Euclid, the Elusive Geometer

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Euclid and His Heritage Meeting

Clay Mathematics Institute, Oxford, October 7–8, 2005

Like most of the books that the Middle Ages transmitted to us from Greco-Roman civilization, the *Elements* is ascribed to an author, Euclid. My intention in this talk is to consider what this ascription meant to ancient readers and should mean to us; or, to put the question differently, what can we know, and what did the ancients think they knew, about the mathematician Euclid? In my subtitle I characterize Euclid as "elusive." We possess a substantial number of writings under his name, enough to fill eight Teubner volumes; his books were read and cited in antiquity, and his name was a byword; yet there is remarkably little that we can confidently assert about him.

We once thought we knew more. In his *Vrais pourtraits et vies des hommes illustres*, first printed in 1584, André Thevet managed to fill four pages with a life of Euclid and adorned it with a portrait that he would have us believe was reproduced from an authentic one that he brought back from Euboea. (This is now the favourite portrait of Euclid on the World Wide Web.) Practically all of Thevet's biographical material derives from Diogenes Laertius' life of the philosopher Euclid of Megara, a contemporary of Plato who was regularly conflated with the mathematician until the preface of Commandino's Latin edition of the *Elements* put things right a dozen years before Thevet's work was printed.

Our modern standard life of Euclid, repeated in numerous popular histories and reference books, is untainted by this confusion, and can be reduced to three assertions: first, that Euclid was born, some say about 365 B.C. and some say as late as 325, perhaps in Athens, where as a young man he was a student in Plato's Academy; secondly, that Ptolemy I of Egypt summoned Euclid to Alexandria about 300 B.C.; and thirdly, that Euclid was one of the scholars employed in the famous Museum, until he died about 275 B.C.

As it happens, not one of these facts can be found explicitly stated in an ancient source, and the more sober historians, such as Heath, are careful to designate them as inferences from the ancient testimonia, while on the whole regarding them as plausible or indeed probable. Such circumspection leaves room for other imaginings of the life of Euclid, of which however there have been surprisingly few. Perhaps the most appealing, if only as a demonstration of how historians project their own environments upon the past, is Jean Itard's mid-twentieth century hypothesis that Euclid was the name adopted by a committee of the leading Alexandrian mathematicians, a Hellenistic Bourbaki.

Let us set all that aside, and consider afresh how one might attempt to construct the biography of Euclid. The first place to look is in his own writings. As everyone knows, Greek writers, especially the philosophers and scientists, were given to personal and agonistic modes of discourse that tend to reveal much about the author. So for example Paul Moraux has put together a most entertaining autobiography of the physician Galen, running to nearly two hundred pages, consisting of passages scattered through his works. A typical instance of an almost trivial autobiographical detail comes from Galen's treatise on simple medicines:

> There exists a stone, black in colour, which gives off a smell like that of bitumen when one brings it close to fire. Dioscorides and other authors say that one finds it in Lycia, near the river called Gagates.... To tell the truth, I have not seen this river myself, though I have travelled along the entire coast of Lycia with a little boat to seek out the products of this district.

Does it ever happen that a mathematical writer drops such asides as this? Practically never, unfortunately, in texts before late antiquity. In a typical Greek mathematical treatise, theorem follows theorem in bald, unrelenting succession. Where a mathematician found room for personalities was in the prefatory letter that usually accompanied a treatise when it was released to a wider readership. Such letters were an opportunity for the author to explain the purpose of a work, its relation to earlier works by the author's predecessors or by himself, and which parts of it were particularly original or occasioned the most pride. The best known are those attached to some of Archimedes' works, from which we get glimpses of a side of Archimedes different from the publicminded Syracusan inventor beloved of the historians, the mathematician who devoted his labours to abstract investigations of curves, surfaces, and volumes of geometrical figures.

Archimedes habitually addressed each work to a particular mathematician in Alexandria, and he chose addressees who are otherwise familiar to us in varying degrees: Eratosthenes, Conon, and Dositheus. From the letters we obtain information about the motivation of the problems that Archimedes solved and the order in which he wrote the books, including some that have not survived. We also glimpse Archimedes' relations with the Alexandrian mathematical community, by turns serious and respectful or teasing and suspicious. Archimedes has also left us a unique instance of a work of popularizing mathematics, the *Sand Reckoner* addressed to Gelon, son of the tyrant Hieron II of Syracuse, which does contain asides of the kind otherwise absent from Greek mathematical writing, including a passing mention of Archimedes' father Phidias.

Through the internal evidence contained in prefatory letters we gain biographical information not only for Archimedes but also for Apollonius, Hypsicles, and Diocles. As for Euclid, the contribution to his biography of such internal evidence from his writings can be summarized as follows: absolutely nil. We can see why if we look at a list of the Euclidean corpus. Three of the treatises bear no prefaces at all, but plunge directly into the stream of definitions, axioms, and theorems. The remaining three do have introductions, at least in some manuscript copies, but these do not take the form of dedicatory letters, and are entirely impersonal expositions and justifications of concepts assumed in the ensuing mathematical reasonings. Euclid's works are not unique in the Hellenistic mathematical literature in lacking prefatory letters—for example, some of Archimedes' books dealing with problems in statics have none—but their consistent absence in a substantial oeuvre strikes one as a bit unusual.

Of course internal evidence need not be the only evidence for an ancient author's life; and even when an author says a lot about himself, references to him in the works of authors, to say nothing of other kinds of documents such as inscriptions, can be valuable, especially when they are contemporary or nearly so with the person in question. Thus we are grateful to Athenaeus for including Galen among the loquatious diners in his *Deipnosophists*, if only as a confirmation that Galen seemed almost as important a personage to his contemporaries as he was in his own eyes; and we are even more grateful for the luck of the spade that has brought to light at Pergamum an inscription erected by Galen's architect father Nikon, commemorating his dabblings in geometry. But we have such a tiny fraction of ancient literature that it is a matter of luck when we get very early mentions of even the most prominent writers. We have no references, for example, to Archimedes older than Polybius. So it is no surprise that there are also no references to Euclid that treat him as a living person.

Well then, what is the earliest reference to Euclid? The question is not quite trivial to answer. The earliest text to *contain* a reference to Euclid is the first book of Archimedes' treatise *On the Sphere and Cylinder*, which was published after 241 B.C. The sentence in question is a cross-reference to a proposition in Book 1 of the *Elements*, which would seem to establish that the *Elements* was already the standard reference for fundamental operations in geometry by the mid third century. But herein lies the difficulty: the operation in question is really very basic, and this reference to the *Elements*, which comes close to the beginning of the treatise by Archimedes that was likely to be the first one that a student of mathematics would study, is the *only* such reference in the entire Archimedean corpus. It is manifestly an interpolation.

Hence the earliest authentic reference is in the prefatory letter to Apollonius' *Conics*. In this letter, which is addressed to a mathematician at Pergamum named Eudemus, Apollonius gives a rather convoluted account of the composition of the treatise, which need not concern us, and briefly describes the contents of each of the eight books that it comprised. Thus he writes:

> The third book of the *Conics* contains many astonishing theorems that are useful for both the syntheses and the determinations of number of solutions of solid loci. Most of these, and the finest of them, are novel. **And when we discovered them we realized that Euclid had not made the synthesis of the locus on three and four lines but only an accidental fragment of it, and even that was not felicitously done.** For it was not possible to complete the synthesis without the things that we discovered for the purpose.

To understand what Apollonius is talking about, we need to know that in Greek geometry a locus meant a theorem demonstrating that a geometrical object, usually a point, that is related in a certain defined way to certain given objects must lie on a geometrical object, usually a line or surface, that can be constructed from the givens. A trivial example is the theorem that a point situated a given length from a given point must lie on the circle that has the given point for its centre and the given length for its radius. "Solid loci" are locus theorems in which the point is proved to lie on a conic section, say an ellipse. A locus could be demonstrated in two manners, called *analysis* and *synthesis*; the analysis of a locus would prove that the curve on which the point lies is in fact constructible but would not yield complete information about how the construction is to be effected or precisely what kind of curve it is, while the synthesis, which could be much more difficult, leads to the actual construction of the curve. According to Apollonius, then, there was a particular solid locus known as the "locus on three and four lines," well enough known so that Apollonius did not have to explain it further, for which Euclid had published a synthesis that Apollonius was now exposing as incomplete.

Setting aside the mathematics, and the interesting fact that the first appearance of Euclid's name in the surviving literature is a criticism, let us focus our attention on the chronological implication of the reference, that Euclid published a work containing his synthesis of the locus before Apollonius published his *Conics*. Apollonius seems to treat Euclid as a figure of the past, though strictly speaking the wording would not be inconsistent with the possibility that Euclid was still living and even still producing mathematics. How much earlier Euclid's synthesis could be is a subjective judgement, depending on our notion of the pace of mathematical work during the decades leading up to Apollonius.

It is fortunate for us, therefore, that the publication of Apollonius' *Conics* can be dated to within a margin of a few years. The argument for this dating is due to G. J. Toomer, and is cogent. It takes its start, not surprisingly, from Apollonius' prefaces to the *Conics*—there are several prefaces, because Apollonius issued the *Conics* in installments, sending the first three books to Eudemus, and the remainder, after Eudemus' death, to a certain Attalus who has sometimes been identified, but very implausibly, as Attalus I, king of Pergamum. Apollonius mentions several predecessors and contemporaries, for example in the preface to Book 1 from which I have already quoted he speaks of one Naucrates who originally encouraged Apollonius to write the *Conics*, but the most significant references for dating the book are in the preface to Book 2. There, Apollonius tells Eudemus that he is sending him Book 2 by way of his own son, also named Apollonius, and he adds,

> And if Philonides the geometer, whom I commended to you at Ephesus, should ever find himself in the vicinity of Pergamum, share it with him.

Now this Philonides is without question the Epicurean philosopher of whom we have fragments of a biography on a papyrus from Herculaneum. Philonides associated himself with several mathematicians, and in particular we are told that his first teacher was none other than Eudemus. Philonides also knew two successive Seleucid kings, Antiochus IV and Demetrius I, so his mature career as a philosopher and sometime diplomat fell around the late 160s B.C. If his schooling under Eudemus took place twenty or thirty years before this, Apollonius' *Conics* would date to some time around 185 B.C., give or take a decade. This leaves more or less the whole third century open for Euclid's career.

After Apollonius, Euclid's name disappears from the extant Greek literature for a long time. Yet we can tell that by the mid first century B.C. he was famed, if perhaps in a rather vague way, as a mathematician, because Cicero has one of the interlocutors in his dialogue *De Oratore* pair Euclid with Archimedes as geometers of the good old days when no one would have suggested subdividing geometry into fields of narrower specialization. Another Latin allusion from a century later is more problematic: Valerius Maximus names Euclid the geometer as the expert to whom Plato referred people who asked him how to solve the Delian problem of doubling the cube, but comparison with Plutarch's telling of the same story reveals that either Valerius or a later scribe has confused Euclid with Eudoxus.

With Valerius Maximus we are *at a minimum* more than two centuries removed from Euclid's death, and except for the spurious cross reference in Archimedes' *Sphere and Cylinder* we have heard not a word of Euclid's *Elements*. This is not to say that there are no mentions or even quotations of material that we find in the *Elements*. For example another Herculaneum papyrus preserves bits of an Epicurean critique of geometry by one Demetrius that cited phrases and demonstrations closely corresponding to parts of *Elements* Book 1. But Euclid is not named in the passages that have been read, and we cannot tell whether Demetrius was reading the *Elements* more or less as we have it or another, perhaps much more limited, text that drew upon the same body of generally accepted fundamentals of plane geometry.

The first securely datable authors who cite Euclid's *Elements* as such are in fact Galen and Alexander of Aphrodisias, neither of them a mathematician but both well versed in the exact sciences as they were taught during the second century of our era. Their citations sometimes specify which book of the *Elements* is meant, and are not restricted to the first book, so we can be sure not only that Euclid's *Elements* had become the standard school text for mathematics but also that it was arranged at least approximately as we know it.

An alert listener familiar with the history of Greek mathematics may be surprised that I have not named another writer ahead of Galen and Alexander, namely Heron of Alexandria. Heron is best known to us as the author of works on mechanical devices, but a commentary by "Heron the mechanician" on the *Elements* was known in late antiquity and is frequently cited in the medieval Arabic commentary of al-Nairizi. Heron's commentary was evidently an important document in the history of diffusion and interpretation of the *Elements*; but it makes some difference whether it preceded or followed the general adoption of the *Elements* as a school text. Now Heron's date, which was long held in doubt, seemed to be definitely established by a reasoning of Neugebauer's. Neugebauer believed that it could be shown that an example of a geographical calculation employing simultaneous observations of a lunar eclipse in two localities that Heron gives in his book *On the Dioptra* refers to a specific historical eclipse that took place in A.D. 62, and he further argued that Heron's choice of this eclipse only makes sense if he was writing shortly after the event. Only very recently Nathan Sidoli has reexamined Neugebauer's analysis and shown that it rests on arbitrary assumptions about which are to be trusted of the various mutually inconsistent data Heron gives for the eclipse; in short, not only is Neugebauer's identification of the eclipse only one of several equally good (or equally bad) matches, but it is open to doubt even whether Heron was illustrating his method with an authentic observation. With this basis for dating him removed, we can only be fairly sure that he wrote after the middle of the first century of our era, and certain that he wrote before about A.D. 320.

Another indication of the *Elements*' popularity around this period is the appearance of parts of it in fragmentary papyrus manuscripts from Roman Egypt. These are not numerous—about five such manuscripts are known—but the *Elements* is the only extant or identifiable treatise of demonstrative mathematics of which we have papyrus fragments at all. The scrap shown here was dated paleographically by Eric Turner to within a few decades of A.D. 100, making it the oldest manuscript of the *Elements* known, and probably a generation earlier than Galen's lessons in geometry at the feet of his father.

The second and third centuries are also the time when we first hear of others among the extant works of Euclid: the *Optics* and the *Phaenomena*. But this is again the bare name of an author. No one says anything *about* Euclid, certainly nothing to prepare us for an astonishing outburst by Pappus of Alexandria, a writer and teacher of mathematics active about A.D. 320.

It comes in the course of a students' guide that Pappus compiled for a corpus of geometrical texts by Euclid, Apollonius, and Eratosthenes that went under the collective title *topos analyomenos* or "Treasury of Analysis." For the most part Pappus sticks to the mathematics, describing the contents and arrangement of each work in the corpus and providing the student with a collection of supplementary theorems. But at one point Pappus is inspired to give a glowing delineation of the personality of Euclid:

> He was extremely fair and kindly to everyone who was capable of helping to add to mathematics to any extent... and not at all offensive, an exact man but not a braggart—like *this* fellow (Apollonius).

The sting in the tail is in fact the real point of Pappus' character sketch. One of the treatises in the Treasury of Analysis, the last one that Pappus intended to introduce, is Apollonius' *Conics*, and Pappus finds it convenient to quote the part of Apollonius' first preface in which he summarized each of the eight books. But Pappus takes offense at

Apollonius' criticism of Euclid's synthesis of the locus on three and four lines, and so he portrays Euclid as a contrasting model of good scholarly manners, even speculating that Euclid *deliberately* left his synthesis incomplete in order not to appear to be outdoing or encroaching upon the work of a still earlier mathematician named Aristaeus.

One may reasonably question whether Pappus had any credible authority for his portrait of Euclid. For my part I believe that we are getting a specimen of the traditional classroom patter of late antiquity, given heightened colour by Pappus' penchant for moralizing about mathematical ethics. But Pappus continues with a more ostensibly factual assertion about the relationship between Apollonius and Euclid:

> Apollonius had acquired the ability to add the remaining parts to the locus... because he had studied with the pupils of Euclid at Alexandria for a very long time; and this is where he also got this great grasp....

Supposing that Apollonius really studied with the pupils of Euclid at Alexandria, this would give us a more precise date for Euclid's maturity, say about forty years before the *Conics* (since Apollonius had a grown son by then). It also seems to associate Euclid with Alexandria, the first indication we have of the place where he worked—although strictly speaking Pappus does not assert that Euclid himself resided at Alexandria.

The trouble with Pappus is this: on the one hand he had access to an enormous quantity of mathematical literature that has since disappeared, and he also had an interest in the stories behind the mathematics; but on the other hand when he did not know something, he was inclined to make guesses and to present those guesses as facts. Apollonius writes in the preface to *Conics* Book 1 that he had formerly spent time in Alexandria, and possibly this is all that is behind Pappus' claim that Apollonius had been a student there. We have no such obvious rationalization of his statement that the mathematicians at Alexandria were Euclid's pupils, with its implication for Euclid's date. What we can say at least is that it made sense to Pappus, who was a mathematician, if not a first-rate one, and who was conversant with a considerably wider range of writings by Euclid, Apollonius, and the other Hellenistic mathematicians than perhaps anyone since his time.

We have no such difficulties with the one remaining ancient biographical notice of Euclid, which is in a strict sense the *only* truly biographical notice among them, since its purpose is to answer some of the same questions we are asking: who was Euclid, when and where did he live, and what was his intellectual background? In the long introductory part of his commentary on Book 1 of the *Elements*, the fifth-century Neoplatonist Proclus lists a long series of Greek mathematicians who contributed to the development of the "Elements," beginning with Thales and ending with people contemporary with and just after Plato, say roughly the middle of the fourth century B.C., at which point Proclus says that his sources stopped. Then, rather awkwardly, he continues,

> Euclid... is not much younger than these. This man lived during the reign of the first Ptolemy.

Since Ptolemy I reigned between 304 and 285 B.C., Proclus is situating Euclid much earlier than Pappus does. But unlike Pappus, Proclus tells us why he believes this. He has two arguments. First, he says:

> Archimedes, who also came immediately after the first [Ptolemy], mentions Euclid.

Modern historians have generally taken Proclus to be referring to the interpolated cross-reference in *Sphere and Cylinder* Book 1. It is by no means certain, however, that the interpolation would have been present in a text of *Sphere and Cylinder* that Proclus would have been able to read, and he might have seen a genuine mention of Euclid in some other work by Archimedes that has not survived. Since Archimedes was an old man when he died about 212 B.C. (Tzetzes' claim that he was 75 years old is not to be trusted as an exact figure), Proclus is approximately correct in saying that his life followed directly after Ptolemy I's reign.[1] Of course he commits a *non sequitur* in concluding that

[1] The meaning of Proclus' verb, ἐπιβάλλω, is evident from his use of the same verb twice in the *Chrestomathy* to signify that the story of one poem in the ancient Epic Cycle began immediately where the preceding poem left off.

Euclid must have been active during that reign. Several of Archimedes' works can in fact be dated to after 241 B.C. so that an authentic reference by Archimedes to Euclid would be compatible with the beginning of Euclid's career falling as late as the reign of Ptolemy III.

Proclus' second argument is the famous anecdote about Euclid telling Ptolemy that there is no royal road to geometry. Anyone can now see that such tales are not evidence that the personages figuring in them ever met; at best this one shows that whoever concocted it thought that Ptolemy was an appropriate king to set against Euclid—and one suspects that the tradition did not bother to specify *which* Ptolemy. As a matter of record, the same anecdote appears elsewhere with a different king and geometer, Alexander the Great and Menaechmus.[2]

Proclus has one more thing to say about Euclid. He points out that the culmination of the *Elements* is the construction and investigation of the five regular polyhedra, the Platonic solids so-called because in the *Timaeus* Plato speculates that they are somehow the building blocks of the four elements and the cosmos. This, he says, was because Euclid was a Platonic philosopher.

Proclus was grasping at straws. He knew nothing about Euclid, and in a manner of history that will always be with us he relies on worthless evidence where better was not to be had. Wishful thinking encouraged Proclus to make Euclid a fellow Platonist, and by dating Euclid to about 300 B.C. Proclus avoided an awkward gap between the last mathematicians mentioned by his sources as contributing to the "Elements" and the perfected *Elements* of Euclid. What I have called the "modern standard life" of Euclid is merely a worked-up version of Proclus' guesses, and deserves to be discarded.

But historiography abhors a vacuum, and Pappus' more youthful Euclid stands in the wings ready to take the place of his indisposed Proclian rival. Should we let him? Familiarity with Pappus' ways advises caution: we ought not to give credence to Pappus' chronology simply because we are in no position to make an independent judgement of his reasons for it.

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[2] Proclus reports that Menaechmus was a pupil of Eudoxus and acquaintance of Plato, so his life could have overlapped Alexander's; but the story is as obviously a fiction when applied to him as when applied to Euclid.

In the end, our only standard for weighing Pappus' authority is to compare Euclid's mathematical work as a whole with the more narrowly datable work of other mathematicians before Apollonius. If mathematical methods and fields of current interest changed through this period, which seems a reasonable assumption, then it should be possible to decide whether Euclid's contributions are plausible or anachronistic for the generation of Eratosthenes, Conon, and Archimedes.

But what if we arrive at a different answer depending on which works of Euclid we take into consideration? In such a case we have to admit the possibility that some of the works attributed to Euclid either are spurious or represent something other than current research, for example didactic texts. Hence I suggest that our starting point should be those works that seem characteristic of the latest stage in the evolution of Hellenistic mathematics; and interestingly, these turn out all to be among the works that were not preserved by the medieval tradition.

First, consider the work of unknown title that Apollonius disparaged in the preface to the *Conics*. According to Pappus' elucidation of Apollonius' terse remark, this was the synthesis of a rather complicated locus in which a variable point is constrained by a fixed relation among its perpendicular distances from three or four given lines. To consider only the four line version of the problem, we can say in modern algebraic terms that if the distances of a point P from the four lines A, B, C, and D are respectively a, b, c, and d, then it is required that $(ab)/(cd)$ is constant. Euclid had to show that any point on a certain conic section determined by the givens satisfies the condition. Apollonius alleges that Euclid's synthesis was not complete; presumably it failed to allow for all the possible arrangements of the givens and for multiple solutions, a kind of comprehensiveness in which Apollonius excelled and apparently prided himself. But even a partial solution calls for some rather sophisticated handling of the geometry of conic sections, for example the determination of a conic given five points that lie on it.

Now the first geometer who worked with conic sections and their basic properties was Menaechmus, whose career fell in the mid to late fourth century B.C. Euclid's synthesis can scarcely be imagined as a work from the early days of this field of investigation; indeed Pappus informs us that it was preceded by a substantial treatise devoted to "solid loci" by Aristaeus.

Two other lost works by Euclid that formed part of Pappus' Treasury of Analysis seem likewise more at home in the second half of the third century than around 300 B.C. We do not know a great deal about the book entitled *Loci on Surfaces*, but it comprised locus theorems in three dimensions, such that a point allowed two degrees of freedom or possibly a curve allowed one degree of freedom was shown to lie on a constructible surface. Euclid's *Porisms* was a large collection of theorems establishing relations among various incompletely determined configurations of points, lines, and circular arcs. Pappus' supplementary theorems to the *Porisms* give us the flavour of its mathematics, which had much in common with modern projective geometry. The famous proposition known as "Pappus' Theorem" was among the more basic results in the *Porisms*.

On the other hand, some of the surviving works attributed to Euclid seem rather archaic or primitive: I include in this group the *Sectio Canonis*, the *Optics*, and the *Catoptrics*. These are works of mathematical reasoning applied to physical problems, and one cannot help being struck by the low level of the mathematics involved and the loose standard of proof, which in the *Optics* often degenerates to an appeal to the diagram—"If you look at the figure, you will see that this must be so." Can this be by the same Euclid who wrote the *Porisms* and the works on loci?

Thus the "Euclidean Question" is at its root not about Euclid's date but about the coherence of the Euclidean corpus. There seem in fact to be three Euclids: the bad, the good, and the great. The bad Euclid, or if we prefer, the archaic Euclid, wrote the crude and logically defective books of applied mathematics. The good Euclid wrote, or skilfully compiled from older sources, the logically satisfactory but professedly elementary elements. The great Euclid wrote books of advanced mathematics, which incidentally assumed concepts for which the *Elements* provides no axiomatic foundation such as compound ratios.

One way of reconciling at least the apparent gradations of logical sophistication in the extant works is to hypothesize that the *Elements* was reworked after Euclid's time. The supposition is that Euclid wrote a version of the *Elements* that more closely resembled the other works in the Euclidean corpus, and that the polish characterizing the text as we have it was added later, perhaps as late as Heron's commentary. This may be true, but it is not a sufficient resolution of the Euclidean enigma.

My best guess, but I am far from certain that it is right, is that the *Optics*, *Catoptrics*, and *Sectio Canonis* were falsely attributed to Euclid in antiquity because they were in a sense foundational works and he was thought of *par excellence* as "the Elementarist" (*ho stoicheiôtês*). But the authentic Euclid is till hard to pin down. He does seem to have lived several decades later than we have been accustomed to say, and he was a more advanced and original mathematician than we would assume from the *Elements* and his other surviving works. But beyond this Euclid remains, as he was already in antiquity, an enigma, the most faceless of the great Hellenistic mathematicians.