

A GREEK PAPYRUS CONTAINING BABYLONIAN LUNAR THEORY

The papyrus presented here, which is owned by Prof. M. L. Colker (University of Virginia), was first described, as a writing exercise, in two dealers' catalogues.¹ Subsequently Dr David Jordan identified the mathematical nature of the text on the reverse, and in turn enlisted the assistance of O. Neugebauer. Neugebauer published an article containing photographs of both sides, a partial transliteration of the reverse, and a discussion demonstrating the relation of the numerical table to Babylonian lunar tables.²

Neugebauer fully appreciated that the papyrus was a document of remarkable importance for the history of astronomy; but because he set out his discovery with characteristic brevity and understatement in a volume devoted ostensibly to assyriology, few who are not specialists in ancient mathematical astronomy know about it. The present article hopes to bring the papyrus to wider attention, and to supplement Neugebauer's article in several respects, most importantly by providing a fuller description and the first proper transcription of the astronomical table.³

Historical background and significance. In its latest period, as represented by the works of Ptolemy (c. A.D. 150), Greek astronomy was very much concerned with analysing the appearances and movements of the moon, the sun, and the five planets visible to the naked eye; and the most noticeable expression of this knowledge took the form of tables of numbers designed to yield predictions of the phenomena of these heavenly bodies. These same problems remained at the centre of astronomical science in the medieval Arabic world, Byzantium, and in western Europe as late as the time of Copernicus, Tycho, and Kepler.

Only in the late 19th century, with the decipherment of cuneiform, did the fact emerge that an astronomical tradition, similar in many respects to that of Ptolemy, had existed in Babylonia during the last three centuries B.C. Like Ptolemy, the Babylonian astronomers broke down complex patterns of phenomena into a combination of simpler components that varied cyclically with precise periods of repetition. But whereas the Greek astronomer treated these individual periodicities as the effects of revolving circular motions in the heavens, the Babylonians described them by patterns of numbers generated by certain arithmetical rules, with no intermediating appeal to geometry.⁴

The products of Babylonian mathematical astronomy were tables of predictions of planetary phenomena, new and full moons, and eclipses.⁵ In a planetary tablet, the rows represent the consecutive occurrences of a specific kind of event such as Venus' first appearance as morning star; the date of the event in the Babylonian lunar calendar and the position of the planet in the zodiac were recorded in separate columns. In order to generate these columns of dates and positions, the scribes followed the principle that the time interval (and likewise the planet's progress through the zodiacal signs) from one event to the next increased and decreased according to a fixed, periodically repeating pattern.

The lunar tablets are much more complex, and testify to far more profound insights on the part of the Babylonian astronomers. These texts were designed to predict such circumstances as the time

¹ Erik von Scherling, *Rotulus* 5, 1949, p. 36, no. 2193; Maggs Bros. catalogue 892, 1964, p. 109, no. 790.

² O. Neugebauer, "A Babylonian Lunar Ephemeris from Roman Egypt," in *A Scientific Humanist: Studies in Memory of Abraham Sachs*, ed. E. Leichty, M. deJ. Ellis, and P. Gerardi, Occasional Publications of the Samuel Noah Kramer Fund 9, Philadelphia, 1988, 301–304.

³ I thank Prof. Colker for permission to publish this papyrus, and for providing me with photographs. Prof. Colker, Dr Jordan, and Dr Revel Coles kindly supplied information that was helpful in preparing this article.

⁴ For concise non-technical treatments of Babylonian and Greek astronomy, see J. Britton and C. Walker, "Astronomy and Astrology in Mesopotamia," in C. Walker, ed., *Astronomy Before the Telescope*, London, 1996, 42–67, and G. J. Toomer, "Ptolemy and his Greek Predecessors," in the same volume, 68–97.

⁵ Most of the known texts are edited in O. Neugebauer, *Astronomical Cuneiform Texts*, London, 1955.

interval between sunset and moonset on the evening when the new moon crescent was first visible. This could not be calculated using a single periodic pattern, but had to be broken down into a computation of many numbers reflecting no less than four distinct periodicities: the solar year, the average lunar month, the period that in modern astronomy would be described as the moon's revolution about its orbit relative to the orbit's apogee, and the average period in which the moon returns to a point furthest north of the plane of the earth's orbit around the sun. (The last three of these periods are referred to respectively as the mean synodic month, the anomalistic month, and the draconic month.) We know of two different sets of rules for the construction of Babylonian lunar tables, called System A and System B in modern discussions. The two systems lead up to the predictions of visibility conditions through a similar sequence of computational stages, so that in tablets following both systems one finds columns of numbers having the same astronomical meaning, often in the same order from left to right. Where the systems chiefly differ is in the rules according to which the columns themselves are generated, and the exact values chosen for periodicities and average and extreme values. Broadly speaking, System A, which appears to be the older system, has better internal consistency, whereas System B assumes more accurate values for the fundamental parameters. Both systems were, however, used concurrently at the two Babylonian sites, Babylon and Uruk, from which we have tablets containing mathematical astronomy.

Not long after the first decipherment of Babylonian astronomical texts, F. X. Kugler noticed that some very precise numerical values for the mean synodic month, the anomalistic month, and the mean draconic month that are quoted by Ptolemy and attributed by him to his predecessor Hipparchus (c. 140 B.C.) were already incorporated in the System B lunar theory well before Hipparchus' time.⁶ Further instances of Babylonian data in Greco-Roman astronomy continued to turn up in the following decades, much of it though by no means all associated in one way or another with Hipparchus.⁷ There were also indications that distorted knowledge of the actual methods of calculating planetary tablets had found its way from Babylonia to India in late antiquity through some sort of Greek intermediary; and B. L. van der Waerden had sought to demonstrate that Babylonian planetary theory also lay behind the computation of planetary positions in certain demotic Egyptian almanacs of the early Roman period.⁸

Nevertheless the presumption up to 1988 was that Babylonian mathematical astronomy, and especially the lunar theory, was much too complex and foreign to the Greek way of visualizing astronomical phenomena for it ever to have become accessible wholesale to Greek astronomers, especially since we only knew of it from tablets originating in a restricted milieu of scribes working in a dead language and in a script that by the Hellenistic period scarcely anyone was trained to read. This state of affairs was overturned by Prof. Colker's papyrus. As Neugebauer showed, the better-preserved column of numbers on the papyrus is a sequence computed precisely according to the rules of the so-called "Column G" of a System B lunar tablet.

Column G is one of a set of four columns that usually appear together in a tablet, and that lead up to the calculation of the precise date and time when the sun and moon are diametrically opposite at full moon (*opposition*) or exactly aligned before new moon (*conjunction*). The moment in question is found as the running total of the time intervals from one opposition or conjunction to the next; and these intervals are found by adding to 29 days two smaller quantities that fluctuate with different periods. Column G is one of these quantities, and has the moon's anomalistic month for its period. The other part, "Column J," takes on alternately positive and negative values with the solar year as its period, and averages zero.

⁶ F. X. Kugler, *Die babylonische Mondrechnung*, Freiburg, 1900, 20–24 and 40.

⁷ G. J. Toomer, "Hipparchus and Babylonian Astronomy," in *A Scientific Humanist* (see note 2), 353–362, and A. Jones, "Evidence for Babylonian Arithmetical Schemes in Greek Astronomy," in H. D. Galter, ed., *Die Rolle der Astronomie in den Kulturen Mesopotamiens*, Grazer morgenländische Studien 3, Graz, 1993, 77–94.

⁸ O. Neugebauer and D. Pingree, *The Pañcasiddhāntikā of Varāhamihira*, 2 vols., Det Kongelige Danske Videnskabskabernes Selskab, Historisk-Filosofiske Skrifter 6.1, København, 1970, vol. 2, 109–128; B. L. van der Waerden, "Aegyptische Planetenrechnung," *Centaurus* 16, 1972, 65–91.

As it happens, Column G is closely connected with two of the three lunar periodicities from System B known to Hipparchus and Ptolemy: its mean value, added to 29 days, is Hipparchus' mean synodic month, and its period is his anomalistic month.⁹ But the definition of Column G could not have been reconstructed by a Greek astronomer from the information that Hipparchus appropriated. Moreover it is obvious, as Neugebauer had already remarked, that Column G is only one element in a complicated scheme of computation, and would not have been useful in isolation. Neugebauer did not succeed in identifying the preceding column of numbers, some traces of which are legible in the papyrus to the left of Column G; but there are strong arguments for believing that it was a version of Column J. When complete, the papyrus must have contained the series of columns leading to the date and time of opposition or conjunction (H, J, G, and L), probably the columns determining the sun's and moon's zodiacal position (A and B), and possibly also some or all of the other columns familiar from the Babylonian System B tablets.

Thus by this single papyrus fragment we learn that one of the two Babylonian systems of lunar theory had spread by the Roman period to Greek-speaking Egypt. So far, it is the unique witness to this transmission; but in the meantime a dozen or so Greek papyri from Oxyrhynchus have come to light that confirm van der Waerden's contention, directly proving that most of the Babylonian planetary theories were in use in Roman Egypt.¹⁰ These discoveries raise profound questions about the origins of the mathematical astronomy of Hipparchus and Ptolemy, and the degree to which it was influenced by its Babylonian counterpart. They also introduce a puzzle of cultural history: by what means did these highly technical methods of prediction spread from the last holdouts of ancient Mesopotamian scholarship to provincial Egypt?¹¹

Description. The fragment, 4.7×16.8 cm, is broken on all sides, although there appears to be approximately 1 cm margin at the bottom on the reverse. The papyrus is darkened and rather tattered along the left edge of the reverse, and one area, which seems to be an attached piece of another sheet, perhaps the remains of a collesis, is entirely stripped of its surface on the reverse.

On the obverse (cf. Tafel IIa), along the fibres, are parts of thirteen lines of a heavily abbreviated register in a first-century A.D. cursive hand.¹² The astronomical table on the reverse (cf. Tafel IIb) is written, within a black tabular framework, in a rather coarse, slightly sloping hand of a type that could belong to the late Ptolemaic period or as late as the first half of the first century A.D.¹³ Letters representing numerals are written without ligatures although the letters standing for tens and units of the

⁹ In the light of the new papyrus table, Neugebauer wrote an article (his last), tracing the history of the System B mean synodic month from Babylonian texts to medieval Books of Hours: O. Neugebauer, "From Assyriology to Renaissance Art," *Proceedings of the American Philosophical Society* 133, 1989, 391–403.

¹⁰ *P. Oxy.* LXI 4152–4161, edited in A. Jones, *Astronomical Papyri from Oxyrhynchus*, Memoirs of the American Philosophical Society, Philadelphia (in press, probably 1999). See also A. Jones, "Studies in the Astronomy of the Roman Period III: Planetary Epoch Tables," *Centaurus* 40, 1997 (in press).

¹¹ Astronomical texts were still being produced in cuneiform at Babylon as late as A.D. 74; see A. Sachs, "The Latest Datable Cuneiform Tablets," in *Kramer Anniversary Volume: Cuneiform Studies in Honor of Samuel Noah Kramer*, ed. B. L. Eichler, Kevelaer and Neukirchen-Vluyn, 1976, 379–398. Activity at Uruk probably came to an end not long after 150 B.C., probably as a consequence of the Parthian conquest; planetary tablets from Uruk covering later dates are probably forecasts. Apropos of the contacts between Greek and Babylonian astronomers, a remarkable discovery of D. Rawlins deserves more attention than it has received: he has shown that an equation representing the exact number of days in a solar year in a Babylonian tablet, no. 210 in Neugebauer, *Astronomical Cuneiform Texts* (note 5), is almost certainly derived from two attested Greek observations of summer solstices, the later one being by Hipparchus in 135 B.C. – again the association with Hipparchus! See D. Rawlins, "Ancient Geodesy: Achievement and Corruption," *Vistas in Astronomy* 28, 1985, 255–268, esp. 256 and 267 note 3; and "Hipparchos' Ultimate Solar Orbit & the Babylonian Tropical Year," *Dio* 1.1, 1991, 49–66.

¹² I am grateful to Dr Coles for providing a description of this side and confirming my estimate of the date of the hand on the reverse.

¹³ It is not unlike *P. Oxy.* II 216, pl. 10a in C. H. Roberts, *Greek Literary Hands*, Oxford, 1955, which belongs to the first half of the first century A.D.

same number occasionally are close enough to touch; the abbreviation signifying “subtractive” in col. ii is written with a ligature and cursive form of phi. Some of the horizontal rulings are discontinuous between cols. i and ii, though this probably only means that the ruler was only so wide and had to be moved to draw lines across the breadth of a wide table. Most likely the fragment comes from a roll, first used for the document, and soon afterwards reused for the table, a pattern that can be paralleled in many other astronomical tables on papyrus.¹⁴

Transcription

Translation

	i	ii	iii		i	ii	iii
			δ κβ λ[δ ι				4 22 3[4 10
			δ ρ δ [ι				4 0 4 [10
			γ λζ λ[δ ι				4 37 3[4 10
			γ ιε δ [ι				3 15 4 [10
5			β νβ λ[δ ι				2 52 3[4 10
			β λ δ [ι				2 30 4 [10
			β ζ λ[δ ι				2 7 3[4 10
			β ρ ε [2 0 5 [
		πρ()	β κβ λ[ε			additive	2 22 3[5
10			β με ε [2 45 5 [
			γ ζ λ[ε				3 7 3[5
			γ λ: ε [3 30 5 [
			γ νβ λ[ε				3 52 3[5
			δ ιε [ε				4 15 [5
15		αφ()	δ κ α ι[θ ι			subtractive	4 21 1[9 10
			γ ν α μ				3 58 4[9 10
			γ μ η ρ				3 36 1[9 10
			γ ι γ μ[θ ι				3 13 4[9 10
			β ν α [ι θ ι				2 51 [19 10
20			β κ η μ[θ ι				2 28 4[9 10
		πρ()	β ζ ι[θ ι			additive	2 6 1[9 10
			β α κ [2 1 20 [
			β κ γ ν [2 23 50 [
			β μ ζ κ [2 46 20 [
25			γ η ν [3 8 50 [
			γ λ α κ [3 31 20 [
		αφ()	γ ν γ ν [subtractive	3 53 50 [
			δ ι ζ κ [4 16 20 [

¹⁴ E.g. *P. Oxy.* LXI 4160 (first century), 4176, 4202 (second century), 4148, 4159, 4165, 4165a, 4166, 4215 (third century), 4213 (fourth century), 4172 (fourth/fifth century), all to appear in Jones, *Astronomical Papyri from Oxyrhynchus* (see note 10).

]	X	δ κ: δ [ι]	X	4 20 4 [10
30]	γ ν ζ λ [δ ι]	3 57 3[4 10
]	γ λ ε [δ ι]	3 35 [4 10
]	γ ι β λ [δ ι]	3 12 3[4 10

Textual notes

col. i:

- 14. The trace at the left edge could belong to γ or ζ. The letter that seems to follow λ is mostly lost to a tear in the papyrus.
- 17. The symbol for zero, which occurs also in iii 2 and 8 and possibly in i 26, is a small circle with a horizontal stroke above it (in the published photograph the circle here falsely appears to be enclosed by vertical strokes on either side). For the various forms taken by this symbol in astronomical papyri, see Jones, *Astronomical Papyri from Oxyrhynchus* (note 9 above), vol. 1, 61–62 and Fig. 14.
- 20. The κ (if that is what it is) is tilted so that the stroke that would normally tend upwards to the right is nearly horizontal.
- 21. ν is very doubtful.

Col. ii:

- 7, 14, 21, and 28. A sideways V resembling a *diple* is written below each of these lines touching the vertical ruling between cols. ii and iii. The mark indicates where the numbers in col. iii change over from increasing to decreasing and *vice versa* (cf. the astronomical commentary below).
- 9 and 21. The resolution of the compendium $\int \uparrow \downarrow$ must be *προσθετικός*, *πρόσθεσις*, or the like.
- 15 and 27. , standing for *ἀφαιρετικός* or *ἀφαίρεσις*.
- 29. I do not know the purpose of the coarsely written cross immediately to the left of the vertical ruling.

Col. iii:

- 12. The two dots to the right of λ are intended to prevent reading λ ε, “30 5”, as λ ε, “35”. The same notation occurs in iii 29. In iii 6, where it is expected, it may have become invisible through darkening of the papyrus.
- 27. ν is written with uncharacteristically curved strokes. Possibly it has been corrected from λ.
- 29. For the double dot following κ see iii 12. The δ at the end of the line is the only unambiguously legible instance of a letter representing the units of the third numeral in col. iii (in the published photograph it is obscured by a fold in the papyrus).

Astronomical commentary. In Babylonian astronomy numbers are written in sexagesimal (base 60) notation, so that the first number to the right of the numeral representing the whole units counts sixtieths, the second counts 3600ths, and so forth. If a number is greater than 60, then the whole number part is itself broken up into larger groups of 60s, 3600s, and so forth to the left of the units place. Although Greek astronomy adopted the Babylonian sexagesimal notation for fractions – of course using Greek alphabetic numerals in place of the cuneiform digits – the whole number parts in Greek texts were conventionally written in the traditional way, as hundreds, tens, and units. I believe that the present papyrus is the first known example of a Greek astronomical text that employs the sexagesimal notation for whole numbers as well as fractions. For clarity with transcribed numerals, we will use the symbol “;” to separate the whole number from the first fractional place, and “,” to separate the other places. In the text a mere space serves both purposes.

Column G in the Babylonian lunar System B is a “linear zigzag function,” that is, a sequence that alternately increases and decreases by a constant increment δ between fixed limits *m* and *M*.¹⁵ In Column G the numbers represent so-called “time-degrees,” such that 360 time-degrees equal one day, and:

¹⁵ For all details of the definitions and properties of the System B columns, see Neugebauer, *Astronomical Cuneiform Texts* (note 5), vol. 1, 69–85.

$\delta = 22;30$ time-degrees

$m = 1,52;34,35$ time-degrees (= 112;34,35 time-degrees)

$M = 4,29;27,5$ time-degrees (= 269;27,5 time-degrees)

When addition of δ would raise the value of Column G above M , one subtracts this sum from twice M and henceforth continues subtracting δ . Conversely when subtraction of δ would make the value drop below m , one subtracts the difference from twice m and henceforth continues adding δ . Wherever this occurs in the papyrus, the scribe marks the change of direction with a sideways V to the left of the numerals.

In the papyrus the rightmost places of Column G (col. iii) are mostly broken off. As Neugebauer showed, the only restoration that is arithmetically consistent with the rules for Column G and agrees with the practice attested in cuneiform tablets is the one presented in the transcription and translation above.¹⁶ In the Babylonian tablets a choice of values for Column G such that ascending and descending branches end alternately in 10 and 0 is characteristic of tables for full moons.

Between the two columns of numbers in the papyrus is a broad space (col. ii), in which the scribe has in some lines written abbreviations signifying “additive” and “subtractive”. Neugebauer’s transcription shows these indications as lining up with the changes of direction in col. iii, but this is not correct. Column G changes from increasing to decreasing and *vice versa* almost always after seven lines (occasionally six), as in the present table where the changes occur after lines 7, 14, 21, and 28. The indications in col. ii, however, occur at intervals of only six lines, on lines 9, 15, 21, and 27. Clearly it is only an accident that they nearly line up with the inflections of col. iii around the middle of the table. This is important, because if they do not have anything to do with col. iii, they must belong instead with the poorly preserved col. i, which Neugebauer was unable to identify. We may remark in passing that indications of directionality in the Babylonian tablets are conventionally written to the right of the numerals.

The periodicity of this column, therefore, would have been close to twelve lines (six additive, six subtractive), i.e. twelve lunar months. In the System B tablets there is only one column that has an appropriate periodicity of close to twelve months, alternating every six lines from being designated “additive” to being “subtractive”, and found – for good reason – in proximity to Column G, and that is Column J. As we have already seen, Column J tabulates a time interval to be added to or subtracted from Column G in order to obtain the excess of time over 29 whole days of the interval from one opposition (or conjunction) to the next. In the majority of System B tablets that have Columns G and J, J appears to the right of G, but there are several in which the order is reversed, as would appear to be the case in our papyrus.¹⁷

The obstacle to this identification is that I have not succeeded in fitting the handful of legible numerals in col. i as the terminal digits of a sequence of values of Column J following the rules of System B. Column J is, in fact, one of the columns with the most complex rules: it is calculated in a manner analogous to a linear zigzag function, with alternately increasing and decreasing stretches between fixed limiting values; but the δ value itself is not constant but assumes the changing values of a second zigzag function. I think it is very likely that col. i contained numbers with the same astronomical meaning as Column J, that is, a correction with an annual periodicity to the time intervals from opposition to opposition, but that the rules for computing it were not precisely the same as in the Babylonian System B tablets.

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¹⁶ In the last line of his transcription, Neugebauer inadvertently writes 3[0 instead of 3[4, an error that vitiates his argument for the restoration of the units in the third place. The restoration is, however, assured by the preservation of one unquestionable delta (4) in line 29.

¹⁷ See texts 170–174 in Neugebauer, *Astronomical Cuneiform Texts* (note 5).

TAFEL II



P. Colker, front and back; A. Jones, pp. 167-172