Household Leverage and the Recession*

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Abstract

A salient feature of the recent recession is that regions that have experienced the largest changes in household leverage have also experienced the largest declines in output and employment. We study a cash-in-advance economy in which home equity borrowing, alongside public money, is used to conduct transactions. Declines in home prices tighten the cash-in-advance constraint, triggering recessions. We parameterize the model to match the key cross-sectional features of the data. The model implies that real activity is very sensitive to liquidity shocks, but not to credit shocks, and that monetary policy can significantly reduce the severity of credit-driven recessions.

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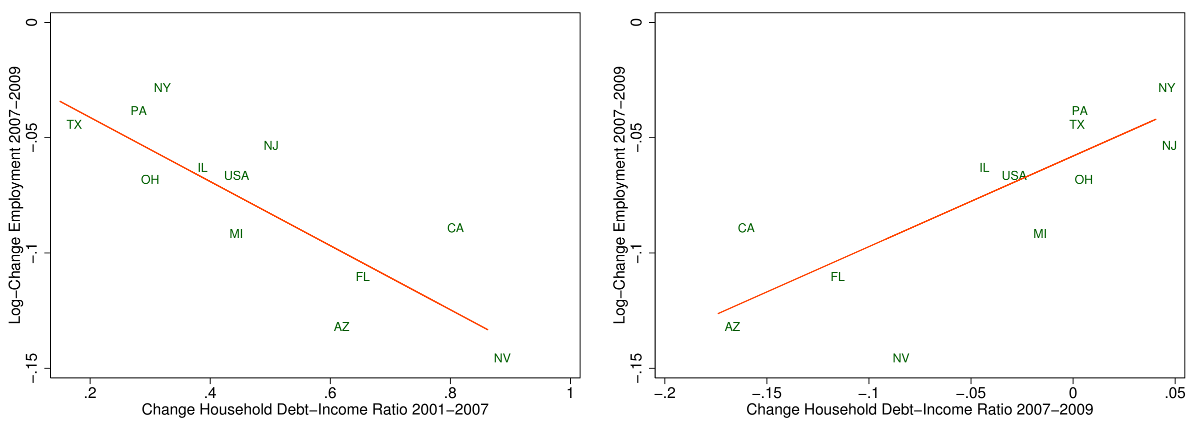
A striking feature of the recent recession is that regions of the U.S. (states or counties) that have experienced the largest swings in household borrowing have also experienced the largest declines in employment and output. Figure 1 illustrates this feature of the data, by plotting the change in employment during the credit crunch (2007-2009) against the change in household debt-to-income ratios during the preceding boom (2001-2007, left panel) and the contemporaneous bust (right panel).[1]

Figure 1: Leveraging, Deleveraging, and Employment across U.S. States



This pattern in the data is at odds with the predictions of standard models of financing frictions. In standard models, a tightening of borrowing constraints leads households to reduce their leverage not only by reducing consumption but also by increasing their labor supply. Hence, the stronger the need to de-lever, the larger the increase in labor supply.[2] Standard models therefore predict a relationship between leverage and employment that is the opposite of that observed in both panels of Figure 1. Non-standard preferences that attenuate wealth effects can mute the counter factual responses of output and employment, but cannot, on their own, reproduce the striking correlation between household debt and employment in the data.

Our goal in this paper is to quantitatively evaluate a theory that can account for the evidence in Figure 1, as well as other important cross-sectional features of the recent U.S. recession. We parameterize the theory to allow it to account for the salient features of the dynamics of leverage and employment in a cross-section of U.S. states and study the aggregate response of our model economy to credit shocks. In doing so we argue that it is important to distinguish between liquidity constraints and credit constraints: while a liquidity

[1] The red lines in Figure 1 are not regression lines, they represented the predicted values from our calibrated model. Data sources are described in the Appendix.

[2] See Chari, Kehoe, and McGrattan (2005). The small open economy "Sudden Stop" literature (see, e.g. Mendoza (2010)) addresses this issue by postulating that the tightening of credit reduces the firms' ability to finance working capital. We discuss these issues below.

crunch in our model can indeed reproduce the relationship between leverage and employment observed in the data, a credit crunch alone cannot.

Our benchmark model focuses solely on the role of liquidity constraints. The model we study is a cash-in-advance economy with a continuum of islands that trade with each other. Each island produces tradable and non-tradable goods subject to a constant-returns technology. Tradable goods produced on different islands are imperfectly substitutable. Our key departure from standard cash-in-advance models is that, in addition to public (government-issued) money, households can draw down a line of credit, which we refer to as home equity borrowing, in order to conduct transactions in the goods market. The amount of home equity borrowing is limited by a collateral constraint: households can only borrow up to a fraction, θ, of the value of their home.

These assumptions have two important consequences. First, homes provide liquidity services in addition to housing services. The liquidity services depend on the price of homes relative to consumption goods, the shadow value of liquidity, and the value of the collateral constraint, θ. Home prices therefore depend on current and expected values of θ. Second, home prices affect the amount of nominal balances that can be used to finance consumption expenditures. From a monetary perspective, an increase in real estate wealth effectively increases the velocity of money. This is the channel through which our model generates business cycles from nominal credit shocks. A decline in borrowing tightens the cash-in-advance constraint and amplifies the transactions frictions, thus leading to a recession. Absent the cash-in-advance constraint such a decline would involve no real transfers of resources from one island to another and would have no effect on real activity.

In addition to the cash-in-advance and liquidity constraints, we introduce two frictions that allow our model to account for the pattern of the data presented in Figure 1. First, nominal wage rigidities translate the decline of nominal consumption expenditures into real consumption spending. Second, we introduce frictions that prevent the immediate re-allocation of labor from the non-tradable to the tradable goods sector. Without this friction a negative credit shock leads to an expansion of the tradable sector which quickly undoes the effect of the credit tightening by increasing the inflow of public money from other islands.

In our model, three parameters determine the aggregate and cross-sectional responses to the large swings in housing wealth observed in the data. The first two parameters are the degree of wage stickiness and the degree of labor mobility. As discussed above, both of these frictions amplify the response of employment to a decline in home equity borrowing. We therefore pin down the size of these parameters by requiring that the model reproduces the relationship between measures of real activity across U.S. states (construction and non-construction employment) and measures of household leverage.

The third key parameter, θ, determines the fraction of consumption expenditures that were financed out of home equity borrowing during the upturn preceding the recession. To pin down the size of this parameter, we turn to the evidence from Mian and Sufi (2010a). These researchers argue that borrowing against the value of one's home accounts for a significant fraction of the rise in U.S. household leverage from 2002 to 2006. They use household-level data for a sample of 74,000 homeowners in different geographic regions of the U.S. and instrument house price growth using proxies for housing supply elasticities at the MSA level. In doing so they find that a 1$ increase in house prices *causes* a $0.25 increase in home equity debt. We use their findings to pin down the third key parameter in our model. To give a sense of the magnitude of this parameter, our calibration implies a marginal propensity to consume out of housing wealth of 6.6 cents on the dollar. This number is in line with existing empirical estimates that range from 5 to 13 cents on the dollar (see Li and Yao (2007) and Case, Quigley, and Shiller (2011) for comprehensive discussions).[3]

We show that the model does a very good job at accounting for the cross-sectional features of the data, and in particular the correlation between changes in measures of real activity during the bust and the change in household leverage in the boom. In addition to matching the relationship between employment and household leverage (by construction), the model also does a good job at reproducing statistics in the data that were not explicitly targeted in our calibration. The model predicts an elasticity of consumption spending to leverage of -22% (-24% in the data), of non-durable consumption spending to leverage of -68% (-69% in the data), and an elasticity of house prices to leverage of 86% (106% in the data). Moreover, the model's fit does not come at the expense of assuming an unrealistic degree of wage stickiness. While in the model wages are actually somewhat sensitive to changes in household leverage (an elasticity of -5%), in the data there is essentially no relationship between wages and changes in household leverage across states.

We use the model to study its predictions about the effect of the credit boom of 2001-2007 (a 50% increase in the debt-to-income ratio) and subsequent bust on measure of aggregate economic activity. We study two experiments. In the first experiment, we assume that monetary policy expands the Fed's balance sheet by 7% of GDP, in line with the Fed's actions in the data. We find that under this path for policy the model's predictions match remarkably well the pattern in the data. The model predicts a decline of non-construction employment of 5.5% (5.3% in the data), non-durable consumption of 3.8% (2.7%) in the data and durable

[3]The marginal propensity is heterogeneous and depends on household characteristics. Li and Yao (2007), for instance, emphasize important life-cycle effects. Our simple model does not capture these features of the data. Another potential concern is that the responses to increases and decreases in housing wealth might be different. Case, Quigley, and Shiller (2011), however, find roughly symmetrical effects.

consumption of 13% (14% in the data). As in the data, the response of durable consumption is a lot more severe due to the much stronger inter-temporal substitution for durables.

In the second experiment we assume away the Fed intervention. Absent the monetary expansion, the model predicts an 8.7% drop in non-construction employment (5.3% in the data), a 7.3% drop in non-durable consumption (2.7% in the data), and a 20% decline in durable goods spending. The model thus over-predicts the decline in real activity observed in the data and suggests that absent the Fed intervention the employment and consumption declines would have been more than 50% larger.

Our final contribution is to extend our analysis to study the role credit constraints in addition to liquidity constraints. Distinguishing between the two types of constraints is important in light of work by Johnson, Parker, and Souleles (2006), Parker, Souleles, Johnson, and McClelland (2011) and Kaplan and Violante (2011) who document empirically that a large fraction of wealthy households – households who are net savers – are nonetheless liquidity constrained.[4]

To introduce a non-trivial credit market we assume that there are two types of households who differ in their rates of time-preference. The proportion of impatient household is the same in each island, and each household can trade a risk-free security in an economy-wide asset market. The amount the household can borrow in this market is restricted by a collateral constraint. We refer to this constraint as the *credit constraint*. To distinguish between the credit and liquidity constraints, we assume that markets are segmented: transferring funds from one market to another entails a one-period delay. The household's balances available for consumption in the goods market are therefore its holdings of (publicly issued) money, as well as the amount it borrows via home equity lines of credit. The cash-in-advance frictions we impose in the goods market implies that, although only impatient households are credit-constrained, both types of households are liquidity-constrained. For this reason shocks to the liquidity constraint have important real effects on output and employment in our setup, whereas credit shocks essentially wash out in the aggregate. The extended model validates our initial approach: We show that the dynamic responses to a liquidity shock in this setup are essentially the same, both in the aggregate and at the island level, as the responses in our parsimonious cash-in-advance economy.

Relation to the literature

Our paper is related to four lines of research: (i) macroeconomic models with credit frictions, (ii) monetary

[4] Johnson, Parker, and Souleles (2006) find that, in 2001, "households spent 20 to 40 percent of their rebates on non durable goods during the three-month period in which their rebates arrived, and roughly two-thirds of their rebates cumulatively during this period and the subsequent three-month period." Parker, Souleles, Johnson, and McClelland (2011) find that, in 2008, "households spent about 12-30% of their stimulus payments on non- durable expenditures during the three-month period in which the payments were received," and that "there was also a substantial and significant increase in spending on durable goods, in particular vehicles, bringing the average total spending response to about 50-90% of the payments."

economics, (iii) real estate wealth; (iv) determinants of consumer spending. We discuss the connections of our paper to each topic. Following Bernanke and Gertler (1989), most macroeconomic papers introduce credit constraints at the entrepreneur level (Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999)). In all these models, the availability of credit limits *corporate* investment. As a result, credit constraints affect the economy by affecting the size of the capital stock. Gertler and Kiyotaki (2010) study a model where shocks that hit the financial intermediation sector lead to tighter borrowing constraints for entrepreneurs. We model shocks in a similar way. The difference is that our borrowers are households, not entrepreneurs, and, we argue, this makes a difference for the model's cross-sectional implications. Models that emphasize firm-level frictions cannot reproduce the strong correlation between household-leverage and employment at the micro-level, unless the banking sector is island-specific, as in the small open economy "Sudden Stop" literature (Mendoza (2010)). This "local lending channel" does not appear to be operative across U.S. states, however, presumably because business lending is not very localized.[5]

On the monetary side, we follow the cash-in-advance literature of Lucas (1980) and Lucas and Stokey (1987). We introduce home equity borrowing and show that velocity becomes a function of home prices. In the model, stricter lending standards lead to a drop in real estate value, which decreases the spending power of consumers. In terms of classical monetary economics our model interprets the recession as a large drop in velocity. We also study monetary responses to the crisis, and in particular non-standard interventions as in Gertler and Karadi (2009) and Curdia and Woodford (2009).

Our paper is related to the literature on housing wealth and consumption. Like Iacoviello (2005) we study a model where housing wealth can be used as collateral for loans. In his model, these are loans to entrepreneurs, while in our model, these are loans to households. Moreover, as emphasized above, the role of credit in our model is to facilitate transactions, not to smooth consumption inter-temporally. Lustig and Van Nieuwerburgh (2005) show that the collateral value of housing plays an important role in shaping asset returns because a decline in house prices undermines risk sharing and increases the market price of risk. Favilukis, Ludvigson, and Van Nieuwerburgh (2009) emphasize the role of time-varying risk premia in the recent increase and declines in housing prices. Burnside, Eichenbaum, and Rebelo (2011) emphasize heterogeneous expectations about long-run fundamentals and "social dynamics." Compared to these papers, our paper is less concerned with the exact source of house price movements, but rather with their their effects on real activity in the aggregate and in the cross-section. An important mechanism in our model

[5]For instance, Mian and Sufi (2010b) find that the predictive power of household borrowing remains the same in counties dominated by national banks. It is also well known that businesses entered the recession with historically strong balanced sheets and were able to draw on existing credit lines (Ivashina and Scharfstein, 2008).

is the feedback from lending standards to house prices. Landvoigt, Piazzesi, and Schneider (2010) provide evidence consistent with this feedback in a detailed analysis of the housing market of San-Diego. They find that easier access to credit for poor households leads to higher house prices at the low end of the housing market.

Most closely connected to our paper is the work of Guerrieri and Lorenzoni (2010) and Eggertsson and Krugman (2011) who also study the responses of an economy to a household-level credit crunch. These researchers find, as we do, that a credit crunch has a minor effect on employment if the economy is away from the zero lower bound. In both of these studies a credit crunch generates a decline in employment essentially because of the zero lower bound constraint. Unlike these researchers, we focus on the effect of a liquidity, rather than credit crunch and show that the former can have a sizable effect on real activity even away from the zero lower bound. Moreover, our focus is on understanding the cross-sectional evidence, in addition to the aggregate responses.

The view that a liquidity crunch can exacerbate transaction frictions is, of course, not novel to our paper. For example, Lucas and Stokey (2011) have argued that a liquidity crisis "has the effect of reducing the supply available to carry out the normal flow of transactions, leading to a reduction in production and employment." Our goal in this paper is to evaluate this mechanism using cross-sectional evidence and study its implications for aggregate dynamics.

Methodologically, we share our emphasis on cross-sectional information with Nakamura and Steinsson (2011). They study the effect of military procurement spending across U.S. regions, and they also emphasize the role of nominal rigidities and the power of cross-sectional evidence for identifying key model parameters. In both models differences in island-level employment dynamics are unaffected by aggregate-level shocks which are difficult to isolate: for example productivity shocks, changes in monetary policy, or foreign capital flows.[6] As a result, both our and their paper argue, cross-sectional statistics impose sharp restrictions on the set of parameter values that allow the model to match the data.

In Section 1 we present the model and we define the equilibrium. In Section 2 we study the qualitative and theoretical properties of the model in simplified setup. In Section 3 we propose a quantitative calibration and we study the response of the economy to various shocks. Section 4 presents the results of our simulation of the boom and bust from 2001 to 2009. Section 6 extends the model and compares liquidity constraints

[6]It is worth emphasizing that these shocks would create important issues in interpreting aggregate data. For instance, Favilukis, Ludvigson, and Van Nieuwerburgh (2009) show that foreign inflows can have a significant impact on aggregate house price dynamics. Similarly, calibrating the model's parameters using only with aggregate data would require to take a stand on controversial issues of monetary policy (Taylor, 2011).

with credit constraints. Section 7 concludes.

1 Model

We study a closed economy with a continuum of islands that trade with each other. Each island produces tradable and non-tradable goods and is populated by a representative household. Means of payment are provided by the government and by private lenders (banks and shadow banks).

Our model can be interpreted as a large country with a collection of regions (e.g., USA), or a monetary union with a collection of states (e.g., EU). The key assumption are that these regions share a common currency, and that agents live and work in only one region.

1.1 Households

The household's preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u\left(\bar{c}_{i,t}, \bar{d}_{i,t}, h_{i,t}, \bar{l}_{i,t}\right)$$

where $\bar{c}_{i,t}$ denotes non durable consumption, $\bar{d}_{i,t}$ and $h_{i,t}$ are the stocks of durable goods and housing owned by the household, and $\bar{l}_{i,t}$ is an index of labor supplied. We motivate the demand for money with a constraint à la Clower (1967). An important feature of our model is that households have two sources of liquidity: cash and private credit. We assume that credit is collateralized by housing wealth while cash is not.

As in all cash-in-advance models, we must specify the timing of trades within a period. We follow the timing proposed by Lucas (1980).[7] Each period is divided into three stages. Money and banking markets open first. Households bring in pre-existing cash balances $X_{i,t-1}$ and obtain a credit line from private lenders, while the government engages in open market operations. We call M_{it} the government-issued cash in the hands of consumers after the open markets operations at time t, and B_{it} the amount of private credit available. In the second stage, each household splits into a worker and a shopper. The shopper can spend no more than $M_{i,t} + B_{i,t}$, while the worker supplies her labor. In the last stage of the period, the household receives its labor income and the profits distributed by the firms, repays the private lenders and carries over $X_{i,t}$ units of currency to the next period. Notice that that $B_{i,t}$ is within-period credit. The timing of the

[7] Sargent and Smith (2009) discuss the importance of the timing of tax collection. This issue does not matter when we perform our cross-sectional analysis since we set taxes to zero. It can matter, however, when we consider various monetary policy responses in the last section of the paper. See also Lucas and Stokey (1987).

model is summarized in Table 1.

Table 1: Timing of Households' Cash and Credit Flows

	Financial Trading	Shopping & Production	Payment Collection
Cash	$M_{i,t} = X_{i,t-1} + T_{i,t}$	$M_{i,t}$	$X_{i,t}$
Credit	0	$B_{i,t}$	$-\left(1+r\right) B_{i,t}$
Spending	0	$\bar{P}_{i,t}\bar{c}_{i,t} + Q_{i,t}y^h_{i,t} + \bar{V}_{i,t}\bar{e}_{i,t}$	0
Income	$T_{i,t}$	0	$\Pi_{i,t} + W_{i,t} \cdot l_{i,t}$

Let $Q_{i,t}$ be the price of houses on island i at time t, and let $y^h_{i,t} = h_{i,t} - \left(1-\delta_h\right) h_{i,t-1}$ denote the purchase of housing. Similarly, let $\bar{V}_{i,t}$ denote the price index for durable goods and $\bar{P}_{i,t}$ denote the price index for non-durable consumption. Let $\bar{e}_{i,t} = \bar{d}_{i,t} - \left(1-\delta_d\right) \bar{d}_{i,t-1}$ denote purchases of durable goods. The consumer spends his balances on non-durables, durables and housing, subject to the cash & credit in advance constraint:

$$\bar{P}_{i,t}\bar{c}_{i,t} + Q_{i,t}y^h_{i,t} + \bar{V}_{i,t}\bar{e}_{i,t} \leq M_{i,t} + B_{i,t}, \tag{1}$$

Equation (1) says that firms accept to sell goods in exchange for bills printed by the government as well as units of credit backed by banks.[8] We assume that private credit for consumption must be collateralized by housing wealth. The amount of private credit is subject to the collateral constraint:

$$B_{i,t} \leq \theta_{i,t}Q_{i,t}h_{i,t}. \tag{2}$$

The parameter $\theta_{i,t}$ is exogenous, potentially island-specific, and the only source of shocks in this economy.

The household supplies three types of labor: to the non-tradable, tradable, and housing sectors. Each is industry-specific and aggregates into a final composite labor supply as:

$$\bar{l}_{i,t} = \left[\alpha_\tau \left(l^\tau_{i,t}\right)^\phi + \alpha_n \left(l^n_{i,t}\right)^\phi + \alpha_h \left(l^h_{i,t}\right)^\phi\right]^{\frac{1}{\phi}} \tag{3}$$

where $\phi \geq 1$ is a parameter that governs how substitutable different types of labor are and determines the degree to which labor can be reallocated across sectors. If $\phi = 1$, we have the model with perfect substitutability (mobility) across sectors, while as ϕ tends to ∞, the total amount of labor supplied is the

[8] An equivalent interpretation of (1) is that houses are purchased with credit, and goods with both cash M_{it} and left-over credit $B_{it} - Q_{i,t}y^h_{i,t}$.

maximum of what is supplied in each sector.

Since labor is sector-specific, wages differ across sectors. Let $W_{i,t}$ denote the vector of nominal wages in each sector and let $\Pi_{i,t}$ be the profits paid by private firms. At the end of the period, the liquidity position of the household is therefore: $X_{i,t} = \Pi_{i,t} + W_{i,t} \cdot l_{i,t} + M_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t} - Q_{i,t}y^h_{i,t} - \bar{V}_{i,t}\bar{e}_{i,t} - rB_{it}$. Finally, government implements monetary policy by printing new bills at the beginning of time t, and distributing them across islands: $M_{i,t+1} = X_{i,t} + T_{i,t+1}$. The flow budget constraint of the consumer is therefore

$$M_{i,t+1} = \Pi_{i,t} + W_{i,t} \cdot l_{i,t} + M_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t} - Q_{i,t}y^h_{i,t} - \bar{V}_{i,t}\bar{e}_{i,t} - rB_{it} + T_{i,t+1}. \quad (4)$$

The total amount printed by the government is simply $T_{t+1} = \int T_{i,t+1}$. In the remaining of the paper, we use the following specification for the utility function:

$$u\left(\bar{c}_{i,t}, \bar{d}_{i,t}, h_{i,t}, \bar{l}_{i,t}\right) = \log \bar{c}_{i,t} + \xi \log \bar{d}_{i,t} + \eta \log h_{i,t} - \frac{\bar{l}_{i,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}.$$

1.2 Credit

Let $B_{i,t}$ be the amount of credit provided by banks. Consumers use this credit, together with their holdings of public money, to purchase goods from firms. As in the search theory of money (see Lagos (2010) for a discussion and references), the idea is that consumers are anonymous to firms, but not to banks. Firms therefore cannot trust consumers to repay but they can go after the banks. Banks can keep track of consumers and seize a fraction $\theta_{i,t}$ of the collateral in case of default.

At the end of the period, the consumer repays $(1+r)\,B_t$ to the bank, and the bank pays B_t to the firm, thus making a profit equal to $\Pi^B_t = rB_t$. We assume free entry in the banking sector, thus in equilibrium we have $r = 0$.[9] Finally, we assume that β and $\theta_{i,t}$ are low enough for the constraints (1) and (2) to bind in all islands at all times.

[9] Also recall that B is within-period credit, i.e. credit flowing from workers to shoppers subject to the cash-advance-constraint. In that sense, B is really private money. The distinction between multi-period credit and within-period credit is not important as long as there are no dead weight losses from default. Analyzing costly defaults is important but clearly beyond the scope of this paper. In our calibration, we assume that home equity loans have a maturity of 5 years and we use the correct accounting to translate stocks into flows.

1.3 Wages

So far we have described the program of households as if there were no frictions in the labor market. In the quantitative experiments below we assume that wages are sticky.[10] The wage in sector k in island i at time t is given by

$$W_{i,t}^{k} = \left(W_{i,t-1}^{k}\right)^{\lambda} \left(W_{i,t}^{k*}\right)^{1-\lambda} \tag{5}$$

where $W_{i,t}^{k*}$ is the the frictionless nominal wage, implicitly defined by the labor-leisure choice:

$$\beta E_t \left[\frac{u_{\bar{c},it+1}}{\bar{P}_{i,t+1}}\right] W_{i,t}^{k*} = -u_{l^k,it}.$$

The parameter λ measures the degree of nominal rigidity. When $\lambda = 1$ wages are fixed, and when $\lambda = 0$ wages are fully flexible. Given the assumptions we have made on preferences, we can write the frictionless wage as:

$$W_{i,t}^{k*} = \alpha_k \left(\bar{l}_{i,t}\right)^{\frac{1}{\nu}} \left(\frac{l_{i,t}^{k}}{\bar{l}_{i,t}}\right)^{\phi-1} \left(\beta E_t \left[\frac{1}{\bar{P}_{i,t+1}\bar{c}_{i,t+1}}\right]\right)^{-1}. \tag{6}$$

Our specification of wage rigidities is thus that of a partial-adjustment model in which a fraction $1-\lambda$ of the gap between the actual and desired wage is closed every period. Note that an alternative would be to explicitly model households as being represented by unions who face a constant hazard of resetting their wages, as in the Calvo model. Since we study the effect of permanent shocks, our conjecture is that this alternative specification, though more notationally burdensome, would produce very similar results. Notice finally that a higher ϕ makes it costlier for sectoral labor to adjust, by increasing the disutility for work and therefore the sectoral wage.

1.4 Housing

We next discuss the housing market. Let $\mu_{i,t}$ be the multiplier on the cash-in-advance constraint. The housing Euler equation is:

$$\frac{\eta}{h_{i,t}} + \mu_{i,t}\theta_{i,t}Q_{i,t} = \frac{Q_{i,t}}{\bar{P}_{i,t}\bar{c}_{i,t}} - \beta\left(1-\delta_h\right) E_t \left[\frac{Q_{i,t+1}}{\bar{P}_{i,t+1}\bar{c}_{i,t+1}}\right] \tag{7}$$

[10] What matters for our cross-sectional result is the stickiness of relative wages across islands. Our interpretation of rigidities as being nominal – denominated in currency common to all islands and controlled by a central bank – only matters in the last part of the paper when we analyze counter-factual monetary experiments.

This equation is intuitive. Without the second term on the LHS, it would be a standard durable demand equation. $\frac{\eta}{h_{i,t}}$ is the marginal benefit of one extra unit of housing, and the RHS is the user cost. In our model, however, houses also provide liquidity services. The value of these services is $\mu_{i,t}$ and each unit of housing provides $\theta_{i,t}Q_{i,t}$ units of liquidity. Note that, using the consumption Euler equation we have that the shadow value of liquidity is $\mu_{i,t} = \frac{1}{P_{i,t}\bar{c}_{i,t}} - \beta E_t \frac{1}{P_{i,t+1}\bar{c}_{i,t+1}}$.

There is a housing construction sector on each island. Firms on each island can produce new houses using a decreasing return technology

$$y_{i,t}^h = \left(l_{i,t}^h\right)^{\chi}, \tag{8}$$

where χ determines the degree of decreasing returns. We allow decreasing returns in order to capture the role of land as a fixed factor in housing production. The aggregate stock of houses evolves according to:

$$h_{i,t} = (1-\delta)\, h_{i,t-1} + y_{i,t}^h \tag{9}$$

Since the price of new housing goods is $Q_{i,t}$, profit maximization by construction firms implies

$$W_{i,t}^h = \chi Q_{i,t}\left(l_{i,t}^h\right)^{\chi-1} \tag{10}$$

Profits of construction firms are simply $\Pi_{i,t}^h = (1-\chi)\, Q_{i,t} y_{i,t}^h$, and we assume for simplicity that construction firms are locally owned, so that $\Pi_{i,t}^h$ is paid to the household of island i.

1.5 Non-Durable Consumption

Household's consumption is an aggregate over the consumption of different varieties of tradable and non-tradable goods. We assume that the aggregation function has a constant elasticity of substitution σ between tradables and non tradables:

$$\bar{c}_{i,t} = [\omega_c^{\frac{1}{\sigma}}\left(\bar{c}_{i,t}^{\tau}\right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_c)^{\frac{1}{\sigma}}\left(c_{i,t}^{n}\right)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}},$$

where $\bar{c}_{i,t}^{\tau}$ is the consumption of the tradable good, $c_{i,t}^n$ is the consumption of the non-tradable good, and $\omega_c \in (0,1)$ is the weight on tradables in the aggregator. The tradable good is itself an aggregate of the goods produced on different islands, with elasticity of substitution γ between goods produced on different islands:

$$\bar{c}^{\tau}_{i,t} = \left(\int\limits_{j} c^{\tau}_{i,t}(j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

where j denotes the island where the good is produced. Let $\bar{P}^{\tau}_t$ denote the price index for tradable goods. It is common to all islands since we assume no trade costs, and it given by $\bar{P}^{\tau}_t \equiv \left(\int_i \left(P^{\tau}_{i,t} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$, where $P^{\tau}_{i,t}$ denotes the price at which the tradables produced on island i are sold. Let $P^{n}_{i,t}$ denote the price of non-tradable goods in island i. The total consumption price index on island i is: $\bar{P}_{i,t} \equiv \left[\omega_c \left(\bar{P}^{\tau}_t \right)^{1-\sigma} + (1-\omega_c) \left(P^{n}_{i,t} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$. Demand for non-tradables is:

$$c^{n}_{i,t} = (1-\omega_c) \left(\frac{P^{n}_{i,t}}{\bar{P}_{i,t}} \right)^{-\sigma} \bar{c}_{i,t} \tag{11}$$

The demand on island i for tradables produced by island j is:

$$c^{\tau}_{i,t}(j) = \omega_c \left(\frac{P^{\tau}_{j,t}}{\bar{P}^{\tau}_t} \right)^{-\gamma} \left(\frac{\bar{P}^{\tau}_t}{\bar{P}_{i,t}} \right)^{-\sigma} \bar{c}_{i,t} \tag{12}$$

1.6 Durables consumption

Investment in durables is also an aggregator over purchases of different varieties of tradable and non-tradable goods. We assume that the aggregation function has the same constant elasticity of substitution σ between tradables and non tradables as for consumption goods:

$$\bar{e}_{i,t} = [\omega_d^{\frac{1}{\sigma}} \left(\bar{e}^{\tau}_{i,t} \right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_d)^{\frac{1}{\sigma}} \left(e^{n}_{i,t} \right)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}},$$

where $\bar{e}^{\tau}_{i,t}$ are purchases of the tradable good to be used for investment, $e^{n}_{i,t}$ are purchases of the non-tradable good to be used for investment, and $\omega_d \in (0,1)$ is the weight on tradables in the investment aggregator. The tradable investment good is itself an aggregate of the goods produced on different islands, with elasticity of substitution γ between goods produced on different islands:

$$\bar{e}^{\tau}_{i,t} = \left(\int\limits_{j} e^{\tau}_{i,t}(j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

where j denotes the island where the good is produced. Let $\bar{V}_t^\tau$ denote the price index for tradable goods. This price index is common to all islands since we assume no trade costs, and it given by $\bar{V}_t^\tau \equiv \left(\int_i \left(P_{i,t}^\tau\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}} = \bar{P}_t^\tau$, where, recall, $P_{i,t}^\tau$ denotes the price at which the tradables produced on island i are sold. Also recall that $P_{i,t}^n$ is the price of non-tradable goods in island i. The total investment price index on island i is: $\bar{V}_{i,t} \equiv \left[\omega_d \left(\bar{P}_t^\tau\right)^{1-\sigma} + (1-\omega_d)\left(P_{i,t}^n\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$. Notice the only reason the durable and non-durable indices may differ is because of the differences in the weight of tradables in the two aggregators. Demand for non-tradable goods used for investment is:

$$e_{i,t}^n = (1-\omega_d)\left(\frac{P_{i,t}^n}{\bar{V}_{i,t}}\right)^{-\sigma} \bar{e}_{i,t} \tag{13}$$

The demand on island i for tradables produced by island j is:

$$e_{i,t}^\tau(j) = \omega_d \left(\frac{P_{j,t}^\tau}{\bar{P}_t^\tau}\right)^{-\gamma} \left(\frac{\bar{P}_t^\tau}{\bar{V}_{i,t}}\right)^{-\sigma} \bar{e}_{i,t} \tag{14}$$

tradable and non-tradable. Finally, investment in durables satisfies the Euler equation

$$\frac{\xi}{\bar{d}_{i,t}} = \frac{\bar{V}_{i,t}}{\bar{P}_{i,t}\bar{c}_{i,t}} - \beta(1-\delta_d)E_t\left[\frac{\bar{V}_{i,t+1}}{\bar{P}_{i,t+1}\bar{c}_{i,t+1}}\right], \tag{15}$$

and durable goods accumulate according to

$$\bar{d}_{i,t} = \bar{e}_{i,t} + (1-\delta_d)\,\bar{d}_{i,t-1} \tag{16}$$

1.7 Production and Market Clearing

We assume perfect competition in both tradables and non-tradables, as well as the housing construction sector. Each island is inhabited by a continuum of firms that produce a tradable good, and a continuum of firms that produce a non-tradable good. We also assume that labor is the only factor and that production is constant returns to scale:

$$y_{i,t}^n = l_{i,t}^n \quad \text{and} \quad y_{i,t}^\tau = l_{i,t}^\tau \tag{17}$$

Because of perfect competition the price of both tradable and non-tradable goods is equal to the nominal marginal cost in each sector on the island: $P_{i,t}^\tau = W_{i,t}^\tau$, and similarly $P_{i,t}^n = W_{i,t}^n$. Tradable and non-tradable goods are used for consumption and investment. Market clearing therefore requires

$$y_{i,t}^{n} = c_{i,t}^{n} + e_{i,t}^{n}, \tag{18}$$

in the non tradable sector, and

$$y_{i,t}^{\tau} = \int_{j\in[0,1]} \left(c_{j,t}^{\tau}(i) + e_{j,t}^{\tau}(i)\right), \tag{19}$$

in the tradable sector.

1.8 Equilibrium

We assume exogenous shocks to the tightness of borrowing constraints $\theta_{i,t}$. We will later discuss the interpretation of these shocks. To complete the description of the economy, we need to specify the monetary and fiscal policy. In equilibrium an island's cash holdings evolve according to (4). The transfers $\{T_{i,t}\}_{i,t}$ and money supplies $\{M_{i,t}\}_{i,t}$ must be consistent with the budget constraints of the government, and island-level money holdings follow the process

$$M_{i,t} = M_{i,t-1} + P_{i,t-1}^{\tau} y_{i,t-1}^{\tau} - \bar{P}_{t-1}^{\tau}\left(\bar{c}_{i,t-1}^{\tau} + \bar{e}_{i,t-1}^{\tau}\right) + T_{i,t}. \tag{20}$$

For most of our analysis we simply assume that the aggregate stock of currency remains constant, and we normalize it to $M_t = 1$ and $T_{i,t} = 0$ for all i and t.[11]

An equilibrium is a collection of prices and allocations. Since the list is long, it is more convenient to use some equilibrium conditions to limit the number of equilibrium objects. From the pricing conditions $P_{i,t}^{\tau} = W_{i,t}^{\tau}$ and $P_{i,t}^{n} = W_{i,t}^{n}$, we can define the tradable price index $\bar{P}_t^{\tau}$ and the island specific price indices $\bar{P}_{i,t}, \bar{V}_{i,t}$, as a function of wages. Therefore we only need to include $Q_{i,t}, W_{i,t}^{\tau}, W_{i,t}^{n}, W_{i,t}^{h}$, in the list of equilibrium prices. Given these prices, real non durable expenditures $\bar{c}_{i,t}$ determine local demand $c_{i,t}^{n}$ and bilateral demands $c_{i,t}^{\tau}(j)$ by (11) and (12). Similarly, real durable expenditures $\bar{e}_{i,t}$ determine $e_{i,t}^{n}$ and $e_{i,t}^{\tau}(j)$ by (13) and (14). Labor inputs determine production in (8) and (17) and the labor index $\bar{l}_{i,t}$ in (3). Finally, the two stock variables $h_{i,t}, \bar{d}_{i,t}$ are simply pinned down by (9) and (16).

The equilibrium is thus defined by the four prices listed above and seven quantities: two for the credit market $B_{i,t}, M_{i,t}$, three for the labor market $l_{i,t}^{n}, l_{i,t}^{\tau}, l_{i,t}^{h}$, and two for the goods market $\bar{c}_{i,t}, \bar{e}_{i,t}$. The intuition

[11] Since in the aggregate we have $\int \left(P_{i,t-1}^{\tau} y_{i,t-1}^{\tau} - \bar{P}_{i,t-1}^{\tau}\left(\bar{c}_{i,t-1}^{\tau} + \bar{e}_{i,t-1}^{\tau}\right)\right) di = 0$ by the resource constraint, we have $\int T_{i,t} = M_t - M_{t-1}$. Nothing pins down transfers to individual islands, however and we use the no transfer case as a benchmark.

for how we pin down the equilibrium is as follows. The three labor supply equations in (5), together with (6), pin down $W^{\tau}_{i,t}, W^{n}_{i,t}, W^{h}_{i,t}$. House prices $Q_{i,t}$ are pinned down by (7). (1), (2) pin down consumption $\bar{c}_{i,t}$ and borrowing $B_{i,t}$. (10), (19), (18) pin down $l^{n}_{i,t}, l^{\tau}_{i,t}, l^{h}_{i,t}$. (15) pins down $\bar{e}_{i,t}$, and (20) pins down $M_{i,t}$.

2 Qualitative Properties of a Simplified Model

We now study a special case to build some intuition about the effect of credit shocks in our model economy. In particular, we explain the difference between aggregate and island-level responses to credit shocks. To do so we consider a model without construction ($h = 1$ given exogenously and $\delta_h = 0$), with perfect labor mobility across sectors ($\phi = 1$), and without durable consumption ($\xi = 0$). Also, let $\omega = \omega_c$ denote the weight on tradables in the consumption basket.

2.1 Nominal Credit and Velocity

Combining the CIA constraint (1) with the collateral constraint equation (2) we obtain a collateralized-credit-in-advance (CCIA) constraint: $\bar{P}_{i,t}\bar{c}_{i,t} = M_{i,t} + \theta_{i,t}Q_{i,t}h_{i,t}$. We define $x_{i,t}$ as nominal consumption spending in island i at time t, $x_{i,t} \equiv \bar{P}_{i,t}\bar{c}_{i,t}$, and $q_{i,t}$ as the housing wealth to spending ratio, $q_{i,t} \equiv \frac{Q_{i,t}h_i}{\bar{P}_{i,t}\bar{c}_{i,t}}$.

Lemma 1. *Nominal credit dynamics in the simplified model are characterized by the velocity equation*

$$x_{i,t} = \frac{M_{i,t}}{1 - \theta_{i,t}q_{i,t}} \tag{21}$$

and the house price equation

$$\eta + \beta E_t\left[q_{i,t+1}\right] = \left(1 - \theta_{i,t}\left(1 - \beta E_t\left[\frac{x_{i,t}}{x_{i,t+1}}\right]\right)\right) q_{i,t} \tag{22}$$

Equations (21) and (22) provide a lot of intuition for the model. Equation (21) combines the cash-in-advance and collateral constraints, while equation (22) replaces equation (7). Given processes for $M_{i,t}$ and $\theta_{i,t}$ we could solve for $x_{i,t}$ and $q_{i,t}$ using (21) and (22). This is what we do in a one-island economy with aggregate money supply M_t controlled by a central bank. Note that $\theta_{i,t}q_{i,t}$ acts as a shock to *velocity* in equation (21).

Across islands, however, $M_{i,t}$ evolves endogenously, for two reasons. First the central bank does not control the allocation of money across industries or locations within a country, and even less across countries in a monetary union. Second, islands accumulate or de-cumulate government money depending on the private credit shocks that they experience. In particular, it would never be optimal for a government to reset $M_{i,t} = 1$ at the beginning of each period. In our benchmark model, we set $T_{it} = 0$. Each island's money holdings are then an island-specific state variable.

The details of the equilibrium are in the appendix. We now explain the qualitative properties of the economy's response to liquidity shocks, first in the aggregate and then for the cross-section of islands.

2.2 Aggregate response: the one island model.

We first consider an economy without heterogeneity. In steady state, the resource constraint is: $\bar{c} = l$ and the labor-leisure condition implies $\bar{c}l^{1/\nu} = \beta$. Therefore $l = \bar{c} = (\beta)^{\frac{\nu}{1+\nu}}$ and the only steady state distortion is the inter-temporal wedge introduced by the cash-in-advance constraint. Equation (22) implies $\bar{q} = \frac{\eta}{(1-\beta)(1-\theta)}$, and (21) implies $x = \frac{M}{1-\bar{\theta}\bar{q}}$, and the price level must be such that

$$\frac{M}{\bar{P}\bar{c}} = 1 - \bar{\theta}\bar{q} \tag{23}$$

The parameters must be such that $\bar{\theta}\bar{q} < 1$, or $(1-\beta)\left(1-\bar{\theta}\right) > \eta\bar{\theta}$. In particular, β, η and θ must all be small enough.

Lemma 2. *Following a permanent tightening of the collateral constraint, aggregate nominal spending and house prices are permanently lower. House prices drop by more than aggregate spending. The persistence of the real effects following a permanent credit shock depends only on the degree of nominal rigidity and on the elasticity of labor supply.*

The key idea is that, given processes $\{M_t\}_t$ and $\{\theta_t\}_t$ for aggregate money supply and credit tightness, the system can be solved for $\{x_t, q_t\}_t$ using (21) and (22) without reference to the rest of the model, i.e., independently of technology, nominal rigidity, and labor supply preferences. After a permanent shock to the borrowing constraint, if monetary policy is unchanged, the economy evolves along a path with constant nominal spending. If the shock is positive, nominal spending jumps up and remains constant. We see that q is increasing in θ: if credit is easier to obtain, housing value must increase relative to consumption spending because the collateral dimension of housing services makes houses more valuable. Spending must go up because of both θ and q. Going back to q, this means that housing prices must also increase so that even though spending goes up, house prices increase more than spending. In the appendix we show that the persistence of the real effects following a permanent credit shock is given by $\frac{\lambda\nu}{1-\lambda+\nu}$.

2.3 Cross-sectional responses

Consider an economy in which islands differ in the tightness of the borrowing constraint, θ_i. Two issues arise at the island level. First, $M_{i,t}$ is endogenous since islands can accumulate more or less public money. Second, $W_{i,t}l_{i,t} \neq \bar{P}_{i,t}\bar{c}_{it}$ since some goods are traded. Both of these issues are reflected in the money accumulation equation: $M_{i,t+1} - M_{i,t} = W_{i,t}l_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t}$. Credit dynamics satisfy (22) and (20). The eight equilibrium conditions have been described earlier. It is easy to check that the steady state allocations satisfy $l_i = \bar{c}_i = (\beta)^{\frac{\nu}{1+\nu}} = \bar{c}$. Since $l_i^n = (1-\omega)\,\bar{c}$ and $l_i^\tau = \omega\bar{c}$, we always have $\frac{l_i^n}{l_i^n + l_i^\tau} = 1-\omega$. All wages are the same and $W_i = \bar{P}$. Therefore all x_i are equal in all islands The following Lemma summarizes the steady state prices and quantities

Lemma 3. *In the steady state, all islands have the same real allocations, the same wages, prices and the same nominal spending. Only house prices differ across islands. The aggregate price level solves*

$$\frac{M}{\bar{P}\bar{c}} = \int_i \left(1 - \frac{\eta\theta_i}{(1-\beta)\,(1-\theta_i)}\right) di. \tag{24}$$

The CIA constraints determine the money balances $M_i = (1-\theta_i q_i)\,\bar{P}\bar{c}$ that implement these allocations. With constant x, we have $q_i = \frac{\eta}{(1-\beta)(1-\theta_i)}$. In the aggregate, we must have, $\int M_i = M$ so the price level must solve Equation (24) which is the generalization of (23) to an economy with heterogeneous nominal credit supplies. The Lemma states that differences in θ_i across islands do not translate into differences in prices or allocations. The reason is that islands with tighter constraints private credit accumulate public money. Since money and private credit are perfect substitutes, both prices and allocations (with the exception of house prices) are unaffected by the cross-sectional dispersion in θ_i.

Consider next the effect of an unanticipated, one-time shock to θ_i in any particular island. We calibrate and solve the system numerically in Section 3, but much intuition can be gained by considering the special case of fixed wages.

Lemma 4. *In the cross section, permanent credit shocks have temporary consequences even when wages are fully rigid.*

The intuition is simple: nominal shocks cannot have permanent effects because money can flow across islands. We present the details of the calculations in the Appendix. The key point is that house prices and nominal spending are linked by the following equation

$$\hat{q}_{i,t} = \tilde{q}_i - a\hat{x}_{i,t},$$

where a is a constant that depends on the parameters of the model and $\tilde{q}_i$ measures the permanent impact of the shock. The intuition comes from the model's implications for aggregate dynamics and the steady state cross section. In the aggregate, we know that permanent shocks to θ lead to constant values for x and q. This is not going to be the case in the cross-section, so x will move, and q will be affected.

The persistence of shocks at the island level does not depend much on the degree of nominal rigidity. This is in sharp contrast with the response of the aggregate economy. The reason is that islands that are hard hit by the nominal credit shock accumulate money balances according to

$$M_{i,t+1} - M_{i,t} = \bar{x}\left(\hat{l}_{i,t} - \hat{x}_{it}\right) = -\omega\bar{x}\hat{x}_{i,t}.$$

This shows again the role of trade in smoothing the cross-sectional shocks.

2.4 Comparison of Time Series and Cross-Section

We finally compare the time-series and cross sectional responses of the economy to permanent shocks to credit supply. In the aggregate we have $q(\theta) = \frac{\eta}{(1-\beta)(1-\theta)}$ and $x(\theta) = \frac{M}{1-\theta q}$. Therefore, on impact, we have $d\ln q = \frac{\bar{\theta}}{1-\bar{\theta}} d\ln\theta$ and thus $\frac{\partial \ln(x)}{\partial \ln(\theta)} = \frac{\bar{\theta}\bar{q}}{(1-\bar{\theta})(1-\bar{\theta}\bar{q})}$. Across islands, relative housing wealth evolves as $d\ln q_{i,t} = \tilde{q}_i - a\hat{x}_{i,t}$. The permanent component, $\tilde{q}_i = \frac{\bar{\theta}}{1-\bar{\theta}}\hat{\theta}_i$, is the same as in the aggregate case. Because of the temporary component, however, the adjustment of relative housing wealth is gradual. Spending reacts according to: $\frac{\partial \ln(x_i)}{\partial \ln(\theta_i)} = \frac{\bar{\theta}\bar{q}}{(1-\bar{\theta})(1-(1-a)\bar{\theta}\bar{q})}$. The response of local spending to local credit is muted by a. For employment, we have $\frac{\partial \ln(l_{i,0})}{\partial \ln(x_{i,0})} = 1-\omega$. We summarize the employment responses in Table 2.

Table 2: Elasticities with Fixed Wages and Permanent Credit Shocks

$\lambda = 1$, $\rho = 1$	Aggregate	Across Islands
Spending to Credit $\frac{\partial \ln(x)}{\partial \ln(\theta)}$	$\frac{\bar{\theta}\bar{q}}{(1-\bar{\theta})(1-\bar{\theta}\bar{q})}$	$\frac{\bar{\theta}\bar{q}}{(1-\bar{\theta})(1-(1-a)\bar{\theta}\bar{q})}$
Labor to Spending $\frac{\partial \log(l)}{\partial \log(x)}$	1	$1-\omega$
Persistence	Permanent	Temporary

With fixed wages, spending is equal to real consumption. So Table 2 also shows that in the cross section, employment reacts by a fraction ω less than consumption, while in the aggregate it responds by as much as consumption does.

We summarize our results in the following Proposition.

Proposition 1. ***Positive Properties***. *Cross sectional responses to credit shocks are muted in three ways relative to aggregate responses: (i) local spending reacts less to local credit because velocity effects are smaller; (ii) employment is less sensitive to local spending because of trade; and (iii) the effects dissipate over time because of endogenous adjustment in money balances.*

The following figures illustrate the proposition. We report some impulse responses to further illustrate the workings of the model. Figure 2 shows impulse responses to a 1% aggregate (common to all islands) drop in θ_t in this economy[12]. W^* drops immediately while actual wages adjust more gradually due to nominal rigidities. As a result consumption and employment drop. House prices drop because nominal spending drops and because the drop in θ_t makes houses less useful in undoing the borrowing constraints. The drop in B is therefore larger than the drop in θ and we have an amplification mechanism.

Figure 2: Aggregate Response to Permanent Credit Tightening

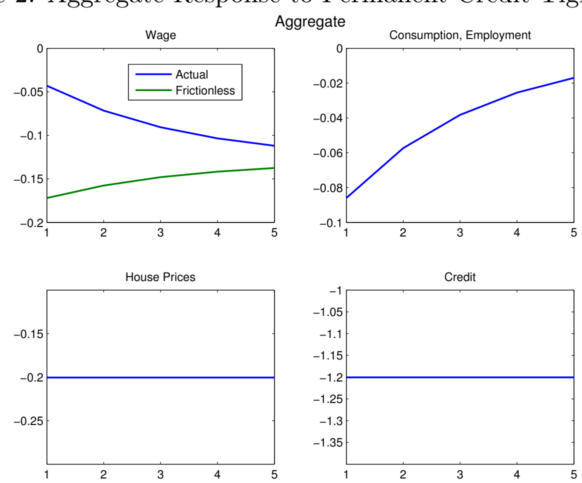


Figure 3 reports similar responses to an island-specific shock, $\theta_{i,t}$, assuming all other islands are at their steady-state values. Consumption responds by more (-0.9% on impact) than employment does (-0.45% on impact) because wages decrease in the island and hence demand for its tradables increases. From the results of the previous section, we know that when shocks are permanent and wages rigid, the ratio of the response

[12]We report the parameter values used in this calculation in Table 3 below.

of l to that of c is equal to $1-\omega$, which is 0.58 for our benchmark value of $\omega = 0.42$. In the actual simulation, the ratio is 0.51, which is close to 0.58 but, as expected, slightly smaller since wages do adjust.

Figure 3: Island Response to Permanent Credit Tightening

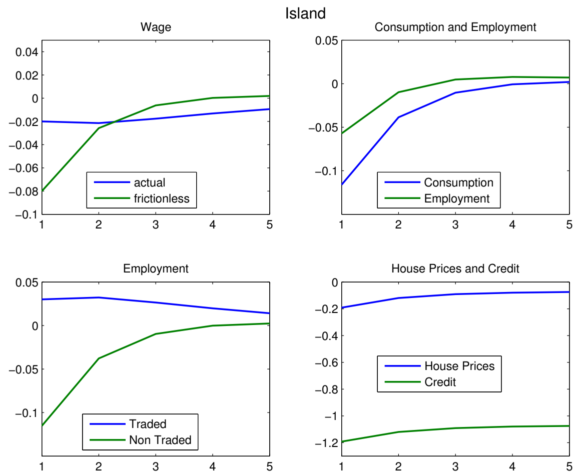
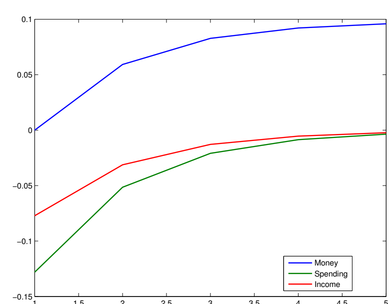


Figure 4 illustrates why all series are less persistent in the cross-section than in the aggregate by showing the evolution of nominal variables. The fact that consumption drops more than employment implies that the island accumulates public money, M, immediately after the shock. This increase in M compensates the decline in private credit, so that nominal spending reverts to the steady-state faster than in the aggregate.

2.5 Some Normative Implications

The focus of our paper is on the positive and quantitative properties of the model. However, in the interest of building intuition, it is useful to state two simple normative propositions.

Figure 4: Island Monetary Response



The first normative proposition is that, absent any frictions on monetary and fiscal policies, the government can always maintain the steady state allocations by targeting nominal spending.

Proposition 2. ***Perfect Stabilization****. Let $\bar{x}$ be the steady state value of nominal spending (common to all islands from Lemma 3). If the government adjusts its island-level transfers and its aggregate money supply so that $M_{i,t} = \bar{x}(1 - \theta_{i,t} q_{i,t})$, then real allocations remain at their steady state levels after any history of credit shocks.*

Proof. Only the path of nominal spending matters for real allocations in equations (5 29, 30, 31, 32, 33). If $x_{i,t} = \bar{x}$ then $\left\{l^n_{i,t}, l^\tau_{i,t}, W_{i,t}, W^*_{i,t}, \bar{P}_{i,t}, \bar{P}^\tau_t\right\}$ all remain constant at their steady state values. Local house prices are pinned down by 22, and $M_{i,t} = \bar{x}(1 - \theta_{i,t} q_{i,t})$ ensures that $x_{i,t} = \bar{x}$. Finally, the implicit transfer payments are given by (20). □

In the one island case, the steady state implementation only requires open market operations to stabilize aggregate nominal spending. With heterogeneity across islands, the implementation requires transfers across islands, presumably involving fiscal authorities.

The second normative proposition concerns corrective taxes on labor income and home construction. Before describing Pigouvian taxes, we note that the Friedman rule would be optimal in our economy without island level shocks. By deflating at rate β the government could reduce the multiplier μ_t to zero and eliminate all distortions in the economy. Our model is silent on the reasons that might make the Friedman rule undesirable, or that might prevent its implementation. Instead, we simply assume that prices are constant in steady state. This creates a wedge in the steady state labor supply. This can be corrected by a labor income subsidy. Now imagine an economy similar to the one we have described, but with endogenous housing supply, and let δ_h be the depreciation rate of houses. In order to understand the nature of optimal taxes, we allow the government to use two separate instruments: a subsidy on labor income, and a specific tax on home construction. We obtain the following results:

Proposition 3. ***Efficient Taxes and Home Construction****. The steady state allocation with constant money supply is efficient when labor income is subsidized at the rate $\beta^{-1} - 1$ and home construction is taxed at the rate $\frac{\theta(1-\beta)}{1-\beta(1-\delta_h)}$.*

Proof. See appendix. □

The subsidy $\beta^{-1}-1$ means that the steady state allocation of a model with exogenous housing would be efficient. The key to understanding the proposition is to see that houses are used as a form of commodity money. For the standard reasons identified in the monetary literature (Sargent and Wallace, 1983), when we introduce a housing construction sector there is excessive production of commodity money, i.e., excessive construction of new houses. The tax rate equals the liquidity services from housing θ times the steady state value of money (the Lagrange multiplier on the CIA constraint, or the opportunity cost of holding money, which is $1-\beta$). The denominator of the tax rate is simply an adjustment for the durability of housing (since $\delta_h < 1$).

3 Calibration

3.1 Complete model

If we combine the cash-in-advance constraint and the collateral constraint we now obtain $\bar{P}_{i,t}\bar{c}_{i,t}+Q_{i,t}y^h_{i,t}+V_{i,t}e_{i,t}=M_{i,t}+\theta_{i,t}Q_{i,t}h_{i,t}$. Defining as in Section 2, $x_{i,t}\equiv\bar{P}_{i,t}\bar{c}_{i,t}$ and $q_{i,t}\equiv\frac{Q_{i,t}h_{i,t}}{x_{i,t}}$, and the corresponding ratio for durable goods $v_{i,t}=\frac{V_{i,t}d_{i,t}}{x_{i,t}}$, we see that equation (21) becomes

$$x_{i,t}\left(1-\left(\theta_{i,t}-\frac{y^h_{i,t}}{h_{i,t}}\right)q_{i,t}+\frac{\bar{e}_{i,t}}{d_{i,t}}v_{i,t}\right)=M_{i,t}. \tag{25}$$

The velocity interpretation still applies, but now we need to take into account housing construction and spending of durable goods. We can write the house price equation (7) as

$$\eta+\beta\left(1-\delta_h\right)E_t\left[q_{i,t+1}\frac{h_{i,t}}{h_{i,t+1}}\right]=\left(1-\theta_{i,t}\left(1-\beta E_t\left[\frac{x_{i,t}}{x_{i,t+1}}\right]\right)\right)q_{i,t}.$$

Similarly, the Euler equation for durables is $\xi+\beta(1-\delta_d)E_t\left[v_{i,t+1}\frac{d_{i,t}}{d_{i,t+1}}\right]=v_{i,t}$. Trade and technology pin down labor demands. Market clearing for non-tradable goods (18) becomes

$$l^n_{i,t}=\left(\left(1-\omega\right)\bar{P}^{\sigma-1}_{i,t}+\left(1-\omega_d\right)v_{i,t}\frac{\bar{e}_{i,t}}{d_{i,t}}\bar{V}^{\sigma-1}_{i,t}\right)\left(W^n_{i,t}\right)^{-\sigma}x_{i,t},$$

and for tradable goods (19) becomes

$$l^\tau_{i,t}=\left(W^\tau_{i,t}\right)^{-\gamma}\left(\bar{P}^\tau_t\right)^{\gamma-\sigma}\int_j\left(\omega\bar{P}^{\sigma-1}_{j,t}+\omega_d v_{j,t}\frac{\bar{e}_{j,t}}{d_{j,t}}\bar{V}^{\sigma-1}_{j,t}\right)x_{j,t}.$$

For convenience, the complete set of equilibrium conditions is provided in the Appendix.

3.2 Steady State and Static Parameters

We consider a steady state in which $\theta_i = \theta$ is the same in all islands. We have two sets of parameters in our model economy. The first set, referred to as *static parameters*, mostly determines the steady state of our model economy. We choose these parameters to ensure that our model matches salient features of the U.S. data. The second set of parameters, referred to as *dynamic parameters*, consists of (λ, ϕ, θ), the parameters that govern the degree of nominal and real labor market rigidities, as well as the size of the collateral constraint. These parameters mostly affect the dynamic responses of the model in response to shocks. We pin down these parameter values by requiring that the model accounts for the cross-sectional dynamics of debt, house prices, and employment in the data.

Here we briefly describe how we have chosen the static parameters of our model . We describe our choice of the dynamic parameters in the next section, after we describe the cross-sectional experiments that we conduct. Table 3 reports the parameter values we use and the moments of the data that pin down each parameter.

We assume that a period is one year. For the borrowing constraints to bind in equilibrium, households must be sufficiently impatient. We therefore set $\beta = 0.95$, at the lower end of the range of values $(0.95-0.98)$ used in the literature.

The ratio of residential investment spending to the housing stock is equal to 3.6% in the data. In the steady state of our model we have that

$$\frac{y^h}{h} = \delta_h = 0.036,$$

so this pins down the rate at which the housing stock depreciates, δ_h. The value of housing stock relative to consumption expenditure, q, is equal to 2.11 in the data. In the steady state of our model we have that

$$q = \frac{\eta}{1 - \beta (1 - \delta_h) - \theta (1 - \beta)},$$

so we choose η accordingly.

In a similar fashion, δ_d pins down the ratio of spending on durables to their stock (equal to 0.27 in the data) since

$$\frac{e}{d} = \delta_d = 0.27$$

in our model. Moreover, the value of the durables relative to consumption expenditures, is equal to 0.5 in the data and equal to

$$v = \frac{\xi}{1 - \beta(1 - \delta_d)}$$

in the steady-state of our model. We thus choose ξ to ensure our model matches the value of v in the data.

We normalize the stock of money, M, equal to 1. Since all nominal variables (including the price of houses, Q) are proportional to M in the model, this is simply a convenient normalization that only determines the price level in this economy. The CCIA constraint (25) therefore gives:

$$x = \frac{M}{1 - (\theta - \delta_h)\, q + \delta_d v} = \frac{1}{1 - (\theta - \delta_h)\, q + \delta_d v}$$

The parameters α_τ and α_n are not separately identified from ω_c and ω_d since we have assumed constant returns to labor and both sets of weights simply pin down the share of each sector's expenditure/labor. We therefore normalize α_τ and α_n to ensure that wages are equal to unity in the two sectors in the steady-state: $W^n = W^\tau = 1$. Given this normalization, goods prices are also equal to 1. We then choose α_h to ensure that $l^h = s_h\,(l^\tau + l^n)$, i.e. so that the steady-state share of labor in construction is s_h that in the goods-producing sectors. In the data, the ratio of labor in construction to 6.6% so we set the value of α_h to hit this target.

Table 3: Parameters

Parameter Name		*Value*	*Source/Target*
Annual Discount Factor	β	0.95	
Home Value over Non Durable Spending	q	2.11	Value in 2001. BEA, Flow of Funds
Home Depreciation Rate	δ_h	0.036	Residential investment spending over housing stock
Labor Share Construction	χ	0.6	Construction Wages over Residential Investment
Durable Stock Value over Non Durable Spending	$\bar{v}$	0.5	Value in 2001. BEA, Flow of Funds
Durable Depreciation Rate	δ_d	0.27	Spending on durables relative to durable stock
Employment Share of Construction	s_h	0.066	Value in 2001. BEA.
Trade weight in non durable consumption	ω_c	0.25	Trade literature. Distribution adjusted.
Trade weight in durable consumption	ω_d	0.6	Trade literature. Distribution adjusted.
Labor Supply Elasticity	ν	2	Hall (2010)
Elasticity of substitution among traded goods	γ	1.5	Trade literature
Elasticity of substitution traded/non traded	σ	0.1	Own estimate

We calibrate the shares of tradable goods as in the international trade literature. We assume that the distribution margin accounts for 40% of the retail price of the good. We assume that all durable goods are tradable. Adjusting for local distribution, this gives $\omega_d = 0.6$. For non durable goods, we use the BEA data on Personal Consumption Expenditure. We identify tradables with "goods" and non-tradables with "services excl. housing". The share of tradables shows a trend decline over time and is around 0.4 in 2002. Adjusting for distribution costs gives $\omega_c = 0.25$. Finally, we choose an elasticity of substitution between tradables and non-tradables, σ, in order to match the comovement of the relative price of tradables to non-tradables and the share of tradables in the data. In the data, there was a substantial decline in the relative price of tradables and only a modest increase in real tradables consumption. A value of σ equal to 0.1 fits this evidence best. It is more difficult to pin down the elasticity of substitution between tradables produced on different islands, γ. In the international trade and macro literature, estimates of trade elasticities range from 0.5 to 4. We consider below a value equal to $\gamma = 1.5$, the typical value used in the international macro literature. It turns out that the exact value of γ is not critical in our model as long as wages are sticky.

Finally, we follow Hall (2010) and set the labor supply elasticity, ν, equal to 2.

4 Quantitative Cross Sectional Experiments

We next describe the cross-sectional experiments we conduct, as well as our choice of the dynamic parameters, λ, ϕ, θ, that allow our model to match the cross-sectional dynamics in the data. We describe the sources of the cross-sectional and aggregate data we use in detail in the Appendix.

4.1 The Experiment

We study an experiment in which all islands start in the (identical) steady-state with the same credit parameter, $\theta = \bar{\theta}$, in 2001. From 2001 to 2007 each island experiences a gradual, equally-sized, island-specific increase in θ. Finally, in 2008 and 2009 the collateral constraint in each island returns to $\bar{\theta}$ in two equally-sized steps. Hence, as in the data, islands that experience the largest booms prior to 2007 also experience the largest busts after 2007.

It turns out that changes in the current value of θ cannot replicate some important features of the cross sectional dispersion that we observe. Specifically, the cross-sectional dispersion of home prices is too large to be explained simply by the current value of θ. The basic issue is the following: θ drives both x and Q, but with reasonable parameters, if the only shock is the current value of θ, the change in house prices cannot be more than 1.5 times the change in nominal spending. This is not a severe constraint with aggregate data, but it is not enough for the cross section. In the Appendix we describe one way to explain the cross-section: news shocks to future values of θ. News shock can change q without changing the current value of θ. As a result Q can move by more that 1.5 nominal income. All our results can be interpreted using this approach.

This "news" interpretation is formally consistent with our model, but is really not crucial for our results.[13] For the sake of simplicity, and since our goal is to study an experiment that accounts simultaneously for the dynamics of credit and housing prices, we simply introduce a wedge in the housing Euler equation that allows us to reproduce the behavior of house prices in the data. In particular, we now have

$$\eta + \beta (1 - \delta_h) E_t \left[q_{i,t+1} \frac{h_{i,t}}{h_{i,t+1}} \right] = \left(1 - \theta_{i,t} \left(1 - \beta E_t \left[\frac{x_{i,t}}{x_{i,t+1}} \right] \right) \right) q_{i,t} + \omega_{i,t}$$

We choose the wedge $\omega_{i,t}$ so that our model reproduces the response of house prices in the data from 2001 to 2007. As with the collateral parameter, each island experiences a gradual, equally-sized, island-specific increase in Q_i, the price of houses, from 2001 to 2007. In 2008 and 2009 house prices revert to the initial steady-state in two equally-sized steps. We continue to assume a one-shock model so that θ_i and Q_i are perfectly correlated. In the news interpretation of the model this implies that current changes in θ are perfectly correlated with expectations of further future increases in θ.

To map the model to the data, we will compute elasticities of island-level employment to island-level changes in debt-to-income ratios. Changes in debt-to-income arise in the model from two sources: changes in the collateral constraint, θ, and changes in house prices. It turns out that in our model the size of these elasticities only depends on the relative size of the change in θ to that of changes in Q, not on the absolute size of these changes, since the model is approximately linear. The relative size matters since changes in house prices affect the returns to construction, and therefore the dynamics of employment, differently than changes in the collateral requirement. The fact that the absolute size of such changes is irrelevant implies that the elasticities we compute are unaffected by the standard deviation and higher order moments of the distribution of changes in debt-to-income in the data.

To pin down the size of changes in θ and Q, we require that the model matches two key moments that describe the credit boom reported in Table 4. The first moment is the average increase in the debt to income ratio of 0.46. (from 0.86 to 1.32 in the cross-section of 12 states in Figure 1 for which data is available). The second moment is the cross-sectional elasticity of house prices to leverage. To compute this moment, we run a regression of the log-change in house prices, $\Delta \log Q_i$, from 2001 to 2007 on the change in the debt-to-income ratio, $\Delta B_i/Y_i$ in this same period and find an elasticity equal to 0.86. Intuitively, the first moment pins down the size of the credit boom (the increase in θ), while the second pins down the size of

[13] To be precise, what is crucial is to set up the experiment with the correct initial conditions, that is, the correct distribution of B/Y and Q/Y. How we obtain these initial conditions does not matter. We could use preference shocks to capture the interactions of demographics (retirement of given age cohorts) and state-level characteristics (weather, etc.). We could use different prices dynamics (Burnside, Eichenbaum, and Rebelo (2011)).

house price increases necessary to allow the model to reproduce the response of house prices to changes in debt-to-income at the state level.

Table 4 reports that the model requires a 21% average increase in the collateral parameter, θ, and a 40% increase in the average house price, in order to match these two moments of the data. Note that the increase in house prices is slightly smaller than in the data (40% vs. 45%). Since our calibration of the dynamic parameters below relies on the model's predictions for cross-sectional elasticities, we prefer a parametrization that accounts for the cross-sectional elasticity of house price changes to changes in leverage in the data, rather than the average change in house prices, though the discrepancy between the model and the data is clearly negligible.

Table 4: Island Credit Boom

	Data	Model
Targets		
$\Delta\,(B/Y)$	0.461	0.461
$\Delta\log(Q)\,/\Delta\,(B/Y)$	0.862	0.861
Parameters		
average $\Delta\log Q$	0.453	0.397
average $\Delta\log\theta$	-	0.211

4.2 Calibration of dynamic parameters

The parameters θ, λ, ϕ are the key parameters in our model since they determine the economy's response to credit shocks. Intuitively, a higher θ implies that a higher fraction of nominal consumption spending is financed out of private credit and is therefore sensitive to credit shocks. A higher degree of wage stickiness, λ, implies a greater extent with which nominal shocks affect real activity. Finally, a greater degree of labor market rigidities, as captured by ϕ, implies that it is costlier to reallocate labor from the non-tradable to the tradable goods sector. Labor market rigidities amplify the island-specific shock by preventing islands from accumulating public money.

4.2.1 Calibration of the steady-state collateral constraint, θ

To pin down the steady-state value of the collateral constraint, θ, we use micro evidence from Mian and Sufi (2010a). Mian and Sufi (2010a) argue that borrowing against the value of home equity accounts for a significant fraction of the rise in US household leverage from 2002 to 2006. They follow from 1997 to 2008 a random sample of 74,000 U.S. homeowners (who owned their homes as of 1997) in 2,300 zip codes located in 68 MSAs. As of 1997, median total debt is \$100,000 of which \$88,000 is home debt (home equity plus mortgages), and the debt to income ratio is 2.5. Total debt grows by 8.6% between 1998 and 2002, and by 34.4% between 2002 and 2006. These changes are accounted for by home debt growth. The debt to income ratio does not change from 1998 to 2002 and then increases by 0.75.

Mian and Sufi argue that there is a causal link from house price growth to borrowing. The critical issue is that house price growth is endogenous. An omitted factor, such as expected income growth, could be driving both house prices and current borrowing (and consumption). To identify a causal link they use instruments for house price growth based on proxies for housing supply elasticities at the MSA level.

In their estimates, a \$1 increase in house prices *causes* a \$0.25 increase in home equity debt. Two issues arise when we map this number into our model. The first issue is maturity. Our model assumes that debt is repaid at the end of each year, while home lines of credit have an average maturity of 5 years. We show in the appendix, using a simple model in which debt has a maturity of N years, households borrow every N years (in a staggered fashion), and repay a fraction $\frac{1}{N}$ of the debt each year, that the conversion factor from the stock measure to the flow measure is $N/2$. This is intuitive, since if the initial amount of debt is equal to B, then the average debt position of all households is equal to $\frac{B}{N/2}$. With an average maturity of 5 years, \$0.25 translates into \$0.1 in our model with one-period debt.

The second issue has to do with the fact that θ changes over time in our model. To see this, note that Mian and Sufi report that a \$1 increase in house prices from 2001 to 2007 leads to a \$0.25 increase in debt, or:

$$B_{2007} - B_{2001} = 0.25\,(Q_{2007} - Q_{2001})$$

If θ were constant, then, since in our model $B = \theta Q$ we would have (ignoring the maturity adjustment) $\theta = 0.25$. We assume however, that θ changes over time, and that the increase in θ is perfectly correlated with changes in house prices. Hence, the Mian-Sufi elasticity does not recover the steady-state collateral constraint, $\theta_{2001} = \theta$. To recover this parameter, note that, according to our model $B_{2007} - B_{2001} = \theta_{2007}Q_{2007} - \theta_{2001}Q_{2001}$. Up to a first-order, we can thus write:

$$B_{2007} - B_{2001} \approx \frac{(\theta_{2007} - \theta_{2001})}{\theta_{2001}}\theta_{2001}Q_{2001} + \frac{(Q_{2007} - Q_{2001})}{Q_{2001}}\theta_{2001}Q_{2001}$$

Since, as shown in Table 4, we assume that θ increases by $1/2$ as much as Q does (0.21 vs. 0.40), we have that

$$B_{2007} - B_{2001} \approx \frac{3}{2}\theta_{2001}\left(Q_{2007} - Q_{2001}\right)$$

This implies that the Mian-Sufi elasticity is related to the steady-state collateral constraint θ by a factor of about 3/2. Accounting for the two sources of bias, we have that $\bar{\theta}_{equity} = 0.25 * 2/3 * 2/5 = 0.067$. Recall also that in our model we allow for housing construction. Assuming a loan-to-value (LTV) ratio of 80% for new mortgages, and noting that in the steady state the annual flow of spending on new homes is equal to δ_h, we have that the collateral on mortgage debt is equal to $\bar{\theta}_{mort} = 0.8\delta_h = .0288$. The total amount of debt (mortgage and home equity lines of credit) is thus bounded above by $\bar{\theta} = \bar{\theta}_{mort} + \bar{\theta}_{equity}$ and is equal to $\bar{\theta} = 0.0955$. This number is quite reasonable a priori. Since $1 = \frac{M}{x} + \theta q$, and since $\theta q \approx 0.2$, the calibration implies that about 20% of consumer spending is sensitive to real estate wealth. Alternatively, 20% of the household consumption spending in our model is financed using debt (private money), and the rest using public money.

4.2.2 Calibration of the degree of labor market rigidities, λ and ϕ

To pin down λ and ϕ, the parameters that govern the degree of real and nominal labor market frictions, we require that the model accounts for the cross-sectional elasticities of changes in employment in the construction and non-construction sectors during the bust (2007 to 2009) to the change in the debt-to-income ratio, $\Delta B/Y$ during the boom (2001-2007), as in Figure 1. We compute these elasticities in the data using the 12 states in Figure 1. Table 5 reports the moments in the model and in the data. We note that the elasticity of changes in non-construction employment during the bust to changes in debt-to-income during the boom is equal to -0.099 in the model and -0.098 in the data. Thus, the large decline in employment in states that have experienced the largest booms is not accounted for by a decline in construction employment alone. Though housing employment was a lot more sensitive to changes in debt (the elasticity is -0.59 in the model and -0.52 in the data), the share of construction employment is fairly small so that declines in non-construction employment account for the bulk, 70% (.098/.139) of the overall drop in employment in the data. The last few rows of the table report several additional predictions of the model which we discuss below.

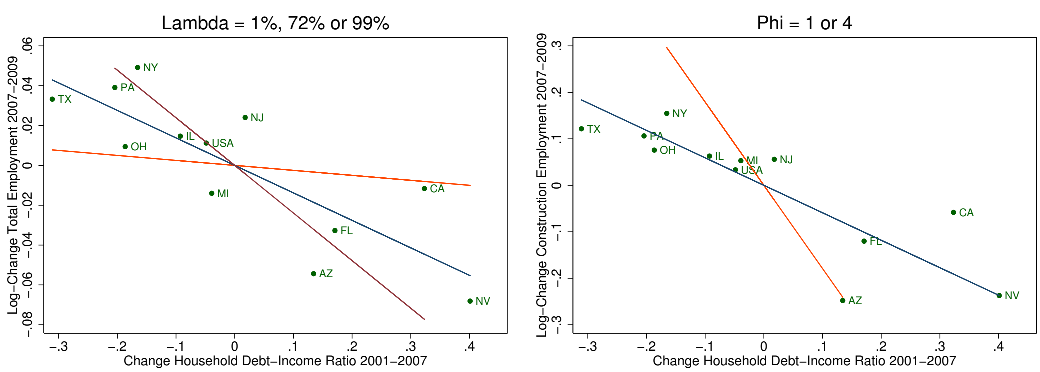
We note that the model, not surprisingly since we use two parameters to fit these facts, does a good job at reproducing the cross-sectional elasticities in the data. The implied parameter values are equal to $\lambda = 0.74$ and $\phi = 4$, suggesting a very large degree of wage stickiness (only a 26% fraction of the gap between the current and frictionless wage is covered each period) and very large costs of reallocating labor across industries.

Table 5: Island Credit Crunch

	Model	Data
Targets (elasticities to debt-income)		
total employment	-0.137	-0.139
non-construction employment	-0.108	-0.098
construction employment	-0.548	-0.524
Additional Testable Predictions		
Leverage	-0.293	-0.248
Home Prices	-0.860	-1.058
Consumption Spending	-0.216	-0.243
Durable Consumption Spend.	-0.683	-0.692
Non Durable Consumption Spend.	-0.187	-0.174
Non Construction Wages	-0.053	0.007
Construction Wages	-0.641	-0.063

Figure (5) gives a sense of how the data identifies the parameters λ and ϕ. The lines are the prediction of the model with the credit boom and bust simulated as in Table 4. Without wage rigidities ($\lambda = 0$), Figure (5) shows that total employment barely moves in the cross section.

Figure 5: Identifying Wage Rigidities and Reallocation Costs from the Cross-Section.



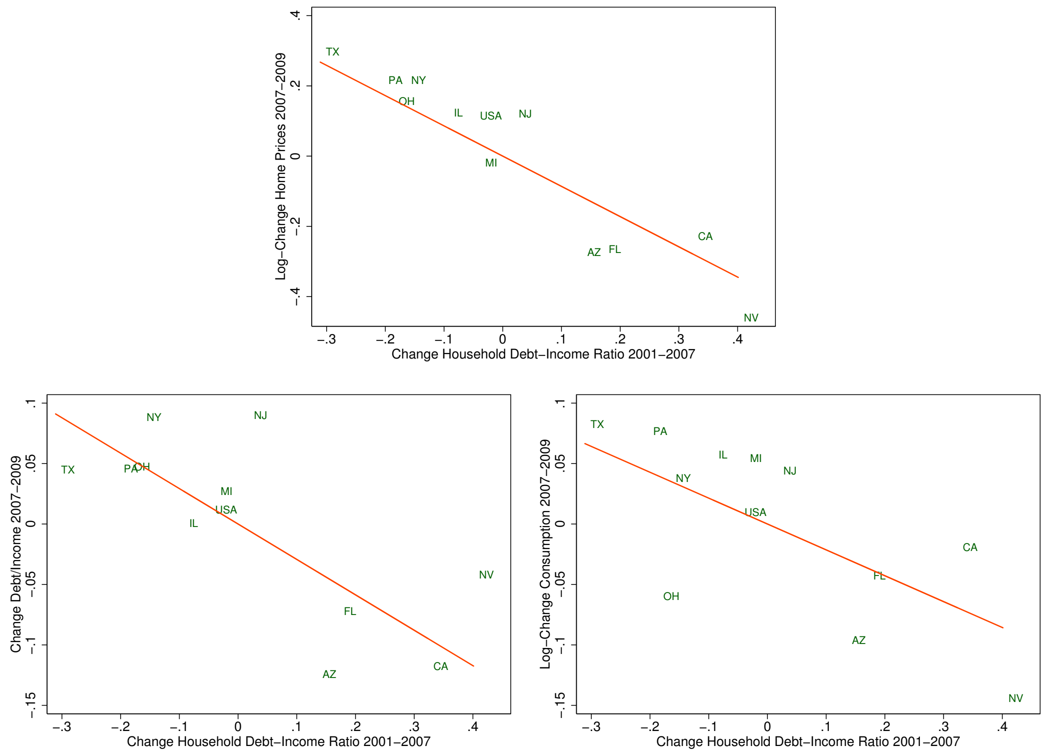
Without sectoral reallocation costs ($\phi = 0$), Figure (5) shows that labor moves too much across sectors. A similar picture emerges if we use durable versus non durable employment.

Consider finally several additional predictions our model makes for the cross section. Figure 6 shows the prediction of the model for the credit and housing markets, also reported in Table 5. The model does an excellent job at reproducing these features of the data. In particular, the elasticity of house price changes in the bust to the change in debt-to-income during the boom is equal to -1.06, thus only slightly higher than the -0.86 in the data. This suggests that our assumption that house prices revert to their steady-state values after the bust is in line with the data. Similarly, our model reproduces well the elasticity of the log-change in debt-to-income, $\Delta \log B/Y$ in the bust to the change in debt-to-income $\Delta B/Y$ in the boom (-0.29 in the model vs. -0.25 in the data), suggesting that our assumption that the collateral constraint returns to its steady-state level is reasonable as well.

Our model also has implications regarding the cross-sectional response of consumption. Testing these implications is difficult, however, since state-level consumption data is unavailable. For lack of a better measure, we construct measures of consumption expenditures across states using the Consumer Expenditure Survey (CEX). The results are reported in Table 5 and in Figure 6. One should be careful in interpreting these results since CEX was never designed to properly measure consumption across states and the state identifier is in some cases coded with noise. For this reasons the cross sectional correlation of consumption with changes in leverage is smaller than for the other variables in the Table. Nonetheless, the model predicts elasticities that are consistent with the data. The elasticity of consumption spending to changes in debt-to-income is equal to -0.21 in the model and -0.24, implying that consumption is 1.56 times more responsive to credit shocks than employment (-0.214/-0.137) in the model, and 1.75 times in the data. Durable goods spending is approximately three times more volatile than total consumption spending, both in the model and in the data.

Notice finally that there is one dimension along which our model does not fit the data well, namely the cross-sectional responses of construction wages (and to a lesser extent non-construction wages). In particular, the model predicts that wages decline more than they do in the data. The elasticity is -5.3% in the non-construction sectors compared to essentially zero in the data. This difference is economically small and statistically insignificant. For construction, the model elasticity is -60% vs. -6.3% drop in the data. Accounting for this large discrepancy would require unreasonable amounts of wage stickiness, and, more importantly, would overstate the employment responses in the data. We also note that, irrespective of the model's prediction, the empirical fit of the cross sectional wage regression is poor. There appears to be a lot of noise in wages that the model cannot replicate because it is essentially uncorrelated with the extent of the credit boom and bust.

Figure 6: Cross-Sectional Predictions for Home Prices, Deleveraging, and Consumption.



5 Quantitative Aggregate Experiments

We next study the model's aggregate implications. We first describe the experiment that replicates the aggregate credit boom. The data sources we use to quantify the size of the aggregate credit boom are slightly different from those used in the state-level analysis, hence there are some minor discrepancies from what we have reported earlier. As in the island experiment, we generate a credit boom by matching key moments of the data. We then let the model return to the initial value of $\bar{\theta}$ in a two-year period.

Table 6 reports the aggregate moments that we use to pin down the dynamics of q_t and θ_t during the boom and bust cycle. We ask the model to replicate the 49% increase in the debt-to-income ratio from 2001 to 2007, as well as the dynamics (an initial 20% increase and a subsequent 30% bust) of the ratio of home values to consumption spending, q. We match these statistics by feeding the model a path for θ_t and housing Euler equation wedges ω_t that match these statistics exactly. As above, we model the bust as a gradual, two-period long, equally-sized decline in θ from its value at the peak (2007) to the steady-state value. As for ω_t, we ask the model to reproduce the dynamics of q_t in each of the years of the bust.

Table 6: Aggregate Experiment

	Data	Model
$d\,(B/Y)$ from 2001 to 2007	0.49	0.49
q in 2001	2.11	2.11
q in 2007	2.53	2.53
q in 2008	2.05	2.05
q in 2009	1.85	1.85

When computing the responses of aggregate variables in these experiments, we entertain two sets of assumptions about monetary policy. The first set of assumptions is meant to capture the observed path of monetary policy and to deliver the model's predictions that can be compared to the data. The second set of assumptions provides counter-factual experiments.

In our first experiment, we allow the central bank to expand its balance sheet by 7% of GDP to capture the first round of non standard monetary policies (see Gertler and Karadi (2009), Gertler and Kiyotaki (2010), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010)). More precisely, we assume here a 7% expansion (relative to the size of GDP) equally distributed across islands, and gradually implemented in 2008 and 2009. Table 7 reports the results of this expansion under alternative implementations. The first column assumes an expansion of the stock of public money (M). The second column assumes that the central bank lends directly to households (B).

Table 7 shows that the model predicts responses that are in line with the actual dynamics in the data. Non-construction employment declines by almost as much as in the data (4.3% versus 5.3%), as does construction employment (20.2% vs. 20.7%). The model slightly overstates the decline in non-durable consumption (a drop of 3.8% vs. 2.7%), and slightly understates the drop in durable consumption (13.1% vs. 13.8%), but overall accounts for the response of consumption and employment remarkably well. In particular, our model is consistent with the observation that durable-goods spending declined by much more than non-durable spending.

Not surprisingly, since public and private money are substitutes in the model, the response of real variables is independent of the exact source of the expansion of the Fed's balance sheet. The only difference between the M-expansion and the B-expansion is the implied change in debt to income, since the increase in monetary aggregates is not counted as an increase in debt at the household level.

Table 7: Aggregate Outcomes with Active Monetary Policy

		QE of 7% of GDP	
Change 2007-2009	Data	with M	with B
Non Construction Employment	-0.053	-0.043	-0.043
Construction Employment	-0.207	-0.202	-0.202
Non Durable Consumption	-0.027	-0.038	-0.038
Durable Consumption	-0.138	-0.131	-0.131
Debt/Income	-0.043	-0.136	-0.006
Home Value/Income	-0.260	-0.289	-0.289
Non Construction Wages	-0.011	0.002	0.002
Construction Wages	-0.007	-0.273	-0.273

In our second set of experiments, we ask: how would have the economy fared absent Fed intervention? We answer this question in Table 8 by reporting the results of a counterfactual experiment in which the Fed leaves its monetary aggregates unchanged at $M = 1$. The first column reports the evolution of aggregate variables in our benchmark model economy, referred to as "Bench". Absent the Fed's intervention, our model predicts a recession that is significantly worse than the one we have observed. For example, non-construction employment would have declined by 8.7% instead of 5.3%. Non-durable consumption would have declined by 7.3% instead of 2.7%. Durable consumption would have declined by 20.0% instead of 13.8%.

The other columns of Table 8 illustrate the role of the key parameters of the model that we have identified using the cross-sectional data. In all of these experiments we continue to assume passive monetary policy so that $M = 1$. When we set $\lambda = 0$, so that wages are flexible, we find that the model produces a much milder recession. For example, non-construction employment declines by about 0.7% compared to 8.7% in the Benchmark economy. The recession is mostly accounted for by the contraction in the construction sector, combined with the frictions on labor mobility that we have assumed. When we set $\bar{\theta} = 0$, credit shocks no longer affect nominal consumption spending. The model once again produces a mild recession, again driven by the contraction in the housing sector, associated with the decline in house prices.

Finally, when we set $\phi = 1$, the model produces a contraction in the housing sector much more severe than in the benchmark model (a 72% decline in employment compared to a 25% decline) or than in the data, but the effect on total employment is muted by the fact that non-construction employment declines by less (5.4% compared to 8.7%) than in the benchmark model. We also note that labor immobility amplifies a bit the response of consumption: absent such frictions (i.e., when $\phi = 1$), non-durable consumption declines by only 4.2% (7.3% in the benchmark model), while non-durable consumption declines by about 15.5% (20% in the benchmark model).

Table 8: Counter-Factual Aggregate Outcomes

	Model Predictions with $M = 1$			
Change 2007-2009	Bench.	$\lambda = 0$	$\bar{\theta} = 0$	$\phi = 1$
Non Construction Employment	-0.087	-0.007	0.007	-0.054
Construction Employment	-0.253	-0.081	-0.129	-0.718
Non Durable Consumption	-0.073	-0.006	0.006	-0.042
Durable Consumption	-0.199	-0.011	0.013	-0.155
Debt/Income	-0.075	-0.080	-0.180	-0.066
Home Value/Income	-0.279	-0.298	-0.279	-0.201
Non Construction Wages	-0.036	-0.126	0.016	-0.032
Construction Wages	-0.324	-0.420	-0.224	-0.032

We thus conclude that the parameters λ and θ play a crucial role in determining the response of our model economy to credit shocks. This reinforces the need to identify such parameters using a richer set of cross-sectional moments, rather than relying solely on aggregate statistics. The latter reflect the stance of monetary and fiscal policy, international capital flows, as well as other real shocks, and use of the aggregate data alone precludes a sharp identification of the key frictions in the model.

To summarize, we find that the Fed intervention can have important consequences for the severity of the recession in our setup. The model predicts that absent the monetary intervention, the drop in non-construction employment would have been about 65% larger, while the drop in non-durable consumption would have been 2.7 times larger and that in durable-goods spending would have been about 65% larger. Of course, in our simple model, a monetary expansion of the right magnitude could completely offset the effect of the decline in household borrowing. An interesting extension of our analysis would consider a more realistic description of monetary policy and the constraints that have prevented the Fed from expanding the supply of public money. We relegate such an extension to future work.

6 Extension: A Model with Credit

In this section we extend our analysis to allow households to save and borrow inter-temporally using a risk-free asset. The purpose of this extension is two-fold. First, we argue that our results in the parsimonious setup considered earlier are robust to allowing inter-temporal borrowing and saving. Second, we can study separately the effects of shocks to the liquidity constraint and shocks to the credit constraint. Distinguishing between the two types of constraints is important in light of work by Parker, Souleles, Johnson, and McClelland (2011) and Kaplan and Violante (2011) who document empirically that a large fraction of wealthy households – households who are net savers – are nonetheless liquidity constrained.

6.1 Setup

Modeling separately credit and liquidity requires some modification to our benchmark model. Each island is inhabited by two types of households, indexed by $z \in \{p, m\}$, where p stands for patient and m for impatient. Let $c_t^z(i)$ denote the consumption of household of type z on island i and use a similar notation for the other island-type-specific allocations. We assume that agents differ in their discount factors, with $\beta_p > \beta_m$. An agent of type z on island i maximizes

$$\sum_{t=0}^{\infty} \beta_z^t u\left(c_t^z(i), l_t^z(i), h_t^z(i)\right) \tag{26}$$

We assume that the goods markets and the asset markets are physically segmented: although funds can be transferred from one market to another with a one-period delay, no such transfers are allowed within a period. We consider separately the two markets.

Consider first the goods market. As earlier, a household's purchases in the goods market are subject to a cash-credit-in-advance (CCIA) constraint. In addition to currency, $M_t^z(i)$, the household can obtain liquidity by borrowing $B_t^z(i)$ units against its home equity. Let θ^b denote the fraction of house value up to which the household can borrow. We have:

$$B_t^z(i) \leq \theta^b Q_t(i) h_t^z(i), \tag{27}$$

where $Q_t(i)$ is the price of houses on the island and $h_t^z(i)$ is the stock of houses owned by the household at the end of period t. We assume home equity borrowing takes the form of one-period risk-free debt that must be repaid at a nominal interest R_t in the following period. We refer to the home equity borrowing constraint (27) as a liquidity constraint.

The household's CCiA constraint is as earlier, given by $P_t(i) c_t^z(i) + Q_t(i)\left(h_t^z(i) - h_{t-1}^z(i)\right) \leq M_t^z(i) + B_t^z(i)$. Consider next the asset market. In addition to money holdings, the household can now save (or borrow) by trading one-period risk-free securities in the asset market. Let $A_t^z(i)$ be the agent's position in the asset market at the end of period t: a positive position indicates that the household is a lender, a negative position indicates that the household is a borrower. The amount the household can borrow in the asset market is constrained as well: the household can only borrow up to a fraction θ^a of the value of its home:

$$-A_t^z(i) \leq \theta^a Q_t(i) h_t^z(i)$$

As with the home equity borrowing, borrowing in the asset market must be repaid at a nominal interest rate R_t in the following period.

Consider next how the agents' balances in the two markets evolve over time. Every period, the household can transfer funds from the asset market to the goods market. Let $X_t^z(i)$ denote the size of the transfer. The evolution of the household's currency holdings is:

$$M_{t+1}^z(i) = M_t^z(i) + B_t^z(i) - P_t(i) c_t^z(i) - Q_t(i)\left(h_t^z(i) - h_{t-1}^z(i)\right) - \left(1 + R_{t-1}\right) B_{t-1}^z(i) + W_t(i) l_t^z(i) + X_t^z(i)$$

This law of motion differs from the one studied in the previous section along two dimensions. First, the household must pay interest $R_{t-1} B_{t-1}^z(i)$ on home equity borrowing. Second, the household can augment its holdings of currency by transferring funds from the asset market. Notice that a transfer at date t is only available for consumption at period $t+1$, thus with a one period delay.

The household's position in the asset market evolves according to:

$$A_t^z(i) = \left(1 + R_{t-1}\right) A_{t-1}^z(i) - X_t^z(i)$$

This says that the household's position at the end of period t is equal to its beginning-of-period balances (which include interest payments on the loans it made in the previous period) net of the transfers it makes to the goods market.

To summarize, the household's problem is to choose $c_t^z(i)$, $l_t^z(i)$, $h_t^z(i)$, $A_{t+1}^z(i)$, $M_{t+1}^z(i)$, $B_t^z(i)$ to maximize (26). The rest of the model is identical to the benchmark model described earlier. Equilibrium in the aggregate credit market requires that the sum of all loans made in the asset market is equal to the sum of all home equity lines of credit:

$$\int A_t^m(i)\, di + \int A_t^p(i)\, di = \int B_t^m(i)\, di + \int B_t^p(i)\, di \tag{28}$$

One way to interpret this equilibrium condition is that competitive financial intermediaries borrow from households in the asset market and lend to households both in the asset market and in the goods market, the latter via home equity lines of credit. The key implicit assumption that underlines equation (28) is that financial intermediaries can partly sidestep the asset market frictions that household face and transfer funds from one market to another without a one-period delay. Their ability to do so is limited however by the liquidity constraint (27). The rest of the equilibrium conditions are straightforward. In the money market we have:

$$\int M_t^p (i)\, di + \int M_t^m (i)\, di = \bar{M}$$

Equilibrium in the labor market for each island i requires that $l_t^p (i) + l_t^m (i) = l_t (i) = l^\tau (i) + l^n (i)$. Final goods consumption is $c_t^m (i) + c_t^p (i) = c_t (i)$. The specification of preferences and technology is otherwise identical in this setup as in the benchmark setup studied in the previous sections.

6.2 Decision Rules

To understand the workings on the model, it is useful to briefly describe the agents' decision rules for how much balances to hold in each market. Let $\lambda_t^z (i)$ be an agent's marginal valuation of its holdings in the asset market and $\gamma_t^z (i)$ denote the multiplier on the CCIA constraint. Moreover, let $\xi_t^z (i)$ denote the multiplier on the credit constraint and $\mu_t^z (i)$ denote the multiplier on the liquidity constraint. These objects satisfy:

$$\lambda_t^z (i) = \beta_z (1 + R_t) E_t \lambda_{t+1}^z (i) + \xi_t^z (i)$$

$$\lambda_t^z (i) + \gamma_t^z (i) = \beta_z (1 + R_t) E_t \lambda_{t+1}^z (i) + \mu_t^z (i)$$

$$\frac{u_{c,t}^z (i)}{P_t (i)} = \lambda_t^z (i) + \gamma_t^z (i)$$

$$\gamma_t^z (i) = \frac{u_{c,t}^z (i)}{P_t} - \beta_z E_t \frac{u_{c,t+1}^z (i)}{P_{t+1} (i)}$$

To understand these expressions, note that $\lambda_t^z (i) + \gamma_t^z (i)$ is the marginal value of one unit of currency in the goods market, while $\lambda_t^z (i)$ is the marginal value of one unit of balances in the asset market. Since it takes one period to transfer funds from the asset market to the goods market, $\gamma_t^z (i)$ is the shadow valuation of liquidity. Subtracting the second expression from the first yields

$$\gamma_t^z (i) + \xi_t^z (i) = \mu_t^z (i)$$

Clearly, if the household is unconstrained in the asset market $(\xi_t^z(i) = 0)$, then the shadow value of liquidity, $\gamma_t^z(i)$ is simply equal to the multiplier on the home equity constraint, $\mu_t^z(i)$. Similarly, absent an upper bound on home equity borrowing, we have $\mu_t^z(i) = 0$ and hence the CCIA constraint does not bind.

Consider next the size of these multipliers in a symmetric steady state. Clearly, the patient household is not constrained in the asset market, so that the nominal interest rate simply reflects the patient household's rate of time preference:

$$R = \frac{1}{\beta_p} - 1$$

Even though patient households are not credit-constrained, they are liquidity constrained. To see this, note that these agents' shadow valuation of liquidity is equal to $\gamma^p(i) = (1 - \beta_p)\frac{u_c^p}{P} > 0$. Finally, impatient household are both credit and liquidity constrained. As with patient households, the severity of their liquidity constraint is determined by the gap between 1 and β_m, since $\gamma^m(i) = (1 - \beta_m)\frac{u_c^m}{P} > 0$. In contrast, the severity of their credit constraint is determined by the gap between the two agents' rates of time preference, since $\xi^m(i) = \left(1 - \frac{\beta_m}{\beta_p}\right)\lambda^m(i)$.

In this economy, the asset holdings of agents on individual islands are non-stationary, as in the small open economy incomplete markets literature. Following Schmitt-Grohe and Uribe (2003) and Neumeyer and Perri (2005), we eliminate this non-stationarity by assuming that patient agents on each island face a quadratic cost of deviating from the aggregate asset market position. These details are discussed in the appendix, together with the calibration. We assign the same parameters to this model as to the original model, and choose θ^a and θ^b to match the Mian-Sufi evidence on the marginal propensity to consume out of housing wealth and the average loan-to-value ratio of mortgages in the U.S. data.

6.3 Quantitative Experiments

We next study the economy's responses to liquidity and credit shocks and also compare the responses to liquidity shocks in this setup with those in the benchmark setup without credit studied in the previous section. To facilitate comparison, we strip down both models of some the elements we have assumed earlier: the housing production sector–housing now is in fixed supply, durable goods consumption, and sectoral labor adjustment costs.

We study separately the responses to a permanent, one-time, unanticipated decrease in each of the collateral constraints, θ^b and θ^a, at both the island and aggregate level. We also contrast the response to liquidity shocks, i.e. shocks to θ^b, in the economies with and without credit.

Consider first the response to a liquidity shock in the benchmark model and in the model with credit. We choose the size of the shocks in the two models so that in both models aggregate nominal consumption expenditure declines on impact by 10%. We then feed through the same shock to an individual island in order to study the island responses. When reporting the island-level responses, we study two separate versions of the model with credit. In the first version the portfolio adjustment cost is infinitesimally small so that it does not affect the model's impulse responses in the first few years after the shock, only the very long-run responses. In the second version we increase the portfolio adjustment cost so as to increase the degree of asset market segmentation and prevent patient households from transferring funds from the asset market to the goods market.

Figure 7 reports impulse responses to an aggregate liquidity shock. Clearly, the responses in our benchmark model and the model with credit are identical. Since the liquidity shock is permanent, nominal spending permanently declines by 10% in both models. The dynamics of real variables are then solely determined by the degree of price rigidities, as captured by the common parameter λ.

Figure 7: Responses to Aggregate Liquidity Shocks

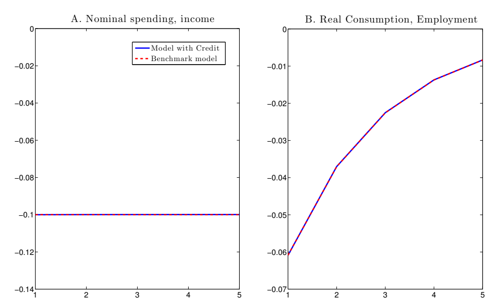
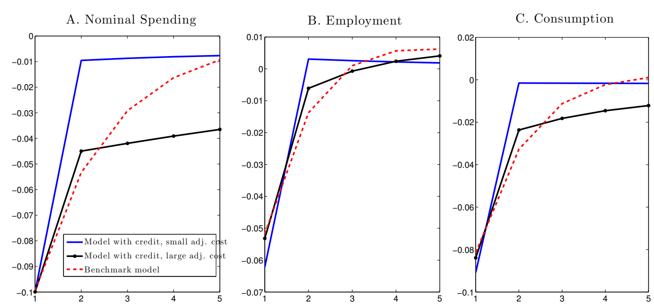


Figure 8 reports impulse responses to an island-level liquidity shock in the models with credit and in the benchmark model. Consider first the credit model without portfolio adjustment costs. Notice that, unlike in the benchmark model, nominal spending declines for only one period and then almost fully returns to the new steady state. The reason for the sharp increase in nominal spending after a period is that patient agents transfer balances from the asset market to the goods market. They use these balances to buy consumption goods, as well as to purchases houses from impatient households. They do so because house prices declines sharply after a liquidity shock (by about 15%) and overshoot their long-run response (which is about 5% lower than the initial steady state). The island's nominal spending thus recovers in essentially one period, as both patient and impatient households are both to increase the amount of currency used for consumption.

The quick recovery in nominal spending implies in turn that real allocations return to essentially their initial values with a one period delay.

Consider next the credit model with portfolio adjustment costs. Such costs restrict in their ability to transfer funds from the asset market to the goods market. As a result the recovery in nominal spending is much slower and the effects on consumption and employment somewhat more persistent. We note however that both the model with and without adjustment costs have broadly similar implications for the responses of employment and consumption as our benchmark model. All models predict a sharp initial drop in real activity and a quick recovery as the island accumulates public money. We thus conclude that our quantitative results in the previous section are robust to allowing for inter-temporal borrowing and lending.

Figure 8: Responses to Island Liquidity Shocks

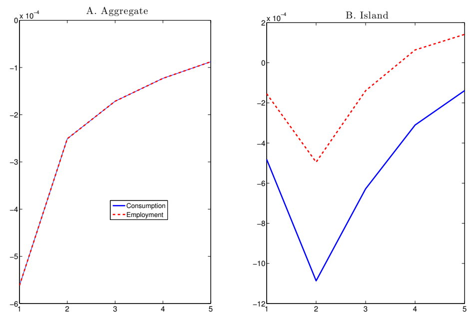


Finally, we consider the model's responses to a credit shock, reported in Figure 9. The size of the shock is equal to the liquidity shocks studied above. Notice that the decline in employment and consumption, both in the aggregate and at the island-level, are about 1/100th the size of these declines in response to liquidity shocks. The reason this decline is small is because credit shocks only affect the constrained agents. Although these agents' nominal balances decline somewhat, the effect of this shock on island and aggregate responses is offset by the fact that unconstrained agents now consume somewhat more and by the fact that impatient agents sell their housing stock and thus smooth out their consumption path. Overall, since credit shocks affect only one subset of households, they wash out in the aggregate and have a negligible effect on real activity.

Comparison with Other Work

Our findings that credit shocks have a small effect on real activity differs from the results of Guerrieri and Lorenzoni (2010) and Eggertsson and Krugman (2011). For aggregate responses, the main reason is that we

Figure 9: Responses to Credit Shocks



study the responses of the economy away from the zero lower bound on interest rates while Guerrieri and Lorenzoni (2010) and Eggertsson and Krugman (2011) focus on the liquidity trap. (In our setup the nominal interest changes very little, by only 5 basis points, after a liquidity shock of the size studied above since impatient households can delever by selling part of their housing stock to patient households). Guerrieri and Lorenzoni (2010) and Eggertsson and Krugman (2011) study only aggregate responses. In the cross-section the zero lower bound is less relevant because islands can borrow and lend at the same interest rate.

Another difference with Guerrieri and Lorenzoni (2010) is that they study a Bewley economy in which a precautionary savings motive leads both constrained as well as some unconstrained households to reduce consumption. Credit shocks have large distributional effects: average labor productivity drops as rich and productive agents cut back on their labor supply. The impact on aggregate employment is ambiguous, but in the benchmark calibration of Guerrieri and Lorenzoni (2010), employment goes up when the zero lower bound does not bind.

We thus think of our paper as complementary to those of Eggertsson and Krugman (2011) and Guerrieri and Lorenzoni (2010) in that we focus on another mechanism–liquidity constraints– that implies sizable real effects from household de-leveraging, and that we emphasize the importance of cross-sectional responses.

7 Conclusion

We have studied a cash-in-advance economy in which home equity borrowing, together with public money, is used to conduct transactions. We calibrate the model to account for the evidence on the dynamics of credit and employment in a cross-section of U.S. states and argue that a model capable of matching the cross-sectional facts implies strong sensitivity of real activity to credit shocks. We interpret these results as suggesting that a sharp reduction in credit at the household level accounts to a non-negligible extent for the collapse of output and employment in the recent recession. Expansionary monetary policy can, in this framework, significantly reduce the severity of a recession.

References

Bernanke, B., and M. Gertler (1989): "Agency Costs, Net Worth and Business Fluctuations," *American Economic Review*, 79, 14–31.

Bernanke, B., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford, vol. 1C. Elsevier Science, North Holland.

Burnside, C., M. Eichenbaum, and S. Rebelo (2011): "Understanding Booms and Busts in Housing Prices," mimeo Northwestern.

Case, K. E., J. M. Quigley, and R. J. Shiller (2011): "Wealth Effects Revisited 1978-2009," NBER WP 16848.

Chari, V., P. Kehoe, and E. McGrattan (2005): "Sudden Stops and Output Drops," *American Economic Review Papers and Proceedings*, 95(2), 381–387.

Clower, R. W. (1967): "A Reconsideration of the Microfoundations of Monetary Theory," *Western Economic Journal*, 6(1), 1–8.

Curdia, V., and M. Woodford (2009): "Conventional and Unconventional Monetary Policy," mimeo Columbia.

Del Negro, M., G. Eggertsson, A. Ferrero, and N. Kiyotaki (2010): "The Great Escape?," mimeo Princeton.

Eggertsson, G., and P. Krugman (2011): "Debt, Deleveraging, and the Liquidity Trap:a Fisher-Minsky-Koo Approach," Working Paper Princeton.

Favilukis, J., S. Ludvigson, and S. Van Nieuwerburgh (2009): "The Macroecononomic Effects of Housing Wealth, Housing Finance, and Limited Risk Sharing in General Equilibrium," Working Paper, NYU.

Gertler, M., and P. Karadi (2009): "A Model of Unconventional Monetary Policy," Working Paper NYU.

Gertler, M., and N. Kiyotaki (2010): "Financial Intermediation and Credit Policy in Business Cycle Analysis," Working Paper, NYU.

Guerrieri, V., and G. Lorenzoni (2010): "Credit Crises, Precautionary Savings and the Liquidity Trap," Chicago-Booth Working Paper.

Iacoviello, M. (2005): "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle," *American Economic Review*, 95, 739–764.

Ivashina, V., and D. Scharfstein (2008): "Bank Lending During the Financial Crisis of 2008," Harvard Business School Working Paper.

Johnson, D. S., J. A. Parker, and N. S. Souleles (2006): "Household Expenditure and the Income Tax Rebates of 2001," *American Economic Review*, 96(5), 1589–1610.

Kaplan, G., and G. Violante (2011): "A Model of the Consumption Response to Fiscal Stimulus Payments," mimeo NYU.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–248.

Lagos, R. (2010): "Asset Prices, Liquidity, and Monetary Policy in the Search Theory of Money," *Federal Reserve Bank of Minneapolis Quarterly Review*, 33(1), 14–20.

Landvoigt, T., M. Piazzesi, and M. Schneider (2010): "The Housing Market(s) of San Diego," Working Paper Stanford University.

Li, W., and R. Yao (2007): "The Life-Cycle Effects of House Price Changes," *Journal of Money, Credit, and Banking*, 39(6), 1375–1409.

Lucas, R. E. (1980): “Equilibrium in a Pure Currency Economy,” in *Models of Monetary Economics*, ed. by J. H. Karaken, and N. Wallace. Federal Reserve Bank of Minneapolis.

Lucas, R. E., and N. L. Stokey (2011): “Liquidity Crises,” Mimeo University of Chicago.

Lucas, R. E. J., and N. L. Stokey (1987): “Money and Interest in a Cash-in-Advance Economy,” *Econometrica,*, 55(3), 491–513.

Lustig, H., and S. Van Nieuwerburgh (2005): “Housing Collateral, Consumption Insurance and Risk Premia: An Empirical Perspective,” *Journal of Finance*, 60(3), 1167–1219.

Mendoza, E. G. (2010): “Sudden Stops, Financial Crises, and Leverage,” *American Economic Review*, 100, 1941–1966.

Mian, A., and A. Sufi (2010a): “House Prices, Home Equity-Based Borrowing, and the U.S. Household Leverage Crisis,” American Economic Review, forthcoming.

——— (2010b): “Household Leverage and the Recession of 2007-2009,” *IMF Economic Review*, 1, 74–117.

Nakamura, E., and J. Steinsson (2011): “Fiscal Stimulus in a Monetary Union: Evidence from U.S. Regions,” mimeo Columbia University.

Neumeyer, P. A., and F. Perri (2005): “Business cycles in emerging economies: the role of interest rates,” *Journal of Monetary Economics*, 52(2), 345–380.

Parker, J. A., N. S. Souleles, D. S. Johnson, and R. McClelland (2011): “Consumer Spending and the Economic Stimulus Payments of 2008,” NBER WP 16684.

Sargent, T. J., and B. D. Smith (2009): “The Timing of Tax Collections and the Structure of "Irrelevance" Theorems in a Cash-in-Advance Model,” mimeo NYU.

Sargent, T. J., and N. Wallace (1983): “A Model of Commodity Money,” *Journal of Monetary Economics*, 12, 189–196.

Schmitt-Grohe, S., and M. Uribe (2003): “Closing small open economy models,” *Journal of International Economics*, 61, 163–185.

Taylor, J. B. (2011): “Historical Evidence on the Benefits of Rules-Based Economic Policies,” in *Address before the Joint AEA/AFA Luncheon Session.*

Appendix

Data

State level population, employment, earnings by place of work, and compensation (by industry) are from the Bureau of Economic Analysis (BEA, Regional Economic Accounts). For wages, we divide compensation by wage & salary employment. Home prices are from the Federal Housing Finance Agency (FHFA). State level household debt comes from the Federal Reserve Bank of New York. It includes Consumer Credit and Mortgage Debt. State level consumption is constructed from the Consumer Expenditure Survey (CEX).

BEA, Income and Employment by State

1. Earnings by Place of Work, $ million. By state of work.
2. Compensation (all industries), $ million. By state of work. Wages + Employer Contributions
3. Wages (all industries), $ million. By state of work.
4. Compensation (construction), $ million. By state of work. Wages + Employer Contributions. NAICS classification.
5. Wages (construction), $ million. By state of work.
6. Total employment (all industries). Persons.
7. Wage employment (all industries). Persons.
8. Total employment (construction). Persons. NAICS classification.
9. Wage employment (construction). Persons. NAICS classification.

NY FED

1. Total Debt, Q1, not seasonally adjusted.
2. Total Debt, Q4, not seasonally adjusted.
3. Total Debt, Average over the year, not seasonally adjusted.

All data in $ million. Total Debt includes mortgage accounts, home equity revolving accounts, auto loans, bank card accounts, student loans and other loans. Excludes population without a social security number.

FHFA

1. House Prices in Q1
2. House Prices in Q4
3. Average House Prices in a year

House prices are based on sales price data. Index (100 in 2001, Q1). The annual average for 2010 is the average of Q1-Q3.

Consumers' First Order Conditions

The first-order conditions for money holdings, consumption, labor, and housing and non-durables are:

$$\begin{aligned}
\frac{u_{\bar{c},it}}{\bar{P}_{i,t}} &= \beta\left(1+r\right)E_t\frac{u_{\bar{c},it+1}}{\bar{P}_{i,t+1}}+\mu_{i,t}, \\
-\frac{u_{l^k,it}}{W_{i,t}^k} &= \beta E_t\frac{u_{\bar{c},it+1}}{\bar{P}_{i,t+1}} \ \text{ for } k=n,\tau,h, \\
u_{h,it}+\mu_{i,t}\theta_{i,t}Q_{i,t} &= \frac{Q_{i,t}}{\bar{P}_{i,t}}u_{\bar{c},it}-\beta(1-\delta_h)E_t\frac{Q_{i,t+1}}{\bar{P}_{i,t+1}}u_{\bar{c},it+1}, \\
u_{\bar{d},it} &= \frac{\bar{V}_{i,t}}{\bar{P}_{i,t}}u_{\bar{c},it}-\beta(1-\delta_d)E_t\frac{\bar{V}_{i,t+1}}{\bar{P}_{i,t+1}}u_{\bar{c},it+1},
\end{aligned}$$

where $\mu_{i,t}$ is the multiplier on the borrowing constraint.

The Simplified Model

Nominal wage setting is given by (5), and labor market clearing in each island implies $l_{i,t}=l_{i,t}^n+l_{i,t}^\tau$. Using $x_{i,t}$, we can rewrite the labor supply (6) as

$$\left(l_{i,t}^n+l_{i,t}^\tau\right)^{\frac{1}{\nu}}=W_{i,t}^*\beta E_t\left[x_{i,t+1}^{-1}\right]. \tag{29}$$

Trade and technology pin down labor demands. For local goods, we have $l_{i,t}^n=c_{i,t}^n$, which we can rewrite as

$$l_{i,t}^n=\left(1-\omega\right)\frac{x_{i,t}W_{i,t}^{-\sigma}}{\bar{P}_{i,t}^{1-\sigma}}. \tag{30}$$

For traded goods, we have $l_{i,t}^\tau=\int_j c_{j,t}^\tau(i)dj$ which we can rewrite as

$$l_{i,t}^\tau=\omega W_{i,t}^{-\gamma}\left(\bar{P}_t^\tau\right)^{\gamma-\sigma}\int\limits_j\frac{x_{j,t}}{\bar{P}_{j,t}^{1-\sigma}} \tag{31}$$

The price indexes are such that

$$\left(\bar{P}_{i,t}\right)^{1-\sigma}=\omega\left(\bar{P}_t^\tau\right)^{1-\sigma}+\left(1-\omega\right)\left(W_{i,t}\right)^{1-\sigma} \tag{32}$$

and

$$\left(\bar{P}_t^\tau\right)^{1-\gamma}=\int\limits_j\left(W_{j,t}\right)^{1-\gamma} \tag{33}$$

In this simplified system, we now have nine equations (5, 20, 21, 22, 29, 30, 31, 32, 33) and nine unknowns $\left\{q_{i,t},x_{i,t},M_{i,t},l_{i,t}^n,l_{i,t}^\tau,W_{i,t},W_{i,t}^*,\bar{P}_{i,t},\bar{P}_t^\tau\right\}$.

Aggregate Equations

Consider the dynamics of credit first. Given processes $\{M_t\}_t$ and $\{\theta_t\}_t$, the system can be solved for $\{x_t, q_t\}_t$ using (21) and (22). When $\theta = 0$, the solution is always $x_t = M_t$ as in the standard cash-in-advance model. When $\theta > 0$, house price or collateral shocks to are transmitted by the collateral constraint. In the one island economy, we have $W_t = \bar{P}_t$ and the equations for the price levels are trivial. We also have $\bar{c}_t = l_t$. Once we have solved for x_t and q_t we can therefore solve for W_t and l_t by using $W_t l_t = x_t$, $W_t = W_{t-1}^{\lambda} \left(W_t^*\right)^{1-\lambda}$, and $(l_t)^{\frac{1}{\nu}} = \beta W_t^* E_t\left[x_{t+1}^{-1}\right]$. Note that the labor shares are constant in the one island economy: since $l_t^n = (1-\omega)\frac{x_t}{P_t}$ and $l_t^{\tau} = \omega \frac{x_t}{P_t}$, we always have $\frac{l_t^n}{l_t^n + l_t^{\tau}} = 1 - \omega$.

Consider next the impact of a permanent, unanticipated shock to θ. When M and θ are constant, we have $q(\theta) = \frac{\eta}{(1-\beta)(1-\theta)}$ and $x(\theta) = \frac{M}{1-\theta q}$. Following a permanent shock, x is constant and since $W_t l_t = x$ and employment is

$$\ln(l_t) = \frac{\lambda\nu}{1-\lambda+\nu}\left(\ln(x) - \ln(W_{t-1})\right) + \frac{(1-\lambda)\nu}{1-\lambda+\nu}\ln(\beta),$$

while W satisfies

$$(1-\lambda+\nu)\ln W_t = \lambda\nu \ln W_{t-1} + (1-\lambda)\left((1+\nu)\log x - \nu\log\beta\right).$$

Without nominal rigidities (i.e., $\lambda = 0$) wages adjust immediately to nominal credit shocks and employment remains constant.[14] The persistence of the real effects following a permanent credit shock is given by $\frac{\lambda\nu}{1-\lambda+\nu}$. Persistence thus depends on the degree of nominal rigidity and on the elasticity of labor supply. If wages are fixed (i.e., $\lambda = 1$) the real impact of *aggregate* nominal credit shocks is permanent. We will show that this result does not hold in the cross-section.

Island Equations

To study the responses to such a shock, we find it useful to study log-linear approximations to the equilibrium conditions. For any variable z_{it} we write $z_{i,t} = \bar{z}(1 + \hat{z}_t + \hat{z}_{it})$, where $\hat{z}_t$ is the solution to the one-island log-linear model, and the total log-change is $d\ln z_{i,t} = \hat{z}_t + \hat{z}_{it}$. We note that, up to a first-order approximation, the evolution of the aggregates in our model with heterogeneous islands is equivalent to the evolution of the one-island economy. Hence, we first characterize the one-island (aggregate) responses and then compute log-deviations of each island from the aggregate responses. From now on, we use the term "one-island" and "aggregate" interchangeably.

Consider first the island-level response of trade and labor demand. In the aggregate, we have that $P_t = W_t$. Around these aggregate dynamics, we have $\hat{l}_{i,t}^n = \hat{x}_{i,t} - \sigma\hat{W}_{i,t} - (1-\sigma)\hat{\bar{P}}_{i,t}$, $\hat{\bar{P}}_{i,t} = (1-\omega)\hat{W}_{i,t}$, and $\hat{l}_{i,t}^{\tau} = -\gamma\hat{W}_{i,t}$. We therefore have that $\hat{l}_{i,t}^n = \hat{x}_{i,t} - (1-\omega(1-\sigma))\hat{W}_{i,t}$. Since $\hat{l}_{i,t} = (1-\omega)\hat{l}_{i,t}^n + \omega\hat{l}_{i,t}^{\tau}$, we obtain

$$\hat{l}_{i,t} = (1-\omega)\hat{x}_{i,t} - \left(\omega\gamma + (1-\omega)(1-\omega(1-\sigma))\right)\hat{W}_{i,t}. \tag{34}$$

This equation links island-level employment to island-level nominal spending on non tradable goods and island-specific wages. Compared to the aggregate economy, employment is less sensitive to (local) spending. The wage elasticity of labor demand depends on both elasticities γ and σ, and on the importance of traded goods ω.

Consider next the island-level responses of labor supply and the dynamics of wages: $\hat{W}_{i,t} = \lambda\hat{W}_{i,t-1} + (1-\lambda)\hat{W}_{i,t}^*$, and $\hat{l}_{i,t} = \nu\left(\hat{W}_{i,t}^* - E_t\left[\hat{x}_{i,t+1}\right]\right)$. Solving for the desired wage, we obtain an equation that

[14] When $\lambda = 0$, we have $(l_t)^{\frac{1+\nu}{\nu}} = \beta E_t\left[\frac{x_t}{x_{t+1}}\right]$ so transitory shocks would still matter. This reflects the inter-temporal distortion coming from the CIA constraint. The model without nominal friction is neutral with respect to permanent nominal credit shocks. It is not super-neutral because θ is not constant, then x moves around, and this creates inter-temporal disturbances in labor supply but these distortions are small.

describes wages dynamics as a function of total spending:

$$(1-\lambda)\left(\hat{l}_{i,t}+\nu E_t\left[\hat{x}_{i,t+1}\right]\right)=\nu\left(\hat{W}_{i,t}-\lambda\hat{W}_{i,t-1}\right). \quad (35)$$

Equation (35) is relevant only when $\lambda < 1$. When $\lambda = 1$, wages are fixed at their steady-state values.

The third and last part of the system describes credit dynamics. In the aggregate, we have $x_t = W_t l_t$. At the island level, we have:

$$\left(1-\bar{\theta}\bar{q}\right)\hat{x}_{i,t}-\bar{\theta}\bar{q}\hat{q}_{i,t}=\hat{W}_{i,t-1}+\hat{l}_{i,t-1}-\bar{\theta}\bar{q}\left(\hat{q}_{i,t-1}+\hat{x}_{i,t-1}\right)+\bar{\theta}\bar{q}\left(\hat{\theta}_{i,t}-\hat{\theta}_{i,t-1}\right), \quad (36)$$

and

$$\beta E_t\left[\hat{q}_{i,t+1}+\bar{\theta}\hat{x}_{i,t+1}\right]=\left(1-(1-\beta)\bar{\theta}\right)\hat{q}_{i,t}+\bar{\theta}\beta\hat{x}_{i,t}-(1-\beta)\bar{\theta}\hat{\theta}_{i,t}. \quad (37)$$

We therefore have a system of four equations (34, 35, 36, 37) in four endogenous unknowns $(\hat{W}_{i,t}, \hat{l}_{i,t}, \hat{x}_{i,t}, \hat{q}_{i,t})$ and one exogenous processes for $\theta_{i,t}$. We calibrate and solve the system numerically in Section 3, but much intuition can be gained by considering the special case of fixed wages.

We consider permanent shocks to $\theta_{i,t}$ so after the initial shock $\theta_{i,0}$ at $t = 0$, we have $\hat{\theta}_{i,t} = \hat{\theta}_{i,t-1}$ for $t = 1, ..., \infty$ and the credit system (36,37) is simplified. We also assume that relative wages do not change: $\hat{W}_{i,t} = 0$.[15] With constant relative wages we have $\hat{l}_{i,t} = (1-\omega)\hat{x}_{i,t}$, and the money accumulation equation (36) becomes:

$$\left(1-\bar{\theta}\bar{q}\right)\hat{x}_{i,t}-\bar{\theta}\bar{q}\hat{q}_{i,t}=\left(1-\omega-\bar{\theta}\bar{q}\right)\hat{x}_{i,t-1}-\bar{\theta}\bar{q}\hat{q}_{i,t-1}.$$

We 'guess and verify' a solution of the type:

$$\hat{q}_{i,t}=\tilde{q}_i-a\hat{x}_{i,t}. \quad (38)$$

In the cross sectional steady state, we have $q_i = \frac{\eta}{(1-\beta)(1-\theta_i)}$ so it is easy to guess that there must be a time invariant component to q. The money accumulation equation implies

$$\hat{x}_{i,t}=\left(1-\frac{\omega}{1-\bar{\theta}\bar{q}\,(1-a)}\right)\hat{x}_{i,t-1}. \quad (39)$$

In the special case $\omega = 0$, we go back to the one island economy with constant x. The house pricing equation becomes

$$\beta\left(\bar{\theta}-a\right)E_t\left[\hat{x}_{i,t+1}\right]+\beta\tilde{q}_i=\left(1-(1-\beta)\bar{\theta}\right)\left(\tilde{q}_i-a\hat{x}_{i,t}\right)+\bar{\theta}\beta\hat{x}_{i,t}-(1-\beta)\bar{\theta}\hat{\theta}_i$$

We can now identify the constant terms and the dynamic terms. For the constant term we get $\tilde{q}_i = \frac{\bar{\theta}}{1-\bar{\theta}}\hat{\theta}_i$. This is what we expected since the long run value for $\hat{q}_i$ implies $d\log q_i = -d\log(1-\theta_i) = \frac{\bar{\theta}}{1-\bar{\theta}}\hat{\theta}_i$. For the dynamic terms we get

$$E_t\left[\hat{x}_{it+1}\right]=\left(1-\frac{a(1-\beta)\left(1-\bar{\theta}\right)}{\beta\left(\bar{\theta}-a\right)}\right)\hat{x}_{it}$$

[15] This could be either because wages are rigid in nominal terms, $\lambda = 1$, or because relative wages are fixed across islands. In the first case, we can drop equation (35). In the second case, we are simply saying $W_{it} = W_t$ in all islands. Empirically, this appears to be a reasonable approximation to the data. Theoretically, we know that $W_{it} = W_t$ in the long run. See below for a discussion of what happens if relative wages move.

Under perfect foresight and using the law of motion (39), we obtain an equation for a:

$$\omega\left(\bar{\theta}-a\right)\beta = a\left(1-\beta\right)\left(1-\bar{\theta}\right)\left(1-\bar{\theta}\bar{q}\left(1-a\right)\right) \tag{40}$$

We can find a solution for a, which validates our initial guess in equation (38). If $\omega = 0$, we have $a = 0$ as in the one-island economy. When $\omega > 0$, the LHS of (40) decreases and reaches zero when $a = \bar{\theta}$, while the RHS is zero when $a = 0$ and increases afterward. There is therefore a unique solution $0 < a < \bar{\theta}$. Equation (39) shows that the system is stable and $\lim_{t\to\infty}\hat{x}_{it} = 0$.

The impact response, assuming we start from steady state with $\hat{\theta}_{i,t-1} = 0$, is $\left(1-\bar{\theta}\bar{q}\right)\hat{x}_{i,0} - \bar{\theta}\bar{q}\hat{q}_{i,0} = \bar{\theta}\bar{q}\hat{\theta}_i$ and since $\hat{q}_{i,0} = \tilde{q}_i - a\hat{x}_{i,0}$ we have

$$\left(1-\left(1-a\right)\bar{\theta}\bar{q}\right)\hat{x}_{i,0} = \frac{\bar{q}}{1-\bar{\theta}}\bar{\theta}\hat{\theta}_i.$$

A positive shock to credit increases spending in the island.

Finally we can come back to our assumption of constant wages. If relative wages can move, they will help smooth the transition by making hard hit islands temporarily more competitive. Without this we force all the adjustment through consumption and nominal spending. But the main intuition should not change much. We can see which way wages want to adjust by looking at equation (35). Since $\hat{l}_{i,t} = \left(1-\omega\right)\hat{x}_{i,t}$ and since $x_{i,t}$ follows an AR(1) process, wages would like to follow an AR(2) process. We thus expect the response of wages to be hump-shaped. Following a negative shock, relative wages fall first, then rise back to one, the long run value. As long as labor supply is somewhat elastic, the response of wages is small, and the dynamics derived under the assumption of fixed wages give a good approximation.

Proofs of Proposition 3

The simplest way to understand the optimal plan is to solve for the plan without CiA constraints, and then to show it can be implemented with the right taxes. Consider for simplicity, a linear technology for home construction. Without CiA, the Lagrangian of the Planner's program is

$$\mathbb{L} = \mathbb{E}\sum\beta^t\left\{u\left(c_t,h_t,l_t\right)+\lambda_t\left(l_t-c_t-h_t+\left(1-\delta_h\right)h_{t-1}\right)\right\}.$$

The optimal labor supply requires $u_c\left(t\right)+u_l\left(t\right)=0$ and the optimal housing investment requires

$$u_h\left(t\right) = u_c\left(t\right)-\beta\left(1-\delta_h\right)\mathbb{E}_t\left[u_c\left(t+1\right)\right].$$

We can compare with the decentralized equilibrium with taxes and constant prices. Let τ^l be the tax on labor income and τ^h be the tax on home construction. Optimal labor supply requires $\left(1-\tau^l\right)\beta = 1$. As expected, the planner would choose a negative labor income tax to correct the inter-temporal distortion. The steady state housing equation of the Planner's program is $\frac{h}{c} = \frac{\eta}{1-\beta(1-\delta_h)}$ while in the decentralized we have $\frac{h}{c} = \left(1-\tau^h\right)q$ and $q = \frac{\eta}{1-\beta(1-\delta_h)-\theta(1-\beta)}$. So optimality requires

$$\tau^h = \frac{\theta\left(1-\beta\right)}{1-\beta\left(1-\delta_h\right)}.$$

Credit and Home Price Dynamics with News

In this section we briefly explain how anticipated changes in θ affect current home prices and credit. This is important to account for the relative volatility of credit and home prices.

Unexpected Shocks (benchmark model)

This is what we have been doing to far. Imagine that we start from a steady state with $\theta = \theta_0$, $q_0 = \frac{\eta}{(1-\beta)(1-\theta_0)}$ and $x_0 = \frac{1}{1-\theta_0 q_0}$. The time line of events is:

- $t = 0$. Steady state with θ_0
- $t \geq 1$. Permanent shock realized, $\theta_t = \theta_1$ remains constant.

For small values of θ, in steady state, we have $x_t \approx 1 + \theta_t q_t$ and $q_t \approx \frac{\eta}{1-\beta}(1+\theta_t)$. Start from $q_0 = 2$ and $\theta_0 = 0$ so $\frac{\eta}{1-\beta} = 2$ while $x_0 = 1$. Consider a small change in θ, then $q_t \approx 2(1+\theta_t)$ and $\hat{q}_t = \frac{q_t - q_0}{q_0} \approx \theta_t$ so that $x_t \approx 1 + 2\theta_t$ and $\hat{x}_t = \frac{x_t - x_0}{x_0} \approx 2\theta_t$. House prices are

$$Q_t = q_t x_t$$

Hence

$$\hat{Q}_1 = \hat{q}_1 + \hat{x}_1 = \frac{3}{2}\hat{x}$$

Say we want x to move up by 10% in our calibration this implies that Q moves up by 15%. In this model, house prices cannot move up more than 1.5 times nominal spending. So spending should move by at least 2/3 of house price appreciation. In the cross section, however, spending moves by 0.14 to 0.18 times the log change in house prices. This suggests we need "anticipated" shocks as well.

Expected Shock (news model)

Now we had a "news shock":

- $t = 0$. Steady state with θ_0
- $t = 1$. News that $\{\theta_t\}$ will permanently jump to $\theta_2 > \theta_0$ at time 2. We still have $\theta_1 = \theta_0$, but $\{q_t\}$ jumps to $q_1 > q_0$, and therefore x will also jump.
- $t \geq 2$. Permanent shock realized, $\theta_t = \theta_2$ remains constant.

It is easy to see that from $t = 2$ onwards, we are back to steady state with $q_2 = \frac{\eta}{(1-\beta)(1-\theta_2)}$ and $x_2 = \frac{1}{1-\theta_2 q_2}$. What is more interesting is what happens at time 1. We have $x_1 = \frac{1}{1-\theta_0 q_1}$ and $\eta + \beta q_2 = \left(1 - \theta_0\left(1 - \beta\frac{x_1}{x_2}\right)\right) q_1$. So we can solve for q_1 exactly using

$$\eta + \beta q_2 = \left(1 - \theta_0\left(1 - \beta\frac{1-\theta_2 q_2}{1-\theta_0 q_1}\right)\right) q_1$$

For small θ, we have $x_1 \approx 1 + \theta_0 q_1$ and $q_1 \approx \frac{\eta + \beta q_2}{1-\theta_0(1-\beta)}$. We cannot literally start from $\theta_0 = 0$ because we would need infinite q_1 to move x_1. Since $x_0 \approx 1 + \theta_0 q_0$, we have $\hat{x}_1 \approx \frac{\theta_0(q_1-q_0)}{1+\theta_0 q_0} = \frac{q_0\theta_0}{1+\theta_0 q_0}\hat{q}_1$. With our usual calibration of $q_0 = 2$ and $\theta_0 = 5\%$, we get $\hat{x}_1 = \frac{1}{11}\hat{q}_1$. Now we get $\hat{Q}_1 = \hat{q}_1 + \hat{x}_1 = 12\hat{x}_1$. If x moves up by 10%, house prices move up by 120%.

Calibrating θ

What is the right value for θ given the Mian-Sufi estimates? We need to map a "5 year" regression estimate into an annual model, taking into account the maturity of HELOCs and the sources of the shock. The Mian-Sufi result says:

$$B_T - B_1 = 0.25\,(Q_T - Q_1)$$

First issue: the source of shock

In our model, normalizing $h = 1$, we have $B = \theta Q$ so

$$B_T - B_1 = \theta_T Q_T - \theta_1 Q_1 \approx (\theta_T - \theta_1)\,Q_1 + (Q_T - Q_1)\,\theta_1$$

In the news model, have $\theta_T = \theta_1$ so we can indeed use the Mian-Sufi estimate of 0.25 to calibrate θ_1. If the shock is a move in current θ, it is not so clear. With current shocks only, we have shown that $\frac{Q_T - Q_1}{Q_1} \approx 3\,(\theta_T - \theta_1)$ so $B_T - B_1 \approx \frac{Q_T - Q_1}{3Q_1} Q_1 + (Q_T - Q_1)\,\theta_1$ and $\Delta B = \left(\theta_1 + \frac{1}{3}\right)\Delta Q$. Thus it is not possible to have a coefficient of 0.25 in this case. It must be at least 0.33. But suppose we think that the current shock has moved by some amount $\theta_T - \theta_1$. The rest is anticipated shocks. The anticipated shocks move house prices. If we assume that anticipated shocks are proportional to realized current ones, then house price movements will be proportional to shocks:

$$\frac{\theta_T - \theta_1}{\theta_1} = m\frac{Q_T - Q_1}{Q_1}$$

which implies for debt

$$B_T - B_1 \approx (\theta_T - \theta_1)\,Q_1 + (Q_T - Q_1)\,\theta_1 = (Q_T - Q_1)\,\theta_1\,(1 + m)$$

So the bias is m. Note that the pure news model has effectively $m = 0$ since it imposes $\theta_T - \theta_1 = 0$. Now we can get a sense of how large m is by looking at macro data. If we write $\theta_T = (1 + g)\,\theta_1$. The macro data suggests $g \approx 0.2$ since debt went up by 20% more than house value (in aggregate 50% versus 40%). Then we have $g = m\frac{Q_T - Q_1}{Q_1}$. Since house value went up 40%, we get $m = 0.5$, which implies that $\theta_1 = 2/3 * 0.25$

Second issue: maturity

Imagine the following economy. There are N households. Household 1 borrow B at the *beginning* of time 1, and spends it immediately. Then it repays B/N at the *end* of time $1, 2..N$. Then at time $N + 1$, it starts the same cycle. Household 2 does it at 2 and $N + 2$ and so on. This economy is stationary and the maturity of debt is 5 years (note that we abstract from interest rates for simplicity, as in our model). Moreover, in any period, the beginning of period spending is B (by the one household who just took out the loan). The total repayment at the end of the period is $N \times B/N = B$. So this matches exactly our model in terms of flows. But the outstanding balances are: (i) Beginning of period outstanding debt: $B + B\left(1 - \frac{1}{N}\right) + B\left(1 - \frac{2}{N}\right) \ldots + \frac{B}{N} = NB - \frac{N(N-1)}{2}B = \frac{N+1}{2}B$; (ii) End of period is beginning of period minus B: $\frac{N-1}{2}B$

So the average balance during the period is exactly $\frac{NB}{2}$. Our θ relates to spending within the period, which is B. If we measure θ as Mian-Sufi, we get an upward bias of $N/2$. Since average maturity is 5 years for HELOC, the bias is 2.5. If we take the "news" model, we want to calibrate θ by scaling it down from 0.25 to 0.1. If we take the mixed model, we obtain our "structural" estimate of θ_1 as

$$\theta_1 = 0.25/2.5 * 2/3 = 0.067.$$

Model with Credit

Stationarity

In this economy, the asset holdings of agents on individual islands are non-stationary, as in the small open economy incomplete markets literature. Following Schmitt-Grohe and Uribe (2003) and Neumeyer and Perri (2005), we eliminate this non-stationarity by assuming that patient agents on each island face a quadratic cost of deviating from the aggregate asset market position. This cost is a fee rebated lump-sum to patient agents. The patient household's budget constraint becomes

$$\Delta M_{t+1}^{p}(i) = B_t^p(i) - P_t(i)\, c_t^p(i) - Q_t(i)\, \Delta h_t^p(i) - (1 + R_{t-1})\, B_{t-1}^p(i) + W_t(i)\, l_t^p(i) + X_t^p(i) - \frac{\xi}{2}\left(A_t^p(i) - A_t^p\right)^2 + T_t(i)$$

where $T_t(i)$ are the lump-sum transfers and $A_t^p = \int A_t^p(i)\, di$ are the aggregate asset holdings of the patient agents.

Calibration

We set identical technology parameters in the two models. As earlier, we choose an elasticity of substitution across different tradables of $\gamma = 1.5$, a tradable-non-tradable elasticity of $\sigma = 0.1$, and a labor supply elasticity of $\nu = 2$. The preference weight on tradables is equal to $\omega = 0.25$. We set the degree of wage stickiness equal to $\lambda = 0.7$. In the benchmark model we choose the preference weight on housing, η so as to ensure a housing to consumption ratio of 2.1, and we set the discount factor equal to $\beta = 0.95$. Finally, as we described earlier, we set the liquidity constraint parameter equal to $\theta = 0.067$.

For the model with credit, we set the discount factor of the two types equal to $\beta_m = 0.92$ and $\beta_p = 0.98$, respectively. We again choose the preference weight on housing, η, to ensure an aggregate housing to consumption ratio of 2.1. Given the heterogeneity in preferences, in equilibrium agents differ in their housing-to-consumption ratio: while patient agent's housing stock is about 3 times greater than their consumption, impatient agent's stock is only about 1 times their consumption.

To calibrate the credit and liquidity constraints in the model with credit, we choose θ^a and θ^b to jointly target two statistics. First, as in the previous model and in the Mian and Sufi (2010a) evidence, we target a marginal propensity to consume out of housing wealth of 0.067. In the model with credit households can consume by borrowing in both the asset and credit market. Hence the marginal propensity to consume out of housing wealth is equal to

$$mpc = \frac{\left(\theta^a + \theta^b\right) h^m + \theta^b h^p}{h^m + h^p} = 0.067$$

Second, standard mortgages have a combined loan-to-value ratio of about 80% and a 15-year total maturity, that is, a $7 = (15-1)/2$ average maturity. Translating the 7-year maturity into our one-period loan in the model implies a loan-to-value ratio equal to $0.8/7 = 0.11$. Hence, our second target is:

$$\theta^a + \theta^b = 0.11$$

These two targets imply that $\theta^a = 0.056$ and $\theta^b = 0.054$, thus not too different then in the benchmark model.