The Works of Archimedes: Translation and Commentary

Volume 1: The Two Books *On the Sphere and the Cylinder*

Reviewed by Alexander Jones

The Works of Archimedes: Translation and Commentary. Volume 1: The Two Books *On the Sphere and the Cylinder*
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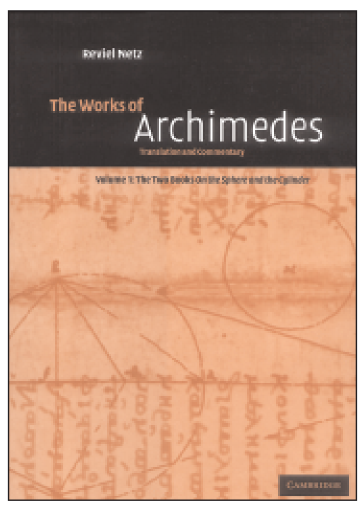
Ancient Greek mathematics is associated in most people's minds with two names: Euclid and Archimedes. The lasting fame of these men does not rest on the same basis. We remember Euclid as the author of a famous book, the *Elements*, which for more than two millennia served as the fundamental introduction to ruler-and-compass geometry and number theory. About Euclid the man we know practically nothing, except that he lived before about 200 B.C. and may have worked in Alexandria. He wrote works on more advanced mathematics than the *Elements*, but none of these have survived, though we have several fairly basic books on mathematical sciences (optics, astronomy, harmonic theory) under his name. All his writings dive straight into the mathematics with no introductions. There are hardly even any unreliable anecdotes about Euclid.

Archimedes, by contrast, is not just an author to us but a personality. He was famous in his time, not only among mathematicians and intellectuals.



He was the subject of a biography—now lost alas!—and stories about him are told by ancient historians and other writers who generally took little interest in scientific matters. The stories of Archimedes' inventions; his solution of the "crown problem"; the machines by which, as an old man, he defended his native city, Syracuse, from the besieging Roman fleet in 212 B.C.; and his death—still doing geometry—at the hands of a Roman soldier when Syracuse at last fell have never lost their appeal. Archimedes became paradoxically emblematic of two stereotypes of the mathematician: a man who could harness reasoning to the seemingly superhuman performance of practical tasks, yet whose preoccupation with abstract problems could make him fatally oblivious to his surroundings.

Alongside the public Archimedes of the stories, we also have the private Archimedes of the writings. In the manner of his time, Archimedes wrote his mathematics in the form of substantial books built up of theorems that cumulatively lead to the

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proof of a series of major results. Copies of these books were sent, in the form of papyrus rolls, to other mathematicians to be read, copied, and appreciated. The third century B.C., when Archimedes lived, was the heyday of the Greek mathematical sciences. Mathematicians were scattered about the Greek-speaking world, but there was a particular concentration of them, as of other intellectuals, in Alexandria. Archimedes sent his books from Syracuse to three Alexandrian mathematicians: Eratosthenes, remembered for his measurement of the Earth and his scientifically based world map; Conon, whose identification of a new constellation in honor of Berenice, the queen of Egypt, was immortalized by the poets Callimachus and Catullus; and one Dositheus.

There was a perverse, teasing streak in Archimedes' relations with his Alexandrian correspondents. At a time when, reflecting the cosmopolitan nature of Greek culture after the conquests of Alexander the Great, writers had embraced a standard form of Greek in place of the many local dialects, Archimedes persisted in writing his mathematics in the provincial Doric dialect of his native city. In the letters that he prefaced to his books, he speaks in less-than-flattering terms of the mathematicians to whom he is sending them, and on one occasion he sent them a list of theorems without proofs, including two false ones that were laid as a trap to catch anyone claiming priority of discovery. In spite of his best efforts to get their backs up, Archimedes' contemporaries evidently thought well of his work, and later people not only preserved them but wrote commentaries on some of them, in some instances also stripping the texts of their dialect for easier reading. We can still read the commentaries of a very late Alexandrian philosopher-mathematician, Eutocius, who lived at the time of the emperor Justinian, in the sixth century of our era.

It was about Eutocius' time that Constantinople began supplanting Alexandria as the focus of Greek learning—the Islamic conquest of Egypt was just a century away—and the selection of ancient Greek literature that was to survive through the Middle Ages was to a large extent determined by which books were brought to Constantinople. There was a very high risk of loss, especially in the seventh and eighth centuries, when classical learning, especially in the sciences, was at a low ebb, but somehow a dozen or so papyrus rolls of works of Archimedes were still around and in copyable condition by the time intellectual conditions were improving, about A.D. 800. (A few works of Archimedes, and apparently some others falsely attributed to him, were translated into Arabic about this time.) During the next two hundred years or so, a large number of manuscripts of ancient philosophical, scientific, and mathematical works were produced. These were codices, manuscripts of parchment sheets bound like modern books, and they could hold the equivalent of many of the old rolls. The parchment codices were exceedingly costly, both because of the materials (good parchment was always expensive) and because of the calligraphy, to say nothing of special skills such as copying geometrical diagrams. Nevertheless, at least three codex collections of Archimedes' works were made, each containing a different selection.

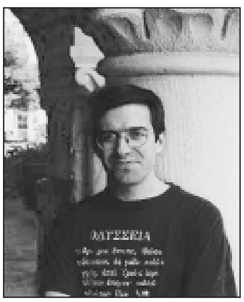
Yet it seems as if no one in the Byzantine Empire read them. Archimedes' name continues to crop up in Byzantine literature, but he is the Archimedes of the old anecdotes, not the mathematical writer. No further copies of the Archimedes codices were made. In fact, by about A.D. 1300 all three codices were in situations where Byzantine scholars could no longer read them. Two of them had somehow made their way to western Europe, where Greek learning was still rather scarce. Perhaps they were part of a royal gift, like the manuscript of Ptolemy's *Almagest* that the Byzantine emperor Manuel Comnenus gave to the Norman king William I of Sicily about 1160. Whatever the story, by 1300 these manuscripts had become part of the small collection of Greek manuscripts in the papal library (one of them was in pretty bad condition). After the papacy was moved to Avignon in 1309, the papal manuscripts seem to have been dispersed, and only one of the Archimedes manuscripts eventually resurfaced in the fifteenth century. Greek humanism was now in full flower in Italy, and good copies were made of this survivor before it again vanished, this time for good. Apparently the other manuscript had never been copied, though a painstaking Latin translation of the contents of both manuscripts had been made by the Dominican scholar William of Moerbeke in 1269, and this Latin version has survived.

The third codex met a different fate: around 1300 it was dismantled, its parchment leaves were cut in two, to some extent cleaned, and rewritten with a Greek prayer book. In this new guise, as a palimpsest (recycled manuscript) it passed through one or more monastic libraries. By the end of the nineteenth century it was in the library of a monastery in Constantinople, and a scholar writing a catalogue of Greek manuscripts in that city noticed and copied a bit of the partially erased text. The Danish classicist Johan Ludvig Heiberg, who had recently published an edition of Archimedes' works based on the manuscripts then known, recognized that the lines copied from the palimpsest were part of Archimedes' *On the Sphere and the Cylinder*, and he made haste to get access to the manuscript. Heiberg succeeded in transcribing a substantial part of the faded Archimedean texts written crossways underneath the prayers; they turned out to include two works that were otherwise entirely unknown and a third

Uncovering New Views on Archimedes

In 1996 when as a postdoc Reviel Netz launched his project of translating the works of Archimedes, his historian colleagues were not encouraging. “They said that obviously it would be a misguided project,” he recalled. “In scholarly terms it would be incomplete because I would not have access to the palimpsest.” That Netz forged ahead anyway proved to be fortuitous. When the long-lost Archimedes palimpsest resurfaced in 1998, his work on translating the first volume, *On the Sphere and Cylinder*, was ideal preparation for working on the palimpsest. “I was incredibly lucky,” he says.

The program of preserving, imaging, and transcribing the Archimedes palimpsest is under way at the Walters Art Museum in Baltimore. Netz, now at Stanford University, and classics scholar Nigel Wilson of Oxford University are leading an international team analyzing the content of the palimpsest. The museum’s work on *Sphere and Cylinder* has been completed. The imaging of *The Method* is also finished, and the Stanford Linear Accelerator is now helping to uncover additional information. Transcription of *The Method* is about one-third finished.



Reviel Netz

The palimpsest is the only extant source for *The Method*, and Netz lost no time in setting about studying it. He has formed some intriguing new insights. For example, in *The Method* Archimedes constructs in the course of a proof a one-to-one correspondence between two infinite (in fact, uncountable) sets. “This was completely unknown,” Netz states. “The assumption was that Greek mathematicians never made this kind of claim” about two infinite sets being the same size.

Another work unique to the palimpsest is *The Stomachion*. This mere half-page fragment has puzzled scholars. “There wasn’t an interpretation of it,” Netz says. “No one really ventured to say what it was.” He has now conjectured that the purpose of *The Stomachion* was to treat a combinatorics problem, a surprising idea, for it was not thought that combinatorics existed in ancient mathematics.

In fact Netz is changing many of the standard views on Archimedes and his work. “The original picture historians had of Archimedes is as a practical engineer,” Netz remarks. “I don’t find any evidence for this.…He was strictly a mathematician.” Even in Archimedes’ works on physics Netz sees mathematics as the ultimate goal. For example, Netz believes that Archimedes invented statics as a way of deriving results in geometry. The strategy was to use imaginary situations involving bodies in equilibrium to derive proportions that lead to measurements of geometric figures.

Netz sees in Archimedes’ work a personality that is “very playful, cunningly and even maliciously playful.” Hellenistic mathematicians produced works that juxtaposed different things in surprising ways and that set challenges and puzzles for readers. Netz says this style of presentation has parallels in the larger cultural tendencies of Alexandrian and Hellenistic society. His next planned book, *Ludic Proof: Greek Mathematics and the Alexandrian Aesthetic*, will examine the playful strands running through Hellenistic mathematics.

“Archimedes, among the truly great, is relatively neglected,” Netz comments. “There is a Newton industry and an Einstein industry, but there isn’t an Archimedes industry, and there ought to be one.” He believes one barrier to the study of Archimedes has been the lack of a complete and faithful translation of his works into English. Netz’s translation is so faithful, he says, that one could use it for serious historical studies. It is not an easy read, but then it is not an easy thing to plunge into the mind of a writer from a completely different culture and time. The translation is “very Greek—it’s a Greek book,” Netz says. He has not transcribed the mathematics into modern notation, preferring instead to let the reader puzzle through the mathematics just as Archimedes’ contemporaries did. Says Netz, “If you are really interested in Archimedes, invest the extra effort to see what he did.”

—Allyn Jackson

that had hitherto only been available in Moerbeke's Latin version. After a rather obscure history in the twentieth century, the Archimedes palimpsest is now in private hands and the subject of an ambitious project of conservation and research intended to recover text that Heiberg was unable to read. In the meantime, Heiberg's second edition, which is very good, remains after a century the basis for studying what Archimedes wrote.

Astonishingly, given Archimedes' fame and the importance of the works, Archimedes' books have never really been translated into English. Thomas Heath, who made an excellent translation of Euclid's *Elements*, published *The Works of Archimedes* in 1897 (before Heiberg's rediscovery of the palimpsest), but except for the prefatory letters this was not a translation so much as a mathematical paraphrase that not only uses modern notation but liberally reorganizes parts of the proofs. E. J. Dijksterhuis's *Archimedes*, first published in 1956, contains mathematical paraphrases of the proofs that are closer to the reasoning of the original than Heath's, and his book, which has been reprinted by Princeton University Press, is still the best general introduction to Archimedes' thought. But neither Heath nor Dijksterhuis gives a complete restatement of all the proofs, and the "feel" of Archimedes' writing is entirely lost.

Now Reviel Netz offers us the first installment of a translation of all the works of Archimedes that survive in Greek. The first volume contains a single large work, *On the Sphere and the Cylinder*, together with the commentary on that work by Eutocius. *On the Sphere and the Cylinder* is the book in which Archimedes proved several relations between the volumes and surface areas of spheres, segments of spheres, and cylinders, including of course the proof that the volume of a sphere is two-thirds the volume of the smallest containing cylinder; a diagram illustrating this relation is reputed to have been inscribed on Archimedes' tomb. The central proofs employ what are traditionally called "exhaustion methods", which are rigorous limit arguments founded on inscribed or circumscribed solids (such as the solids of rotation of polygons). Because exhaustion arguments are not easy to transfer from one geometrical figure to another, the proofs have a certain virtuosic character, very different from the more readily generalizable approaches of European mathematicians to such problems leading up to the calculus. In another book, *The Method* (to appear in a subsequent volume), Archimedes presented a more heuristic and less rigorous strategy for demonstrating volume and area relations by comparison of infinitesimal slices of figures. This work, however, was unknown to modern mathematicians until Heiberg discovered it in the palimpsest.

As well as the intrinsic interest of Archimedes' mathematics in *On the Sphere and the Cylinder*, there are two "loose ends" that provoked Eutocius to collect for us some remarkable specimens of Greek geometry that we would not otherwise know about. These are the solutions by several geometers of the problem of finding two mean proportionals between given magnitudes (i.e., given A and D, to find B and C such that $A : B = B : C = C : D$) and another problem mathematically equivalent to solving a cubic equation. Both problems fall into the class that Greek mathematicians sensed from experience, though they could not prove, to be insoluble using only the postulates of Euclid's *Elements*, and the solutions that Eutocius preserves illustrate how they extended their "toolbox" by allowing certain mechanical constructions, intersections of conic sections, or special curves.

In due course Netz will follow this volume with two more. Of the books awaiting translation, some are, like *On the Sphere and the Cylinder*, works of pure mathematics, while others give a mathematical treatment of problems in statics such as centers of gravity of figures and conditions of stability of floating solids. Netz does not intend to include in his scope the medieval Arabic tradition of Archimedes' works. In fact, it is likely that none of the works that pass under Archimedes' name in Arabic, aside from those that we also have in Greek, are authentic, although several contain interesting mathematics. An edition and translation collecting these would be a worthwhile project.

Netz's English Archimedes could hardly be more different from Heath's. To begin with, it is ruthlessly literal. Archimedes, like all Greek geometers, wrote his mathematics in continuous prose, using words to represent concepts and relations, and letters of the Greek alphabet to name them. The vocabulary and sentence structures of Greek geometrical writing were highly standardized and formulaic by the third century B.C., a quality that gives the arguments something of the same clarity and freedom from ambiguity that notation provides in modern mathematics. But a mathematical argument written out as prose (sometimes referred to as "rhetorical" mathematics) has two characteristics in contrast to notation: the relation to the spoken word is much more immedate, so that one can read the argument aloud correctly even if one does not understand the mathematics at all, but it takes much more space on the page to write—and more time for the eye to take in—each step of a proof. Mathematicians are often more comfortable with translations of early mathematics that employ notations to compress the argument, whereas nonmathematicians interested in the history of science may find the rhetorical style more approachable, though in the case of Archimedes they soon find that the

The first of three views (see next page) of a page from the Archimedes Palimpsest (fols. 93v-92r). The photograph at right is taken in regular light.

Photographs taken by the Rochester Institute of Technology and Johns Hopkins University. Copyright: The owner of the Archimedes Palimpsest.

easy flow of words expresses some very difficult mathematics.

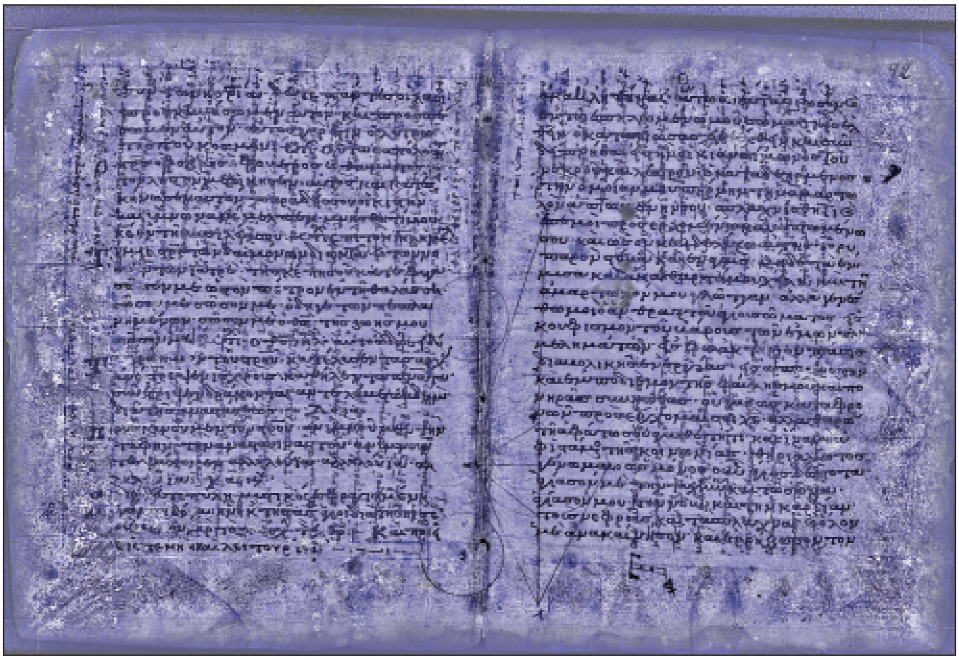
Netz does not hold that it is the translator's business to cater to the comfort of the reader; he writes (p. 3) that "the purpose of a scholarly translation as I understand it is to remove all barriers having to do with the foreign language itself, leaving all other barriers intact." And whatever one may say of Netz's Archimedes, it cannot in any way be charged with looseness. He maintains a close and consistent correspondence between the Greek terminology and his English equivalents. The formulaic character of Greek geometrical prose makes it often possible to omit certain common words without ambiguity, and the geometers took full advantage of this means of shortening their sentences: for example, the conventional Greek way of saying "the angle contained by *AB* and *BC*" would translate word-for-word as "the contained by *ABC*." Netz carefully marks all the implied words by enclosing them in angle brackets, as in this case "the <angle> contained by *ABC*." He also adheres more than any other translator I know to the Greek word order, even when this goes against natural habits of English. This is particularly apparent when ratios are being expressed and manipulated: for example, a sentence that might be represented by the notation

$$Z : H = AC : CB = AD^2 : DB^2$$

is rendered: "it is: as *CD* to *CB*, that is *Z* to *H*, the <square> on *AD* to the <square> on *DB*." Reading long stretches of this is hard going, though one soon gets used to the oddness, and the reader will probably find that writing out each step symbolically as one works through the proofs makes them easier to follow and verify. It is a pity that Netz has not provided such a step-by-step synopsis to accompany the translation.

For Archimedes' text Netz has relied on Heiberg's edition. (In subsequent volumes he intends to take account of new readings arising from study of the palimpsest.) The figures are a different matter. Heiberg, like most scholars editing works of Greek mathematics, preferred to redraw the figures that accompany each theorem according to the sense of the text rather than reproduce with necessary corrections the drawings that appear in the manuscripts. In many cases the resulting reconstructed figures look quite different from the transmitted ones, though they are usually mathematically equivalent. Netz has made a close study of the role of figures in Greek mathematics, and in his earlier book, *The Shaping of Deduction in Greek Mathematics* (1999), he showed how the figures were intended as an integral and indispensible part of the proofs, not mere adjuncts to aid visualization. So here Netz has gone back to the manuscripts and produced the first "edition" of the figures, reporting all significant variations in the manuscript versions. Like his close translation, this brings the reader a step closer to seeing Archimedes' mathematics as an ancient reader would have seen it.

Netz's intention, which I have quoted, of "leaving all other barriers intact" sounds forbidding, but it applies only to the bare translation; taking the book as a whole, he provides the reader with a great deal of help in getting over those barriers. This help takes several forms. There are frequent footnotes clarifying and justifying unobvious steps in Archimedes' arguments and cross-referencing them with the relevant parts of Eutocius' commentary, which appears later in the volume. After each theorem Netz gives two sections of commentary: one discussing textual matters (this will be of



The photograph at left is taken in ultraviolet light, showing some of the underlying text of Archimedes' *Sphere and Cylinder*.

The photograph below is a processed image, making the Archimedes undertext and drawing appear in red.

particular interest to people comparing the translation to Heiberg's edition of the Greek), the other offering more general remarks. The general comments are not, for the most part, mathematical in scope, but discuss linguistic and stylistic aspects of the text. For mathematical commentary, Netz refers us to Eutocius and Dijksterhuis, who are certainly worthy and sensible guides, though Dijksterhuis is difficult to use in close comparison with Netz's translation, because the lettering of the diagrams is different, and Eutocius is writing for a reader more conversant in the idioms of ancient mathematics than most modern readers are likely to be. A possible consequence of Netz's decision not to provide his own mathematical commentary is that there may be places where he has not detected that the text as transmitted by the manuscripts and edited by Heiberg is incorrect. I have noticed one place where this seems to be the case: in the fourth theorem of Book 2 (p. 203, step 24) a statement about ratios in Heiberg's text is both false and different from what Eutocius seems to have read in his copy of the same passage.

In general the translation has been made consistently and with care, and I have noticed very few misprints, except in the bibliography, where the typesetters seem to have been uncharacteristically creative (and, oddly, Lewis Carroll appears disguised as Carol wherever he turns up). The typography, layout, and draftsmanship of the diagrams are of a standard that one can unfortunately no longer take for granted in scholarly books. A regrettable corollary is the high price; individuals planning to own all three volumes will need deep pockets.