

Long-term debt and hidden borrowing*

Heski Bar-Isaac and Vicente Cuñat[†]

New York University and London School of Economics

2012

Abstract

We consider borrowers with the opportunity to raise funds from a competitive banking sector that shares information, as well as from other hidden lenders. The presence of hidden lenders allows borrowers to conceal poor results from their banks and, thus, restricts the contracts that can be obtained from the banking sector. In equilibrium, borrowers obtain funds from both the banking sector and inefficient hidden lenders simultaneously, so that different types of borrowers cannot be distinguished by banks. This generates cross-subsidies between different borrowers that are observationally equivalent to the banking sector. We show that the cheaper the cost of hidden borrowing, the lower is welfare and the lower is the variety of funding arrangements in the banking sector. In particular, while high costs of hidden borrowing allow each different (viable) type of borrower to access different terms from the banking sector, as the cost of hidden borrowing falls, more and more borrowers face identical terms up to the point where all borrowers who access the banking sector (which may include inefficient ones) face identical terms. We generalize the model to allow for partially-hidden lenders and obtain qualitatively similar results.

Keywords: Long-term debt, hidden borrowing, debt contracts, adverse selection

JEL Codes: D82, D14, G21, D86

*We thank Ken Ayotte, Alberto Bisin, Patrick Bolton, Leonardo Felli, Charles Goodhart, John Moore, Tomasz Piskorski, Larry White and audiences at the Econometric Society World Congress, ESSFM Gerzensee, FMG, and Columbia and the editor and an anonymous referee for their comments. Cuñat thanks the AXA Foundation for financial support.

[†]heski@nyu.edu, Department of Economics, Stern School of Business, NYU, 44 West 4th street 7-73, NYC, NY 10012 USA; and v.cunat@lse.ac.uk, Room A450, Department of Finance, London School of Economics, Houghton Street, London WC2A 2AE, U.K.

1 Introduction

Firms and consumers have access to different sources of borrowing. Since loans may differ in their seniority, covenants, and interest rates, this may induce an apparent pecking order among them. However, loans may also differ in the extent of their informational opacity with respect to other lenders. While some lenders perfectly share information—through a public credit registry, for example—other lenders may have no involvement in such information sharing. Borrowers may choose more-opaque loans in order to conceal information from others.

This paper investigates the consequences of the presence of opaque loans for formal bank loans in terms of the types of loans offered and adopted, liquidation decisions, and welfare. We argue that the presence of opaque lenders limits the contracting options of other lenders: If all lenders perfectly share information, loans induce borrowers to reveal their solvency at all times by setting interest rates that are highly responsive to repayment schedules. However, if borrowers can secretly obtain funds, loan repayments might reflect not only a borrower’s creditworthiness, but also her access to alternative loans. For this reason, loans become less responsive to interim payments. A borrower may simultaneously access both opaque and transparent loans even though more-opaque loans may be more costly in terms of higher interest rates for the lenders. Consequently, different types of borrowers—that is with different abilities to repay—might appear indistinguishable to the formal banking sector and face the same borrowing terms. Furthermore, the presence of opaque loans generates concealment costs and inefficient liquidation policies driven by cross-subsidies between borrowers. Overall welfare, can diminish with the availability and affordability of hidden loans, both as a result of inefficient liquidation, and borrowers accessing relatively expensive opaque funds.

Our results provide one explanation for the empirical observation that borrowers get loans from apparently costly lenders without fully exhausting cheaper sources. Firms use costly trade credit and personal loans from the owner before exhausting their credit lines and while having free collateral.¹ On the consumer side, Gross and Souleles (2002), for example, report that in a large sample of credit card holders, almost 70 percent of those borrowing on bankcards have positive housing equity. Our model suggest that a rationale for this behavior is that by using alternative sources of borrowing that are not perfectly

¹For example, in the 1998 National Survey of Small Business Finance (NSSBF), among the firms with bank debt not exceeding the value of their land (a conservative estimate of firms with free collateral), 14.7 percent used trade credit and 13.5 percent used lines of credit.

observable to their main lenders, borrowers can conceal their liquidity shocks.²

For example, missing a repayment can trigger a renegotiation with the bank and lead to a higher future interest rate. This reflects the bank's renewed assessment of the borrower's ability to repay. An effort to renegotiate the loan may be costly for the borrower because it would reveal information about current and future cash flows. In order to avoid this penalty, an entrepreneur might borrow from elsewhere, taking a personal loan, for example, to conceal the bad news that the enterprise has suffered a negative shock. In turn, this makes missing a payment even worse news, as it reflects a negative shock so large that it is prohibitively costly to conceal. The resulting overall cost of renegotiation may be sufficiently high that the financier would repossess the asset or foreclose following a missed payment.

We illustrate the interaction between publicly-observable and hidden borrowing more formally in a two-period model where agents have access to an investment project that yields cash flows correlated across time. They can fund the project through two sources: a competitive banking sector that shares information, and an opaque lending sector. Banks are senior claimants and seek to obtain information regarding borrowers through interim payments. While most of our discussion views banks as providing flexible long-term (two-period) financing, one could also interpret the banking sector as providing a sequence of short-term loans.

Our first result shows that if the alternative source of borrowing is sufficiently expensive (or is absent), banking contracts will achieve first-best. By rewarding higher interim payments with lower future interest rates, the optimal contract gives borrowers incentives to reveal their intermediate cash flows perfectly. However, with a viable alternative hidden lender, a borrower might be tempted to borrow from that source in order to disguise her type. The original lender in the banking sector anticipates this possibility. In general, this will lead to a more-limited menu of repayment schedules in the optimal contract. Further, we show that borrowers borrow from the opaque sector to make the interim repayment. Thus, in equilibrium, borrowers are simultaneously borrowing from both the banking and the opaque sectors. This is a well-documented phenomenon and, in our model, it is not a result of behavioral biases. By imposing a distributional assumption on borrowers' types,

²Other explanations have been posited to explain this apparent puzzle; for example, Laibson et al. (2003) calibrate a model of life-cycle borrowing with time-inconsistent preferences, and Bertaut, Haliassos, and Reiter (2009) discuss a model of separate mental accounts. The results of this paper assume fully rational consumers and need not contradict such explanations, but can be seen as complementary to them.

we fully characterize the menu of contracts. We show that as the cost of hidden borrowing falls, the equilibrium changes from a continuum of contracts that fully separate borrowers to a countable set of contracts each attracting a pool of borrowers. The pools get larger when the cost of hidden borrowing falls, leading eventually to a single pooling contract offered in the banking sector. We also allow for partially-hidden lending and obtain similar results.

We perform comparative statics exercises that lead to some empirical predictions. We find that more-expensive hidden lending improves the sorting of borrowers by banks. This allows for greater variety of lending arrangements in the formal banking sector in the sense that different types of borrowers are more likely to face different terms in the banking sector. When hidden borrowing is sufficiently cheap, all borrowers face identical terms in the banking sector. Thus, one could think of the sophistication of the banking sector as depending on the cost of hidden lending. The effect of increasing the cost of hidden lending is, in general, similar to making it more transparent, so technological and regulatory changes that improve information-sharing should have similar effects. Furthermore, more-expensive (or more-transparent) hidden lending leads to improved terms (that is, lower interest rates) from the formal sector because it is more expensive for borrowers to conceal their creditworthiness. In the presence of hidden lending, borrowers may liquidate projects too seldom since there are cross-subsidies induced by concealment. Increasing the cost of borrowing from hidden sources increases welfare, and naturally one might also suppose that this increases the volume of loans initiated. Influencing and regulating obscure or opaque sources of funds can, therefore, help to improve flexibility and the credit conditions in more-formal sectors.

Related Literature and Supportive Evidence The historical and international evidence on information sharing in the financial sector is consistent with the predictions of our model (see, for example, the edited volume Miller (2003), Hunt (2006), Jentzsch, (2007), and Jappelli and Pagano (2006)). In particular, Brown, Jappelli, and Pagano (2009), in an investigation of firms in Eastern Europe and the former Soviet Union, conclude that “information sharing is associated with improved availability and lower cost of credit to firms” (p. 1). In Cowen and De Gregorio (2003) show evidence from Chile that information sharing increased the volume of lending. There are alternative explanations for these

facts;³ however, the previous literature has discussed the complexity of contracts very little. Our model suggests that higher availability of creditor information should lead to debt contracts with more-flexible repayment terms and schedules, rather than, say, fixed repayment levels at fixed dates. This seems consistent with anecdotal evidence on the development of mortgage contracts in the U.K., for example, where there has been growth in flexible (or “lifestyle”) mortgages at a time when consumer credit-scoring has developed. In contrast, at the firm level, the empirical accounting literature has recently paid attention to the increasing use of financial innovations such as off-balance sheet lease financing as a form of opaque borrowing (Cornaggia et al, 2010; Zechman, 2011). This opaque borrowing has contributed to making the analysis of balance sheets by creditors and ratings less precise (Franzen et. al., 2009).

In this paper, the banking sector cannot write contracts that make payments depend on the amount borrowed from the hidden lender. This is a natural consequence of the assumption that the banking sector cannot observe this borrowing. This paper is, therefore, related to a growing literature on non-exclusive contracts and hidden savings.

Our focus on *different lending sectors* that vary in the information that they have, as well as the simple comparative statics analysis that this allows, distinguishes our paper from the literature on exclusivity. For example, there are models of non-exclusivity with simultaneous contracting (Bisin and Guaitoli, (2004), Jaynes (1978), and Arnott and Stiglitz (1991) in the context of insurance markets), with sequential access to loans (Bizer and DeMarzo (1992)), and with financial intermediaries who are *ex-ante* identical.

In the optimal contracts that we characterize, interim payments provide useful information that can allow for more-efficient outcomes. This mirrors observations in Allen (1985) and Dionne and Lasserre (1985). Hidden borrowing or savings (as in Cole and Kocherlakota (2001)) can, therefore, create inefficiency in these environments by reducing the information available from interim payments.

A feature of our analysis is that we vary the cost of borrowing from the hidden source. Allen (1985) and others focus on the case where this cost is equal to the social planner’s rate.⁴ Innes (1990), in order to generate monotonicity in repayment schedules, considers

³See Padilla and Pagano, 1997; Jappelli and Pagano, 1993; and, more broadly, the discussion in Jappelli and Pagano, 2006 for alternative, but related models of information sharing.

⁴The general model of Doepke and Townsend (2004), as illustrated in their example in Section 7.1, allows for this more general interest rate; however, as in Cole and Kocherlakota (2001) and Ljungqvist and Sargent (2003), they consider hidden saving and insurance rather than hidden borrowing and focus on numerical rather than analytical solutions.

the case where money can be repaid immediately so that the cost of borrowing is essentially zero.

Finally, a key element of the model is that a lender may not perfectly observe all the loans that a borrower may hold. Empirically, this is certainly the case. For example, although information sharing takes place through credit bureaus, many lenders choose neither to pay for access to credit bureaus nor to provide information to them. Trade credit, informal black-market lending, and personal loans to entrepreneurs subsequently used in their firms are clear examples. Further examples include consumer credit, store credit, payday lenders, and other sources that do not participate in organized information-gathering credit bureaus, both in developing countries and elsewhere, both currently and historically.⁵ For instance, Barron and Staten (2003) highlight that in some Latin American countries, there are “comprehensive credit histories on consumers but only on loans held by commercial banks”(pp.273-4). Note, further, that even when a lender has access to a credit bureau, the costs associated with accessing and processing the relevant information may lead lenders to obtain and use this information only in particular circumstances. Such circumstances would include the loan-approval stage, missed payments, and renegotiation; otherwise, there is unlikely to be continual monitoring. In this paper, we simply take it for granted that some types of borrowing are not commonly observed by all lenders.

2 The Model

Although the underlying economic mechanisms have wider applicability, we focus the model on the particular example of a small business that is raising funds for a capital-investment project that will generate an interim and a final return. Because these pay-outs are positively correlated, there is additional information at the interim stage that is useful for assessing creditworthiness. The firm has access to both a competitive banking sector and a hidden lender. One can think of the hidden lender as a personal loan to the entrepreneur secretly diverted to the firm.

We introduce a two-period model to consider the interaction between alternative sources of borrowing: a transparent banking sector and an opaque hidden lender (or lending sector).

⁵For example, in the U.S., payday lenders do not share information with banks (Elliehausen and Lawrence (2001) and Mann and Hawkins (2007)). However, it has been shown that their presence alters the borrowers’ payment of other loans. In particular, mortgage delinquency after an aggregate liquidity shock is significantly lower in areas where there are payday lenders (Morse (2007)).

2.1 Lending Sectors

In the transparent sector, credit is provided by a continuum of agents that we call banks. Banks are risk-neutral deep pockets, and there is competition among them. Banks share information, and so the borrowing position of any borrower with a bank is perfectly observable and verifiable among all banks. We normalize the gross riskless market interest rate of this banking sector to one.

The key assumptions concerning the banking sector are that it is competitive and that it shares information. More formally, we assume that banks compete in two stages (i) to initiate the loan, and (ii) at an interim stage following the first repayment. To make competition at the interim stage relevant, we assume that, after the first repayment, refinancing the loan at the interim stage (that is, repaying the full amount borrowed or repaying whatever is left following the first repayment and taking out another loan from another bank) entails no penalty for the borrower.⁶ Further, we assume that banks perfectly share the information about the borrower's payments and outstanding loans. In particular, this implies that they cannot simply replicate hidden lending, as they have no means to hide such contracts from other banks.

Banks compete with each other in the first stage by making sequential offers which consist of menus $\{p, q(p)\}$ of first-period repayments, p , and associated second-period repayments, $q(p)$. In the first stage, when the loan is initiated, a bank is picked at random to make an offer. If it makes no offer, the process ends and the lender receives no loan. If the borrower holds the bank's offer, an alternative bank is selected at random to observe the borrower's current contract and make a second offer. If the borrower strictly prefers the first bank's offer, no further offers are made and the lender either takes the first offer or no offer at all. If the borrower prefers the second bank's offer, then he holds that offer provisionally and, again, another bank is selected at random to observe the current offer and make an alternative offer, and so on. This structure of competition ensures that there is never an offer that is shielded from a counteroffer as in Bernardo et al (2010).

Competition at the second interim stage is similar, except that initially the borrower has chosen an existing repayment schedule, from the menu against which future offers may

⁶The opting out assumption seems empirically reasonable in many markets of long-term debt. Mortgages, for example, have small or no penalties for early payment (see Green and Wachter, 2005). Lines of credit are often used for long-term borrowing and are also fully pre-payable with no penalty. Bonds are often callable (see Sundaresan, 2009). Private debt agreements are, in general, not explicitly callable, but they are frequently renegotiated (Roberts and Sufi, 2009; report a 90-percent renegotiation rate among publicly traded companies and financial institutions).

be compared. In the second stage, after observing the menu $\{p, q(p)\}$ offered by the existing bank and the particular repayment schedule — p and associated $q(p)$ — that any particular borrower has adopted, banks compete by making sequential offers of an alternate first- and second-period payments $\tilde{p}(p, q(p), \{p, q(p)\})$ and $\tilde{q}(p, q(p), \{p, q(p)\})$ that, as suggested by the notation, depend on the initial schedule and the option chosen. In both stages, we model the sequence of offers as open-ended. The next bank observes the existing contract of the borrower in the first-stage bank and can offer an alternative first and second payment to each borrower. If the next bank's offer is taken, a further offer can arise from the next competing bank; which observes the original menu offered, the schedule that was picked from that menu and the last alternative offer taken. The process continues until no further offers are made.

Note that if banks could lock borrowers into long-term contracts with no opportunity to renegotiate, then information-sharing would be irrelevant in this model. The assumption that banks compete at each stage and that loans can be refinanced without penalty rules this out. The model can, alternatively, be understood as characterizing a sequence of two short-term contracts. In this latter interpretation, the counterpart to the flexibility of the long-term contract would be the variety of short-term contracts offered in the second stage.

In addition to the transparent banking sector, we introduce an alternative opaque, or hidden, lending sector that lends at a flat repayment rate $r > 1$; for now, we take the rate as exogenous. We discuss endogenizing this interest rate below.

A key feature of this alternative borrowing source is that it does not share information with the rest of the financial system. That is, the borrowing position of any borrower in the opaque sector is not observable by banks. Further, we model the opaque sector as a junior lender. This is certainly consistent with an interpretation as a concealed loan from the firm owner to the firm.⁷ In our model, lenders exogenously belong to either the banking sector or the opaque sector.

2.2 Borrowers

Demand for funds comes from borrowers who require funds for an investment project and who are heterogeneous in the quality of their projects. They are risk-neutral and maximize total consumption across periods.

The timing of the model is as follows:

⁷In terms of seniority, it is also consistent with trade credit or credit cards. Other types of hidden lending, including black-market lending, may be more ambiguous with respect to seniority.

At $t = 0$, each borrower does not know her type. In order to raise D units of funding necessary to invest in the project, the borrower can choose among the different offers available of first- and associated second-period debt repayment schedules $\{p, q(p)\}$. Second-period payments may be contingent on first-period ones.

At $t = \frac{1}{2}$, each borrower privately learns the type of her project, which is parametrized by α , where α is distributed on $[0, 1]$. At this point, the borrower can either costlessly liquidate the project for D , and fully repay the loan, or continue with the project and choose a repayment schedule.⁸

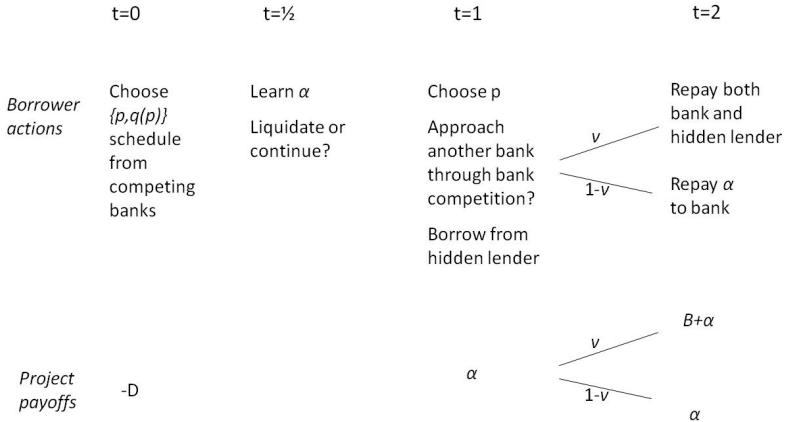
At $t = 1$, a borrower realizes a cash flow α that corresponds to her type. At this stage, the borrower may also switch to a competing bank offer (\tilde{p}, \tilde{q}) where the nature of competition is outlined above. She can choose to borrow funds from the opaque lending source that is hidden from the banks. A loan of the opaque lender is junior to the bank loan, and banks do not observe it. The borrower can use these funds either to consume or to choose one of the repayment schedules from the menu and repay p to the bank.

At $t = 2$, the project is successful and delivers $B + \alpha$ with probability ν . Otherwise, the project fails and delivers only α . In both cases, seniority of debt is such that the borrower repays $q(p)$ to the bank first and then repays opaque lenders up to rd . The borrower consumes all the remaining funds.

The parameter α represents the creditworthiness of the borrower since the expected final cash flow of the project is positively correlated with its interim cash flow. Note that, overall, a project of type α generates a net present value of $-D + \alpha + \nu(B + \alpha) + (1 - \nu)\alpha = -D + \nu B + 2\alpha$. In particular, the best potential project, a project of type $\alpha = 1$, generates an expected net present value $-D + \nu B + 2$. Low values for the overall net present value suggest (though, obviously, depending on the distribution of types) that a high proportion of projects are inefficient. In particular, $D \geq 2 + \nu B$ implies that no projects should be funded, while $\nu B \geq D$ implies that all projects are efficient and should be funded. With intermediate values of the net present value of a project, only projects with $\alpha \geq l^* := \frac{D - \nu B}{2}$ are efficient.

The following diagram summarizes both the borrower's actions and the payoffs required and generated by the investment project.

⁸We model this option to stop the project as a costless liquidation at a very early stage; but supposing that the agent were able to recover a sufficiently large salvage value at an early stage would generate similar qualitative results.



Summary of Timing

Borrowers and lenders are risk-neutral, and every agent seeks to maximize the sum of their first- and second-period incomes.

2.3 Simplifying assumptions

We add several auxiliary assumptions that help to simplify the analysis.

First, we assume that banks weakly prefer renegotiation-proof contracts. In the absence of such an assumption, more-general contracts could arise in period 0, but renegotiation would lead to the same outcomes characterized by the model.⁹

Second, for expositional purposes, we suppose that any particular borrower obtains all of his loans from the banking sector from a single bank; that is, each borrower only uses one bank at a time. This can be justified by small transaction costs that ensure that a borrower will borrow from a single bank when otherwise indifferent. In terms of borrowers' outcomes and welfare, this is without loss of generality. Given that information is shared, different banks offer contracts that depend on the overall borrowing position in the banking sector. All banks have the same seniority and in case of default are compensated proportionately to their outstanding loans. Borrowing from multiple banks does not affect consumption or

⁹For example, banks could offer a repayment schedule with an extremely high second-period interest rate that would surely be renegotiated at $t = 1$. Since banks are competitive, the new renegotiated interest rate would make banks break even, conditioning on this new information. The outcomes and payoffs under renegotiated and renegotiation-proof contracts are identical, so the role of this assumption is to emphasize the long-term nature of the contract.

liquidation decisions for borrowers, or profits for banks (given the nature of competition, banks make zero profits).

Next, we assume that a borrower cannot owe more than she can possibly repay in the best possible state (that is, no more than $B + 2\alpha$). It can be understood as a “no fraud” condition that prevents borrowers from consuming in the interim period with the intention of defaulting for sure in the future.¹⁰ This is a reasonable assumption, since most legal systems allow for punishment above limited liability (i.e., prison or personal liability) if it is found (perhaps with some probability) that a borrower did not intend to repay in any possible state of the world.¹¹

Finally, we make parametric restrictions that preclude some trivial and uninteresting cases. Specifically, we assume that $D > 2$, which ensures that no borrower can repay for sure, and $D > \nu B > D - 2$, which ensures that all types of borrowers will default to a different extent if the project is unsuccessful (so, from the lenders’ point of view, they really are different types). In particular, the second restriction implies that some projects are efficient and should be funded, while others are not.

3 Equilibrium

A bank offer at $t = 0$ consists of a menu of repayment schedules $\{p, q(p)\}$. A bank strategy is an offer that could depend on the full history of offers up to that point. After the initial stage of bank competition is concluded and the borrower has accepted an offer, she has to decide whether to pursue the project at $t = \frac{1}{2}$, or to liquidate. If the borrower does not liquidate, she has to decide, at $t = 1$, which schedule from the current menu to choose. As described above, the borrower can choose to stay with her current bank for the final period or switch to another bank. Note that, if on the final schedule chosen, $p > \alpha$, the borrower needs to fund any shortfall for the first payment by borrowing from the hidden source.

Proposition 1 provides a preliminary result that puts limits on the contracts that the banking sector might offer in equilibrium. It is a consequence of the potentially unlimited sequence of offers and the structure of competition between banks, described in Section

¹⁰Since the hidden sector is more expensive than bank borrowing, no borrower accesses funds from the hidden sector to consume in the interim period and repay in the good state. Therefore, borrowing to consume in the interim would be worthwhile only if the borrower intended to default for sure.

¹¹Note that such a borrowing limit requires the payoff to become verifiable in case of default. We believe, that it is plausible that if the project fails, triggering liquidation and investigation, α becomes verifiable but in the absence of a liquidation proceedings, it is not. Introducing a small verification cost in Period 2, in the spirit of the costly state verification literature (Townsend, 1979; Gale and Hellwig, 1985), would not affect the qualitative results.

2.1.

Proposition 1 *In equilibrium the menu offered can have no observable cross subsidies; that is, a bank must break even on each $p, q(p)$ schedule taken in equilibrium.*

Proof. The proof of this and all subsequent results appear in the Appendix. ■

Proposition 1 shows that conditional on the information known in the banking sector every contract must break even for each observationally distinct type of borrower. That is, there can be no cross-subsidies between borrowers who are perceived as different by the banking sector. The intuition here is that if during a round of competition a bank observed a set of borrowers that were subsidizing other borrowers, it could offer improved terms only to them. These borrowers would switch banks, leaving their previous bank with only subsidized borrowers and, thus, losses. This is the case in both stages of competition and implies that, overall, contracts must break even. Note that, given that banks compete sequentially, the argument of Proposition 1 can be easily extended from the menu offered in equilibrium to any deviation offer.

Among contracts that satisfy these restrictions, the assumption that borrowers do not know their own type at stage 0 selects the contract that maximizes ex-ante welfare. The full equilibrium configuration turns out to depend crucially on the interest rate at which the hidden sector lends. In particular, if the interest rate is sufficiently high ($r > \frac{2-\nu}{\nu}$), then opaque lending is too expensive to be used to conceal a bad realization of α , making it irrelevant. Instead, if the interest rate is very low, then it is easy for lower-type borrowers to mimic higher-type ones.

Note that, regardless of the amount borrowed, the opaque lender will always be repaid if the good state is realized and will always face default in the bad state. This follows from the seniority of bank debt. Thus, the break-even rate for r is $\frac{1}{\nu}$, regardless of the pool of borrowers that the hidden lender attracts. This would be the endogenous rate for the opaque sector if there were no other frictions or inefficiencies. However, whether we think of the opaque lending sector as trade credit, a credit card, personal loans to an entrepreneur, or an informal lender, it is reasonable to believe that the interest rate charged could be above this break-even rate—for example, if there are other uses or users of this source of lending. Therefore, we study situations in which $r \geq \frac{1}{\nu}$.¹²

¹²Note that the model leads to a break-even rate that is independent of the amount borrowed, and we simply assume that the markup that the hidden lender charges above it is also independent. In application,

3.1 Expensive hidden borrowing

In this section, we explore the implications of an expensive opaque sector. When the interest rate r is bigger than $\frac{2-\nu}{\nu}$, borrowing from the opaque lender is so expensive that it is irrelevant. As a result, there is full separation among those types that borrow—that is, each different type repays the banking sector a different interim payment. The intuition is that there is no opportunity for banks, at stage 1, to offer a more attractive contract to any borrower—contingent on observable information, a borrower’s surplus is maximized; here, the observable information *is*, in effect, the agent’s type. Moreover, since this outcome leads to efficient liquidation decisions, and borrowers retain all the (maximized) surplus, this is the outcome that they prefer at the ex-ante stage where contracts are determined.

Proposition 2 *When the opaque sector lends at a sufficiently high interest rate ($r > \frac{2-\nu}{\nu}$), there exists a fully separating equilibrium where all banks offer the same equilibrium contract. This contract is a contingent one, where the interim payment is equal to the first-period cashflow, and the corresponding final payment fully reflects the information implied by the revealed first-period cashflow. Liquidation at $t = \frac{1}{2}$ is at the efficient level, $l^* := \frac{D-\nu B}{2}$ and the equilibrium achieves first-best.*

Note that if there is no hidden lender, then, as a corollary of Proposition 2, the outcome is first-best. This follows since the absence of a hidden lender corresponds to an infinitely high interest rate ($r \rightarrow \infty$) from the opaque sector.

It is also worth noting that borrowers obtain all the surplus generated since banks are competitive and earn no profits. Since hidden lenders are prohibitively expensive, they are inactive. Thus, with expensive hidden lending, the first-best is achieved, and borrowers retain all the surplus from projects that are financed.

Formally, beyond the equilibrium described in Proposition 2, there are many other essentially observationally equivalent equilibria. That is, the offered menu could include many other redundant $\{p, q(p)\}$ schedules that are never taken up and that have no effect on outcomes (for example, schedules with very high p ’s and q ’s), or where some banks (that, in any case, earn no expected profits) offer menus that are never taken up. Henceforth, we ignore such equilibria.

this is justified because borrowers may be able to obtain several small loans from different lenders (e.g. different credit cards, different payday lenders or both simultaneously). Given that they do not share information, they cannot condition on other loans.

Finally, except for these observationally equivalent equilibria, there are no other equilibria. This follows since the fully separating equilibrium achieves the first-best and maximizes the surplus for borrowers at $t = 0$; instead, any pooling equilibrium must involve inefficient liquidation and/or hidden borrowing and so less surplus for borrowers. This would leave an opportunity for a rival bank to offer a more attractive contract.

3.2 Cheap hidden borrowing

In this section, we explore equilibrium outcomes when the opaque sector is relatively cheap—that is, when $r < \frac{2-\nu}{\nu}$. Note, in particular, that this regime includes the case where there are no frictions in the opaque sector and r is equal to $\frac{1}{\nu}$.

We start by describing some general features of any equilibria. In particular, we demonstrate that there will be some pooling among different types of borrowers with regard to their interim payments. Further, banks cannot distinguish the different types within a pool of borrowers who all make the same interim repayment. Given that banks break even within each observable pool of borrowers, it follows that, within such a pool, borrowers will cross-subsidize each other.

Proposition 3 *When the hidden lender’s interest rate is sufficiently low ($r < \frac{2-\nu}{\nu}$), there cannot be an equilibrium where a continuum of borrowers are able to fully separate. Further, in any equilibrium with borrowing, borrowers’ types can be partitioned, with each pool of borrowers paying a different interim payment; that is, there is some countable n and $\alpha_0 = 1 > \alpha_1 > \dots > \alpha_{n-1} > \alpha_n \geq 0$ where all types $\alpha \in (\alpha_{i-1}, \alpha_i)$ make the same interim payment.*

The intuition here is that if two similar types can fully separate, then, by borrowing “a little” from the hidden lender, a lower type can mimic a higher type and will be better off overall. That is, by borrowing marginally, the borrower can affect the interest rate on infra-marginal outstanding debt. As a result, Proposition 3 implies that when $r < \frac{2-\nu}{\nu}$, in any equilibrium, all borrowers belong to some pool—i.e., no borrower is able to fully separate.

Thus, compared to the outcome in Proposition 2, which shows that when $r > \frac{2-\nu}{\nu}$ a continuum of contract contingencies arises, here, with a cheaper lender, only a countable number of contingencies arises. That is, contracts are simpler or, equivalently, less flexible when hidden borrowing is relatively cheap.

We can further characterize equilibrium contracts. In particular, we argue that no borrower consumes in the interim period. The intuition here is that if some borrowers are consuming in the interim period, it is less costly for them to pay a higher interim payment than it is for a type who is borrowing to access the same contract. A rival bank can therefore cherry-pick these higher types of borrowers at the interim period by offering a slightly higher interim payment and a lower final payment—this cannot be the case in equilibrium, since this entails losses for the original lending bank.

Lemma 1 *No borrowers consume in the interim period (that is, $p(\alpha) \geq \alpha$ for all α).*

Following Propositions 3 and Lemma 1, the presence of a relatively cheap hidden lender restricts the banks' contractual options, forcing the contract to be less contingent on intermediate payments. As the interest rate of the hidden lender falls, banks find it harder to distinguish between borrowers. Note that, within a pool of indistinguishable borrowers, the interest rate between $t = 1$ and $t = 2$ is the same for all borrowers, regardless of their creditworthiness. Within this pool of indistinguishable borrowers, higher-quality borrowers cross-subsidize lower-quality borrowers.

In general, the lower the cost of borrowing from the hidden sector, the more easily a lower type of borrower can imitate a marginally better type and, intuitively, an infra-marginally better type of borrower. This intuition suggests that the top pool of borrowers, between α_1 and 1, as described in Proposition 3, must get broader as the cost of borrowing from the hidden lender decreases. Equivalently, a lower cost of hidden borrowing leads to a greater range of different types of borrowers all adopting the same repayment schedule. As we describe below, this intuition is consistent with results for uniformly distributed types.

The general case can be further characterized. In particular, interim competition suggests that in equilibrium the highest type in a pool of borrowers cannot be borrowing from the hidden lender. This can be established through a proof by contradiction. If every type in a pool of borrowers obtains funds from the hidden borrower, another bank at the interim stage could offer this pool of borrowers a more attractive contract (with a lower interim payment) that requires less hidden borrowing and is profitable for the bank. This proof is formally stated in the Appendix and allows us to establish the following result.

Proposition 4 *Suppose that there is an equilibrium with n pools of borrowers, then for the i^{th} pool—that is for types $\alpha \in (\alpha_{i-1}, \alpha_i)$ —the interim payment, p , is equal to α_{i-1} and*

the final period payment is $\frac{D-\alpha_{i-1}-(1-\nu)E(\alpha|\alpha \in [\alpha_i, \alpha_{i-1}])}{\nu}$. Moreover, α_i satisfies

$$\frac{\nu r - 1}{1 - \nu}(\alpha_{i-1} - \alpha_i) = E(\alpha|\alpha \in [\alpha_i, \alpha_{i-1}]) - E(\alpha|\alpha \in [\alpha_{i+1}, \alpha_i]) \quad (1)$$

for $i = 1, \dots, n-1$ and $\alpha_n = \max\{0, \tilde{\alpha}_n\}$ where

$$\frac{\nu r - 1}{1 - \nu}(\alpha_{n-1} - \tilde{\alpha}_n) = E(\alpha|\alpha \in [\tilde{\alpha}_n, \alpha_{n-1}]) - \frac{D - \nu B}{1 - \nu} + \tilde{\alpha}_n \frac{1 + \nu}{1 - \nu}. \quad (2)$$

Proposition 4 defines a system of n equations in the n unknowns $\alpha_1, \dots, \alpha_n$. For given parameter values, it is straightforward to determine the solution (or possibly solutions) for all values of n , assess the feasibility of these candidate equilibria (that is ensure that the solutions are in the range $1 > \alpha_1 > \dots > \alpha_n \geq 0$). In principle, it then remains to compare the feasible solutions to determine the equilibrium contract, noting that competition among banks at stage 1 implies that the equilibrium contract must maximize welfare amongst all possible candidate contracts that satisfy (1) and (2).

Equation (1) is a direct implication of the incentive compatibility constraint of the lower member of a pool. It has an intuitive interpretation. A large pool is easier to sustain when r is low –so imitating better types is cheap– and when the average quality of the borrowers in the next-lower pool is worse. Given one pool, equation (1) determines the size of the pools immediately above or below it, generating $n-1$ conditions for n pools. Equation (2) uses the indifference of the last borrower of the last pool between investing or liquidating and is the condition that closes the system.

In order to provide a complete characterization we assume from now on that types are uniformly distributed.

Proposition 5 *If $\alpha \sim U[0, 1]$, formal and hidden lending in equilibrium is uniquely determined.*

1. If $r \geq \frac{2-\nu}{\nu}$ there is full separation of borrower types with each type $\alpha > l^* = \frac{D-\nu B}{2}$ paying α at the interim stage and $q = \frac{D-(2-\nu)\alpha}{\nu}$ as a final payment.
2. If $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$ there are infinite countable pools where

$$\alpha_i = 1 - \frac{2 - r\nu - \nu}{1 - \nu} \left(1 - \frac{D - \nu B}{2}\right) \frac{1 - \left(\frac{\nu+2r\nu-3}{1-\nu}\right)^i}{1 - \frac{\nu+2r\nu-3}{1-\nu}}$$

for $i = 1, 2, \dots$ where all borrowers of type $(\alpha_i, \alpha_{i-1}]$ make an interim payment of α_{i-1} and a final payment $q = \frac{2D-2\alpha_{i-1}-(1-\nu)(\alpha_i+\alpha_{i-1})}{2\nu}$.

3. If $\frac{3-\nu}{2\nu} > r$ there is a single pool of borrowers of type $\alpha \in (\frac{\nu+2D-2B\nu+2r\nu-3}{\nu+2r\nu+1}, 1]$ who make an interim payment of 1 and a final payment of $q = \frac{B-4r+\nu+2D-B\nu+2r\nu+2rD-3}{\nu+2r\nu+1}$.

The regime in the range $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$ contains multiple pools. The regime converges smoothly to the other two. In the range $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$, equation (1) requires that the mass of borrowers accessing the n^{th} contract is a fraction $\frac{\nu+2r\nu-3}{1-\nu} \in (0, 1)$ of the mass of borrowers accessing the $n - 1^{th}$ contract. As $r \rightarrow \frac{2-\nu}{\nu}$ the term $\frac{\nu+2r\nu-3}{1-\nu}$ converges to 1 leading to an equilibrium with infinite almost-equally sized small pools, covering the borrowers in the range $(1, \frac{D-B\nu}{2})$; which resembles the fully separating equilibrium. As r grows, each pool n becomes bigger relative to the next lower pool $n + 1$. When $r \rightarrow \frac{3-\nu}{2\nu}$ the term $\frac{\nu+2r\nu-3}{1-\nu}$ converges to zero and the first pool covers most of the borrowers. When $r > \frac{3-\nu}{2\nu}$ only one pool can exist; the cross subsidies between borrowers lead to $\alpha_n < \frac{D-B\nu}{2}$ so liquidation is inefficient.

In the range $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$, the top contract (high quality borrowers) accounts for the greatest fraction of overall borrowing and pools become smaller towards the bottom. There are infinite pools and the bottom pool can be considered as arbitrarily small. This implies no cross subsidies at the very bottom and therefore efficient liquidation decisions, i.e. as $i \rightarrow \infty$, $\alpha_i \rightarrow \frac{D-B\nu}{2}$. Thus throughout this range for the cost of hidden borrowing, only efficient projects are conducted, and all inefficient projects are liquidated. The following corollary to Proposition 5 demonstrates that the lower the cost of hidden borrowing, the lower is α_1 and so the higher the fraction of borrowers who pool in the top contract by the banking sector. Indeed, it shows more generally that as the cost of hidden borrowing falls, the “highest” contracts account for a larger share of borrowers. It is in this sense that a high cost of hidden borrowing is associated with a greater proportion of borrowers accessing a greater variety of contracts or with increased financial complexity in the formal banking sector.

Corollary 1 If $\alpha \sim U[0, 1]$, and $\frac{2-\nu}{\nu} > r \geq \frac{3-\nu}{2\nu}$ then $\frac{d\alpha_i}{dr} > 0$ for all i .

Given the characterization in Proposition 5, it is simple to characterize welfare and show that it increases as the cost of hidden borrowing increases.

Corollary 2 If $\alpha \sim U[0, 1]$, welfare is given by

$$W := \begin{cases} \left(1 - \frac{D-\nu B}{2}\right)^2 & \text{for } r \geq \frac{2-\nu}{\nu} \\ \frac{1+r}{2}\nu \left(1 - \frac{D-\nu B}{2}\right)^2 & \text{for } \frac{2-\nu}{\nu} \geq r \geq \frac{3-\nu}{2\nu} \\ 8\nu \frac{r+1}{(\nu+2r\nu+1)^2} \left(1 - \frac{D-\nu B}{2}\right)^2 & \text{for } \frac{3-\nu}{2\nu} \geq r \geq \frac{1}{\nu} \end{cases} \quad (3)$$

Thus it is non-decreasing in r and strictly increasing in r for $\frac{2-\nu}{\nu} > r$.

Within the fully separating equilibrium the first-best is achieved and welfare is constant. In the range where multiple pools exist, liquidation is efficient, but pools become smaller, the larger the r . The reduction in borrowing dominates the higher cost of borrowing and welfare grows with r . In the single-pool equilibrium, lower r leads to more borrowing and less efficient liquidation, these two effects again dominate the lower cost of borrowing, so welfare is increasing in r . Consistent with the convergence of the different regimes, there is smooth pasting of welfare across them.

4 Partially-Hidden Borrowing

Next, we modify the model slightly to allow for a partially-hidden lender. We introduce the possibility that the banking sector observes the level of hidden borrowing by the borrower with some probability $(1 - h)$. With probability h , borrowing from the non-banking sector remains hidden. A rationale for this modeling assumption is that the banking sector investigates each of its borrowers and obtains full information about the borrowing position of each with some probability $(1 - h)$.

Once a borrower is successfully investigated, her borrowing position with all possible alternative lenders is perfectly known by the whole banking sector. In this case, the banking sector will learn the borrower's type perfectly by viewing her borrowing position, and in the continuation, full separation is achieved with certainty. However, if the borrower is found to be borrowing from the opaque sector, then she must incur a cost, s per-unit of hidden borrowing.

Thus, the model with probabilistic observability of the hidden borrowing is like a switching model in which, with probability $(1 - h)$, full separation is achieved with certainty, and, with probability h , looks like the model of the previous sections. In this latter case, the only difference is that, from the borrower's point of view, the costs and benefits of the hidden borrowing need to be recalculated since, with probability $(1 - h)$, hidden borrowing is useless and entails a penalty s per-unit of hidden borrowing.

In fact, once the alternative borrowing remains hidden, the rest of the model with probabilistic observation of the hidden borrowing can be fully solved by realizing that, in effect, the cost of borrowing from the hidden source is now $\frac{hr+(1-h)s}{h}$ instead of just r . Borrowing one unit from the hidden source costs r with probability h and costs s with probability $(1 - h)$. It produces some concealment benefit to the borrower only with probability h , so the whole cost has to be re-scaled by $1/h$.

We write $r(h, s) := \frac{hr+(1-h)s}{h}$ as the *effective* interest rate when borrowing from the opaque sector remains hidden with probability h ; the rate of interest is r when borrowing remains hidden; and the penalty cost, or cost of early repayment, when the banking sector observes the borrowing is s . With this notation, we obtain the following results, which are similar to those in the fully-hidden case:

Proposition 6 *If $\alpha \sim U[0, 1]$, equilibrium formal and hidden lending is uniquely determined.*

1. *If the opaque sector lends at a sufficiently high effective interest rate $r(h, s) \geq \frac{2-\nu}{\nu}$ there is full separation of borrower types with each type $\alpha > l^* = \frac{D-\nu B}{2}$ paying α at the interim stage and $q = \frac{D-(2-\nu)\alpha}{\nu}$ as a final payment.*
2. *If the opaque sector lends at an intermediate effective interest rate $\frac{2-\nu}{\nu} > r(h, s) \geq \frac{3-\nu}{2\nu}$ there is a continuum of pools where*

$$\alpha_i = 1 - \frac{2 - r(h, s)\nu - \nu}{1 - \nu} \left(1 - \frac{D - \nu B}{2}\right) \frac{1 - \left(\frac{\nu + 2r(h, s)\nu - 3}{1 - \nu}\right)^i}{1 - \frac{\nu + 2r(h, s)\nu - 3}{1 - \nu}}$$

for $i = 1, 2, \dots$ where all borrowers of type $(\alpha_i, \alpha_{i-1}]$ make an interim payment of α_{i-1} and a final payment $q = \frac{2D - 2\alpha_{i-1} - (1-\nu)(\alpha_i + \alpha_{i-1})}{2\nu}$.

3. *If the opaque sector lends at a sufficiently low effective interest rate $\frac{3-\nu}{2\nu} > r(h, s)$ there is a single pool of borrowers of type $\alpha \in (\frac{\nu + 2D - 2B\nu + 2r(h, s)\nu - 3}{\nu + 2r(h, s)\nu + 1}, 1]$ who make an interim payment of 1 and a final payment of $q = \frac{B - 4r(h, s) + \nu + 2D - B\nu + 2r(h, s)\nu + 2rD - 3}{\nu + 2r(h, s)\nu + 1}$.*

The functional form of the welfare equation and the incentive-compatibility conditions are similar to those of the basic model, so comparative statics are analogous to those of Section 3.2. First, note that $r(h, s)$ is increasing in r and s , but decreasing in h ; that

is, it increases as hidden lending becomes more transparent. As in Section 3.2, and, in particular, Corollary 2, welfare is increasing in $r(h, s)$.

Transparency, as captured by $(1-h)$, has both direct and indirect effects on welfare. The direct effect prevents borrowers from accessing costly hidden lending, and unambiguously benefits welfare. The indirect effect operates through $r(h, s)$ in the same way that raising the cost of borrowing from the hidden sector operates in Section 3.2, where the lower aggregate level of hidden borrowing and more efficient liquidation decisions (when $r(h, s)$ is sufficiently low) outweigh the costs of more expensive hidden borrowing. Overall, therefore, more transparency raises welfare. Similarly, increasing the cost of hidden borrowing (either through r or s) operates through changing the effective cost of hidden borrowing $r(h, s)$ and so increases welfare, analogous to the results on increasing r in Section 3.2.

Note that our analysis is related to the literature on the interactions between direct screening of lenders by actively investigating them and indirect screening that can be achieved by offering them a menu of contracts, as in Manove et al. (2001). While in most models these are seen as substitutes, in our model they are complements. That is, an increase in $(1 - h)$ leads to more information about some borrowers directly and also to a more informative equilibrium with regard to the other borrowers, who may have loans from the alternative sector that remain hidden.¹³

Our results show that changing the transparency of the alternative hidden lenders can have an impact on the types of contracts that the banking sector provides and in general on borrowing patterns and the efficiency of loans. For example, if payday lenders are a major source of funds used for concealment, forcing them to disclose information would reduce the use of payday loans and lead to a better sorting of risks from banks. Conversely, information-protection regulation that limits information sharing across lenders should lead to a higher fragmentation of the sources that a given borrower uses and, in general, harms welfare when compared to full disclosure.

5 Conclusions

We present a model of financing for an investment project, with central mechanisms that have wide applicability—in particular, the interaction of different sources of borrowing and the implications for contractual form. Our results highlight a possible reason that

¹³Even though, so far, we have considered h as an exogenous parameter, endogenizing it seems relatively straightforward. We could allow banks to choose their monitoring effort h at a cost. Higher transparency (lower h) would be more costly, and competition among banks should equalize the marginal cost of additional monitoring (reducing h) with its marginal gain in terms of borrowers' surplus in equilibrium.

long-term debt contracts are inflexible with respect to interim payments: The information that long-term lenders would extract from these interim payments would be corrupted by additional borrowing from hidden sources of funds. Our results also suggest an explanation for simultaneous borrowing from different sources, even when there is an apparently clear pecking order among them, and the borrowing from the cheaper source is not fully exhausted (for example, firm loans and trade credit, or mortgages and credit card borrowing when both trade credit and credit card borrowing are not costlessly observable by the bank).

Existing literature has drawn a distinction between informal and formal lending and highlighted that the informal sector may increase credit availability through different information and enforcement technologies. In this paper, we focus on opaque lending and, to the extent that informal lending may be opaque, we highlight an indirect channel through which it may diminish welfare. In particular this indirect channel is its effect on lending in the formal sector. Even though this indirect channel may be strong enough to generate a net welfare loss, borrowers, who would (*ex-ante*) prefer to commit to not access informal lending, have no means to do so, and so might find themselves compelled to access informal lending.

The model makes several empirical predictions. Specifically, we highlight that changes in the efficiency or observability of alternative lending sources affect the form and nature of bank lending. The results on contractual form in the banking sector (where higher costs of hidden borrowing lead to more variety) suggest that, as the informational transparency of the financial sector as a whole improves, more consumers take up a wider range or a more-sophisticated set of financial instruments from banks. Finally, we predict that borrowers might simultaneously use expensive but hidden sources of credit, and cheaper credit from the banking sector.

Most of the empirical predictions of the model relate to the levels of efficiency and informational transparency of alternative lenders. Cross-country comparisons show substantial differences in the effective level of information sharing across countries (Miller (2003), Jentzsch, (2007)). In some countries, such as France, restrictions such as privacy-protection laws have precluded the creation of credit bureaus. In others, the existence of cheap-to-access and centralized public credit registers (that do not cover borrowing sources such as small credits, credit cards or consumer credit) has also crowded private ones. The model predicts that these differences should affect debt-market contracts. Along these same lines, it is suggestive that a higher level of innovations in mortgage markets in Anglo-

Saxon countries has not had a counterpart in continental Europe (as suggested in Green and Wachter (2005)).

References

- [1] Allen, Franklin (1985): "Repeated Principal-Agent Relationships with Lending and Borrowing," *Economics Letters*, 17, 27-31.
- [2] Arnott, Richard J. and Joseph E. Stiglitz (1991): "Equilibrium in Competitive Insurance Markets with Moral Hazard," NBER Working Paper No. W3588.
- [3] Bennardo, Alberto, Pagano, Marco and Piccolo, Salvatore (2009), "Multiple-Bank Lending, Creditor Rights and Information Sharing, SSRN - working paper.
- [4] Bertaut, Carol, Michael Haliassos, and Michael Reiter (2009): "Credit Card Debt Puzzles and Debt Revolvers for Self Control," *Review of Finance*, 13(4), 657-692
- [5] Bisin, Alberto and Danilo Guaitoli (2004): "Moral Hazard and Nonexclusive Contracts Production: Theories and Evidence from Joint Ventures" *RAND Journal of Economics*, 35, 306-328.
- [6] Bisin, Alberto and Adriano A. Rampini (2006) "Exclusive Contracts and the Institution of Bankruptcy," *Economic Theory*, 27, 277-304.
- [7] Bizer, David S. and Peter M. DeMarzo (1992): "Sequential Banking," *Journal of Political Economy*, 100, 41-61.
- [8] Brown, Martin, Jappelli, Tullio, and Marco Pagano (2009), "Information sharing and credit market performance: firm-level evidence from transition countries," *Journal of Financial Intermediation*, 18(2), 151-172.
- [9] Cole, Harold L. and Narayana R. Kocherlakota (2001): "Efficient Allocations with Hidden Income and Hidden Storage," *Review of Economic Studies*, 68, 523-542.
- [10] Cowen, Kevin and Jose De Gregorio (2003): "Credit Information and Market Performance: The Case of Chile," Chapter 4 in Miller, Margaret J. ed. (2003) *Credit Reporting Systems and the International Economy*, MIT Press, 273-310.
- [11] Cornaggia, Kimberly J. Laurel A. Franzen, and Timothy T. Simin (2011) "Manipulating the balance sheet? Implications of off-balance-sheet lease financing". SSRN - working paper
- [12] Dionne, Georges and Pierre Lasserre (1985): "Adverse Selection, Repeated Insurance Contracts and Announcement Strategy," *Review of Economic Studies*, 52, 719-723.
- [13] Doepke, Matthias and Townsend, Robert M., (2006) "Dynamic mechanism design with hidden income and hidden actions," *Journal of Economic Theory*, Elsevier, vol.

126(1), 235-285.

- [14] Elliehausen, G. and E. C. Lawrence, (2001). “Payday advance credit in America: An analysis of consumer demand (Monograph No. 35)”. Washington, D.C.: Georgetown University, McDonough School of Business, Credit Research Center.
- [15] Franzen, Laurel, Cornaggia, Kimberly Rodgers and Simin, Timothy T., (2009) “Capital Structure and the Changing Role of Off-Balance-Sheet Lease Financing” SSRN - working paper
- [16] Gale, Douglas and Martin Hellwig (1985) “Incentive-Compatible Debt Contracts: The One-Period Problem,” *Review of Economic Studies*, 647-663.
- [17] Green, Richard K. and Susan M. Wachter (2005) “The American Mortgage in Historical and International Context,” *Journal of Economic Perspectives*, 19(4), 93-114.
- [18] Gross, David and Nicholas S. Souleles (2002) “Do Liquidity Constraints and Interest Rates Matter for Consumer Behavior? Evidence from Credit Card Data,” *Quarterly Journal of Economics*, 149-185.
- [19] Hunt, Robert M. (2006) “The Development and Regulation of Consumer Credit Reporting in America,” in Bertola, Giuseppe and Charles Grant, eds., *The Economics of Consumer Credit: European Experience and Lessons from the US*, Boston: MIT Press.
- [20] Innes, Robert (1990) “Limited Liability and Incentive Contracting With Ex-Ante Action Choices,” *Journal of Economic Theory*, 52, 45-68.
- [21] Jaynes, Gerald D. (1979) “Equilibria in Monopolistically Competitive Insurance Markets,” *Journal of Economic Theory*, 19, 394-422.
- [22] Jappelli, Tulio and Marco Pagano (1993) “Information Sharing in Credit Markets,” *Journal of Finance*, 43(5), 1693-1718.
- [23] Jappelli, Tulio, and Marco Pagano (2006) “Role and Effects of Credit Information Sharing,” in *The Economics of Consumer Credit*, edited by G. Bertola, R. Disney and C. Grant. Cambridge: MIT Press, 347-371.
- [24] Jentzsch, Nicola (2007) *Financial Privacy - An International Comparison of Credit Reporting Systems* (Springer-Verlag, Heidelberg), 2nd. edition.
- [25] Laibson, David, Andrea Repetto, and Jeremy Tobacman (2003) “A Debt Puzzle” in eds. Philippe Aghion, Roman Frydman, Joseph Stiglitz, Michael Woodford *Knowledge, Information, and Expectations in Modern Economics: In Honor of Edmund S. Phelps*, Princeton University Press.
- [26] Ljungqvist, Lars and Thomas Sargent (2004) *Recursive Macroeconomic Theory*, MIT

Press.

- [27] Mann R. J. and J. Hawkins (2007) “Just Until Payday,” *UCLA Law Review*, 54 – 4.
- [28] Manove, Michael, A. Jorge Padilla and Marco Pagano (2001), “Collateral vs. Project Screening: A Model of Lazy Banks,” *Rand Journal of Economics*, 32 (4), 726-744.
- [29] Miller, Margaret J. ed. (2003) *Credit Reporting Systems and the International Economy*, MIT Press, 273-310.
- [30] Morse, Adair (2007) “Payday Lenders: Heroes or Villains?” Mimeo University of Chicago.
- [31] Padilla, A. J., and M. Pagano. “Endogenous Communication Among Lenders and Entrepreneurial Incentives,” *Review of Financial Studies*, 10(1), 205-23.
- [32] Roberts, Michael R. and Amir Sufi, (2009) “Renegotiation of financial contracts: Evidence from private credit agreements” *Journal of Financial Economics*, 93(2), 159-184.
- [33] Sundaresan, Suresh M. (2009) *Fixed income Markets and their Derivatives*, Elsevier, Oxford, UK.
- [34] Townsend, Robert M. (1979) “Optimal Contracts and Competitive Markets with Costly State Verification,” *Journal of Economic Theory*, 21(2), 265-293.
- [35] Zechman, Sarah L. C. (2011), “The relation between voluntary disclosure and financial reporting: Evidence from synthetic leases” *Journal of Accounting Research* (forthcoming)

A Omitted Proofs

Proof of Proposition 1

Proof. The proof is by contradiction.

Consider a set of borrowers, A , that comprises those borrowers who in equilibrium adopt the same repayment schedule p, q . Suppose that this schedule is strictly profitable for the bank. That is,

$$p + \nu q + (1 - \nu)E(\alpha|\alpha \in A) - D > 0.$$

In the interim stage, $t = 1$, a rival bank can offer only to consumers in A (that is, to those that have chosen that particular repayment schedule) a contract with the same p and with q set just above the break even level. That is, $q' = \frac{D-p-(1-\nu)E(\alpha|\alpha \in A)}{\nu}$ this would attract all the borrowers in A and be profitable for the rival bank. Thus the original outcome could not have been an equilibrium providing a contradiction. ■

Proof of Proposition 2

Proof. In order to characterize the equilibrium, we can draw on the revelation principle at $t = 1$ and think of the borrower’s choice from the menu $\{p, q(p)\}$ as a function of her type—that is, we could think of offering a menu $\{p(\alpha), q(\alpha)\}$.

Formally, the Proposition claims that $p(\alpha) = \alpha$, $q(\alpha) = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$, and that all types $\alpha < \frac{D-\nu B}{2}$ liquidate at $t = \frac{1}{2}$. This last follows since the marginal type that liquidates is indifferent between liquidating and receiving 0, or continuing the project and expecting a payoff of

$$\begin{aligned}\nu(B + \alpha - q(\alpha)) &= \nu(B + \alpha - \frac{D - \alpha - (1 - \nu)\alpha}{\nu}) \\ &= \nu B - D + 2\alpha.\end{aligned}$$

Turning to the characterization of $p(\alpha)$ and $q(\alpha)$: As discussed above, Proposition 1, ensures that any meaningful contract on the menu—that is, any contract that is ever taken up in equilibrium—will break even at each stage of the contract; and so will not contain any observable cross-subsidies. The break-even condition, given that the first payment $p = \alpha$ reveals the type of the borrower as α , is that $D = \alpha + \nu q + (1 - \nu)\alpha$, so that in expectation the bank recovers its investment. This determines that the break-even second payment is $q = \frac{D-p-(1-\nu)p}{\nu}$.

Further, incentive-compatibility must be satisfied; that is, a borrower of type α prefers to make a first-period payment $p(\alpha)$ than any other $p(\alpha')$. We analyze the incentive-compatibility condition by considering two deviations: imitating a lower type and imitating a higher type.

Incentive-compatibility condition 1: The contract needs to guarantee that no borrower wants to imitate a lower-quality borrower. Suppose that a borrower of quality α claims to be a lower-quality borrower $\alpha' < \alpha$ by paying a first payment $p = \alpha'$; in that case, her total utility would be

$$(\alpha - \alpha') + \nu(B - \frac{D - \alpha' - (1 - \nu)\alpha'}{\nu} + \alpha).$$

Note that $(\alpha - \alpha')$ is the additional consumption at $t = 1$ from reporting a lower type, while $(B - \frac{D-\alpha'-(1-\nu)\alpha'}{\nu} + \alpha)$ is the net consumption in the good state (which occurs with probability ν) after repaying $q(\alpha')$. Instead, by revealing her own type, she would get $\nu(B - \frac{D-\alpha-(1-\nu)\alpha}{\nu} + \alpha)$. The difference between these two terms is

$$-(1 - \nu)(\alpha - \alpha') < 0,$$

and so it cannot be optimal to claim to be a borrower of a lower type.

Incentive-compatibility condition 2: The contract also needs to guarantee that no borrower wants to imitate a higher-quality borrower by borrowing from the hidden source and paying a first payment $p > \alpha$. Suppose, for contradiction, that a borrower claims to be a higher-quality borrower by paying a first payment $p = \alpha'' > \alpha$ and borrowing $\alpha'' - \alpha$ from the hidden source to fund this payment. The total utility of the borrower would be $\nu(B - \frac{D-\alpha''-(1-\nu)\alpha''}{\nu} - r(\alpha'' - \alpha) + \alpha)$ instead of $\nu(B - \frac{D-\alpha-(1-\nu)\alpha}{\nu} + \alpha)$. The difference between the two is:

$$(2 - \nu - \nu r)(\alpha'' - \alpha),$$

which is negative if and only if $r > \frac{2-\nu}{\nu}$, so this is the necessary and sufficient condition for this incentive-compatibility condition to hold.

Finally, in the first-best, a borrower should be funded if and only if the expected NPV of the project is positive—that is, if and only if

$$-D + \alpha + \nu(B + \alpha) + (1 - \nu)\alpha \geq 0.$$

In the candidate equilibrium described above, banks perfectly sort borrowers and offer break-even deals, so borrowers fully internalize the proceeds of their projects. Therefore, the marginal borrower is precisely the one with $\text{NPV}=0$ that is at the efficient level, and there is no costly, inefficient borrowing from hidden lenders, so overall efficiency is maximized.

Since the contract breaks even on a type-by-type basis, there is no scope for rival banks to make profitable offers at the interim stage. In addition, given that there is efficient liquidation and that the bank breaks even, there is no scope for rival banks to profitably offer any more attractive menus at the initial stage. Indeed, the existence of this menu as a feasible contract offer, suggests that any equilibrium must maximize surplus at the initial stage—in particular, this must involve efficient liquidation and no (inefficient) borrowing from the hidden sector. As a result, the candidate equilibrium is indeed an equilibrium.

It is immediate that the equilibrium involves a borrower of type α paying α to the banking sector in the interim period and $\frac{D-\alpha-(1-\nu)\alpha}{\nu}$, and all types $\alpha < \frac{D-\nu B}{2}$ liquidating at $t = \frac{1}{2}$. ■

Lemma 2 (*Monotonicity of p*) *A borrower that earns a higher interim-period return will pay a (weakly) higher interim repayment. (More formally, for all types $\alpha > \beta$ that do not liquidate, $p(\alpha) \geq p(\beta)$).*

Proof of Lemma 2

Proof. Note that this lemma is instrumental to the proof of Proposition 3.

Suppose that borrowers face the choice between two generic contracts a and b and without loss of generality, we label them so that $p_a > p_b$. The following possibilities are exhaustive: (i) $\alpha > p_a > \beta > p_b > \gamma$; (ii) $p_a > p_b > \alpha > \beta > \gamma$; (iii) $\alpha > \beta > \gamma > p_a > p_b$; (iv) $\alpha > p_a > p_b > \beta > \gamma$; and (v) $\alpha > \beta > p_a > p_b > \gamma$.

In cases (ii), (iii), and (v), the conditions for a borrower of type α to prefer a repayment of schedule a to one of type b are identical to the conditions for a borrower of type β . It remains to consider cases of type (i) and (iv).

In Case (i), a borrower of type β prefers schedule a to schedule b whenever

$$\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu) + r\nu(p_a - \beta), \quad (4)$$

and a borrower of type α prefers schedule b to schedule a whenever the following condition is satisfied:

$$\alpha - p_b + \nu(B + \alpha - p_b - q_b) \geq \alpha - p_a + \nu(B + \alpha - p_a - q_a),$$

or, equivalently,

$$\nu(q_b - q_a) \leq (p_a - p_b)(1 + \nu),$$

which contradicts (4).

Finally, in Case (iv), the condition for a type α borrower to prefer the b schedule is that

$$\nu(q_b - q_a) \leq (p_a - p_b)(1 + \nu),$$

and the condition for a type β borrower to prefer the a schedule is that

$$\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu + r\nu).$$

These two conditions are mutually incompatible.

In all cases, therefore, it cannot be that a borrower of type $\alpha > \beta$ strictly prefers the schedule with the first payment $p_b < p_a$, and the borrower of type β prefers the schedule with the first payment p_a . This completes the proof. ■

Proof of Proposition 3

Proof. To show that with $r < \frac{2-\nu}{\nu}$ there cannot be an equilibrium where a continuum of borrowers are able to separate, we proceed in a similar fashion as with the proof of Proposition 2 and show that if two borrowers that are arbitrarily close to each other are able to separate, we reach a contradiction.

We start by conjecturing an equilibrium menu that achieves the separation of some borrowers in a continuum and then pick two arbitrarily close borrowers α and α' with $\alpha < \alpha'$ and $p(\alpha) \neq p(\alpha')$. By proposition 1 the corresponding second payments must break even, and are given by $q(\alpha) = \frac{D-p(\alpha)-(1-\nu)\alpha}{\nu}$ and $q(\alpha') = \frac{D-p(\alpha')-(1-\nu)\alpha'}{\nu}$. We know by Lemma 2 that $p(\alpha) < p(\alpha')$. These payment schedules have to fulfill similar incentive-compatibility conditions to the ones shown in Proposition 2.

In particular, we can define the two conditions as:

IC1: No borrower of a higher type (α') wants to imitate a borrower of a lower type (α).

IC2: No borrower of a lower type (α) wants to imitate a borrower of a higher type (α').

If there is a continuum of borrowers that can individually separate, at least one of the following situations must be true:

- a) At least two arbitrarily close borrowers are neither consuming nor borrowing from a hidden lender at $t = 1$.
- b) At least two arbitrarily close borrowers are both consuming $t = 1$.
- c) At least two arbitrarily close borrowers are both borrowing from a hidden lender at $t = 1$.

We analyze each of these situations in turn.

a) This part of the equilibrium is characterized by Proposition 2, and we know that IC2 cannot hold in this situation if $r < \frac{2-\nu}{\nu}$.

b) Suppose that there is a borrower α' that fully separates from the rest and is able to consume at $t = 1$ (that is, $p(\alpha') < \alpha'$). Then, there must be a borrower α , such that $\alpha < \alpha'$, that is also able to pay $p(\alpha')$ without borrowing. The utility of borrower α of claiming his own type is

$$\nu(B - \frac{D - p(\alpha) - (1 - \nu)\alpha}{\nu} + \alpha) + (\alpha - p(\alpha)),$$

and the utility of imitating borrower α' is

$$\nu(B - \frac{D - p(\alpha') - (1 - \nu)\alpha'}{\nu} + \alpha) + (\alpha - p(\alpha')).$$

The necessary and sufficient condition for IC2 to hold is, therefore:

$$\nu(B - \frac{D - p(\alpha) - (1 - \nu)\alpha}{\nu} + \alpha) + (\alpha - p(\alpha)) > \nu(B - \frac{D - p(\alpha') - (1 - \nu)\alpha'}{\nu} + \alpha) + (\alpha - p(\alpha')),$$

which simplifies to: $(1 - \nu)(\alpha - \alpha') > 0$, which is always false, so we reach a contradiction.

c) In this case, we start by exploring IC2.

A borrower of a lower type would have a utility of

$$\nu\left(\left(B - \frac{D - p(\alpha) - (1 - \nu)\alpha}{\nu} + \alpha\right) - r(p(\alpha) - \alpha)\right),$$

while claiming to be a higher-type borrower would yield her a utility of

$$\nu\left(B - \frac{D - p(\alpha') - (1 - \nu)\alpha'}{\nu} + \alpha - r(p(\alpha') - \alpha)\right).$$

Subtracting the first term from the second, we get a condition that must be smaller than zero for IC2 to hold.

$$\nu\left(B - \frac{D - p(\alpha') - (1 - \nu)\alpha'}{\nu} - r(p(\alpha') - \alpha) + \alpha\right) - \nu\left(\left(B - \frac{D - p(\alpha) - (1 - \nu)\alpha}{\nu}\right) - r(p(\alpha) - \alpha) + \alpha\right) < 0,$$

which can be simplified as

$$(1 - \nu)(\alpha' - \alpha) + (1 - r\nu)(p(\alpha') - p(\alpha)) < 0.$$

However, in this case, IC1 becomes:

$$\nu\left(B - \frac{D - p(\alpha') - (1 - \nu)\alpha'}{\nu} - r(p(\alpha') - \alpha') + \alpha'\right) - \nu\left(\left(B - \frac{D - p(\alpha) - (1 - \nu)\alpha}{\nu}\right) - r(p(\alpha) - \alpha') + \alpha'\right) > 0.$$

This expression simplifies to

$$(1 - \nu)(\alpha' - \alpha) + (1 - r\nu)(p(\alpha') - p(\alpha)) > 0,$$

which is exactly the opposite condition to the one necessary for IC2. Therefore, when two arbitrarily close borrowers borrow and achieve separation, IC1 and IC2 are mutually incompatible, which poses a contradiction.

This last part of the proposition follows by noting that Lemma 2 implies that for every three borrowers with types α , β , and γ such that $\alpha > \beta > \gamma$ where $p(\alpha) = p(\gamma)$, it must be the case that $p(\alpha) = p(\beta) = p(\gamma)$. ■

Proof of Lemma 1

1) Auxiliary results

First, we define l as the type that is “just indifferent” between liquidating and continuing the project with the $(p(l), q(l))$ repayment schedule that corresponds to the lowest pool of borrowers.

We begin by arguing that $l \leq p(l)$ and do so by contradiction: Conditional on $l > p$, the utility of the indifferent borrower l can be expressed as $l - p(l) + \nu(B + l - q(l))$. Given that liquidating provides utility equal to zero and that the borrower is indifferent, this implies that

$$\nu(B + l - q(l)) + l - p(l) = 0.$$

As $l > p$, then $l - p(l) > 0$. This implies, jointly with the condition above, that $B + l - q(l) < 0$, which violates Assumption 5.

2) Proof by induction

Next we proceed inductively, showing that if the borrower at the bottom of a pool is not consuming, the one at the top of the pool must also not be consuming (otherwise there is a profitable deviating contract at the interim stage). Given that by Proposition 3, that there are a finite number of different pools of borrowers, and, as above $l \leq p(l)$ the result follows by induction.

It is convenient to introduce some notation: We denote the expected surplus to a borrower of type x choosing contract \tilde{p}, \tilde{q} as:

$$S_{\tilde{p}, \tilde{q}}(x) := (x - \tilde{p})1_{x > \tilde{p}} + \nu(B + x - \tilde{q} - r(\tilde{p} - x)1_{\tilde{p} > x}).$$

Consider any interval of types, $(\alpha_{i+1}, \alpha_i]$, that in equilibrium at the interim stage chooses a contract p, q where $\alpha_{i+1} \leq p$. Then, we claim that $\alpha_i \leq p$. Note that we have shown that for the lowest pool $l \leq p(l)$, so that the infimum of the lowest pool satisfies this condition.

For contradiction, consider an alternative contract p', q' with $\alpha_i > p' > p$ and $q' = q - \frac{1}{\nu}(p' - p) + \varepsilon$ then $S_{p', q'}(\alpha_i) = S_{p, q}(\alpha_i) + \varepsilon$ but

$$S_{p', q'}(\alpha_{i+1}) - S_{p, q}(\alpha_{i+1}) = -(\nu r - 1)(p' - p) - \nu \varepsilon.$$

For ε small and positive, $S_{p', q'}(\alpha_i) > S_{p, q}(\alpha_i)$ but $S_{p', q'}(\alpha_{i+1}) < S_{p, q}(\alpha_{i+1})$, it follows that higher types prefer the contract (p', q') to the contract (p, q) , and so the contract (p', q') is strictly profitable.

This completes the inductive step and the result. ■

Proof of Proposition 4

Proof. Following Lemma 1, any equilibrium must be such that for the i^{th} pool the first period payment, p_i , is such that $p_i \geq \alpha_{i-1}$.

We argue that $p_i = \alpha_{i-1}$.

Suppose that the equilibrium includes an existing contract p_i, q_i with an associated pool of borrowers $(\alpha_i, \alpha_{i-1}]$ such that $p_i > \alpha_{i-1}$. We argue that a rival bank could propose an offer p', q' to borrowers $(\alpha_i, \alpha_{i-1}]$ that is strictly profitable, leading to a contradiction.

For a generic agent, β , in the pool,

$$\begin{aligned} S_{p_i, q_i}(\beta) - S_{p', q'}(\beta) &= \nu(B + \beta - q_i) - \nu r(p_i - \beta) - \nu(B + \beta - q') + \nu r(p' - \beta) \\ &= \nu(q' - q_i + r(p' - p_i)). \end{aligned}$$

Note that $\nu(q' - q_i + r(p' - p_i))$ is independent of β . So either all of the agents switch or none of them do. In particular, all of them switch if $q_i + r(p_i - p) > q'$. There is a profitable deviation as long as $q' > \frac{D - p' - (1 - \nu)E(\alpha | \alpha \in (\alpha_{i-1}, \alpha_i])}{\nu}$. In particular, a necessary condition for everyone switching and the deviating contract being profitable is

$$\frac{D - p_i - (1 - \nu)E(\alpha | \alpha \in (\alpha_{i-1}, \alpha_i])}{\nu} + r(p_i - p') - \frac{D - p' - (1 - \nu)E(\alpha | \alpha \in (\alpha_{i-1}, \alpha_i])}{\nu} > 0,$$

or, equivalently,

$$(p_i - p') \frac{r\nu - 1}{\nu} > 0.$$

Therefore whenever $p_i > \alpha_{i-1}$ we can find a p' such that $p_i > p' > \alpha_{i-1}$ that is a profitable deviation. Since $p_i \geq \alpha_{i-1}$, the only possibility is that $p_i = \alpha_{i-1}$ and the associated breakeven $q_i = \frac{D - \alpha_{i-1} - (1 - \nu)E(\alpha | \alpha \in [\alpha_i, \alpha_{i-1}])}{\nu}$.

Next, note that each threshold type α_i must be indifferent between the i^{th} pool and the $i + 1^{th}$ pool (for $i = 1, \dots, n - 1$) and between the n^{th} pool and liquidating for α_n (if α_n , otherwise the α_n must strictly prefer taking the loan). Thus for $i = 1, \dots, n - 1$

$$\nu(B + \alpha_i - q_i - r(\alpha_{i-1} - \alpha_i)) = \nu(B + \alpha_i - q_{i+1})$$

and

$$\begin{aligned} \nu(B + \alpha_n - q_n - r(\alpha_{n-1} - \alpha_n)) &= 0. \\ \text{or } \nu(B + \alpha_n - q_n - r(\alpha_{n-1} - \alpha_n)) &> 0 \text{ if } \alpha_n = 0. \end{aligned}$$

Substituting for the second period payments q_i and q_{i+1} and rearranging the expressions leads to the expressions in the statement of the Proposition. ■

Proof of Proposition 5

Proof. Part 1 follows immediately from Proposition 2.

Next, note that if the equilibrium involves more than one pool of borrowers then (1) from Proposition 4 applies and so

$$\frac{\nu r - 1}{1 - \nu}(1 - \alpha_1) = E(\alpha | \alpha \in [\alpha_i, \alpha_{i-1}]) - E(\alpha | \alpha \in [\alpha_{i+1}, \alpha_i]) = \frac{1 + \alpha_1}{2} - \frac{\alpha_1 + \alpha_2}{2}, \quad (5)$$

where the second equality follows from the uniform distribution assumption. This can be re-written as $\frac{1 - \alpha_1}{1 - \alpha_2} = \frac{1 - \nu}{2(\nu r - 1)}$. By definition $\alpha_1 > \alpha_2$ so $1 < \frac{1 - \nu}{2(\nu r - 1)}$ or equivalently $\frac{3 - \nu}{2\nu} > r$.

It is immediate, therefore, that if $r < \frac{3 - \nu}{2\nu}$ assuming two pools of borrowers with $1 > \alpha_1 > \alpha_2$ we reach a contradiction. Consequently, there can be at most a single pool of borrowers and (2) determines the lowest borrower in this pool as $\alpha_n = \frac{\nu + 2D - 2B\nu + 2r\nu - 3}{\nu + 2r\nu + 1}$. It is easy to verify that $\frac{D - \nu B}{2} > \frac{\nu + 2D - 2B\nu + 2r\nu - 3}{\nu + 2r\nu + 1} > 0$ in this range of r so some inefficient project do not get liquidated. It is also easy to calculate the break-even value of q as $\frac{B - 4r + \nu + 2D - B\nu + 2r\nu + 2rD - 3}{\nu + 2r\nu + 1}$. This is the only feasible solution and breaks even establishing Part 3.

Finally, turning to part 2, first note that following Proposition 4, a candidate must satisfy (1) which can be written as $\frac{\alpha_i - \alpha_{i+1}}{\alpha_i - \alpha_{i+2}} = \frac{1 - \nu}{2(\nu r - 1)}$ and (2) which can be written as: $\frac{\nu r - 1}{1 - \nu}(\alpha_{n-1} - \tilde{\alpha}_n) = \frac{\tilde{\alpha}_n + \alpha_{n-1}}{2} - \frac{D - \nu B}{1 - \nu} + \tilde{\alpha}_n \frac{1 + \nu}{1 - \nu}$.

We proceed to characterizing a proposed n -tranche candidate equilibrium, and then show that welfare is increasing in n so that competition in the first stage leads to a candidate with $n \rightarrow \infty$. Finally we show that the analysis in the limit corresponds to the expressions in the statement of the proposition.

Characterizing an n -tranche equilibrium

We introduce some additional notation and denote the i^{th} threshold in the n -tranche equilibrium by $\alpha_i^{(n)}$ where there is ambiguity about the number of tranches considered.

Following Proposition 4, an n -tranche equilibrium must satisfy (1) which can be written as $\frac{\alpha_i^{(n)} - \alpha_{i+1}^{(n)}}{\alpha_i^{(n)} - \alpha_{i+2}^{(n)}} = \frac{1 - \nu}{2(\nu r - 1)}$ for $i = 0, 1, \dots, n - 2$ and (2) which can be written as: $\frac{\nu r - 1}{1 - \nu}(\alpha_{n-1}^{(n)} - \tilde{\alpha}_n^{(n)}) = \frac{\tilde{\alpha}_n^{(n)} + \alpha_{n-1}^{(n)}}{2} - \frac{D - \nu B}{1 - \nu} + \tilde{\alpha}_n^{(n)} \frac{1 + \nu}{1 - \nu}$.

It is convenient to simplify expressions a little by setting $\rho := \frac{1 - \nu}{2(\nu r - 1)}$ and $\gamma := \frac{1 - \rho}{\rho} = \frac{\nu + 2r\nu - 3}{1 - \nu}$.

The (1) equations can be written as $\frac{\alpha_i - \alpha_{i+1}}{\alpha_i - \alpha_{i+2}} = \rho$

Note that $\alpha_i^{(n)} - \alpha_{i+2}^{(n)} = \alpha_i^{(n)} - \alpha_{i+1}^{(n)} + \alpha_{i+1}^{(n)} - \alpha_{i+2}^{(n)}$ so that (1) can be written as:

$$\alpha_i^{(n)} - \alpha_{i+1}^{(n)} = \frac{1 - \rho}{\rho} (\alpha_{i-1}^{(n)} - \alpha_i^{(n)}) = \left(\frac{1 - \rho}{\rho}\right)^i (1 - \alpha_1^{(n)}) = \gamma^i (1 - \alpha_1^{(n)}) \quad (6)$$

In addition, we can write

$$\begin{aligned} 1 - \alpha_n^{(n)} &= 1 - \alpha_1^{(n)} + (\alpha_1^{(n)} - \alpha_2^{(n)}) + \dots + (\alpha_{n-1}^{(n)} - \alpha_n^{(n)}) \\ &= 1 - \alpha_1^{(n)} + \gamma(1 - \alpha_1^{(n)}) + \dots + \gamma^{n-1}(1 - \alpha_1^{(n)}) \\ &= (1 - \alpha_1^{(n)}) \frac{1 - \gamma^n}{1 - \gamma} \end{aligned} \quad (7)$$

Following Proposition 4, the remaining condition that defines the solution to the equations characterizing an n -tranche equilibrium is that either $\alpha_n^{(n)} = 0$ with $B + \alpha_n^{(n)} - q_n^{(n)} - r(\alpha_{n-1}^{(n)} - \alpha_n^{(n)}) > 0$ or $B + \alpha_n^{(n)} - q_n^{(n)} - r(\alpha_{n-1}^{(n)} - \alpha_n^{(n)}) = 0$.

We first prove by contradiction that $\alpha_n^{(n)} > 0$. Suppose that $\alpha_n^{(n)} = 0$ then $B + \alpha_n^{(n)} - \frac{D - \alpha_{n-1}^{(n)} - (1-\nu)\frac{\alpha_n^{(n)} + \alpha_{n-1}^{(n)}}{2}}{\nu} - r(\alpha_{n-1}^{(n)} - \alpha_n^{(n)}) > 0$ so $B - \frac{D - \alpha_{n-1}^{(n)} - (1-\nu)\frac{\alpha_{n-1}^{(n)}}{2}}{\nu} - r\alpha_{n-1}^{(n)} > 0$ or, equivalently, $\nu B - D - \alpha_{n-1}^{(n)}(\frac{\nu+2r\nu-3}{2}) > 0$ which is impossible since $\nu B - D < 0$, $r \geq \frac{3-\nu}{2\nu}$ and $\alpha_{n-1}^{(n)} > 0$.

Thus $\alpha_n^{(n)}$ satisfies $B + \alpha_n^{(n)} - \frac{D - \alpha_{n-1}^{(n)} - (1-\nu)\frac{\alpha_n^{(n)} + \alpha_{n-1}^{(n)}}{2}}{\nu} - r(\alpha_{n-1}^{(n)} - \alpha_n^{(n)}) = 0$. Following (6) and (7) $\alpha_{n-1}^{(n)} = \alpha_n^{(n)} + \gamma^{n-1}(1 - \alpha_1^{(n)}) = \alpha_n^{(n)} + (1 - \alpha_n^{(n)}) \frac{\gamma^{n-1} - \gamma^n}{1 - \gamma^n}$. Substituting for $\alpha_{n-1}^{(n)}$ in the previous equation allows us to characterize $\alpha_n^{(n)}$.

$$\alpha_n^{(n)} = \frac{2(D - \nu B) + (\nu + 2r\nu - 3) \frac{\gamma^{n-1} - \gamma^n}{1 - \gamma^n}}{4 + (\nu + 2r\nu - 3) \frac{\gamma^{n-1} - \gamma^n}{1 - \gamma^n}} \quad (8)$$

Welfare in an n -tranche equilibrium

The expressions in (2), (6) and (7) fully characterize any n -tranche candidate equilibrium. However, bank competition in the first stage ensures that whichever value of n maximizes welfare is the equilibrium value of n . We therefore proceed by calculating welfare associated with an n -tranche candidate equilibrium.

First, the welfare loss of hidden borrowing in each tranche i is

$$(\nu r - 1) \int_{\alpha_i^{(n)}}^{\alpha_{i-1}^{(n)}} (\alpha_{i-1}^{(n)} - x) dx = \frac{\nu r - 1}{2} \left(\alpha_i^{(n)} - \alpha_{i-1}^{(n)} \right)^2. \quad (9)$$

It follows that the total welfare loss of hidden borrowing in the n -tranche equilibrium is

$$\begin{aligned} \frac{r\nu - 1}{2} \sum_{i=0}^{n-1} (\alpha_i^{(n)} - \alpha_{i+1}^{(n)})^2 &= \frac{r\nu - 1}{2} \sum_{i=0}^{n-1} \gamma^{2i} (1 - \alpha_1^{(n)})^2 \\ &= \frac{r\nu - 1}{2} \frac{1 + \gamma^n}{1 - \gamma^n} \frac{1 - \gamma}{1 + \gamma} (1 - \alpha_n^{(n)})^2. \end{aligned} \quad (10)$$

Overall, therefore, the welfare associated with an n-tranche candidate equilibrium is given by

$$W(n) = \int_{\alpha_n^{(n)}}^1 (\nu B - D + 2x) dx - \frac{r\nu - 1}{2} \frac{1 + \gamma^n}{1 + \gamma} \frac{1 - \gamma}{1 - \gamma^n} (1 - \alpha_n^{(n)})^2. \quad (11)$$

Note, that following (8),

$$\frac{d\alpha_n^{(n)}}{dn} = (\ln \gamma) \frac{2\gamma^{n-1} (1 - \gamma) (2 - D - \nu B) (\nu + 2r\nu - 3)}{(\gamma^n - \gamma^{n-1}\nu + 3\gamma^{n-1} + \gamma^n\nu - 2r\gamma^{n-1}\nu + 2r\gamma^n\nu - 4)^2} < 0 \quad (12)$$

where the inequality follows since $\gamma < 1$ and so $\ln \gamma < 0$. Note also that as $n \rightarrow \infty$ then $\alpha_n^{(n)} \rightarrow \frac{D-\nu B}{2}$. $\int_A^1 (\nu B - D + 2x) dx$ is decreasing in A for $1 > A > \frac{D-\nu B}{2}$ (reflecting that above the point of efficient liquation more surplus is generated by increasing the range of projects conducted) and since $\frac{d\alpha_n^{(n)}}{dn} < 0$, it follows that $\int_{\alpha_n^{(n)}}^1 (\nu B - D + 2x) dx$.

Consequently a sufficient condition that ensures $W(n)$ increases in n is that $\frac{1+\gamma^n}{1-\gamma^n} (1 - \alpha_n^{(n)})^2$ is decreases in n .

$$\begin{aligned} \frac{d}{dn} \left(\frac{1 + \gamma^n}{1 - \gamma^n} (1 - \alpha_n^{(n)})^2 \right) &= \frac{d}{dn} \left(\frac{4(2 - D + \nu B)^2 (1 - \gamma^{2n})}{(4(1 - \gamma^n) + (\nu + 2r\nu - 3)(\gamma^{n-1} - \gamma^n))^2} \right) \\ &= 8 (\ln \gamma) (2 - K)^2 \frac{4\gamma^n (1 - \gamma^n) - \gamma^{n-1}(\nu + 2r\nu - 3)(1 - \gamma)}{(\gamma^{n-1}(\nu + 2r\nu - 3)(1 - \gamma) + 4(1 - \gamma^n))^3} \\ &< 0 \end{aligned} \quad (13)$$

where the last inequality follows on substituting for γ in the denominator of the fraction to obtain that $4\gamma^n (1 - \gamma^n) - \gamma^{n-1}(\nu + 2r\nu - 3)(1 - \gamma) = 2 \left(\frac{\nu+2r\nu-3}{1-\nu} \right)^n \left(\nu + r\nu - 2 \left(\frac{\nu+2r\nu-3}{1-\nu} \right)^n \right)$ and noting that $\nu + r\nu - 2 \left(\frac{\nu+2r\nu-3}{1-\nu} \right)^n$ increases in n and that at $n = 1$, $\nu + r\nu - 2 \left(\frac{\nu+2r\nu-3}{1-\nu} \right) = \frac{(\nu+3)(2-\nu-r\nu)}{1-\nu} > 0$.

This establishes that $W(n)$ increases in n .

Characterizing the limiting equilibrium

The expressions in (2), (6) and (7) fully characterize an n-tranche equilibrium. Taking the limit as $n \rightarrow \infty$, $\alpha_n^{(n)} \rightarrow \frac{D-\nu B}{2}$ and so (7) can be written as $1 - \frac{D-\nu B}{2} = (1 - \alpha_1) \frac{1}{1-\gamma}$ and so

$$\alpha_1 = 1 - \frac{2(2 - \nu - r\nu)}{1 - \nu} \left(1 - \frac{D - \nu B}{2} \right), \quad (14)$$

substituting this in (7) and substituting for γ then completes the characterization in the statement of the proposition. ■

Proof of Corollary 1

Proof. It is immediate that

$$\frac{d\alpha_i}{dr} = \left(1 - \frac{D - \nu B}{2} \right) 2n\nu \frac{\left(\frac{(\nu+2r\nu-3)}{1-\nu} \right)^i}{\nu + 2r\nu - 3} > 0.$$

■ **Proof of Corollary 2**

Proof. In the range $r > \frac{2-\nu}{\nu}$ welfare is constant and equal to

$$W = \int_{\frac{D-\nu B}{2}}^1 (\nu B - D + 2x) dx = \left(1 - \frac{D - \nu B}{2}\right)^2$$

In the range $\frac{2-\nu}{\nu} \geq r > \frac{3-\nu}{2\nu}$ then

$$\begin{aligned} W &= \lim_{n \rightarrow \infty} (1 - \alpha_n^{(n)})(\alpha_n^{(n)} + 1 - (D - \nu B)) - \frac{r\nu - 1}{2} \frac{1 + \gamma^n}{1 + \gamma} \frac{1 - \gamma}{1 - \gamma^n} (1 - \alpha_n^{(n)})^2 \\ &= (1 - \frac{D - \nu B}{2})(\frac{D - \nu B}{2} + 1 - (D - \nu B)) - \frac{r\nu - 1}{2} \frac{1 - \frac{\nu+2r\nu-3}{1-\nu}}{1 + \frac{\nu+2r\nu-3}{1-\nu}} (1 - \frac{D - \nu B}{2})^2 \\ &= \frac{1+r}{2} \nu \left(1 - \frac{D - \nu B}{2}\right)^2 \end{aligned}$$

Note that at $\frac{2-\nu}{\nu}, \frac{1+r}{2} = 1$.

Finally in the range $\frac{3-\nu}{2\nu} \geq r \geq \frac{1}{\nu}$, we have

$$\begin{aligned} W &= \int_{\frac{\nu+2D-2B\nu+2r\nu-3}{\nu+2r\nu+1}}^1 (\nu B - D + 2x) dx - (\nu r - 1) \int_{\frac{\nu+2D-2B\nu+2r\nu-3}{\nu+2r\nu+1}}^1 (1-x) dx \\ &= 8\nu \frac{r+1}{(\nu+2r\nu+1)^2} \left(1 - \frac{D - \nu B}{2}\right)^2. \end{aligned}$$

Note that $r = \frac{3-\nu}{2\nu}, 8\nu \frac{r+1}{(\nu+2r\nu+1)^2} = \frac{1+r}{2} \nu = \frac{3+\nu}{4}$.

Trivially, W is constant when $r > \frac{2-\nu}{\nu}$ and increasing in r when $\frac{2-\nu}{\nu} > r > \frac{3-\nu}{2\nu}$. In the range $\frac{3-\nu}{2\nu} > r \geq \frac{1}{\nu}$ then $\frac{d}{dr} W = \frac{d}{dr} (8 \frac{(\nu+r\nu+2)}{(\nu+2r\nu+3)^2} (1 - \frac{D - \nu B}{2})^2) = 2\nu (1 - 3\nu - 2r\nu) \frac{(2-D+B\nu)^2}{(\nu+2r\nu+1)^3}$ which has the same sign as $(1 - 3\nu - 2r\nu)$ and is positive when $\frac{1-3\nu}{2\nu} > r$. Finally $\frac{1}{\nu} > \frac{1-3\nu}{2\nu}$ establishing the monotonicity of W throughout the relevant range. ■