicts that the likelihood that a selected portfolio beats the benchmark portfolio increases with its mean, the benchmark's variance, and the correlation between these two portfolios. On the other hand, these odds decrease with the increase in the variance of the selected portfolio and the benchmark's mean.

Most research on ambiguity focuses on preferences towards ambiguity.[2] Ellbserg (1961), for example, demonstrates that individuals have a preference towards ‘games’ with known probabilities and are willing to pay in order to avoid ‘games’ with unknown (ambiguous) probabilities. Bossaert, Ghirardato, Guarneschelli and Zame (2009) study the impact of ambiguity aversion on equilibrium asset prices, and the relationship between attitudes toward risk and attitudes toward ambiguity.[3] Our paper focuses on decision makers' beliefs rather than

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[1] Theoretical models of decision making assuming ambiguity have been suggested by Gilboa and Schmeidler (1989), Schmeidler (1989), Ghirardato, Maccheroni and Marinacci (2004), Klibanoff, Marinacci and Mukerji (2005) and Epstein and Schneider (2007). Most of them assume that the decision maker has a subjective set of plausible priors regarding the distribution of returns, rather than a single prior.

[2] There is a long list of papers that examine the impact of preferences toward ambiguity on asset pricing. To mention a few; Epstein and Wang, (1994), Uppal and Wang (2003), Maenhout (2004), Cao, Wang and Zhang (2005), Leippold, Trojani and Vanini (2008), Anderson, Ghysels and Juergens (2009), Hansen and Sargent (2008), Easley and O'Hara (2009).

[3] A study by Hsu, Bhatt, Adolphs, Tranel and Camerer (2005), using fMRI technology, shows that risky and ambiguous choices are treated in different parts of the brain.

on their preferences. We investigate the effect of ambiguity on the subjective estimation of odds. Early work on parameter uncertainty (beliefs) was done by Brown (1979) and Bawa, Brown and Klien (1979).

In our paper we extract the decision maker's beliefs from his observed choices, i.e. it abstracts from the impact of the decision maker's preferences and concentrates on his perception of chances, and shows that those beliefs are related to overconfidence. Our results, using a behavioral experiment, are consistent with the above mentioned studies.

The behavioral experiment shows that indeed individuals assign a lower likelihood that their portfolio will outperform a benchmark portfolio when they are exposed to a higher degree of ambiguity.[4] Yet, our experiment demonstrates that individuals tend to overestimate their ability to make correct choices and exhibit significant overconfidence.[5]

In recent years the finance literature has incorporated overconfidence into financial models. In the asset pricing context the literature investigates the impact of overconfidence on trading volume, price volatility or momentum.[6] In the corporate finance context the literature investigates the impact that decisions by overconfident managers have on the corporation’s performance.[7]

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[4] There is growing experimental literature that investigates and provides insights about various aspects of ambiguity aversion. To name a few, Borghans, Golsteyn, Heckman and Meijers (2009), Chen, Katuščák and Ozdenoren (2007), Ho, Keller and Keltyka (2002). Ahn, Choi, Gale and Kariv (2009).

[5] Overconfidence is individuals’ tendency to overestimate their abilities and / or the quality of the information they have.

[6] See for example, Odean (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998) among others.

[7] See for example, Malmendier and Geoffrey (2008).

Unlike papers that investigate the outcome of overconfidence on financial decisions, our paper focuses on the sources of overconfidence ("better than average" phenomenon) and shows that ambiguity is negatively correlated with overconfidence. The "better than average" phenomenon which is the focus of this paper occurs when agents think that their own personal attributes and achievments are better on average than what the actual percentage is, by definition. For example, Svenson (1981) asked groups of subjects to compare their driving ability to their peers in a group. Around 70–80% rated themselves above the median of the group.[8] In the financial literature context Glaser and Weber (2007) find that overconfidence (in the form of "better than average" aspect) is associated with higher levels of online trading.

A paper that is related to our paper, Kogan (2009), investigates the source of overconfidence. It suggests that agents rationally "overweigh" their information; they are discounting the information signaled by others since it may be affected by non rational behavior.[9] Our approach, however, is different; we rely on the foundation of the ambiguity literature and develop a model that explains what may affect overconfidence and its magnitude. While we do not claim that ambiguity is the sole determinant of the level of overconfidence, we do provide a model and experimental evidence that suggest that ambiguity is indeed an important factor.

Our paper is also closely related to the "competence hypothesis" by Heath and Tversky (1991). People prefer to act in situations where they feel knowledgeable or competent than in

[8] Overconfidence ("better than average") phenomenon has been widely documented in various business and non business areas like health, Larwood (1978), managerial skills, Larwood and Whittaker (1977) and business success, Cooper, Woo, and Dunkelberg (1988) and Camerer and Lovallo (1999).

[9] Overconfidence is sometimes interpreted as "the investor's exaggeration of her information-processing ability", see Ping and Xiong (2006).

situations where they feel ignorant or incompetent. Our findings are consistent with Heath and Tversky (1991).

The paper is organized as follows. In section 2 we present a mathematical model of the expected likelihoods in an economy typified by ambiguity. In section 3 we provide an illustrative numerical example. Section 4 describes the behavioral experiment, the results and the insights it provides. Summary and conclusions are given in section 5.

2. The model

The decision maker's objective function is to select a portfolio (or single asset) that maximizes the probability that this portfolio will beat a benchmark portfolio (e.g. fund managers). We propose a model of a decision maker's subjective estimation of likelihoods. We first assume an economy without ambiguity (the odds are known) and then extend the model to the case of an economy typified by ambiguity (the odds are unknown).

2.1 A Non ambiguous environment

Consider, first, an environment without ambiguity and a decision maker who has to estimate the likelihood that his selected portfolio X will beat a benchmark portfolio B. Assume that the value S_i of each portfolio $i \in \{X, B\}$ follows a geometric Brownian motion dynamics with a drift μ_i and diffusion σ_i:

$$dS_i = S_i \mu_i dt + S_i \sigma_i dW_i, \qquad (1)$$

where t stands for the time and dW_i is a standard Wiener process.

Theorem 1

In an environment without ambiguity, a-priori, the likelihood that portfolio X will beat a benchmark portfolio B is

$$\Pr\left(X \geq B\right) = \begin{cases} \dfrac{1}{2} & \left| \begin{array}{l} \mu_X = \mu_B \ AND \\ \sigma_X = \sigma_B \ AND \\ \rho_{XB} = 1 \end{array} \right. \\ \mathrm{N}\left(\dfrac{\left(\mu_X - \mu_B\right) - \frac{1}{2}\left(\sigma_X^2 - \sigma_B^2\right)}{\sqrt{\sigma_X^2 + \sigma_B^2 - 2\rho_{XB}\sigma_X\sigma_B}} \sqrt{t} \right) & \mid otherwise \end{cases},$$

where $\mathrm{N}(\cdot)$ is the standard normal cumulative probability distribution, μ_X and μ_B are the means of the selected portfolio and the benchmark portfolio, respectively, σ_X^2 and σ_B^2 are their variances and ρ_{XB} is the correlation between these two portfolios.

Theorem 1 shows the probability of beating a benchmark portfolio typified by mean μ_B and variance σ_B^2. Clearly, the odds that a selected portfolio X beats the benchmark portfolio increases with its mean, μ_X , the benchmark's variance, σ_B^2, and the correlation between these two portfolios. On the other hand, these odds decrease with the variance of the selected portfolio, σ_X^2 , and the benchmark's mean, μ_B. Notice that in Theorem 1 we model beliefs and not preferences for risk as in the classical mean-variance model (Markowitz (1952, 1959), Sharpe (1970)). However, the decision maker's selection process is akin to the process that he would follow if he was maximizing a mean-variance preferences, where his optimal-portfolio maximizes $\mu_X - \frac{1}{2}\sigma_X^2$.

2.2. An ambiguous environment, max-min settings

Now, we consider an ambiguity-averse decision maker of the max-min type. We assume that this decision maker follows the Gilboa and Schmeidler (1989) max-min ambiguity model with multiple priors. This ambiguity approach assumes that investor's (decision maker's) beliefs about uncertainty are represented not as a single probability measure on the set of states (or outcomes) but instead as a subjective set of probability measures (multiple priors).

In selecting his portfolio, the decision maker is making his choice based on the worst possible prior. Since, as discussed above, maximizing probabilities is equivalent to maximizing mean-variance preferences, his choice follows the condition

$$\max_{\lambda} \min_{\Pi \in \Psi} \Pr\left(X \geq B \,|\, \Pi, \lambda \right), \tag{2}$$

where λ stands for the composition of the selected portfolio and Π is a probability measure out of the set of possible priors Ψ. That is, the decision maker chooses the portfolio λ which maximizes his chances to outperform the benchmark, conditional on the worst prior in his subjective set of possible priors. [10]

Since the decision maker is unsure about the probability distribution that governs the outcome of his selected portfolio and the probability distribution that governs the benchmark portfolio, his beliefs on the outcomes of the two portfolios are captured by two closed and convex sets of probability measures. Ψ_X is the set of possible priors regarding the outcome of

[10] Garlappi, Uppal and Wang (2007) also extend the mean-variance model to multiple priors using the Gilboa-Schmeidler (1989) max-min model. However, they use it to find an optimal portfolio under uncertainty, while we use a similar extension to evaluate its effect on beliefs, i.e. the odds of possible outcomes.

the decision maker's selected portfolio X and Ψ_B is the set of priors the investor thinks are possible for the benchmark portfolio B. To simplify matters, we assume that all probability distributions are normally distributed and distinguished by their mean and variance.

From the decision maker's point of view the worst possible scenario occurs when his selected portfolio follows the worst prior in his subjective set of priors Ψ_X and the benchmark portfolio follows the best possible prior in Ψ_B. While "worst" refers to the portfolio that is most likely to have a low mean and high variance, "best" refers to the portfolio that is most likely to have a high mean and low variance. Notice that those two priors are determined simultaneously by their mean and the variance and the correlation between the two comparative portfolios.

By definition**,** imposing the max-min settings on Theorem 1 gives the decision maker's estimation of his chances to beat a benchmark portfolio in an ambiguous environment as presented in the following proposition.

Proposition 1

In an ambiguous environment the likelihood that portfolio X will beat a benchmark portfolio B is

$$\Pr\left(X \geq B\right)=\begin{cases} \dfrac{1}{2} & \left| \begin{array}{l} \underline{\mu}_X = \bar{\mu}_B \ AND \\ \underline{\sigma}_X = \bar{\sigma}_B \ AND \\ \rho_{XB} = 1 \end{array} \right. \\ \mathrm{N}\left(\dfrac{\left(\underline{\mu}_X - \bar{\mu}_B\right) - \frac{1}{2}\left(\underline{\sigma}_X^2 - \bar{\sigma}_B^2\right)}{\sqrt{\underline{\sigma}_X^2 + \bar{\sigma}_B^2 - 2\rho_{XB}\underline{\sigma}_X\bar{\sigma}_B}} \sqrt{t} \right) & \left| \ otherwise \right. \end{cases},$$

where $\underline{\mu}_X$ and $\underline{\sigma}_X^2$ characterize the worst prior in Ψ_X and, $\bar{\mu}_B$ and $\bar{\sigma}_B^2$ characterize the best *prior in* Ψ_B.[11]

The level of ambiguity is determined by the breadth of the set of possible priors. We say that environment Y is more ambiguous than environment X if $\Psi_X \subseteq \Psi_Y$, where Ψ_i is the set of priors which are considered relevant to environment $i \in \{X, Y\}$. That is, the subjective set of priors attributed to environment Y consist of priors that are considered implausible in environment X.

Proposition 1 states that when the investor is more ambiguous regarding the probabilities of outcomes on his portfolio, say $\Psi_X \subseteq \Psi_Y$, then he evaluates chances conditional on the worst prior of the set Ψ_Y. That is, he calculates the chances using $\mu_Y \leq \mu_X$ or $\sigma_Y \geq \sigma_X$ or both, such that $\Pr(Y \leq B) \leq \Pr(X \leq B)$. Furthermore, if the probabilities of the benchmark portfolio become more ambiguous, $\Psi_B \subseteq \Psi_{B'}$, then he calculates his chances conditional on the best prior in B', i.e. $\mu_{B'} \geq \mu_B$ or $\sigma_{B'} \leq \sigma_B$ or both, such that $\Pr(X \leq B') \leq \Pr(X \leq B)$.

Figure 1 gives a diagrammatic representation of our model. In this diagram, as in our behavioral experiment there are two comparative groups, i.e. two benchmarks: the decision maker's class and his school (class and school will be defined later). In that case $\Psi_{\text{class}} \subset \Psi_{\text{school}}$, which means that the decision maker is more ambiguous about (feels less familiar with) the performance of school compared to the performance of his class. Accordingly, the model

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[11] We implicitly assume that the decision maker assigns a unique correlation for each pair of priors (one of his portfolio and the other of the benchmark portfolio). We can generalize our model to the case of ambiguous correlation, where in that case the decision maker maximizes his objective function conditional on the lowest possible correlation.

predicts that such a decision maker will give a higher likelihood that his portfolio will outperform the "class benchmark" than outperform the "school benchmark": $\mathrm{P}\left(R_{\mathrm{X}} \geq R_{\mathrm{Class}}\right) \geq \mathrm{P}\left(R_{\mathrm{X}} \geq R_{\mathrm{School}}\right)$.

[[INSERT FIGURE 1]]

3. An illustrative example

The purpose of the numerical example is to link the model to the experiments, which are described later. The experiments were conducted in business school classes attended by graduate and undergraduate students. In the experiments, the students play the role of investors.

Consider an investor who exhibits ambiguity-aversion. For simplicity, assume that the investor is not exposed to ambiguity regarding his own selected portfolio but only regarding the benchmark portfolio. This means that he knows precisely the characteristics of the random process that governs his selected portfolio, $\left\{\left(\mu_X = 4, \sigma_X = 3\right)\right\}$, but is ambiguous with regard to the parameters of the benchmark portfolio.

Assume that the investor is asked to estimate his chances to beat a benchmark-portfolio which is the 9th highest decile portfolio of all portfolios composed by the investors in a reference group. Assume first that the reference group is his classmates. The portfolios of the reference group are allocated by deciles based on the max-min criterion where we choose the portfolios with the best possible performance so that the chances to outperform this benchmark are the lowest. The investor is ambiguous about the characteristics of the benchmark portfolio (the portfolio which performs better than 90% of the portfolios of his classmates). Lets say that the set of plausible probability distributions is $\left\{\left(\mu_B=5,\sigma_B=2\right),...,\left(\mu_B=4,\sigma_B=3\right)\right\}$. That is, the closed and convex set of priors contains all the distributions "between" the worst distribution $\left(\mu_B=4,\sigma_B=3\right)$, and the best distribution, $\left(\mu_B=5,\sigma_B=2\right)$.

The investor assumes that his portfolio is uncorrelated with the benchmark portfolio, $\rho_{XB}=0$. According to Proposition 1, the investor evaluates his chances by his worst prior (in this case, it is the only one), i.e. $\left(\mu_X=4,\sigma_X=3\right)$, and the best prior of the reference group, i.e. $\left(\mu_B=5,\sigma_B=2\right)$. Thus, his subjective odds of outperforming the benchmark portfolio are 36.44%.

Now assume that the reference group is not the investor's class but his school. He feels more ambiguous with regard to the performance of the portfolios chosen by the school population than by the portfolios chosen by his class mates. In that case he is considering additional probability distributions as plausible ones, such that the school's benchmark-portfolio is characterizes by the set of possible parameters: $\left\{\left(\mu_B=6,\sigma_B=1\right),...,\left(\mu_B=5,\sigma_B=2\right),...,\left(\mu_B=4,\sigma_B=3\right)\right\}$. Formally, $\Psi_{\text{class}}\subset\Psi_{\text{school}}$. The school's set of priors consists of priors which, from the investor's point of view, are worse than all the previous priors. His own prior is the same as before, $\left(\mu_X=4,\sigma_X=3\right)$, but the benchmark’s best possible prior is now $\left(\mu_B=6,\sigma_B=1\right)$. Accordingly, the investor’s subjective chances are now 22.39%. Our model predicts that $\text{P}\left(R_{\text{X}}\geq R_{\text{Class}}\right)=36.44\%\geq 22.39\%=\text{P}\left(R_{\text{X}}\geq R_{\text{School}}\right)$, which means that the investor assigns a higher probability that his portfolio will outperform the "class benchmark" compared with the "school benchmark".

The model predictions are tested next in an experimental setting.

4. Experimental design and results

In this section we are describing the experiments that we conducted, the tests and the results.

4.1 The experimental design

The experiments were conducted in a leading private university in the US and in a leading public university in Israel. The participants were undergraduate (4 groups; 2 in each university) and MBA students (2 groups in the US); see Table 1. The purpose of the experiments was to find out the students' assessment of their ability to make investment decisions that provide better results than their peers (classmates, schoolmates). [12]

[[INSERT TABLE 1]]

[12] Initially the questions were part of the homework assignments in the "introduction of finance" / "introduction of financial markets" courses and were not intended to be used in research. In all courses the students received homework credit as part of their grade. We assume that most students treated the experiment seriously and devoted time and attention to their answers. When we later decided that their responses could be used for research we asked the students for permission. No students have objected that we use their answers for our research.

The students were asked to answer three different questions: in the first question we used a local, well known, stock index (the Dow in the U.S and the TA25 in Israel[13]) as the reference index in the investment exercise and the comparison group for each student was his or her classmates.[14] The students answered the following question: "Estimate (according to your own judgment) the probability that the adjusted return of your portfolio will be at the highest 10% of the class".[15]

In the second question the index was again a local, well-known, stock index but the comparison group was the total population of students that studied at the same time the same core class (not his/ her classmates). In the third question the index was a foreign index (the German DAX) while the comparison group was his or her classmates. We can classify our experiments by the following combinations of the reference index and comparison group: (local index, classmates), (local index, schoolmates) and (foreign index, classmates).

Several studies have examined the notion of 'familiarity', or 'home bias', and its effect on financial decision making. Kilka and Weber (2000), for example, conducted an experiment investigating the attitude toward investments in local stocks versus foreign stocks. They found that people feel more competent making investment decisions regarding local stocks than foreign stocks.[16] Huberman (2001) finds that "people invest in the familiar while often ignoring the principles of portfolio theory" (p. 659).

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[13] TA25 is a stock index consisting of the 25 largest companies in Tel-Aviv stock exchange.

[14] The full text of the questions is given in Appendix II.

[15] The adjusted return was with respect to systematic risk using Treynor's ratio (in one class the students used the Sharp ratio).

[16] See also Coval and Moskowitz (1999), Kang and Stulz (1997).

Our experiments are related to these studies yet are different in two aspects. First we ask the participants to assess their performance relative to their peers and not just making a comparison of local vs. foreign assets. Second, as our experiments try to link competence with the level of ambiguity, we presented the participants with two different foreign vs. local questions. In the local index vs. the foreign index one may think that it is not a matter of ambiguity, but rather the illusion of having better information regarding the local market. Yet, in comparing a given section (class) of the course with the total student population at the same school (same cohort), there is no reason to assume that the abilities of the students to pick stocks in one particular section of the core classes are different from the abilities of other students of the core courses. Hence, finding differences in the assessment of the classmates abilities to beat the market vs. the abilities of the school wide cohort could be interpreted as evidence that ambiguity plays a role in the confidence evaluations.

Our paper is also closely related to ‘competence hypothesis’ by Heath and Tversky, (1991): People prefer to act in situations where they feel knowledgeable or competent than in situations where they feel ignorant or incompetent. While the comparison between local index versus foreign index can be attributed to the competence hypothesis, the comparison between the classmates and the school population does not fall into this category. Our findings support Heath and Tversky (1991) but suggest that ambiguity aversion is beyond competence illusion but rather "familiarity" per-se.

To sum, our experiments are designed such that in one setting the ambiguity can be associated with ‘familiarity’ with the investment (local index vs, foreign index) while in the second setting we test ambiguity associated with ‘familiarity’ with other investors. The behavioral experiments indicate that when investors are asked to estimate the likelihood that they will perform better than a comparison group, their estimation is affected by their perceived familiarity regarding the financial asset and regarding the comparison group.

4.2. The experimental results

Table 2 provides summary statistics of the results of all the experiments that were conducted. In general, on average, the participants in the experiments estimate their chance of being at the top 10% of the class to be significantly higher than 10%. Sometimes the overestimation is, on average, three times larger than 10%. By definition, only 10% can be at the top 10% of the population.

[[INSERT TABLE 2]]

Figure 2 provides the frequency distribution of the likelihood of outperforming one's classmates when the stocks in the portfolio are local stocks. Figure 3 provides the frequency distribution of the likelihood of outperforming one's schoolmates when the stocks in the portfolio are local stocks while figure 4 provides the frequency distribution of the likelihood of outperforming one's classmates when the stocks in the portfolio are foreign stocks. The distribution is divided into deciles. The mode is between 10 and 20 percent.

[[INSERT FIGURES 2-4]]

Table 3 summarizes the results of the 3 questions that were presented to the participants in the experiments. The mean estimate of the likelihood of being in the top 10% of the class, when the investor selects from local stocks (question 1) is between 28% and 43% (across the 6 classes) which is significantly different than 10%. The mean estimate of the likelihood of being in the top 10% of the school, when the investor selects from local stocks (question 2) is between 25% and 35% (across the 6 classes). The mean estimate of the likelihood of being in the top 10% of the class, when the investor selects from foreign stocks (question 3) is between 24% and 37% (across the 6 classes) which is significantly different than 10%.

[[INSERT TABLES 3]]

In table 4 we present the results of the test that compare the responses of the participants to the questions. In particular, we are interested in the difference between their response to question 2 vs. 1 and 3 vs. 1. First, we test whether they are more confident of their performance when compared to their classmates, rather than the wider population of their schoolmates. Second, we test whether they are more confident of their performance, relative to their classmates, when they pick stocks from the local-familiar market, rather than from a foreign market.

On the average, across all participants, the estimated likelihood of being in the top 10% was higher, and significant, by about 4.6% (it was between 2.8% and 7.5% across the 6 classes and statistically significant in 5 out of 6 classes) when the class used a local index compared with the rest of the core students (school) using the local index as the reference index. In the third column of table 4 we present the results of the difference between the estimated likelihood of being in the top 10% when one selects from local stocks compared with foreign stocks. We find that in all classes the participants have exhibited more confidence choosing from local stocks compared to foreign stocks. The difference ranges from 3.6% to 9.5%, all statistically significant.

This was true regardless if the questions were answered by undergraduate students or M.B.A students and if it was answered by students at a leading US private university or at a leading public Israeli university. While in comparing the responses to question 3 with the responses to question 1, it could be argued that information may have affected the students' choices, this can't be argued about question 2 versus question 1. There was no reason to assume that the students that attended a particular section of the foundation of finance class were any different than those that attended other sections. The only difference is that they were familiar with the faces or names of their classmates while they are probably much less familiar with the students that took the same course in a different class.

[[INSERT TABLES 4]]

Figure 5 and 6 are a diagrammatic representation of the experimental results presented in tables 3 and 4. Each blue diamond in the graphs represents a specific group. The horizontal (vertical) red line represents the unbiased estimate of being in the highest 10% of the school (class). The pink line is a simple 45 degree line, and the black solid line is the linear trend-line. It can be observed that all 6 groups are above the 45 degree line.

The x-axis (y-axis) in Figure 5 describes the average estimated (subjective) likelihood that members of a group will be at the highest 10% of their school (class) when their reference market is the local one. That is, the average of the estimated (subjective) likelihood of being at the highest 10% of the class was higher than the average likelihood of being at the highest 10% of the school in all the 6 tested groups. The trend-line demonstrates that the "level of overconfidence" is preserved, from class to school. That is, if the magnitude of the bias of the estimated chances to be at the highest 10% of the class is high (low) then the magnitude of bias of the estimated chances to be at the highest 10% of the school is also high (low) and vice versa.

[[INSERT FIGURES 5]]

The x-axis (y-axis) in Figure 6 describes the average estimated (subjective) likelihood that members of a group will be at the highest 10% of their class when their reference market is the foreign market (local market).

[[INSERT FIGURES 6]]

Our experimental results indicate that ambiguity and not just knowledge affects the level of overconfidence, which in turn affect many of our decision making processes.

5. Summary and Conclusions

There are two phenomena in behavioral finance and economics which have been studied separately; overconfidence and ambiguity aversion. The objective of this paper is to investigate the effect of ambiguity on the level of overconfidence in financial decision making. The theoretical contribution of the paper is in modeling investment performance under conditions of ambiguity. The decision maker in our settings obeys the Gilboa and Schmeidler (1989) Max-Min model. He holds two subjective sets of priors: one about his portfolio and the other about the benchmark portfolio. That is, he acts as if his portfolio follows the worst prior while the benchmark portfolio follows the best possible prior.

We show that ambiguity affects the decision maker's expected performance, thereby affecting his investment decisions. The model predicts that the likelihood that a selected portfolio beats the benchmark portfolio increases with its mean, the benchmark's variance, and the correlation between these two portfolios. These odds also decrease with the variance of the selected portfolio and the benchmark's mean.

To test the model's predictions we have conducted experiments with graduate and undergraduate students in two countries. The behavioral experiments show that indeed individuals assign a lower likelihood that their portfolio will outperform a benchmark portfolio when they are exposed to a higher degree of ambiguity. Yet, our experiments demonstrate that individuals tend to overestimate their ability to make correct choices and tend to exhibit significant overconfidence. The results that we obtained are consistent with the predictions of our theoretical model.

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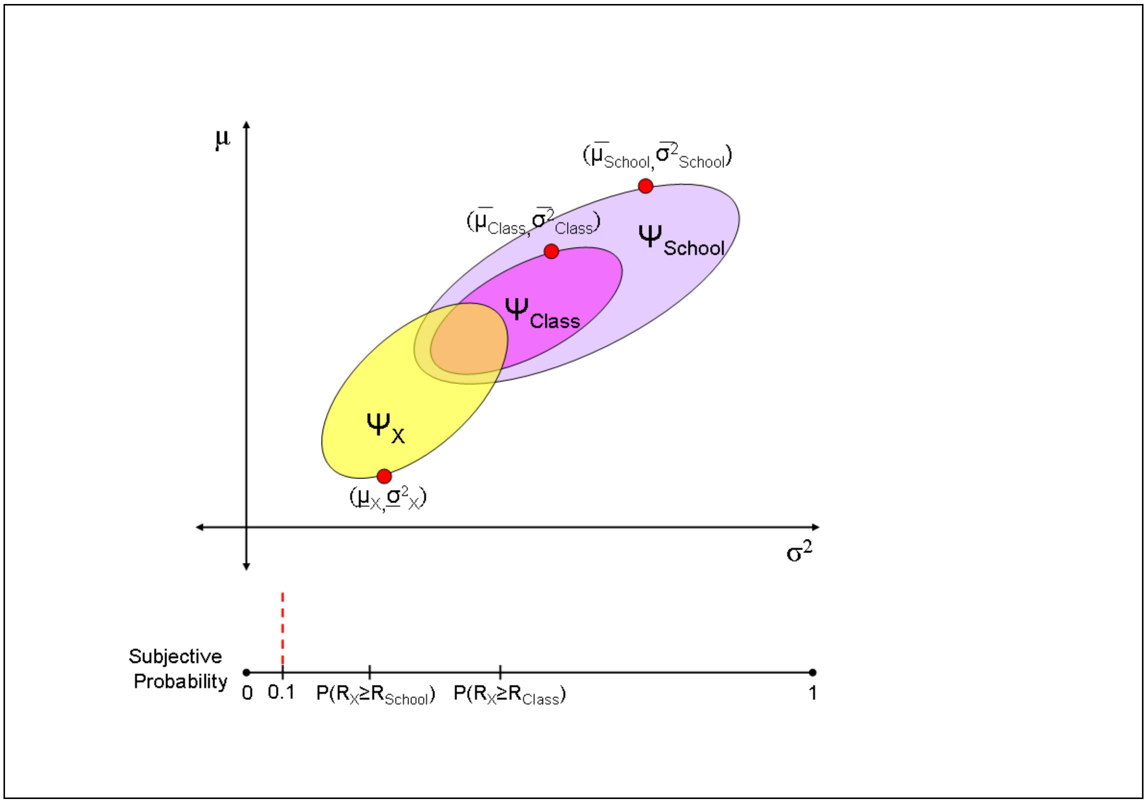

Fig. 1. Diagrammatic representation of the model; the sets of priors (believes). The yellow region contains all the priors that the decision maker considers plausible, regarding his own portfolio. The pink region contains all the priors that the decision maker considers plausible, regarding the class benchmark portfolio. The purple region contains all the priors that the decision maker considers plausible, regarding the school benchmark portfolio. The red dots indicate the priors that the decision maker uses to make his decisions.

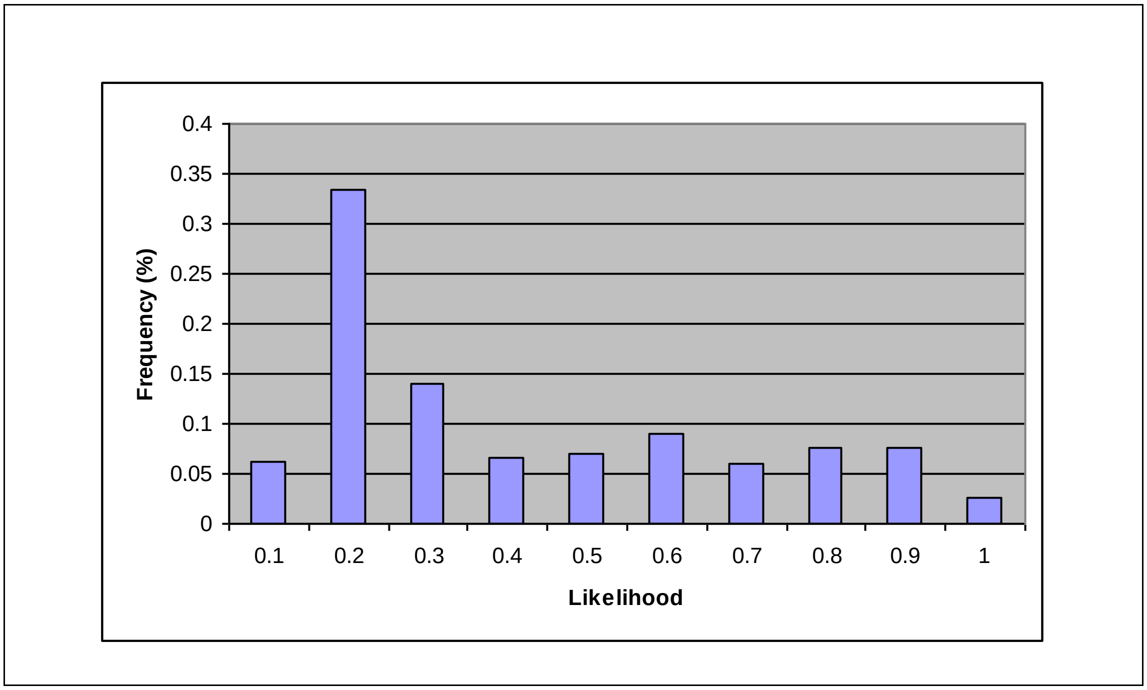

Fig. 2. The frequency distribution of the likelihood of outperforming one's classmates when the stocks in the portfolio are local stocks (i.e. Dow Jones for U.S. investors, Tel-Aviv 25 for Israeli investors). The distribution is divided into deciles. The mode is between 10 and 20 percent.

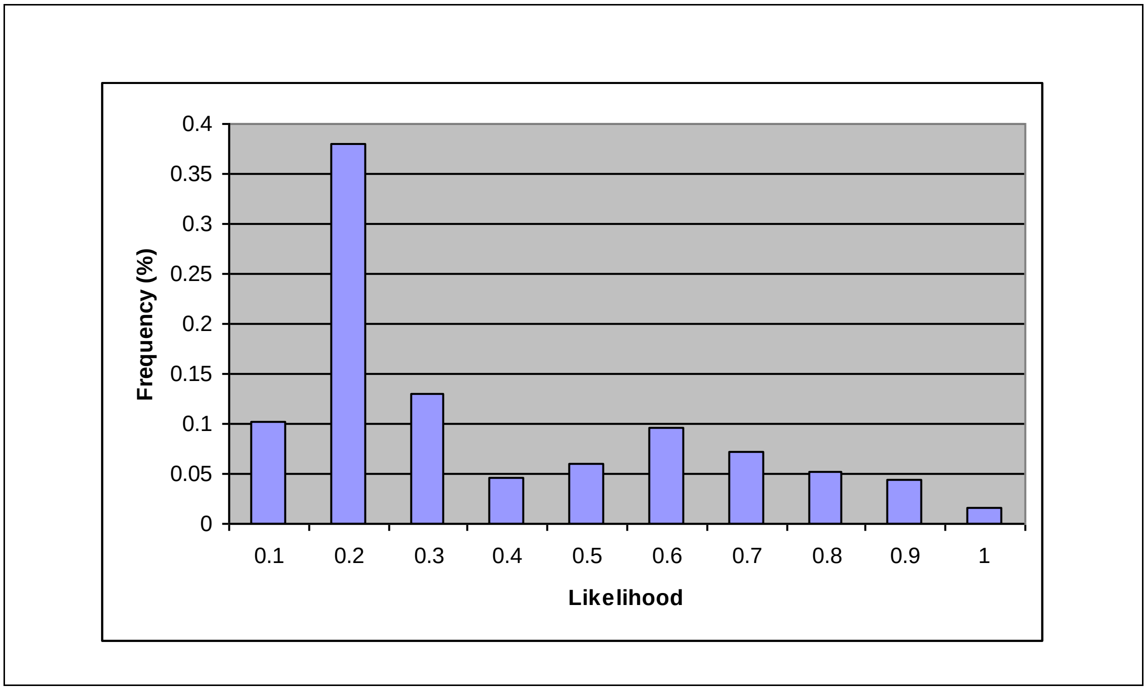

Fig. 3. The frequency distribution of the likelihood of outperforming one's schoolmates when the stocks in the portfolio are local stocks (i.e. Dow Jones for U.S. investors, Tel-Aviv 25 for Israeli investors). The distribution is divided into deciles. The mode is between 10 and 20 percent.

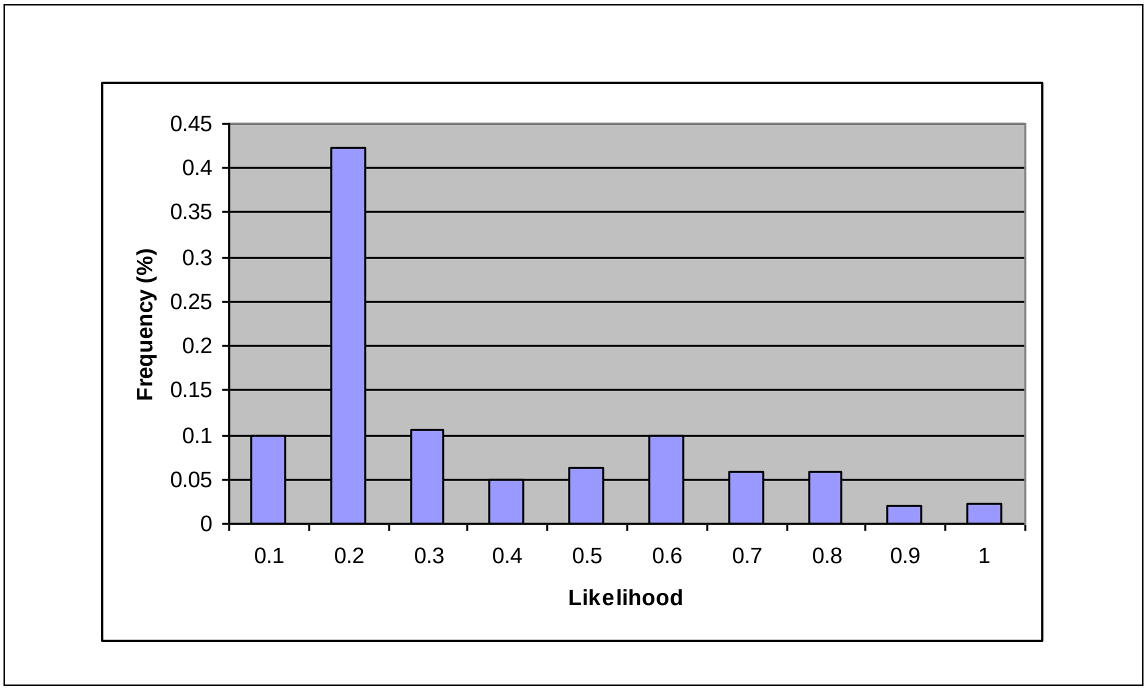

Fig. 4. The frequency distribution of the likelihood of outperforming one's classmates when the stocks in the portfolio are foreign stocks (i.e. The German DAX for non German investors). The distribution is divided into deciles. The mode is between 10 and 20 percent.

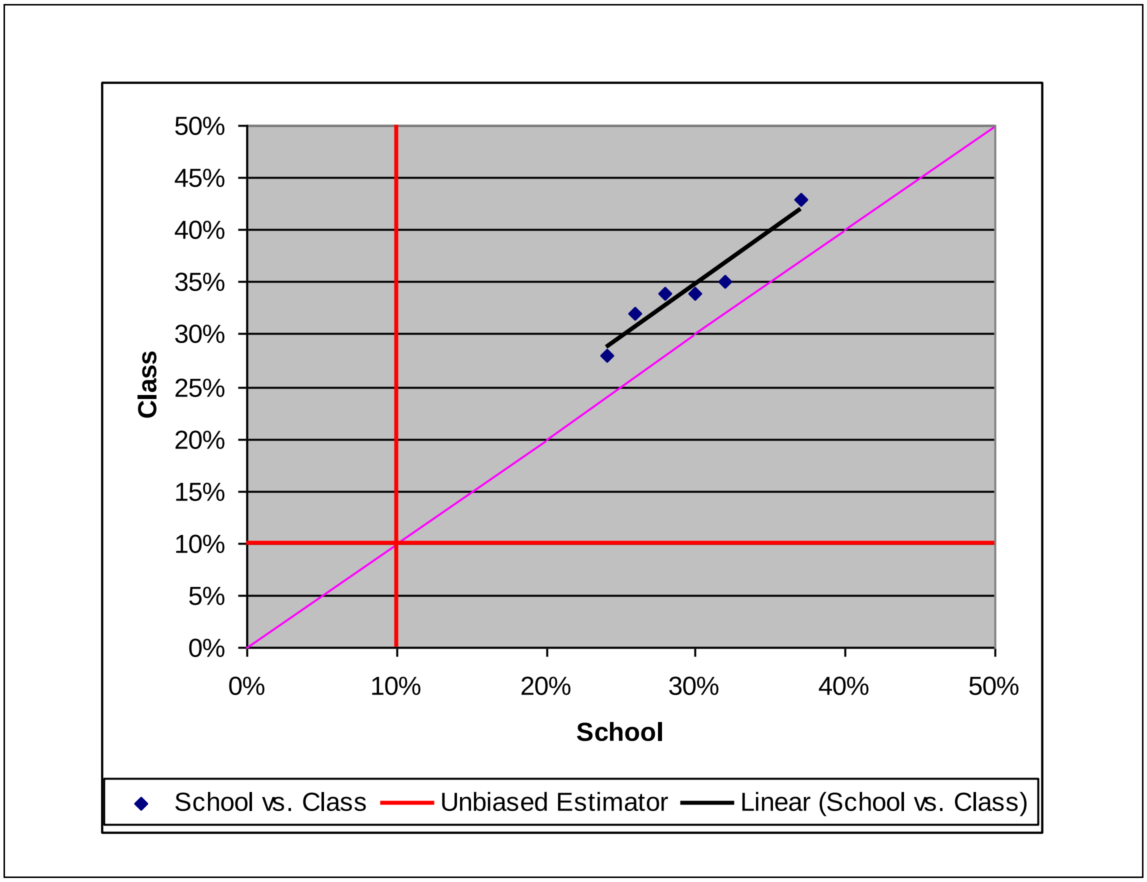

Fig. 5. Diagrammatic representation of the experimental results. The x axis is the average estimated probability of being in the highest 10% of the school. The y axis is the average estimated probability of being in the highest 10% of the class. The two red lines (horizontal and vertical) stand for the unbiased estimator (10%). The pink, diagonal line, represents the anticipated result if there is no difference in ambiguity between class and school. The blue dots are the results of the 6 experiments.

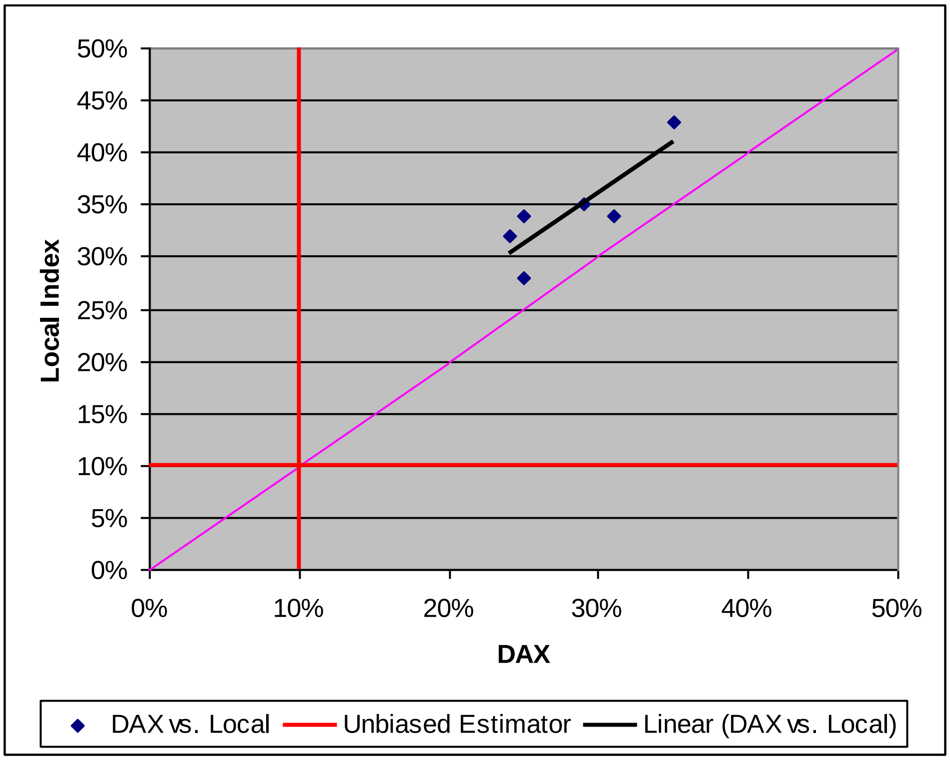

Fig. 6. Diagrammatic representation of the experimental results. The x axis is the average estimated probability of being in the highest 10% of the class when the investment objects are foreign stocks (DAX). The y axis is the average estimated probability of being in the highest 10% of the class when the investment objects are local stocks. The two red lines (horizontal and vertical) stand for the unbiased estimator (10%). The pink, diagonal line, represents the anticipated result if there is no difference in ambiguity between class and school. The blue dots are the results of the 6 experiments.

Table 1

Experiment participants by school and degree

The participants in the experiment where undergraduate and graduate students. The graduate students come from a leading US business school. The undergraduate business students come from leading schools in the US and Israel. There were a total of 303 participants who formed 6 groups.

Group #	Degree	Location	# Students
1	Undergraduate	Leading Private University US	50
2	Undergraduate	Leading Private University US	52
3	MBA	Leading Private University US	45
4	MBA	Leading Private University US	31
5	Undergraduate	Leading Public University Israel	37
6	Undergraduate	Leading Public University Israel	88

Table 2

Summary Statistics of the Survey Results

The table reports the mean, median, standard deviation, skewness and kurtosis of the answers that 6 groups of students gave regarding their relative expected performance in making investment decisions. The students were asked 3 questions about their ability to perform better than their peers. The answers were given in percentages.

Question	Group	# Students	Median	Mean	Variance	Skewness	Kurtosis
1	1	50	0.2500	0.3423	0.0751	0.8565	-0.3929
	2	52	0.2250	0.3199	0.0616	1.0247	-0.0265
	3	45	0.2500	0.3508	0.0862	0.5314	-1.3861
	4	31	0.1200	0.2779	0.0819	1.3001	0.1737
	5	37	0.4500	0.4286	0.0661	0.2023	-1.1508
	6	88	0.2250	0.3378	0.0644	0.5861	-1.0844
	Total	303	0.2500	0.3424	0.0711	0.6870	-0.8780
2	1	50	0.1750	0.2763	0.0621	1.3189	0.9013
	2	52	0.1500	0.2643	0.0567	1.0682	-0.1892
	3	45	0.1500	0.3175	0.0811	0.6919	-1.1207
	4	31	0.1000	0.2496	0.0796	1.3883	0.2574
	5	37	0.3500	0.3538	0.0448	0.4687	-0.3975
	6	88	0.2000	0.3067	0.0592	0.8015	-0.6106
	Total	303	0.2000	0.2959	0.0627	0.8998	-0.4527
3	1	50	0.1500	0.2469	0.0427	1.2252	0.6888
	2	52	0.1500	0.2396	0.0434	1.3321	0.7941
	3	45	0.1000	0.2873	0.0695	0.8675	-0.6221
	4	31	0.1000	0.2418	0.0683	1.6156	1.2330
	5	37	0.4000	0.3685	0.0693	0.4343	-1.0748
	6	88	0.2000	0.2968	0.0575	0.8201	-0.5113
	All	303	0.1500	0.2805	0.0577	0.9808	-0.2504

Table 3

Test Statistics of the Hypothesis regarding their expected performance, relative to their peers.

The table reports the results of the significance tests of the hypothesis that investors are overconfident regarding their performance in making investment decisions. The null hypothesis is that their chance to perform better than the highest 10% is no more than 10%.

The top number in each cell is the **mean** estimated likelihood of being in the top 10 % of the class/school. The number below the mean, is the **t-statistic**. All the results are significant at the 5 percent level.

	Mean (t-stat)		
Group #	**Question 1**	**Question 2**	**Question 3**
1	0.3423 (6.1899)	0.2763 (4.9520)	0.2469 (4.9779)
2	0.3199 (6.3291)	0.2643 (4.9267)	0.2396 (4.7880)
3	0.3508 (5.6676)	0.3175 (5.0662)	0.2873 (4.7142)
4	0.2779 (3.4058)	0.2496 (2.9033)	0.2418 (2.9718)
5	0.4286 (7.6701)	0.3538 (7.1908)	0.3685 (6.1195)
6	0.3378 (8.7451)	0.3067 (7.9255)	0.2968 (7.6514)
All	0.3424 (15.7940)	0.2959 (13.5923)	0.2805 (13.0614)

Table 4

Test statistics of the hypothesis regarding their comparative performance, class vs. school

The table reports the results of the significance tests of the hypothesis that investors are more overconfident, regarding their performance in making investment decisions, when his/her peers are classmates rather than his/her schoolmates. The null hypothesis is that their confidence to perform better than their classmates is not different than their performance relative to their schoolmates.

The top number in each cell is the mean estimated likelihood of the difference between their performance vis-a-vis their classmates and their performance vis-à-vis their schoolmates. The number below the mean is the t-statistic. All the results, except for one, are significant at the 5 percent level.

	Mean (t-stat)	
Group #	**Question 2 vs. Question 1**	**Question 3 vs. question 1**
1	0.0660 (5.7289)	0.0954 (2.6865)
2	0.0556 (4.3328)	0.0803 (4.0246)
3	0.0333 (1.6365)	0.0635 (2.4337)
4	0.0284 (2.0029)	0.0361 (2.0559)
5	0.0748 (2.8371)	0.0601 (3.9817)
6	0.0311 (2.2581)	0.0411 (3.9273)
All	0.0465 (6.8781)	0.0619 (7.1107)

Appendix I

Proof of Theorem 1

Identifying the dynamics of $Z = \frac{S_X}{S_B}$ using Ito's Lemma gives

$$dZ = Z\mu dt + Z\sigma dW\left(t\right) \tag{3}$$

where

$$\mu = \mu_X - \mu_B - \rho_{XB}\sigma_X\sigma_B + \sigma_B^2$$

and

$$\sigma^2 = \sigma_X^2 + \sigma_B^2 - 2\rho_{XB}\sigma_X\sigma_B .$$

After a time period t the value of Z conditional on a sample random path, W is

$$Z = e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}W}$$

Portfolio X beats the benchmark when $Z = \frac{S_X}{S_B} \geq 1$. This happens with probability

$$\begin{aligned}
\Pr\left(Z \geq 1\right) &= 1 - \Pr\left(Z \leq 1\right) \\
&= 1 - \Pr\left(e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}W} \leq 1\right) \\
&= 1 - \Pr\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}W \leq 0\right)
\end{aligned} .$$

Since W is normally distributed

$$\Pr\left(Z \geq 1\right) = 1 - \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-\frac{\left(y - \left(\mu - \frac{1}{2}\sigma^2\right)t\right)^2}{2\sigma^2 t}} dy$$

$$= 1 - \int_{-\infty}^{-\frac{\left(\mu - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \quad .$$

$$= 1 - \mathrm{N}\left(-\frac{\mu - \frac{1}{2}\sigma^2}{\sigma} \sqrt{t} \right)$$

The standard normal distribution is symmetric, thus

$$\Pr\left(Z \geq 1\right) = \mathrm{N}\left(\frac{\mu - \frac{1}{2}\sigma^2}{\sigma} \sqrt{t} \right).$$

Substituting for μ and σ gives the requested result. ■

Appendix II: Instructions

Assume that you have $100,000 that you decided to invest in either 1 company or 2 companies from the 30 companies that are listed below (these are the companies that the Dow index is constructed from). Your performance in problem set 5 will be evaluated according to: (return of your portfolio minus Rf)/ Beta of the portfolio

In the evaluation, we will make few simplifying assumptions:

1. The Beta of each share will be taken from Yahoo finance.
2. For simplicity you will be given a constant rate for the Rf.

Yet the evaluation is in problem set 5 – so what do you need to do for THIS problem set?

For this problem set you need to:

0. Write a short report that describes your investment strategy. Explain your choices. Remember: only 1 or 2 stocks from the list that is provided below.
1. Estimate (according to your own judgment) the probability that the adjusted return of your portfolio will be at the highest 10% of the class _________________.
2. Imagine that the same exercise is done not only in our section of the course but also in all other sections that study "Foundation of Finance" this semester. If this was the case (which is not), predict (according to your own judgment) the probability that the adjusted return of your portfolio will be at the highest 10% of the students that attend all sections of the foundation of finance course __________________.

3. Imagine that instead of asking you to invest in either 1 company or 2 from the Dow – I asked you to invest in 1 company or 2 from the Dax 100 (Germany) – you do not need to pick any particular firms but rather to answer the following question according to your own judgment: Estimate (according to your own judgment) the probability that the adjusted return of your portfolio (one or 2 stocks from the Dax) will be at the highest 10% of our class___________.