Standing on the Shoulders of Babies: Dominant Firms and Incentives to Innovate

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Abstract. Critics of Microsoft and Google's dominance claim these companies are nothing but "giants standing on the shoulders of babies," whose dominance destroys the incentives for entrants to innovate. By contrast, pro-Microsoft and pro-Google analysts stress the benefits of large, innovative firms. We analyze the validity of these competing claims in a model of R&D and product market competition between a dominant firm and a small rival. An increase in firm dominance, which we measure by a premium in consumer valuation, increases the dominant firm's incentives but decreases the rival firm's incentives for R&D. We provide sufficient conditions such that the positive effect on the dominant firm is mostly infra-marginal, whereas the negative effect on the rival firm is mostly marginal. As a result, the R&D encouragement effect is lower than the R&D discouragement effect; and if innovation is sufficiently important then firm dominance also decreases consumer and social surplus. We also provide conditions such that an increase in firm dominance increases the probability of innovation, essentially because the transfer of innovation incentives form the rival firm to the dominant firm reduces the probability of duplicative R&D efforts.

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1. Introduction

The economic analysis of innovation incentives has received considerable attention in both theoretical and empirical industrial organization. Some of the main questions that have been addressed include: (a) the relation between market structure and innovation incentives; (b) the relation between the private and the social optimum innovation rates; and (c) the importance of innovation spillovers.[1]

The recent evolution of various high-tech industries has brought forth a new set of issues, namely the positive and normative implications of market dominance in terms of innovation incentives, especially in a context where imitation is relatively easy. For example, one might argue that much of Microsoft's economic advance has primarily resulted from imitation and gradual innovation, not from drastic innovation: Excel owes much to Lotus 123; Microsoft Word followed the lead of several other popular word processors; Microsoft's Power Point was inspired by programs such as Harvard Graphics or Freelance; and so forth.

In some respects, Microsoft's products have brought about a significant benefit to consumers. If consumers buy them, it is largely because they value them more than competing products. However, one fears that, in the long run, such imitation-and-improvement strategy may decrease the incentives for other firms to innovate:

> Today, Microsoft so completely dominates each of these markets that few venture capitalists would even consider funding new programs that would seek to dislodge it. Microsoft is not only successful, it seems unbeatable in the PC applications markets.[2]

Nor is this problem limited to Microsoft:

> In some niches of the software business, Google is casting the same sort of shadow over Silicon Valley that Microsoft once did. "You've got people who don't even feel they can launch a product for fear that Google will get in."[3]

What the Microsoft and the Google examples have in common is the negative effect of firm dominance on the innovation incentives by small entrants: it's a case of "standing on the shoulders of babies."[4]

However, innovation incentives by small entrants is not the only relevant effect from a consumer and welfare point of view. One might argue that firm dominance is the result of a superior offering by the dominant firm, in which case what leads to firm dominance also leads to higher consumer and social welfare. For example, by controlling the operating system and other applications, Microsoft is better able to integrate each piece in the whole PC/Windows platform in a way that adds value to its applications. In the words of Jim Allchin, Senior Microsoft Vice President,

1. See Reinganum (1989) for a survey of the main theoretical approaches and Cohen and Levin (1989) for a survey of the empirical evidence. Below we provide more detailed references.
2. "What To Do About Microsoft?" by Ralph Nader and James Love, *Le Monde Diplomatique*, November 1997.
3. "Microsoft And Google Set to Wage Arms Race," by Steve Lohr and Saul Hansell, *The New York Times*, May 2, 2006.
4. We are unaware of the origins of this paraphrase; it was heard recently at a roundtable that included the chief economists of Microsoft and Google. The original quote goes back (at least) to Sir Isaac Newtown: "If I have seen further it is by standing on the shoulders of giants." See also Scotchmer (1991).

> Innovation through integration is the engine that drives the computer industry, bringing the benefits of computing to hundreds of millions of people.[5]

Similarly, search engines algorithms are subject to considerable learning economies: the more searches are performed, the better the algorithm becomes. For this reason, Google's dominance produces a positive effect that may increase consumer and social welfare. Finally, the ability that a dominant firm has to "stand on the shoulders of babies" may give it additional incentives to innovate.

Our purpose in this paper is to evaluate the relative importance of these various claims against and in favor of firm dominance in an innovation context. We consider a two-stage model where two firms invest in R&D and then compete in prices. Our model has three important features. First, we assume that property rights are imperfect, so that a lagging firm can (imperfectly) imitate the leader. Second, firms' R&D levels are substitutes from a social point of view and strategic substitutes from a firm point of view. Third, we assume that one of the firms is dominant, in the sense that, everything else constant, its product is worth more in the eyes of consumers.

Modeling of a dominant firm is the most distinctive feature of our paper with respect to previous approaches to innovation and product market competition; it thus warrants some additional discussion. We do not assume that the dominant firm is better at performing R&D or imitating other firms' R&D. Rather, we assume that, for a given set of product quality outcomes resulting from innovation and imitation, one of the firms (the dominant firm) is able to add value to its product in a way that other firms are not. We model this in a reduced form by assuming consumers are willing to pay an extra amount, b, for the dominant firm's product (everything else constant).

There may be various underlying sources of the value b, the parameter that defines firm dominance. It may be that the dominant firm has a more popular brand or greater reputation for product quality. It may also be the case that the dominant firm possesses complementary products which better interact with its product than with the rivals' products. For example, Microsoft's knowledge of the Windows operating systems allows the company to better integrate new software into the Windows platform. Similarly, Google's superior search algorithm allows it to add more value to existing applications.[6]

While we are ultimately interested in the welfare effects of firm dominance, we start by looking at the effects of firm dominance on the probability of innovation (Section 3). An increase in firm dominance, which we measure by a premium in consumer valuation, increases the dominant firm's incentives and decreases the rival firm's incentives for R&D. These changes influence the probability of innovation through two effects: changes in total R&D effort by the two firms and changes in how this total is distributed between the two firms.

For a given total level of research effort, the shift in research effort from the rival firm to the dominant firm is a good thing as it decreases the likelihood of duplicate innovation: in the limit, it is better to have one firm innovating with probability r than two firms innovating with probability $r/2$ each (from the point of view of the innovation probability). However, the shift in research effort is not one-to-one. The dominant firm's benefit from

5. Allchin direct testimony in Microsoft case, para 41.

6. In Section 6 we contrast the case when consumers pay a premium b for the dominant firm's product to the case when they place a value discount d on the rival firm's product. Some of our result remain valid, some do not.

increased dominance is more infra-marginal than marginal when compared to the rival firm's disincentive. As a result, total research effort decreases when firm dominance increases.

Which of the two effects dominates depends on the degree of size of the barriers to imitation (which we summarize in one parameter that reflects barriers such as obstacles to imitation, strength of property rights, and so on). When the barriers to imitation are high, the equilibrium levels of research are high. We show this implies that the total effort effect is low, whereas the duplication effect is high. As a result, the probability of innovation increases with firm dominance. When imitation barriers are low, the equilibrium levels of research are low. This implies that the duplication effect is negligible, whereas the total effort effect is high. As a result, the probability of innovation decreases with firm dominance.

These effects on the probability of innovation carry over to consumer and social surplus (Sections 4 and 5). In fact, if the gains from innovation are sufficiently high, then consumer and social surplus increase if and only if the probability of innovation increases. However, there are cases when the probability of innovation increases but consumer surplus does not. The reason is that firm dominance implies a lower probability of duplication. Reducing duplication is good in terms of probability of innovation but bad from the consumers' point of view (a second innovator increases consumer surplus through price competition). There are also cases when the probability of innovation does not increase but social surplus does. The reason is that firm dominance has a positive direct effect on social value, in addition to the indirect effect through changes in the probability of innovation.

■ **Policy implications.** Our model and our results depend on two critical parameters: the dominant firm's advantage, b; and the level of imitation barriers, l. Our results pertain to the comparative statics with respect to changes in b (both marginal and discrete changes). We provide necessary and sufficient conditions such that an increase in b helps innovation, consumers surplus, or social surplus. Broadly speaking, if imitation barriers are low, then an increase in firm dominance has a negative effect on innovation and welfare. This corresponds to the view that firms like Microsoft and Google "stand on the shoulders of babies," thus reducing overall innovation incentives. In other words, if the dominant firm can easily copy an innovation brought to market by a small entrant, then it will — and such threat creates significant disincentives for innovation. In this context, policies that reduce the value of b may have a positive impact on innovation and welfare. For example, in the European Microsoft case the European Commission ordered Microsoft

> As regards interoperability, Microsoft is required, within 120 days, to disclose complete and accurate interface documentation which would allow non-Microsoft work group servers to achieve full interoperability with Windows PCs and servers.[7]

In the case of Google, similar policies might be imposed that create greater transparency in the search and rank algorithms.

Public policy is also very important with respect to the level of imitation barriers. Some imitation barriers result from the nature of the technology (how easy it is to reverse engineer) as well as other "natural factors." However, intellectual property rights also play an important role. For example, the recent debate over the appropriate level of patent protection for some of Apple's patents (can you patent a "look and feel"?) directly reflects

7. European Commission, "Commission concludes on Microsoft investigation, imposes conduct remedies and a fine," http://europa.eu/rapid/press-release_IP-04-382_en.htm, visited October 2012.

on the value of imitation barriers. Our results do not address the effects of changes in such imitations barriers. However, the value of these barriers plays an important role in our assessment of the effects of firm dominance. Our main point is that, if replicating the "look and feel" of a given app is easy (from an engineering and legal point of view), then not only are innovation incentives low but also firm dominance creates additional problems from an innovation and welfare point of view.[8]

■ **Related literature.** As mentioned earlier, there is an extensive theoretical literature that examines the relation between market structure and innovation incentives. Most authors consider a winner-take-all model.[9] Others consider two-stage models, with independent R&D investments in the first period and product market competition in the second period.[10] We follow the latter tradition, though we add an intermediate stage of (possible) imitation. We examine some of the questions that were the focus in the previous literature, namely the relation between market structure and innovation incentives; and the possibility of spillovers through imitation. The main difference in our approach is that we explicitly consider the level of firm dominance and the extent of imitation barriers, in particular how these two measures interact in terms of welfare implications.

There is a recent related literature looking at R&D incentives when a dominant firm owns an essential component. Farrell and Katz (1999) show that the dominant firm may have excessive incentives to invest in the complementary component and thus squeeze out the competition. Choi and Stefanadis (2001) argue that tying the essential and complementary components may create a barrier to entry: it requires a firm to be successful in both markets in order to successfully enter. Gilbert and Riordan (2006) show that tying leads to foreclosure and possibly an increase in social welfare. In these papers, the focus is on the interaction between tying decisions and the incentives to innovate. By contrast, we focus on the effects of increased firm dominance. We abstract from the source of the dominant firm's advantage (or rival firm's disadvantage), assuming instead that consumers are willing to pay more for the dominant firm's product. As in Farrell and Katz (1999), Choi and Stefanadis (2001), Gilbert and Riordan (2006), we consider a framework with R&D decisions followed by price competition.[11]

Also related to our paper is the literature on imitation and innovation incentives: Gallini (1992), Cadot and Lippman (1995), Bessen and Maskin (2000). These papers focus primarily on the effect of imitation barriers on the incentives to innovate. For example, Cadot and Lippman show that innovation incentives may be non-monotonic on imitation barriers. By contrast, our focus is on the effects of firm dominance on innovation incentives for a given

8. Regarding the comparative statics with respect to the level of imitation barriers, our model indicates that an increase in such barriers increases innovation incentives and consumer welfare. We do not wish to put much weight on these results because (a) the comparative statics with respect to innovation barriers is not the main issue in the paper, and (b) many previous papers have addressed this issue in more complete models of R&D and product market competition.
9. In some cases, the winner is the firm that pays the most (e.g., Gilbert and Newbery, 1982); in other cases, the probability of winning is smoothly increasing in investment, either as a reduced form (Futia, 1980) or through the explicit modeling of a race in time (Loury, 1979; Dasgupta and Stiglitz, 1980a,b; Reinganum, 1981, 1982, 1983).
10. See for example D'Aspremont and Jacquemin (1988).
11. One difference is that we assume uncertain R&D success and obtain a unique equilibrium. By contrast, Gilbert and Riordan (2006) consider a deterministic R&D technology and obtain multiple equilibria, some in mixed strategies.

level of imitation barriers (including, among other, innovation lags).

We conclude our literature review with two papers that are closely related to ours: Miller (2006) and Segal and Whinston (2007). Miller (2006) considers a natural monopoly with uncertainty about demand and asymmetric potential entrants. In this situation, the disadvantaged potential entrant may have very low incentives to enter. In fact, if entry is successful — thus signalling that demand is favorable — then the advantaged entrant will follow suit. This may result in an equilibrium with too little entry. The structure of Miller's model is quite different from ours. However, the qualitative implications of the two models are similar.

Segal and Whinston (2007) study the effects of antitrust policy in industries with continual innovation. They show that antitrust policies that restrict incumbent behavior toward new entrants may have conflicting effects on innovation incentives: they raise the profits of new entrants but lower those of continuing incumbents. They show that in some cases the trade-off may not arise, so that policies that protect entrants necessarily increase innovation. Our paper is related to theirs in that there is a trade-off between incumbent and entrant incentives. The main difference is that we explicitly model a "permanent" asymmetry given by one of the firm's dominance.

2. Model

Suppose that two firms, 0 and 1, simultaneously invest in R&D. In order to achieve a probability of success r_i a firm must pay a cost $\frac{1}{2}\,r_i^2$.[12] If successful (probability r_i), firm i gets a product of quality q_H. If unsuccessful (probability $1 - r_i$) firm i's product is worth q_L. Let $g \equiv q_H - q_L$ be the gain from technical progress. The success probabilities of the two firms are independent.

Consumers are willing to pay q_1 for firm 1's product and $q_0 + b$ for firm 0's, where $q_i$is firm i's quality level. That is, firm 0 has an advantage, perhaps because it controls a complementary product such as an operating system. After learning the value of q_i, each firm has the option of imitating its rival. By imitating firm j, firm i's quality becomes $q_i = q_j - l$. The value of l measures imitation barriers. Some of these barriers correspond to the nature of technology (how easy it is to reverse engineer), some to legal barriers (how strong is IP protection).

Once the values of q_i have been determined (including, possibly, imitation), firms competein prices. For simplicity, we assume there is one consumer buying one unit from one of thefirms, whichever firm maximizes the difference between valuation and price. We normalizeproduction costs to zero.

Throughout the paper, we make the following assumptions about the parameters b, l, and g.

Assumption 1. $b < l < 1 < g$

The assumption that l is lower than g corresponds to the assumption that barriers to imitation are less then infinite. In other words, it is optimal for a firm with a lower interim

12. Given our assumption of a quadratic cost function, there is no additional loss of generality in assuming it to be $\frac{1}{2}\,r_i^2$. By an appropriate choice of units, we can turn a more general function $\frac{k}{2}\,r_i^2$ into the one we consider.

Table 1
Timing of the model

1. Firms simultaneously invest in R&D	
	Firms observe R&D outcome
	Laggard imitates leader (if applicable)
2. Firm simultaneously set prices	
	Consumer chooses one of the firms and buys one unit

Table 2
Notation

Variable	Description
q	Quality level
g	Innovation gain: $g \equiv q_H - q_L$
l	Imitation barrier
b	Firm 0's extra value
r_i	Firm i's level of R&D, $i = 0, 1$
V_i	Firm i's expected value, $i = 0, 1$
CS	Consumer surplus
SS	Social surplus

quality q to imitate its rival. We therefore simplify the second stage of the game by assuming that, in the case where only one firm is successful in R&D, the unsuccessful firm imitates the successful one.[13]

The assumption that $b < l$ allows for the possibility that firm 1 be the market leader (in product quality). If $b > l$, then firm 1 could never be the leader, even if firm 1's R&D were successful and firm 0's failed. If this were the case, then firm 1 would not invest in R&D at all.[14]

The assumption that $l < 1$ ensures that our equilibria are interior.[15] Finally, the assumption $g > 1$ reflects the idea that innovation is important.

The timing of the game is summarized in Table 1, whereas Table 2 lists the model's notation.

13. In equilibrium, the imitator makes zero profits. This raises the issue of robustness with respect to an infinitesimal but strictly positive imitation cost. In the Appendix, we show that the subgame equilibrium we choose can be seen as the limit of a sequence of games with some uncertainty about consumer preferences. Along the sequence of games that converge to the game we consider (no uncertainty), the non-innovating firm strictly prefers to imitate (and receives a strictly positive expected profit).

14. In this case, firm 0 would also underinvest in R&D from the point of view of social surplus maximization, but this would just be because intellectual property protection is imperfect ($l < 1 < g$). There would not be any strategic effects.

15. If $l > 1$ then there is a corner solution in which $r_0 = 1$ and $r_1 = 0$. If $l - b > 1/(1 - b)$ then there is a second corner solution in which $r_1 = 1$ and $r_0 = b$.

Table 3
Consumer surplus, social surplus and firm gross profit as a function of R&D outcome

	R&D success event			
	Both	Firm 0 only	Firm 1 only	Neither
Probability	$r_0 r_1$	$r_0 (1 - r_1)$	$(1 - r_0) r_1$	$(1 - r_0)(1 - r_1)$
Probability notation	P_{both}	P_{inc}	P_{riv}	P_{none}
Firm 0's willingness to pay	$q_L + g + b$	$q_L + g + b$	$q_L + g - l + b$	$q_L + b$
Firm 1's willingness to pay	$q_L + g$	$q_L + g - l$	$q_L + g$	q_L
Price	b	$l + b$	$l - b$	b
Firm 0's gross profit	b	$l + b$	0	b
Firm 1's gross profit	0	0	$l - b$	0
Consumer surplus	$q_L + g$	$q_L + g - l$	$q_L + g - l + b$	q_L
Gross Social Surplus	$q_L + g + b$	$q_L + g + b$	$q_L + g$	$q_L + b$

As usual, we solve the game backwards, beginning with the second-stage pricing game and then solving the R&D stage. There are four possible events: both firms' R&D are successful, just firm 0 is successful, just firm 1 is successful, or neither is successful. For each such event, Table 3 summarizes its probability, the willingness of consumers to pay for each firm's product, the equilibrium price, each firm's profit (gross of R&D costs), consumer surplus and social surplus (again gross of R&D costs).

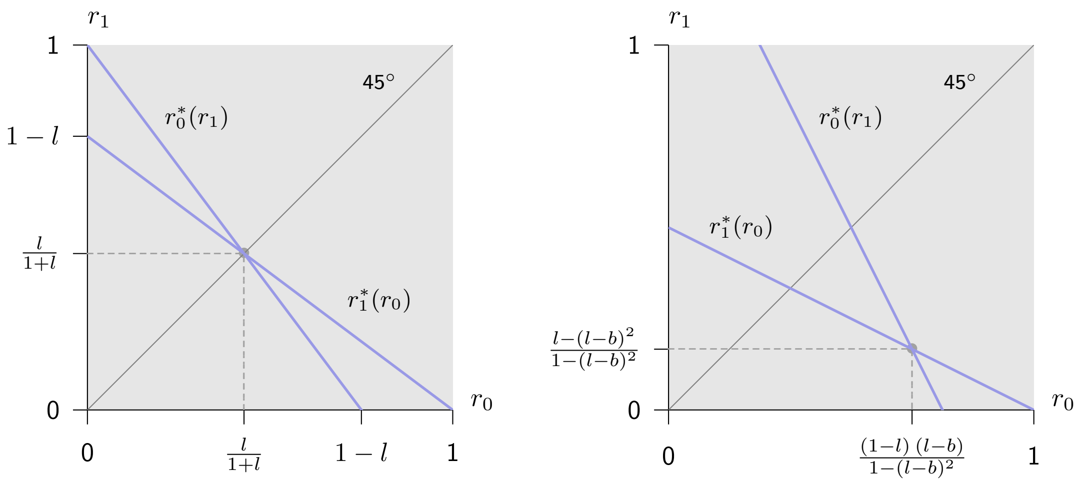
For example, the probability that Firm 0 is successful and Firm 1 is not (the event in the second column) is given by$r_0 (1 - r_1)$. In this case, consumers' willingness to pay for firm 0's product is $q_H + b = q_L + g + b$. And so on. More generally, equilibrium prices are as follows: the firm with higher willingness to pay sets a price equal to the difference to the willingness to pay for the rival's product; the rival firm, in turn, sets price equal to zero. In equilibrium, consumers choose the firm charging a positive price (which we designate as equilibrium price). Consumer surplus is the difference between willingness to pay and price for the firm with higher willingness to pay; and gross social surplus is the willingness to pay for the firm with higher value of willingness to pay.

3. Firm dominance and innovation

We first solve for the equilibrium of the R&D game. Suppose initially that the game is symmetric ($b = 0$). In this case, firm i only makes any profits in the event that it is the only successful innovator. This event occurs with probability $r_i (1 - r_{-i})$ (where r_{-i} is the rival's research effort); and the gross profit in this event is just equal to the innovation barrier l. Therefore, in the symmetric case, the marginal benefit to firm i of research effort is $(1 - r_{-i}) l$. Given quadratic costs $\frac{1}{2} r_i^2$, the marginal cost to firm i of increasing its research effort is equal to that effort level, r_i. Therefore, firm i's reaction curve is given by

$$r_i^*(r_{-i}) = (1 - r_{-i}) l$$

Figure 1
Reaction curves for $l = \frac{3}{4}$, and $b = 0$ (blue lines), $b = \frac{1}{4}$ (red lines)



In other words, each firm's reaction curve is linear with intercepts at $r_i = l$ when $r_{-i} = 0$ and $r_i = 0$ when $r_{-i} = 1$. This symmetric case is shown by the thin lines in Figure 1. The corresponding equilibrium is at

$$r_i^e = r_{-i}^e = \bar{r} = \frac{l}{1+l}$$

As one would expect, as the imitation barrier l increases, each firm's equilibrium R&D effort increases. Notice, however, that $\bar{r} < \frac{1}{2}$ for all $l < 1$.

Now consider the case where the dominant firm has an advantage $b > 0$. Not surprisingly, the dominant firm's expected profit is now larger and the rival's is smaller. However, in terms of marginal incentives for R&D, the changes implied by b are not symmetric. For the dominant firm's marginal incentives, the advantage b only matters when the rival firm's R&D succeeds. In fact, if the rival does not succeed, then firm 0 collects the benefit b regardless of whether it innovates or not (as can be seen from Table 3). Specifically, the dominant firm's marginal revenue from R&D is now $(1-r_1)\,l + r_1\,b$, an increase of $r_1\,b$ from the previous $(1-r_1)\,l$. This corresponds to a anti-clockwise rotation of firm 0's reaction curve around its $r_1 = 0$ intercept, as shown in the right panel of Figure 1. For the rival firm, the dominant firm's advantage b implies a lower marginal benefit. Specifically, the rival's firm marginal benefit is now given by $(1-r_0)\,(l-b)$, a decrease of $(1-r_0)\,b$ with respect to the initial $(1-r_0)\,l$.

The right panel in Figure 1 illustrates the new equilibrium when b increases from $b = 0$. It is given by

$$r_0^e = \frac{l-(l-b)^2}{1-(l-b)^2} \quad \text{and} \quad r_1^e = \frac{(1-l)\,(l-b)}{1-(l-b)^2}$$

It is easily checked that the sum of the two firms' R&D efforts, $r_0^e + r_1^e$, is increasing in the innovation barrier l, but that this sum is still less than 1 for all $l < 1$; that is, the equilibrium always lies below the second diagonal in Figure 1. The following proposition formalizes this discussion.

Proposition 1. *An increase in b implies an increase in firm 0's equilibrium R&D effort, r_0, and a decrease in firm 1's equilibrium R&D effort, r_1.*

A formal proof of this and the following results is included in the Appendix. Proposition 1 describes a fundamental trade-off that permeates much of the analysis in the paper. In addition to the direct effect of firm dominance on consumer and social surplus, we must also consider the indirect effect through the firms' innovation incentives. The good news is that one of the firms, the dominant firm, has greater incentives; the bad news is that the other firm has lower incentives.

The natural follow-up question is: how does firm dominance influence the likelihood of innovation? This is a complex question, for two reasons: First, as Proposition 1 states, an increase of b has opposite effects on the likelihood of innovation by each individual firm. Second, to the extent that innovation requires only one firm to innovate, the likelihood of innovation is more than just the sums of each firm's probability of innovation: we must also take into consideration the likelihood of duplicated R&D successes.

To better understand the various components of the probability of innovation, we can write the overall probability of innovation success as the sum of the probabilities that each firm's R&D is successful minus the probability that these successes duplicate each other:

$$\begin{aligned} P_{innov} &= 1 - (1 - r_0^e)(1 - r_1^e) \\ &= (r_0^e + r_1^e) - r_0^e r_1^e \\ &= R_{tot} - P_{both} \end{aligned}$$

where R_{tot} is the total research effort of the two firms and P_{both} is the probability of duplication. Thus, we can divide the impact of an increase in b into two effects: a total effort effect, ΔR_{tot}, and a duplication effect, ΔP_{both}. The following lemmas describes these effects.

Lemma 1. *Total research effort R_{tot} is decreasing in b. The absolute size of this effect is decreasing in l. Moreover,*

$$\lim_{l \to 0} \frac{dR_{\text{tot}}}{db} = -1$$
$$\lim_{l \to 1} \frac{dR_{\text{tot}}}{db} = 0$$

In order to understand the intuition for this result, it is helpful to consider the impact of b on the firms' profits as well as on the firms' marginal benefit from innovation. From Table 3, we see that

$$\begin{aligned} \pi_0 &= b \Big(1 - r_1 (1 - r_0)\Big) + l\, r_0 (1 - r_1) \\ \pi_1 &= (l - b)\, r_1 (1 - r_0) \end{aligned}$$

In words, firm 1's only hope is to be the only firm innovating, in which case it gets $l - b$. Firm 0, by contrast, receives b in every state except when firm 1 is the only firm innovating. Moreover, if firm 0 is the only innovator then it gets l as well. Based on these profit functions, it is easy to see that the positive effect of b on π_0 is greater than the negative effect of b on π_1; however, the positive effect of b on firm 0's marginal benefits from innovation, $d\pi_0 / dr_0$, is lower than the negative effect of b on firm 0's marginal benefits from innovation, $d\pi_1/dr_1$. Formally,

$$\frac{\partial \pi_0}{\partial b} > -\frac{\partial \pi_1}{\partial b}$$

whereas

$$\frac{\partial^2 \pi_0}{\partial b\, \partial r_0} < -\frac{\partial^2 \pi_1}{\partial b\, \partial r_1}$$

In words, the effect of b on the dominant firm is relatively more infra-marginal, whereas the effect of b on the rival firm is relatively more marginal. Specifically, in the limit when the values of r_0, r_1 tend to zero, the effect of an increase is totally infra-marginal on firm 0 and totally marginal on firm 1. As a result, the negative effect of an increase of b (lower r_1) dominates the positive effect of an increase in b (higher r_0). In words: if barriers to imitation are small then marginal benefits from innovation are small — and an increase in the dominant firm's dominance only makes matters worse.

So far, we have dealt with the effect of b on total R&D effort. Next we consider the effect of b on the probability of duplication of R&D efforts.

Lemma 2. *The probability of duplicate successes, P_{both}, is decreasing in b. The absolute size of this effect is small when l is small:*

$$\lim_{l \to 0} \frac{dP_{\text{both}}}{db} = 0$$

To understand this result, note that, controlling for the level of total effort, a shift from firm 1 to firm 0 decrease the likelihood of duplication. For example, if both firms innovate with probability 50%, then there is a 25% probability of duplication; whereas if firms innovate with probabilities 100% and 0% (same total effort) the probability of duplication is zero. In other words, there is a certain quality of "natural monopoly" in the R&D process; in this sense, an increase in b brings us closer to the monopoly solution where all of the research effort comes from one firm only.

Together, Lemmas 1 and 2 show that the total research effort effect and the duplication effect work in opposite directions on the probability of innovation.

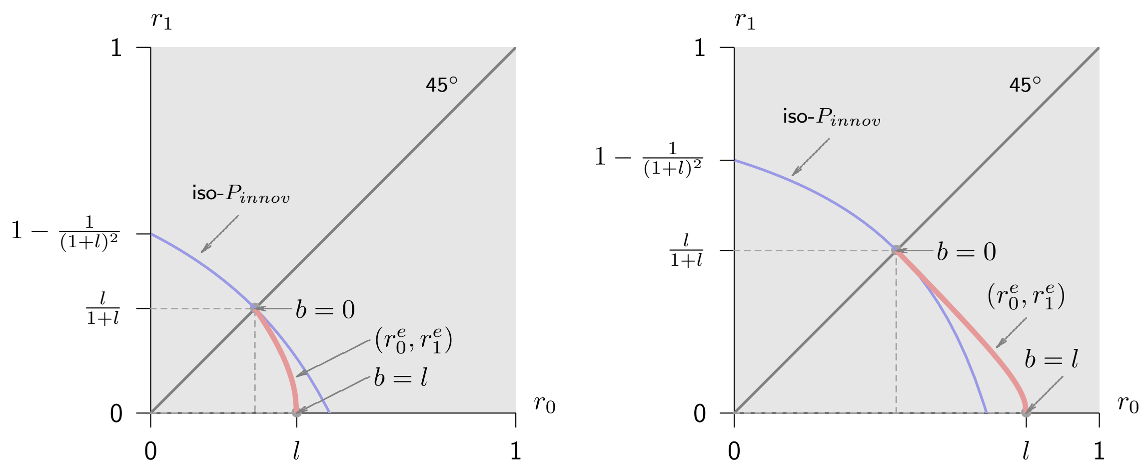
We are now ready to state formally the effect of changes in b on the probability of innovation. Proposition 2 is concerned with small changes in the advantage of the dominant firm.

Proposition 2. *Consider a small increase in firm dominance b. There exists a threshold $\bar{l}(b)$ such that the probability of innovation decreases as we raise b if and only if the innovation barrier l is smaller than $\bar{l}(b)$, where $0 < \bar{l}(b) < 1$ if $b \in (0, 1)$.*

The intuition follows directly from the lemmas above: when l is small, duplication effects are small and the total effort effect dominates; when l is large, total effort effects are small and the duplication effect dominates.

Figure 2 illustrates this idea. Specifically, we consider various levels of the imitation barriers parameter l. The bullets on the 45° line show equilibrium values for (r_0, r_1) when $b = 0$ at different levels of l (l=.4 in the left panel, l=.8 in right panel). Higher values of l induce more R&D, so higher values of l imply equilibria that are further out along the 45° line. The thick lines show how these equilibria change as we increase b holding l

Figure 2
Effect of an increase in b (from 0 to l) for l=.4 (left panel) and l=.8 (right panel)



fixed. Consistent with Proposition 2, increasing b always moves the equilibria south-east reaching the $r_1 = 0$ axis when $b = l$. The thinner lines in the figure represent points at which the probability of a successful innovation is equal, that is, iso-P_{innov} curves. Recall that $P_{innov} = (r_0^e + r_1^e) - r_0^e\, r_1^e$, so these iso-innovation-probability are concave to the origin. Consistent with Proposition 2, we can see that increasing b reduces the overall probability of innovation when innovation barriers are small, but increasing b increases the overall probability of innovation when innovation barriers are large.

Figure 3 also illustrates Proposition 2. For each value of b (horizontal axis) there exits a threshold $\bar{l}(b)$ dividing the quadrant into two regions: above the threshold we have $d P_{innov} \,/\, db > 0$, whereas below the threshold we have $d P_{innov} \,/\, db < 0$. Notice that the critical level of l, $\bar{l}(b)$, is equal to 1 for $b = 0$ or $b = 1$. When $b = 0$, that is, around symmetry, small reallocations of effort (leaving total effort constant) have only a second-order effect on the probability of duplication (the iso-P_{innov} curves all have slope -1 at the 45°-line). Hence the duplication effect is second order around $b = 0$; the total effort effect dominates, and so $d P_{innov} \,/\, db < 0$. In terms of Proposition 3, this means that $\bar{l}(0) = 1$.

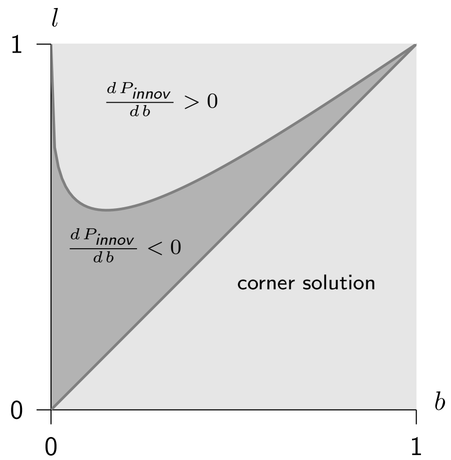
For $b = 1$, notice that the rival firm's expected payoff is given by $\pi_1 = (l - b)\,(1 - r_0)$. At $l = b = 1$, both terms in brackets are zero, and so a small decrease in b has no effect on r_1. The dominant firm's expected payoff, in turn, is given by $(1 - r_1)\, l + r_1\, b$. Since $r_1 = 0$ and r_1 does not change with b, it follows that r_0 does not change either. We conclude that $d P_{innov} \,/\, db = 0$ at $l = b = 1$, as Figure 3 indicates.

In addition to this local result, we are interested in how the probability of innovation changes as we change from $b = 0$ to $b = l$, the two extreme values consistent with Assumption 1: At $b = 0$, we are in a symmetric duopoly. At $b = l$, the advantage of the dominant firm is such that we are in an effective monopoly in which only the dominant firm undertakes R&D.

Proposition 3. *Consider an increase in b from 0 (symmetric duopoly) to $b = l$ (monopoly). The probability of innovation increases if and only if $l > \frac{\sqrt{5}-1}{2}$.*

The exact cut-off value for l stems from the details of the model, but the intuition is

Figure 3
Proposition 2 illustrated



the same as before. When l is small, the total effort effect dominates. When l is large, the duplication effect dominates. For example, as l approaches 1, at monopoly ($b = l$), only the dominant firm undertakes R&D. Its efforts, and hence the probability of an innovation, approach 1. At symmetry ($b = 0$), both firms are active and their R&D efforts each approach 1/2, so the total R&D effort is the same. But now the probability of a successful innovation is only 3/4. This is the effect of duplication.

4. Consumer surplus

What is the effect of an increase in firm dominance b on consumer surplus? As before, it depends on the size of the innovation barrier, l, but now also on the social gains to innovations, g. From Table 3, we can write consumer surplus as

$$\begin{aligned} CS &= q_L + P_{both}\, g + P_{inc}\,(g - l) + P_{riv}\,(g - l + b) \\ &= q_L + P_{innov}\,(g - l) + P_{both}\, l + P_{riv}\, b \end{aligned}$$

The last term is a direct effect. Consumers gain directly from the quality improvement b but only in the event in which only the rival firm's R&D is successful. In that event, the advantage of the dominant firm limits the price that the rival firm can charge. In all other events, the dominant firm wins the sale and, though the good sold incorporates the quality improvement b, the price fully captures this increase in quality. Thus the size of the direct effect of an increase in b on consumer surplus depends on $P_{riv} := (1 - r_0)r_1$. We already know from the previous section that this probability is small when l is small and when b is large. In both these cases, the rival firm has low incentive to undertake R&D.

In addition, increasing b has three indirect effects on consumer surplus. First: whenever there is a successful innovation, consumers gain (at least) the difference between the quality improvement and the innovation barrier, $g - l$. An immediate implication is that, if the quality gain of innovation g is large, then consumer surplus increases if and only if the probability of innovation increases. That is, If g is large, the qualitative results of Propositions 2 and 3 apply to consumer surplus.

When g is small or intermediate, other indirect effects can matter. Second: from Lemma 2, we already know that increasing b decreases the probability of duplicate R&D successes, P_{both}. In the last section, we were only concerned with increasing the probability of innovation, and duplicate R&D successes were no help. But consumers gain from duplicate successes since increased competition reduces prices. Thus, decreasing P_{both} is bad for consumers. Third: we also know from the previous section that increasing b decreases P_{riv}, reducing consumer surplus. Since the second and third indirect effects are negative, the sum of the indirect effects are negative whenever $\Delta P_{innov} < 0$, which again returns us to Propositions 2 and 3. For example, when l is small $\Delta P_{innov} < 0$. The indirect effects on consumer surplus, however, are also negative when l is large relative to g. To see this, recall that $P_{innov} = R_{tot} - P_{both}$. Thus we can write consumer surplus as

$$\begin{aligned} CS &= R_{tot}\,(g-l) - P_{both}\,(g-l) + P_{both}\,l + P_{riv}\,b \\ &= R_{tot}\,(g-l) + P_{both}\,(2\,l-g) + P_{riv}\,b \end{aligned}$$

The probability of duplication appears twice. It hurts surplus (by an amount equal to $g-l$) by reducing the probability of innovation, but it helps consumers (by an amount equal to l) by increasing competition. Thus, when $l > 2\,g$ duplication is a net gain to consumers. By Lemmas 1 and 2 and Proposition 2, we know that total effort, the probability of duplication and the probability that only the rival succeeds are all decreasing in b. Thus, if $2\,l > g$, all indirect effects of changing b on consumer surplus are negative.

The following proposition combines the effects from the above discussion for a small changes in the advantage of the dominant firm.

Proposition 4. *Consider a small increase in firm dominance b. If the social gains to innovation g are large, then consumer surplus moves in the same direction as the probability of innovation as we raise b (see Proposition 2). For lower values of g, consumer surplus may decrease even as the probability of innovation increases as we raise b if both b and l are large.*

As in the previous section, we are also interested in large changes in firm dominance from symmetry ($b = 0$) to effective monopoly ($b = l$).

Proposition 5. *Consider an increase in b from 0 (symmetric duopoly) to $b = l$ (monopoly). If the social gains to innovation $g < 2$, then consumer surplus declines.*

For an intuition, first notice that at both $b = 0$ and $b = l$, the term $P_{riv}\,b$ is zero. That is, the direct effects are irrelevant to this comparison. Second, recall from Proposition 3 that, when l is large, the large reduction in duplication as we move to effective monopoly leads to an increase in the probability of innovation. For consumer surplus, however, duplication has both positive and negative effects. As l approaches and then exceeds $g/2$, the net negative effect of duplication first becomes small and then reverses sign. Thus, once l is large, the reduction in duplication as we move to effective monopoly has either a small effect and eventually a negative effect on consumer surplus.

5. Social surplus

We now consider how changes in firm dominance affect social surplus. There are two important differences with respect to our discussion of consumer surplus. First, we now ignore how surplus is split between consumers and firms. Second, we must now take into account the costs of R&D.

From Table 3, we can write social surplus as

$$\begin{aligned} SS &= q_L + P_{both}\,(g+b) + P_{inc}\,(g+b) + P_{riv}\,(g) + P_{none}\,b - c_0 - c_1 \\ &= q_L + P_{innov}\,(g) + (1 - P_{riv})\,b - c_0 - c_1 \end{aligned}$$

where c_i is the cost of firm i's R&D efforts. Notice the contrast between consumer and social surplus. From a social point of view, total benefit is the same when both firms innovate as when only the dominant firm innovates. The only difference is how that surplus is split between consumers and firms. For consumers, however, a second innovator increases competition and brings about an extra benefit of l, as we saw in the previous section.

The contrast between consumer and social surplus is particularly drastic with respect to the effects of b. As we saw in the previous section, consumers only benefit from b when it is only the rival firm whose R&D succeeds. In this case, a higher b limits the extent of the rival firm's market power. From a social welfare point of view, however, the opposite is true: an increase in b benefits society in all states except that in which only the rival firm's R&D succeeds.

When we change b, as before, we get direct and indirect effects. The direct effect is $(1-P_{riv})$ and it is always positive. As before, the indirect effects works through r_0 increasing and r_1 decreasing. Increasing firm 0's R&D effort moves probability from the event that neither firm succeeds to the event that only firm 0 succeeds, and moves probability from the event that only firm 1 succeeds to the event that both succeed. It also increases costs. Thus, marginal net social benefit through changes in r_0 is given by

$$(1 - r_1)\,g + r_1\,b - c_0'$$

where c_0' represents marginal cost. (Table 3 may prove useful in understanding these values and the ones that follow.) Compare this with the marginal net private benefit of increasing r_0 to firm 0 (from Section 3):

$$(1 - r_1)\,l + r_1\,b - c_0'$$

The difference between these terms is just $(1 - r_1)\,(g - l)$: the difference between the public and private gains when firm 0 is the only innovator. But in equilibrium, we know that this marginal net private benefit is zero (this is the envelope theorem). Thus, the net marginal social benefit is just $(1 - r_1)\,(g - l)$.

Similarly, increasing firm 1's R&D effort moves probability from the event that neither firm succeeds to the event that only firm 1 succeeds, and moves probability from the event that only firm 0 succeeds to the event that both succeed, and also affects costs. Thus, the marginal net social benefit through changes in r_1 is given by

$$(1 - r_0)\,(g - b) - c_1'$$

while the marginal net private benefit is given by

$$(1 - r_0)\,(l - b) - c_1'$$

Again, the difference is just $(1-r_0)\,(g-l)$: the difference between the public and private gains when firm 1 is the only innovator. And again, in equilibrium, we know that the marginal net private benefit is zero. Thus, the net marginal social benefit is just $(1-r_0)\,(g-l)$.

Putting this together, we find that the total *indirect* marginal effect of increasing b on social surplus is given by

$$(g-l)\left((1-r_1)\,\frac{dr_0}{db} + (1-r_0)\,\frac{dr_0}{db}\right) = (g-l)\,\frac{dP_{innov}}{db}$$

Recall that the direct effects are positive. Thus, for small increases in b, social surplus is increasing in b whenever the probability of innovation is increasing in b; for example, when the innovation barrier l is large. As the innovation barrier l approaches zero, the weight $(1-P_{riv})$ on the direct effect converges to one: it is almost never the case that the rival firm innovates. On the other hand, since $dP_{innov}\,/\,db = dR_{tot}\,/\,db - dP_{both}\,/\,db$, we know from our lemmas that the indirect effect converges to $-g$. Thus, when l is mall (that is, for very small innovation barriers), social surplus is decreasing in b (like innovation and consumer surplus). To summarize,

Proposition 6. *Consider a small increase in firm dominance b. If the social gains to innovation g are large, then social surplus moves in the same direction as the probability of innovation as we raise b (see Proposition 2). Even if g is small, if the probability of innovation is increased as we raise b, then so is social surplus.*

Once again, we are also interested in large changes in firm dominance from symmetry $(b=0)$ to effective monopoly $(b=l)$. For social surplus, these mirror the local results.[16]

Proposition 7. *Consider an increase in b from 0 (symmetric duopoly) to $b=l$ (monopoly). If the probability of innovation is increased as we raise b, then so is social surplus.*

6. Summary and extensions

Table 4 summarizes the main results in the paper. First, we have shown that an increase in firm dominance b increases the probability of innovation if and only if imitation barriers l are large. Second, if the gains from innovation are sufficiently large, then the effects on consumer and social surplus have the same sign as the effects on the probability of innovation. Third, an increase in b has a positive direct effect on social surplus, and so an increase in the probability of innovation is a sufficient condition for an increase in social surplus. Finally, an increase in b may have a negative effect on consumer surplus through a decrease in the probability of duplication, which in turn may lead to a decrease in consumer surplus even when the probability of innovation increases.

One important extension of our model concerns the nature of firm dominance. While most agree that Microsoft has a competitive advantage over rivals, there is wide disagreement as to what the source of such advantage is. In the previous sections, we have assumed that firm 0, the dominant firm, is able to add value to its product. For example, in the Introduction we mentioned that, by controlling the operating system and other applications,

16. Propositions 6 and 7 follow from the above discussion. Moreover, Proposition 7 can be seen as a corollary of the second half of Proposition 6.

Table 4
Summary of main results

	Small change in b	Change from $b = 0$ to $b = l$
Probability of innovation	$+$ iff $l > \bar{l}(b)$	$+$ iff $l > \frac{\sqrt{5}-1}{2}$
Consumer surplus	$\propto$ innovation probability if g large	
Consumer surplus	$+$ if small g, large b, l	$-$ if $g < 2$
Social surplus	$\propto$ innovation probability if g large	
Social surplus	$+$ if ΔP_{innov} $+$	

Table 5
Consumer surplus, social surplus and firm gross profit as a function of R&D outcome

	R&D success event			
	Both	Firm 0 only	Firm 1 only	Neither
Probability	$r_0\, r_1$	$r_0\,(1 - r_1)$	$(1 - r_0)\, r_1$	$(1 - r_0)\,(1 - r_1)$
Willingness to pay 0	$q_L + g$	$q_L + g$	$q_L + g - l$	q_L
Willingness to pay 1	$q_L + g - d$	$q_L + g - l - d$	$q_L + g - d$	$q_L - d$
Price	d	$l + d$	$l - d$	d
Firm 0's gross profit	d	$l + d$	0	d
Firm 1's gross profit	0	0	$l - d$	0
Consumer surplus	$q_L + g - d$	$q_L + g - l - d$	$q_L + g - l$	$q_L - d$
Gross Social Surplus	$q_L + g$	$q_L + g$	$q_L + g - d$	q_L

Microsoft is better able to integrate each piece in the whole PC/Windows platform. But Microsoft may also prevent rival firms from integrating their software with the Windows operating system, thereby reducing the rival firm's value.[17] In practice, it may be relatively easy to see that a firm is dominant but, in the absence of a smoking gun, it may be very hard to distinguish whether the dominant firm is adding value to its own product or reducing the value of its rival's product. It is therefore useful to know which of our results are robust to the nature of firm dominance.

To model the idea of reducing rival's value, we set $b = 0$ and assume a negative premium d on the rival's value. reducing the willingness to pay for the rival firm's product by d. Table 5 is then the analog of Table 3. An immediate observation is that each firm's profits (and hence its marginal incentive to undertake R&D) are the same as before except that d has replaced b. In particular, this means that all our results from Section 3 about the probability of innovation still hold. For example, we learned in Section 3 that increasing firm dominance

17. "Microsoft has ... engaged in practices which are often described as ... anti-competitive, such as the continual manipulation of the proprietary operating system to undermine rival's products, selective dissemination of information regarding the operating system's current and future functionality, ... pre-announcements of non-existent products to discourage consumer purchases of rival goods (sometimes referred to as "vaporware")" (Nader and Love, 1997).

increases the probability of innovation if the innovation barrier l is large, and reduces the probability of innovation if the innovation barrier is small. This result still applies.

Corollary 1. *If the dominant firm advantage comes in the form of a reduction $-d$ in its rival's product value, then equilibrium R&D effort levels of the two firms are exactly as if the advantage came in the form of an increase b in the dominant firm's product value. Hence the comparative statics results of Propositions 2 and 3, and Lemmas 1 and 2 still apply (with $b = d$).*

Just as before, if the gains to innovation g are large then the effect of increasing firm dominance on both consumer and social surplus follow the effect on the probability of innovation. Thus, for example, if g is large, even when firm dominance is achieved by reducing a rival's value, such a reduction still increases social and consumer surplus when the innovation barrier l is large.

When g is relatively small, however, we get different welfare effects of increasing dominance if dominance is in the form of reducing rival's value. Analogously to Section 4, we can write consumer surplus as

$$CS = q_L + P_{innov}\,(g - l) + P_{both}\,l - (1 - P_{riv})\,d$$

As before, the last term is a direct effect, but now it is negative. Moreover (using the fact that comparative statics are unchanged from Section 3) the term $-(1 - P_{riv})\,d$ is decreasing as we increase d. The probability of duplication, P_{both}, is also decreasing in firm dominance. Thus, whenever increasing firm dominance d reduces the probability of innovation, it also reduces consumer surplus. Also as before, we can also write consumer surplus as

$$CS = q_L + R_{tot}\,(g - l) + P_{both}\,(2\,l - g) - (1 - P_{riv})\,d$$

Thus, if $g < 2l$, increasing firm dominance d always reduces consumer surplus.

Moving to social surplus, analogously to before, we can write social surplus as

$$SS = q_L + P_{innov}\,g - P_{riv}\,d - c_0 - c_1$$

Once again, we can divide the effect of increasing d into direct and indirect effects. The direct effect, $-P_{riv}$, is negative. As in Section 5, we can reduce the indirect effects to

$$(g - l)\,\frac{dP_{innov}}{dd}$$

Thus, whenever innovation is decreased by raising d, social surplus is also. To summarize:

Corollary 2. *If g is large, then (as before) consumer and social surplus move in the same direction as the probability of innovation as we raise d. If g is relatively small and the probability of innovation decreases as we raise d, then so do both consumer and social surplus. If $g < 2\,l$ then increasing d always reduces consumer surplus.*

It is not a surprise that reducing a rival value has worse welfare effects than increasing the dominant firm's value. What is perhaps more surprising is that the qualitative effects are so similar. In other words, our results are surprisingly robust with respect the nature of firm dominance.

Another robustness test is the nature of product market competition. We have assumed price competition with vertical differentiation only. While this implies profit functions that are not smooth (at the point where rivals have equal valuations), we believe our results are not knife-edged and would hold with some degree of horizontal product differentiation. The important feature is that the gap between dominant firm and rival firm in terms of marginal incentive for research effort increases as the value of b increases.

Our assumption of a quadratic cost function is made for analytical convenience. It is clear from the proofs that our qualitative results do not depend on the exact functional form. Finally, we consider the case of one rival firm only. We believe the main results and intuitions would extend to the n case as well.

Appendix

This appendix includes all proofs not included in the text as well as a discussion of the subgame where firms decide whether or not to imitate.

Proof of Proposition 1: Solving for the Nash equilibrium, we get

$$\begin{aligned} r_0^e &= \frac{l-(l-b)^2}{1-(l-b)^2} \\ r_1^e &= \frac{(1-l)(l-b)}{1-(l-b)^2} \end{aligned}$$

Straightforward differentiation implies that

$$\begin{aligned} \frac{dr_0^e}{db} &= 2\,\frac{(1-l)\,(l-b)}{\left(1-(l-b)^2\right)^2} \\ \frac{dr_1^e}{db} &= -\frac{(1-l)\left(1+(l-b)^2\right)}{\left(1-(l-b)^2\right)^2} \end{aligned}$$

The result then follows from Assumption 1. ■

Proof of Lemma 1: As seen before, the best-response functions are given by

$$\begin{aligned} r_0^* &= l-(l-b)\,r_1 \\ r_1^* &= (l-b)\,(1-r_0) \end{aligned}$$

Summing and differentiating (at the equilibrium values) yields

$$\begin{aligned} \left(\frac{dr_0^e}{db}+\frac{dr_1^e}{db}\right) &= -(l-b)\left(\frac{dr_0^e}{db}+\frac{dr_1^e}{db}\right)+r_1^e-(1-r_0^e) \\ \left(\frac{dr_0^e}{dl}+\frac{dr_1^e}{dl}\right) &= -(l-b)\left(\frac{dr_0^e}{dl}+\frac{dr_1^e}{dl}\right)+(1-r_1^e)+(1-r_0^e) \end{aligned} \tag{1}$$

Rearranging the top expression, we get

$$(1+l-b)\left(\frac{dr_0^e}{db}+\frac{dr_1^e}{db}\right)=-(1-r_0^e-r_1^e)$$

Since $r_0+r_1<1$, the right-hand side is negative, and the first part of the result follows, that is, $\partial R_{tot}/\partial b<0$.

Next we determine how the derivative $\partial R_{tot}/\partial b$ varies with the value of l

Notice that $l=1$ and $b>0$ imply $r_0=1, r_1=0$, so the right-hand side is zero.

Rearranging (1) yields

$$(1+l-b)\left(\frac{dr_0^e}{dl}+\frac{dr_1^e}{dl}\right)=(1-r_1^e)+(1-r_0^e)$$

Thus, total effort is increasing in l. Taking the cross derivative, we get

$$\left(\frac{\partial^2 r_0^e}{\partial b\,\partial l}+\frac{\partial^2 r_1^e}{\partial b\,\partial d}\right)=-(l-b)\left(\frac{\partial^2 r_0^e}{\partial b\,\partial l}+\frac{\partial^2 r_1^e}{\partial b\,\partial d}\right)-\left(\frac{dr_0^e}{db}+\frac{dr_1^e}{db}\right)+\left(\frac{dr_0^e}{dl}+\frac{dr_1^e}{dl}\right)$$

or

$$(1+l-b)\left(\frac{\partial^2 r_0^e}{\partial b\,\partial l}+\frac{\partial^2 r_1^e}{\partial b\,\partial d}\right)=\frac{dR_{tot}}{dl}-\frac{dR_{tot}}{db}>0$$

Regarding the limit resuts, notice that, as $l\to 1$, we have two possibilities: if $b=0$, then $r_0^e=r_1^e\to\frac{1}{2}$; if instead $0<b<l$, then $r_0^e\to 1$ and $r_1^e\to 0$; in either case, $(r_1^e+r_0^e)\to 1$ and hence $dR_{tot}/db\to 0$. Finally, as $l\to 0$ (with $b<l$), $(r_1^e+r_0^e)\to 0$ and hence $dR_{tot}/db\to -1$. ■

Proof of Lemma 2: Differentiating P_{both} we get

$$\begin{aligned}\frac{d(r_0^e r_1^e)}{db}&=r_1^e\,\frac{dr_0^e}{db}+r_0^e\,\frac{dr_1^e}{db}\\&=r_1^e\left(\frac{dr_0^e}{db}+\frac{dr_1^e}{db}\right)+(r_0^e-r_1^e)\left(\frac{dr_1^e}{db}\right)\end{aligned}$$

The first term is negative since total effort is decreasing in b (Lemma 1). The second term is non-positive since r_1^e is decreasing and $r_0^e\geq r_1^e$ (Proposition 1). For the limit, as $l\to 0$, both $r_1^e\to 0$ and $r_0^e\to 0$, so it is enough to show that neither of the derivatives dr_0^e/db or dr_1^e/db explode as $l\to 0$. This is easily checked by inspection of the equilibrium expressions. ■

Proof of Proposition 2: Computation establishes that

$$\begin{aligned}\left.\frac{dP_{innov}}{db}\right|_{b=0}&=-\frac{1-l}{(1+l)^3}<0\\\left.\frac{dP_{innov}}{db}\right|_{b=l}&=-(1-l)^2<0\end{aligned}$$

Moreover, solving $dP_{innov}/db=0$ with respect to l yields only two real roots: $l=1$ and

$$l=\bar{l}(b)=1-\sqrt[3]{b\,(2-b)^2}+\sqrt[3]{b^2\,(2-b)}$$

Notice that $\bar{l}(0)=\bar{l}(1)=1$ and $\bar{l}(b)<1$ for $b\in(0,1)$. By Lemmas 1 and 2 we know that $dP_{innov}/db<0$ when l is close to zero. Since dP_{innov}/db is continuous in l,b, we conclude that $dP_{innov}/db<0$ if and only if $l<\bar{l}(b)$. ■

Proof of Proposition 3: At $b=0$, the equilibrium is symmetric with $r_0^e=r_1^e=r=l/(1+l)$. Thus, $P_{innov}|_{b=0}=1-(1-r)^2=1-1/(1+l)^2$. At $b=l$, only the dominant firm is active and $r_0=l$. Thus, $P_{innov}|_{b=l}=l$. The result follows by direct calculation. ■

Proof of Proposition 4: Consumer surplus may be written as

$$\begin{aligned} CS &= q_L + P_{both}\, g + P_{inc}\,(g-l) + P_{riv}\,(g-l+b) \\ &= q_L + P_{innov}\,(g-l) + P_{both}\, l + P_{riv}\, b \end{aligned}$$

If g is large enough, then $d\,CS^e\,/\,db > 0$ if and only if $d\,P_{innov}\,/\,db > 0$. This proves the first part of the proposition.

Computing the derivative of CS^e with respect to b and equating b to zero we get

$$\left.\frac{d\,CS^e}{db}\right|_{b\,=\,0} = \frac{2\,l - l^2\,(1-l) - g(1-l)}{(1+l)^3}$$

Immediate inspection reveals that the derivative ranges from $-g < 0$ for $l = 0$ to $2 > 0$ for $l = 1$. Moreover, the derivative of the numerator with respect to l is given by $3\,l^2{+}2\,(1{-}l){+}g$, which is positive under Assumption 1. It follows that, for small values of b, there exists an $\bar{l}(b)$ such that consumer surplus is decreasing in b if and only if $l < \bar{l}(b)$.

Consider now the case when $b = l$. Computation establishes that

$$\left.\frac{d\,CS^e}{db}\right|_{b\,=\,l} = -(1-l)\left(g\,(1-l) + l^2\right) < 0$$

Together with the previous result, this implies that for high values of l, consumer surplus is maximal for an intermediate value of b. ■

Proof of Proposition 5: Straightforward computation yields

$$\begin{aligned} CS\,|_{\,b\,=\,l} &= q_L + (g-l)\,l \\ CS\,|_{\,b\,=\,0} &= \frac{l\big(g\,l + 2\,(g-l)\big)}{(1+l)^2} \end{aligned}$$

We now show that, given $l < g < 2$, we have $CS|_{b=0} > CS|_{b=l}$. If fact, if $l^2 + l - 1 > 0$ then

$$\begin{aligned} CS\,|_{\,b\,=\,0} \;-\; CS\,|_{\,b\,=\,l} &= \frac{l\Big(l\,(l^2+2\,l-1) - g\,(l^2+l-1)\Big)}{(1+l)^2} \\ &> \frac{l\Big(l\,(l^2+2\,l-1) - 2\,(l^2+l-1)\Big)}{(1+l)^2} \\ &= \frac{l(1-l)^2\,(2+l)}{(1+l)^2} \\ &> 0 \end{aligned}$$

On the other hand, if $l^2 + l - 1 < 0$ then

$$\begin{aligned} CS\,|_{\,b\,=\,0} \;-\; CS\,|_{\,b\,=\,l} &= \frac{l\Big(l\,(l^2+2\,l-1) - g\,(l^2+l-1)\Big)}{(1+l)^2} \\ &> \frac{l\Big(l\,(l^2+2\,l-1) - l\,(l^2+l-1)\Big)}{(1+l)^2} \\ &= \frac{l^3}{(1+l)^2} \\ &> 0 \end{aligned}$$

which concludes the proof. ■

Proof of Proposition 6: In text. ■

Proof of Proposition 7: In text. ■

■ **The imitation subgame.** Consider the subgame where firm 1 is successful in innovating but firm 0 is not. In the paper we assumed that firm 0 imitates firm 1's innovation, even though firm 0 is indifferent between imitating and not imitating. Moreover, firm 0's market share is zero. The first feature of the model is potentially bothersome (an equilibrium based on indifference), while the second feature is rather extreme.

We now show that our model can be seen as the limit case where there is only vertical product differentiation. In a model with horizontal product differentiation as well, we would obtain equilibria with strict preferences and strictly positive market shares. Specifically, consider the above subgame and consider a more general case where the consumer's utility includes a horizontal product differentiation component x, which without loss of generality denotes preference for firm 0 (everything else constant, including the dominance parameter b).

We assume that x is distributed according to $F(x/\sigma)$, where σ is the standard devotion of x (an index of product differentiation). Moreover, we assume that $F(0) = \frac{1}{2}$; $f(x/\sigma) := dF/dx$ is continuous and strictly positive in $I\!R$; and $F(x)/f(x)$ is strictly increasing.

We now solve for the subgame where firm 0 must decide whether to imitate or not. Let γ be the imitation cost. Suppose first that firm 0 decides to imitate firm 1. Then the indifferent consumer's type, x', is given by

$$x' + b + g - l - p_0 = g - p_1$$

Demand for firm 0 is therefore given by

$$D_0 = 1 - F(x') = 1 - F(p_0 - p_1 + l - b)$$

The profit functions (excluding the imitation cost) are given by

$$\begin{aligned} \pi_0 &= p_0\left(1 - F(p_0 - p_1 + l - b)\right) \\ \pi_1 &= p_1\, F(p_0 - p_1 + l - b) \end{aligned}$$

The first-order conditions for profit maximization are given by

$$\begin{aligned} 1 - F(p_0 - p_1 + l - b) - f(p_0 - p_1 + l - b)\, p_0 &= 0 \\ F(p_0 - p_1 + l - b) - f(p_0 - p_1 + l - b)\, p_1 &= 0 \end{aligned}$$

or simply

$$\begin{aligned} p_0 &= \frac{1 - F(p_0 - p_1 + l - b)}{f(p_0 - p_1 + l - b)} \\ p_1 &= \frac{F(p_0 - p_1 + l - b)}{f(p_0 - p_1 + l - b)} \end{aligned} \tag{2}$$

Substituting in the profit functions and simplifying, we get equilibrium profits

$$\begin{aligned} \pi_0^e &= \frac{\big(1 - F(p_0 - p_1 + l - b)\big)^2}{f(p_0 - p_1 + l - b)} \\ \pi_1^e &= \frac{\big(F(p_0 - p_1 + l - b)\big)^2}{f(p_0 - p_1 + l - b)} \end{aligned} \tag{3}$$

Subtracting the two first-order conditions (2) and simplifying we get

$$p_0 - p_1 = \frac{1 - 2\, F(p_0 - p_1 + l - b)}{f(p_0 - p_1 + l - b)}$$

or

$$\Delta\, p = \frac{1 - 2\, F(\Delta\, p + l - b)}{f(\Delta\, p + l - b)} \tag{4}$$

where $\Delta\, p := p_0 - p_1$. If $F(x)/f(x)$ is strictly increasing then $\big(1 - 2\, F(x)\big)\,/f(x)$ is strictly decreasing in x. The left hand side of (4) is strictly increasing in $\Delta\, p$, ranging from $-\infty$ to $+\infty$. The right hand side of (4) is strictly decreasing in $\Delta\, p$. It follows that there exists a unique solution to (4). From, (2), this also implies a unique solution to p_0, p_1.

Let $\Omega(\Delta\, p + l - b)$ denote the right-hand side of (4). By applying the implicit function theorem we have

$$\frac{d\,\Delta p}{dl} = \frac{\Omega'}{1 - \Omega'}$$

This implies that

$$\frac{d\,(\Delta p + l - b)}{dl} = \frac{1}{1 - \Omega'} > 0$$

since $\Omega' > 0$. Finally, from (3) we conclude that firm 0's equilibrium profit is strictly decreasing in l.

Suppose now that firm 0 decides not to imitate. The indifferent consumer x'' is now given by

$$x'' + b - p_0 = g - p_1$$

Demand for firm 0 is now given by

$$D_0 = 1 - F(x'') = 1 - F(p_0 - p_1 + g - b)$$

The analysis proceeds as before except that we have $g - b$ instead of $l - b$. Noting that $g > l$, we conclude that firm 0's profits are strictly lower if it does not imitate than if it does.

Next we consider the link between our differentiated product model and the case considered in the paper. Suppose that the distribution of x is given by $F(x/\sigma)$, where σ is a measure of product differentiation (for example, the standard deviation of x). The case we considered in the text corresponds to the limit at $\sigma \to 0$. The analysis above implies that, for a given σ, there exists a threshold value of the imitation cost γ, call it $\gamma'(\sigma)$ such that firm 0 prefers to imitate if and only if $\gamma < \gamma'(\sigma)$. Consider a sequence of values σ_n and γ_n such that (a) $\sigma_n \to 0$ and $\gamma_n < \gamma'(\sigma_n)$. Then, for every equilibrium along this sequence firm 0's strict best response is to imitate and its market share is strictly positive. Moreover, the limit of this sequence is the subgame equilibrium considered in the text.

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