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Collusion and the political differentiation of newspapers

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Abstract

We analyse a newspaper market where two editors compete for advertising as well as for readership. They first choose the political position of their newspaper, then set cover prices and advertising tariffs. We build on the work of Gabszewicz, Laussel and Sonnac (2001, 2002), whose model we take as the stage game of an infinitely repeated game, and investigate the incentives to collude and the properties of the collusive agreements in terms of welfare and pluralism. We analyse and compare two forms of collusion: in the first, publishers cooperatively select both prices and political position; in the second, publishers cooperatively select prices only. Whereas the first leads to intermediate product differentiation, the second leads, as in Gabszewicz, Laussel and Sonnac (2001, 2002), to minimal product differentiation. However, in the latter case, differently from Gabszewicz, Laussel and Sonnac (2001, 2002), cover prices are positive and the minimal differentiation outcome does not depend on the size of the advertising market. We thus show that collusion on prices reinforces the tendency towards a *Pensée Unique* discussed in Gabszewicz, Laussel and Sonnac (2001).

JEL classification: L41, L82, D43, K21

Keywords: collusion, newspapers, two-sided markets, indirect network effects, pluralism, spatial competition

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1. Introduction

Media industries are well known examples of two sided markets.[1] Newspaper publishers sell their products to two different categories of buyers, namely readers and advertisers. Readers are interested in news while advertisers aim at reaching potential consumers by buying advertising space in the newspaper. Newspaper publishers know that the more readers their newspaper has the higher the willingness to pay of advertisers for a slot in the newspaper. Vice versa readers may be affected by advertising in the newspaper.[2] Publishers therefore choose cover prices and advertising tariffs taking into account this link between the demands on the two sides of the market. Yet, differently from the case of complement products, this interdependency among the two demands and the resulting link between prices is not recognised by advertisers or readers as they buy only one of the two products sold by the publisher.

Whereas this particular characteristic of media markets has always been known to those working in the field and has thus been recognised in the economic literature ever since the first studies of these industries[3], the literature on two-sided markets itself has developed only in the last ten years, as economists became aware of the fact that other, apparently very different, markets share this basic features with media markets.[4] It is the case for instance of payment cards, auction houses and game-consoles.

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[1] See Anderson and Gabszewicz (2008) on media markets as two-sided markets.

[2] Whether and to what extent readers/viewers/listeners are instead negatively or positively affected by the amount (or concentration) of advertising in a given media is a debated issue. Some theoretical models assume that consumers are advertising-averse, e.g. Gabszewicz, Laussel and Sonnac (2004), Kind, Nillssen and Sorgard (2007) and Peitz and Valletti (2008) for the TV industry. Others specify a (variable) proportion between ad-lovers and ad-haters, e.g. Gabszewicz, Laussel and Sonnac (2005) for the press industry. Yet other models assume that consumers are advertising indifferent, e.g. Gabszewicz, Laussel and Sonnac (2001, 2002) for newspapers, or advertising-lovers, e.g. Gabszewicz, Garella and Sonnac (2007) again for newspapers.

[3] See already Corden (1953) and Reddaway (1963). More recently, but still before the theory of two-sided markets was developed, Blair and Romano (1993) and Chaundri (1998).

[4] The seminal papers in the field are those by Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003, 2006), Evans (2003), Parker and Van Alstyne (2005), and Armstrong (2006). More recently,Weyl(2010) proposed a new way to model firms pricing in two-sided markets. Whereas Caillaud and Jullien (2001, 2003) and Parker and Van Alstyne (2005) respectively talk about indirect network effects and two-sided network effects, the term two-sided market appears to have been used first by Rochet and Tirole (2002, 2003, 2006) as well as Armstrong (2006). Evans (2003) instead, preferred to talk about markets with two-sided platforms. In the media literature, Chaundri (1998) talks about "duality in the product space" of a newspaper publisher, as the publisher serves both advertisers and readers. Gabszewicz, Laussel and Sonnac (2001, 2002) discuss instead cross-market network effects. See Evans and Schmalensee (2007) for an overview of different types of two-sided markets and Filistrucchi (2010) for a discussion of how many markets are two-sided.

The issue of concentration in media markets has been debated both in academic circles as well as among policy makers since at least the 1960's. On the one hand the debate centred on the reasons why a high concentration has often been observed in the industry[5], on the other hand it focused on whether high concentration in the market is detrimental to pluralism, i.e. on whether high concentration leads to duplication of content or, on the contrary, higher product differentiation.[6] With regard to the latter question, results are somewhat ambiguous but surprisingly it has been shown that competition can lead to duplication of content when media outlets are mainly financed through advertising.[7]

Despite the fears of a possible lack of pluralism in the media due to high concentration in the market, not the same attention has been devoted to the possible effects of collusion. Yet in principle joint profit maximization by colluding publishers is able to reproduce the same market outcome of a merger among those same publishers.

On the contrary, the US Newspaper Preservation Act allowed so-called joint operating agreements, which permitted newspapers within the same market area to jointly set cover prices and advertising rates. Hence, under a JOA newspapers would appear to be able to collude on both sides of the market.[8] The idea behind this block-exemption was to help newspapers to survive, given the trend of market exit initiated by the appearance of radio and continued with the introduction of TV.

Also in the academia relatively little attention has been paid to the topic of collusion in media markets. More generally, despite the rapid growth of the literature on two-sided markets, only very recently the issue of collusion has been investigated in empirical as well as in theoretical works and the literature on the topic is still scarce.

Rhumer (2010) shows how in two-sided markets the presence of indirect network externalities affects the incentives to collude and the welfare implications of

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[5] See Reddaway (1963) and Rosse (1967), Rosse, Owen and Dertouzos (1975), Rosse (1977), Rosse (1978), Rosse (1980) and Bucklin, Caves and Lo (1989) for studies, which highlight the importance of economies of scale. See instead Gabszewicz, Garella and Sonnac (2007) and Häckner and Nyberg (2008) for the role played by the indirect network effects.

[6] See Polo (2007) for a discussion.

[7] See Steiner (1952) for the radio industry and both Beebe (1977) and Spence and Owen (1977) for the TV industry. More recently, see Gabszewicz, Laussel and Sonnac (2001, 2002) for the newspaper industry and Gabszewicz, Laussel and Sonnac (2004) for the TV industry. See Anderson and Coate (2005) for why the relationship between competition and the amount of different programming is ambiguous.

[8] Note that indeed, Fan (2010) models newspapers engaged in JOAs as cooperatively setting cover prices and advertising tariffs.

collusion. She uses the single-homing model in Armstrong (2006) as a stage game of an infinitely repeated game to model a two-sided market where firms are differentiated on both sides and simultaneously choose both prices. Assuming firms adopt grim trigger strategies she finds that higher network externalities have two opposite effects: on the one hand they tend to raise incentives to collude as they increase the gain from collusion (collusive profits increase and competitive profits decline); whilst on the other hand they tend to lower incentives to collude as they increase the gain from deviation. In her model the latter effect is always found to dominate. As a result, collusion becomes harder to sustain as indirect network effects between the two sides of the market increase. Furthermore, she finds that a higher asymmetry in the indirect network effects reduces the incentives to collude.

Dewenter, Haucap and Wenzel (2010) analyse instead the welfare consequence of collusion on the advertising tariffs only in a duopoly newspaper market where firms first choose the advertising quantity and then the cover prices, while readers dislike advertising. Under these assumptions and the additional assumption of a linear demand for differentiated products, they find that collusion on the advertising tariffs may not only lead to an increase in readers' welfare (since it may reduce readers' prices more than it reduces the value of the newspaper to readers by decreasing the quantity of ads) but it can also lead to a higher advertiser welfare (as it increases advertising tariffs less than it increases the newspaper's value to advertisers due to higher circulation).

Most recently, Boffa and Filistrucchi (2011) discuss an interesting particular case in which firms in a two-sided market raise prices above the monopoly price on one side of the market in order to be able to sustain collusion when perfect joint profit maximization is not sustainable.

In all these theoretical works however product differentiation is exogenous.

Among empirical works, Argentesi and Filistrucchi (2007) provide econometric evidence that daily newspapers in Italy have been colluding on the cover price (but not on the advertising tariffs). They do not address however the issue of whether collusion affected the political position of newspapers.

Whereas the issue of endogenous product positioning in two-sided markets is per se interesting, concerns about pluralism imply that product differentiation plays a much more crucial role in media markets than in standard markets, at least in as much as pluralism plays the role of a positive externality in the political process.[9]

Indeed, Gabszewicz, Laussel and Sonnac (2001, 2002) develop a model of oligopolistic competition among publishers who choose first political position, then cover prices and advertising tariffs. They show that advertising financing can lead to minimum product differentiation. Behringer and Filistrucchi (2011) extend their model to more than two publishers and show that concerns for minimum product differentiation as a result of advertising financing diminish as the number of firms increases.

Recently, Fan (2010) analysed the market for daily newspapers in the US. Her structural econometric model allows for endogenous product differentiation. However, it focuses on vertical rather than horizontal product differentiation. As such it does not address the issue of the political differentiation of newspapers.

All in all, no paper so far has looked at the impact of collusion on product differentiation in two-sided markets, not even in media markets.

We fill the gap by building on a non-cooperative sequential game developed by Gabszewicz, Laussel and Sonnac (2001, 2002), which we take as the stage game of an infinitely repeated game, modelling a newspaper market where two publishers compete for advertising as well as for readership and decide whether and how to collude. Publishers first choose the political position of their newspaper, then set cover prices and advertising tariffs. Whereas readers single-home, i.e. they buy only one copy of only one newspaper, advertisers may multi-home, i.e. they can buy ad spaces from one, both or none of the two newspapers.

We investigate the incentives to collude using grim trigger strategies and report the properties of the potential collusive agreements in terms of welfare and pluralism. Two kinds of collusion are analysed and compared: in the first, publishers cooperatively select both prices and political position; in the second, publishers cooperatively select

[9] While Gentzkow (2006) shows that the introduction of TV decreased voter turnout, Gentzkow, Shapiro and Sinkinson (2009) find that entry of newspapers has a robust positive effect on political participation, but newspaper competition is not a key driver of turnout as the effect is driven mainly by the first newspaper to enter the market, and the effect of a second or third paper is significantly smaller. Della Vigna and Kaplan (2007) show instead that the introduction of the Fox News in the US lead to a significant increase in votes for Republicans.

prices only. Whereas the first leads to intermediate product differentiation, the second leads, as in Gabszewicz, Laussel and Sonnac (2001, 2002), to minimal product differentiation. However, in the latter case, differently from Gabszewicz, Laussel and Sonnac (2001, 2002), equilibrium prices are positive. Whatever the type of collusion, our findings confirm the traditional idea that the more competition there is in the market, the better off the consumers will be. The effects on total welfare are instead ambiguous.

2. Competition in the newspaper market

We first introduce the model of competition in the newspaper market developed by Gabszewicz, Laussel, and Sonnac (2001, 2002), which we take as the stage game of an infinitely repeated game. We also make explicit the condition on the demand parameters which guarantees that the market is, as in their work, always covered. Such a condition is necessary to compare the competitive outcome with the collusive ones.

2.1. The stage game

The model of Gabszewicz, Laussel and Sonnac (2002) consists of three steps:

- first, publishers choose the political orientation of their newspaper out of a unit interval representing the political spectrum from extreme left to extreme right;
- second, given the political position of their newspaper, publishers compete for readers; readers are assumed to buy one copy of one newspaper, i.e. they single-home;
- third, publishers compete in the advertising market[10]; advertisers are assumed to buy ad spaces from one, two or neither newspaper, i.e. they multi-home.

At any step, choices are made simultaneously and are common knowledge at every subsequent step.

Such a game corresponds to a Hotelling spatial duopoly with a further step for the advertising competition.[11]

2.1.1. Readers demand

[10] Note that, in fact, given the particular assumptions on the advertising side, whether firms first set cover prices and then advertising tariffs or vice versa is irrelevant.
[11] See Hotelling (1929) as corrected by D'Aspremont, Gabszewicz and Thisse (1979).

Readers have political opinions ranging from extreme left to extreme right; they are thus located uniformly on a unit interval [0,1] with every reader ideally corresponding to a point on this line. The market size is 1.

As readers care both about the price of the newspaper and about the political orientation, utility of reader r is defined as follows

$$U_r = \bar{u} - tx_{ir}^2 - p_i \qquad (1)$$

where x_{ir} is the distance between the political orientation of newspaper i with $i = 1,2$, and the political opinion of reader r and p_i denotes the price to be paid for newspaper i. Readers therefore bear a cost when buying a newspaper, which is proportional to the square of the distance between their political opinion and the political line of the newspaper. Thus the sum $tx_{ir}^2 + p_i$ is the *total cost* sustained by reader r when buying newspaper i. The parameter $\bar{u}$ represents instead the reservation price of readers when buying a copy of a newspaper, i.e. the maximum willingness to pay for a copy of the newspaper. In other words, this parameter is interpreted as the *intrinsic value* of the newspaper. Such a value is assumed equal across consumers and newspapers.

It is also assumed that readers are indifferent to advertising on daily newspapers.[12]

Without loss of generality, let us assume that publisher 1 chooses a political orientation denoted by a on the unit interval, where a will be the distance between the selected point and 0, while publisher 2 chooses a political opinion denoted by b on the unit interval, where b will be the distance between the selected point and 1.

[12] Empirical evidence on the effect of advertising can be found in Sonnac (2000), who reports that the effect of advertising on readers depends on the type of media and on the country, Kaiser and Wright (2006), who find a positive but small effect of advertising on the sales of daily newspapers in Germany and Argentesi and Filistrucchi (2007), who find no effect of advertising on the sales of daily newspapers in Italy. A similar finding is reported by Fan (2010) for US daily newspapers and by van Cayselee and Vanormelingen (2010) for Belgian daily newspapers. Kaiser and Song (2009) find that readers of magazines do not dislike advertising but depending on the type of magazine may also like it. Finally, Wilbur (2008) and Jeziorski (2011) respectively find that TV viewers and radio listeners in the US dislike advertising.
The conclusion we draw from the literature above is that on average consumers like advertising in magazines (when it is relatively targeted and can be avoided), dislike it on TV (when it is not targeted and cannot be avoided) and are indifferent to it on daily newspapers (where it is not targeted but can be avoided). As a consequence, we maintain the assumption in Gabszewicz, Laussel and Sonnac (2001, 2002) that readers are indifferent to advertising on daily newspapers.

Gabszewicz, Laussel and Sonnac (2001, 2002) assume that every consumer in the market buys a copy of a newspaper or, in other words, that the reader market is always entirely covered; more formally, the following condition is assumed to be always satisfied at equilibrium:

$$\bar{u} - t x_{ir}^2 - p_i \geq 0 \tag{2}$$

From now on, we will refer to this inequality as *market coverage condition*.

This condition implies the following restrictions on the parameters:

$$\begin{cases} \bar{u} \geq \dfrac{t}{4} \\ \bar{u} \geq \dfrac{5t}{4} + c - k \end{cases} \tag{3}$$

which, as we will see, ensures that the market coverage condition holds for the two possible outcomes of the stage game.

As in a standard Hotelling model with quadratic transportation costs, one can derive the demand from readers by first identifying the indifferent consumer y for each couple of prices p_1 and p_2 and locations a and b.

Figure 1 is a diagrammatic representation of the problem as in Economides (1984). The horizontal axis represents the unit interval on which the readers' opinions are listed. Publisher 1 and 2 locate respectively at a and $1 - b$. Instead, the vertical axis displays the reservation price, newspaper cover prices and the total costs faced by readers. The intersection between the total cost curves gives the location of the consumer y who is indifferent between buying newspaper 1 or newspaper 2. Consumers to the left of y will sustain a lower total cost when buying newspaper 1 while consumers to the right of y will sustain a lower total cost when buying newspaper 2. In other words, y splits the market into the demand for newspaper 1, n_1, and the demand for newspaper 2, n_2.

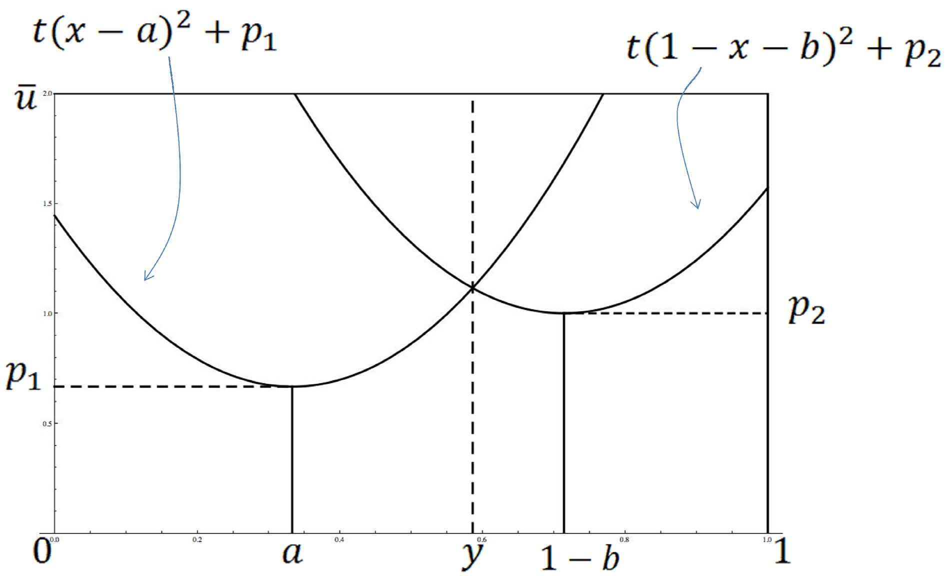

Figure 1

In Figure 1, $y = a + \frac{p_2 - p_1}{2t(1-a-b)} + \frac{1-a-b}{2}$

One can thus obtain the demand of newspapers as functions of prices p_1 and p_2 and location a and b[13]:

$$n_1 = \begin{cases} 0, & if\ a + \dfrac{p_2 - p_1}{2t(1-a-b)} + \dfrac{1-a-b}{2} < 0 \\ a + \dfrac{p_2 - p_1}{2t(1-a-b)} + \dfrac{1-a-b}{2}, & if\ \ 0 \leq a + \dfrac{p_2 - p_1}{2t(1-a-b)} + \dfrac{1-a-b}{2} \leq 1 \\ 1, & if\ a + \dfrac{p_2 - p_1}{2t(1-a-b)} + \dfrac{1-a-b}{2} > 1 \end{cases} \quad (4)$$

$$n_2 = \begin{cases} 0, & if\ b + \dfrac{p_1 - p_2}{2t(1-a-b)} + \dfrac{1-a-b}{2} < 0 \\ b + \dfrac{p_1 - p_2}{2t(1-a-b)} + \dfrac{1-a-b}{2}, & \text{if } 0 \leq b + \dfrac{p_1 - p_2}{2t(1-a-b)} + \dfrac{1-a-b}{2} \leq 1 \\ 1, & b + \dfrac{p_1 - p_2}{2t(1-a-b)} + \dfrac{1-a-b}{2} > 1 \end{cases} \quad (5)$$

13 Note that, as the reservation price is the same for both newspapers, it does not alter the decision of which newspaper to buy.

Having normalized the mass of readers to 1, n_i can also be seen as the market share or the mass of readers of publisher i.

2.1.2. Advertising demand

For convenience, the size of the advertising market is assumed to be $4k$, i.e. there are $4k$ advertisers in the market. Each advertiser's preferences depend on the price of ad spaces in a newspaper and on the mass of readers of that newspaper. The intensity of preference for the mass of readers is assumed to depend on a parameter, which characterizes each advertiser. Formally, this intensity is represented by the parameter θ, $\theta \in [0,1]$ and is referred to an ad space in a newspaper; the advertisers' population is uniformly distributed on $[0,1]$.

Thus, the utility of buying an ad in newspaper i for an advertiser a of type θ is measured by:

$$U_a = n_i\theta - s_i, \qquad i = 1,2;\ \theta \in [0,1], \tag{6}$$

where n_i represents the amount of readers of newspaper i, and s_i is the advertising tariff applied by publisher i.

Each advertiser is willing to buy an ad space in a newspaper as long as her utility is higher or equal to 0. Therefore, each advertiser has three possible choices: i) not to place an ad in any newspaper; ii) to place an ad in a single newspaper; iii) to place an ad in both. Advertisers can therefore multi-home. Each newspaper can carry as many ad spaces as demanded. Since each advertiser can place only one ad in each newspaper, the quantity of ads in a newspaper will equal the number of advertisers placing ads in that newspaper.

If an advertiser a of type θ multi-homes, her utility is measured by:

$$U_a = n_1\theta - s_1 + n_2\theta - s_2 \tag{7}$$

that is each advertiser buys an ad space in a newspaper if her utility to do so is positive. It is important to notice that the newspaper market is split into readers of newspaper 1 and readers of newspaper 2 because of single-homing on that side.

Accordingly, advertisers' utility from advertising on one newspaper is independent of whether or not she advertises also on the other.[14]

2.2. The competitive equilibria

Gabszewicz, Laussel and Sonnac (2002) solve the model by backward induction to find sub-game perfect equilibria. Firstly, they identify the optimal pricing in the advertising market; the results are shown in the previous paragraph. Secondly, by deriving the profits in (8) with respect to cover prices, they obtain the reaction functions for the second step of the game; from them they derive the equilibrium prices as functions of the locations. Thirdly, they demonstrate that both a minimal opinion differentiation equilibrium and a maximal opinion differentiation equilibrium exist for given sets of the parameters.

2.2.1. Optimal advertising tariffs

Starting from equation (6) and (7), Gabszewicz, Laussel and Sonnac (2002) show that publishers select $s^* = n_i/2$ as equilibrium tariff, leading to revenues equal to kn_i. Thus, the equilibrium revenues are directly proportional to the mass of readers; in other words, for any newspaper sold, the publisher receives a fixed sum from the advertisers. Hence, k also identifies the *advertising revenues per reader*.

As shown in Gabszewicz, Laussel and Sonnac (2002), the demand for ad spaces in a newspaper is independent from the demand of ad spaces in the other newspaper. Hence, publishers enjoy monopoly power on the advertising side for providing access to their readers. A similar situation is represented in Armstrong's (2006) competitive bottleneck model. However, here, it is assumed that advertising does not affect readers' utility. As such the model is a particular case of the one in Armstrong (2006).

2.2.2. Optimal cover prices

From the previous analysis and supposing that both publishers produce newspapers at a unit cost per copy $c > 0$, we can easily derive the profit functions, namely:

$$\pi_i = (p_i + k - c)n_i \quad i = 1,2 \tag{8}$$

[14] Note that this assumption on the advertising side, though restrictive and surely debatable, has been verified empirically by Rysman (2004) for yellow pages in the US and Fan (2010) for US daily newspapers.

where n_i is defined in either equation (4) or (5).

The best reply functions are then:

$$p_1 = \max\left\{0, \frac{1}{2}(c - k + p_2 + t - 2tb + tb^2 - ta^2\right\}$$
$$p_2 = \max\left\{0, \frac{1}{2}(c - k + p_1 + t - 2ta + ta^2 - tb^2\right\}$$

which, depending on the parameters c, t and k and on the political positions chosen in the first step, lead to the optimal prices reported in Table 1a. Substituting these into the profit equation allows to write profits as a function of political locations, as shown in Table 1b.

Parameter regions		**Optimal prices** (as a function of political locations a and b and parameters c, k and t)
Region 1	$\begin{cases} c - k + t(1 - a - b)(1 + \frac{a-b}{3}) \geq 0 \\ c - k + t(1 - a - b)(1 + \frac{b-a}{3}) \geq 0 \end{cases}$ $\boldsymbol{c - k + t(1 - a - b) \geq 0}$	$p_1^* = c - k + t(1 - a - b)(1 + \frac{a-b}{3})$ $p_2^* = c - k + t(1 - a - b)(1 + \frac{a-b}{3})$
Region 2	$\begin{cases} c - k + t(1 - a - b)\left(1 + \frac{a-b}{3}\right) < 0 \\ c - k + t(1 - a - b)\left(1 + \frac{b-a}{3}\right) < 0 \end{cases}$ $c - k + t(1 - a - b) < 0$	$p_1^* = 0$ $p_2^* = 0$
Region 3	$\begin{cases} c - k + t(1 - a - b)\left(1 + \frac{a-b}{3}\right) < 0 \\ c - k + t(1 - a - b)\left(1 + \frac{b-a}{3}\right) > 0 \end{cases}$ $a < b$	$p_1^* = 0$ $p_2^* = 1/2(c - k + t - 2ta + ta^2 - tb^2)$
Region 4	$\begin{cases} c - k + t(1 - a - b)\left(1 + \frac{a-b}{3}\right) \geq 0 \\ c - k + t(1 - a - b)\left(1 + \frac{b-a}{3}\right) < 0 \end{cases}$ $a \geq b$	$p_1^* = 1/2(c - k + t - 2tb - ta^2 + tb^2)$ $p_2^* = 0$

Table 1a

Profits as a function of location	
Region 1	$\pi_1^*(a,b) = \frac{t}{18}(1-a-b)(a-b-3)^2$ $\pi_2^*(a,b) = \frac{t}{18}(1-a-b)(b-a-3)^2$
Region 2	$\pi_1^*(a,b) = (k-c)(\frac{1+a-b}{2})$ $\pi_1^*(a,b) = (k-c)(\frac{1+b-a}{2})$
Region 3	$\pi_1^*(a,b) = \frac{1}{4}(k-c)(\frac{-3t+2ta+4tb+ta^2-tb^2-c+k}{t(a+b-1)})$ $\pi_2^*(a,b) = -\frac{1}{8}\frac{(t-2ta+ta^2-tb^2-c+k)^2}{(-1+a+b)}$
Region 4	$\pi_1^*(a,b) = -\frac{1}{8}\frac{(t-2tb+tb^2-ta^2-c+k)^2}{(-1+a+b)}$ $\pi_2^*(a,b) = \frac{1}{4}(k-c)(\frac{-3t+2tb+4ta+tb^2-ta^2-c+k}{t(a+b-1)})$

Table 1b

2.2.3. Equilibrium political positions

Gabszewicz, Laussel and Sonnac (2002) demonstrate that both a minimal opinion differentiation equilibrium and a maximal opinion differentiation equilibrium exist for given sets of the parameters.

Minimal opinion differentiation equilibrium

For $k \geq c + 25t/72$, both publishers choose to locate in the middle of the political spectrum ($a^* = b^* = 1/2$) and to set a common price equal to

$$p^* = 0 \tag{9}$$

Consequently, they split the market in two equal parts and equilibrium profits are:

$$\pi_{N1} = \frac{(k-c)}{2} \tag{10}$$

From now on, π_{N1} identifies profits arising from a one-shot competition in which publishers decide to locate in the middle of the political spectrum. Figure 2 displays the minimal differentiation equilibrium.

Maximal opinion differentiation equilibrium

For $k \leq c + t/2$, both publishers choose to locate at the endpoints of the political spectrum ($a^* = b^* = 0$) and to set a common price equal to

$$p^* = c - k + t \tag{11}$$

Consequently, they split the market into two equal parts and equilibrium profits are:

$$\pi_{N2} = \frac{t}{2} \tag{12}$$

From now on, π_{N2} identifies profits arising from a one-shot competition in which publishers decide to locate at the endpoints of the political spectrum. Figure 3 displays the maximal differentiation equilibrium.

One can check that if $c + \frac{25t}{72} \leq k \leq c + \frac{t}{2}$, both equilibria exist. By comparing (12) and (10), we can see that for this subset of the parameters $\pi_{N2} > \pi_{N1}$: profits made in the maximal differentiation equilibrium are higher than profits made in the minimal differentiation equilibrium. In other words, the maximal differentiation equilibrium Pareto dominates the minimal differentiation equilibrium when both equilibria exist.

Note that the maximal differentiation equilibrium is the classical outcome of a one-sided Hotelling model with quadratic transportation costs, as first proposed by D'Aspremont, Gabszewicz and Thisse (1979). It can be explained in the same way: firms relax competition by locating at the endpoints. On the other hand, the existence of the minimal differentiation equilibrium is the main contribution of Gabszewicz, Laussel and Sonnac (2002) and the one on which Gabszewicz, Laussel and Sonnac (2001) focus their discussion: this equilibrium arises because of the presence of the advertising side, which makes stealing customers from the rival more profitable. Indeed, it can be seen that the equilibrium is sustainable if the advertising market is large enough: in this case gaining a reader is more profitable because advertising revenues per reader are higher. Furthermore, if the advertising market is very large, only the minimal differentiation equilibrium remains: due to advertising, competition for readers is very harsh and the publishers do not choose to locate at the endpoints anymore.

Thus, Gabszewicz, Laussel and Sonnac (2002) conclude that convergence in the political orientation of newspapers could result from a rise in the importance of advertisements as source of revenues. In other words, as argued by Gabszewicz, Laussel and Sonnac (2001), the growth of advertising as a source of revenues for newspapers can help explain the emergence of the so-called *Pensée Unique.*

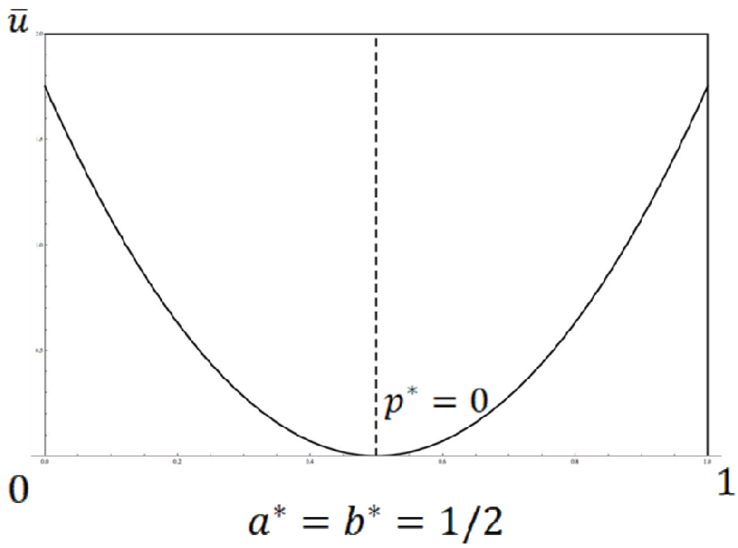

Figure 2

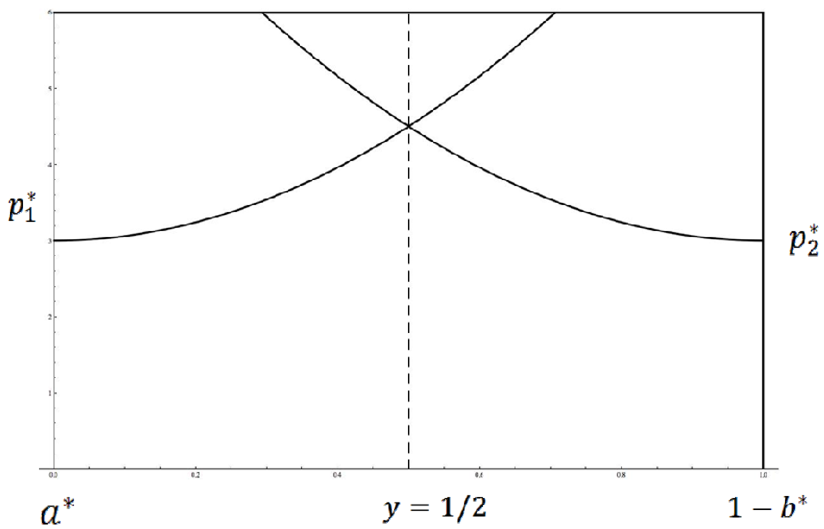

Figure 3

Condition (3) guarantees that the reservation price is high enough for the market to be entirely covered in both equilibria of the Gabszewicz, Laussel and Sonnac (2002) model. It is important to make this condition explicit as we move to analyse the repeated game and the possibility of collusive behaviour. Indeed, without a finite reservation price, publishers could collude at an indefinitely high price at no cost.

3. Collusion in the newspaper market

We employ the sequential game described above as a stage game of an infinitely repeated game. In this multi-period framework, publishers may choose to cooperate in order to obtain higher profits.

We assume that publishers take into account as collusive agreements only Pareto optimal agreements, i.e. pairs of strategies that cannot be changed without decreasing at least one of the two publishers' payoffs.

We then assume that the agreement is implemented over time by using grim trigger strategies, i.e. each publisher cooperates as long as the other publisher cooperates and punishes forever any defection from the agreement and believes the other publisher will behave in the same way.

As already discussed, in our model publishers enjoy monopoly power on advertisers for access to their readers. Accordingly, acting cooperatively cannot improve this already optimal behaviour. As a result, any optimal strategy of the repeated game cannot include a different advertising tariff than the competitive one. In practice, publishers can only collude on the cover price and on the political orientation. As we will see, this does not imply that advertising is not relevant anymore, but that publishers will not need to collude on that side of the market.

We thus take into consideration two kinds of agreements: in the first one, publishers coordinate both the political orientation and the prices of the newspapers; in the second one, publishers coordinate prices only. Whereas the first is a case of full collusion[15], the second one is a case of semi-collusion and fits well with an environment in which publishers find it difficult to coordinate on the political orientation of their newspapers.

When publishers are colluding on both political orientation and prices, we assume that if a publisher defects when choosing political position for its newspaper, punishment starts already at the next price stage (and goes on forever).

[15] To be precise, to the extent that collusion does not take place on the advertising tariff both are cases of semi-collusion. However, as noted above, in this model collusion on the advertising tariff does not make sense as publishers already enjoy monopoly power over access to the readers of their newspapers and, in addition, readers are assumed to be indifferent to advertising.

The main aims of the analysis are: firstly, investigating the factors facilitating collusion; secondly, inspecting the properties of the collusive agreements in terms of welfare and pluralism in the media.

Given the assumptions above, following Friedman and Thisse (1993), we first look for possible collusive agreements among the Pareto optimal outcomes of the stage game. When more than one Pareto optimum is identified, publishers select the Pareto optimum that implies setting common prices; if none of the Pareto optima imply common prices, publishers select the one which implies splitting the market in the middle (this will not happen anyway). Indeed, setting a common price is a plausible outcome since the firms are symmetric. Furthermore, coordination on a common price is easier to achieve and a defection from a common price is easier to detect, thus facilitating collusion.

Publishers adhere to a collusive agreement supported by grim trigger strategies as long as the discounted sum of profits associated with collusion is higher than the discounted sum of profits associated with defection.

Let us assume that π_C are the collusion profits, π_N are the Nash profits, and π_D are the defection profits, following Motta (2004), this problem can be formalized as follows:

$$\frac{\delta}{1-\delta}(\pi_C - \pi_N) \geq (\pi_D - \pi_C) \qquad joint \qquad (13)$$

The left hand side of the inequality represents the sum of discounted future losses due to defection, while the right hand side represents the one-time gain from optimal defection. The critical discount factor is easily derived as:

$$\hat{\delta} = \frac{(\pi_D - \pi_C)}{(\pi_D - \pi_N)} \qquad (14)$$

For all discount factors above $\hat{\delta}$, publishers will find it more profitable to collude than to defect. This critical discount factor will depend on the parameters of the game; as a consequence, any change in one of the parameters implies altering the profitability and sustainability of collusion. For example, if a change in parameters makes the critical discount factor increase, the set of discount factors supporting collusion is smaller and therefore collusion will be harder to sustain.

3.1. Collusion on prices and political orientation

In this paragraph collusion on prices and political orientation is analysed; firstly, the collusive agreement is investigated; secondly, the optimal defection strategy is found. Collusion profits and defection profits are then obtained.

3.1.1. Collusive agreement

As stated above, we first characterize the Pareto optima of the game. Then we select as possible collusive agreement the Pareto optimum with common price.

We will call *state* a particular pair of strategies $(a, p_1), (b, p_2)$.[16] A state can be improved if a different state implies increasing the payoff of one publisher without decreasing the payoff of the other. When a state can be improved, such state cannot be a Pareto optimum.

Lemma 1. Any state for which all readers obtain a utility strictly higher than 0 can be improved.

Proof: Take a state $(a, p_1), (b, p_2)$ and let y denote the marginal consumer who splits the market in the demand for 1, y, and the demand for 2, $1 - y$, in such state. Take now the lowest utility consumer as the reader with the lowest utility associated with such state: this consumer can only be in 0, 1 or y because they are the most distant from the location of the newspaper she buys. She is paying a total price lower than her reservation price. The publishers can mutually increase their payoff by simply increasing their prices by the same amount $\Delta p_1 = \Delta p_2$ so that the lowest utility consumer still buys a copy: thus, all the readers are buying a copy in this new state $(a, p_1 + \Delta p_1), (b, p_2 + \Delta p_2)$, while y is constant because the change in prices is the same for both newspapers. To conclude, the demands are unchanged but prices are higher and, as a consequence, payoffs increase. QED.

Figure 4 summarizes what happens in Lemma 1.

[16] Advertising tariffs are kept out of the definition of state for the sake of simplicity, as the optimal tariff does not change during the analysis.

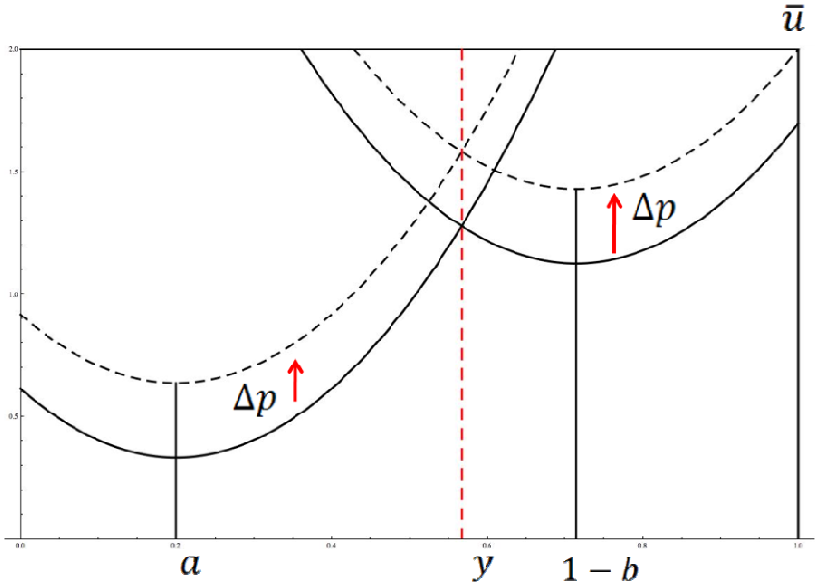

Figure 4

Lemma 1 is useful not only because it helps to characterize the Pareto optima of the game, but also because it suggests how publishers can easily improve a state: they just need to increase prices so that the lowest utility consumer is taken from positive utility to utility 0.

Definition 1. A state for which at least one consumer obtains utility 0 is called touch state. This consumer is called reservation price consumer and can be in 0, 1 or y.

Therefore at least one of the following conditions is to be satisfied by the touch state:

if reservation price consumer is at 0: $\bar{u} - ta^2 - p_1 = 0,$

if reservation price consumer is at y: $\bar{u} - t(y-a)^2 - p_1 = \bar{u} - t(1-y-b)^2 - p_2 = 0,$

if reservation price consumer is at 1: $\bar{u} - tb^2 - p_2 = 0.$

Pareto optima are to be found among the touch states. Lemma 2 helps to select some touch states.

Lemma 2. Any touch state for which only one consumer obtains utility 0 can be improved.

Proof: The proof consists in finding a state, which makes both the publishers better off by taking consumers in 0, y and 1 to utility 0.

To begin with, let us consider how each publisher tends to relocate when she takes into account that her demand is kept fixed by the other publisher. For example, when publisher 1's demand y is fixed, we can derive her profits from (8):

$$\pi_1 = (p_1 + k - c)y.$$

It is easy to see how the publisher will set the highest price compatible with the coverage of her demand; this optimum price will be:

$$p_1^* = \begin{cases} \bar{u} - t(y-a)^2, & \text{if } 0 \leq a < y/2 \\ \bar{u} - ta^2, & \text{if } y/2 \leq a \leq y \end{cases}$$

Her profits can be written as:

$$\pi_1 = \begin{cases} (\bar{u} - t(y-a)^2 + k - c)y, & \text{if } 0 \leq a < y/2 \\ (\bar{u} - ta^2 + k - c)y, & \text{if } y/2 \leq a \leq y \end{cases}$$

Taking the first derivative with respect to a, we find that

$$\begin{cases} \dfrac{\partial \pi_1}{\partial a} > 0, & \text{if } 0 \leq a < y/2 \\ \dfrac{\partial \pi_1}{\partial a} < 0, & \text{if } y/2 \leq a \leq y \end{cases} \tag{15}$$

The optimal location is therefore $a^* = y/2$. The same result can be similarly derived for publisher 2, her optimal location is $b^* = (1-y)/2$.

Expression (15) also states that each publisher tends to relocate to the middle of her demand when the other publisher simply accommodates this relocation in order to maintain the indifference at y. In fact both publishers can relocate following this tendency: the final state will imply that the demand is still at y while prices are higher and therefore payoffs are bigger. In this final state, consumers in 0, y and 1 are kept at utility 0. To conclude, a state in which consumers in 0, y and 1 are kept at utility 0 improves a state in which only one of these consumers is kept at utility 0 and the marginal consumer is at y.[17] QED.

Like Lemma 1, Lemma 2 helps to restrict the set of eligible Pareto optima. Lemma 2 also suggests how the publishers can relocate to improve a touch state. This can be understood as follows. If in a touch state both newspapers are located in the middle of their readerships, it is actually a complete touch state. Instead, if in a touch state a newspaper is not located in the middle of its demand, a relocation of this newspaper permits to both publishers to increase their price. Indeed, it can be noticed

[17] In the proof, the respective demands are kept constant to the touch state levels. This is only a fiction that helps to characterize Pareto optima: once they are fully characterized, different Pareto optima will imply different market shares and the publishers will be able to choose among them.

that a reservation price consumer is located at one of the endpoints of the demand of that newspaper (see Figure 5 for a visual representation). If the publisher of this newspaper shifts her location slightly towards the reservation price consumer, she can increase her price so that the reservation price consumer obtains again utility zero after the relocation. At the same time, the other publisher can increase her price so that the marginal consumer is kept at the same location: the demands are the same and the prices have increased. The procedure can be repeated until the newspaper converges towards the middle of its readership: in that case both the consumers on the endpoints are reservation price consumers. Since the utility of readers depends on prices and transportation costs (the reservation price is constant), decreasing the transportation costs of the readers permits to increase the price. In fact, each publisher relocates in the middle of her demand in order to reduce the transportation costs sustained by her readers; this permits to impose higher prices without losing consumers.

Thanks to Lemma 2 we can introduce the following definition.

Definition 2. A state for which consumers in 0, 1 and y are reservation price consumers is called complete touch state.

Therefore the following conditions are to be satisfied by the complete touch state:

$$
\begin{gathered}
\bar{\mathrm{u}} - \mathrm{ta}^2 - \mathrm{p}_1 = 0, \\
\bar{\mathrm{u}} - \mathrm{t}(\mathrm{y}-\mathrm{a})^2 - \mathrm{p}_1 = \bar{\mathrm{u}} - \mathrm{t}(1-\mathrm{y}-\mathrm{b})^2 - \mathrm{p}_2 = 0, \\
\bar{\mathrm{u}} - \mathrm{tb}^2 - \mathrm{p}_2 = 0.
\end{gathered}
$$

The strategies supporting this kind of state are the following

$$
\begin{gathered}
\text{publisher 1 applies: } \left(\frac{y}{2}\ ,\ \bar{u} - t\frac{y^2}{4}\right) \\
\text{publisher 2 applies: } \left(\frac{1-y}{2}\ ,\ \bar{u} - t\frac{(1-y)^2}{4}\right)
\end{gathered}
\tag{16}
$$

Publishers locate in the middle of their demand and set prices that keep consumers at the extremes of their demands at utility 0. Every complete touch state is characterized only by the marginal consumer y, $0 \le y \le 1$.

Lemma 2 and Definition 2 can be better understood in the light of the graphical representation in Figure 5. The dashed curves correspond to the new complete touch state.

To find the Pareto optima of the game, one last question has to be answered: can a complete touch state be improved? Lemma 3 identifies the Pareto optima of the game by answering this question.

Lemma 3. Every complete touch state is a Pareto optimum.

Proof. The proof consists in checking that starting from any point on the unit interval, any change in y does not increase the payoff of one publisher without decreasing the payoff of the other publisher.

First, profits can be written as:

$$\pi_1 = \left(\bar{u} - t\frac{y^2}{4} + k - c\right)y$$
$$\pi_2 = \left(\bar{u} - t\frac{(1-y)^2}{4} + k - c\right)(1-y)$$

Differentiating the profit functions with respect to y we find that, given the parameters compatible with the assumptions of this model:

$$\frac{\partial \pi_1}{\partial y} > 0 \ \ \forall y \in [0,1]$$
$$\frac{\partial \pi_2}{\partial y} < 0 \ \ \forall y \in [0,1]$$

Therefore in all cases the profit of one publisher cannot be increased without the profit of the other to decrease. QED.

Once Pareto optima are identified, publishers select the state, which implies a common price: in practice, the following expression has to be satisfied:

$$\bar{u} - t\frac{y^2}{4} = \bar{u} - t\frac{(1-y)^2}{4}$$

which in turn implies:

$$y = \frac{1}{2}.$$

Thus, the complete touch state with $y = 1/2$ forms the agreement between the publishers when they cooperate on both prices and locations. Besides, this agreement implies common prices and profits and splitting the market in the middle. These features suggest that it is actually plausible for the publishers to find an agreement on such a pair of strategies.

Taking (16) with $y = 1/2$, we can verify that publishers agree to locate at ¼ and at ¾ of the unit interval and to set a common price equal to $\bar{u} - \mathrm{t}/16$.

This leads to

Proposition 1. When collusion takes place on both the political orientation and the cover price of the newspaper, the collusive agreement implies medium political differentiation. Publishers choose $a = b =$ *¼ and set a common price* $p_i = \bar{u} - \frac{t}{16}$.

The corresponding collusive profits for each publisher will be:

$$\pi_{CLP} = \pi_1 = \pi_2 = \frac{\left(\bar{u} - \frac{t}{16} + k - c\right)}{2} \tag{17}$$

From now on, we will refer to expression (17) as collusion profits π_{CLP} when publishers are allowed to collude on prices and locations. The collusion outcome is graphically represented in Figure 6.

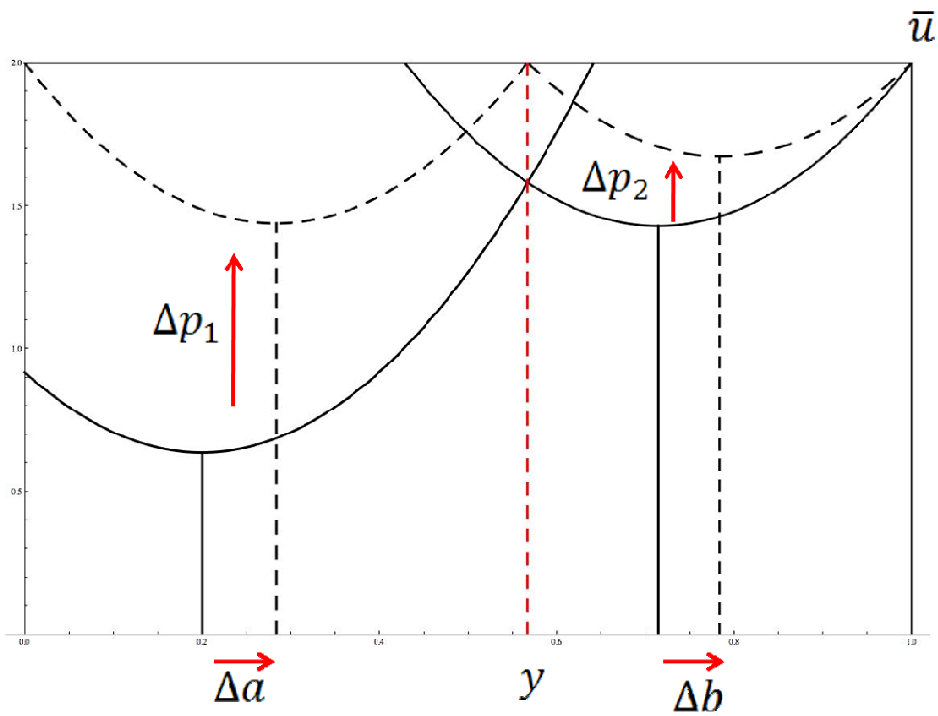

Figure 5

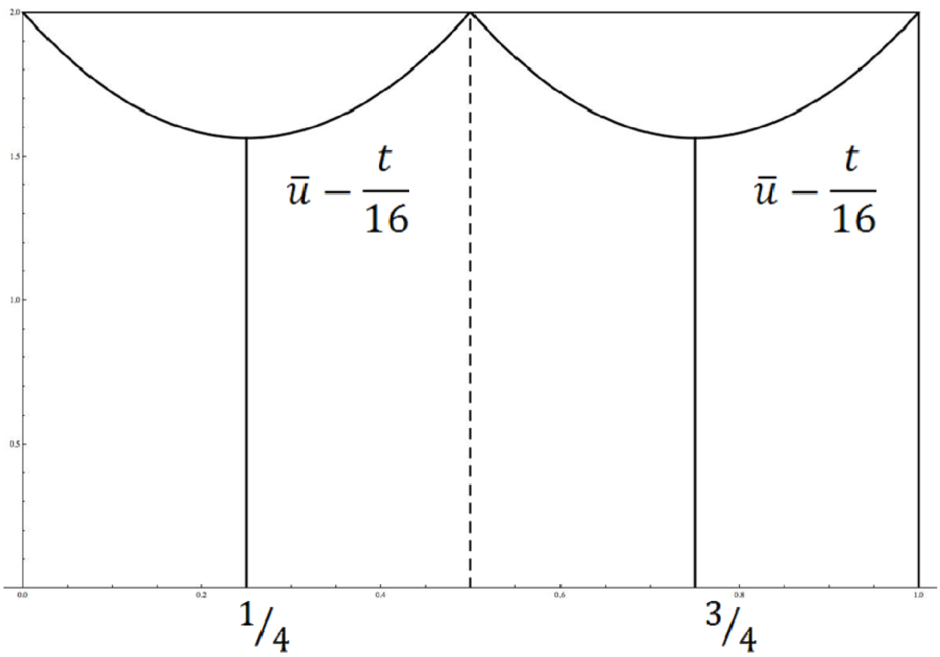

Figure 6

During the analysis, we have assumed the market to be covered. It is not difficult to check that.

Corollary 1. If the market is covered in competition, it will be covered also under collusion on both prices and political location.

Let us consider the profits made by a publisher located in ¼ who increases its price above $\bar{u} - t/16$. The readership associated with this new price cannot be identified through the normal functions in (4). Nevertheless, it is easy to note that the readership shrinks and is distributed for one half to the left of the location and for the other half to the right. The marginal consumer on the right endpoint obtains utility zero[18]; recalling (1), we find the location of this reader is

$$x_{mc} = \frac{1}{4} + \sqrt{\frac{\bar{u} - p}{t}}$$

Thus, it is easy to observe that the readership is now

$$n = 2\sqrt{\frac{\bar{u} - p}{t}}$$

Recalling the profit function in (8), we can observe that the new profit function is

[18] In this case, between buying a copy and not buying.

$$\pi = 2\sqrt{\frac{\bar{u}-p}{t}}(p+k-c) \tag{18}$$

We can check that the profits in (18) are never higher than the collusive profits in (17) when the parameters set is restricted as in (3) and $p > \bar{u} - t/16$.[19] The argument is parallel for the publisher located at ¾. Accordingly, if the market is covered in competition, it will be covered also under collusion. QED

3.1.2. Defection

Since punishment starts after any defection at any step of the game, each publisher has two alternatives to defect: first, she could select a political orientation different from the agreed one at the first step of a stage game and then compete in prices in the second step; second, she could stick with the agreed political orientation and defect on the cover price at the second step. Clearly she will select the defection that offers the higher payoff.

In order to find the optimal defection strategy it is necessary to find the optimal defection strategies of the two cases separately and then compare the payoffs. We should keep in mind that the non-deviant publisher applies the strategy

$$\left(\frac{1}{4}, \bar{u} - \frac{t}{16}\right) \tag{19}$$

Proposition 2. The optimal defection strategy consists of keeping political position unchanged and setting a price equal to:

$$p^* = \begin{cases} \tilde{p} = \dfrac{\bar{u}+c-k+\frac{7}{16}t}{2}, & if\, \bar{u} \leq c-k+\dfrac{25}{16}t \\ \bar{p} = \bar{u} - \dfrac{9}{16}t, & if\, \bar{u} > c-k+\dfrac{25}{16}t \end{cases} \tag{20}$$

Defection profits are then

$$\pi(\tilde{p}) = \frac{(\bar{u}+k-c+\frac{7}{16}t)^2}{4t} \tag{21a}$$

$$\pi(\bar{p}) = \bar{u} - \frac{9}{16}t + k - c \tag{22a}$$

[19] It should be remembered that the restrictions in (3) ensures the competitive equilibria exist once the market coverage condition is assumed.

Proof. See Appendix A.

3.2. Collusion on prices only

3.2.1. Collusive agreement

When publishers are allowed to collude on prices only, at the first step of the stage game, they locate independently while at the second step they set prices cooperatively. Therefore, publishers locate on the political opinion spectrum taking into account that they will apply an agreed rule on prices.

Like the collusive agreement on prices and political orientations, the collusive agreement on prices only has to be a state for which it is not possible to increase the payoff of one publisher without decreasing the payoff of the other publisher, i.e. a Pareto optimum. However, here publishers cannot choose a state within the entire set available, because that would mean they cooperatively select locations as well. Instead they choose a rule on prices that will take the pair of locations as given. Hence, the rule identifies a state for every possible pair of locations.

Thus we first identify the cooperative rule on prices for the second stage and then find the equilibrium behaviour for the first stage taking into account this rule.

The cooperative rule for prices is to be optimal and one could start by characterizing the Pareto optima. It is important to remember, however, that publishers are assumed to set a common price when more than one Pareto optimum is identified. Therefore, we can proceed in a different way: we first take the best common-price-rule into account and then check if it selects Pareto optima.

The best common-price-rule can be identified starting from Lemma 1: indeed, Lemma 1 shows how every state for which all the readers obtain a utility higher than 0 can be improved. This is valid also for the current case in which locations are held constant.[20] Consequently, given the pair of locations (a, b), publishers have to choose among all the touch states compatible with such pair of locations. This result is summed up in Lemma 4.

Lemma 4. Given a pair of locations (a, b) *publishers choose a state among the touch states.*

[20] In the proof of Lemma 1, locations are held constant.

Lemma 4 however does not fully characterize Pareto optima for the case of cooperative selection of prices only. Yet, it selects the only candidate states for Pareto optima: applying the same-price-rule to touch states we therefore obtain the best possible same-price-rule.

The common price has to be set according to the consumer who pays the maximal transportation cost in the market; indeed, this consumer will be the one facing utility 0 in the outcome. As stated above, her location can be 0, $(1+a-b)/2$ or 1. The most distant one among them from the opinion a or b of the newspaper bought by that consumer, characterizes the price.[21]

Definition 3. Any touch state $(a,\tilde{p}),(b,\tilde{p})$ *with common pricing is characterized as follows:*

$$\tilde{p}=\bar{u}-t\frac{(1-a-b)^2}{4} \quad \mathit{if} \begin{cases} \frac{1-a-b}{2}>a \\ \frac{1-a-b}{2}>b \end{cases} \tag{22}$$

$$\tilde{p}=\bar{u}-ta^2 \quad \mathit{if} \begin{cases} \frac{1-a-b}{2}\leq a \\ a\geq b \end{cases} \tag{23}$$

$$\tilde{p}=\bar{u}-tb^2 \quad \mathit{if} \begin{cases} \frac{1-a-b}{2}\leq b \\ b>a \end{cases} \tag{24}$$

Figure 7 summarizes case 3) of the pricing rule.

[21] Once the pair (a,b) is constant, publishers can set a common price, select a touch state, and cover the entire market at the same time, only by setting the price according to the largest values among a,b, and $(1-a-b)/2$, which are the distance from the newspaper bought by respectively consumer $0,1$, and $(1+a-b)/2=y$.

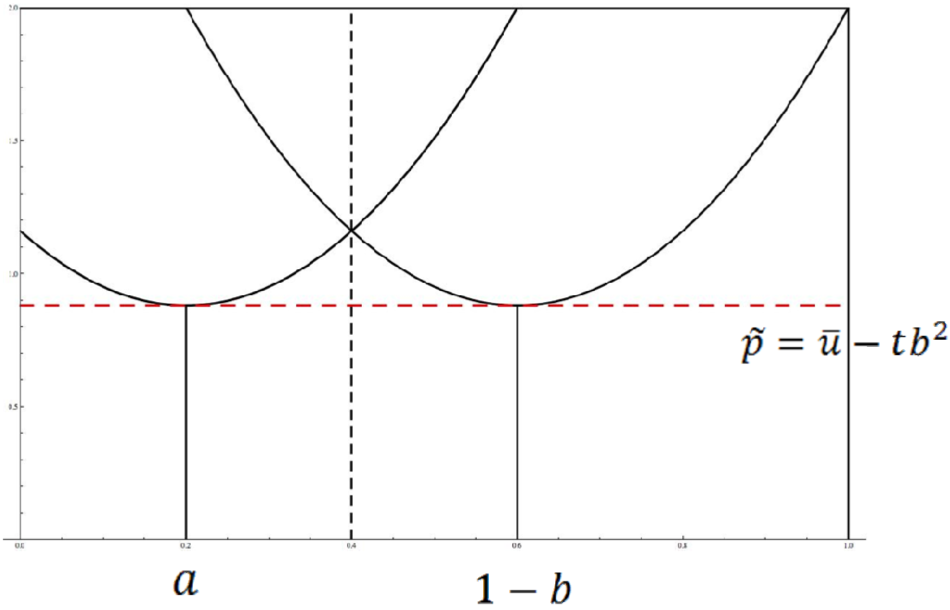

Figure 7

In brief, once (a, b) is given from the first step, publishers cooperatively select the touch state with common price compatible with such couple (a, b); it is easy to note that such a state is unique and therefore the pricing rule is well defined. What remains to be demonstrated is that this pricing rule, i.e. the best common-price-rule, always selects Pareto optima. In other words, it should not be possible to find a state for which a publisher increases her profits without the profits of the other to decrease starting from the touch state selected and changing prices only. This result is derived in Proposition 3.

Lemma 5. The best common-price-rule is Pareto optimal.

Proof. See Appendix A.

We now have all the instruments to find the Nash equilibrium.

Proposition 3. When collusion takes place on the cover price of the newspaper only, the collusive agreement implies minimal political differentiation. Publishers choose $a = b = 1/2$ *and set a common price* $p_i = \bar{u} - \frac{t}{4}$.

The corresponding collusive profits for each publisher will be:

$$\pi_{CP} = \pi_1 = \pi_2 = \frac{\left(\bar{u} - \frac{t}{4} + k - c\right)}{2} \qquad (25)$$

Proof. See Appendix A.

The result is rather straightforward: once a rule on pricing has been agreed, publishers behave like the two firms of a differentiated duopoly with common price (the famous ice-cream sellers on a beach). The only difference is that this common price changes with the resulting pair of locations but the incentive to gain market share is very similar: the only possible Nash equilibrium consists in the pair of locations (½, ½).

From now on, we will refer to the expression in (25) as collusion profits π_{CP} when publishers are allowed to collude on prices only.

The equilibrium is represented in Figure 8. For the sake of simplicity, the reader market is split in the middle, i.e. newspaper 1 caters to readers to the left of ½ and newspaper 2 caters to readers to the right of ½.

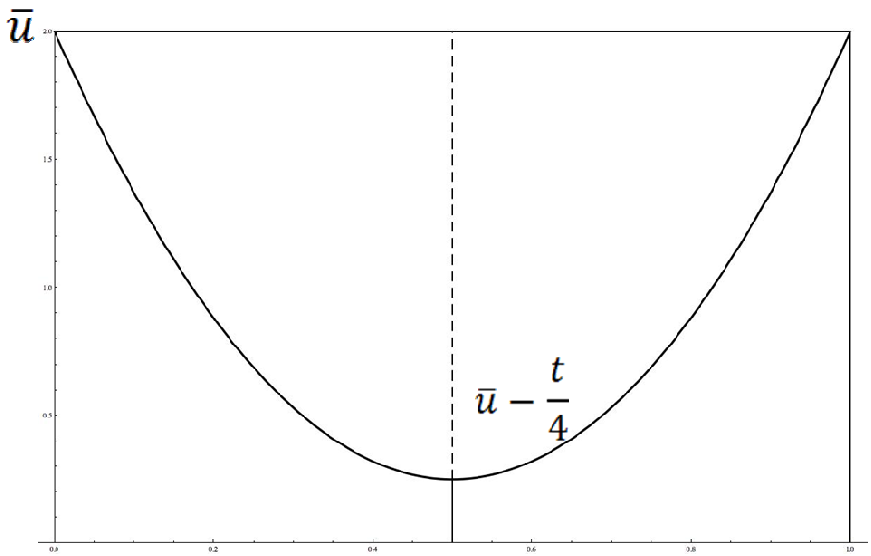

Figure 8

During the analysis, we assumed the market to be covered, exactly like in the case of collusion on both prices and political orientation.

It is not difficult to check

Corollary 2 - If the market is covered in competition, it will be covered also under collusion on prices.

The proof is similar to the one given in Paragraph 3.2.1.

Let us consider the joint profits made by publishers located at ½ and setting a common price above $\bar{u} - t/4$. The respective readerships associated with this new price cannot be identified through the functions in (4) and (5). Nevertheless, it is easy to note that publisher 1 sells to readers located to the left of ½ and publisher 2 sells to readers located to the right of ½. Naturally, the readerships are equal. The marginal consumer on the right endpoint obtains utility zero[22]; recalling (1), we find the location of this reader is

$$x_{mc} = \frac{1}{2} + \sqrt{\frac{\bar{u} - p}{t}}$$

Thus, it is easy to observe that the readership of each newspaper is now

$$n = \sqrt{\frac{\bar{u} - p}{t}}$$

Recalling the profit function in (8), we can observe that the new profit function is

$$\pi = \sqrt{\frac{\bar{u} - p}{t}}(p + k - c) \tag{26}$$

We can check that the profits in (26) are never higher than the collusive profits in (25) when the parameters set is restricted as in (3) and $p > \bar{u} - t/4$.[23]

Accordingly, if the market is covered in competition, it will be covered also under collusion. QED

3.2.2. Defection

At the first stage, the non-deviant publisher chooses location ½: whatever the location of the defecting publisher, the common price will be set at $\bar{u} - \frac{t}{4}$ since the most distant consumers will be ½ distant from the non defecting publisher. Therefore, the non-deviant publisher plays the equilibrium strategy in any case.

In order to find the optimal defection strategy we can use the procedure employed in the previous paragraph. The same reasoning on punishment and expectations is assumed to hold for this case as well. We should keep in mind that the non-deviant publisher applies the strategy:

[22] In this case, between buying a copy and not buying.
[23] It should be remembered that the restrictions in (3) allow the competitive equilibria to exist once the market coverage condition is assumed.

$$\left(\frac{1}{2}, \bar{u} - \frac{t}{4}\right) \tag{27}$$

Proposition 4. When the publishers collude on prices only, a publisher optimally defects by locating at the same point of the collusive strategy and applying a slightly lower price.

Proof. See Appendix A.

The optimal defection consists in undercutting. The location is held at ½ so that both the newspapers remain located at the same point: readers simply buy the less expensive newspaper.

Profits related to this strategy are:

$$\pi_{DP} = \bar{u} - \frac{t}{4} + k - c \tag{28}$$

The optimal defection strategy is displayed in Figure 9:

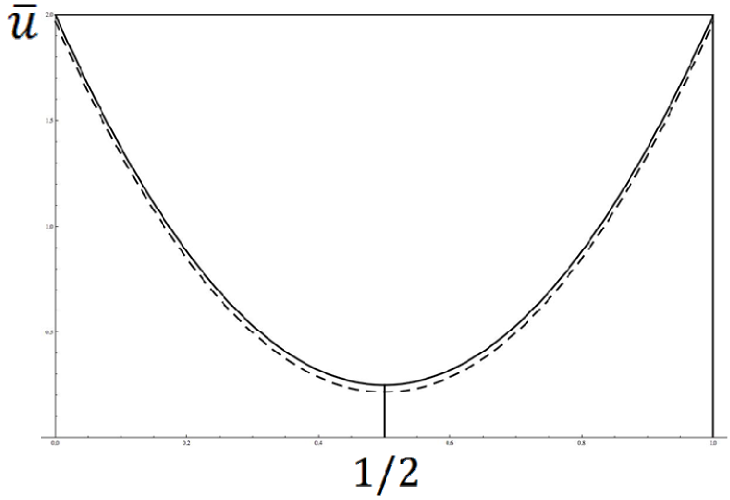

Figure 9

3.3. Welfare analysis

In normal one-sided markets, collusion usually implies a loss in both consumer welfare and in total welfare. Only firms gain.

In a two-sided market, we have two distinct groups of consumers (readers and advertisers in our case) and the firms. As discussed at the beginning, it is possible that consumers on one side of the market gain even if they pay a higher price. This is the case when the benefit those consumers enjoy from interacting with a higher number of people on the other side of the market is higher than the loss due to the price increase.

Given the assumption that readers are indifferent to advertising, which we have extensively justified above, the particular outcome just described could take place only on the advertiser side of the market.

Yet, in our model, as in Gabszewicz, Laussel and Sonnac (2001, 2002), advertisers always pay the monopoly price, even in competition. Thus, only the total mass of readers affects the utility of advertisers; however, since the market is always covered, the mass of readers is always 1 and the advertisers' surplus does not change throughout the analysis.[24]

As a consequence, we will focus on the component of total welfare which is equal to the sum of the readers' surplus plus the publishers' profits. For the sake of simplicity we will redefine this to be total welfare.

3.3.1. Readers' and publishers' surplus

The effects of collusive behaviour on readers and publishers are measured hereafter.

The readers' surplus can be defined as follows:

$$W_r = \int_0^y [\bar{u} - t(x-a)^2 - p_1]dx + \int_y^1 [\bar{u} - (1-x-b)^2 - p_2]dx \qquad (29)$$

Joint profits are:

$$W_p = \pi_1 + \pi_2 = (p_1 + k - c)y + (p_2 + k - c)(1-y) \qquad (30)$$

Total welfare can therefore be identified as:

$$W_r + W_p = W_t \qquad (31)$$

It is easy to derive readers', publishers' and total welfare for the several cases analysed in this work:

Competition - Minimal political differentiation equilibrium

Readers' W_r	$\bar{u} - \frac{t}{12}$

[24] An analytical reason for this can be found by observing the utility of any advertiser in (6).

Publishers' W_p	$k - c$
Total W_t	$\bar{u} - \frac{t}{12} + k - c$

Table 2

Competition - Maximal political differentiation equilibrium

Readers' W_r	$\bar{u} - \frac{2t}{3} + c - k$
Publishers' W_p	t
Total W_t	$\bar{u} + \frac{t}{3} + c - k$

Table 3

Collusion on prices and locations (medium political differentiation equilibrium)

Readers' W_r	$\frac{t}{24}$
Publishers' W_p	$\bar{u} - \frac{t}{16} + k - c$
Total W_t	$\bar{u} - \frac{t}{48} + k - c$

Table 4

Collusion on prices only (minimal political differentiation equilibrium)

Readers' W_r	$\frac{t}{6}$
Publishers' W_p	$\bar{u} - \frac{t}{4} + k - c$
Total W_t	$\bar{u} - \frac{t}{12} + k - c$

Table 5

3.3.2. Welfare implications

Using the tables above, several comparisons can be made. They will be analysed firstly from a consumer welfare perspective and secondly from a total welfare perspective.

However, before starting to compare the results above, it is worth noticing that in this framework prices are never so high that they prevent any consumer from buying a newspaper, or to put it simply, the market is always covered. Consequently, no possible increase in prices can reduce the total demand of newspapers, meaning that any shift in prices only implies a redistribution of surplus between readers and publishers. Nevertheless, shifts in prices are often associated with shifts in the political orientations of newspapers, with positive or negative effects on each reader's utility and in turn on readers' surplus.

From a reader's perspective, the competitive outcomes outperform both collusive outcomes; moreover, collusion on prices only is better than collusion on both prices and locations. For the subset of the parameters for which both competitive equilibria are sustained, the equilibrium with minimal differentiation is better than the equilibrium with maximal differentiation. Thus no gain in the readers' welfare due to relocation of newspapers in the political spectrum is large enough to offset the price increase. These results confirm the idea that the more competition there is in the market, the better off readers are.

Therefore, in our model, the two-sided nature of the market, albeit there, is such that it makes no exception to the general rule that competition is good for consumers.

From a total welfare perspective, results are not as clear as from a readers' perspective. It is easy to check that collusion on prices and political orientations outperforms collusion on prices only.

Compared to the minimal differentiation equilibrium, collusion outcomes show interesting results: the collusion-on-prices-only outcome provides the same welfare while the collusion-on-everything outcome does even better. This means that the gains provided to the publishers by collusive agreements either exceed or just offset the losses in the readers' surplus. Indeed:

$$W_{t(Collusion\ on\ prices\ only)} = \bar{u} - \frac{t}{12} + k - c = W_{t(Min\ diff\ eqilibrium)}$$

$$W_{t(Collusion\ on\ everything)} = \bar{u} - \frac{t}{48} + k - c > \bar{u} - \frac{t}{12} + k - c = W_{t(Min\ diff\ eqilibrium)}$$

These results represent an exception to the acknowledged fact that collusion brings about allocative efficiency losses. This particular effect is due to high competition in the minimal differentiation equilibrium and to the fact that once the market is covered in competition, it will be covered in collusion as well.

On the contrary, considering the maximal differentiation equilibrium, comparisons with the collusive agreements show that the former can outperform as well as underperform the latter, depending on the parameters of the game. In detail, total welfare associated with the collusion-on-everything outcome exceeds total welfare associated with maximal differentiation equilibrium if

$$c + \frac{17}{96}t < k \leq \mathrm{c} + \frac{t}{2}$$

Similarly, total welfare associated with the collusion-on-prices-only outcome exceeds total welfare associated with the maximal differentiation equilibrium if

$$c + \frac{5}{24}t < k \leq c + \frac{t}{2}$$

It is worth observing that each of these two new parameters sets splits the parameters set $k \leq \mathrm{c} + t/2$ for which the maximal differentiation equilibrium is sustained. The situation is summarized in Figure 10: the black line represents $k = \mathrm{c} + t/2$, the red line represents $k = c + \frac{17}{96}t$, and the blue line represents $k = c + \frac{5}{24}t$, with $c = 1$. Thus, the red and blue lines delimit the sets of parameters for which total welfare is higher under collusion or under competition.

In conclusion, when the competitive outcome is the minimal differentiation equilibrium, i.e. when the advertising market is very large and the per-reader advertising revenues are high:

- a collusive agreement on prices and locations provides a perfect trade-off between consumers' welfare and total welfare, i.e. consumers' welfare decreases while total welfare increases;

- a collusive agreement on prices only brings about no consequences for total welfare but harms consumers.

In the latter case, prices go up while locations remain constant, so only a redistribution of surplus takes place. Instead in the former case, prices go up but locations improve from the point of view of readers. Indeed, it is easy to note that the new location pair minimizes the sum of the transportation costs sustained by all readers. Furthermore, the pair of collusive strategies permits the publishers to make the most from the market and to maximize their joint profits.

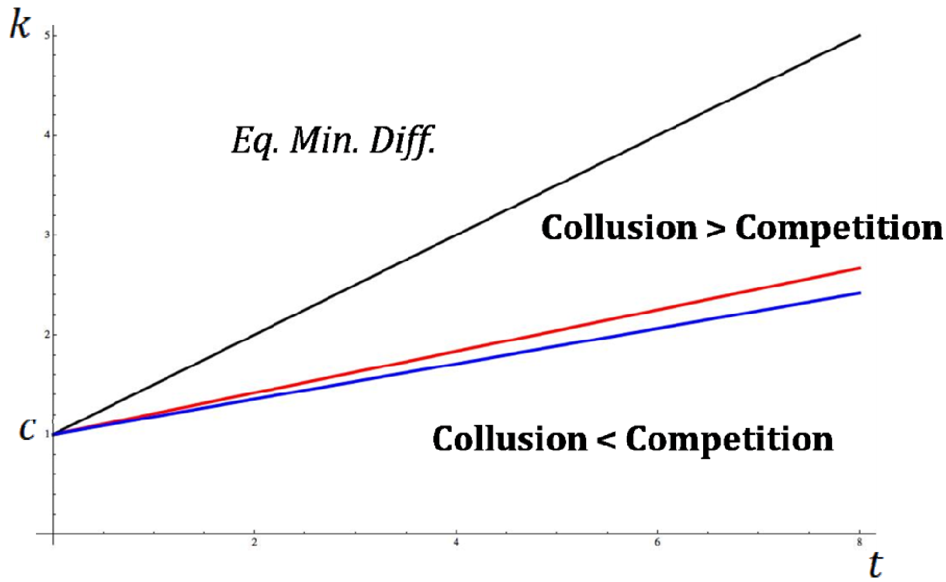

Figure 10 Welfare Implications of Collusion with Maximal Differentiation Equilibrium

Similarly, when the competitive outcome is the maximal differentiation equilibrium, i.e. when the advertising market is small and per-reader advertising revenues are low:

- both forms of collusion harm consumers;

- but total welfare can increase or decrease, depending on the parameters; in particular, whatever the form of collusion, total welfare is likely to decrease the lower the ratio of the advertising market parameter k to the political sensitiveness of the readers t.

3.4. Incentives to collude

We now recover the critical discount factor for each type of collusion and each of the stage game equilibria, which would be used as a punishment in case of defection.

It is then possible to analyse the change in the incentives to collude by differentiating each discount factor with respect to each parameter. Any change in a parameter which decreases a critical discount factor, enlarges the set of discount factors supporting collusion and therefore makes collusion more likely.

3.4.1. Collusion on prices only

The critical discount factors when publishers collude on prices only are

Punishment triggered	Critical discount factor
Minimal differentiation equilibrium $k \geq c + 25t/72$	$\hat{\delta}_{\min P}$
Maximal differentiation equilibrium $k \leq \mathrm{c} + \mathrm{t}/2$	$\hat{\delta}_{\max P}$

Table 6

where, recalling that $\hat{\delta} = \frac{(\pi_D - \pi_C)}{(\pi_D - \pi_N)}$ and substituting the relevant expressions for profits as a function of the parameters,

$$\hat{\delta}_{\max P} = \frac{4c - 4k + t - 4\bar{u}}{4c - 4k + 2t - 8\bar{u}}$$

$$\hat{\delta}_{\min P} = \frac{4c - 4k + t - 4\bar{u}}{8c - 8k + 6t - 8\bar{u}}$$

Below, we report the effect of an increase in the exogenous parameters on the critical discount factor. Clearly, a higher discount factor makes collusion less likely while a lower one makes collusion more likely.

Parameter	**Punishment triggered**	
	Minimal differentiation	**Maximal differentiation**
Political sensitiveness (t)	positive	Positive

Advertising market dimension (k)	positive	Negative
Marginal cost (c)	negative	Positive
Reservation price ($\bar{u}$)	negative	Negative

Table 7

The results shown in Table 7 can be summarized and explained as follows.

Political sensitiveness. As readers become more sensitive to political messages, i.e. the transportation cost increases, collusion among publishers is less likely. The reason for this is not straightforward: defection as well as collusion profits decrease, but the former effect outweighs the latter so that the one-time gain from defection decreases. However, punishment profits are either increasing or constant in the political sensitiveness: future losses due to current defection are thus enlarged. This effect cancels out the previous one.

The intuition is that the more readers are politically conscious, the easier it is for newspapers to find their own readership: in fact readers will be more attached to their "closer" newspaper. Indeed the equilibrium of maximal differentiation is sustained by high transportation parameters, everything else being equal: readers are easier to be targeted and competition can be relaxed by locating at the endpoints of the unit interval. Therefore, the newspapers do not need to coordinate decisions to cater to different segments of the market.

Reservation price. Collusion is in general easier to sustain if the reservation price increases. Two countervailing effects take place: on one hand, defection profits increase more than the collusion profits, exactly like in the case of the transportation cost; on the other hand the punishment profits are steady in respect to the reservation price while the collusion profits increase. The second effect offsets the first one. In fact, this confirms the idea that when two firms collude they can exploit the market more than they could do competing. An increase in the reservation price shifts the demand outward and allows publishers to gain more from collusion.[25]

Advertising market dimension. Interestingly the effect of a larger advertising market (and thus higher revenues per reader) on collusion strongly depends on the punishment triggered and, in turn, on the parameters. This means that a shift in the parameter can foster as well as discourage collusive behaviour depending on the starting value of the parameter itself.

In order to obtain a complete picture of how the incentives to collude depend on the advertising parameter k, a graphical representation of the results of Table 7 is provided below in Figure 11. The graph displays the critical discount factor (on the vertical axis) as a function of the dimension of the advertising market (on the horizontal axis).[26] Note that, given the reservation price, the marginal cost of the newspaper, and the political sensitiveness, the advertising market dimension k can take values above k_0 only, as also the conditions in (3) need to be satisfied.[27] For low values of k, $k_0 \leq k < k_1$, only the equilibrium of maximal differentiation is sustainable in one-shot competition. For intermediate values of k, $k_1 \leq k < k_2$, both equilibria are sustainable. For high values of $k, k \geq k_2$, only the equilibrium of minimal differentiation is sustainable.[28] The green curve shows the critical discount factor when the punishment triggered is the maximal differentiation equilibrium ($\hat{\delta}_{2P}$,); instead, the red curve shows the critical discount factor when the punishment triggered is the minimal differentiation equilibrium ($\hat{\delta}_{1P}$).

[25] Note that here the effect is reinforced by the assumption that the market is always covered: publishers can cooperatively set prices as functions of the reservation price itself (see the prices under collusion at (17) and (25)) while, when they are competing, they simply guarantee that the prices are not too high so that everyone is buying a newspaper.

[26] The chosen values for the other parameters are: $\bar{u} = 2,\ c = 2,\ t = 1$, which satisfy the assumptions of the model.

[27] These conditions allow the market to be covered in the competitive outcomes of the stage game.

[28] $k_0 = \frac{5}{4}, k_1 = \frac{169}{72}, k_2 = \frac{5}{2}$.

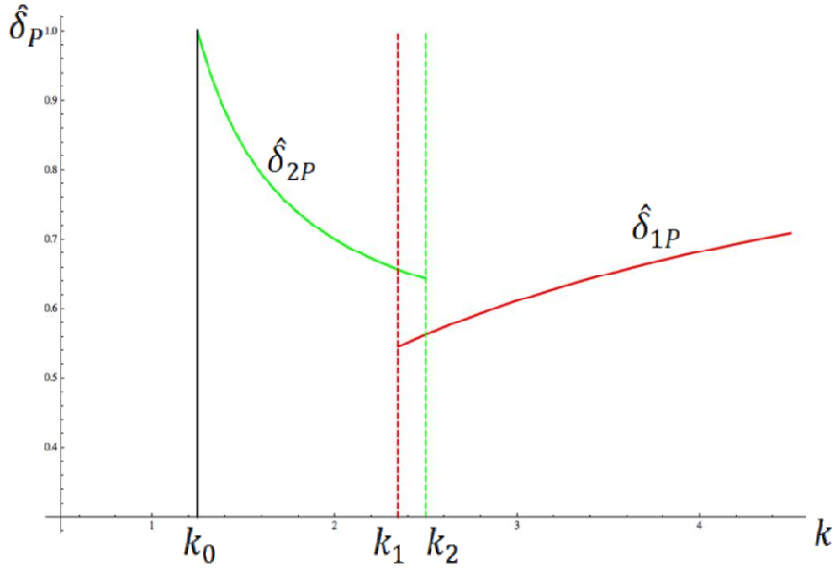

Figure 11

As we can observe, the critical discount factor decreases as long as the starting value is low, i.e. when the punishment triggered can only be the maximal differentiation equilibrium; it increases from intermediate values on, i.e. when the punishment triggered can only be the minimal differentiation equilibrium; not surprisingly, for intermediate values, i.e. when both equilibria are sustainable, the effects depend on the punishment chosen. To sum up, the relative magnitude of k will determine which punishment is triggered and which is therefore the effect of a higher k.

When k is high enough, so that the punishment is the minimal differentiation equilibrium, any further increase in the advertising market increases the critical discount factor and makes collusion less likely. First, it should be noticed that collusion, defection and punishment profits are all increasing functions of k; in fact, defection profits increase more than collusion profits so that the one-time gain increases; collusion profits and punishment profits increase in the same way so that future losses will be steady. Thus, collusion is less feasible when k grows. An explanation of this can be offered by noting that the advertising parameter represents the sum paid to each publisher for any copy sold: the market is split in the middle when publishers collude as well as when they compete but not when one of them defects; thus, if k rises, defecting becomes more profitable than colluding and than competing because the deviant has more readers than in the other cases. Thus, the trend exhibited for high levels of k

seems to be explained by the fact that each publisher gains more and more for any additional reader she can cater to by defecting; this makes defection more likely.

Once could argue that the same reasoning applies when the punishment triggered is a maximal differentiation equilibrium; indeed, defection and collusion profits are identical to the previous case; hence, gaining market share in the defection turn is more profitable when the advertising market dimension is higher. Nevertheless, as shown in the graph, when the one-stage equilibrium is the maximal differentiation one, punishment profits do not depend on the dimension of the advertising market. This may sound surprising. However, recall from (11) that competitive prices are in that case:

$$p^* = t + c - k$$

So that the revenue per reader k is entirely *passed on* to readers in form of a discount on the cover price: the publishers internalize the indirect network effects running from the advertisers to the readers; they bridge the two sides perfectly and make no money on it. Indeed, the respective profits as in (12) are:

$$\pi_{N2} = \frac{t}{2}$$

Conversely, collusion prices do not depend on the advertising market dimension, while the profits do (see (17) and (25)). Thus, when the publishers collude, they do not subsidise the readers with the advertising receipts they earn. When passing from competition to collusion, the publishers pass from a situation in which the advertising receipts are totally passed on to readers to a situation in which the advertising receipts are totally retained. Therefore, if the advertising revenues per reader increase, collusion becomes more and more desirable and therefore more likely.

Marginal cost of a copy. The analysis of the change in the collusive incentives due to a change in the marginal cost is in fact symmetric to the previous one. The reason is that, although marginal cost and the dimension of the advertising market are conceptually different, due to the assumptions about the advertising market, they enter the profit function in (8) in additive way. The only difference is the sign: advertising represents a source of revenues while the unit cost is a burden to sustain. In addition both parameters are *per copy*.

First and foremost it is true that, as with advertising, the effect of the marginal cost on collusion strongly depends on the punishment triggered and, in turn, on the parameters of the game.

The graph in Figure 12 shows the critical discount factor (on the vertical axis) as a function of the per copy cost of the newspapers (on the horizontal axis).[29] Given the reservation parameter, the advertising market dimension, and the transportation cost, the unit cost c can have values only below c_2 as the conditions in (3) need to be satisfied. The admissible values are therefore $0 < c < c_2$. For low values of c, $0 < c < c_0$, only the equilibrium of minimal differentiation is sustainable in one-shot competition. For intermediate values of c, $c_0 < c < c_1$, both equilibria are sustainable. For relatively high values of c, $c_1 \leq c \leq c_2$, only the equilibrium of maximal differentiation is sustainable.[30] The green curve shows the critical discount factor when the punishment triggered is the maximal differentiation equilibrium ($\hat{\delta}_{2P}$,); instead, the red curve shows the critical discount factor when the punishment triggered is the minimal differentiation equilibrium ($\hat{\delta}_{1P}$,).

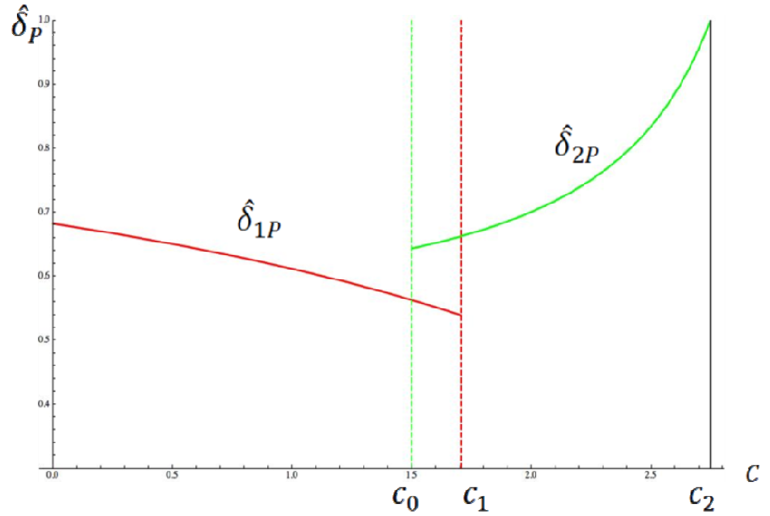

Figure 12

The critical discount factor decreases as long as the starting value is low, i.e. when the punishment triggered can only be the minimal differentiation equilibrium; it increases from intermediate values on, i.e. when the punishment triggered can only be

[29] As before, the values of the other parameters are: $\bar{u} = 2$, $k = 2$, $t = 1$, which are consistent with the assumptions of the model.

[30] Note that $c_0 = \frac{3}{2}, c_1 = \frac{123}{72}, c_2 = \frac{11}{4}$.

the maximal differentiation equilibrium; not surprisingly, for intermediate values, i.e. when both equilibria are sustainable, the effects depend on the punishment chosen.

Therefore, once again, the relative magnitude of c will determine which punishment is triggered and therefore which is the effect of a further increase in c on the likelihood of collusion. An increase in the unit cost facilitates collusion when the punishment triggered is the minimal differentiation equilibrium but discourages it when the punishment triggered is the maximal differentiation equilibrium.

When the punishment is the maximal differentiation equilibrium, collusion and defection profits are both decreasing functions of c; in fact, defection profits decrease more than collusion profits so that the one-time gain decreases. However, punishment profits are constant with respect to the unit cost: hence, the future loss decreases as well. The second effect offsets the first, and the overall impact is to discourage collusion. From (11), it is easy to check that the equilibrium cover price in one-shot competition is:

$$p^* = t + c - k$$

Indirectly, the readers pay the whole production cost. On the other hand, when the publishers collude, they bear the production costs themselves and make the readers pay a price based on their reservation price. Accordingly, the publishers find collusion relatively less desirable in respect to competition when the unit cost increases.

When instead the punishment triggered is the minimal differentiation equilibrium, an increase in the unit cost facilitates collusion. Collusion and defection profits decrease but the latter decrease more than the former; thus, the one-time gain from defection drops. Nevertheless, as the equilibrium cover prices are zero and all production costs are sustained by publishers, the punishment profits decrease by the same amount as the collusion profits.[31] As a result, future losses are constant as unit costs change. To conclude, a marginal cost increase facilitates collusion because it makes defection less profitable with respect to collusion; in other words, coordination is easier.

[31] One could easily note that the market is split in the middle in both cases and therefore the production costs are the same in the two cases.

3.4.2. Collusion on both prices and political position

The critical discount factors when publishers collude on both prices and the political position are

Punishment triggered	Defection	Critical discount factor
Minimal differentiation equilibrium $k \geq c + 25t/72$	Defection type 1 $\bar{u} \geq c - k + 25t/16$	$\hat{\delta}_{\min 1LP}$
	Defection type 2 $\bar{u} < c - k + 25t/16$	$\hat{\delta}_{\min 2LP}$
Maximal differntiation equilibrium $k \leq \mathrm{c} + \mathrm{t}/2$	Defection type 1 $\bar{u} \geq c - k + 25t/16$	$\hat{\delta}_{\max 1LP}$
	Defection type 2 $\bar{u} < c - k + 25t/16$	$\hat{\delta}_{\max 2LP}$

Table 8

where, recalling that $\hat{\delta} = \frac{(\pi_D - \pi_C)}{(\pi_D - \pi_N)}$ and substituting the relevant expressions for profits as a function of the parameters,

$$\hat{\delta}_{\min 1LP} = \frac{c - k + \frac{17}{16}t - \bar{u}}{c - k + \frac{9}{8}t - 4\bar{u}}$$

$$\hat{\delta}_{\min 2LP} = \frac{c - k + \frac{\mathrm{t}}{16} - \bar{u} + \frac{t}{2}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}{c - k + \frac{t}{2}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}$$

$$\hat{\delta}_{\max 1LP} = \frac{1}{2}$$

$$\hat{\delta}_{\max 2LP} = \frac{c - k + \frac{\mathrm{t}}{16} - \bar{u} + \frac{t}{2}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}{-t + \frac{t}{2}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}$$

In this case the study of the effect of an increase in the exogenous parameters on the critical discount factors are much more complex. As an example, we thus report only the effect of an increase in the dimension of the advertising market, which is the parameter most directly related to the two-sided nature of the market.

		Effect on the critical discount factor
Minimal differentiation	**Defection type 1**	Always positive
	Defection type 2	• if $\boldsymbol{\bar{u}t} - \boldsymbol{ct} + \frac{7}{16}\boldsymbol{t^2} - \boldsymbol{1} > 0$, positive $\forall \boldsymbol{k}$ admissible • if $\boldsymbol{\bar{u}t} - \boldsymbol{ct} + \frac{7}{16}\boldsymbol{t^2} - \boldsymbol{1} < 0$ $\begin{cases} \text{negative when} \quad \boldsymbol{c} + \frac{\mathbf{25}}{\mathbf{72}}\boldsymbol{t} < k < c + \frac{\mathbf{1}}{\boldsymbol{t}} - \frac{\mathbf{7}}{\mathbf{16}}\boldsymbol{t} - \boldsymbol{\bar{u}}, \\ \text{positive when} \quad \boldsymbol{k} \geq \boldsymbol{c} + \frac{\mathbf{1}}{\boldsymbol{t}} - \frac{\mathbf{7}}{\mathbf{16}}\boldsymbol{t} - \boldsymbol{\bar{u}} \end{cases}$
Maximal differentiation	**Defection type 1**	None
	Defection type 2	see next paragraph

Table 1

The critical discount factor associated with maximal differentiation and defection type 2 in the collusion-on-everything environment is the following:

$$f(k) = \frac{c - k + \frac{t}{16} - \bar{u} + \frac{t}{2}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}{-t + \frac{t}{2}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}$$

This is a complicated function of k: it can be upward or downward sloping in the interval in which the analysis applies to[32] depending on the other parameters

Though in theory possible, the conditions apply for very small ranges of the parameters. Accordingly, we decided not to take the study of the critical discount factor for this second type of defection (it would ambiguous and complicated to interpret anyway).

Observing Table 5, we can see that in case the minimal differentiation equilibrium prevails, collusion is not favoured by the advertising market dimension, though the effect is controversial. Nonetheless, we can think the relationship is negative for most of the times since the subset of parameters for which collusion is actually favoured by the advertising market dimension selects intermediate values of k; what we are more interested in is the case in which k is high.

About the maximal differentiation case we cannot say a lot: advertising does not affect collusion when the deviant would use a defection of type 1.

3.5. Relocation costs and the form of collusion

A possible weakness of the discussion above is represented by the assumption that publishers can change the political orientation of their newspapers at any repetition of the stage game without any time constraint and without incurring any cost. However, in reality, changing political orientations may in fact be more complicated than changing prices. A publisher wishing to move the newspaper from left to right might for instance need to substitute part of its left-wing journalists who are not ready to change their articles' line.

To obviate the issue, let us assume that every time a publisher changes her political orientation, she has to bear a non-negative sunk cost F, no matter how much it is changed or the direction of the change. This cost can well represent the expenses linked to the recruitment of new signatures and to marketing campaigns. The higher the sunk cost, the more difficult will be changing the political orientation with respect to

[32] This interval is $\frac{5}{4}t + c - \bar{u} < k < \frac{25}{16}t + c - \bar{u}$,

prices. Since it is a cost associated with relocation of the newspaper on the political line, we hereafter call it *relocation cost*.

First of all, let us consider two extreme cases: in the first one the relocation cost is zero, in the second it is infinite. In fact the first case is represented by the model studied until now; instead, the second corresponds to a repeated Gabzsewicz, Laussel and Sonnac (2002) where locations are selected at the beginning of time, once forever. In this sense it is easy to note that the form of collusion on prices is the only one remaining in the second case (see Friedman and Thisse, 1979): no agreement on political orientations could arise since a defection at the very first step could not be punished. Furthermore, it can be easily derived that the final outcome and the defection strategy found for collusion on prices only and $F = 0$ can apply in this new case with F infinite too.

We should now consider the intermediate and more general case in which F is finite and positive. Both forms of collusion can be sustained in theory and do not change their characteristics in terms of outcomes. However, the sustainability of collusion is now altered by the relocation cost: in some cases, punishment strategy implies relocation and therefore leads to different payoffs. Accordingly, sustainability of collusion should be analyzed looking at different critical discount factor.

As usual, this critical discount factor depends on the kind of collusion and on the prevailing equilibrium. If locations change between collusion, defection and/or punishment, publishers pay a relocation cost. It is easy to notice that in one case, locations do not change: it is when publishers collude on prices only and the prevailing equilibrium is the minimal differentiation one. In all the other cases, locations change in the first punishment turn. The critical discount factor can be derived starting from the streams of payoffs associated with defection and cooperation. We found that cooperating is more likely if :

$$F\delta^2 + (\pi_D - \pi_N + F)\delta + \pi_C - \pi_D \geq 0$$

$$\hat{\delta} = \frac{(\pi_D - \pi_N + F) - \sqrt{F^2 + (\pi_D - \pi_N)^2 + F[4\pi_C - 2(\pi_D + \pi_N)]}}{2F}$$

$$\pi_D - \pi_N > F$$

The last condition guarantees that the relocation cost is not so high to cover the benefits of defecting. If it actually does, the case is similar to the one with infinite relocation cost.

Unsurprisingly, it can be shown that the critical discount factor decreases in F so that the bigger the F, the more likely is collusion, no matter which is the type of collusion. Accordingly, as long as F is less than $\pi_D - \pi_N$, both kinds of collusion make sense, with collusion being more likely as F grows. When F approaches $\pi_D - \pi_N$, only collusion on prices remains a possibility.

4. Conclusions

We analysed a newspaper market where two editors compete for advertising as well as for readers. They first choose the political position of their newspaper, then set cover prices and advertising tariffs.

We built on the work of Gabszewicz, Laussel and Sonnac (2001, 2002), who show that advertising financing tends to reduce political differentiation among newspapers and can explain the ascent of the so-called *Pensée Unique.* This is more likely the larger the advertising market and therefore the larger the per reader revenues from advertising.

We took their model as the stage game of an infinitely repeated game, and investigated the incentives to collude using grimm trigger strategies and the properties of the collusive agreements in terms of welfare and pluralism.

As in Gabszewicz, Laussel and Sonnac (2001, 2002), newspapers are assumed to enjoy monopoly power over advertisers, who may multi-home, for access to their readers, who are instead assumed to single-home. It is further assumed that readers are not affected by the quantity of advertising on the newspaper. We justify this assumption referring to the empirical literature on newspaper markets and, more generally, media markets. In such a situation there is in fact no gain from collusion on the advertising market.

We thus analysed and compared two types of collusion: in the first, publishers cooperatively select both prices and political position; in the second, publishers cooperatively select prices only.

Whereas full collusion leads to intermediate product differentiation, collusion on prices only leads, as in Gabszewicz, Laussel and Sonnac (2001, 2002), to minimal product differentiation. However, in the latter case, differently from Gabszewicz, Laussel and Sonnac (2001, 2002), cover prices are positive and the minimal differentiation outcome does not depend on the size of the advertising market. So that collusion on prices reinforces the tendency towards a Pensée Unique discussed in Gabszewicz, Laussel and Sonnac (2001). Indeed, if advertising revenues are so low that maximal differentiation would be the competitive outcome, the higher they are the more likely is collusion. Yet we also showed that, whenever advertising revenues are so high that minimum differentiation arises as a competitive equilibrium, then the higher advertising revenues the less likely is collusion.

Our analysis shows that despite the two-sided nature of the market the more competition there is the better it is for consumers. In particular, competition yields higher welfare than collusion on prices only, which in turn outperforms collusion on both prices and the political position of newspapers. Note that this is true even if we assumed the market to be covered. Despite this assumption, no gain in the readers' welfare due to relocation of newspapers in the political spectrum is large enough to offset the price increase.

From a total welfare point of view instead, when the competitive outcome is the minimal differentiation equilibrium, i.e. when the advertising market is very large and the per-reader advertising revenues are high:

- a collusive agreement on prices and locations decreases consumers' welfare while increasing total welfare; the reason is that newspapers chose locations that minimize the sum of the political costs sustained by all readers but then extract all the extra surplus of readers through higher prices.

- a collusive agreement on prices only brings about no consequences for total welfare but harms consumers, because prices increase while locations remain constant, so only a redistribution of surplus takes place.

Similarly, when the competitive outcome is the maximal differentiation equilibrium, i.e. when the advertising market is small and per-reader advertising revenues are low:

- both forms of collusion harm consumers;

- but total welfare can increase or decrease, depending on the parameters; in particular, whatever the form of collusion, total welfare is likely to decrease the lower the ratio of the advertising market parameter k to the political sensitiveness of the readers t.

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Appendix A.

Proof of Proposition 2. The proof consists of threes different stages: a) finding the best defection strategy in case the location is shifted from ¼; b) finding the best defection strategy keeping location fixed at ¼; c) comparing the two defection strategies to find the overall optimal one.

a) When a publisher defects at the location step, the other publisher punishes from the subsequent step on; as a result, in the defection turn the deviant selects a location different from ¼ while the other one actually plays ¼ and they play a Nash equilibrium in prices given locations at the second step. If the Nash equilibria are unique, when choosing the location the deviant knows which Nash equilibrium will follow in the second step; in other words, she will select the preferred Nash equilibrium.

First of all, it is necessary to find the Nash equilibria given the pair of political orientations $(a, ¼)$. Following Gabsewicz, Laussel and Sonnac (2002) and starting from the profit function in (8), we can find the best prices of publisher 1 and 2 respectively:

$$p_1 = \max\left\{0, \frac{1}{2}(c - k + p_2 + \frac{9}{16}t - a^2 t)\right\} \tag{32}$$

$$p_2 = \max\left\{0, \frac{1}{2}(c - k + p_1 + \frac{15}{16}t - 2at + ta^2)\right\} \tag{33}$$

By substitution we can find Nash equilibria in prices. The non-negativity constraint on price levels leads us to four regions, as in Gabzewicz, Laussel and Sonnac (2002).

Region 1.

$$p_1 = c - k + \frac{11}{32}t - \frac{1}{3}at - \frac{1}{6}a^2 t \tag{34}$$

$$p_2 = c - k + \frac{13}{32}t - \frac{2}{3}at - \frac{1}{6}a^2 t \tag{35}$$

Region 2.

$$p_1 = \frac{1}{2}\left(c - k + \frac{9}{16}t - a^2 t\right) \tag{36}$$

$$p_2 = 0 \tag{37}$$

Region 3.

$$p_1 = 0 \tag{38}$$

$$p_2 = \frac{1}{2}\left(c - k + \frac{15}{16}t - 2at + a^2 t\right) \tag{39}$$

Region 4.

$$p_1 = 0 \tag{40}$$

$$p_2 = 0 \tag{41}$$

One should notice that newspaper demands in (4) and (5) and so the profit function in (8) assume that publisher 1 is located on the left hand side of publisher 2; accordingly, the four regions apply in $0 \leq a \leq \frac{3}{4}$. On the other hand, publisher 1 would never locate in the interval $\frac{3}{4} < a \leq 1$ since she would be located in the smaller segment of the political spectrum.

The four regions are admissible for the following sets of parameters:

- If $c > k$, only Region 1 is admissible.
- If $k > c$ and
  - $0 < t \leq \frac{32}{13}(k - c)$
    - ✓ only Region 4 is admissible in $[0, \frac{3}{4}]$
  - $\frac{32}{13}(k - c) < t \leq \frac{32}{11}(k - c)$
    - ✓ Region 3 is admissible for $0 \leq a < a_1$
    - ✓ Region 4 is admissible for $a_1 \leq a \leq 3/4$
  - $\frac{32}{11}(k - c) < t \leq 4(k - c)$
    - ✓ Region 1 is admissible for $0 \leq a < a_2$
    - ✓ Region 3 is admissible for $a_2 \leq a < a_1$
    - ✓ Region 4 is admissible for $a_1 \leq a < 3/4$
  - $t > 4(k - c)$

- ✓ Region 1 is admissible for $0 \leq a < a_1$
- ✓ Region 2 is admissible for $a_1 \leq a < a_2$
- ✓ Region 4 is admissible for $a_2 \leq a \leq 3/4$

Let us note that

$$a_1 = 2 - \frac{1}{4}\sqrt{96\frac{(k-c)}{t} + 25}$$

$$a_2 = -1 + \frac{1}{4}\sqrt{96\frac{(k-c)}{t} + 49}$$

Once the Nash equilibria are fully characterized, we can go backwards to the first step and find the best location possible. Therefore, we should find the maximum of the profit function for every possible set of parameters; first of all, it is useful to inspect the sign of the derivative of the profits with respect to location a in every possible region. We find that

In Region 1

$$\frac{\partial \pi}{\partial a} < 0 \qquad \text{when applicable}$$

In Region 2: this region is meaningful if $t > 4(k-c)$ and $a_1 \leq a < a_2$

If $4(k-c) \leq t < \frac{578}{99}(k-c)$, $\frac{\partial \pi}{\partial a} > 0 \;\; \forall a \in [a_1, a_2)$

If $\frac{578}{99}(k-c) \leq t < \frac{722}{177}(k-c)$, $\begin{cases} \frac{\partial \pi}{\partial a} > 0 \;\; \forall a \in [a_1, a_3) \\ \frac{\partial \pi}{\partial a} < 0 \;\; \forall a \in (a_3, a_2) \end{cases}$

$$\text{where } \; a_3 = \frac{1}{2} - \frac{1}{4\sqrt{3}}\sqrt{\frac{16(k-c)}{t} + 3}$$

If $t \geq \frac{722}{177}(k-c)$, $\frac{\partial \pi}{\partial a} < 0 \;\; \forall a \in [a_1, a_2)$

In Region 3

$$\frac{\partial \pi}{\partial a} > 0 \qquad \text{when applicable}$$

In Region 4

$$\frac{\partial \pi}{\partial a} > 0 \qquad \text{when applicable}$$

It is therefore easy to see that the critical points are $a = 0$, $a = 3/4$ and $a = a_3$. After calculating the profits made by the deviant publisher in the critical points and checking which of them is actually optimal in any possible subset of parameters, we can conclude that the optimal defection strategy consists of selecting

$$a = 0 \qquad \text{if } c > k \bigvee (k > c \bigwedge t > \frac{288}{5}(k-c))$$

$$a = \frac{3}{4} \qquad \text{if } k > c \bigwedge 0 < t \leq \frac{288}{5}(k-c)$$

at the first step of the turn and then applying the Nash equilibrium strategy in the prices step.

Let us name $\pi(a = 0)$ the profits made by the deviant in the defection turn when she selects $a = 0$ in the first step, and $\pi(a = 3/4)$ the profits made by the deviant in the defection turn when she selects $a = 3/4$ in the first step. These profits do not include the relocation cost F paid due to shifting location from ¼ to 0 or to ¾. In fact, the payoffs would be $\pi(a = 0) - F$ and $\pi(a = 3/4) - F$ respectively. However, we can pursue the demonstration as if there was no relocation cost. The reason will be clear shortly.

$$\pi(a = 0) = \frac{55}{384}t$$

$$\pi(a = 3/4) = \frac{3}{4}(k-c)$$

b) Optimal defection with location fixed at ¼ is easier to obtain since it simply consists of finding a defection price for the second step of the stage game. To sum up, the pair of locations is fixed at (1/4,1/4) and the non-deviant publisher applies a price equal to $\bar{u} - \frac{t}{16}$. The deviant publisher maximizes the profits in the price.

By maximising the profits in (8) with respect to p_1 with $a = \frac{1}{4}$, $b = \frac{1}{4}$ and $p_2 = \bar{u} - \frac{t}{16}$, we obtain that

$$1)\quad \frac{\partial \pi}{\partial p} > 0 \text{ for } p < \frac{1}{2}\left(\bar{u} + c - k + \frac{7}{16}t\right)$$

$$2)\quad \frac{\partial \pi}{\partial p} < 0 \text{ for } p > \frac{1}{2}\left(\bar{u} + c - k + \frac{7}{16}t\right)$$

$$3)\quad \frac{\partial \pi}{\partial p} = 0 \text{ for } p = \frac{1}{2}\left(\bar{u} + c - k + \frac{7}{16}t\right)$$

Therefore, the best price is

$$p_1^* = \frac{1}{2}\left(\bar{u} + c - k + \frac{7}{16}t\right) \tag{42}$$

Nevertheless, it should be noticed that this best price is meaningful as long as it implies a total demand lower than 1. When defecting, the deviant publisher gets part of the demand of the non-deviant publisher; if she gets all the demand of the other, her demand will be 1 and any smaller price would not imply more demand. It is easy to check that the threshold price is

$$p = \bar{u} - \frac{9}{16}t \tag{43}$$

for which the deviant's demand is exactly 1. There is no reason why the publisher should make any lower price; hence, if the price in (42) is smaller than the threshold price in (43), the best price is the threshold price itself. The following condition summarizes the fact and gives the expression of the optimal defection strategy for this case:

$$p_1^* = \frac{1}{2}\left(\bar{u} + c - k + \frac{7}{16}t\right) \quad \text{if} \quad \bar{u} \le c - k + \frac{25}{16}t \tag{44}$$

$$p_1^* = \bar{u} - \frac{9}{16} \quad \text{if} \quad \bar{u} > c - k + \frac{25}{16}t \tag{45}$$

Let us name profits made applying the best price in (44) $\pi(\tilde{p})$ and profits made with the best price in (45) $\pi(\bar{p})$.

$$\pi(\tilde{p}) = \frac{(\bar{u} + k - c + \frac{7}{16}t)^2}{4t}$$

$$\pi(\bar{p}) = \bar{u} - \frac{9}{16}t + k - c$$

c) We have now to compare profits made in the two alternative defections. In other words, we have to compare $\pi(\tilde{p})$ and $\pi(\bar{p})$ with $\pi(a = 0)$ and $\pi(a = 3/4)$.

It is easy to check that in every possible subset of parameters, $\pi(\tilde{p})$ and $\pi(\bar{p})$ are higher than $\pi(a = 0)$ and $\pi(a = 3/4)$ respectively.

Proof of Lemma 4. We can start analysing the second and third case of Definition 3. The demonstration is parallel and will be given for case 2 only. When this is the case, the consumer located at 0 is the reservation price consumer: for the market to be entirely covered in any different state with (a, b) given, $p_1 \leq \bar{u} - ta^2$. For any $p_1 < \bar{u} - ta^2$, the state can be improved for Proposition 1 (it is not a touch state anymore). Therefore, only p_2 can be modified: it cannot be decreased because with p_1 fixed at $\bar{u} - ta^2$ for the just mentioned reasons, this would lead to a decrease in publisher 1's profits; it can be increased on condition that publisher 2's profits increase. In fact, in this case publisher 1's profits would increase. With (a, b) given and $p_1 = \bar{u} - ta^2$, publisher 2's profits can be written as

$$\pi_2 = (p_2 + k - c)\left(\frac{1 + b - a}{2} + \frac{\bar{u} - ta^2 - p_2}{2t(1 - a - b)}\right).$$

Deriving such function on p_2, we find

$$\frac{\partial \pi_2}{\partial p_2} = \frac{c - k - 2p_2 + t - 2at - b^2 t + \bar{u}}{2t(1 - a - b)}$$

We can check that this derivative is negative for $p_2 = \bar{u} - ta^2$ and therefore increasing p_2 would lead to decrease publisher 2's profits. Consequently, no change in prices can make any publisher better off without making the other publisher worse off for case 2.

In case 1, the reservation price consumer is located in the middle between a and $1 - b$; this means that any different price pair has to set the consumer on y to be the reservation price consumer; otherwise, either the market is not covered, or the state is not a touch state and can be improved. The new price pair has so to satisfy the following

$$\begin{cases} p_1 = \bar{u} - t(y - a)^2 \\ p_2 = \bar{u} - t(1 - y - b)^2 \end{cases}$$

If you give y, the new price pair is given as above in (33). Accordingly, profits can be rewritten as

$$\begin{cases} \pi_1 = (\bar{u} - t(y-a)^2 + k - c)y \\ \pi_2 = (\bar{u} - t(1-y-b)^2 + k - c)(1-y) \end{cases}$$

Deriving such profits functions on y, we obtain:

$$\begin{cases} \dfrac{\partial \pi_1}{\partial y} = -c + k - a^2 t + u + 4aty - 3ty^2 \\ \dfrac{\partial \pi_2}{\partial y} = c - k + 3t - 4bt + b^2 t - u - 6ty + 4bty + 3ty^2 \end{cases}$$

We can check that with $y = \frac{1+a-b}{2}$

$$\begin{cases} \dfrac{\partial \pi_1}{\partial y} > 0 \\ \dfrac{\partial \pi_2}{\partial y} < 0 \end{cases}$$

Therefore, it is not possible to increase one's profits without decreasing the other one's profits. As a consequence, the best same-price-rule is optimal, i.e. selects Pareto optima only.

To conclude, the publishers agree to follow this pricing rule at any second stage every turn when they are allowed to collude on prices only. Going backward, at the first stage they take into account this rule in order to select the optimal location. As a result, we can investigate a Nash equilibrium outcome for the stage game in order to fully characterize the collusion outcome.

Without loss of generality, it can be assumed that $\leq 1 - \mathrm{b}$. This does not mean that, for example, publisher 1 cannot select a location on the right hand side of publisher 2's one, but only that when she does it, she can be thought to have changed his profits to that of publisher 2.

Before moving to equilibrium behaviour, it is important to note that each case identifies a different region on $[0,1] \times [0,1]$ when $\mathrm{a} \leq 1 - \mathrm{b}, (\mathrm{a}, \mathrm{b}) \in [0,1] \times [0,1]$.

To do this, we firstly derive the profit functions for every of the three regions identified by the three cases of Definition 3:

[33] In fact, the starting price pair verifies the condition and represents the case of $y = \frac{1+a-b}{2}$.

$$\text{region 1)} \quad \begin{aligned} \pi_1 &= \left(\bar{u} - \frac{t}{4}(1-a-b)^2 + k - c\right)\left(\frac{1+a-b}{2}\right) \\ \pi_2 &= \left(\bar{u} - \frac{t}{4}(1-a-b)^2 + k - c\right)\left(\frac{1-a+b}{2}\right) \end{aligned} \tag{44}$$

$$\text{region 2)} \quad \begin{aligned} \pi_1 &= (\bar{u} - ta^2 + k - c)\left(\frac{1+a-b}{2}\right) \\ \pi_2 &= (\bar{u} - ta^2 + k - c)\left(\frac{1-a+b}{2}\right) \end{aligned} \tag{45}$$

$$\text{region 3)} \quad \begin{aligned} \pi_1 &= (\bar{u} - tb^2 + k - c)\left(\frac{1+a-b}{2}\right) \\ \pi_2 &= (\bar{u} - tb^2 + k - c)\left(\frac{1-a+b}{2}\right) \end{aligned} \tag{46}$$

Given this set of profits functions depending on locations only, equilibrium behaviour is easily derived for publisher 1. Given symmetry, the same result holds for publisher 2. It can be shown that given b, deriving the profits functions in (44), (45) and (46) on a and comparing the results with the set of parameters compatible with the model and with the respective set of variables represented by each region as in (22), (23) and (24) results in:

$$\frac{\partial \pi_1}{\partial a} > 0 \quad \text{and} \quad \frac{\partial \pi_2}{\partial b} > 0, \ \ \forall (a, b) \in [0,1] \times [0,1], \text{with } a \leq 1 - b \tag{47}$$

Therefore the rule only selects Pareto optima. QED.

Proof of Proposition 3. The expression in (27) implies that publisher 1, given an expectation of publisher 2's location b, whatever the region is, tends to locate at $1 - b$ because it is the maximum value of a compatible with the limit $a \leq 1 - b$; the same can be said for publisher 2: she tends to locate at $1 - a$. This behaviour derives from the fact that setting a common price at the second stage of the game implies the split of the demand between a and $1 - b$ in two equal parts: moving towards the other's location, each publisher gains demand; this is a common outcome when the prices are fixed (beach). This happens in every region and in particular, between the regions so that publishers face the same incentives moving their location towards the one of the other publisher.

Every pair of locations $(a, 1-a)$ (or $(1-b, b)$, that is the same), is a Nash equilibrium candidate. Only $(1/2,1/2)$ is left when we consider that when the publisher 1 locate in $1 - b$, she faces the incentive to jump on the other side of the $1 - b$ if $1 - b < 1/2$; indeed, given the demand function when the publishers locate at the same point of the unit interval, publisher 1 takes the demand on the left hand side of the point while publisher 2 takes the demand on the right hand side. The symmetric incentive is faced by publisher 2 when $1 - a < 1/2$. Therefore, the publishers can expect the exact location of the other only locating in the middle of the unit interval. QED.

Proof of Proposition 4. The best reply to $(1/2, \bar{u} - t/4)$ can be found taking the profit function of publisher 1 when publisher 2 plays such strategy, deriving firstly on the price and then on the location taking into account the optimal price.

First, the profit function from (8) can be found for $(b, p_2) = \left(\frac{1}{2}, \bar{u} - \frac{t}{4}\right)$; if we take

$$\tilde{n} = \frac{a}{2} + \frac{1}{4} + \frac{\bar{u} - \frac{t}{4} - p_1}{2t\left(\frac{1}{2} - a\right)} \tag{48}$$

we obtain:

$$\pi_1 = \begin{cases} 0, & \tilde{n} < 0 \\ (p_1 + k - c)\left(\frac{a}{2} + \frac{1}{4} + \frac{\bar{u} - \frac{t}{4} - p_1}{2t\left(\frac{1}{2} - a\right)}\right), & 0 \leq \tilde{n} \leq 1 \\ (p_1 + k - c), & \tilde{n} > 1 \end{cases} \tag{49}$$

Deriving (42) on $p_1$we find that:

$$\frac{\partial \pi_1}{\partial p_1} = \begin{cases} 0, & \tilde{n} < 0 \\ \frac{c - k - 2p_1 - a^2 t + \bar{u}}{t - 2at}, & 0 \leq \tilde{n} \leq 1 \\ 1, & \tilde{n} > 1 \end{cases} \tag{50}$$

We can analyse the optimal behaviour in the sub-interval $\left[0, \frac{1}{2}\right]$; indeed, the optimal behaviour in the subinterval $\left[\frac{1}{2}, 1\right]$ is exactly specular to the former.

As long as the demand of newspapers is between 0 and 1 (second row in (49) and (50)), publisher 1 chooses an optimal price equal to:

$$\tilde{p}_1^* = \frac{c - k - a^2 t + \bar{u}}{2} \tag{51}$$

It is important to observe that $\frac{\partial \tilde{p}_1^*}{\partial a} < 0$ in the entire unit interval, except 1/2.

This pricing rule is valid as long as the demand of newspapers is between 0 and 1. Indeed, recalling (49) with $p_1 = \tilde{p}_1^*$, we can easily obtain that:

$$0 \leq \tilde{n} \leq 1 \text{ if and only if } a \in [0, \bar{a}], \text{ where } \bar{a} = \frac{2t - \sqrt{-ct + kt + 2t^2 + t\bar{u}}}{t} \quad (52)$$

We can also obtain that:

$$0 \leq \bar{a} < 1/2 \quad (53)$$

Using $\tilde{p}_1^*$ as pricing rule, publisher 1 chooses the best location $\tilde{a}^*$ deriving the profit function on a; we obtain that:

$$\frac{\partial \pi_1}{\partial a} > 0, \qquad \forall a \in [0, \bar{a}] \quad (54)$$

As explained in Proposition 5 for a similar case, (54) makes sense as long as the pricing in (51) leads to a demand between 0 and 1.

Therefore, for $a \in \left[\bar{a}, \frac{1}{2}\right]$, the pricing $\tilde{p}_1^*$ leads to a demand equal to 1 for which this pricing is not optimal anymore: as can be seen from the third row of (43), in this case the publisher optimally selects the highest price possible. We can start identifying the price for which the demand is exactly 1 for $a \in \left[\bar{a}, \frac{1}{2}\right)$:

$$\bar{p}_1^* = \bar{u} - t(1 - a)^2 \quad (55)$$

It is easy to observe that any lower price will bring lower profits and hence is not optimal. Any higher price does not guarantee demand equal to 1: in this case we can check from (28) and (29) that profits increase when p decreases. As a consequence, $\bar{p}_1^*$ is the optimal pricing for $a \in [\bar{a}, 1/2]$. Using $\bar{p}_1^*$ as pricing rule we can check that

$$\frac{\partial \pi_1}{\partial a} > 0, \qquad \forall a \in \left[\bar{a}, \frac{1}{2}\right) \quad (56)$$

Summarizing results in (44) and (56),

$$\frac{\partial \pi_1}{\partial a} \begin{cases} > 0, & \forall a \in [0, \bar{a}) \\ > 0, & \forall a \in \left[\bar{a}, \frac{1}{2}\right) \end{cases} \quad (57)$$

It remains to see what happens when $a = 1/2$; here the pricing $\bar{p}_1^*$ leads exactly to the collusion strategy where demand for publisher 1 is ½ while the pricing $\tilde{p}_1^*$ leads to a demand equal to 1 for which $\tilde{p}_1^*$ is not optimal.

In fact, with $a = 1/2$ publisher 1 locates at the same point as publisher 2 or, from a different perspective, she does not differentiate her product: in this framework publisher 1 takes the whole market simply applying a price slightly lower than the price applied by publisher 2, in this case equal to $\mathrm{p_2} = \bar{\mathrm{u}} - \frac{\mathrm{t}}{4}$.

We can now compare this result with the one in (57): this hints that publisher 1 is incentivized to locate as close as possible to ½ applying a price that sets the consumer in 1 to utility 0; such pricing would lead to a higher price in a = ½, but the relative demand there would be ½ only because publisher 2 is located at the same point and applies the same price. As just observed, here publisher 1 optimally applies a slightly lower price. Therefore, publisher 1's optimal defection strategy consists of $a = 1/2$ and $p = \bar{u} - \frac{t}{4} - \varepsilon$, with $\varepsilon > 0, as$ small as wanted. QED.