Activism's Impact on Diversified Investors and the Market

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Abstract

We model activism as it affects the future distribution of prices in a portfolio context with risk-averse expected utility of end-of-period wealth maximizing investors. We characterize activism as affecting the mean, the variance, and/or the covariance of the target firm's price with the prices of the other firms. This characterization allows us to investigate conditions under which the activist would choose to become an activist and, subsequently, to derive the sequence of equilibria that begins with the surreptitious acquisition of shares by the activist and ends at the moment of the activist's divestiture of these shares. We investigate the impact of activism not only on the target firm's price over time and the activist's profit, but also on the redistribution of portfolio holdings of all market participants that this activism induces. We propose a method to evaluate activism and show that, while activism may augment the share price of the target firm, there exist conditions under which activism would not necessarily increase the value of the market. Furthermore, we show that the profit of the activist is at the expense of the group of other investors. We compare our results to recent empirical findings.

1 Introduction and Literature Review

1.1 Setting the Context

Recent empirical literature suggests a description of hedge fund activism that differs in a number of ways from the type of activism that has been studied extensively in the past. In particular, this new form may be characterized as a sequence of equilibria, elements of the sequence being provoked by the legal requirements of the release of information. Also, at each time point in the sequence, all activity is determined in a competitive market. Two other points pertinent to the distinction of hedge fund activism are that the percentage of shares acquired by the activist in order to pursue his agenda is relatively small, and that these excessive shares are sold within a relatively short period of time.

We begin by describing these equilibria in more detail and noting differences with earlier models. Specifically, we model activism as it affects the future distribution of prices in a portfolio context with risk-averse expected utility of end-of-period wealth maximizing investors. We characterize activism as affecting the mean, the variance, and/or the covariance of the target firm's price with the prices of other firms. This characterization allows us to derive the sequence of equilibria that begins with the surreptitious acquisition of shares by the activist, and ends at the moment of the activist's divestiture of these shares.

A key feature of the new empirical research is that the hedge fund activist (or the entrepreneurial activist, for example, an individual investor or a private equity firm) must acquire shares in the marketplace in order to become an activist.[1] That is, the starting point is not from that of a large blockholder as, for example, in

[1] See, for example, Kahan and Rock (2007, p. 1069) where they state "... it is noteworthy that activist hedge funds usually accumulate stakes in portfolio companies *in order to engage in activism.*" Italics in original.

the models of Shleifer and Vishny (1986), Huddart (1993), Admati, Pfleiderer and Zechner (1994), Kahn and Winton (1998) and Maug (1998). Rather, the initial focus is on the surreptitious accumulation of shares to enable the potential activist to acquire a sufficient equity stake in the target firm to be able to accomplish his goal of causing management to alter its behavior. This share acquisition occurs in a competitive market equilibrium. When that stake reaches the critical 5% percent level of ownership, the potential activist has the obligation to file Schedule 13D with the Securities and Exchange Commission.[2] This filing announces the activist's intent to the marketplace. Importantly, since the filing of Schedule 13D within 10 days of acquiring a 5% stake of the voting power of a registered security with the intent to alter the policies of the current management is a legal requirement, the information in the filing is not released for strategic purposes. But even when a hedge fund decides to enter the market to acquire shares to pursue an activist agenda, it does not tend to acquire a controlling block of shares (see Brav et al. (2008) and Klein and Zur (2009)). Moreover, it does not maintain even its non-controlling additional equity stake for a lengthy period of time: the median holding period from filing Schedule 13D to exit (i.e., selling of excess shares) is approximately one year in duration (see Brav et al. (2008) and Klein and Zur (2009)). Thus, the extensive literature on activists' monitoring, found in many models with large blockholders, does not offer insights into this alternative behavior. Also, hedge fund and other entrepreneurial investors are quite successful, accomplishing their goals roughly two-thirds of the time (see Klein and Zur (2009) and Brav et al. (2008)), or even more (see Bratton (2010)). Earlier studies did not find such success (see Black (1998), Karpoff (2001)

[2] See, for example, Briggs (2007, p. 706): "...hedge fund activism with an ownership level below the 5% Schedule 13D threshold only happens relatively rarely." Also note Greenwood and Schor (2009, p. 2) who state "the activists need to acquire at least a 5% stake in order to most effectively push for a takeover with the public filing of the mandatory 13D."

and Gillian and Starks (2007); see also Klein and Zur (2009) and Brav et al. (2008) for a discussion of those earlier findings). Finally, the shedding of excess shares when the activism is concluded is typically via sales in the market place (see Brav et al. (2008)). When the level of ownership sinks below the 5% level, another reporting requirement is triggered and an amended Schedule 13D (Schedule 13D/A) must be filed, again alerting the market to this information.

In sum, it appears that hedge funds and other entrepreneurial activists currently start from a position without significant ownership in a firm they later target. Then, they come to believe that they could profit by improving some aspect of the way that firm operates. They subsequently commence accumulating a larger position in that firm by purchasing its shares in the marketplace. Generally, hedge funds attain somewhat more than the 5% ownership level that necessitates a filing of Schedule 13D. This filing makes the activist's intent public. The activist then undertakes various tactics involving attempts to change the board of directors, change the business strategy of the firm, change the capital structure, alter the dividend policy, divest assets, etc. Within a year or so, the activist sheds his excess holdings in the target, regardless of whether he has had success in achieving change. Generally this requires a filing of Schedule 13D/A.

1.2 Related Literature

Brav et al. (2008), studying hostile and non-hostile hedge funds from 2001 to 2006, and Klein and Zur (2009), studying confrontational hedge funds and other entrepreneurial activists from 2003 to 2005, provide surveys of the evolution of activism and present, in many cases, similar empirical findings. Both find that the price of the target firm increases around the time of the Schedule 13D filing (in the range of 7% (Brav et al.) and 10% (Klein and Zur)), and that for a period of about a year afterwards the boost in returns remains. Both conclude that the market tends to assess hedge fund activism as increasing shareholder value and suggest this is due to a lessening of agency problems. Also, Brav et al. find that the target firm exhibits price declines only if the hedge fund fails in its efforts. On the other hand, Klein and Zur find that when activists are unsuccessful within the year horizon, the price of the target firm increases less than when the activist is successful. A main empirical finding of both Brav et al. and Klein and Zur is to show the reaction of a target firm to activism, in particular, highlighting the increase in the average excess return that is found around the time of the Schedule 13D filing and its persistence. It is primarily on this basis that both studies posit activism benefits target firm shareholders.

In a recent study of a single fund, the Hermes U.K. Focus Fund (HUKFF), over the period 1998-2004, Becht et al. (2010) find activism produces substantial benefits not only for HUKFF but also for the shareholders.[3] The differences in institutional environments between the U.K. and the U.S. are described, and the U.K. threshold level of 3% for disclosure is noted. Becht et al. do not find the same type of behavior of the target price around the announcement date that Brav et al. or Klein and Zur find.[4] But Becht et al. do find that, when the private "behind the scenes" negotiations between the HUKFF and its targets are successful, abnormal returns net of fees of 4.9% against a benchmark were achieved. Furthermore, they find that over 90% of the excess returns can be attributed to activism.

Boyson and Mooradian (2011) also study the impact of activism on the performance of both the activist fund and the target firms. They construct a data set to permit such a study for the period 1994-2005 and, using Schedule 13D filings, find that hedge funds are successful in their activism at rates similar to those found by Brav et al. and Klein and Zur. Boyson and Mooradian find a longer average holding

[3] In their paper, the HUKFF is described as closer to a hedge fund or private equity investor than a traditional institutional activist investor.

[4] They suggest this may be an artifact of reporting as HUKFF announcements may be difficult to separate from overall British Telecommunications pension fund announcements.

period of the target firm by the activist than Brav et al. and Klein and Zur, while finding that the target firm price increases in the short term. This compares with Brav et al. and Klein and Zur who find a sustained positive response to the Schedule 13D filing. Boyson and Mooradian also find activist hedge funds earn 7-11% more per year, risk-adjusted, than non-active hedge funds, and both activist hedge funds and shareholders benefit from activism.

Clifford (2008), too, finds hedge fund activism benefits shareholders and hedge funds themselves. He introduces data on the filings of Schedule 13G (required when the 5% threshold is reached but the stated intent is passive ownership) by hedge funds and finds that, over the period 1998-2005, target firms for which Schedule 13D have been filed have 1.07% higher excess returns than those for which Schedule 13G have been filed.[5] Clifford's finding that firms targeted for activist hedge fund involvement earn larger excess returns than those targeted for passive hedge fund investment is in line with the findings of positive excess returns at Schedule 13D filings found by Brav et al., Klein and Zur and Boyson and Mooradian. Again comparing Schedule 13D and 13G filings, Clifford determines that when a hedge fund changes its stance toward a target firm, necessitating a filing of Schedule 13D subsequent to a prior filing of 13G, the target's excess return increases. The largest gains to hedge fund activism, according to Clifford, are associated with the sale of assets or the entire firm.

The role of the subsequent sale of assets of the target firm (including mergers and the sale of the entire firm to the activist that targeted it for improvements in corporate governance) is studied further by Greenwood and Schor (2009). For the period 1993-2006, they consider Schedule 13D filings as well as definitive proxy statements filed with the SEC by non-management (DFANs) announcing the initiation of a

[5] Since the timing of Schedule 13G filing is more lax (45 days after the end of the calendar year in which the 5% stake is acquired), Clifford compares 13D to 13G filings made within 10 days of the acquisition; see his Table IV, panel B.

proxy battle. They show higher returns at the announcement of Schedule 13D filings, with targets for which a merger or sale of all or part of its assets was announced or completed within an 18-month period earning, on average, more than 5%, with the complement group earning, on average, only 2.4%. These increases surrounding the Schedule 13D announcements comport with those found by Brav et al., Klein and Zur, Boyson and Mooradian, and Clifford. Looking further at the grouping with no takeovers announced or completed within an 18-month window, Greenwood and Schor find the only characteristic associated with a significant positive abnormal return at announcement is that of "spinoffs," and the return in this case is 6.4%. Switching to a monthly rather than daily event analysis reveals that only the targets for which announcements of either acquisition or completed acquisition have occurred within an 18-month period experience a positive abnormal announcement return. In the monthly analysis, the complement group (and all of its sub-categories) earns no significant positive abnormal returns. Greenwood and Schor also show that the presence of an activist increases the probability of a takeover.

Six other studies, Bratton (2007), Bratton (2010), Briggs (2007), Kahan and Rock (2007), Stowell (2010, chapter 13), and Brenner (2008), provide valuable additional depth and background on the phenomenon of hedge fund activism. They also supply details on particular activist engagements and their outcomes.

In a related work on the initial behavior of a sovereign fund tasked with improving corporate governance, Lee and Park (2009) take a different tact: they investigate the impact of the announcement of a 5% stake in a target firm by the Korean Corporate Governance Fund in August 2006 on the prices of non-target Korean firms.[6] Lee and Park document that there is no leakage before the announcement date, after which

[6] Although a regulatory filing must be made at the 5% level, no distinction between active and passive intent need be specified in Korea. Lee and Park note, however, the activist intent of the Korean Corporate Governance Fund was obvious.

cumulative returns increase in the two chosen targets (see their Figure 1, and also note that one target is the parent company of the other). They go on to show that, while all non-target firms are positively affected by the announcement, the non-target firms having poorer governance provisions subsequently have larger abnormal returns.

1.3 Our Contributions

By their nature, however, the empirical studies just cited, with the exception of Lee and Park, examine the marginal impact of hedge fund and other entrepreneurial activism on specific targets, but all the studies have difficulty addressing a number of broader issues. For example, under what conditions would an activist choose to increase his risk exposure by devoting time and money to his activism? Why and in what manner does the price of the target firm change over the course of the activist's involvement? How would activism affect the entire portfolio holdings not only of the activist but also of all other market participants? Why, if the activist should succeed in his agenda, would he pare back his holdings within a relatively short period of time? How does the value of the market change as a consequence of activism? How might we evaluate the benefits of activism in the presence of diversified shareholders?

In order to provide some answers to these questions, in this paper we present a model in which we embed the activist and his target firm within a general market of correlated firms over time. Our model permits us to trace the impact of activism through the period of involvement of the activist in the target firm. This allows us to address the preliminary decision of the activist to become an activist. Our results exhibit the changes that occur not only on the target firm's price and the activist's profit, but also the changes that occur in all investors' portfolios as a consequence of activism. Furthermore, we can show how the market's value changes over the course of activism and how the group of other investors fare over the same period. We find that the benefits of activism expressed in the empirical literature have been broadly overstated.

We present our model in the Section 2. In Section 3 we analyze the criteria that the activist uses in his decision to become an activist. We derive and compare the forecasts of our model with the price movements described in the empirical literature in Section 4. Section 5 analyzes the impact of activism on the market and other diversified investors. Discussion and conclusions follow in Section 6.

2 The Model

The model that we consider specifies four moments in time at which investors gather together to compete for shares in firms for their portfolios. These moments are distinguished by the information sets available to investors at each of these points in time. At time $t = 0$, all participants hold the same view regarding the future values of the firms, and come together to buy shares in these firms based on that commonly held information. We refer to the set of portfolios determined in this manner as the benchmark portfolios. We assume that the benchmark portfolios remain the same until one of those investors, called the activist, comes to believe that his involvement can improve the performance of a firm. With this new private information, the activist must first decide whether it would be profitable for him to act on his private information. If not, activism obviously does not occur. Should the decision to act be taken, then the activist moves at time $t = 1$ to acquire shares to facilitate his objective. This move precipitates a new competitive market equilibrium where the activist acts on his private information while the views of all other investors concerning the future values of the set of firms remain unchanged. If the activist is successful in acquiring a sufficient number of shares then, at time $t = 2$, the activist files

Schedule 13D.[7] At the time of the filing, the investors become informed of the activist's intent to improve the performance of the firm. Having gained knowledge of the activist's intent, the remaining investors enter into a new competitive equilibrium for shares. Here, the activist refrains from entering into trading since he needs the shares he has already acquired to carry out his activist program. Subsequently, at time $t = 3$, it becomes known to all market participants whether or not the activist has been successful in his plans to improve the firm. This new information acquired by all market participants induces a new competitive equilibrium with all investors participating. Should the activist's holdings fall sufficiently, he files an amended Schedule 13D (Schedule 13D/A).[8] Finally, at time T, all uncertainty concerning the firms is resolved and all the firms are liquidated. In this section, we investigate these equilibria and some of their consequences.

In each competitive market equilibrium, we assume that there exists the same set of N risky assets and a riskless one. Each of the M risk-averse investors is a price-taker and a von Neumann-Morgenstern expected utility of end-of-period wealth maximizer. We now introduce some notation. Let $\mathbf{x}_{it}$ be the N x 1 vector of shares held by investor i, $i = 1, ..., M$, at time t, $t = 0, 1, 2, 3$, in the N firms. Let y_{it} be the amount investor i borrows (lends) at time t to facilitate purchases. Let $\widetilde{\mathbf{p}}_{it}$ be an N x 1 vector of random prices per share of the N firms that would prevail at T as perceived by investor i at time t, and let p_0 be the price of the riskless asset. Let u_i be the utility function of investor i, w_{it} be the wealth of investor i at time t and, for convenience, let $p_0 = 1$.

At time t, $t = 0, 1, 2, 3$, the equilibrium process is defined as follows. Taking the N

[7] When an owner acquires 5% or more of the voting power of a registered security, and has the intent to attempt to alter the policies of the current management, SEC rules require that Schedule 13D (the so-called beneficial ownership report), be filed within 10 days.

[8] According to SEC regulations, if the activist's holdings fall below the 5% threshold, Schedule 13D/A must be filed promptly.

x 1 vector $\mathbf{p}_t$ as given, investor i determines $\mathbf{x}_{it}^*$ which satisfies $\arg\max_{\mathbf{x}_{it}} E_{it}u_i(y_{it} + \mathbf{x}_{it}'\widetilde{\mathbf{p}}_{it})$ $s.t.$ $y_{it} + \mathbf{x}_{it}'\mathbf{p}_t = w_{it}$ where E_{it} is the expectation of investor i at time t with respect to the distribution of $\widetilde{\mathbf{p}}_{it}$ and a prime denotes a transpose operation. The equilibrium price vector at time t, $\mathbf{P}_t$, yields the demands $\mathbf{x}_{it}^*$ so that all shares are sold, i.e., $\sum_{i=1}^{M} \mathbf{x}_{it}^* = \mathbf{Q}$ where $\mathbf{Q}$ is the N x 1 vector whose elements are the total number of shares in each of the N risky firms. For convenience, we normalize $\mathbf{Q}$ and represent it by $\mathbf{1}$, an N x 1 vector whose elements are 1, so that $\mathbf{x}_{it}$ represents the vector of proportional ownership of investor i at time t in the N risky firms. We assume that each investor has an exponential utility function with Pratt-Arrow coefficient of absolute risk aversion a_i. We further assume that the random vector $\widetilde{\mathbf{p}}_{it}$ is normally distribution with mean vector $\boldsymbol{\mu}_{it}$ and positive definite covariance matrix $\boldsymbol{\Omega}_{it}$. With these assumptions, the equilibrium solution at each time t is the solution to the nonhomogeneous portfolio problem derived in Rabinovitch and Owen (1978). Furthermore, the trading that determines these equilibria takes place only when new information becomes available. We use the results in Rabinovitch and Owen to determine the explicit prices and holdings in the four competitive equilibria we study here.

At time $t = 0$, all investors agree on their assessments of the distribution of prices that will occur at T. Thus, in this case, $\boldsymbol{\mu}_{i0} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i0} = \boldsymbol{\Omega}_0$. We state this well-known equilibrium solution result without proof in the next proposition.

Proposition 1. At time $t = 0$, $\boldsymbol{\mu}_{i0} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i0} = \boldsymbol{\Omega}_0, i = 1, ..., M$. Then the equilibrium solution yielding the benchmark is $\mathbf{x}_{i0}^* = \frac{d_i}{d}\mathbf{1}$, $i = 1, ..., M$, and $\mathbf{P}_0 = \boldsymbol{\mu}_0 - \frac{1}{d}\boldsymbol{\Omega}_0\mathbf{1}$ where $d_i = \frac{1}{a_i}$ and $d = \sum d_i$.

Following this market exchange, one of the investors comes to believe that, with sufficient shares in a particular firm, he can improve its performance and thereby profit from his activism. We designate this activist as investor 1, and the single firm that is the target of his interest as firm 1.[9] Since we have assumed that all investors can borrow, lend, as well as sell short, our activist must have these capabilities as well. Thus, our model necessarily excludes mutual funds as activists, but includes both hedge fund activists and other entrepreneurial activists such as individual investors, and private equity funds.[10]

If the activist proceeds with his plan to acquire additional shares, it is done surreptitiously, and it forces a new round of trading. This leads to a heterogeneous information equilibrium whose solution is given in Proposition 2. The proof of this proposition, and all following propositions and the lemma, can be found in the Appendix.

Proposition 2. Let the distributional parameters for investor 1, the activist, be $\boldsymbol{\mu}_{11} = \boldsymbol{\mu}_0 + \Delta\boldsymbol{\mu}$ and $\boldsymbol{\Omega}_{11} = \boldsymbol{\Omega}_0 + \Delta\boldsymbol{\Omega}$ and let those for investor i, $i = 2, ..., M$, be $\boldsymbol{\mu}_{i1} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i1} = \boldsymbol{\Omega}_0$. Then, at $t = 1$, the equilibrium solution is given by

$$\mathbf{x}_{11}^* - \mathbf{x}_{10}^* = (d - d_1)\boldsymbol{\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0)$$

$$\mathbf{x}_{i1}^* - \mathbf{x}_{i0}^* = -d_i\boldsymbol{\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0) \text{ for } i = 2, ..., M, \text{ and}$$

$$[d\mathbf{I} + (d - d_1)\Delta\boldsymbol{\Omega}\boldsymbol{\Omega}_0^{-1}](\mathbf{P}_1 - \mathbf{P}_0) = d_1(\boldsymbol{\Delta\mu} - \boldsymbol{\Delta\Omega}\mathbf{1}/d).$$

We have assumed here that the intent of activism is to alter the future distribution of prices of the risky assets. We next sharpen this assumption further. Since we restrict activism to firm 1, we assume that activism may alter the future expected value of the price of this target firm as well as the variance of this price and its

[9]The activist has only one target firm in our model. This assumption is made for convenience of exposition.

[10]Mutual funds are subject to the Investment Company Act of 1940 which, among other things, prevents them from selling short, borrowing, and holding concentrated positions. Hedge funds, by having a small number of high net worth investors, are not subject to this Act, and, accordingly, are not governed by the regulation of fees specified in the Act. See Brav et al. (2008, pp. 1734-1736) for a discussion of differences between mutual funds and hedge funds.

covariance with prices of the other firms. In particular, we assume at time $t = 1$ that the activist believes that the expected price per share of firm 1 will increase by $m > 0$ if he succeeds and remain the same otherwise. The expected values of the remaining firms are unchanged. The variance of the price of firm 1, as well as the covariances of the price of firm 1 with the other firms, might also change. Thus, we assume that the covariance matrix of prices might change in the first row and first column if the activist succeeds and would remain the same otherwise.[11]

We introduce the following notation. The subscript -1 is used for a vector or matrix to denote that vector or matrix without its first element or first row, respectively, e.g., the N x 1 vector $\mathbf{v}$, with first element v_1, is written as $\mathbf{v}'= (v_1, \mathbf{v}'_{-1})$. We let $\boldsymbol{\Omega}_0^{-1} = (\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^N) = \begin{pmatrix} \omega_1^1 & \boldsymbol{\omega}_{-1}^{1\prime} \\ \boldsymbol{\omega}_{-1}^1 & \mathbf{R} \end{pmatrix}$ where $\mathbf{R}$ is a positive definite $N-1$ x $N-1$ symmetric matrix. The omission of the first row of the matrix $\boldsymbol{\Omega}_0^{-1}$ will be written as $\boldsymbol{\Omega}_{-1,0}^{-1}$. If we define the N x N matrix $\mathbf{V} = \begin{pmatrix} v_1 & \mathbf{v}'_{-1} \\ \mathbf{v}_{-1} & \mathbf{0} \end{pmatrix}$ and π as the probability that the activist will succeed in his plans, the activist approaches the market at $t = 1$ with parameters $\boldsymbol{\mu}_{11}= \boldsymbol{\mu}_0 + \pi m \mathbf{e}_1$ and $\boldsymbol{\Omega}_{11} = \boldsymbol{\Omega}_0 + \pi \mathbf{V}$ where $\mathbf{e}_1$ is an N x 1 vector with 1 in the first position and zeros elsewhere. The other investors remain with their previous information, i.e., $\boldsymbol{\mu}_{i1}= \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i1} = \boldsymbol{\Omega}_0$, $i = 2, ..., M$. We next present a lemma that permits us to solve explicitly for the inverse needed to determine the equilibrium price changes in Proposition 2.

In what follows, we let $(\mathbf{P}_1 - \mathbf{P}_0)' = ((\mathbf{P}_1 - \mathbf{P}_0)_1, ..., (\mathbf{P}_1 - \mathbf{P}_0)_N)$, where $(\mathbf{P}_1 - \mathbf{P}_0)_j$ is the j^{th} component of $\mathbf{P}_1 - \mathbf{P}_0$. Scalar components for other vectors are indicated in a similar manner.

Lemma. The N x 1 vectors $\mathbf{x}' = (x_1, \mathbf{x}'_{-1})$ and $\mathbf{z}' = (z_1, \mathbf{z}'_{-1})$ and the matrix

[11] For example, Clifford (2007), Bratton (2007) and Boyson and Mooradian (2011) find evidence that hedge fund activism is concentrated in particular industries.

$\mathbf{M} = [\mathbf{I} - \alpha\binom{\mathbf{x}'}{\mathbf{v}_{-1}\mathbf{z}'}]$ satisfy $\mathbf{M}[\mathbf{I}+\alpha\mathbf{V}\boldsymbol{\Omega}_0^{-1}] = \mathbf{I}$ where

$$\begin{aligned}
x_1 &= \frac{1}{c}[\mathbf{v}'\boldsymbol{\omega}^1 - \alpha\omega_1^1(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)] \\
\mathbf{x}_{-1} &= \frac{1}{c}[\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v} - \alpha\frac{(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})}{(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)}\boldsymbol{\omega}_{-1}^1] \\
z_1 &= \frac{\omega_1^1}{c(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)} \\
\mathbf{z}_{-1} &= \frac{1}{c(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)}[(1+\alpha\mathbf{v}'\boldsymbol{\omega}^1)\boldsymbol{\omega}_{-1}^1 - \alpha\omega_1^1\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}] \\
c &= 1+\alpha\mathbf{v}'\boldsymbol{\omega}^1 - \alpha^2\omega_1^1(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1) \text{ and} \\
0 &< \alpha \leq 1.
\end{aligned}$$

Since $\mathbf{M}[\mathbf{I}+\alpha\mathbf{V}\boldsymbol{\Omega}_0^{-1}] = \mathbf{I}$, it follows that $\boldsymbol{\Omega}_0^{-1}\mathbf{M}$ is the inverse of $[\boldsymbol{\Omega}_0+\alpha\mathbf{V}]$. Because this latter matrix is assumed to be positive definite, its inverse must have positive diagonal elements. It follows that the upper left diagonal element of $\boldsymbol{\Omega}_0^{-1}\mathbf{M}$ must be positive and this can only happen if $c(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1) > 0$. For the remainder of the paper we assume that the parameters satisfy $c > 0$ and $1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1 > 0$.

This lemma allows us to present the equilibrium prices at $t = 1$ explicitly. We do this in the next proposition.

Proposition 3. At time $t = 1$, $\boldsymbol{\mu}_{11} = \boldsymbol{\mu}_0 + \boldsymbol{\pi} m\mathbf{e}_1$ and $\boldsymbol{\Omega}_{11} = \boldsymbol{\Omega}_0 + \pi\mathbf{V}$, and $\boldsymbol{\mu}_{i1} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i1} = \boldsymbol{\Omega}_0$, $i = 2, ..., M$. Then the equilibrium prices can be written as

$$(\mathbf{P}_1 - \mathbf{P}_0)_1 = g_1$$

and

$$(\mathbf{P}_1 - \mathbf{P}_0)_{-1} = -g_2\mathbf{v}_{-1}$$

where

$$
\begin{aligned}
g_1 &= \frac{d_1\pi}{cd}[m - \mathbf{v}'\mathbf{1}/d + \alpha\frac{1}{d}(\mathbf{v}'_{-1}\boldsymbol{\Omega}^{-1}_{-1,0}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^{1}_{-1})] \\
g_2 &= \frac{d_1\pi}{cd}[\frac{\alpha\omega^1_1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}(m-\mathbf{v}'\mathbf{1}/d) + \frac{1}{d}(\frac{1+\alpha\mathbf{v}'\boldsymbol{\omega}^1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}})] \text{ and} \\
\alpha &= \frac{d-d_1}{d}\pi.
\end{aligned}
$$

Propositions 2 and 3 demonstrate the result of the surreptitious acquisition of shares by the activist.[12] The activist's predictions of the changes that his activism would produce caused him to seek to alter his portfolio holdings consistent with his predictions. Because this alteration of shares had to be acquired in the market and because his view of future prices was different from other investors, the market exchange was characterized by a heterogeneous information equilibrium. Under these conditions, the two propositions establish the relationship between his predictions and their impact on prices and the holdings of all market participants at $t = 1$. In particular, Proposition 3 shows how changes in the variance or covariances effect both the price change of firm 1 and all prices connected to firm 1. Furthermore, Proposition 2 extends this observation to the holdings themselves.

In our model, we have chosen to abstract from the normal activities of the activist, for example, from attempting to acquire representation on the board, changing dividend policy, changing CEO salary, and/or selling parts of the firm, etc. Instead, we have chosen to characterize activities into ways in which they alter the future distribution of prices. Specifically, some activities will affect the mean, others the variance and still others the covariance of the target firm with other firms. Indeed, some activities will affect these three features in various combinations.

Should the activist believe that the result of his activism would have no additional

[12] Note that hedge funds with less than $100 million in assets are not subject to quarterly reports, needing only to file Schedule 13D as required.

effect on the covariances between firm 1 and the remaining firms, i.e., $\mathbf{v}_{-1} = \mathbf{0}$, then from Proposition 3, it follows immediately that prices other than the price of shares of the first firm would not change. However, using Proposition 2 under the condition that $\mathbf{v}_{-1} = \mathbf{0}$, we note that holdings for all investors change nevertheless. That is, a rebalancing of portfolios occurs for all investors even though only the price of the shares of the target firm changes.

Examining the change in the price of the shares of firm 1 exhibited in Proposition 3, it is not clear, in general, that the price increases without imposing some further conditions. These conditions will be clarified when, after discussing the remaining two equilibria, we address the preliminary decision that the activist would have had to have made to become an activist in the first place. We will then show how these conditions impose further constraints on our model. We need to delay the discussion for the following reason. Under the assumptions that the activist will have acquired additional shares, he will be able to begin his efforts to alter the direction of the firm. This, however, has come at a cost of acquiring these additional shares that can be written as $\mathbf{P}_1'(\mathbf{x}_{11}^* - \mathbf{x}_{01}^*)$. In the initial decision as to whether to become an activist, the activist must consider this cost against the expected revenue he will subsequently receive when he has finished his activist activities and sells his extra shares on the market at $t = 3$. Indeed, this would be the necessary condition for his activism. Obviously, this initial decision will depend on the fact that the expected revenue at time $t = 3$ exceeds this cost plus whatever other costs the activist may incur in connection with his activist agenda.

Assuming the activist has acquired sufficient shares at $t = 1$, then at $t = 2$, he files Schedule 13D. With the release of information contained in his filing of Schedule 13D, all investors, except for the activist, institute a trading round based on this new information. The activist will not be involved in this trading round since we assume his acquisition of additional shares was predicated on the fact that he would continue to hold shares long enough to execute his plan.[13] Thus, the trading round at time $t = 2$ is again one of homogeneous information, but with the number of shares held by the activist excluded from the competition.

More precisely, at time $t = 2$, the activist does not trade and each of the other investors learns of the information held by the activist. Thus, at this time we have $M - 1$ investors sharing the same information $\boldsymbol{\mu}_{i2} = \boldsymbol{\mu}_0 + \pi m \mathbf{e}_1$ and $\boldsymbol{\Omega}_{i2} = \boldsymbol{\Omega}_0 + \pi \mathbf{V}$, $i = 2, ..., M$. The result of this competition is contained in the next proposition.

Proposition 4. At time $t = 2$, the activist does not trade, and $\boldsymbol{\mu}_{i2} = \boldsymbol{\mu}_0 + \pi m \mathbf{e}_1$ and $\boldsymbol{\Omega}_{i2} = \boldsymbol{\Omega}_0 + \pi \mathbf{V}$, $i = 2, ..., M$. Then the equilibrium solution yields $\mathbf{x}^*_{i2} = \mathbf{x}^*_{i1}$ for $i = 2, ..., M$, and $\mathbf{P}_2 = \boldsymbol{\mu}_0 + \pi m \mathbf{e}_1 - \frac{1}{d - d_1}(\boldsymbol{\Omega}_0 + \pi \mathbf{V})(\mathbf{1} - \mathbf{x}^*_{11})$.

Proposition 4 establishes the fact that the new information acquired by the remaining investors when the activist abstains from the trading has no impact on their holdings. The intuition for this result is as follows. The activist would not wish to sell his recently acquired shares in firm 1 since this would undermine his purpose as an activist. Given this point, he would not wish to trade his shares in other firms either, since he already optimized his holdings in these firms in conjunction with his purchase of additional shares in firm 1 using his private information. In fact, he would be at a disadvantage to trade in a market in which all investors had the same information as he did. On the other hand, the other investors, having been alerted to the possible activism by the Schedule 13D filing and now perhaps wanting more of the shares of firm 1, can only get those shares from among themselves. In their attempt to get more shares, the prices will change. At these changed prices, however, it becomes optimal for these other investors to end up with portfolios identical to the ones they

[13] See Clifford (2007) who finds that hedge funds do not seem to buy or sell additional shares when they change from a passive status to an active one, although that change in status necessitates a filing of Schedule 13D.

selected at time $t = 1$.

Subsequently, at time $t = 3$, there is new information since it becomes known as to whether or not the activist was successful. The distributional parameters held by all market participants then are either $\boldsymbol{\mu}_0 + m\mathbf{e}_1$ and $\boldsymbol{\Omega}_0 + \mathbf{V}$ if the activist was successful, or $\boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_0$ otherwise. At this point, the activist is involved in the trading since there is no further informational advantage from holding the portfolio acquired at $t = 1$. Thus, all investors participate in a homogenous information equilibrium. Should this equilibrium result in the sale of shares in firm 1, then at this time Schedule 13D/A is filed by the activist, acknowledging the change in his ownership. The next proposition provides the results.

Proposition 5. At $t = 3$, if the activist is successful, $\boldsymbol{\mu}_{i3} = \boldsymbol{\mu}_0 + m\mathbf{e}_1$ and $\boldsymbol{\Omega}_{i3} = \boldsymbol{\Omega}_0 + \mathbf{V}$. At $t = 3$, if the activist is not successful, $\boldsymbol{\mu}_{i3} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i3} = \boldsymbol{\Omega}_0$, $i = 1, ..., M$. In either case, $\mathbf{x}^*_{i3} = \mathbf{x}^*_{i0} = \frac{d_i}{d}\mathbf{1}$. If the activist is successful, the equilibrium price $\mathbf{P}_3 = \boldsymbol{\mu}_0 + m\mathbf{e}_1 - \frac{1}{d}(\boldsymbol{\Omega}_0 + \mathbf{V})\mathbf{1}$; if the activist is unsuccessful, the equilibrium price is $\mathbf{P}_3 = \mathbf{P}_0$.

One interesting feature of Proposition 5 is that whether successful or not at $t = 3$, the activist chooses to sell the additional shares he acquired at $t = 1$ in firm 1. That is, there is no way for the activist, if successful, to take advantage of the improved distribution of prices. This follows since all investors now know of his success, and thus the combined demand forces this result in equilibrium.

The derivation of the equilibria was predicated on an initial decision made by the activist: the decision to become an activist or not. In the next Section we discuss how this preliminary decision was made.

3 The Decision to Become an Activist

In our model, the activist approaches the decision to become an activist with a presumption of how the value of the target firm, as well as the value of other firms, would change as a result of his activism. This is summarized by the parameters of his subjective probability distribution of the future value of the target as well as other firms in the market. Under what conditions does this distribution warrant activism?

We assume that the activist needs to be convinced before proceeding of two issues. In order to be a successful activist, he must expect to acquire a foothold in the target firm from which to act. Thus, before time $t = 1$, he must believe that at time $t = 1$, when surreptitiously seeking additional ownership in the target firm, he will indeed be able to acquire such additional ownership. Using the notation established above, the first condition that we impose for proceeding to activism, *CA1*, is that $(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_1 > 0$.[14] Believing *CA1* to be satisfied, the activist must further expect that the cost of acquiring the foothold will be offset by the gain made when he exits the target firm at time $t = 3$. Thus, the second condition that we impose, *CA2*, is that the expected cash flow over the life of the activist's involvement in the target firm be positive. Using our notation, we next evaluate the expected cash flows.

From Proposition 5, it follows that the cash flow to the activist resulting from the equilibrium at time $t = 3$ is $\mathbf{P}'_3(\mathbf{x}^*_{11} - \mathbf{x}^*_{13})$. Since $\mathbf{x}^*_{13} = \mathbf{x}^*_{10}$, this cash flow can be written as $\mathbf{P}'_3(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$. Recall that the cost to the activist at $t = 1$ for acquiring additional shares in the target firm was $\mathbf{P}'_1(\mathbf{x}^*_{10} - \mathbf{x}^*_{11})$. Thus, the cash flow to the activist from $t = 1$ to $t = 3$ is $(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$. This can be written as $[(\mathbf{P}_3 - \mathbf{P}_0) - (\mathbf{P}_1 - \mathbf{P}_0)]'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$. Since $(\mathbf{P}_3 - \mathbf{P}_0)$ can take one of two values, the cash flow equals $(m\mathbf{e}_1 - \frac{1}{d}\mathbf{V1})'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10}) - (\mathbf{P}_1 - \mathbf{P}_0)]'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$ if the activist is successful, and equals $-(\mathbf{P}_1 - \mathbf{P}_0)]'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$ otherwise. Thus, the second condition

[14] We could have imposed the requirement $(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_1 > \tau > 0$ but for convenience chose $\tau = 0$.

for proceeding to activism, *CA2,* is that the activist would proceed only if the expected cash flow were positive, that is, if

$$E(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*) = \pi(m\mathbf{e}_1 - \frac{1}{d}\mathbf{V}\mathbf{1})'(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*) - (\mathbf{P}_1 - \mathbf{P}_0)'(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*) > 0.$$

Together, we call these two conditions for activism CA, and note that CA places a constraint on the parameters that the potential activist brings to the problem. Only when CA is satisfied, will the activist proceed. We now rewrite CA to exhibit its dependency on the fundamental parameters. Using Propositions 2 and 3, we can rewrite *CA1* as $[g_1\omega_1^1 - g_2\mathbf{v}_{-1}'\boldsymbol{\omega}_{-1}^1] > 0$. Using Propositions 2 and 3, we can also rewrite *CA2* as $[\pi(m - \mathbf{v}'\mathbf{1}/d) - g_1)](\mathbf{x}_{11}^* - \mathbf{x}_{10}^*)_1 + (g_2 - \pi/d)\mathbf{v}_{-1}'(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*)_{-1} > 0$. Since $(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*)_1 > 0$ by *CA1*, we can write *CA2* as $\pi(m - \mathbf{v}'\mathbf{1}/d) - g_1 - (\frac{\pi}{d} - g_2)\theta > 0$ where $\theta = \frac{\mathbf{v}_{-1}(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*)_{-1}}{(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*)_1}$.

To illustrate CA, we offer the following example. Suppose the potential activist intends to alter the target firm in ways that increase shareholder value by influencing the way management decisions are made. If, in our example, we exclude those decisions of management that involve selling all or part of the firm, or merging it with another firm, decisions that alter the covariance of the target with other firms, then our example is one in which the activist's activities are those that will affect the future mean and variance of the value of the target. Examples of these types of activist activities include cost cutting, general operational efficiency, obtaining board representation, and increased dividend payouts. Each of these activities might increase the expected future value of the target. Corresponding to an increase in expected value is a change in the precision of the estimate of this increase, i.e., the change in the variance of the forecast. This change in forecast accuracy can yield a more precise overall prediction if, for instance, the existing management agrees with the activities proposed by the activist. On the other hand, if, in order to carry forth his intentions, the activist must come into conflict with management, the outcome might become more uncertain. The salient feature of this example, put in terms of the parameters of the probability distribution of the future price of the target firm as well as other firms, is that $\mathbf{v}_{-1} = \mathbf{0}$. This leaves two distributional parameters to be specified: m, the increase in future value, and v_1, the increase or decrease in the variance of that future value. The condition *CA* determines the relationship between these two values that permits the activist to proceed, as we now demonstrate.

We first evaluate the functions g_1 and g_2 under the condition that $\mathbf{v}_{-1} = \mathbf{0}$. Using Proposition 3, $g_1 = \frac{d_1\pi}{cd}(m - v_1/d)$ and $g_2 = \frac{d_1\pi}{cd}[\alpha\omega_1^1(m - v_1/d) + \frac{1}{d}(1 + \alpha v_1\omega_1^1)]$. It now follows that if $c = 1 + \alpha v_1\omega_1^1 > 0$ and if $(m - v_1/d) > 0$, then *CA1* is satisfied and $g_1 > 0$. Combining these two inequalities, it follows that if $-\frac{1}{\alpha\omega_1^1} < v_1 < md$, then *CA1* is satisfied. Examining inequality *CA2*, this inequality, when combined with *CA1* is satisfied if $\pi(m - v_1/d)(1 - \frac{d_1}{cd}) > 0$ or if $c > d_1/d$. Since $c = 1 + \alpha v_1\omega_1^1$, we have that $c > d_1/d$ if $v_1 > -\frac{1}{\pi\omega_1^1}$. Finally, since $-\frac{1}{\alpha\omega_1^1} < -\frac{1}{\pi\omega_1^1}$, condition *CA* is satisfied if m and v_1 satisfy $-\frac{1}{\pi\omega_1^1} < v_1 < md$.

The term $(m - v_1/d)$ can be interpreted as the marginal risk-adjusted value that the activist could create. Also, from Proposition 3, $(\mathbf{P}_1 - \mathbf{P}_0)_1 = g_1 > 0$ and since $\theta = 0$, *CA2* implies that $(\mathbf{P}_3 - \mathbf{P}_1)_1 = \pi(m - v_1/d) - g_1 > 0$. Thus, in order to proceed in this simple example, the activist must believe that the changes that he could induce will produce a risk-adjusted increase in value sufficient to yield an increase in the price of the target firm at time $t = 1$, and, at a minimum, to further increase the price of the target firm at time $t = 3$. The intuitive simplicity of these requirements follow from allowing the changes due to activism to affect the target firm and that firm alone.

When the impact of activism changes the relationship of the target to other firms in the market, then the constraints imposed by *CA* become more complicated. We will investigate this situation below. However, we point out that the condition *CA* can be used by the potential activist as a criterion for the selection of the target firm.

In considering various targets and calculating the expected cash flows that would result from activism in each of these firms, the activist would obviously choose the one which yields the highest cash flow. In our example above, the activist would choose the target associated with the largest value of $m_k - v_{1k}/d$, where k ranges over the potential target firms and m_k and v_{1k} represent the corresponding parameters.

4 Price Movements over the Period of Activism

We now return to the general case and consider how the satisfaction of CA affects the subsequent prices. We begin by deriving the forecasts of our model and then compare them with the price changes described in the empirical literature.

The next proposition establishes the relationship between acquiring a foothold in the target firm and the change in price of that target firm from time $t = 0$ to $t = 1$.

Proposition 6. $(\mathbf{P}_1 - \mathbf{P}_0)_1 > \frac{d_1 \pi}{\omega_1^1 d^2} \mathbf{v}'_{-1} \boldsymbol{\omega}^1_{-1}$.

An important consequence of this proposition is that, depending on the sign of $\mathbf{v}'_{-1} \boldsymbol{\omega}^1_{-1}$, the target firm price change from time $t = 0$ to $t = 1$ may or may not be positive. Furthermore, a negative price change, while still leading to a foothold in the target firm, is not necessarily a deterrent from proceeding with activism. As can be seen from the lower bound in Proposition 6, whether or not $(\mathbf{P}_1 - \mathbf{P}_0)_1$ must be positive depends both on the changes in covariances of the target firm with other firms and on the initial covariances between these firms. For instance, as illustrated in the example in Section 3 above, when $\mathbf{v}_{-1} = \mathbf{0}$, it follows that $(\mathbf{P}_1 - \mathbf{P}_0)_1 > 0$. We now continue by investigating implications of CA on the remaining prices.

Proposition 7.

(1) $(\mathbf{P}_2 - \mathbf{P}_1)_1 = \pi[m - \mathbf{v}'\mathbf{1}/d + \frac{(x^*_{11} - x^*_{10})_1}{d - d_1}(v_1 + \theta)]$,

$$(2)\ (\mathbf{P}_3-\mathbf{P}_1)_1=\begin{cases} m-\mathbf{v}'\mathbf{1}/d-g_1 & \text{if } \mathbf{P}_3=\mathbf{P}_0+m\mathbf{e}_1-\frac{1}{d}\mathbf{V}\mathbf{1} \\ -g_1 & \text{if } \mathbf{P}_3=\mathbf{P}_0 \end{cases},$$

$$(3)\ (\mathbf{P}_3-\mathbf{P}_2)_1=\begin{cases} m-\mathbf{v}'\mathbf{1}/d-g_1-(\mathbf{P}_2-\mathbf{P}_1)_1 & \text{if } \mathbf{P}_3=\mathbf{P}_0+m\mathbf{e}_1-\frac{1}{d}\mathbf{V}\mathbf{1} \\ -g_1-(\mathbf{P}_2-\mathbf{P}_1)_1 & \text{if } \mathbf{P}_3=\mathbf{P}_0 \end{cases}.$$

Notice that the sign of $(\mathbf{P}_3-\mathbf{P}_1)_1$ could be positive or negative. Also, from Proposition 6, the sign of $(\mathbf{P}_1-\mathbf{P}_0)_1$ could be negative. Why would it be desirable for the activist to proceed to gain a foothold when he expects the price at time $t=1$ to decline over that at time $t=0$ and faces the possibility that the price at time $t=3$ would subsequently fall? The answer, since *CA2* is satisfied, is that in this instance the activist gains at time $t=1$, and the losses he incurs at time $t=3$ are less than the gains. The situation just described may not be typical and we impose further conditions on our parameters so that $(\mathbf{P}_1-\mathbf{P}_0)_1>0$, also allowing us to sign all the prices. Additionally, we focus on the case where $(\mathbf{P}_1-\mathbf{P}_0)_1>0$ since this comports with that studied in the empirical literature. Since we are assuming that the activist has proceeded, for the remainder of the paper we assume that condition *CA* is satisfied.

Proposition 8. Let $\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}>0$, $(\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})^2\geq\omega^1_1\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1}$ and $v_1\geq-\frac{\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1}}{\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}$. Then

(1) $(\mathbf{P}_1-\mathbf{P}_0)_1>0$,

(2) $(\mathbf{P}_2-\mathbf{P}_1)_1>0$,

$$(3)\ \begin{cases} \text{There exists a constant } k_1>0 \text{ such that} & \\ (\mathbf{P}_3-\mathbf{P}_1)_1\geq 0 & \text{if } \mathbf{P}_3=\mathbf{P}_0+m\mathbf{e}_1-\frac{1}{d}\mathbf{V}\mathbf{1} \text{ and } \frac{d_1\pi}{dc}\leq k_1 \\ (\mathbf{P}_3-\mathbf{P}_1)_1<0 & \text{if } \mathbf{P}_3=\mathbf{P}_0 \end{cases},$$

$$(4)\ \begin{cases} \text{There exists a constant } k_2>0 \text{ such that} & \\ (\mathbf{P}_3-\mathbf{P}_2)_1\geq 0 & \text{if } \mathbf{P}_3=\mathbf{P}_0+m\mathbf{e}_1-\frac{1}{d}\mathbf{V}\mathbf{1} \text{ and } \pi\leq k_2 \\ (\mathbf{P}_3-\mathbf{P}_2)_1<0 & \text{if } \mathbf{P}_3=\mathbf{P}_0 \end{cases}.$$

We see from Propositions 7(1) and 8(2) that the increase in the price of the target firm at time $t = 2$ over $t = 1$ depends on what we call the risk adjusted mean, $m - \mathbf{v}'\mathbf{1}/d$, the change in the variance of the price of the target firm and a function dependent on all of the changes in the covariances, i.e., θ.

Propositions 6 through 8 describe the sequence of changes in the price of the target firm conditional upon the constraints imposed by the activist in order to proceed. The usefulness of the generality of these results can be seen in the relationship to the empirical work on activism mentioned above. For example, in one-quarter to one-third of the instances of activism examined by Brav et al. (see p. 1755), Klein and Zur (see Table IV) and Clifford (see Table IV), the price of the target firm declines around the time of the filing of Schedule 13D (our times $t = 1$ and $t = 2$), consistent with our Proposition 6. Our model supplies a possible explanation for these cases in which the activist proceeds to become an activist despite the expectation of a decline in the price of the target firm at time $t = 1$. However, in the majority of cases considered in the literature, the target price at this time increases. It is based on this increase that the literature broadly concludes that the impact of activism on the market is positive.

The results of Proposition 8 exhibit a sequence of prices for the target firm derived from our model. Each of the prices in this sequence is a result of an equilibrium generated by new information. Initially the activist brought new information (surreptitiously) to the market at time $t = 1$. Under the assumed conditions, this caused the price of the target firm to rise. Then, after the filing of Schedule 13D, the activist's intentions were revealed to the market place yielding the equilibrium price at time $t = 2$. This equilibrium caused another increase in the price of the target firm. Finally, when information about the activist's success or failure became known, the equilibrium price at time $t = 3$ was determined. No further change in price occurred between $t = 2$ and $t = 3$ since we assumed that no further information was forthcoming in that interval. With the information available at time $t = 3$, the equilibrium price of the target firm was shown to exceed the equilibrium price at time $t = 1$ when the activist was successful, and exceed the equilibrium price at time $t = 2$ in this case as well. However, if the activist were unsuccessful, the equilibrium price of the target firm at time $t = 3$ falls below the equilibrium prices at both $t = 1$ and $t = 2$. Proposition 8 predicts that the profile of prices of the target firm over the period of activism due to activism has the following shape: a stark increase around the time of the Schedule 13D filing ($t = 1$ and $t = 2$), the price remaining at that elevated level until knowledge of the success or failure of the activist becomes available ($t = 3$), and the price subsequently rising or falling depending on the success or failure of the activist.

We next show that the forecast of the sequence of prices given in Proposition 8 reasonably coincides with the average performance of target firm prices that forms the focus of the recent empirical literature. In that literature, we observe that significant average abnormal returns for the target firm around the filing of Schedule 13D are prominent findings in the studies of Brav et al. (see Figure 1 for an illustration), Klein and Zur, Boyson and Mooradian, Clifford, and Greenwood and Schor.[15] While earlier studies, for example, Black, Karpoff, and Gillian and Starks, found scant evidence of the impact of activism, the newer empirical studies focus on the release of information by hedge funds (and other entrepreneurial activists) via their Schedule 13D filings. Viewed from the prospective of our theoretical model, it is this shift in focus of the newer empirical work to the release of new information by the hedge funds and others to the market place that permits these studies to discover the positive average

[15] To our knowledge, the only recent study not providing evidence of this price increase at the time of announcement is Becht et al. in the context of the Hermes U.K. Focus Fund that, as noted above, does not announce its acquisitions as a separate entity, but rather has its acquisitions announced, along with others, by a parent group.

abnormal returns around the filing event. Furthermore, from the perspective of our model, the relative constancy of positive abnormal returns of the targets during the year following the Schedule 13D filing event noted in recent studies (in particular, see Brav et al., and Klein and Zur), also is quite natural as no new information has been released during this time period; hence, our model would predict no price changes.

When new information later becomes available, specifically the knowledge of whether the activist has been successful or not, recent empirical findings suggest that the target firm's price declines with evidence of failure and rises with evidence of success. In particular, see, Brav et al., Figure 2, where they show that, for activist hedge funds that exit the target (and file Schedule 13D/A) after failure, prices decline. Although not shown explicitly in their Figure 2, the subset of activists that exit (filing Schedule 13D/A) after success exhibit price increases. This comports with our model's prediction that when success or failure of the activist becomes known and the activist exits the target firm, the price of the target will increase in instances of success and fall (revert to the original benchmark price) in instances of failure.[16]

In sum, our model is able to predict both the positive and negative price movements of target firms exhibited in the empirical literature. Under appropriate assumptions (Proposition 8), our model is able to predict the average price movements that form the basis of discussion in that literature. In fact, the behavior of the average price (mean or median) is the basis used in the literature for arguing the benefits of activism. These benefits, it is argued, follow from the increase in the target firm's price from time $t = 0$ to $t = 1$, which implies an immediate increase in shareholder value. Our approach, which focuses on diversified investors over the entire period of

[16] Slightly different findings are described in Klein and Zur: they find that when the activist exits with success, the price of the target increases more than when he exits with failure. Perhaps these differences in the findings arise as Klein and Zur study only confrontational hedge funds and other entrepreneurial investors while Brav et al. study both confrontational and non-confrontational hedge funds.

the activist's involvement, raises questions about ascribing benefits of activism based only on the movements of the target firm's price at time $t = 1$. It is with this in mind that we turn to the question of evaluating the benefits of activism. Having demonstrated the reasonableness of the predictions of Proposition 8 in describing the average price movements of the target firm, we retain the assumptions of that proposition in evaluating the benefits of activism.

5 Assessing Activism's Impact on the Market and other Diversified Investors

Up to this point we have established the conditions that permit an activist to proceed, and have shown the way in which information has affected equilibrium prices over the course of the activist's involvement with the target firm. We have also shown the conditions under which our model predicts target price movements consistent with those found in the empirical literature. We now propose to use our model to investigate several features that we believe ought to be considered in an evaluation of activism. These features reflect the notion that this evaluation should depend on the way in which the outcome of the activist's activities affect the market as a whole.

Our model permits us to calculate the relationship between the outcome of activism and the outcome on the market as a whole in two different ways. First, we can compute the total change in market value between times t and τ, $t > \tau$, resulting from activism, i.e., $\mathbf{1}'(\mathbf{P}_t - \mathbf{P}_\tau)$. A positive assessment of activism, especially at the time of the filing of Schedule 13D, $t = 1$, might require that an increase in the target price at that time should yield $\mathbf{1}'(\mathbf{P}_1 - \mathbf{P}_0) > 0$. Additionally, the assessment might include that the total change in the market value from $t = 0$ to $t = 3$ be positive under the assumption that the activist were successful. In the context of diversified portfolio holders, these assumptions include consideration of the impact of activism on other firms held in the portfolios.[17]

Second, an evaluation of activism should consider the impact of activism on all other participants in the market. The activist's involvement in the firm is predicated on the assumption that it will earn an expected profit over the course of his activism. Indeed, our condition *CA* is an expression of this expectation. *CA2* implies that the activist would expect to earn $E(\mathbf{P}_3-\mathbf{P}_1)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10})$. But what would the expectation be for the group of other investors? If we let $\mathbf{x}_{Gt}=\sum_{j=2}^{M}\mathbf{x}^*_{jt}$, $t=0,1,2,3$, then the expected cash flow of the group of other investors over the period of activism is $E(\mathbf{P}_3-\mathbf{P}_1)'(\mathbf{x}^*_{G1}-\mathbf{x}^*_{G0})$. A positive assessment of activism over the period might require that when this expectation of the activist is positive, so should the expectation of the group of other investors. We next apply these criteria to activism as described in our model, beginning with the first criterion.

Proposition 9. Let $g_1>0$.

(1) At time $t=1$, there exists a constant β, $\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}<\beta<\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}+\frac{1}{\alpha}$, such that

(a) $(\mathbf{P}_1-\mathbf{P}_0)_1>0$ and $\mathbf{1}'(\mathbf{P}_1-\mathbf{P}_0)>0$ if $\boldsymbol{\omega}^1_{-1}\mathbf{v}'_{-1}\mathbf{1}_{-1}<\beta$

(b) $(\mathbf{P}_1-\mathbf{P}_0)_1>0$ and $\mathbf{1}'(\mathbf{P}_1-\mathbf{P}_0)\le 0$ if $\boldsymbol{\omega}^1_{-1}\mathbf{v}'_{-1}\mathbf{1}_{-1}\ge\beta$.

(2) At time $t=3$, if the activist is successful

(a) $\mathbf{1}'(\mathbf{P}_3-\mathbf{P}_0)>0$ if $\frac{1}{d}\mathbf{v}'_{-1}\mathbf{1}_{-1}<\frac{1}{2}(m-v_1/d)$

(b) $\mathbf{1}'(\mathbf{P}_3-\mathbf{P}_0)\le 0$ if $\frac{1}{d}\mathbf{v}'_{-1}\mathbf{1}_{-1}>\frac{1}{2}(m-v_1/d)$.

(3) At time $t=3$, if the activist is not successful, $\mathbf{1}'(\mathbf{P}_3-\mathbf{P}_0)=0$.

Since $\mathbf{v}'_{-1}\mathbf{1}_{-1}$ is proportional to the average change of the covariances due to activism, Proposition 9 demonstrates the importance of the size of this average in

[17] We note that in their model of diversified investors, Hansen and Lott (1996) emphasize that, in the presence of externalities, the appropriate objective of analysis is the portfolio, in which spillovers can be incorporated, rather than the individual stock prices and their responses to announcements.

determining the direction of change of the market value. Note that when $\mathbf{v}_{-1} = \mathbf{0}$, the first of the cases at $t = 1$ prevails, and it follows that at that time, a rise in the price of the target firm is coincident with a rise in the market. It suffices for this conclusion to hold if $\mathbf{v}'_{-1}\mathbf{1}_{-1} = 0$ and $\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1} \geq 0$. From the proof of part (1) of Proposition 9, we conclude that, if $\mathbf{v}'_{-1}\mathbf{1}_{-1}$ is sufficiently negative, then increases in the price of the target firm and that of the market will both occur. A similar conclusion follows from part (2) of Proposition 9, where at time $t = 3$, these two positive price changes will occur for small values of $\mathbf{v}'_{-1}\mathbf{1}_{-1}$. However, the importance of this proposition is to show that, whether at the time of the filing of Schedule 13D, $t = 1$, or at the exit of the activist, $t = 3$, the direction of the change in the market cannot be determined only by the direction of the price change in the target firm. Specifically, the changes in the covariance structure caused by activism must also be taken into account in order to ascertain the impact of activism on the market.

A general conclusion of recent empirical studies of hedge fund and other entrepreneurial activism (see, for example, Brav et al., Klein and Zur, Boyson and Mooradian, and Clifford) is that shareholders benefit from activism. The evidence presented to support this claim is that at the time of the Schedule 13D filing, the price of the target firm increases and stays elevated for about a year afterwards. But shareholders are not typically shareholders in the target firm exclusively, but rather are typically diversified investors. From the standpoint of diversified investors, the benefits of activism should be analyzed in the context not only on their impact on the price of the target firm, but also in conjunction with their impact on the value of all the holdings of all of the investors, that is, in conjunction with the value of the market. Our Proposition 9 shows that judging that activism benefits shareholders simply because it increases the price of the target firm is not sufficient for making an assessment that activism benefits diversified shareholders. In order to make that assessment, it is necessary to know the changes in the covariances of the prices of the other firms with the target firm. As described above, on the basis of the average change of the covariances due to activism, we can determine whether an increase in target price due to activism is associated with an increase in the values of all the portfolios of all the investors in the market. Only when the average change in the covariances due to activism is below a positive constant β, can we conclude that an increase in the price of the target firm ensures an increase in the value of the market. Thus, from our point of view, we find that while there is theoretical support for the empirical claim that a higher target firm price due to activism means that shareholders benefit from activism, this conclusion is only true under specific conditions on the average change of the covariances due to activism. When these conditions are not met, an increase in the price of the target firm will be associated with a fall in the value of the market, suggesting the conclusion that activism in that case has a negative impact on the shareholders.

It is not surprising that the average changes in the covariances of the prices of the other firms in the market relative to the target firm play a key role in assessing whether the impact of activism is positive or negative on diversified shareholders. Indeed, it would be odd if the covariance structure were irrelevant in this context. Moreover, there are numerous instances in which the covariance structure becomes meaningful in the strategy of the activist. See, for example Kahan and Rock's (pp. 1073-1075) description of AXA's proposed acquisition of MONY, and Lee and Park who demonstrate the impact of activist behavior on the prices of other firms.[18]

[18] Other strategies that may exploit the covariance structure include those that make use of hidden ownership (that is, economic ownership held without voting rights) and empty voting (that is, voting exceeding economic ownership); see Hu and Black (2007). However, the use of derivatives, and in particular, equity swaps, to mask the accumulation of shares that would necessitiate a 13D filing, has been challenged in Federal Court in connection with a case involving The Children's Investment Fund and CSX; see, for example, Stowell (2010). According to Stowell (2010, p. 249), the 2008 ruling in the CSX vs. The Children's Investment Fund Management case "represents a strong challenge to hedge funds who attempt to conceal their true economic position through the use of derivatives."

But it is also appropriate to recall at this juncture that, as we showed in Proposition 6 above, activism can proceed when the price of the target firm falls at the time of the Schedule 13D announcement. This result, too, depends on the changes in covariances of the target firm with other firms, as well as on the initial covariance between the firms. Thus, although we do not explore the impact on the market value when the price of the target firm falls at the Schedule 13D announcement (since all the empirical conclusions are drawn with reference to an initial increase in the target firm price), we conjecture that there would also be conditions under which the initial fall in the target price would be associated with an increase in market value, thus providing an additional situation in which activism might be said to benefit shareholders. This conclusion cannot be drawn from the empirical literature that focuses solely on the increase in the price of the target firm at time $t = 1$. Thus, whether the initial target price increases or decreases, the relationship of that change to the change in the value of the market is contingent on changes in the covariance structure.

Even in cases where the increase in the price of the target firm is coincident with an increase in the value of the market, the source of this increase is undetermined. Since the market is made up of both the activist and the group of other investors, the increase in the market value could be caused by both increasing or either one increasing. To separate these possibilities, we next evaluate the impact of activism on the portfolios of the group of other investors.

Proposition 10.

(1) $E(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{G1} - \mathbf{x}^*_{G0}) = -E(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$.

We note that more recently, the 2nd Circuit U.S. Court of Appeals, considering the same case, left unsettled exactly under what circumstances cash-settled total equity swap agreements provide beneficial ownership. In the context of the Dodd-Frank Act, since section 766(b) amends Sections 13(d) and 13(g) of the Exchange Act, this issue remains not fully settled. See Cuillerier and Hall (2011).

(2) If, at time $t = 3$, the activist is successful, $(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{G1} - \mathbf{x}^*_{G0}) = -(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$ where $\mathbf{P}_3$ is given by Proposition 5.

As we see from Proposition 10, regardless of the change in market value, if the activist's expected cash flow is positive over the period of activism, then the expected cash flow to the group of other investors over the same period is negative. Furthermore, this result holds at time $t = 3$ when it is known that the activist is successful. Though Proposition 10 is disheartening, it does not imply that every other investor loses money over the period. It is only the group as a whole for which this is true.

Proposition 10 is seemingly at odds with a major conclusion drawn in the empirical literature surveyed above. This conclusion, which is that activism benefits shareholders, is based on the observation that the price of the target firm increases around the time of the Schedule 13D filing ($t = 1$) and does not decrease for approximately a year ($t = 3$). While this conclusion might be valid for a shareholder in the target firm who had shares in the target firm and held those shares throughout the period of activism, it cannot hold for the group of shareholders. Some of that group had to have sold shares in order that the activist could acquire shares and proceed with his plan, and some of that group had to have repurchased shares when the activist exited the target firm. What Proposition 10 demonstrates is that, as a group, shareholders pay for any profit that the activist might make.

6 Discussion and Conclusions

The types of activities that activists assert they will undertake in their Schedule 13D filings are varied. In this paper we abstract from these activities, considering instead the manner in which these activities alter the joint distribution of the future value of the target firm as well as other firms in the market. For instance, an activity that leads to cost reduction is interpreted in our model as an increase in the expected value of the target firm. If this cost reduction were accompanied by antagonism with the existing board, this increase in expectation could be accompanied by an increase in the variance of the target firm price. An activity such as the sale of part of the target firm is interpreted as a change in the covariance of the target firm within other firms correlated with it. From the onset of activism until the departure of the activist, this type of abstraction permits us to derive, at points of release of new information, the sequence of equilibria prices of all firms in the market as well as the sequence of equilibria holdings of all participants in the market.

By examining the sequence of equilibria, we find that the activist induces changes in the price of the target firm and possibly in the prices of other firms as well. These changes lead to a redistribution of holdings of all shareholders in those firms. In fact, we find that at each point of new information, equilibrium is accompanied by a redistribution of shares of all market participants. An illustration of the consequences of an activist's activity on other firms is found in Lee and Park who describe the impact of the behavior of a target firm on the prices of non-target firms.

We next focus on the sequence of equilibrium prices corresponding to the target firm. We show that, while it is possible for the price of the target firm to increase around the 13D announcement date and remain elevated for the period of the activist's involvement in the firm, it is also possible that activism leads to a decline in the price of the target firm around the date of the 13D announcement and that the price may remain lower for the remainder of the period. Though not discussed in the literature, this latter case has been observed in Brav et al., Klein and Zur, and Clifford. Changes in prices other than in the target firm lead to the possibility that the activist could profit in his portfolio due to activism, but not improve the value of the target firm. We then focus on the case discussed in the literature and show the conditions under which an increase in target firm price would prevail.

Focusing on the interests of the activist, we next consider the criteria used by an activist to become an activist. We assume that to proceed an activist must believe (1) that he can acquire sufficient ownership in the target firm and (2) that the expected cost of acquiring the needed additional shares will be offset by the expected gain he will make when he exits the target firm. In fact, these criteria can be used by the activist to select among potential target firms by choosing the one that maximizes the expected cash flow. The application of these criteria might explain why, in the preponderance of cases when the activist proceeds, he is successful (see Brav et al., Klein and Zur, and Bratton (2010)). However, it does not follow that the satisfaction of these criteria implies that the price of the target firm will increase over the period of activism.

Recognizing that an activist might or might not increase the value of the target firm, but will change the portfolios of all investors as well as his own portfolio, the question arises as to how to evaluate the impact of activism. We answer this by assuming that such an evaluation must depend on two relationships: first, the relationship between changes in the target firm price due to activism and changes in the market, and second, the relationship between the cash flows accruing to the activist and those accruing to the remainder of the market investors. We establish that the increase in the price of the target firm does not, by itself, determine the direction of change of the market.

It can be shown that when the activities of the activist lead to a sufficient reduction of the covariance between the target firm and other firms, then an increase in the price of the target firm will lead to an increase in the value of the market. Some evidence of this occurring can be found in Clifford (Table V) and Brav et al. (p. 1731), where they observe that the highest abnormal returns occur when the activist spins off non-core assets, changing the covariance structure. Conversely, we establish conditions where the activities yield an increase in the target firm's price but a decrease in the corresponding market value.

Even in the case in which the value of the market increases with activism, it is not clear who benefits from this increase. We next show that if the activist makes a profit over the course of his involvement with the target firm, this profit occurs at the expense of the remaining group of investors over that period. Although this does not imply that each investor would lose when the activist gains, it does deny the claim that successful activism benefits shareholders.

Why does the activist seem to have an advantage over other investors? The answer seems to be two-fold. First, the activist must have sufficient funds to acquire the necessary foothold in the target firm. Second, the activist must believe that his behavior would alter the future value of the firm. As a result, the activist approaches the market surreptitiously with this private information. Thus his acquisition of ownership comes at the expense of the ignorance of the other market participants, giving the activist an advantage. This advantage is similar in flavor to insider information.

It has been observed that the activist does not succeed in his plans approximately one-third of the time (see Brav et al., Klein and Zur, and Bratton (2010)). When he does not succeed, we show that neither the activist nor the other investors benefit financially. However, the volatility of the price of the target firm, as well as possibly other prices, increases over the period of activism.

In sum, we are left with no clear-cut argument showing the benefits of activism beyond those accruing to the activist. Since the impact of the Dodd-Frank Act on Section 13d requirements is currently under discussion, we offer the following proposals suggested by our model. First, the 10 day delay time before filing Schedule 13D should be shortened. Second, no exemptions from filing Schedule 13D announcements should be allowed. Third, the use of derivatives etc., to obscure beneficial ownership should be precluded. Finally, restrictions on coalitions of activists should be implemented to prevent gaming.

7 Appendix

Proof of Proposition 2. In Rabinovitch and Owen (1978; hereafter RO) they proved that, under the assumptions we have made concerning utilities and distributions, the equilibrium solution for the general heterogeneous portfolio problem can be written as $\mathbf{x}_{i1}^* = d_i \boldsymbol{\Omega}_{i1}^{-1}[\boldsymbol{\mu}_{i1} - \mathbf{P}_1]$ where $d_i = 1/a_i$ and where $\mathbf{P}_1$ is chosen to satisfy $\sum_{i=1}^{M} \mathbf{x}_{i1}^* = \mathbf{1}$. Evaluating the RO solution under the present assumptions yields $\boldsymbol{\Omega}_{11}\mathbf{x}_{11}^* = d_1(\boldsymbol{\mu}_{11} - \mathbf{P}_1)$ and for $i > 1$, $\boldsymbol{\Omega}_0 \mathbf{x}_{i1}^* = d_i(\boldsymbol{\mu}_0 - \mathbf{P}_1)$. It follows that

$$\begin{aligned}
\mathbf{x}_{11}^* - \mathbf{x}_{10}^* = \mathbf{x}_{11}^* - \tfrac{d_1}{d}\mathbf{1} &= d_1 \boldsymbol{\Omega}_{11}^{-1}(\boldsymbol{\mu}_{11} - \boldsymbol{\Omega}_{11}\mathbf{1}/d - \mathbf{P}_1) \\
&= d_1 \boldsymbol{\Omega}_{11}^{-1}(\boldsymbol{\mu}_0 + \Delta\boldsymbol{\mu} - (\boldsymbol{\Omega}_0 + \Delta\boldsymbol{\Omega})\mathbf{1}/d - \mathbf{P}_1) \\
&= d_1 \boldsymbol{\Omega}_{11}^{-1}(\mathbf{P}_0 - \mathbf{P}_1 + \Delta\boldsymbol{\mu} - \Delta\boldsymbol{\Omega}\mathbf{1}/d).
\end{aligned}$$

Also,
$$\begin{aligned}
\mathbf{x}_{i1}^* - \mathbf{x}_{i0}^* = \mathbf{x}_{i1}^* - \tfrac{d_i}{d}\mathbf{1} &= d_i \boldsymbol{\Omega}_0^{-1}(\boldsymbol{\mu}_0 - \boldsymbol{\Omega}_0\mathbf{1}/d - \mathbf{P}_1) \\
&= d_i \boldsymbol{\Omega}_0^{-1}(\mathbf{P}_0 - \mathbf{P}_1).
\end{aligned}$$

Summing over $i = 1, ..., M$, we have

$$0 = (d - d_1)\boldsymbol{\Omega}_0^{-1}(\mathbf{P}_0 - \mathbf{P}_1) + d_1 \boldsymbol{\Omega}_{11}^{-1}(\mathbf{P}_0 - \mathbf{P}_1 + \Delta\boldsymbol{\mu} - \Delta\boldsymbol{\Omega}\mathbf{1}/d) \text{ or}$$
$$[(d - d_1)\boldsymbol{\Omega}_0^{-1} + d_1 \boldsymbol{\Omega}_{11}^{-1}](\mathbf{P}_1 - \mathbf{P}_0) = d_1 \boldsymbol{\Omega}_{11}^{-1}(\Delta\boldsymbol{\mu} - \Delta\boldsymbol{\Omega}\mathbf{1}/d) \text{ or}$$
$$[(d\mathbf{I} + (d - d_1)\Delta\boldsymbol{\Omega}\boldsymbol{\Omega}_0^{-1}](\mathbf{P}_1 - \mathbf{P}_0) = d_1(\Delta\boldsymbol{\mu} - \Delta\boldsymbol{\Omega}\mathbf{1}/d).$$

From above,
$$\begin{aligned}
\mathbf{x}_{11}^* - \mathbf{x}_{10}^* &= d_1 \boldsymbol{\Omega}_{11}^{-1}(\mathbf{P}_0 - \mathbf{P}_1) + d_1 \boldsymbol{\Omega}_{11}^{-1}(\Delta\boldsymbol{\mu} - \Delta\boldsymbol{\Omega}\mathbf{1}/d) \\
&= -d_1 \boldsymbol{\Omega}_{11}^{-1}(\mathbf{P}_1 - \mathbf{P}_0) + [(d - d_1)\boldsymbol{\Omega}_0^{-1} + d_1 \boldsymbol{\Omega}_{11}^{-1}](\mathbf{P}_1 - \mathbf{P}_0) \\
&= (d - d_1)\boldsymbol{\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0).
\end{aligned}$$

Since we established above that for $i > 1$, $\mathbf{x}_{i1}^* - \mathbf{x}_{i0}^* = -d_i \boldsymbol{\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0)$, the proposition is proved.■

Proof of Lemma. We must show that $\left[\mathbf{I} - \alpha\left(\begin{smallmatrix}(x_1, \mathbf{x}'_{-1}) \\ \mathbf{v}_{-1}(z_1, z'_{-1})\end{smallmatrix}\right)\right]\left[\mathbf{I} + \alpha\left(\begin{smallmatrix}\mathbf{v}'\boldsymbol{\Omega}_0^{-1} \\ \mathbf{v}_{-1}\boldsymbol{\omega}^{1\prime}\end{smallmatrix}\right)\right] = \mathbf{I}$. We begin by solving for $\mathbf{x}$. It must satisfy $-\left[\begin{smallmatrix}(x_1, \mathbf{x}'_{-1}) \\ \mathbf{v}_{-1}(z_1, \mathbf{z}'_{-1})\end{smallmatrix}\right] + \left[\begin{smallmatrix}\mathbf{v}'\boldsymbol{\Omega}_0^{-1} \\ \mathbf{v}_{-1}\boldsymbol{\omega}^{1\prime}\end{smallmatrix}\right] - \alpha\left[\begin{smallmatrix}x_1\mathbf{v}'\boldsymbol{\Omega}_0^{-1} + (\mathbf{x}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}^{1\prime} \\ \mathbf{v}_{-1}(z_1\mathbf{v}'\boldsymbol{\Omega}_0^{-1} + (\mathbf{z}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}^{1\prime})\end{smallmatrix}\right] = \mathbf{0}$. It follows that

$$x_1 = \mathbf{v}'\boldsymbol{\omega}^1 - \alpha x_1 \mathbf{v}'\boldsymbol{\omega}^1 - \alpha(\mathbf{x}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}_1^1$$
$$\mathbf{x}_{-1} = \boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v} - \alpha x_1 \boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v} - \alpha(\mathbf{x}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}_{-1}^1.$$

Similarly,

$z_1 = \omega_1^1 - \alpha z_1 \mathbf{v}'\boldsymbol{\omega}^1 - \alpha(\mathbf{z}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}_1^1$

$\mathbf{z}_{-1} = \omega_{-1}^1 - \alpha z_1 \boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v} - \alpha(\mathbf{z}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}_{-1}^1.$

Pre-multiplying $\mathbf{x}_{-1}$through by the vector $\mathbf{v}'_{-1}$, we have

$(\mathbf{v}'_{-1}\mathbf{x}_{-1}) = (1-\alpha x_1)(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\omega_{-1}^1)$ which implies

$\mathbf{x}_{-1} = (1-\alpha x_1)\left[\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v} - \alpha\frac{\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1}\boldsymbol{\omega}_{-1}^1\right]$, and

$x_1 = \mathbf{v}'\boldsymbol{\omega}^1 - \alpha x_1 \mathbf{v}'\boldsymbol{\omega}^1 - \alpha(1-\alpha x_1)\frac{\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1}\boldsymbol{\omega}_{-1}^1.$

Solving for x_1 yields

$x_1 = \frac{\mathbf{v}'\boldsymbol{\omega}^1 - \alpha\omega_1^1(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)}{1+\alpha[\mathbf{v}'\boldsymbol{\omega}^1 - \alpha\omega_1^1(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)]}.$

We let $c = 1+\alpha[\mathbf{v}'\boldsymbol{\omega}^1 - \alpha\boldsymbol{\omega}_1^1(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)]$.

The development of $\mathbf{z}$ proceeds in the same fashion yielding

$z_1 = \frac{\omega_1^1}{c(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)}$ and

$\mathbf{z}_{-1} = (\frac{1}{c(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)})[(1+\alpha\mathbf{v}'\boldsymbol{\omega}^1)\boldsymbol{\omega}_{-1}^1 - \alpha\omega_1^1\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}].\blacksquare$

Proof of Proposition 3. From Proposition 2, we have $[\mathbf{I}+\frac{d-d_1}{d}\Delta\boldsymbol{\Omega}\boldsymbol{\Omega}_0^{-1}](\mathbf{P}_1-\mathbf{P}_0) = \frac{d_1}{d}(\boldsymbol{\Delta\mu} - \boldsymbol{\Delta\Omega}\mathbf{1}/d)$. From the Lemma, there exists an $\mathbf{M}$ such that $\mathbf{P}_1-\mathbf{P}_0 = \frac{d_1}{d}\mathbf{M}(\boldsymbol{\Delta\mu} - \boldsymbol{\Delta\Omega}\mathbf{1}/d)$ for $\alpha = \frac{d-d_1}{d}\pi$. Substituting the values for $\Delta\boldsymbol{\mu}$ and $\Delta\boldsymbol{\Omega}$**,** we have

$$
\begin{aligned}
\mathbf{P}_1-\mathbf{P}_0 &= \frac{d_1\pi}{d}\mathbf{M}[me_1 - \frac{1}{d}\begin{pmatrix}\mathbf{v}'\mathbf{1}\\ \mathbf{v}_{-1}\mathbf{q}_1\end{pmatrix} \\
&= \frac{d_1\pi}{d}\mathbf{M}\begin{pmatrix}m-\mathbf{v}'\mathbf{1}/d\\ -\frac{q_1}{d}\mathbf{v}_{-1}\end{pmatrix} \\
&= \frac{d_1\pi}{d}\{\begin{pmatrix}m-\mathbf{v}'\mathbf{1}/d\\ -\frac{q_1}{d}\mathbf{v}_{-1}\end{pmatrix} - \alpha\begin{pmatrix}(x_1,\mathbf{x}'_{-1})\\ \mathbf{v}_{-1}(z_1,\mathbf{z}'_{-1})\end{pmatrix}\begin{pmatrix}m-\mathbf{v}'\mathbf{1}/d\\ -\frac{q_1}{d}\mathbf{v}_{-1}\end{pmatrix}\} \\
&= \frac{d_1\pi}{d}[\begin{pmatrix}(1-\alpha x_1)(m-\mathbf{v}'\mathbf{1}/d)+\alpha\frac{q_1}{d}\mathbf{x}'_{-1}\mathbf{v}_{-1}\\ -\mathbf{v}_{-1}[\alpha z_1(m-\mathbf{v}'\mathbf{1}/d)]+\frac{q_1}{d}(1-\alpha\mathbf{z}'_{-1}\mathbf{v}_{-1})\end{pmatrix}].
\end{aligned}
$$

Substituting from the proof of the Lemma, we have

$$\mathbf{P}_1-\mathbf{P}_0 = \frac{d_1\pi}{dc}\begin{bmatrix} m-\mathbf{v}'\mathbf{1}/d+\alpha\frac{q_1}{d}(\mathbf{v}'_{-1}\mathbf{\Omega}^{-1}_{-1,0}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}) \\ -\mathbf{v}_{-1}[\frac{\alpha\omega_1^1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}(m-\mathbf{v}'\mathbf{1}/d)+\frac{q_1}{d}(c+\frac{\alpha^2\omega_1^1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}(\mathbf{v}'_{-1}\mathbf{\Omega}^{-1}_{-1,0}\mathbf{v}))] \end{bmatrix}$$
$$= \frac{d_1\pi}{dc}\begin{bmatrix} m-\mathbf{v}'\mathbf{1}/d+\alpha\frac{q_1}{d}(\mathbf{v}'_{-1}\mathbf{\Omega}^{-1}_{-1,0}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}) \\ -\mathbf{v}_{-1}[\frac{\alpha\omega_1^1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}(m-\mathbf{v}'\mathbf{1}/d)+\frac{q_1}{d}(1+\alpha\mathbf{v}'\boldsymbol{\omega}^1)] \end{bmatrix}$$
$$= \begin{bmatrix} g_1 \\ -\mathbf{v}_{-1}g_2 \end{bmatrix}.\blacksquare$$

Proof of Proposition 4. Given the homogeneous information set, the equilibrium solution would be $\mathbf{x}^*_{i2}=\frac{d_i}{d-d_1}(\mathbf{1}-\mathbf{x}^*_{11})$ and $\mathbf{P}_2=\boldsymbol{\mu}_0+\pi m\mathbf{e}_1-\frac{1}{d-d_1}(\mathbf{\Omega}_0+\pi\mathbf{V})(\mathbf{1}-\mathbf{x}^*_{11})$. But from the proof of Proposition 2, for $i>1$, $\mathbf{\Omega}_0\mathbf{x}^*_{i1}=d_i(\boldsymbol{\mu}_0-\mathbf{P}_1)$ which implies that $(\mathbf{1}-\mathbf{x}^*_{11})/(d-d_1)=\mathbf{\Omega}_0^{-1}(\boldsymbol{\mu}_0-\mathbf{P}_1)$. Thus, after substitution, $\mathbf{x}^*_{i2}=\mathbf{x}^*_{i1}$.$\blacksquare$

Proof of Proposition 5. At time $t=3$, we have one of two possible homogeneous equilibria. Again, Proposition 1 yields the result for the respective parameter values.$\blacksquare$

Proof of Proposition 6. *CA* implies that $\mathbf{x}^*_{11}-\mathbf{x}^*_{10}>0$. From Propositions 2 and 3, we have $\mathbf{x}^*_{11}-\mathbf{x}^*_{10}=(d-d_1)[\omega_1^1g_1-g_2(\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})]$. Thus, *CA* implies $\omega_1^1g_1>g_2(\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})$. From Proposition 3, g_2 can be written in terms of g_1 as follows:

$$g_2 = \frac{\alpha\omega_1^1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}g_1+\frac{d_1\pi}{cd^2(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})}(1+\alpha v'\boldsymbol{\omega}^1-\alpha^2\omega_1^1\mathbf{v}'_{-1}\mathbf{\Omega}^{-1}_{-1,0}\mathbf{v}/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})$$
$$= \frac{\alpha\omega_1^1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}g_1+\frac{d_1\pi}{d^2(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})}.$$

It follows that $\omega_1^1g_1>g_2(\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})$ if $\omega_1^1g_1>\frac{\alpha\omega_1^1(\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}g_1+\frac{d_1\pi(\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})}{d^2(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})}$ or if $\omega_1^1g_1>\frac{d_1\pi(\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})}{d^2}$. Since, by Proposition 3, $g_1=(\mathbf{P}_1-\mathbf{P}_0)_1$, the result follows.$\blacksquare$

Proof of Proposition 7.

(1) From Proposition 4, $\mathbf{P}_2=\boldsymbol{\mu}_0+\pi m\mathbf{e}_1-\frac{1}{d-d_1}(\mathbf{\Omega}_0+\pi\mathbf{V})(\mathbf{1}-\mathbf{x}^*_{11})$. From the proof of Proposition 2 it follows that $(1-\mathbf{x}^*_{11})=-(d-d_1)\mathbf{\Omega}_0^{-1}(\mathbf{P}_1-\boldsymbol{\mu}_0)$ so $\mathbf{P}_2=$

$\mathbf{P}_1 + \pi[m\mathbf{e}_1 - \mathbf{V1}/d + \mathbf{V\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0)] = \mathbf{P}_1 + \pi[m\mathbf{e}_1 - \mathbf{V1}/d + \frac{1}{d-d_1}\mathbf{V}(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})]$. So the first component of $\mathbf{P}_2 - \mathbf{P}_1$, $(\mathbf{P}_2 - \mathbf{P}_1)_1 = \pi[m - \mathbf{v}'\mathbf{1}/d + \frac{(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_1}{d-d_1}(v_1 + \theta)]$.

(2) From Proposition 5, $\mathbf{P}_3 - \mathbf{P}_0 = me_1 - \mathbf{V1}/d$ where the activist is successful and $\mathbf{0}$ otherwise. So $\mathbf{P}_3 - \mathbf{P}_1 = \mathbf{P}_3 - \mathbf{P}_0 - (\mathbf{P}_1 - \mathbf{P}_0)$. Thus, the first component of $\mathbf{P}_3 - \mathbf{P}_1$, $(\mathbf{P}_3 - \mathbf{P}_1)_1 = m - \mathbf{v}'\mathbf{1}/d - g_1$ where the activist is successful and $-g_1$ otherwise.

(3) From parts (1) and (2), $(\mathbf{P}_3 - \mathbf{P}_2)_1 = (\mathbf{P}_3 - \mathbf{P}_1)_1 - (\mathbf{P}_2 - \mathbf{P}_1)_1$ and the result follows.■

Proof of Proposition 8.

(1) This part follows from Proposition 6 since $\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1} > 0$.

(2) We first show that $\theta > \frac{\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1}}{\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}$. The difference

$$\begin{aligned}
\theta - \frac{\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1}}{\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}} &= \frac{g_1(\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}) - g_2(\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1})}{\omega^1_1 g_1 - g_2\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}} - \frac{\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1}}{\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}} \\
&= \frac{g_1}{\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}(\omega^1_1 g_1 - g_2\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})}[(\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})^2 - \omega^1_1(\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1})].
\end{aligned}$$

Because $\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1} > 0$ and from part (1) $g_1 > 0$ and by *CA1*, $\omega^1_1 g_1 - g_2\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1} > 0$, the last expression will be positive if $\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1} > \frac{\omega^1_1(\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1})}{\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}$ which is assumed. It then follows that $\theta > \frac{\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1}}{\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}} > 0$.

From *CA2*, the parameters must satisfy $\pi(m - \mathbf{V1}/d)(\mathbf{x}^*_{11} - \mathbf{x}^*_{10}) > \frac{1}{d}\mathbf{v}'_{-1}(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_{-1} + (\mathbf{P}_1 - \mathbf{P}_0)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$. The right hand side of this inequality is positive since $\theta = \frac{\mathbf{v}'_{-1}(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_{-1}}{(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_1} > 0$ and $(\mathbf{P}_1 - \mathbf{P}_0)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10}) = (d - d_1)(\mathbf{P}_1 - \mathbf{P}_0)'\mathbf{\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0)$ with $\mathbf{\Omega}_0$ being a positive definite matrix. Finally, since $m - \mathbf{v}'\mathbf{1}/d > 0$, from Proposition 7, $(\mathbf{P}_2 - \mathbf{P}_1)_1 > \frac{(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_1}{d-d_1}(v_1 + \theta) = \frac{(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_1}{d-d_1}[(v_1 + \frac{\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1}}{\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}) + (\theta - \frac{\mathbf{v}'_{-1}\mathbf{R}\mathbf{v}_{-1}}{\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}})]$. Our assumptions make each term in parentheses positive, proving part (2).

(3) From Proposition 7(2), it follows that $(\mathbf{P}_3 - \mathbf{P}_1)_1 = m - \mathbf{v}'\mathbf{1}/d - g_1$ if the activist succeeds. Using Proposition 3, $(\mathbf{P}_3 - \mathbf{P}_1)_1 = m - \mathbf{v}'\mathbf{1}/d - \frac{d_1\pi}{dc}[m -$

$\mathbf{v}'\mathbf{1}/d + \frac{\alpha}{d}(\mathbf{v}'_{-1}\mathbf{\Omega}^{-1}_{-1,0}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})]$. It follows that if $\frac{d_1\pi}{dc}$ is sufficiently small, that $(\mathbf{P}_3 - \mathbf{P}_1)_1 > 0$. Also, from Proposition 7(2). when the activist does not succeed, $(\mathbf{P}_3 - \mathbf{P}_1)_1 = -g_1$, which is negative by part (1).

(4) From Propositions 7(1) and (2), when the activist is successful, $g_1 + (\mathbf{P}_2 - \mathbf{P}_1)_1$ is a positive multiple of π. Thus, for π sufficiently small, $(\mathbf{P}_3 - \mathbf{P}_2)_1$ will be positive from Proposition 7(3). When the activist is not successful $(\mathbf{P}_3 - \mathbf{P}_2)_1 < 0$, also from Proposition 7(3).■

Proof of Proposition 9.

(1) Because of *CA1* and Proposition 6, we require that $g_1 > \frac{d_1\pi}{d^2\omega^1_1}\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}$. In order for the market to increase at time $t = 1$ at the same time that the target firm price increases, we further require that $\mathbf{1}'(\mathbf{P}_1 - \mathbf{P}_0) = g_1 - g_2\mathbf{v}'_{-1}\mathbf{1}_{-1} > 0$ and $g_1 > 0$. We showed in the proof of Proposition 6 that $g_2 = \frac{1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}(\alpha\omega^1_1 g_1 + \frac{d_1\pi}{d^2})$. Thus, the requirement that the market increases can be written as $g_1[1 + \alpha(\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1} - \omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1})] > \frac{d_1\pi}{d^2}\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1}$. We consider the three mutually exhaustive cases $\omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1} \leq \mathbf{v}'_{-\mathbf{1}}\boldsymbol{\omega}^1_{-1}$; $\mathbf{v}'_{-\mathbf{1}}\boldsymbol{\omega}^1_{-1} < \omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1} < \mathbf{v}'_{-\mathbf{1}}\boldsymbol{\omega}^1_{-1} + \frac{1}{\alpha}$; and $\omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1} \geq \mathbf{v}'_{-\mathbf{1}}\boldsymbol{\omega}^1_{-1} + \frac{1}{\alpha}$.

For the case $\omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1} \leq \mathbf{v}'_{-\mathbf{1}}\boldsymbol{\omega}^1_{-1}$ it easily follows that the requirement that the market increases is implied by *CA1*. Thus, for this case, the market increases when the target firm price increases. For the case $\omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1} \geq \mathbf{v}'_{-\mathbf{1}}\boldsymbol{\omega}^1_{-1} + \frac{1}{\alpha}$, the left hand side of the requirement that the market increases becomes negative. This implies that the market would increase only if $g_1 < 0$, which violates our assumption. Thus, in this case, the market falls. For the case $\mathbf{v}'_{-\mathbf{1}}\boldsymbol{\omega}^1_{-1} < \omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1} < \mathbf{v}'_{-\mathbf{1}}\boldsymbol{\omega}^1_{-1} + \frac{1}{\alpha}$, when the value of $\omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1}$ is close to the left end of the interval, both inequalities can be satisfied if $\frac{\omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1}}{1+\alpha(\mathbf{v}'_{-\mathbf{1}}\boldsymbol{\omega}^1_{-1} - \omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1})} > \mathbf{v}'_{-\mathbf{1}}\boldsymbol{\omega}^1_{-1}$. However, when the value of $\omega^1_1\mathbf{v}'_{-\mathbf{1}}\mathbf{1}_{-1}$ is close to the right end of the interval, the value that g_1 needs to exceed for the market to rise goes to infinity, implying that here the value of the market must fall. Therefore, there is a point in this interval where the two inequalities cannot simultaneously hold. We let β be that point.

(2) At time $t = 3$, if the activist is successful, it follows from Proposition 5 that $\mathbf{1}'(\mathbf{P}_3 - \mathbf{P}_0) = m - \mathbf{v}'\mathbf{1}/d - \frac{1}{d}\mathbf{v}'_{-1}\mathbf{1}_{-1} = m - v_1/d - 2\mathbf{v}'_{-1}\mathbf{1}_{-1}$ and the result follows.

(3) At time $t = 3$, if the activist is not successful, it follows from Proposition 5 that $\mathbf{1}'(\mathbf{P}_3 - \mathbf{P}_0) = 0$ and the result follows.■

Proof of Proposition 10.

(1) Earlier we showed that

$$E(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10}) = \pi(m\mathbf{e}_1 - \frac{1}{d}\mathbf{V1})'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10}) - (\mathbf{P}_1 - \mathbf{P}_0)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10}) > 0.$$

Since $\sum_{i=1}^{M} \mathbf{x}^*_{ij} = 1$ for all j, $(\mathbf{x}^*_{11} - \mathbf{x}^*_{10}) = -(\mathbf{x}^*_{G1} - \mathbf{x}^*_{G0})$ and the result follows.

(2) For $\mathbf{P}_3$ determined at time $t = 3$ if the activist is successful, the cash flow to the activist is $(m\mathbf{e}_1 - \frac{1}{d}\mathbf{V1})'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10}) - (\mathbf{P}_1 - \mathbf{P}_0)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$. The same argument as above completes the proof.■

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