The Cross-Section and Time-Series of Stock and Bond Returns*

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ABSTRACT

We propose a three-factor model that jointly prices the cross-section of returns on portfolios of stocks sorted on the book-to-market dimension, the cross-section of government bonds sorted by maturity, and time series variation in expected bond returns. The main insight is that innovations to the nominal bond risk premium price the book-to-market sorted stock portfolios. We argue that these innovations capture business cycle risk and show that dividends of the highest book-to-market portfolio fall substantially more than those of the low book-to-market portfolio during NBER recessions. We propose a structural model that ties together the nominal bond risk premium, the cross-section of book-to-market sorted stock portfolios, and recessions. This model is quantitatively consistent with the observed value, equity, and nominal bond risk premia.

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As long as some investors have access to both stock and bond markets, the absence of arbitrage opportunities imposes cross-market restrictions on the stochastic discount factor, henceforth SDF. Despite tremendous progress in the separate modeling of SDFs for bond markets and stock markets, the cross-market restrictions between stocks and bonds are typically not imposed. As a result, the state-of-the art bond pricing model does not price stocks and the state-of-the-art equity pricing model does not price bonds. We propose a parsimonious no-arbitrage SDF model that exploits these cross-market restrictions to learn about the pricing of risk across stock and bond markets.

We find that three factors are able to account for most of the average return differences between book-to-market sorted equity portfolios, the aggregate stock market, and maturity-sorted government bond portfolios. The first factor is a stock market return factor familiar from the Capital Asset Pricing Model, the second factor is the level of the nominal term structure, and the third factor is a proxy for the nominal bond risk premium, the Cochrane and Piazzesi (2005) factor (CP factor).

The first contribution of the paper is to document that value portfolio returns have a higher covariance with innovations in the nominal bond risk premium than growth portfolio returns; see Figure 1. Combined with the positive price of risk we estimate on innovations to the CP factor, this differential exposure results in a value premium. Differential exposure to level shocks accounts for the difference between returns on long-term and short-term bonds, consistent with Cochrane and Piazzesi (2008), while exposure to the market return accounts for the aggregate equity premium. The three-factor model reduces mean absolute pricing errors from 4.7% per year in a risk-neutral benchmark economy to 0.4% per year (Section II). In Section V, we present similar results for different sub-samples and for different equity and bond portfolios. In particular, our model does a very good job pricing corporate bond portfolios differing by credit rating, in addition to equities and Treasuries. We also present stock-level evidence that exposure to the CP shocks is priced in the cross-section.

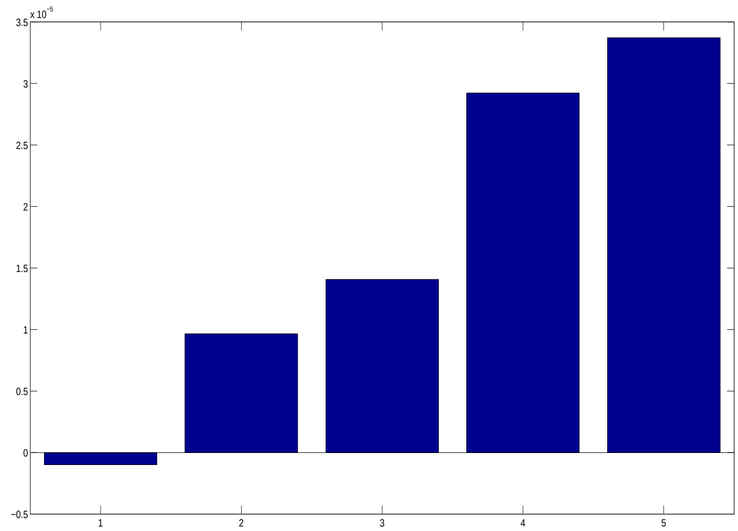

Figure 1. Exposure of value and growth portfolio returns to CP innovations.
The figure shows the covariance of innovations to the nominal bond risk premium (CP factor) and innovations to returns on five quintile portfolios sorted on the BM ratio. Portfolio 1 is the lowest book-to-market (growth) portfolio; portfolio 5 the highest book-to-market (value) portfolio. Innovations to CP are constructed as described in Section II.

The factor model leaves two important questions unanswered. Why are value portfolios more exposed to bond risk premium innovations than growth portfolios, and why is the price of bond risk premium (CP) innovations positive? We provide new empirical evidence and a new structural model to shed light on these questions.

The second contribution of our paper is to document that the dividends of value portfolios are substantially more cyclical than those of growth portfolios (Section III). During the average recession, dividends on value stocks (fifth book-to-market quintile) fall 21% while dividends on growth stocks (first quintile) rise by 2%. This 23% gap is much higher in some recessions than in others. For example, during the Great Recession of 2007-2009, the fall in value-minus-growth dividends was 37%. During the Great

Depression the relative log change was -360%. The fall during the NBER recession months underestimates the fall during the broader bust period because the NBER dates may neither coincide with the peak nor the trough for real dividends. Focussing on periods with a protracted fall in real dividends on the market portfolio, we find that real dividends on the highest book-to-market portfolio fall by 53% more than those on the lowest book-to-market portfolio. This decline in the relative dividend on value-minus-growth is twice as high as the fall in the dividends on the market portfolio itself. Thus, our paper provides new evidence that value stocks suffer from bad cash-flow shocks during aggregate bad times. Since those are times of high marginal utility growth for the representative investor, a value risk premium naturally follows. As Lakonishok, Schleifer, and Vishny (1994) and Cochrane (2006) point out, empirical evidence that value stocks suffer from terrible shocks during aggregate bad times has been elusive. We provide a fresh look at the data and find clear patterns in dividend growth differences between value and growth stocks during (and surrounding) NBER recessions. This evidence also answers the question why the price of CP risk is positive. Since the risk that value stocks are more exposed to than growth stocks is that associated with cyclical movements in real economic activity, its price is naturally positive (higher activity indicates good times for the representative investor). Section III also document the connection between the bond risk premium and the business cycle.

The third contribution of the paper is to develop a structural asset pricing model to understand the link between the value risk premium and changes in the nominal bond risk premium (Section IV). In particular, we model cyclical variations in real economic activity as an AR(1) process, whose innovations carry a positive price of risk. Mimicking the backward-looking NBER dating procedure, we define recessions ex-post as a sequence of negative cyclical shocks combined with a low level of economic activity, followed by a sequence of positive shocks. The model replicates the observed duration of recessions. Dividend growth of value stocks has a higher loading on the state of the business cycle than that of growth stocks. The heterogeneity in this exposure is chosen to match the observed fall in dividends on value and growth stocks over the course of a recession. It is the source of the value premium in the model. The model also features aggregate dividend growth risk, which is permanent, and expected inflation shocks. The price of aggregate dividend (positively) and expected inflation risk (negatively) depend on the state of the business cycle. The former ensures that the return difference between value and growth stocks does not arise from different exposure to aggregate dividend risk (market beta), while the latter causes bond risk premia to vary over time. We construct the CP factor inside the model; it captures fluctuations in the business cycle. Hence, differential exposure to innovations in the CP factor is what generates the value premium. The model is able to replicate the exposure of nominal bond and stock portfolio returns for market return, level, and CP shocks that we document in the data. Despite its simplicity, it generates the same size value premium, nominal bond risk premium, equity premium, and 1- through 5-year nominal term structure as what we see in the data for reasonably calibrated dividend growth and inflation processes.

I. Related Literature

This paper relates to several strands of the literature. The last twenty years have seen dramatic improvements in economists' understanding of what determines differences in yields (e.g., Cox, Ingersoll, and Ross (1985), Duffie and Kan (1996), Dai and Singleton (2000, 2002), Duffee (2002), Ang and Piazzesi (2003), Ang, Bekaert, and Wei (2008), and Cochrane and Piazzesi (2008)) and returns on bonds (Campbell and Shiller (1991), Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009)), as well as what determines heterogeneity in stock returns that differ in terms of size and book-to-market value (e.g., Fama and French (1992, 1993)). Yet, these two literatures have developed largely separately and employ largely different asset pricing factors. This is curious from the perspective of either no-arbitrage or equilibrium asset pricing models. As long as some investors have access to both markets, stock and bond prices ought to equal the expected present discounted value of their future cash-flows, discounted by the *same* SDF. This paper contributes to both literatures and helps to bridge the gap between them. It speaks to a large empirical literature and a small but fast-growing theoretical literature.

On the empirical side, the nominal short rate or the yield spread is routinely used either as a predictor of the aggregate return on the stock market or as a conditioning variable in an estimation of a conditional beta model of the cross-section of stock returns. Chen, Roll, and Ross (1986) were the first to study the connection between stock returns and bond yields. Ferson and Harvey (1991) study stock and bond returns' sensitivity to aggregate state variables, among which the slope of the yield curve. They conclude that time variation in equity risk premia is important for understanding the cross-sectional variation in size and industry equity portfolios, and that interest rate risk premia are important for understanding the cross-sectional variation in bond return portfolios. Similarly, Fama and French (1993) find that three factors (market, size, and book-to-market) account for the cross-sectional variation in stock returns and that two bond factors (the excess return on a long-term bond over the short rate and a default spread) explain the variation in government and corporate bond returns. However, all of their stock portfolios load in the same way on their term structure factors. Ang and Bekaert (2007) find some predictability of nominal short rates for future aggregate stock returns. Brennan, Wang, and Xia (2004) write down an intertemporal-CAPM model where the real rate, expected inflation, and the Sharpe ratio move around the investment opportunity set. They show that this model prices the cross-section of stocks. Similarly, Petkova (2006) studies the connection between the Fama-French factors and innovations in state variables such as the default spread, the dividend-price ratio, the yield spread, and the short rate. In contrast to this literature, our focus is on the joint pricing of stock and bond returns, the link with dividend growth on equity portfolios, and the connection of bond and value-minus-growth returns with the business cycle. Baker and Wurgler (2007) show that government bonds comove most strongly with "bond-like stocks," which are stocks of large, mature, low-volatility, profitable, dividend-paying firms that are neither high growth nor distressed. They propose a common sentiment indicator driving stock and bond returns. Finally, Lustig, Van Nieuwerburgh, and Verdelhan (2010) price both nominal bond yields and the aggregate stock market return in a no-arbitrage model in order to measure the wealth-consumption ratio in the data; they do not study the cross-section of bond nor stock returns.

On the theory side, several representative-agent models have been developed that are successful in accounting for many of the features of both stocks and bonds. Examples are the external habit model of Campbell and Cochrane (1999), whose implications for bonds were studied by Wachter (2006) and whose implications for the cross-section of stocks were studied separately by Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006). Likewise, the implications of the long-run risk model of Bansal and Yaron (2004) for the term structure of interest rates were studied by Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2007), while Bansal, Dittmar, and Lundblad (2005) and Bansal, Dittmar, and Kiku (2007) study the implications for the cross-section of equity portfolios. A small but growing literature models stock and bond returns jointly. Examples are the affine models of Bakshi and Chen (1996, 1997) in a square-root diffusion setting and Bakshi and Chen (2005) and Bekaert, Engstrom, and Grenadier (2005), Bekaert, Engstrom, and Xing (2008) in a Gaussian setting, and the linear-quadratic model of Campbell, Sunderam, and Viceira (2008) all of which explore the relationship between aggregate stock and bond markets. Lettau and Wachter (2009) and Gabaix (2009) additionally study the cross-section of stock returns. The former is a model with common shocks to the risk premium in stock and bond markets, while the latter is a time-varying rare disasters model.

Our paper contributes to the large literature on the value risk premium. Value strategies call for buying stocks that have low prices relative to fundamental measures of value, such as dividends or book assets and are associated with superior returns, unexplained by the CAPM (e.g., Basu (1977) and Fama and French (1992)). The profession has hotly debated whether these superior returns reflect a fair compensation for other sources of systematic risk or a behavioral anomaly. Extrapolative investors may push up the prices of "glamour" (growth) stocks that performed well in the recent past, allowing contrarian investors to profit from their over-optimism by investing in out-of-favor value stocks and/or shorting the growth stocks (De Bondt and Thaler 1985). To settle this debate, Cochrane (2006) points out that "Our lives would be so much easier if we could trace price movements back to visible news about dividends or cash flows." But early attempts to connect the cash flows of value and growth firms to macro-economic sources of risk came up empty handed (Lakonishok, Schleifer, and Vishny 1994). Relative to LSV, we study a longer sample (1926-2009 compared to 1968-1989, or 15 recessions compared to 4), we focus on dividends, and trace those dividends more finely over the course of NBER recessions (from the exact peak month to the trough month). We find large differences in the cyclical behavior of value and growth dividends. Complementary work in production-based asset pricing has linked investment behavior of value and growth firms during a recession to the value premium (Zhang 2005).

Finally, there is a related literature that studies the temporal composition of risk in asset prices, including Cochrane and Hansen (1992), Kazemi (1992), Bansal and Lehman (1997), Hansen, Heaton, and Li (2008), Alvarez and Jermann (2004, 2005), Hansen and Scheinkman (2009), Borovicka, Hansen, Hendricks, and Scheinkman (2009), Martin (2008), Backus, Routledge, and Zin (2008), and Backus, Chernov, and Martin (2009). Our model connects to this discussion because it features both permanent shocks (to aggregate dividend) and transitory shocks (to real economic activity). Relative to this literature, we emphasize the importance of transitory cash-flow risk, as reflected in business cycle variation in bond risk premia, and its relationship to the value premium.

II. Empirical Link Between Stocks and Bonds

In this section we develop and estimate a reduced-form, no-arbitrage SDF model which achieves consistent risk pricing across stocks and bonds. The model suggests that three priced sources of risk are necessary to account for the market equity risk premium, the value premium, and the risk premia on nominal bonds of various maturities. Section IV presents a structural asset pricing model that provides an economic intuition for the empirical connection between stocks and bonds we document here.

A. Setup

Let P_t be the price of a risky asset and D_{t+1} its (stochastic) cash-flow. Then the nominal SDF implies $P_t = E_t[M^{\$}_{t+1}(P_{t+1} + D_{t+1})]$. Lowercase letters denote natural logarithms: $m^{\$}_t = \log\left(M^{\$}_t\right)$. We propose a reduced-form SDF, akin to that in the empirical term structure literature (Duffie and Kan 1996):

$$-m_{t+1}^{\$} = y_t^{\$} + \frac{1}{2}\Lambda_t'\Sigma\Lambda_t + \Lambda_t'\varepsilon_{t+1}, \tag{1}$$

where $y_t^{\$}$ is the nominal short-term interest rate, ε_{t+1} is a $N \times 1$ vector of shocks to the $N \times 1$ vector of demeaned state variables X_t, and where Λ_t is the $N \times 1$ vector of market prices of risk associated with these shocks at time t. The state vector in (2) follows a first-order vector-autoregression with intercept γ_0, companion matrix Γ, and conditionally normally, i.i.d. distributed innovations, $\varepsilon_t \sim \mathcal{N}(0, \Sigma)$:

$$X_{t+1} = \Gamma X_t + \varepsilon_{t+1}, \tag{2}$$

$$\Lambda_t = \Lambda_0 + \Lambda_1 X_t. \tag{3}$$

The market prices of risk are affine in the state vector, where Λ_0 is an $N \times 1$ vector of constants and Λ_1 is an $N \times N$ matrix that governs the time variation in the prices of risk.

Log returns on an asset j can always be written as the sum of expected and unexpected returns: $r_{t+1}^j = E_t[r_{t+1}^j] + \eta_{t+1}^j$. Unexpected log returns η_{t+1}^j are assumed to be conditionally normally distributed. We denote the covariance matrix between shocks to returns and shocks to the state variables by Σ_{Xj}. We define log excess returns to include a Jensen adjustment:

$$rx_{t+1}^j \equiv r_{t+1}^j - y_t^{\$}(1) + \frac{1}{2}V[\eta_{t+1}^j].$$

The no-arbitrage condition then implies:

$$E_t\left[rx_{t+1}^j\right] = Cov_t\left[rx_{t+1}^j, -m_{t+1}^{\$}\right] = Cov\left[\eta_{t+1}^j, \varepsilon_{t+1}'\right]\Lambda_t \equiv \Sigma_{Xj}\left(\Lambda_0 + \Lambda_1 X_t\right). \tag{4}$$

Unconditional expected excess returns are computed by taking the unconditional expectation of (4):

$$E\left[rx_{t+1}^{j}\right] \;=\; \Sigma_{Xj}\Lambda_0. \tag{5}$$

The main object of interest is Λ_0, which we will estimate below. Equation (5) suggests an interpretation of our model as a simple three-factor model. In Section D, we estimate how the market prices of risk vary with X_t (Λ_1).

B. Data and Implementation

We aim to explain the average excess returns on the five value-weighted quintile portfolios sorted on their book-to-market ratio from Fama and French (1992), the value-weighted stock market return from CRSP (NYSE, Amex, and Nasdaq), and five zero-coupon nominal government bond portfolios with maturities 1, 2, 5, 7, and 10 years from CRSP. The return data are monthly from July 1952 until December 2009.

We propose three asset pricing factors in X_t: the CP factor, the LVL factor, and the $Market$ factor. First, a substantial bond return predictability literature shows that bond risk premia vary over time. Cochrane and Piazzesi (2005) combine bond yields of maturities one to five years to form the CP factor and show that it does a good job predicting future excess bond returns. Our first asset pricing factor is the CP, constructed as in Cochrane and Piazzesi (2005).[1] We construct the unexpected bond

[1] In particular, we use monthly Fama-Bliss yield data for nominal government bonds of maturities one- through five-years. These data are available from June 1952 until December 2009. We construct one- through five-year forward rates from the one- through five-year bond prices. We then regress the equally-weighted average of the one-year excess return on bonds of maturities of two, three, four, and five years on a constant, the one-year yield, and the two- through five-year forward rates. The yields are one-year lagged relative to the return on the left-hand side. The CP factor is the fitted value of this predictive regression. The R^2 of this regression in our sample of monthly data is 23.8%, roughly twice

returns in η as the residuals from a regression of each bond portfolio's log excess return on the lagged CP factor. Similarly, we assume that stock returns are also predictable by the CP factor, and construct the unexpected stock returns in η as the residual from a regression of each bond portfolio's log excess return on the lagged CP factor.[2] Second, we construct the level factor LVL as the first principal component of the one- through five-year Fama-Bliss forward rates. Third, the market factor (MKT) is the value-weighted stock market return from CRSP.

We use a monthly VAR(1) with the CP, LVL, and MKT factors to extract innovations ε in these factors. Innovations to the state vector ε follow from equation-by-equation OLS estimation of the VAR model in (2). The innovation correlations between our three factors are close to zero: 0.05 between LVL and CP, 0.03 between MKT and CP, and -0.12 between *Level* and MKT.[3]

The first column of Table I shows the expected excess returns, expressed in percent per year, on our 11 test assets we wish to explain. They are the pricing errors resulting from a model where all prices of risk in Λ_0 are zero, that is, from a risk-neutral SDF

the 12.3% R^2 of the fiver-year minus one-year yield spread, another well-known bond return predictor.

[2]Cochrane and Piazzesi (2005) provide evidence of predictability of the aggregate market return by the lagged CP factor. In addition, we could include the aggregate dividend-price ratio as a predictor of the stock market. However, given the low R^2 of these monthly predictive regressions, the resulting unexpected returns are almost the same whether we assume predictability by CP, DP, both, or no predictability at all. In an earlier version of the paper, we had the DP ratio as a factor instead of the market return (and with stock return predictability by the lagged PD ratio), with very similar results.

[3]In the context of an annual model, Cochrane and Piazzesi (2008) argue that the CP factor is not well described by an AR(1) process. In addition to the level of the term structure, they include the slope and the curvature (second and third principal components of the Fama-Bliss forward rates) as predictors in their VAR. The second difference is that they project forward rates on the CP factor before taking principal components of the forward rates. Our results (in a monthly VAR) are not sensitive to either including slope and curvature factors in our VAR to form innovations or to computing level, slope, and curvature in the alternative fashion, or to making both changes at once. Results are available upon request. The only difference is that the VAR innovations for CP, LVL, and MKT are nearly uncorrelated in our procedure, whereas the correlation between CP shocks and LVL shocks is highly negative when forward rates are orthogonalized on CP before taking principal components. We focus on the three-factor structure because it is simpler and it maps more directly into the structural model of Section IV. The latter also implies a MKT, LVL, and CP factor structure whose innovations are nearly uncorrelated.

model (*RN SDF*). Average excess returns on bonds are between 1.0 and 1.8% per year and generally increase in maturity. The aggregate excess stock market return is 6.3%, and the risk premia on the book-to-market portfolios range from 5.7% (BM1, growth stocks) to 10.1% (BM5, value stocks), implying a value premium of 4.4% per year.

C. Estimation Results

We estimate the three price of risk parameters in $\hat{\Lambda}_0$ by minimizing the root mean-squared pricing errors on our $J = 11$ test assets. This is equivalent to regressing the $J \times 1$ average excess returns on the $J \times 3$ covariances in Σ_{XJ}. The results from our model are in the second column of Table I (Our Model). The top panel shows the pricing errors. Our model succeeds in reducing the mean absolute pricing errors (MAPE) on the 11 stock and bond portfolios to a mere 40 basis points per year. The model largely eliminates the value spread: The spread between the fifth and the first book-to-market quintile portfolios is 80 basis points per year. We also match the market equity risk premium and the average bond risk premium. Pricing errors on the stock and bond portfolios are an order of magnitude lower than in the first column and substantially below those in several benchmark models we discuss below. In sum, our three-factor pricing model is able to account for the bulk of the cross-sectional variation in stock and bond returns with a single set of market price of risk estimates.

The bottom panel of the table shows the point estimates for $\hat{\Lambda}_0$. We estimate a positive price of CP risk, while the price of *Level* risk is negative and that of $Market$ risk is positive. The signs on the last two are as expected. A positive shock to the level factor leads to a drop in bond prices and negative bond returns. A negative shock to bond returns increases the SDF (the representative agent's marginal utility of wealth) and, hence, carries a negative risk prices. A positive shock to the market factor increases stock returns and the SDF and should carry a positive risk price. We return at length to the CP factor and its positive risk price below. We also report asymptotic standard errors on the Λ_0 estimates using GMM with an identity weighting matrix. The standard errors are 37.23 for the CP factor price (98.10), 9.20 for the LVL factor price (-19.45), and 1.42 for the MKT factor price (2.11). Hence, the first two risk prices are statistically different from zero (with t-stats of 2.6 and -2.1), whereas the last one is not (t-stat of 1.5).

To help us understand the separate roles of each of the three risk factors in accounting for the risk premia on these stock and bond portfolios, we switch on only one risk factor and set the other risk prices to zero. Column 3 of Table I minimizes the pricing errors on the same 11 test assets but only allows for a non-zero price of level risk (Column *LVL*). This is the bond pricing model proposed by Cochrane and Piazzesi (2008). They show that the cross-section of average bond returns is well described by differences in exposure to the level factor. Long-horizon bonds have returns that are more sensitive to interest rate shocks than short-horizon bonds; a familiar duration argument. However, this bond SDF is unable to jointly explain the cross-section of stock and bond returns; the MAPE is 4.23%. All pricing errors on the stock portfolios are large and positive, there is a 4.4% value spread, and all pricing errors on the bond portfolios are large and negative. Clearly, exposure to the level factor alone does not help to understand the high equity risk premium nor the value risk premium. Value and growth stocks have similar exposure to the level factor, that is, a similar "bond duration." The reason that this model does not do better pricing the bond portfolios is that the excess returns on stock portfolios are larger in magnitude and therefore receive most attention in the estimation. Consequently, the estimation concentrates its efforts on reducing the pricing errors of

Table I
Unified SDF Model for Stocks and Bonds - Pricing Errors

Panel A of this table reports pricing errors on five book-to-market sorted quintile stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. They are expressed in percent per year (monthly numbers multiplied by 1200). Each column corresponds to a different stochastic discount factor (SDF) model. The first column contains the risk-neutral SDF and therefore reports the average pricing errors that are to be explained. The second column presents our SDF model with three priced factors (Our Model). The third column presents the results for a bond pricing model, where only the level factor is priced (LVL). In the fourth column, we only use the bond returns as moments to estimate the same SDF as in the third column (LVL-only bonds). The SDF model of the fifth column has the market return as the only factor, and therefore is the CAPM model (MKT). The sixth column allows for both the prices of LVL and MKT risk to be non-zero. The last column refers to the three factor model of Fama and French (1992). The last row of Panel A reports the mean absolute pricing error across all 11 securities (MAPE). Panel B reports the estimates of the prices of risk. The first six columns report market prices of risk Λ_0 for (a subset) of the following pricing factors : ε^{CP} (CP), ε^{L} (Level), and ε^{M} (MKT). In the last column, the pricing factors are the innovations in the excess market return (MKT), in the size factor (SMB), and in the value factor (HML), where innovations are computed as the residuals of a regression of these factors on the lagged dividend-price ratio on the market. Panel C reports asymptotic p-values of χ^2 tests of the null hypothesis that all market prices of risk in Λ_0 are jointly zero, and of the null hypothesis that all pricing errors are jointly zero. The data are monthly from June 1952 through December 2009.

				Panel A: Pricing Errors (in % per year)				
	RN SDF	Our SDF	LVL	LVL-only bonds	MKT	$LVL+MKT$		FF
1-yr	1.00	-0.54	-0.12	0.64	0.89	0.61		0.79
2-yr	1.21	-0.61	-0.98	0.50	1.04	0.49		0.83
5-yr	1.52	-0.22	-2.84	0.09	1.25	0.14		0.78
7-yr	1.78	0.42	-3.35	0.11	1.38	0.08		0.80
10-yr	1.39	0.12	-4.37	-0.49	0.83	-0.63		0.16
Market	6.32	-0.72	4.77	5.82	-1.37	-1.30		-0.05
BM1	5.69	-0.08	4.20	5.21	-2.44	-2.32		0.47
BM2	6.63	-0.18	4.84	6.05	-0.86	-0.85		-0.56
BM3	7.64	0.62	5.84	7.06	0.66	0.63		-0.28
BM4	8.36	-0.32	6.59	7.78	1.55	1.51		-0.68
BM5	10.11	0.72	8.66	9.64	2.69	2.78		0.88
MAPE	4.70	0.41	4.23	3.94	1.36	1.03		0.57
				Panel B: Prices of Risk Estimates Λ_0				
MKT	0	2.11	0	0	3.43	3.22	MKT	5.12
LVL	0	-19.45	-32.74	-10.66	0	-8.45	SMB	-6.29
CP	0	98.10	0	0	0	0	HML	6.40
				Panel C: P-values of χ^2 Tests				
$H_0 : \Lambda_0 = 0$	–	0.94%	0.02%	–	0.08%	0.07%		0.02%
H_0 : Pr. err. $= 0$	–	12.64%	0.00%	–	0.00%	0.00%		0.00%

stocks. To illustrate that this bond SDF is able to price the cross-section of bonds, we estimate the same model by minimizing only the bond pricing errors (the first five moments in the table). The fourth column of Table I (*LVL - only bonds*) confirms the Cochrane and Piazzesi (2008) results that the bond pricing errors are small. However, the overall MAPE remains high at 3.94%. The canonical bond pricing model offers one important ingredient for the joint pricing of stocks and bonds, bonds' heterogeneous exposure to the level factor, but this ingredient does not help to account for equity returns.

Another natural candidate is the canonical equity pricing model: the Capital Asset Pricing Model. The only non-zero price of risk is the one corresponding to the MKT factor. The fifth column of Table I (*MKT*) reports pricing errors for the CAPM. This model is again unable to jointly price stock and bond returns. The MAPE is 1.36%. One valuable feature is that the aggregate market portfolio is priced well and the pricing errors of book-to-market portfolio returns go through zero. This means that the model gets the common level in all stock portfolio returns right. However, the pattern of pricing errors contains a 5.11% value spread. Pricing errors on bond portfolios are sizeable as well and are all positive. In the language of our model, neither book-to-market nor bond portfolios display interesting heterogeneity in their exposure to shocks to the permanent component of the SDF. Models such as the canonical CAPM or the consumption-CAPM, that feature only permanent shocks, cannot jointly explain stock and bond returns.

So, while the LVL factor helps to explain the cross-sectional variation in average bond returns and the MKT factor helps to explain the level of equity risk premia, neither factor is able to explain why value stocks have much higher risk premia than growth stocks. The sixth column of Table I indeed shows that having both the level and market factor priced does not materially improve the pricing errors and leaves the value premium puzzle in tact.

This is where the CP factor comes in. Figure 2 decomposes each asset's risk premium into its three components: risk compensation for exposure to the CP factor, the level factor, and the DP factor. The top panel is for the five bond portfolios, organized from shortest maturity on the left (1-year) to longest maturity on the right (10-year). The bottom panel shows the decomposition for the book-to-market quintile portfolios, ordered from growth to value from left to right, as well as for the market portfolio (most right bar). This bottom panel shows that all book-to-market portfolios have about equal exposure to both MKT and LVL shocks. If anything, growth stocks (G) have slightly higher market (CAPM) betas than value stocks (V), but the difference is small. The spread between value and growth risk premia entirely reflects differential compensation for CP risk. Value stocks have a large and positive exposure to CP shocks while growth stocks have a low or even negative exposure; recall Figure 1. The differential exposure between the fifth and first book-to-market portfolio is statistically different from zero. Multiplying the spread in exposures by the market price of CP risk delivers a value premium of 0.34% per month or 4.1% per year. That is, the CP factor's contribution to the risk premia accounts for most of the 4.4% value premium. Given the monotonically increasing pattern in exposures of the book-to-market portfolios to CP shocks, a positive price of CP risk estimate is what allows the model to match the value premium. The key economic questions that are left unanswered at this point are: what economic source of risk do CP shocks capture, and why do value stocks have higher exposure to these shocks than growth stocks? Below we present new empirical evidence from dividend data and a new structural model to shed light on these questions.

The top panel of Figure 2 shows the risk premium decomposition for the five bond portfolios. Risk premia are positive and increasing in maturity due to their exposure to LVL risk. The exposure to level shocks is negative and the price of level risk is negative, resulting in a positive contribution to the risk premium. This is the duration effect mentioned above. But bonds also have a negative exposure to CP shocks. Being a measure of the risk premium in bond markets, positive shocks to CP lower bond prices and realized returns. This effect is larger the longer the maturity of the bond. Given the positive price of CP risk, this exposure translates into an increasingly negative contribution to the risk premium. Because exposure of bond returns to the equity market shocks MKT is positive but near-zero, the sum of the level and CP contributions delivers the observed pattern of bond risk premia that increase in maturity.

The last but one row of Table I tests the null hypothesis that the market price of risk parameters are jointly zero. This null hypothesis is strongly rejected for all models, including ours. The asymptotic p-value for the χ^2 test, computed by GMM using the identity weighting matrix, is less than 1% for our model. The last row reports the p-value for the χ^2 test that all pricing errors are jointly zero. Interestingly, ours is the only model for which the null hypothesis cannot be rejected; the p-value is 13%. These tests lend statistical credibility to our results.

We also study book-to-market decile portfolios instead of quintile portfolios, alongside the same bond portfolios and the stock market portfolio. Table II shows that the value spread between the tenth and first book-to-market portfolios is 5.42% per annum (Column 1), about one percentage point higher than between the extreme quintile portfolios. The mean absolute pricing error among these 16 assets is 5.67% per year. Our model's residual pricing error is a mere 0.48% (Column 2). It eliminates all but 0.24% of the 5.42% value premium. The market price of risk estimates are nearly identical to those obtained with the quintile portfolios. Again, the null hypothesis that all market prices of risk are jointly zero is strongly rejected, while the null that all pricing errors are

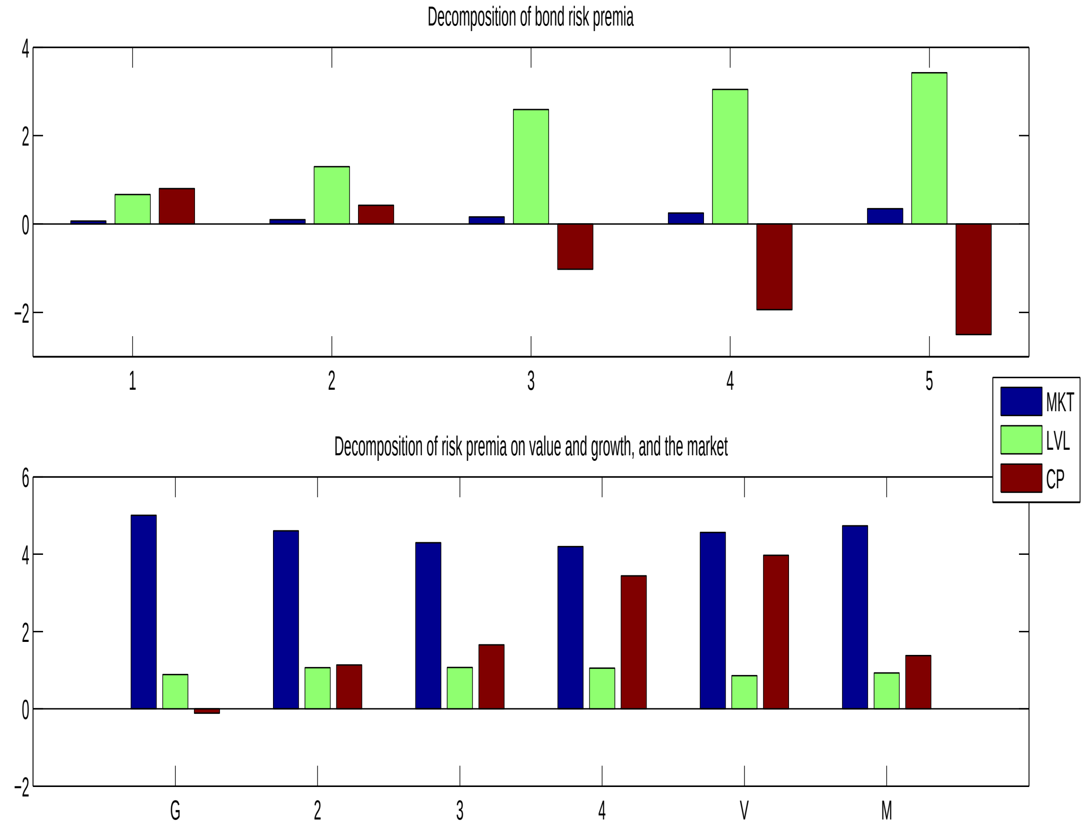

Figure 2. Exposure of portfolio excess returns to priced innovations.
The figure plots the risk premium decomposition into risk compensation for exposure to the MKT, LVL, and CP factors. The top panel is for the five bond portfolios: one-, two-, five-, seven-, and ten-year maturities from left to right, respectively. The bottom panel is for for the book-to-market decile quintile portfolios, from growth (G) to value (V), and for the market portfolio (M). The three bars for each asset are computed as $\Sigma'_{XR}\Lambda_0$. The data are monthly from June 1952 until December 2009.

jointly zero cannot be rejected. Section V studies other sets of test assets for robustness.

One might be tempted to conclude that any model with three priced risk factors can always account for the three salient patterns in our test assets. To highlight that such a conjecture is false and to highlight the challenge in jointly pricing stocks and bonds, Appendix B develops a simple model where (1) the CP factor is a perfect univariate pricing factor for the book-to-market portfolios (it absorbs all cross-sectional variation), (2) the LVL factor is a perfect univariate pricing factor for the bond portfolios, and (3)

Table II
Decile Book-to-Market Portfolios

See Table I. The book-to-market quintile portfolios are replaced by decile portfolios.

	Panel A: Pricing Errors (in % per year)			
	RN SDF	Our SDF		FF
1-yr	1.00	-0.53		0.83
2-yr	1.21	-0.60		0.92
5-yr	1.52	-0.18		1.02
7-yr	1.78	0.46		1.13
10-yr	1.39	0.16		0.48
Market	6.32	-0.74		-0.17
BM1	5.40	0.15		0.52
BM2	6.30	-0.33		0.20
BM3	6.86	-0.23		0.13
BM4	6.49	-0.24		-1.06
BM5	7.49	0.64		-0.13
BM6	7.59	0.17		-0.22
BM7	7.42	-1.35		-0.84
BM8	9.34	0.71		0.03
BM9	9.83	0.81		0.79
BM10	10.82	0.39		0.45
MAPE	5.67	0.48		0.56
	Panel B: Prices of Risk Estimates Λ_0			
MKT	0	2.13	MKT	4.47
LVL	0	-19.19	SMB	-2.92
CP	0	97.88	HML	5.95
	Panel C: P-values of χ^2 Tests			
$H_0 : \Lambda_0 = 0$	–	1.26%	0.03%	
H_0 : Pr. err. $= 0$	–	26.28%	0.03%	

the CP and the *Level* factor are uncorrelated. It shows that such a model generally fails to price the stock and bond portfolios jointly. This failure arises because the bond portfolios are exposed to the CP factor, while the stock portfolios are not exposed to the LVL factor. Consistent risk pricing across stocks and bonds only works if the exposures of maturity-sorted bond portfolios to CP are linear in maturity, with the same slope (in absolute value) as the level exposures. The data happen to approximately satisfy the three assumptions underlying the stark model, but this was not a foregone conclusion. Appendix B thus underscores the challenges in finding a model with consistent risk prices across stocks and bonds.

D. Time-varying Risk Prices

Having estimated the constant market prices of risk, Λ_0, we turn to the estimation of the matrix Λ_1, which governs the time variation in the prices of risk. We allow the price of level risk $\Lambda_{1(2)}$ and the price of market risk $\Lambda_{1(3)}$ to depend on the CP factor. We use two predictive regressions to pin down this variation in risk prices. We regress excess returns on a constant and lagged CP:

$$rx^j_{t+1} = a_j + b_j CP_t + \eta^j_{t+1},$$

where we use either excess returns on the stock market portfolio or an equally-weighted portfolio of all bond returns used in estimation. Using equation (4), it then follows:

$$\begin{pmatrix} \Lambda_{1(2)} \\ \Lambda_{1(3)} \end{pmatrix} = \begin{pmatrix} \Sigma_{X,Market(2:3)} \\ \Sigma_{X,Bonds(2:3)} \end{pmatrix}^{-1} \times \begin{pmatrix} b_{Market} \\ b_{Bonds} \end{pmatrix}.$$

Following this procedure, we find that $\hat{\Lambda}_{1(2)} = -796$ and $\hat{\Lambda}_{1(3)} = 47$. This implies that equity and bond risk premia are high when CP is high, consistent with the findings of Cochrane and Piazzesi (2005).

III. Documenting Two Empirical Links

Before we present the structural model, we turn to the data and document two important linkages that help connect the reduced form from the previous section to the structural model of the next section. First, we show that dividends on value stocks fall considerably more than those of growth stocks during recessions. Second, we connect bond risk premia, as measured by the CP factor, to business cycles.

A. Value Stocks' Dividends Fall More in Recessions

We use monthly data on dividends and inflation from January 1926 until December 2009. Inflation is measured as the change in the Consumer Price Index from the Bureau of Labor Statistics. Dividends on five book-to-market sorted portfolios are based on cum-dividend and ex-dividend returns from Kenneth French' data library. To eliminate seasonality in dividends, we construct annualized dividends by adding the current month's dividends to the dividends of the past 11 months, where the latter are reinvested at the risk-free rate.[4] We form log real dividends by subtracting the log change in the CPI from the log of nominal dividends. We are left with monthly time series of 997 observations. We define recessions following the NBER's Business Cycle Dating committee.

[4]Not reinvesting dividends yields similar results. Binsbergen and Koijen (2010) show that reinvesting monthly dividends at the market return severely impact properties of dividend growth.

Figure 3 plots log real dividends on book-to-market quintile portfolios 1 (G) and 5 (V). For ease of readability, the sample is split in two: before and after 1952. The raw data show strong evidence that dividends on value stocks fall substantially more in recessions than in expansions. Value stocks show strong cyclical fluctuations whereas dividends on growth stocks are, if anything, slightly pro-cyclical. The two most obvious examples of the differential cash-flow behavior of value and growth are the Great Depression in the left panel (1929-1933) and the Great Recession in the right panel (December 2007-June 2009), but the same pattern holds during most recession periods (e.g., 1973, 1983, 1991, 2001). During the Great Depression, the log change in real dividends from the peak is -400% for Value, -60% for the Market, and -40% for Growth. In the Great Recession, dividends fell 31% for Value, 13% for the Market, while Growth dividends rose 6%.

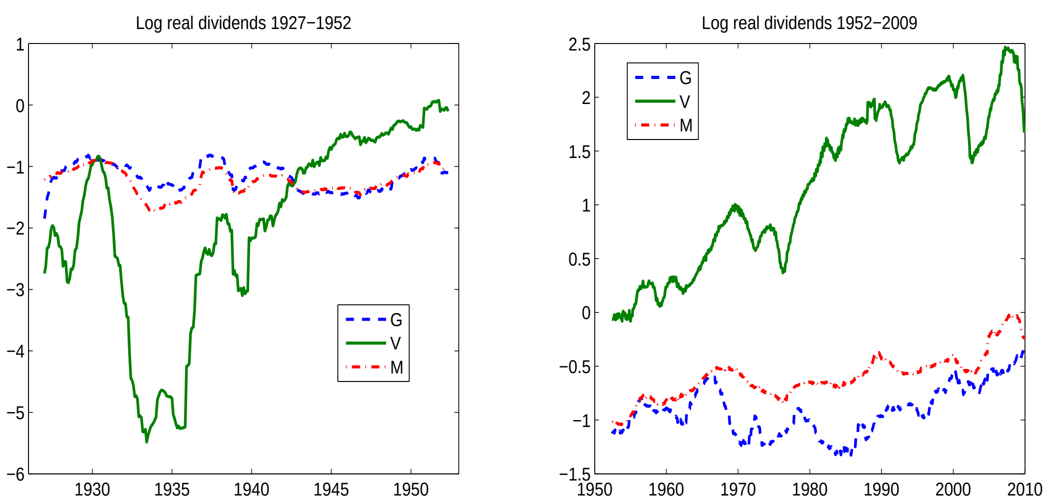

Figure 3. Dividends on value, growth, and market portfolios.
The figure plots the log real dividend on book-to-market quintile portfolios 1 and 5 and on the CRSP value-weighted market portfolio. Dividends are constructed form cum- and ex-dividend returns on these portfolios. Monthly dividends are annualized by reinvesting the dividends received during the year at the risk-free rate. The data are monthly from December 1926 until December 2009 and are sampled every three months in the figure.

Strictly adhering to the NBER recession dates understates the change in dividends from the highest to their lowest point over the cycle. For example, dividends on Value-minus-Growth fall by 37% during the December 2007-June 2009 recession, but by 103% during the May 2007-December 2009 period. The difference arises mostly because V-G dividends fall another 54% between June and December 2009. Similarly, the Value-minus-Growth dividends fall by 80% (88%) in the period surrounding the 1991 (2001) recession compared to a 13% (26%) drop between the NBER peak and the last month of the recession. Figure 4 plots the log difference between V and G portfolios (right axis) as well as NBER recessions (bars).

To get at these broader boom-bust cycles in dividends more systematically, we alternatively define busts as periods where real dividends on the market portfolio drop by 5% or more over a protracted period (6 months or more). There are 11 such periods in the 1926 to 2009 sample. They last an average of 32 months and real dividends on the market portfolio fall by 25%, on average. Real dividends on the growth portfolio fall by 17% on average, while those on the value portfolio fall by 70%, a difference of 53%. In all but two of these periods (starting in 1941 and in 1951), dividends on value stocks fall by more than those on growth stocks. The average ratio of the fall in the V-G dividend to the fall in the market dividend is 2.0. In other words, the periods with large sustained decreases in real dividends on the market are associated with much larger declines in the dividends on value than on growth.

In the calibration of the model below, we -conservatively- match the behavior of dividends as measured over the course of the official NBER recession months. In particular, we calibrate to an observed fall in log annual real dividends on value stocks of 21.0% (fifth book-to-market quintile), on the market portfolio of 5.2%, and a rise in dividends on growth stocks of 2.1% (first quintile).

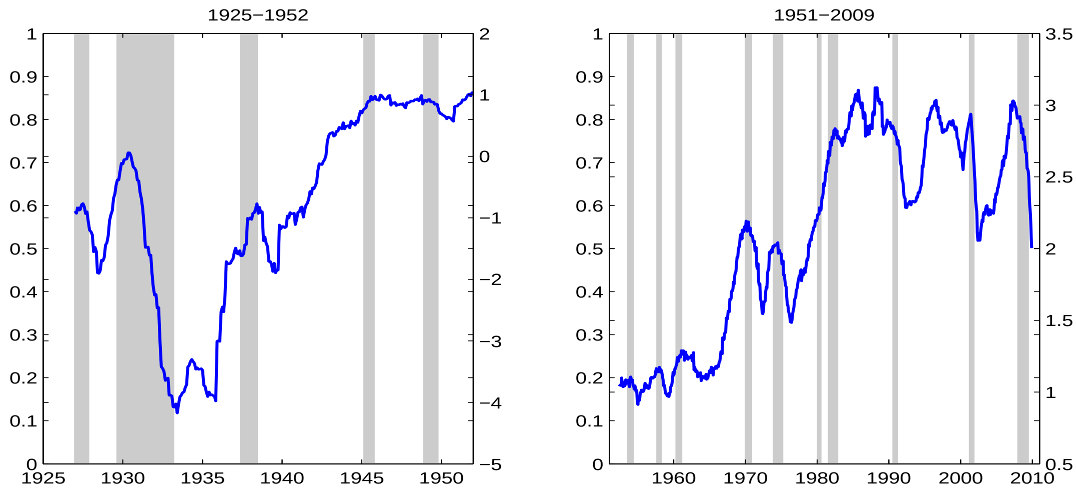

Figure 4. Dividends on value-minus-growth and NBER recessions.
The figure plots the log real dividend on book-to-market quintile portfolios 5-1, plotted against the right axis. The yellow bars indicate official NBER recession dates. Dividends are constructed form cum- and ex-dividend returns on these portfolios. Monthly dividends are annualized by reinvesting the dividends received during the year at the risk-free rate. The data are monthly from December 1926 until December 2009 and are sampled every three months in the figure.

B. Bond Risk Premia and Business Cycles

The second empirical link we document is between bond risk premia and the business cycle. We show that the CP factor is not only a strong forecaster of future bond returns, but also of future economic activity. We will use this evidence in our model of the next section where the bond risk premium will reflect compensation for business cycle risk, and the CP factor will forecast the next year's aggregate real cash flow growth.

We consider the following predictive regression in which we forecast future economic activity, measured by the Chicago Fed National Activity Index ($CFNAI$),[5] using the

[5] The CFNAI is a weighted average of 85 existing monthly indicators of national economic activity. It is constructed to have an average value of zero and a standard deviation of one. Since economic activity tends toward trend growth rate over time, a positive index reading corresponds to growth above trend and a negative index reading corresponds to growth below trend.

current CP factor:

$$CFNAI_{t+k} = c_k + \beta_k CP_t + \varepsilon_{t+k}, \quad (6)$$

where k is the forecast horizon expressed in months. The regressions are estimated by OLS and we calculate Newey-West standard errors with $k-1$ lags. The sample runs from March 1967 until December 2009 because that is when the $CFNAI$ is available. Figure 5 shows the coefficient β_k in the top panel, its t-statistic in the middle panel, and the regression R-squared in the bottom panel. The forecast horizon k is displayed on the horizontal axis and runs from 1 to 36 months. The key finding is the strong predictability of the CP factor for future economic activity. All three statistics display a hump-shaped pattern, gradually increasing until approximately 18 months and gradually declining afterwards. The maximum t-statistic is about 4.2, which corresponds to an R-squared value of 15%. The results suggest that a high CP reading strongly predicts higher future economic activity 12 to 24 months ahead.

The positive relationship between CP and better economic prospects suggests why the price of CP risk is positive: innovations to CP are good news and lower the marginal utility of wealth for investors.[6] This finding also explains intuitively why value stocks are riskier than growth stocks. We showed that value stocks have a higher exposure to CP shocks than growth stocks. This implies that value returns are high, exactly when economic activity is expected to increase, that is, when the marginal value of wealth for investors is low. This makes value stocks riskier than growth stocks and results in the value premium. We formalize this intuition in the next section.

[6] This requires the income effect to dominate the substitution effect, which could be obtained for example from Epstein-Zin preferences with a preference for early resolution of uncertainty. In the next section, we model preferences in a reduced-form way.

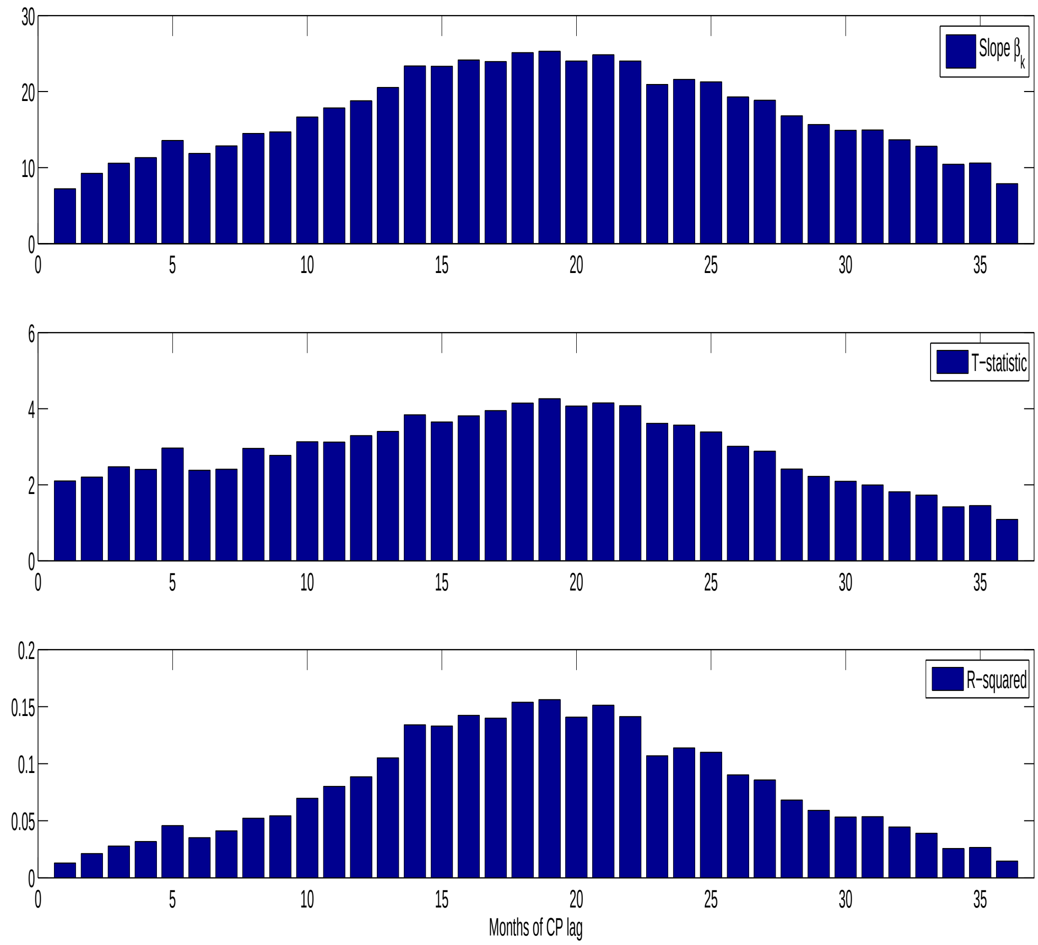

Figure 5. Economic activity predicted by the CP factor.
The top panel displays the predictive coefficient β_k in (6), the middle panel the t-statistic, and the bottom panel the corresponding R-squared value. We consider $k = 1, \ldots, 36$ months of lags, displayed on the horizontal axis in each panel, and the t-statistics are computed using Newey-West standard errors with $k - 1$ lags. The sample is March 1967 until December 2009.

Figure 6 plots the CP factor (right axis) against NBER recessions (shaded areas). Consistent with the predictability regressions, the CP factor is low before the start of most recessions in the post-1952 sample. It subsequently increases over the course of a recession, especially towards the end of the recession when better times are around the corner. Indeed, if we split each recession in three equal-length phases, then the CP

factor typically falls from the last month of the first phase to the last month of the second phase, but rebounds strongly between the last month of the second phase and the last month of the third phase (which is the last month of the recession).[7] The relapse in CP in late 2008, during the Great Recession of 2007-2009, suggests that the bond markets feared that the recession was far from over.

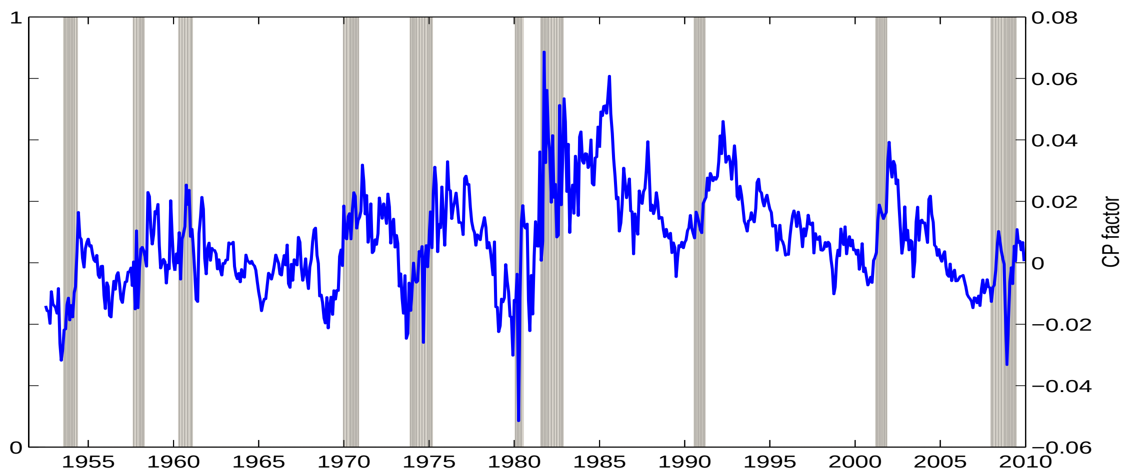

Figure 6. CP factor and NBER recessions.
The figure plots the CP factor (solid line, against the right axis) and the NBER recessions (shaded areas). The sample is July 1952 until December 2009.

A similar result to the one above appears in the working paper version of Cochrane and Piazzesi (2005), where CP is shown to forecast real gross domestic product growth. Brooks (2010) shows that the CP factor has a 35% contemporaneous correlation with news about unemployment, measured as deviations of realized unemployment from the consensus forecast.[8]

[7] The mean across the 10 NBER recessions in the post-1952 sample for the CP factor is 0.007 in the last month of the first phase, -0.002 in the last month of the second phase, and 0.013 in the last month of the third phase.

[8] A related literature studies the reverse predictability of macro-economic factors for future bond returns. Cooper and Priestly (2008) show that industrial production in deviation from its trend forecasts future bond returns; Joslin, Priebsch, and Singleton (2010) incorporate this finding in an affine term structure model. Ludvigson and Ng (2009) shows that a principal component extracted from many

IV. Structural Model with Business Cycle Risk

This section provides a simple structural asset pricing model to shed light on the empirical results from Section II. In particular, it proposes an answer to the two key questions: what economic source of risk do CP shocks capture, and why do value stocks have higher exposure to these shocks than growth stocks? The model connects the nominal bond risk premium to the state of the business cycle. In turn, it connects the state of the business cycle to the cash flow properties of value and growth stocks. Both links are based on the evidence presented in Section III. The fact that value stocks have dividends that fall substantially more over the course of a recession than those of growth stocks naturally results in a value premium because recessions are times a high marginal utility growth (a high SDF). As in the reduced-form model, value stocks are more exposed to innovations in the bond risk premium than growth stocks. Hence, the model provides structural labels for the reduced-form shocks: The CP shock corresponds to a cyclical shock to the real economy, the LVL shock captures an expected inflation shock, and the MKT shock captures a (permanent) dividend growth shock. A numerical illustration shows that market prices of risk can be found that deliver equity, book-to-market, and nominal bond risk premia of the same magnitude as the data for dividend growth processes that match the data. All derivations are relegated to the appendix.

macroeconomic series also forecasts future bond returns. While macro-economic series do not fully soak up the variation in bond risk premia, there clearly is an economically meaningful link between them.

A. Setup

The main driving force in the model is the mean-reverting process for s_t, which describes the state of the business cycle:

$$s_{t+1} = \rho_s s_t + \sigma_s \varepsilon^s_{t+1}.$$

Since this variable moves at business cycle frequency, the persistence ρ_s is moderate. Real dividend growth on asset $i = \{G, V, M\}$ (Value, Growth, and the Market) is given by:

$$\Delta d^i_{t+1} = \gamma_{0i} + \gamma_{1i} s_t + \sigma_{di} \varepsilon^d_{t+1} + \sigma_i \varepsilon^i_{t+1}. \tag{7}$$

If $\gamma_{1i} > 0$, dividend growth is pro-cyclical. The shock ε^d_{t+1} is an aggregate dividend shock, while ε^i_{t+1} is an (non-priced) idiosyncratic shock; the market portfolio has no idiosyncratic risk; $\sigma_M = 0$. The key parameter configuration is $\gamma_{1V} > \gamma_{1G}$ so that value stocks are more exposed to cyclical risk than growth stocks. Inflation is the sum of a constant, a mean-zero autoregressive process which captures expected inflation, and an unexpected inflation term:

$$\begin{aligned} \pi_{t+1} &= \bar{\pi} + x_t + \sigma_\pi \varepsilon^\pi_{t+1}, \\ x_{t+1} &= \rho_x x_t + \sigma_x \varepsilon^x_{t+1}. \end{aligned}$$

All shocks are cross-sectionally and serially independent and standard normally distributed.

To simplify our analysis, we assume that market participants' preferences are summarized by a real stochastic discount factor (SDF), whose log evolves according to the process:[9]

$$-m_{t+1} = y + \frac{1}{2}\Lambda_t'\Lambda_t + \Lambda_t'\varepsilon_{t+1}.$$

where the vector $\varepsilon_{t+1} \equiv \left(\varepsilon_{t+1}^d, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s\right)'$ and y is the real interest rate. The risk price dynamics are affine in the state of the economy s_t:

$$\Lambda_t = \Lambda_0 + \Lambda_1 s_t$$

As in the reduced form model, the structural model features three priced sources of risk: aggregate dividend growth risk, which carries a positive price of risk ($\Lambda_0(1) > 0$), inflation risk ($\Lambda_0(2) < 0$), and cyclical risk ($\Lambda_0(3) > 0$). Choosing $\Lambda_1(2) < 0$ makes the price of inflation risk counter-cyclical. As we show below, this makes bond risk premia increase pro-cyclical. We also set $\Lambda_1(1) > 0$ resulting in a pro-cyclical price of aggregate dividend risk. The log nominal SDF is given by $m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}$.

B. Asset Prices

We now study the equilibrium bond and stock prices in this model.

B.1. Bond Prices

It follows immediately from the specification of the real SDF that the real term structure of interest rates is flat at y. Nominal bond prices are exponentially affine in

[9] For similar approaches see Bekaert, Engstrom, and Grenadier (2005), Bekaert, Engstrom, and Xing (2008), Lettau and Wachter (2009), Campbell, Sunderam, and Viceira (2008), and David and Veronesi (2009).

the state of the economy and in expected inflation:

$$P_t^{\$}(n) = \exp\left(A_n^{\$} + B_n^{\$} s_t + C_n^{\$} x_t\right),$$

with coefficients that follow recursions described in the appendix. As usual, nominal bond yields are $y_t^{\$}(n) = -\log(P_t^{\$}(n))/n$. The appendix shows that nominal interest rates increase (bond prices fall) with inflation: $C_n^{\$} < 0$. Nominal interest rates also increase with the state of the economy s_t ($B_n^{\$} < 0$) when $\Lambda_1(2) < 0$. These signs are consistent with intuition.

The nominal bond risk premium, the expected excess log return on buying an n-period nominal bond and selling it one period later (as an $(n-1)$-period bond), is given by:

$$\begin{aligned} E_t\left[rx_{t+1}^{\$}(n)\right] &= -cov_t\left(m_{t+1}^{\$}, B_{n-1}^{\$} s_{t+1} + C_{n-1}^{\$}\pi_{t+1}\right) \\ &= \underbrace{\Lambda_0(2) C_{n-1}^{\$}\sigma_x + \Lambda_0(3) B_{n-1}^{\$}\sigma_s}_{\text{Constant component bond risk premium}} + \underbrace{\Lambda_1(2) C_{n-1}^{\$}\sigma_x s_t}_{\text{Time-varying component bond risk premium}}, \end{aligned}$$

In this model, all of the variation in bond risk premia comes from cyclical variation in the economy, s_t. This lends the interpretation of CP factor to s_t. Because $C_{n-1}^{\$} < 0$, $\Lambda_1(2) < 0$ generates lower bond risk premia when economic activity is low ($s_t < 0$). The constant bond risk premium contains compensation for inflation shocks and cyclical shocks. Inflation exposure results in a positive risk compensation (first term); it increases in maturity. Since most of the common variation in bond yields is driven by the inflation shock, we can interpret it as a shock to the level of the term structure (LVL). Long bonds are more sensitive to level shocks, the traditional duration effect. Exposure to the cyclical shock subtracts from the risk premium (second term). Indeed, a positive shock to the bond risk premium lowers bond prices and returns, and more so for long than for short bonds.

B.2. Stock Prices

We show in the appendix that the log price-dividend ratio on stock (portfolio) i is affine in s_t:

$$pd_t^i = A_i + B_i s_t,$$

where

$$B_i = \frac{\gamma_{1i} - \Lambda_1(1)\sigma_{di}}{1 - \kappa_{1i}\rho_s}.$$

and the expression for A_i is given in the appendix. Stock i's price-dividend ratios is pro-cyclical ($B_i > 0$) when dividend growth is more pro-cyclical than the risk premium for the aggregate dividend risk of asset i: $\gamma_{1i} > \sigma_{di}\Lambda_1(1)$. The equity risk premium on portfolio i can be computed to be:

$$\begin{aligned} E_t\left[rx_{t+1}^i\right] &= cov_t\left(-m_{t+1}^{\$}, r_{t+1}^i + \pi_{t+1}\right) \\ &= \underbrace{\Lambda_0(1)\sigma_{di} + \Lambda_0(3)\kappa_{1i}B_i\sigma_s}_{\text{Constant component equity risk premium}} + \underbrace{\Lambda_1(1)\sigma_{di}s_t}_{\text{Time-varying component equity risk premium}}. \end{aligned}$$

The equity risk premium provides compensation for (permanent) aggregate dividend growth risk (first term) and for cyclical risk (second term). Risk premia vary over time with the state of the economy (third term). As we showed above, the data suggest that value stocks' dividends fall more in recessions than those of growth stocks ($\gamma_{1V} > \gamma_{1G}$). With $\sigma_{dV} \approx \sigma_{dG}$, this implies that $B_V > B_G$. Because the price of business cycle risk $\Lambda_0(3)$ is naturally positive, the second term delivers the value premium.

B.3. Link with Reduced-form Model

To make the link with the reduced-form model of Section II clear, we study the link between the structural shocks and the reduced form shocks. In the model, shocks to the market return (MKT) are given a linear combination of ε^d and ε^s shocks:

$$\varepsilon_{t+1}^{MKT} \equiv r_{t+1}^M - E_t[r_{t+1}^M] = \sigma_{dM}\varepsilon_{t+1}^d + \kappa_{1M}B_M\sigma_s\varepsilon_{t+1}^s$$

We construct the CP factor in the same way as in the data, from yields on 1- through 5-year yields and average excess bond returns.[10] The model's CP factor is perfectly correlated with the process s, and has a innovations that differs by a factor σ^{CP}: $\varepsilon_{t+1}^{CP} = \varepsilon_{t+1}^s\sigma^{CP}$. Finally, since expected inflation drives most of the variation in bond yields in the model, LVL shocks in the model are proportional to expected inflation shocks: $\varepsilon_{t+1}^{LVL} = \varepsilon_{t+1}^x\sigma^L$. Denote $\tilde{\varepsilon} = [\varepsilon^{MKT}, \varepsilon^{LVL}, \varepsilon^{CP}]'$. Associated with $\tilde{\varepsilon}$, we can define market prices of risk $\tilde{\Lambda}$, such that SDF innovations remain unaltered: $\Lambda_t'\varepsilon_{t+1} = \tilde{\Lambda}_t'\tilde{\varepsilon}_{t+1}$. It is easy to verify that $\tilde{\Lambda}_0(1) = \Lambda_0(1)/\sigma_{dM}$, $\tilde{\Lambda}_0(2) = \Lambda_0(2)/\sigma^L$, and $\tilde{\Lambda}_0(3) = \Lambda_0(3)/\sigma^{CP} - \kappa_{1M}B_M\sigma_s\Lambda_0(1)/(\sigma_{dM}\sigma^{CP})$.

For each asset, we can compute covariances of unexpected returns with the MKT, LVL, and CP shocks inside the model. In the data, Figure 2 showed that the covariance of value and growth stocks returns with MKT shocks was similar across portfolios. In the model that covariance is given by:

$$cov(r_{t+1}^i - E_t[r_{t+1}^i], \varepsilon_{t+1}^{MKT}) = \sigma_{dM}\sigma_{di} + \kappa_{1M}B_M\kappa_{1i}B_i\sigma_s^2.$$

[10] See footnote 1. Since the model has a two-factor structure for bond yields and forward rates, we use only the two- and the five-year forward rates as independent variables in the CP regression of average excess returns on forward rates.

A calibration where $B_M \approx 0$ and $\sigma_{dV} \approx \sigma_{dG}$ will replicate the observed pattern (the linearization constant κ_{1i} will be close to 1 for all portfolios). Second, the covariance of stock portfolio returns with CP shocks is given by:

$$cov(r^i_{t+1} - E_t[r^i_{t+1}], \varepsilon^{CP}_{t+1}) = \kappa_{1i} B_i \sigma_s \sigma^{CP}.$$

The model generates a value premium because of differential exposure to CP shocks when $B_V > B_G$. When $\sigma_{dV} \approx \sigma_{dG}$, the stronger loading of expected dividend growth of value stocks to s_t ($\gamma_{1V} > \gamma_{1G}$) makes $B_V > B_G$. Put differently, in the model -as in the data- returns on value stocks are more exposed to bond risk premium shocks than returns on growth stocks. Third, stock return innovations have a zero covariance with LVL shocks in the model by construction, similar to the small exposures in the data.

Likewise, we can compute covariances of bond return innovations with the MKT, LVL, and CP shocks. In that order, they are:

$$B^\$_n \kappa_{1M} B_M \sigma_s, \quad C^\$_n \sigma_x \sigma^L, \quad B^\$_n \sigma_s \sigma^{CP}.$$

When $B_M \approx 0$, exposure of bond returns to the market factor shocks is close to zero. Exposure to level shocks is negative: an increase in the level of interest rates reduces bond prices and returns. Exposure to CP shocks is also negative: an increase in the bond risk premium reduces bond prices and returns. Both exposures become more negative with the horizon because $B^\$_n$ and $C^\$_n$ increase in absolute value with maturity n.

C. Calibration

To illustrate the model's quantitative implications for bond risk premia and equity risk premia on various book-to-market sorted portfolios, we conduct a calibration and model simulation. The crucial new ingredient in the model is the differential cyclicality of value and growth dividends. In the interest of space, Appendix A.3 discusses the calibration in detail. Here we focus on the new aspects of the calibration and the main results.

The most important parameter is γ_{1i}, which measures how sensitive dividend growth is to changes in real economic activity. In light of the empirical evidence in Section III.A, we choose γ_{1i} to match the log change in annual real dividends between the peak of the cycle and the last month of the recession for the Growth, Value, and Market portfolios to the observed change in the 1927-2009 data (the average over all 15 recessions in our sample). The model matches these changes for $\gamma_{1G} = -.00004$, $\gamma_{1V} = .00976$, and $\gamma_{1M} = .00248$. Note that $\gamma_{1V} > \gamma_{1G}$ delivers a greater fall of dividends on value stocks in recessions, the central mechanism behind the value premium.

In order to measure how dividends change over the recession, we have to define recessions in the model. Our algorithm mimics several of the features of the NBER dating procedure: (i) The recession is determined by looking back in time at past real economic activity (s_t in the model) and its start is not known in real time, (ii) there is a minimum recession length, and (iii) it captures the notion that the economy went through a sequence of negative shocks and that economic activity is at a low level. We split each recession into three equal periods and refer to the last month of each period as the first, second, and third stage of the recession. The s process is negative at the start of the recession, falls considerably in the first stage of a recession, continues to fall in the second stage, and partially recovers in the last stage. Our recession dating procedure, described in more detail in the appendix, is novel, matches the empirical distribution of recession duration, and generates interesting asset pricing dynamics during recessions, to which we return to below.

The rest of the dividend growth parameters are chosen to match the observed mean and volatility. Inflation parameters are chosen to match mean inflation, and the volatility and persistence of nominal bond yields. The market price of expected inflation risk is chosen to match the 0.6% slope of the term structure (5-year minus 1-year), while the market price of aggregate dividend risk and the market price of CP risk are chosen to generate the 7.28% equity risk premium and the 5.22% value premium observed in the 1927-2009 sample. The model matches also the average excess bond return, averaged across maturities, and its volatility. Put differently, the model matches the mean, volatility, and twelve-month autocorrelation of the CP factor.

The main result from the calibration exercise is that the model is able to replicate the three-factor risk premium decomposition we uncovered in Section II. Figure 7 is the model's counterpart to Figure 2 in the data. There is a match in terms of the relative contribution of each of the three sources of risk to the equity and bond risk premium.

Finally, our model implies interesting asset pricing dynamics over the cycle. The CP factor, or nominal bond risk premium, starts out negative at the start of the recession, falls substantially in the first stage of the recession, falls slightly more in the second stage, before increasing substantially in the third stage of the recession. This pattern for bond risk premia is reflected in realized bond returns. In particular, the negative risk premium shocks at the start of a recession increase bond prices and returns, and more so on long-term than short-term bonds. An investment of $100 made at the peak in a portfolio that goes long the 30-year and short the 3-month nominal bond gains $8.0 in

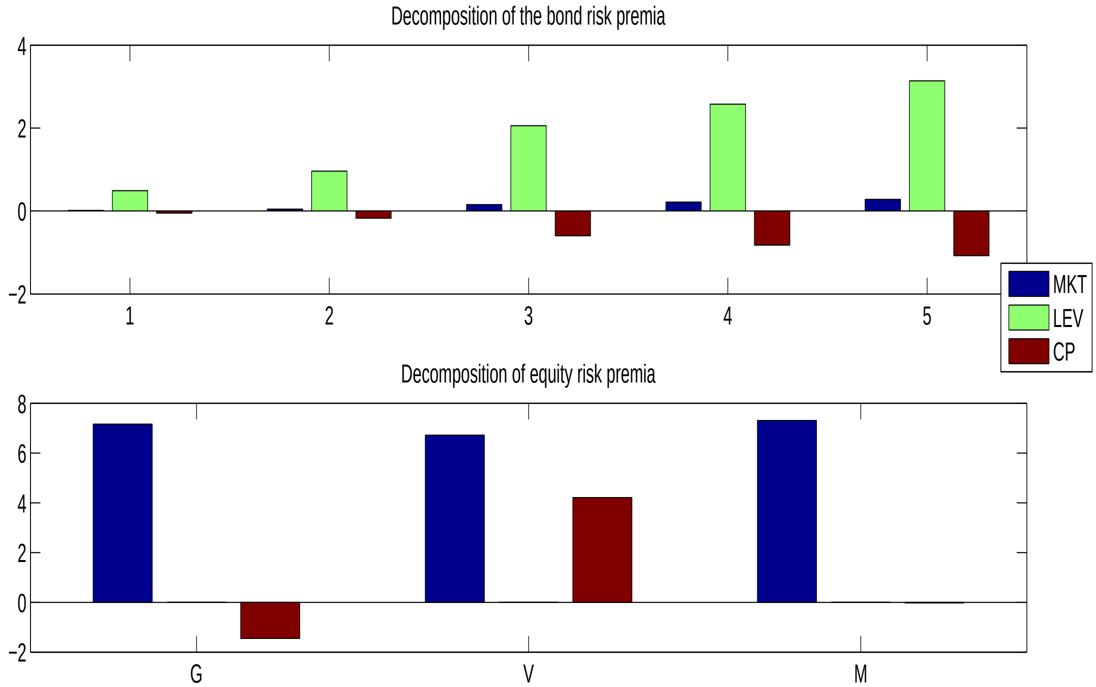

Figure 7. Exposure of portfolio excess returns to priced innovations in model.
The figure plots the risk premium decomposition into risk compensation for exposure to the CP factor, the $Level$ factor, and the MKT factor. The top panel is for the five bond portfolios (1-yr, 2-yr, 5-yr, 7-yr, and 10-yr) whereas the bottom panel is for growth (G), value (V), and market (M) stock portfolios. The results are computed from a 10,000 month model simulation under the calibration described in detail in Section C.

the first stage of the recession. The gain further increases to \$11.7 in the second stage, before falling back to a \$7.4 gain by the last month of the recession. The latter increase occurs as consequence of the rising bond risk premium. Taken over the entire recession, long bonds gain in value so that they are recession hedges (Campbell, Sunderam, and Viceira 2008). The same is true in the data, where the gain on long-short bond position is \$6.1 by the last month of the recession. Value stocks are risky in the model. Their price-dividend ratio falls by 21% in the first stage compared to peak, continues to fall to -34%, before recovering to -29% by the end of the recession. In the data, the pd ratio on value stocks similarly falls by 16% in the first stage, falls further to -26%, before recovering to +4%. Value stocks perform poorly, losing more during the recession than growth stocks, both in the model and in the data.

One important feature the model (deliberately) abstracts from are discount rate shocks to the stock market. As a result, the price-dividend ratio and stock return are insufficiently volatile and reflect mostly cash-flow risk. While obviously counter-factual, this assumption is made to keep the exposition focussed on the main, new channel: time variation in the bond risk premium, the exposure to cyclical risk, and its relationship to the value risk premium.[11]

We conclude that the model delivers a structural interpretation for the MKT, LVL, and CP shocks. CP shocks reflect cyclical shocks to the real economy, which naturally carry a positive price of risk. The model quantitatively replicates the unconditional risk premium on growth, value, and market equity portfolios, and bond portfolios of various maturities, as well as the decomposition of these risk premia in terms of their MKT, LVL, and CP shock exposures. Furthermore, it delivers a realistic term structure of interest rates and bond risk premia. It does so for plausibly calibrated dividend growth and inflation processes.

V. Robustness

This section considers several exercises investigating the robustness of our empirical results in Section II. First, we use a different weighting matrix in the market price of

[11] One could write down a richer model to address this issues, but only at the cost of making the model more complicated. Such a model would feature a market price of aggregate dividend risk which varies with some state variable z. The latter would follow an AR(1) process with high persistence, as in Lettau and Wachter (2009). All price-dividend ratios and expected stock returns would become more volatile and more persistent, generating a difference between the business-cycle frequency behavior of the bond risk premium and the generational-frequency behavior of the pd ratio. This state variable could differentially affect value and growth stocks, potentially lead to a stronger increase in the pd ratio of value than that of growth in the last stage of a recession. This would shrink the cumulative return gap between value and growth stocks during recessions, which the model now overstates.

risk estimation. Second, we compare our results to alternative pricing models. Third, we do a subsample analysis. Fourth, we study additional stock and bond portfolios. Fifth, we study individual-level stock returns.

A. Weighted Least-Squares

Our cross-sectional estimation results in Table I assume a GMM weighting matrix equal to the identity matrix. This is equivalent to minimizing the root mean-squared pricing error across the 11 test assets. The estimation devotes equal attention to each pricing error, an leads to a RMSE of 47bp per year. A natural alternative to the identity weighting matrix is to give more weight to the assets that are more precisely measured. We use the inverse covariance matrix of excess returns, as in Hansen and Jagannathan (1997). This amounts to weighting the bond pricing errors more heavily than the stock portfolio pricing errors in our context. When using the Hansen-Jagannathan distance matrix, we find a MAPE of 52bp per year compared to 41bp per year. However, the weighted RMSE drops from 47bp to 23bp per year. The reason for the improvement in RMSE is that the pricing errors on the bonds decrease substantially, from a MAPE of 38bp to 13bp per year. Finally, the price of risk estimates in $\hat{\Lambda}_0$ are comparable to those in the benchmark case. The price of CP risk remains positive and is estimated to be somewhat lower than in the benchmark case, at 57.27 (with a t-statistic of 4.3). The market price of level risk remains statistically negative (-14.29 with t-statistic of -2.1), and the price of market risk remains positive (2.58 with a t-statistic of 2.2). The null hypothesis that all pricing error parameters are jointly zero continues to be strongly rejected. We conclude that our results are similar when we use a different weighting matrix.

B. Alternative Pricing Models

In Section II, we compare our SDF model to a SDF model that has been designed to price bonds, as proposed by Cochrane and Piazzesi (2008), and the CAPM, the most basic model for pricing stocks. It might also be interesting to also compare the model to the three-factor model of Fama and French (1992), which offers a better-performing alternative to the CAPM for pricing the cross-section of stocks. We ask how well it prices the cross-section of book-to-market stocks and government bonds over our monthly sample from June 1952 until December 2008. We use the market return (MKT), the size (SMB), and the value factor (HML) as pricing factors and price the same 11 (16) test assets as in Tables I (II). The last column of each table contains the pricing errors for the Fama-French models. The MAPE is 57 (56) basis points per year with 11 test assets (16 test assets), which is somewhat higher than the 41 (48) basis points of our model in the second column. In both cases, the slightly worse fit in the last column is due to higher pricing errors on the bond portfolios. Tests of the null hypothesis that all pricing errors are jointly zero are rejected at conventional levels. We have verified that this rejection is due to the higher pricing errors on the 1-, 2-, and 5-year bond moments. This finding is consistent with the findings in Fama and French (1993) who introduce additional pricing factors beyond MKT, SMB, and HML to price bonds. Our results suggest that three factors suffice. In unreported results, we find that the difference between the MAPE of our model and the Fama-French model increases when we weight the 11 Euler equation errors by the inverse of their variance as opposed to equally. In addition, there remains a statistical difference between the p-values of χ^2 tests of the null that all pricing errors are jointly zero between our model (5%) and the FF model (<1%) with the alternative weighting matrix. The reason is that our model fits the bond return moments better.

C. Subsample Analysis

Table I shows that our main empirical results are robust for various subsamples. When we start the analysis in 1963, an often-used starting point for cross-sectional equity analysis (e.g., Fama and French (1993)), we find very similar results. The left columns of Table III shows a MAPE of 39 basis points per year, close to the 41 basis points MAPE in the full sample. Our model improves relative to the Fama-French three-factor model, which has a pricing error of 71 basis points in this subsample. There are no monotone patterns in the pricing errors on bonds or book-to-market quintile portfolios left. In the right columns, we investigate the results in the second half of our sample, 1980-2009. Mean absolute pricing errors fall further to 36 basis points, while the MAPE under the Fama-French model rises to 91 basis points. Panel B of Table III shows that the price of risk estimates are similar to the ones from the benchmark estimation. Finally, Panel C shows that in both subsamples, we reject the null that the prices of risk are zero, but we fail to reject the null that all pricing errors are jointly zero. These subsample results use LVL and CP factors which are estimated over the entire sample. We have also re-estimated the state vector (e.g., the CP factor) over the subsample in question, with similar results.

D. Other Test Assets

Given that we found a unified SDF that does a good job pricing the cross-section and time-series of book-to-market sorted stock and maturity-sorted bond returns, a natural question that arises is whether the same SDF model also prices other stock or bond portfolios. In addition, studying more test assets allows us to address the Lewellen, Shanken, and Nagel (2009) critique. They argue that explanatory power of many risk-

Table III
Other Sample periods - Pricing Errors

This table reports robustness with respect to different sample periods. It is otherwise identical to Table I. The data are monthly from January 1963 through December 2009 in the left columns and from January 1980 until December 2009 in the right columns.

	Panel A: Pricing Errors (in % per year)					
	1963-2009			1980-2009		
	RN SDF	Our SDF	FF	RN SDF	Our SDF	FF
1-yr	1.06	-0.48	0.82	1.36	0.06	0.99
2-yr	1.23	-0.74	0.80	1.90	-0.13	1.07
5-yr	1.69	0.00	0.85	2.82	0.04	0.94
7-yr	2.02	0.46	0.93	3.40	0.09	0.77
10-yr	1.71	-0.02	0.30	3.41	-0.01	0.16
Market	5.16	-0.77	-0.12	6.47	-0.86	0.54
BM1	4.52	0.07	0.75	6.19	-0.01	0.53
BM2	5.62	-0.02	-0.59	7.65	0.72	-0.63
BM3	6.07	0.22	1.00	7.00	0.10	-1.46
BM4	7.64	-0.50	-0.47	7.73	-0.95	-1.05
BM5	9.67	1.00	1.14	9.94	0.99	1.86
MAPE	4.22	0.39	0.71	5.78	0.36	0.91
	Panel B: Market Prices of Risk					
MKT		0.96	4.37		1.73	5.35
LVL/SMB		-19.77	-4.36		-20.21	-14.66
CP/HML		71.21	6.14		46.73	2.96
	Panel C: P-values of χ^2 Tests					
$H_0 : \Lambda_0 = 0$	–	3.48%	0.20%	–	4.37%	1.47%
H_0 : Pr. err. $= 0$	–	14.13%	0.01%	–	36.23%	0.53%

based models for the cross-section of (size and) value stocks may be poorly summarized by the cross-sectional R^2.

One of their proposed remedies is to use more test assets in the evaluation of asset pricing models. Our benchmark results address this criticism already by adding maturity-sorted government bond portfolios to the cross-section of book-to-market stock portfolios. In addition, we now study several other sets of test assets. We start by adding corporate bond portfolios. Then we study replacing ten decile book-to-market portfolios by ten size decile portfolios, 25 size and book-to-market portfolios, and ten earnings-price portfolios.

D.1. Adding Corporate Bond Portfolios

One asset class that deserves particular attention is corporate bonds. After all, at the firm level, stocks and corporate bonds are both claims on the firm's cash flows albeit with a different priority structure. We ask whether, at the portfolio level, our SDF model is able to price portfolios or corporate bonds sorted by ratings class. Fama and French (1993) also include a set of corporate bond portfolios in their analysis but end up concluding that a separate credit risk factor is needed to price these portfolios. Instead, we find that the same three factors we used so far are able to price the cross-section of corporate bond portfolios.

We use data from Citi's Yield Book for four investment-grade portfolios: AAA, AA, A, and BBB. Return data for these portfolios are available monthly from October 1980 until December 2009. Their annualized excess returns are listed in the first column of Table IV. In a first exercise, we calculate Euler equations errors for these four portfolios, using our SDF model presented in Section II. That is, we do not re-estimate the market price of risk parameters $\hat{\Lambda}_0$, but simply calculate the pricing errors for the corporate bond portfolios. The resulting annualized pricing errors are listed in the second column of Table IV. The model does a reasonable job pricing the corporate bonds: pricing errors are on average below 1% per year, compared to excess returns of more than 3.5% per year. The mean absolute pricing error among all fifteen test assets (five BM portfolios, the market portfolio, five Treasury bond portfolios, and four corporate bond portfolios) is 61 basis points per year.

Equally interesting is to re-estimate the market price of risk parameters of the SDF model when the corporate bond portfolios are included in the set of test assets. We do not allow for additional priced factors; the CP, LVL, and MKT factors remain the only three priced risk factors. The third column of Table IV shows that the corporate bond pricing errors are now below 60 basis points per year on average. The overall MAPE on all 15 assets is 51 basis points per year, a mere 8 basis points above the MAPE when corporate bonds were not considered and 10 basis points less than when the corporate bonds were not included in the estimation. Finally, comparing Columns 2 and 3, the point estimates for the market prices of risk Λ_0 in Panel B are similar for the models with or without corporate bonds. The last column reports results for the Fama-French three-factor model. Its pricing errors are higher than in our three-factor model; the MAPE is 110 basis points. Average pricing errors on the corporate bond portfolios are more than 1% per year, and monotonically declining in credit quality. We fail to reject the null that all pricing errors are jointly zero, while the FF model continues to strongly reject it.

Table IV
Unified SDF Model for Stocks, Treasuries, and Corporate Bonds

Panel A of this table reports pricing errors on 10 book-to-market-sorted stock portfolios, the value-weighted market portfolio, five Treasury bond portfolios of maturities 1, 2, 5, 7, and 10 years, and four corporate bond portfolios sorted by S&P credit rating (AAA, AA, A, and BBB). They are expressed in percent per year. The estimation period for stock and Treasury bond portfolios is June 1952 through December 2009, while the corporate bond portfolio data are available only from October 1980 until December 2009.

Panel A: Pricing Errors (% per year)				
	RN SDF	Our SDF not re-estimated	Our SDF re-estimated	FF
1-yr	1.37	-0.13	0.05	0.93
2-yr	1.97	-0.34	-0.07	0.93
5-yr	3.04	0.48	0.79	0.66
7-yr	3.72	0.18	0.60	0.36
10-yr	3.64	-0.30	0.15	-0.46
Market	6.15	-0.84	-0.88	0.95
BM1	5.80	-0.21	-0.41	0.55
BM2	7.31	0.80	0.73	-1.27
BM3	6.71	0.00	-0.01	-1.93
BM4	7.63	-1.08	-0.78	-1.39
BM5	9.93	1.35	1.58	2.21
Credit1	3.58	-1.09	-0.60	0.76
Credit2	3.79	-0.90	-0.41	0.84
Credit3	4.03	-0.91	-0.41	1.18
Credit4	4.42	-0.60	-0.13	2.10
MAPE	4.87	0.61	0.51	1.10
Panel B: Prices of Risk Estimates				
MKT		2.04	2.14	5.76
LVL/SMB		-34.33	-30.18	-20.83
CP/HML		96.22	85.55	2.86
Panel C: P-values of χ^2 Tests				
$H_0 : \Lambda_0 = 0$	–	–	9.35%	1.46%
H_0 : Pr. err. $= 0$	–	–	18.71%	0.21%

D.2. Different Stock Portfolios

Table V shows three exercises where we replace the five book-to-market sorted portfolios by other sets of stock portfolios. In the first three columns we use ten market capitalization-sorted portfolios alongside the bond portfolios and the market. The first column shows the risk premia to be explained (risk neutral SDF). Small firms (S1) have about 3.8% higher risk premia than large stocks (S10). Our model in the second column manages to bring the overall mean absolute pricing error down from 6.0% per year to 0.42% per year, comparable to the 41% we obtained with the book-to-market quintile portfolios and the 0.48% with the book-to-market decile portfolios. This MAPE is comparable to that in the Fama-French model in the third column. The Fama-French model does better eliminating the spread between small and large stocks, whereas our model does better pricing the bond portfolios alongside the size portfolios.

The next three columns use earnings-price-sorted quintile stock portfolios. The highest earnings-price portfolio has an average risk premia that is 6.8% higher per year than the lowest earnings-price portfolio. Our model reduces this spread in risk premia to 1.9% per year, while continuing to price the bonds reasonably well. The MAPE is 89 basis points per year compared to 61 in the Fama-French model.

The last three columns use the five-by-five market capitalization- and book-to-market-sorted portfolios. Our three-factor model manages to bring the overall mean absolute pricing error down from 7.53% per year to 1.28% per year. This is again comparable to the three-factor Fama-French model of 1.13%. Relative to the FF model, ours reduces the pricing errors on the hard-to-explain S1B1 portfolio, but makes a larger error on the S1B4 and S1B5 portfolios.

The market price of risk estimates Λ_0 in Panel B of Table V are comparable to those we found for the book-to-market portfolios in Table I. Panel C shows that we reject the null hypothesis that all market prices of risk are zero, albeit at the 10% level for the size portfolios. We fail to reject the null hypothesis that all pricing errors are zero on the size and earnings-price portfolios, and marginally reject the null (at the 4% level) for the twenty-five portfolios.

E. Individual Firm Returns

As a final robustness check, we investigate whether exposure to CP shocks is associated with higher risk premia not only among stock and bond portfolios, but also among individual stocks.

Our first sample is the CRSP universe, which contains 2.88 million firm-month observations on 22,811 firms and 689 months between August 1952 and December 2009, the same sample period as our results in Section II. Our second sample is the CRSP/Compustat universe, which contains 2.04 million firm-month observations on 17,673 firms and 558 months between July 1963 and December 2009. For each stock-month pair, we estimate the covariance between monthly CP innovations and the stock's return based on 60-month rolling windows. We consider both monthly and annual rebalancing. In the monthly rebalancing exercise, we sort stocks into five portfolios based on their CP-exposure and calculate the quintile portfolio returns over the next month, value-weighting stocks within each portfolio. In the annual rebalancing exercise, we sort stocks each year in December based on their CP-exposure and calculate the quintile portfolio returns over the next 12 months, again value-weighting stocks within each portfolio. Annual rebalancing is what is done to construct the book-to-market sorted portfolios we used in Section II. The latter data set contains accounting information,

Table V
Other Stock Portfolios - Pricing Errors

This table reports robustness with respect to different stock market portfolios, listed in the first row. Panel A of this table reports pricing errors (in % per year) on various stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. Each column corresponds to a different stochastic discount factor (SDF) model. The first column contains the risk-neutral SDF. The second column presents our SDF model with three priced factors (Our). The third column refers to the three factor model of Fama and French (FF). The last row of Panel A reports the mean absolute pricing error across all securities (MAPE). Panel B reports the estimates of the prices of risk. The data are monthly from June 1952 through December 2009.

Panel A: Pricing Errors (in % per year)											
10 Size Portfolios				5 Earnings-Price Portfolios				25 Size and Value Portfolios			
Assets	RN SDF	Our SDF	FF	Assets	RN SDF	Our SDF	FF	Assets	RN SDF	Our SDF	FF
1-yr	1.00	-0.12	0.71		1.00	-1.36	0.78		1.00	-1.24	0.85
2-yr	1.21	-0.17	0.79		1.21	-1.42	0.84		1.21	-1.30	1.02
5-yr	1.52	0.07	0.98		1.52	-0.53	0.89		1.52	-0.48	1.31
7-yr	1.78	0.51	1.28		1.78	0.58	1.00		1.78	0.58	1.58
10-yr	1.39	0.10	0.34		1.39	0.53	0.28		1.39	0.52	0.90
Market	6.32	-0.75	-0.09		6.32	-1.10	-0.10		6.32	-0.08	0.21
S1	9.46	1.56	-0.13	EP1	4.83	-0.33	0.61	S1B1	3.37	-3.07	-5.02
S2	8.98	0.07	0.02	EP2	6.07	-0.86	-0.55	S1B2	9.47	-0.39	0.49
S3	9.49	0.82	0.24	EP3	7.53	-0.32	-0.92	S1B3	9.61	-0.19	0.45
S4	8.86	0.28	0.17	EP4	9.38	1.08	-0.05	S1B4	11.99	3.72	2.25
S5	9.04	0.01	0.41	EP5	11.63	1.62	0.72	S1B5	13.45	3.57	1.82
S6	8.36	-0.17	-0.12					S2B1	5.18	-1.57	-1.77
S7	8.37	-0.40	0.25					S2B2	8.63	0.00	0.45
S8	7.83	-0.85	-0.47					S2B3	10.90	1.39	1.67
S9	7.25	-0.23	-0.82					S2B4	11.09	1.26	0.89
S10	5.68	-0.60	0.35					S2B5	15.51	2.96	0.11
								S3B1	6.30	-1.70	0.44
								S3B2	8.97	-0.20	0.82
								S3B3	9.49	-0.08	0.39
								S3B4	10.63	0.41	0.73
								S3B5	11.99	1.31	0.37
								S4B1	6.87	0.02	1.84
								S4B2	7.16	-1.27	-0.76
								S4B3	9.28	-0.36	0.23
								S4B4	9.77	-2.14	0.13
								S4B5	10.02	-1.67	-1.66
								S5B1	5.80	2.07	2.20
								S5B2	6.37	1.14	0.27
								S5B3	6.82	0.87	-0.13
								S5B4	6.74	-2.16	-2.21
								S5B5	7.77	-2.03	-2.56
MAPE	6.03	0.42	0.45		4.79	0.89	0.61		7.53	1.28	1.13
Panel B: Market Prices of Risk											
MKT		2.43	6.39			1.68	5.28			1.31	3.77
Level/SMB		-14.65	-19.19			-27.41	-3.63			-26.53	1.98
CP/HML		66.12	20.67			166.57	9.85			157.48	8.01
Panel C: P-values of χ^2 Tests											
$H_0 : \Lambda_0 = 0$	–	1.72%	2.30%		–	0.84%	0.00%		–	1.12%	0.00%
H_0 : Pr. err. = 0	–	9.76%	0.47%		–	11.05%	0.01%		–	3.97%	0.00%

such as the book-to-market ratio, and is used in most empirical work at the stock level. Therefore, the Compustat, annual rebalancing case is our main case.

Table VI reports post-formation CP-exposures, average returns, and CAPM alphas of the quintile portfolios. The results from the Compustat sample are in the left columns, those from the CRSP sample in the right columns. The annual rebalancing results are in the top panels, while the monthly rebalancing is reported in the bottom panels. In both samples, firms with high CP betas have higher average returns, CAPM alphas, and KLN alphas. The difference between the highest and lowest quintile is 4.1% per year (2.6%) in the Compustat sample under annual (monthly) rebalancing and 3.5% (3.0%) in the CRSP sample under annual (monthly) rebalancing. This represents a substantial fraction of the 4.4% annual value premium in our sample. We note that portfolio sorting based on rolling-window regressions can dampen the true spread in returns between high- and low-exposure stocks.[12] In light of this attenuation bias, the spreads we find between CP-exposure sorted portfolios seem economically substantial. The 5-1 spread in CAPM alphas is 0.4% per year lower than that of returns. Applying our CP market price of risk estimate of 98 from Table I to the differential CP-exposure of the CP-sorted portfolios leads to a 1.7-2.7% annual return spread, close to the reported difference in average returns. We conclude that CP-sorted portfolios of individual stocks lend support to our earlier findings.[13]

[12]To illustrate, we regress the returns on the five book-to-market quintile portfolios on the three Fama-French factors using 120-month rolling-window regressions. In each month, we assign each quintile portfolio to an HML-quintile portfolio by sorting on the estimated rolling-window HML exposure. The resulting spread between the highest and lowest HML-quintile portfolio is only 1.6%, compared to the 4.4% value spread between these portfolios.

[13]We obtain similar results using longer rolling windows to compute the CP-exposure, using expanding windows, or using equal-weighting instead of value-weighting to compute portfolio returns. We also obtain similar results when we control for differential exposure of the quintile portfolios to MKT and LVL innovations.

Table VI
CP-beta Sorts

For each stock-month pair, we estimate the covariance between monthly CP innovations and the stock's return based on 60-month rolling windows. In the monthly rebalancing exercise, we sort stocks based into five portfolios based on their CP-exposure and calculate the quintile portfolio returns over the next month, value-weighting stocks within each portfolio (bottom panels). In the annual rebalancing exercise (top panels), we sort stocks each year in December based on their CP-exposure and calculate the quintile portfolio returns over the next 12 months, again value-weighting stocks within each portfolio. We winsorize the CP-exposures at the 1% and 99% levels and exclude stocks with a market capitalization below \$100,000. The table reports the CP-exposures, average returns, and CAPM alphas of the quintile portfolios. All entries are multiplied by 1200 so as to express them as a percentage per year. The left panels are for the CRSP/Compustat universe from 1963.7 to 2009.12. The right panels are for the universe of CRSP stocks from 1952.8 to 2009.12.

	CP-exp	avg ret	capm alpha		CP-exp	avg ret	capm alpha
	Compustat/annual rebalancing				CRSP/annual rebalancing		
1	0.023	3.325	-1.514	1	0.016	4.443	-1.758
2	0.030	4.391	0.244	2	0.025	5.227	-0.074
3	0.042	5.893	1.765	3	0.026	6.758	1.422
4	0.045	6.229	1.600	4	0.030	7.245	1.455
5	0.041	7.441	2.262	5	0.032	7.907	1.263
5-1	0.017	4.116	3.776	5-1	0.017	3.464	3.021
	Compustat/monthly rebalancing				CRSP/monthly rebalancing		
1	0.022	3.648	-1.243	1	0.013	4.766	-0.999
2	0.026	4.348	0.170	2	0.018	4.645	-0.237
3	0.029	5.295	1.014	3	0.018	6.780	1.746
4	0.039	6.207	1.622	4	0.024	6.950	1.641
5	0.049	6.268	0.997	5	0.036	7.727	1.530
5-1	0.027	2.620	2.240	5-1	0.023	2.962	2.528

VI. Conclusion

This paper makes three contributions. First, we estimate a parsimonious three-factor model that explains return differences between average excess returns on book-to-market sorted stock portfolios, the aggregate stock market portfolio, government bond portfolios sorted by maturity, and corporate bond portfolios. The first factor is the traditional CAPM market return factor, the second one is the level of the term structure, and the third factor captures fluctuations in the nominal bond risk premium. We show that the value portfolio returns have a higher exposure to bond risk premium shocks than the returns on the growth portfolio. With a positive estimate for the market price of bond risk premium shocks, this differential exposure delivers the value risk premium. Second, we provide and calibrate a structural model that links nominal bond risk premia shocks to the returns on value and growth stocks. The main state variable in the model is the state of the real economy. When economic activity is low and suffers a sequence of negative shocks, the economy is in recession. Bond risk premia fall, thereby increasing bond prices; bonds are a recession hedge. During recessions, dividends on the value portfolio fall by more than those on growth, and so do their returns. Value stocks' cash flows are more exposed to cyclical shocks, the same shocks that drive the bond risk premium. The model replicates the empirical finding that the value premium is due to higher exposure of value returns to bond risk premium shocks. Third, we provide empirical evidence that dividends on value portfolios fall by more in recessions than those on growth stocks. This evidence supports the main driving force in the model. Taken together, this paper provides a new mechanism linking the properties of stock and bond prices, obviating the need for a behavioral explanation of the value anomaly. Rather, it resuscitates a central role for business cycle risk in asset pricing.

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Appendix

Appendix A. Derivations Structural Model

In this appendix we provide derivations of the asset pricing expressions given in the main text. We also list the parameters used in the numerical example, and how they were chosen. It is the simplest structural model that provides the link between the state of the economy, the nominal bond risk premium, and value/growth stocks.

Appendix A.1. Nominal Bond Prices and Risk Premia

The nominal SDF is given by:

$$\begin{aligned} m^{\$}_{t+1} &= m_{t+1} - \pi_{t+1} \\ &= -y - \bar{\pi} - x_t - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1} - \sigma_\pi \varepsilon^{\pi}_{t+1} \end{aligned}$$

The price of an n-period bond is given by:

$$P^n_t = \exp\left(A^{\$}_n + B^{\$}_n s_t + C^{\$}_n x_t\right).$$

The recursion of nominal bond prices is given by:

$$
\begin{aligned}
P_t^n &= E_t\left(P_{t+1}^{n-1} M_{t+1}^{\$}\right) \\
&= E_t\left(\exp\left(A_{n-1}^{\$} + B_{n-1}^{\$} s_{t+1} + C_{n-1}^{\$} x_{t+1} - y - \bar{\pi} - x_t - \frac{1}{2}\Lambda_t' \Lambda_t - \Lambda_t' \varepsilon_{t+1} - \sigma_\pi \varepsilon_{t+1}^{\pi}\right)\right) \\
&= \exp\left(A_{n-1}^{\$} - y - \bar{\pi} - x_t - \frac{1}{2}\Lambda_t' \Lambda_t^{+} B_{n-1}^{\$} \rho_s s_t + C_{n-1}^{\$} \rho_x x_t\right) \times \\
&\quad E_t\left(\exp\left(B_{n-1}^{\$} \sigma_s \varepsilon_{t+1}^{s} + C_{n-1}^{\$} \sigma_x \varepsilon_{t+1}^{x} - \Lambda_t' \varepsilon_{t+1} - \sigma_\pi \varepsilon^{\pi} t + 1\right)\right) \\
&= \exp\left(A_{n-1}^{\$} - y - \bar{\pi} - x_t + B_{n-1}^{\$} \rho_s s_t + C_{n-1}^{\$} \rho_x x_t\right) \times \\
&\quad \exp\left(\frac{1}{2}[B_{n-1}^{\$}]^2 \sigma_s^2 + \frac{1}{2}[C_{n-1}^{\$}]^2 \sigma_x^2 - B_{n-1}^{\$} \sigma_s \Lambda_t(3) - C_{n-1}^{\$} \sigma_x \Lambda_t(2) + \frac{1}{2}\sigma_\pi^2\right),
\end{aligned}
$$

which implies:

$$
\begin{aligned}
A_n^{\$} &= A_{n-1}^{\$} - y - \bar{\pi} + \frac{1}{2}[B_{n-1}^{\$} \sigma_s]^2 + \frac{1}{2}[C_{n-1}^{\$} \sigma_x]^2 + \frac{1}{2}\sigma_\pi^2 - B_{n-1}^{\$} \sigma_s \Lambda_0(3) - C_{n-1}^{\$} \sigma_x \Lambda_0(2), \\
B_n^{\$} &= B_{n-1}^{\$} \rho_s - C_{n-1}^{\$} \sigma_x \Lambda_1(2), \\
C_n^{\$} &= -1 + C_{n-1}^{\$} \rho_x.
\end{aligned}
$$

The starting values for the recursion are $A_0^{\$} = 0$, $B_0^{\$} = 0$, and $C_0^{\$} = 0$. The expression for $C_n^{\$}$ can be written more compactly as:

$$
C_n^{\$} = -\frac{1 - \rho_x^n}{1 - \rho_x} \rho_x < 0,
$$

implying that bond prices drop (nominal interest rates increase) when inflation increases. If $\Lambda_1(2) < 0$ then $B_n^{\$} < 0$, implying that nominal bond prices fall (interest rates rise) in business cycle expansions ($s_t > 0$). Both signs seem consistent with intuition.

The nominal bond risk premium, which is the expected excess return in logs and corrected for a Jensen term:

$$\begin{aligned}
E_t\left[rx^{\$}_{t+1}(n)\right] &= -cov_t\left(m^{\$}_{t+1}, p^{n-1}_{t+1} - p^n_t\right) \\
&= cov_t\left(\Lambda'_t \varepsilon_{t+1}, B^{\$}_{n-1} s_{t+1} + C^{\$}_{n-1} x_{t+1}\right) \\
&= \Lambda_t(2) C^{\$}_{n-1}\sigma_x + \Lambda_t(3) B^{\$}_{n-1}\sigma_s \\
&= \Lambda_0(2) C^{\$}_{n-1}\sigma_x + \Lambda_0(3) B^{\$}_{n-1}\sigma_s + \Lambda_1(2) C^{\$}_{n-1}\sigma_x s_t.
\end{aligned}$$

Appendix A.2. Stock Return, Price-Dividend Ratio, and Equity Risk Premium

The return definition implies:

$$\begin{aligned}
r_{t+1} &= \ln\left(\exp\left(pd_{t+1}\right) + 1\right) + \Delta d_{t+1} - pd_t \\
&\simeq \ln\left(\exp\left(\overline{pd}\right) + 1\right) + \frac{\exp\left(\overline{pd}\right)}{\exp\left(\overline{pd}\right) + 1}\left(pd_{t+1} - \overline{pd}\right) + \Delta d_{t+1} - pd_t \\
&= \kappa_0 + \kappa_1 pd_{t+1} + \Delta d_{t+1} - pd_t,
\end{aligned}$$

where:

$$\begin{aligned}
\kappa_0 &= \ln\left(\exp\left(\overline{pd}\right) + 1\right) - \kappa_1 \overline{pd}, \\
\kappa_1 &= \frac{\exp\left(\overline{pd}\right)}{\exp\left(\overline{pd}\right) + 1}.
\end{aligned}$$

We conjecture that the log price-dividend ratio is of the form:

$$pd_t = A + B s_t,$$

The price-dividend ratio coefficients are obtained by solving the Euler equation:

$$E_t\left(M^{\$}_{t+1}R^{\$}_{t+1}\right) = 1.$$

We suppress the dependence on i in the following derivation:

$$\begin{aligned}
1 &= E_t\left(\exp\left(m_{t+1} - \pi_{t+1} + \kappa_0 + \kappa_1 pd_{t+1} + \Delta d_{t+1} - pd_t + \pi_{t+1}\right)\right) \\
0 &= E_t\left(m_{t+1}\right) + \frac{1}{2}V_t\left(m_{t+1}\right) + E_t\left(\kappa_0 + \Delta d_{t+1} + \kappa_1 pd_{t+1} - pd_t\right) \\
&\quad + \frac{1}{2}V_t\left(\Delta d_{t+1} + \kappa_1 pd_{t+1}\right) + Cov_t\left(-\Lambda_t'\varepsilon_{t+1}, \Delta d_{t+1} + \kappa_1 pd_{t+1}\right) \\
&= -y + \kappa_0 + \gamma_0 + \gamma_1 s_t + \left(\kappa_1 - 1\right)A + \left(\kappa_1\rho_s - 1\right)Bs_t \\
&\quad + \frac{1}{2}\sigma_d^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\kappa_1^2 B^2\sigma_s^2 - \Lambda_t(1)\sigma_d - \Lambda_t(3)\kappa_1 B\sigma_s.
\end{aligned}$$

This results in the system:

$$\begin{aligned}
0 &= -y + \kappa_0 + \gamma_0 + \left(\kappa_1 - 1\right)A + \frac{1}{2}\sigma_d^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\kappa_1^2 B^2\sigma_s^2 - \Lambda_0(1)\sigma_d - \Lambda_0(3)\kappa_1 B\sigma_s, \\
0 &= \left(\kappa_1\rho_s - 1\right)B - \Lambda_1(1)\sigma_d + \gamma_1,
\end{aligned}$$

Rearranging terms, we get the following expressions for the pd ratio coefficients, where we makw the dependence on i explicit:

$$\begin{aligned}
A_i &= \frac{\frac{1}{2}\sigma_{di}^2 + \frac{1}{2}\sigma_i^2 + \frac{1}{2}\kappa_{1i}^2 B_i^2\sigma_s^2 - \Lambda_0(1)\sigma_{di} - \Lambda_0(3)\kappa_{1i}B_i\sigma_s - y + \kappa_{0i} + \gamma_{0i}}{1 - \kappa_{1i}}, \\
B_i &= \frac{\gamma_{1i} - \Lambda_1(1)\sigma_{di}}{1 - \kappa_{1i}\rho_s}.
\end{aligned}$$

Note that B_i can be positive or negative depending on the importance of dividend growth predictability (γ_{1i}) and fluctuations in risk premia ($\Lambda_1(1)\sigma_{di}$).

The equity risk premium on portfolio i can be computed as follows:

$$\begin{aligned} E_t\left[rx_{t+1}^i\right] &= cov_t\left(-m_{t+1}^{\$}, r_{t+1}^i + \pi_{t+1}\right) \\ &= cov\left(\Lambda_t'\varepsilon_{t+1}, \kappa_{1i}B_i\sigma_s\varepsilon_{t+1}^s + \sigma_{di}\varepsilon_{t+1}^d\right) \\ &= \Lambda_0(1)\sigma_{di} + \Lambda_0(3)\kappa_{1i}B_i\sigma_s + \Lambda_1(1)\sigma_{di}s_t. \end{aligned}$$

Appendix A.3. Calibration in Detail

In this appendix, we provide the details of our calibration. We start by describing how we define recessions in the model. Second, we describe the calibration of dividends and inflation processes. Third, we describe the choice of market price of risk parameters. A summary of this discussion is found in the main text.

Recessions in the Model Our calibration of recessions mimics the NBER dating procedure. The parameters we chose generate a simulated distribution over the length of recessions that matches several moments of the empirical distribution. We now describe this procedure in detail.

Recessions in the model are determined by the dynamics of the state process s_t. Define the cumulative shock process $\chi_t \equiv \sum_{k=0}^{K} \varepsilon_{t-k}^s$, where the parameter K governs the length of the backward-looking window. Let $\underline{\chi}$ and $\overline{\chi}$ be the p_1^{th} and p_2^{th} percentiles of the distribution of χ_t, respectively, and let $\underline{s}$ be the be the p_3^{th} percentile of the distribution of the s process. Whenever $\chi_t < \underline{\chi}$, we find the first negative shock between $t-K$ and t; say it occurs in month $t-j$. If, in addition, $s_{t-j} < \underline{s}$, we say that the recession started in month $t-j$. We say that the recession ends the fist month that $\chi_{t+i} > \overline{\chi}$, for $i \geq 1$. We assume that a new recession cannot start before the previous one has ended.

We find the recession parameters (K, p_1, p_2, p_3) by matching features of the fifteen recessions in the 1926-2009 data. In particular, we consider the fraction of recession months (19.86% in the data), the average length of a recession (13.3 months), the minimum length of a recession (6 months), the 25^{th} percentile (8 months), the median (11 months), the 75^{th} percentile (14.5 months), and the maximum length (43 months). We simulate the process for s_t for 10,000 months, determine recession months as described above, and calculate the weighted distance between the seven moments in the simulation and in the data. We iterate on the procedure to find the four parameters that minimize the distance between model and data.[14] The best fit has 19.70% of months in recession, an average length of 12.0 months, a minimum of 6, 25^{th} percentile of 8, median of 11, 75^{th} percentile of 14, and maximum of 43 months. The corresponding parameters are $K = 7$ months, $p_1 = 17$, $p_2 = 37$, and $p_3 = 29$.

To describe how the variables of interest behave over the course of a recession, it is convenient to divide each recession into three equal stages, and to keep track of the value in the last month of each stage. More precisely, we express the variable in percentage difference from the peak, which is the month before the recession starts. For example, if a recession lasts 9 (10) months, we calculate how much lower dividends are in months 3, 6, and 9 (10) of the recession, in percentage terms relative to peak. Averaging these numbers over recessions indicates the typical change of the variable of interest in three stages of a recession. The third-stage number summarizes the behavior of the variable over the entire course of the recession. We apply this procedure equally to the data and the model simulation.

We set $\rho_s = .9355$ to exactly match the 12-month autocorrelation of the CP factor of .435. This low annual autocorrelation is consistent with the interpretation of s as a business-cycle frequency variable. We set $\sigma_s = 1$; this is an innocuous normalization. The s process is negative at the start of the recession (1.6 standard deviations below the mean), falls considerably in the first stage of a recession (to 3.2 standard deviations below the mean), continues to fall in the second stage (to -3.9 standard deviations), and partially recovers in the last stage (to -2.9

[14] The weighting matrix is diagonal and takes on the following values: .9, .9, .7, .5, .7, .5, and .5, where the weights are described in the same order as the moments in the text. We use an extensive grid search and limit ourselves to integer values for the parameters.

standard deviations).

Dividend and Inflation Parameters We calibrate parameters to match moments of real dividend growth on the market portfolio, value portfolio (fifth book-to-market quintile), and growth portfolio (first quintile) for 1927-2009 (997 months). Since nominal bond yields are unavailable before 1952, we compare our model's output for nominal bond yields and associated returns to the average for 1952-2009. In our model simulation, we reinvest monthly dividends at the risk-free rate to compute an annual real dividend series, replicating the procedure in the data. We calculate annual inflation as the twelve-month sum of log monthly inflation, as in the data.

The most important parameter is γ_{1i}, which measures how sensitive dividend growth is to changes in real economic activity. In light of the empirical evidence presented above, we choose γ_{1i} to match the log change in annual real dividends between the peak of the cycle and the last month of the recession. In the data, the corresponding change is -21.0% for value stocks (the fifth BTM portfolio), + 2.2% for growth stocks (first BTM portfolio), and -5.2% for the market portfolio (CRSP value-weighted portfolio). Given the parameters governing the s dynamics and the recession determination described above, the model matches these changes exactly for $\gamma_{1G} = -.4e - 4$, $\gamma_{1V} = 97.6e - 4$, and $\gamma_{1M} = 24.8e - 4$. Note that $\gamma_{1V} > \gamma_{1G}$ delivers the differential fall of dividends on value and growth stocks.

We choose $\gamma_{0G} = .0010$, $\gamma_{0V} = .0044$, and $\gamma_{0M} = .0010$ to exactly match the unconditional mean annual log real dividend growth of 1.23% on growth, 5.26% on value, and 1.23% on the market portfolio. We choose $\sigma_{dM} = 2.09\%$ to exactly match the unconditional volatility of annual log real dividend growth of 10.48%. We set $\sigma_{dG} = 1.94\%$ and $\sigma_{dV} = 2.23\%$ in order to match the fact that the covariance of growth stocks with market return innovations is slightly higher than that of value stocks. However, the difference needs to be small to prevent the value premium from being due to differential exposure to market return shocks. To be precise, this difference makes the contribution of the market factor to the value premium equal to 0.44% per year, the same as in the data. We set the idiosyncratic volatility parameter for growth $\sigma_G = 3.48\%$ to match exactly the 13.75% volatility of dividend growth on growth stocks, given the other parameters. We set $\sigma_V = 10.94\%$ because the volatility of dividend growth on value stocks of 48.93%. The 12-month autocorrelation of annual log real dividend growth in the model results from these parameter choices and is -.01 for G, .21 for V, and .29 for M, close to the observed values of .11, .16, and .29, respectively.

We choose $\bar{\pi} = .0026$ to exactly match average annual inflation of 3.06%. We choose $\rho_x = .989$ and $\sigma_x = .03894\%$ to match the unconditional volatility and 12-month autocorrelation of nominal bond yields of maturities 1- through 5-years (1952-2009 Fama-Bliss data). In the model, volatilities decline from 3.13% for 1-year to 2.58% for 5-year bonds. In the data, volatilities decline from 2.93% to 2.72%. The 12-month autocorrelations of nominal yields range from .88 to .84 in the model, and from .84 to .90 in the data. Our parameters match the averages of the autocorrelations and volatilities across these maturities. We choose the volatility of unexpected inflation $\sigma_\pi = .7044\%$ to match the volatility of inflation of 4.08% in the data. The 12-month autocorrelation of annual inflation is implied by these parameter choices and is .59 in the model , close to the .61 in the data. We set the real short rate $y = .0018$, or 2.1% per year, to match the mean 1-year nominal bond yield of 5.37% exactly, given all other parameters.

Market Prices of Risk We set $\Lambda_0(1) = .2913$ to match the unconditional equity risk premium on the market portfolio of 7.28% per year (in the 1927-2009 data). The market price of expected inflation risk $\Lambda_0(1) = -.0986$ is set to match the 5-1-year slope of the nominal yield curve of 0.60%. The term structure behaves nicely at longer horizons with 10-year yields equal to 6.27% per year, and 30-year yields equal to 6.49% per year. The average of the annual bond risk premium on 2-year, 3-year, 4-year, and 5-year bond returns, which is the left-hand side variable of the CP regression, is 0.75% in the model compared to 0.87% in the data. The mean CP factor is .0075 in model and .0075 in the data. We set the market price of cyclical risk $\Lambda_0(3) = .0249$ in order to match the 5.22% annual value premium (in the 1927-2009 data).

We set $\Lambda_1(1) = .1208$ in order to generate a slightly negative $B_M = -6.24e-4$. As argued above, the near-zero B_M prevents the value premium from arising from exposure to market return shocks, and it prevents bond returns from being heavily exposed to market risk. The slight negative sign delivers a slightly positive contribution of exposure to market return shocks to bond excess returns, as in the data (recall Figures 2 and 7). In particular, it generates a 15 basis point spread between ten-year and 1-year bond risk premia coming from market exposure, close to the 30 basis points in the post-1952 data. Finally, we set $\Lambda_1(1) = -0.0702$ in order to exactly match the volatility of the CP factor of 1.55%. The volatility of the average annual bond risk premium on 2-year, 3-year, 4-year, and 5-year bonds is 3.93% in the model and 3.72% in the data. As mentioned above, ρ_s is chosen to match the persistence of CP. Thus the model replicates the mean, volatility, and persistence of the CP factor and the nominal bond risk premium. The maximum annualized log Sharpe ratio implied by the model, $E[\sqrt{\Lambda_t'\Lambda_t}]\sqrt(12)$ is 1.44. Unfortunately, there is no easy comparison with the numbers in the empirical section (bottom panel of Table I).

Appendix B. How Pricing Stocks and Bonds Jointly Can Go Wrong

Consider two factors F_t^i, $i = 1, 2$, with innovations η_{t+1}^i. We normalize $\sigma\left(\eta_{t+1}^i\right) = 1$. Let $cov\left(\eta_{t+1}^1, \eta_{t+1}^2\right) = \rho = corr\left(\eta_{t+1}^1, \eta_{t+1}^2\right)$. We also have two cross-sections of test assets with excess, geometric returns r_{t+1}^{ki}, $i = 1, 2$ and $k = 1, ..., K_i$, with innovations ε_{t+1}^{ki}. We assume that these returns include the Jensen's correction term. Suppose that both cross-sections exhibit a one-factor pricing structure:

$$E\left(r_{t+1}^{ki}\right) = cov\left(\varepsilon_{t+1}^{ki}, \eta_{t+1}^i\right)\lambda_i,\ i = 1, 2.$$

The first factor perfectly prices the first set of test assets, whereas the second factor prices the second set of test assets. We show below that this does *not* imply that there exists a single SDF that prices both sets of assets.

Consider the following model of unexpected returns for both sets of test assets:

$$\begin{aligned}
\varepsilon_{t+1}^{k1} &= E\left(r_{t+1}^{k1}\right)\eta_{t+1}^{1}, \\
\varepsilon_{t+1}^{k2} &= E\left(r_{t+1}^{k2}\right)\eta_{t+1}^{2} + \alpha_{2k}\eta_{t+1}^{3},
\end{aligned}$$

with $cov\left(\eta_{t+1}^{2}, \eta_{t+1}^{3}\right) = 0$. Unexpected returns on the first set of test assets are completely governed by innovations to the first factor, whereas unexpected returns on the second set of test assets contain a component $\alpha_{2k}\eta_{t+1}^{3}$ that is orthogonal to the component governed by innovations to the second factor. These η^3 shocks are not priced (by assumption). We assume that they are correlated with the η^1 shocks: $cov\left(\eta_{t+1}^{1}, \eta_{t+1}^{3}\right) \neq 0$.

This structure implies:

$$cov\left(\varepsilon_{t+1}^{ki}, \eta_{t+1}^{i}\right) = E\left(r_{t+1}^{ki}\right) var\left(\eta_{t+1}^{i}\right) = E\left(r_{t+1}^{ki}\right),$$

and hence $\lambda_i = 1$, $i = 1, 2$. Then we have:

$$\begin{aligned}
cov\left(\varepsilon_{t+1}^{k1}, \eta_{t+1}^{1}\right) &= E\left(r_{t+1}^{k1}\right), & cov\left(\varepsilon_{t+1}^{k1}, \eta_{t+1}^{2}\right) &= E\left(r_{t+1}^{k1}\right)\rho, \\
cov\left(\varepsilon_{t+1}^{k2}, \eta_{t+1}^{1}\right) &= \left(r_{t+1}^{k2}\right)\rho + \alpha_{2k} cov\left(\eta_{t+1}^{1}, \eta_{t+1}^{3}\right), & cov\left(\varepsilon_{t+1}^{k2}, \eta_{t+1}^{2}\right) &= E\left(r_{t+1}^{k2}\right).
\end{aligned}$$

The main point is that, if α_{2k} is not proportional to $E\left(r_{t+1}^{k2}\right)$, then there exist no Λ_1 and Λ_2 such that:

$$E\left(r_{t+1}^{ki}\right) = cov\left(\varepsilon_{t+1}^{ki}, \eta_{t+1}^{1}\right)\Lambda_1 + cov\left(\varepsilon_{t+1}^{ki}, \eta_{t+1}^{2}\right)\Lambda_2.$$

On the other hand, if there is proportionality and $\alpha_{2k} = \alpha E\left(r_{t+1}^{k2}\right)$, then we have:

$$cov\left(\varepsilon_{t+1}^{k2}, \eta_{t+1}^{1}\right) = E\left(r_{t+1}^{k2}\right)\left(\rho + \alpha cov\left(\eta_{t+1}^{1}, \eta_{t+1}^{3}\right)\right) = E\left(r_{t+1}^{k2}\right)\xi,$$

and Λ_1 and Λ_2 are given by:

$$\Lambda_1 = \frac{1-\rho}{1-\xi\rho}, \text{and } \Lambda_2 = \frac{1-\xi}{1-\xi\rho}.$$

This setup is satisfied approximately in our model, where the first set of test assets are the book-to-market portfolios, η^1 are CP innovations, the second set of test assets are the bond portfolios, and η^2 are level innovations. Unexpected bond returns contain a component η^3 that is uncorrelated with level innovations, but that is correlated with CP innovations. Unexpected book-to-market portfolio returns, in contrast, are largely uncorrelated with level innovations. The result above illustrates that consistent risk pricing is possible because unexpected bond returns' exposure to CP shocks has a proportionality structure. This can also be seen in the top panel of Figure 2.