A Greek Saturn Table

by

ALEXANDER JONES*

We know very little about the methods Greek astronomers before Ptolemy devised or adapted for computing the positions, above all the zodiacal longitudes, of the planets. Literary sources give us scarcely any help at all. Several papyrus and other archeologically recovered documents from the first three centuries A.D. state dated planetary longitudes; these were certainly computed rather than observed, but generally they are either too few or too imprecise to allow a sure induction of the underlying mathematical principles.[1]

Far more informative, at least potentially, are texts that give either rules or tables required for computation. In the Moon's case a lucky preservation of both kinds of text, together with the comparative simplicity of its motion, has made it possible to approach a complete restoration of one widely used scheme for lunar motion.[2] With the planets we have been less fortunate so far. No "procedure texts" of any substance have turned up, and only a couple of auxiliary tables are known from whose fragments we can recover at least part of their derivation and purpose.

One of these is a demotic papyrus table, P. Carlsberg 32, that gives a second order (quadratic) sequence of four place sexagesimal numbers that, as Neugebauer has shown, describe the daily motion of Mercury between its first and last morning appearances.[3] Since the longitudes start from 0;0,0,0°, the table is clearly intended to describe not a set of absolute positions associated with specific dates, but rather a repeatable "template" pattern that can be applied to any predetermined first morning visibility of Mercury.

The other table is similar but slightly more complex.[4] The end of the tabulation survives, and it consists of three columns: a sequence of four place sexagesimal numbers, an intermittently recorded index

* History of Mathematics, Box 1900, Brown University, Providence, R.I. 02912, USA.

number (from which we find that the last line of the table was number 378), and a second four place sequence that is the summation of the first. In lines 346 to 378 the first column remains constantly 0,8,15,18, and hence the third increases linearly, from 8,7,20,24 (line 346) to 12,31,30,0 (line 378). Counting backwards from line 346 (after which is written ΔΥΣΙΣ or "setting") the first column diminishes linearly by 0,0,4,14 per line, while the third becomes accordingly a second order sequence. Extrapolating off the surviving fragment, we find the first column at 0,0,0,0 on line 229, with the third at 0,0,17,42.

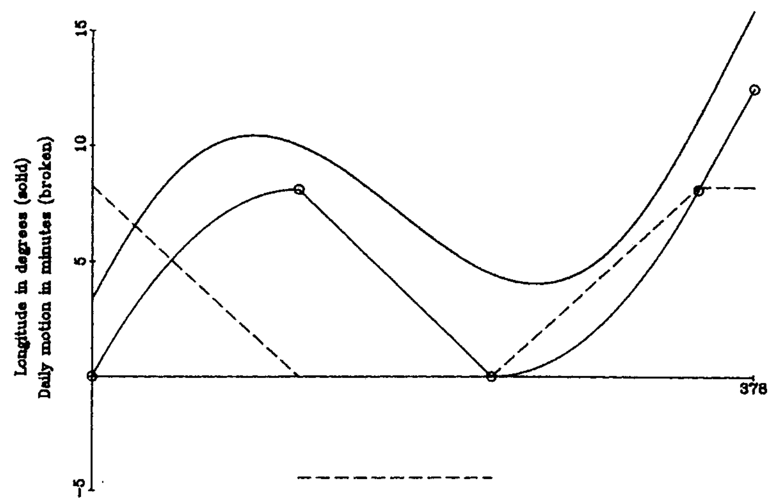
Again Neugebauer has demonstrated that this directly reconstructible portion conforms to Saturn's daily motion in longitude, and the accumulated increment in longitude since the preceding heliacal rising, from second station to the following heliacal rising.[5] We now ask whether the lost first part of the table can be recovered. If the original pattern was asymmetrical around the central phases of solar conjunction and opposition, the attempt will be vain. We therefore assume symmetry.

We postulate an increment in longitude (column 3) of 0;0,0,0° on day "zero" and apply a mirror image of the known part of the table from second station to heliacal setting to obtain the symmetrically equivalent motion from heliacal rising to first station.[6] The first column (daily motion in longitude) then decreases linearly, reaching 0;0,0,0° at line 118, where the increment in longitude is 8;7,2,42°. Therefore the hypothetical retrogradation is 8;7,2,42° – 0;0,17,42° or 8;6,45,0° in 228 – 118 or 110 days. The test of the hypothesis will be whether a plausible arithmetical pattern, restricted to four sexagesimal places, can produce this retrogradation.

One can imagine several ways of bridging this kind of gap by arithmetical methods. A single third order sequence could connect both the column 1 and column 3 termini. A second order sequence could join each terminus with the central solar opposition. Or the already established second order sequences could be continued until the column 1 values equal a (subtractive or negative) number that exactly matches the average daily motion required to bridge the remaining space between; and then a linear patch finishes the pattern. Each of these postulates strictly determines a single solution, and in every case the solution will not work with integer days and four place figures.

Reconstruction of PSI 1492: Synodic Motion of Saturn.

		Template for Saturn			
Daily Motion	Day	Longitude	Daily Motion	Day	Longitude
0 8 15 18		0 8 15 18	0 8 15 18		8 7 20 24
0 8 11 4		0 16 26 22	0 8 15 18		8 15 35 42
0 8 6 50		0 24 33 12	0 8 15 18		8 23 51 0
0 8 2 36		0 32 35 48	0 8 15 18		8 32 6 18
0 7 58 22	5	0 40 34 10	0 8 15 18	350	8 40 21 36
			0 8 15 18		8 48 36 54
0 0 12 42	115	8 6 50 0	0 8 15 18		8 56 52 12
0 0 8 28		8 6 58 28	0 8 15 18		9 5 7 30
0 0 4 14		8 7 2 42	0 8 15 18		9 13 22 48
0 0 0 0		8 7 2 42	0 8 15 18	355	9 21 38 6
	STATION		0 8 15 18		9 29 53 24
0 4 25 30		8 2 37 12	0 8 15 18		9 38 8 42
0 4 25 30	120	7 58 11 42	0 8 15 18		9 46 24 0
0 4 25 30		7 53 46 12	0 8 15 18		9 54 39 18
0 4 25 30		7 49 20 42	0 8 15 18	360	10 2 54 36
0 4 25 30		7 44 55 12	0 8 15 18		10 11 9 54
0 4 25 30		7 40 29 42	0 8 15 18		10 19 25 12
0 4 25 30	125	7 36 4 12	0 8 15 18		10 27 40 30
			0 8 15 18		10 35 55 48
0 4 25 30	225	0 13 34 12	0 8 15 18	365	10 44 11 6
0 4 25 30		0 9 8 42	0 8 15 18		10 52 26 24
0 4 25 30		0 4 43 12	0 8 15 18		11 0 41 42
0 4 25 30		0 0 17 42	0 8 15 18		11 8 57 0
	STATION		0 8 15 18		11 17 12 18
0 0 0 0		0 0 17 42	0 8 15 18	370	11 25 27 36
0 0 4 14	230	0 0 21 56	0 8 15 18		11 33 42 54
0 0 8 28		0 0 30 24	0 8 15 18		11 41 58 12
0 0 12 42		0 0 43 6	0 8 15 18		11 50 13 30
0 0 16 56		0 1 0 2	0 8 15 18		11 58 28 48
0 0 21 10		0 1 21 12	0 8 15 18	375	12 6 44 6
0 0 25 24	235	0 1 46 36	0 8 15 18		12 14 59 24
			0 8 15 18		12 23 14 42
0 7 49 54	340	7 18 52 6	0 8 15 18		12 31 30 0
0 7 54 8		7 26 46 14		RISING	
0 7 58 22		7 34 44 36			
0 8 2 36		7 42 47 12			
0 8 6 50		7 50 54 2			
0 8 11 4	345	7 59 5 6			
	SETTING				



Longitude and Daily Motion of Saturn for 378 days (PSI 1492).

There remains the simplest solution, which does not, however, preserve smoothness of motion, since it requires a small but abrupt jump in the daily motion column: a purely linear retrograde motion. And if we apply a subtractive 0;4,25,30° constantly in column 1 from line 118 to line 227, we will effect the needed retrogradation exactly. Hence the whole Saturn template can be derived by a simple alternation of linear and second order sequences. The appended table displays the reconstructed template, abbreviated, with the part attested in the papyrus in bold-face (a few scribal errors are not noted). The graph illustrates the longitude and daily motion functions over one synodic period. Above the curve of the papyrus longitude function, I have put for comparison a longitude function based on a simple epicycle (in other words we ignore annual anomaly), using Ptolemy's ratio of 6;30 to 60 for the radii.

The text's reliance on arithmetic sequences points surely to Babylonian ancestry. We may wish at this point to compare it with what we know of Babylonian methods for generating ephemerides of planetary motion. Two surviving texts use sequences of order higher than linear to bridge positions at phases. One (ACT nos. 654 and 655)[7] uses, essentially, two alternating third order sequences for direct and retrograde sections of Jupiter's daily motion. The other text, for Mercury (ACT no. 310),[8] follows the pattern of PSI 1492, alternating linear and second order sequences. These texts differ in an important respect from the Greek table, since they are applied to pre-established phase dates (although these may have been adjusted to allow neat bridging by the arithmetic sequences). The scribes of the cuneiform texts seem to have extrapolated forward line by line, not using templates. Hence we find some unexpected irregularities: when the Jupiter scribe reaches the date of opposition, he forces the second-order differences to pass from negative to positive at once by means of a clumsy sign change, while the Mercury scribe, approaching each new phase, introduces abrupt and wild deviations in the first-order differences, and once in the middle of the linear part of the direct motion he changes the constant difference slightly, making it equal to that of the preceding cycle. Thus the pattern of PSI 1492 may likely be originally Babylonian, but we are not justified in assuming the same source for its being set in a reusable template.

Our template for Saturn's motion over one synodic period cannot by itself adequately describe the planet's motion, since it cannot account for the effect of the planet's eccentric orbit. If one repeatedly applied a fixed average pattern of motion, the predicted longitudes, as well as the expected dates of phase, would usually be in error, in accordance with the planet's elongation from its apsidal line. That some kind of correction was applied, for this and the other planets, is suggested by another papyrus fragment.

This text is very imperfectly preserved,[9] but one can tell that it described some sort of table in seven or more columns, plausibly containing corrections for the eccentric anomalies of the planets, as we shall see. Between the gaps, several data are mentioned with regard to the later columns:

(IV) Aries and Libra
(V) Capricorn and Cancer
(VI) Virgo and Pisces
(VII) a "limit" (ὅρος) of 4,25,27.

The pairings of signs in (V) and (VI) suggest the apsidal lines of Mars and Jupiter, while (IV) more likely represents Mercury than Venus (apsidal line Taurus/Scorpio). Venus exhibits only a very small eccentric anomaly, and for this reason may not have required a column. Column (III) then would probably belong to the Sun, and the first two columns would be the arguments. The description of the seventh column, probably by an accident of preservation, mentions no apsidal signs, but does allude to a "limit" of 4,25,27, which is very reminiscent of the hypothetical retrograde daily motion derived above, 0;4,25,30° per day.

The fragmentary text often uses the words προσθαφαίρεσις, μέγιστος, ἐλάχιστος, that is, "addition/subtraction" (the standard Greek term for a quantity to be added or subtracted to a base amount, hence "equation"), "greatest", "least". This terminology, together with the emphasis on apsidal signs, hints at a table of corrections to solar and planetary longitudes, probably on an arithmetical rather than trigonometric basis. Thus to compute a planetary longitude for a specific date, one would first determine a preceding epoch date (for example the preceding heliacal rising); the rules yielding the epoch date would also give the corresponding "mean" longitude of the planet at epoch. Then, entering with the elapsed time since epoch in the synodic template table, one would get the necessary increment since epoch. Last, one would enter the table of corrections with this longitude to get an equation for the anomaly.

Without further testimony to the numerical components of this kind of planetary scheme, it would be pointless to speculate further on its operation. But at the least in PSI 1492, a probably symmetrical and schematized representation of synodic planetary motion independent of anomaly, together with the combined equation tables that PSI 1491 seems to have described, we have evidence for a necessary stage in Greek mathematical astronomy between the Babylonian planetary theory that computed the phase dates independently and interpolated for daily motion, and Ptolemy's methodically correct geometrical analysis of the separate components of the variations in planetary motion that allowed a legitimate computation of longitude as a function of time.

ABBREVIATIONS

ACT: O. Neugebauer, *Astronomical Cuneiform Texts*, London 1955.
HAMA: O. Neugebauer, *A History of Ancient Mathematical Astronomy*, Berlin etc., 1975.
PSI: Pubblicazioni della Società Italiana per la ricerca dei papiri greci e latini in Egitto.

NOTES

1. In general, see O. Neugebauer, *HAMA*, pp. 781–808.
2. Neugebauer, *HAMA*, pp. 808–817, 822–823. A general survey of these and related texts from Mesopotamia and India is Jones, "The Development and Transmission of 248 Day Schemes for Lunar Motion in Ancient Astronomy", *Archive for History of Exact Sciences*, 29 (1983), pp. 1–36.
3. First published, R. Parker, "Two Demotic Astronomical Papyri in the Carlsberg Collection", *Acta Orientalia*, 26 (1962), pp. 143–147. Edition with commentary, O. Neugebauer and R. Parker, *Egyptian Astronomical Texts*, v. 3, Providence 1969, pp. 240–241 and pl. 79B.
4. Awaiting publication as *PSI*, v. XV as no. 1492. Neugebauer gives a description and commentary in *HAMA*, pp. 790–791.
5. A synodic period for Saturn of 378 days is well attested in Greek and Greek-derived Indian sources. See *HAMA*, p. 783; O. Neugebauer and D. Pingree, "The Pañcasiddhāntikā of Varāhamihira", *Danske Vidensk. Selskab, Histor.-Filos. Skrifter*, 6 (1970–71). v 2, pp. 110–111; Pingree, "History of Mathematical Astronomy in India", *Dictionary of Scientific Biography*, 15, New York 1978, pp. 601–602.
6. The standard of assuming zero increment in longitude for the day before the first line of a table is attested in the Mercury fragment described above and in the lunar template *PSI* 1493 (see Jones, "248-Day Schemes", pp. 17–18).
7. O. Neugebauer, *ACT*, pp. 354–56 and plates 202–205. The lost portions were reconstructed by P. Huber, "Zur täglichen Bewegung des Jupiter nach babylonischen Texten", *Zeitschrift für Assyriologie*, N.F. 18 (1957), pp. 265–303.
8. *ACT*, pp. 326–28 and plates 168–69a.
9. Awaiting publication as *PSI* 1491. See Neugebauer, "Astronomical Papyri and Ostraca: Bibliographical Notes", *Proc. Am. Phil. Soc.*, 106 (1962), p. 388 and *HAMA*, p. 946. Like *PSI* 1492, it cannot be dated accurately by contents, but paleographically 2nd century A.D. seems likely. Professor Neugebauer has kindly allowed me to examine a photograph and his unpublished commentary on this text.