# **b** The Front Cover Inscription

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### Abstract

The bronze plate known as the "Front Cover" of the Antikythera Mechanism had inscriptions on its outside face. This paper describes the reconstruction of the surviving parts of this text from the Mechanism's fragments, giving transcriptions and translations. The texts give data on synodic cycles for the five planets, and it may be conjectured that lost lines described the behaviour of the Sun and Moon. The data strongly support the idea that planetary motions were displayed on the front face of the Mechanism using simple epicyclic or eccentric models. Previously unattested long and accurate period relations are given for Venus and Saturn, which are favourable for geared representation and probably of Greek, rather than Babylonian, origin.

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# 6.1 Introduction

Whereas during much or all of the time that it was immersed in the sea the Mechanism's back face was partly covered by an inscribed plate (the Back Cover Plate),<sup>1</sup> the front face was covered by *two* layers of inscribed plate. Fragment C, in its "original" 1902 state, comprised these two layers fused to parts of the front face (the dial plate and the casing of the Moon's phase display). Immediately superimposed on the front face were the two Parapegma plates, displaced from their proper locations above and below the dial plate and oriented in what seems to be a random manner.<sup>2</sup> The Parapegma plates were themselves overlaid by the plate that we conventionally call the Front Cover Plate, though we cannot be certain that it really was meant to serve as a cover or that the position in which it was found reflects where it was meant to be when the Mechanism was intact. On its outside face, the Front Cover Plate bore an inscription, oriented upright with respect to the Mechanism's top and bottom. Like the Back Cover Plate, the Front Cover Plate accumulated a hard layer of accretion over its inscribed face that retained mirror-reversed offsets of the inscription. Patches of the accretion layer subsequently became detached from the corresponding surfaces of the plate and became fused again in somewhat shifted positions.

As part of the c. 1905 conservation work, the accretion layer and the Front Cover Plate were painstakingly removed from Fragment C in many small pieces, and most of the pieces of the Front Cover were later reassembled as the present Fragment G. Besides G, two smaller pieces of the plate exist as separate fragments; and additionally we have many small fragments of the accretion layer bearing offsets, most but not all of which overlap with extant parts of the Front Cover. (Also some bits of the accretion layer remain on the surface of G.) In all we have a vertical extent of a little over 110 mm preserving parts of 43 consecutive lines of the inscription, and there were certainly more lines at the beginning and likely also more at the end. Supposing that the plate was truly a cover, it could have held about sixty lines of text if its height matched that of the front dial plate, and double that if it protected the entire front face. The aggregate width of the surviving plate is about 115 mm. If, as seems probable, the plate was originally about the same width as the Mechanism's faces (i.e. just over 170 mm), the average line would have contained about 70 letters. We can thus estimate that the complete inscription contained well over three hundred words, and likely on the order of five hundred to a thousand words.

The surviving portion of the inscription consists of descriptions of the cycles of apparent motion (*synodic cycles*) of the five planets through the zodiac. Each planet is discussed individually in a passage of eight to twelve lines, in the order Mercury, Venus, Mars, Jupiter, Saturn. (The planets' pointers were described in the same order in the Back Cover

<sup>1</sup> IAM 5.1.

<sup>2</sup> IAM 3.1-2.

Inscription, with the Sun inserted between Venus and Mars.)<sup>3</sup> The first part of each planet's section states a long time interval that is supposed to contain exact whole numbers of synodic cycles, periods of the planet's revolution around the zodiac, and solar years, followed by the approximate length of a single synodic cycle in days. The remainder of the section breaks down the synodic cycle into intervals of specified durations in days, characterized by whether the planet is moving eastward or westward in the zodiac and towards or away from the Sun.

Texts providing such information about planetary synodic cycles are attested in both Greek and Babylonian astronomy. For a close parallel, comprising five sections giving breakdowns of each planet's synodic cycle into stages of specific durations, we have to wait until late antiquity. The text in question is transmitted in various Byzantine astrological manuscripts, some of which ascribe it to Heliodoros, the brother of the sixth century AD Neoplatonist philosopher Ammonios; it is based in a rather haphazard way on Ptolemy's astronomical models and tables, but debases Ptolemy by treating each planet's synodic cycle as a constant period subdivided into constant stages.<sup>4</sup> In the planetary theories of both Babylonian and Roman-period Greek astronomy the synodic cycles were modelled as variable and dependent on the planet's position in the zodiac. It is probably significant that the only instances currently known of texts on cuneiform tablets or Greco-Egyptian papyri that prescribe a nonvarying subdivision of a planet's synodic cycles pertain to Venus, the planet with the least pronounced zodiacal anomaly.<sup>5</sup>

The Front Cover Inscription is not simply an astronomical text, but an astronomical text accompanying an astronomical mechanism. The reader of such a text in such a setting would receive it not only as a description of astronomical reality but at the same time as a description of the behavior of the device: the theoretical assumptions built into it as well as the phenomena that it simulated. The Front Cover Inscription and the Back Cover Inscription thus have complementary roles as "captions" for the Mechanism, with the Back Cover Inscription giving the viewer a guide to the meaning of the many exterior features, and the Front Cover Inscription directing the viewer's attention to the astronomical "facts" that these features displayed when the Mechanism was in operation. Since there is no reason why the text should have been limited to describing the behavior of the planets (or, if we prefer, the behavior of the planetary pointers on the front dial), we may conjecture that lost lines were devoted to the phenomena of the Sun and Moon as represented by the gearwork.

<sup>3</sup> IAM 5.5, note to I 18.

<sup>4</sup> Neugebauer 1958.

<sup>5</sup> Babylonian cuneiform tablet BM 33552, in Britton & Walker 1991; Greek papyrus *POxy astron.* 4135 in Jones 1999, 1.81-84 and 2.10-13.

Just as the Back Cover Inscription supplies the modern investigator with information about aspects of the Mechanism's exterior that cannot be reconstructed from the physical remains, the Front Cover Inscription provides us with clues to the lost planetary gearwork as well as some measure of the understanding of planetary motion that the designers of the Mechanism possessed. In this last respect it is especially valuable, despite its many lacunae, because we have extremely few documents from the Hellenistic period that present any aspect of planetary theory beyond an elementary level.

# 6.2 Fragments preserving parts of the Front Cover Inscription

Three fragments are parts of the original inscribed plate of the Front Cover Inscription. In addition, we have many identified fragments of the accretion layer bearing offsets of the inscription. With the exception of Fragments 42 and 51, the original relative locations of all the fragments are known (Figs 6.1-6.2).<sup>6</sup>

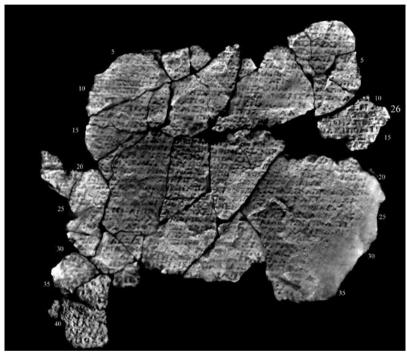


Figure 6.1: CT composite image of the plate fragments of the Front Cover Inscription (Image: Antikythera Mechanism Research Project)

The locations on G of offset fragments 23, 37-41, 43-44, and the fragments with numbers above 45 were found by A. Jones; T. Freeth found the location of 27, while Jones and Freeth independently located 21. Freeth conjectured the locations of 26 and 29 in relation to G before they were established by study of the text and the photographic evidence for 29 mentioned in the next note. Most of these juxtapositions of fragments were shown visually in a video animation prepared by Images First Ltd. which was displayed as part of the National Archaeological Museum's temporary exhibition, "The Antikythera Shipwreck: The Ship, The Treasures, the Mechanism" (April 5, 2012-June 29, 2014).

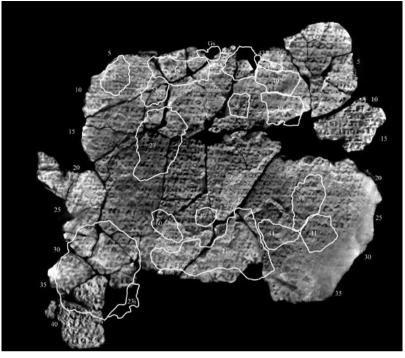


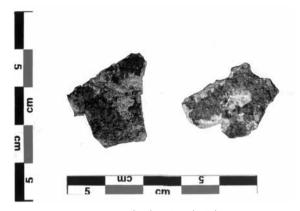
Figure 6.2: Locations of offset fragments of the Front Cover Inscription. For notations with appended letters see the introduction to the apparatus in section 6.4 (Image: Antikythera Mechanism Research Project)

#### Fragments of the Front Cover Plate

Fragment G (supplementary Fig. S5), 115 mm (width) by 94 mm (height), containing parts of thirty-six text lines (1-36, though the remains of 36 are just illegible traces). This fragment, our principal witness for the Front Cover Inscription, was assembled by museum technicians from about twenty pieces of plate, the largest of which, constituting its lower right portion, is approximately 48 mm by 51 mm. Most of the pieces bear visible writing, though some patches are concealed behind a thin layer of accretion. The engraving is everywhere shallow and blurred owing to corrosion and perhaps also early chemical cleaning, and even in CT images the legibility varies from mediocre to poor. The surface towards the lower right edge tapers to complete smoothness. The continuity of text as established in the present transcription confirms that the pieces have been fitted together correctly.

The average baseline-to-baseline spacing in G is approximately 2.6 mm as measured between the baselines of lines 2 and 36. Typical letter height is about 2.0 mm. The average letter width, from left edge to left edge of consecutive letters, is approximately 2.2 mm, though from line to line the average can deviate by as much as roughly 10% from this value. Thus while the letter heights and horizontal spacing of the Front Cover Inscription are about the same as those of the Back Cover Inscription, the line spacing is considerably tighter than

the Back Cover Inscription's 3.5 mm baseline-to-baseline.<sup>7</sup>



Fragment 26 (Fig. 6.3, left), 26 mm by 20 mm, containing parts of seven text lines (10-16).

Figure 6.3: Fragments 26 (left) and 29 (right) (Images: National Archaeological Museum, Athens, photographer: Kostas Xenikakis, copyright: Hellenic Ministry of Culture and Sports/Archaeological Receipts Fund)

Fragment 29 (Fig. 6.3, right), 23 mm by 23 mm, containing parts of ten text lines (lines 34-43, the last of which is just illegible traces). In some of Price's photographs taken during his visit to the National Archeological Museum in 1958, Fragment 29 is visible as an attached part of G, joining the present bottom edge at its left end.<sup>8</sup> The correctness of this join is confirmed by Fragment 23's offsets, which overlap parts of both 29 and G. Continuity of text establishes that Fragment 26 belongs in the large inlet of the right side of G. The configuration of the three plate fragments is shown in Fig. 6.1.

### Fragments of the accretion layer containing offsets of the inscription

We list below the offset fragments with their approximate dimensions and the line numbers of the text lines that they partially preserve. Figs. 6.4-6.7 show photographs and PTM

<sup>7</sup> IAM 5.2.

Adler Planetarium collection, color negative in Envelope 2, showing all fragments in the cardboard boxes; black-and-white photograph in Price family collection showing Price measuring Fragment A with G and other fragments visible on his work table. A blackand-white photograph of G in Adler Folder 1, reproduced as Price 1974, 50, fig. 40, shows the fragment missing not only Fragment 29 but also two small bits that are at present attached immediately to the left and right of where 29 was. Black-and-white negatives in Adler Negative Roll 2 show G in its present state together with several small fragments. Unfortunately we do not know the relative chronology of the various photographs.

images of these fragments, and Fig. 6.2 shows their original locations in relation to the plate fragments where these are known.

21: 45 mm by 26 mm, 9 lines (25-33). 23: 28 mm by 35 mm, 13 lines (27-39). 27: 18 mm by 25 mm, 9 lines (13-21). 37: 23 mm by 38 mm, 7 lines (20-26). 38: 36 mm by 18 mm, 3 lines (6-8). 39: 27 mm by 20 mm, 4 lines (10-13). 40: 28 mm by 16 mm, 4 lines (6-9). 41: 23 mm by 23 mm, 5 lines (25-29). 42: 20 mm by 14 mm, 3 lines (not placed). 43: 22 mm by 21 mm, 5 lines (4-8). 44: 26 mm by 17 mm, 4 lines (26-29). 49: 09 mm by 08 mm, 3 lines (25-27). 51: 13 mm by 14 mm, 5 lines (not placed). 54: 10 mm by 12 mm, 4 lines (8-11). 55: 10 mm by 14 mm, 5 lines (6-10). 56: 07 mm by 09 mm, 3 lines (10-12). 60: 10 mm by 11 mm, 3 lines (25-27).

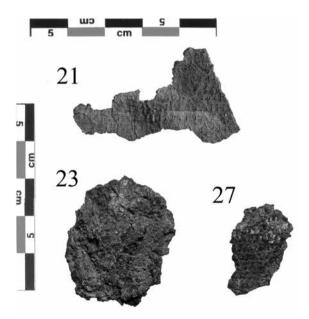


Figure 6.4: Fragments 21, 23, and 27, mirror-reflected (Images: National Archaeological Museum of Athens (K. Xenikakis), copyright: Hellenic Ministry of Culture and Sports/Archaeological Receipts Fund)

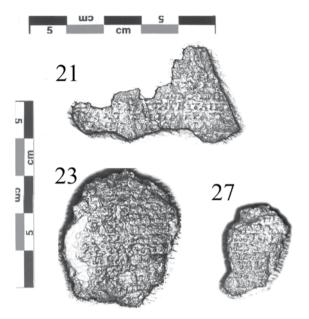


Figure 6.5: Mirror-reflected PTM images of Fragments 21, 23, and 27 with specular enhancement (Image: Antikythera Mechanism Research Project)

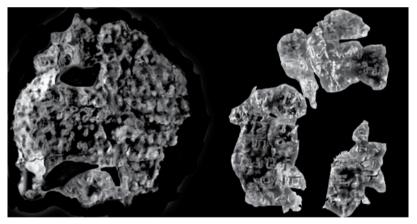


Figure 6.6: Fragment 23, CT composite images of offsets on surface (left) and flakes in interior (right) (Image: Antikythera Mechanism Research Project)

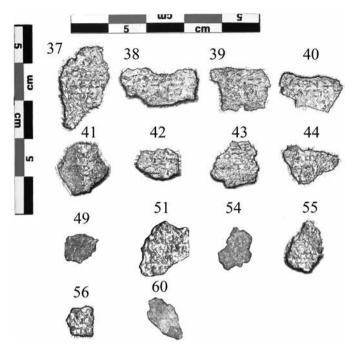


Figure 6.7: Mirror-reversed PTM images of small offset fragments with specular enhancement (Image: Antikythera Mechanism Research Project)

Most of the offset fragments are thin plates, but 23 and 27 are comparatively thick, and their interiors contain jumbled flakes of accretion including some that bear offsets legible in CT. The interior offsets in 23 are particularly helpful for reconstituting the inscription (Fig. 6.6). All the offset fragments are presumed to have been separated from Fragment C during the conservation work of c. 1905. The 1903 published photograph of C-1 (supplementary Fig. S9) shows only the Front Cover plate and the layer of accretion, featureless and indistinguishable from each other. Rediadis reported that C-1 bore "traces of an illegible (forwards-running) inscription", and it is possible that a region indicated by the letter "b" in the photograph was where these letters could be seen.<sup>9</sup>

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<sup>9</sup> This is according to the German language edition, Svoronos 1903b, 46. In the Greek edition, Svoronos 1903a, 46, Rediadis mistakenly asserted that C-1 bore the part of the Back Plate Inscription that Svoronos had in fact transcribed from A-2. The letters on Plate 10 were intended to mark features discussed in Rediadis's text, but there is no reference to "b". The region marked by "b" corresponds to the upper right corner of the present Fragment G, where the lettering is comparatively clearly preserved.

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By the time that Rehm saw C-1 in September, 1905, the Front Cover plate and accretion layer had been entirely removed (supplementary Fig. S10).<sup>10</sup> The first record of the Front Cover Inscription fragments as separate entities is Rehm's notebook of 1906.<sup>11</sup> On the pages numbered 86 and 87 of this notebook (Fig. 6.8), Rehm drew the outlines and what he could make out of the text of ten small fragments, labelled with the Greek letters I through  $\sigma$ .<sup>12</sup> A transcript of Rehm's copies, without the Greek letter identifiers, also exists among Price's manuscript notes on the inscriptions (Fig. 6.9), and this includes three more fragments which, for continuity, we designate  $\tau$  through  $\phi$ .<sup>13</sup> Rehm's copies are interesting as showing that Fragment G had not yet been assembled from the pieces that had been separated from C. His ι, ο, and ξ are easily recognized as three of the larger pieces now in G. Among the offset fragments that Rehm copied,  $\kappa$  is the present 41 joined to the bottom half (only!) of 37;  $\lambda$  is 40, but some letters copied by Rehm have since broken off;  $\mu$  is 43, but again some letters have since broken off; v is a piece of 21;  $\sigma$  is 44;  $\tau$  is probably 23;  $\mu$  is another piece of 21; and  $\phi$  is 27.<sup>14</sup> It thus appears that some joining of small offset fragments, as well as minor breakage, took place between 1906 and 1958, when Price saw the fragments in essentially the form that they have now (except for the detachment of 29 from G).<sup>15</sup>

14 Rehm's  $\pi$  is 25 (offsets of the Back Plate Inscription); we have not been able to identify  $\rho$ .

<sup>10</sup> Rehm 1905, 17-18.

<sup>11</sup> Rehm 1906a.

<sup>12</sup> Rehm must therefore have previously made a collection of eight inscription fragments labelled α through θ, which is not known to survive. These likely included the inscriptions on Fragments A-2 (Back Plate Inscription), B-1 (Back Cover Inscription offsets), and 19 (Back Cover Inscription) previously published by Svoronos and Stais, the Egyptian calendar month and the Parapegma Inscription that Rehm had found on C-1 in 1905, and perhaps also the Back Cover Inscription offsets on A-2 and the isolated inscribed letters on A-2 and C-2.

<sup>13</sup> Price collection at the Adler Planetarium. This sheet must have duplicated a set of Rehm's notes different from the 1906 notebook.

<sup>15</sup> Theofanidis's transcription of text read on "certain oxidized fragments of inscribed plates" (Theofanidis [1927-1930], "99" [correct pagination 91]) is from the part of G that Rehm copied as his fragment omicron, but it is not possible to tell whether it was still a separate fragment in the 1920s. It seems improbable that a conservator would have known how to fit the pieces of G together at a date so remote from when they were separated from C.

26 1) 25-28 Prochoft 2 den Uler Rand TAXAN Van ION AQN. 67] NKA 10 0 6 100 405 26 anthatt ; Deckel mut Tarepegnie , 44 of Rehmen & Nonius verstäckt durch einen Reit, der gehörig P 23 100 1222 Aire UMALON theriver Thega 5 Troh in

Figure 6.8: Pages from Rehm's 1906 notebook with copies of small inscription fragments (Bayerische Staatsbibliothek)

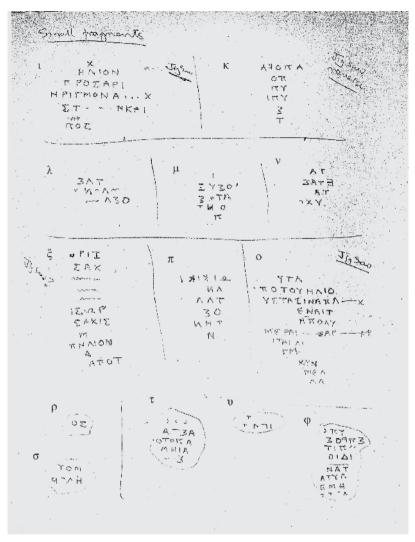


Figure 6.9: *Price's transcription of Rehm's small inscription copies* (Adler Planetarium)

# 6.3 Previous transcriptions and study of the Front Cover Inscription

Rehm's copies of fragments of the Front Cover Inscription plate and offsets, preserved in his 1906 notebook and in a transcription among Price's papers, have been referred to above in section 2. The first published transcription of any part of the inscription appeared in Theofanidis's encyclopedia article on the voyages of St. Paul; it comprises a few letters and traces from lines 21-31.<sup>16</sup> Price gave disjointed readings (fewer than two hundred letters, few complete words) from the more legible parts of thirty lines of G in *Gears from the Greeks*, as well as six lines from Fragment 21.<sup>17</sup>

The 2006 AMRP paper presented a far more extensive provisional text of Fragment G, comprising nearly a thousand letters read from CT.<sup>18</sup> A revised and extended text by A. Tselikas was reported by M. Zafeiropoulou in 2012.<sup>19</sup>

<sup>16</sup> Theofanidis [1927-1930], "99" [correct pagination 91]. The first five lines are reprinted in Theofanidis 1934, 146.

<sup>17</sup> Price 1974, 49, Fig. 38 and 48, Fig. 37. The caption of the latter figure seems to imply that Price thought that Fragment 21 belonged to the Back Plate Inscription.

<sup>18</sup> Freeth et al. 2006, Supplementary Information, 8.

<sup>19</sup> Zapheiropoulou 2012, 245.

# 6.4 Transcription and translation

The text presented here combines readings from fragments G, 26, and 29 of the Front Cover Plate, read from CT, readings from the offset fragments 21, 23, 27, 37-44, 49, 51, 54-56, and 60, read from both CT and PTMs, and occasional readings from Rehm's 1906 copies of lost portions of offset fragments. The apparatus reports details of the contributions of the individual fragments.

Техі	
1	ὑπολει]πόμενος [
2	μεγίσ]του άποστήμ[ατος
3	] .ον έξ άρχῆς Π[
4	]Σ[]. είς δὲ [τ]ὰ ἑπόμενα. ν ὁ δὲ Φώ[σφορος
5	] ζωιδ[ί]ου, έν δὲ ἴσοις ν υξβL ν ἀποκαταστάσ[εις
6	]ΥΣυζβ, ἑκάστην δ'ἀποκατάστασιν ἐν ἡμέραις φ[πδ
7	]. ΝΑΣ. καὶ ἀπὸ μὲν [τ]ῆς πρὸς τὸν Ἡλιον συνόδου ὑπολε[ίπεται
8	]Ν απόστημα έν ημέραις σκδ. προσάγει δὲ πρὸς τὸν Ἄλ[ιον
9	παρ]αγίνεται έπὶ τὸν [ἐ]σπερινὸν στηριγμὸν, ἀπέχων ἀπὸ το[ῦ Ἡλίου
10	πρ]οσάγει πρὸς τὸν Ἄλιον ἐκ προηγήσεων καὶ σύνοδον Α.Ω.[
11	] έπὶ τὸ μέγιστον ἀπόστημα ἐν ἄλλαις ἡμέραις ν ξη ν [
12	στηριγ]μόν. Σπροηγούμενος, άποστὰς δ'άπ[ὸ τ]οῦ Ἡλίου .Μ[
13	ημέραις μθ ύπολειπόμενος έπὶ τὸ μέγιστ[ον έ]ῶιον ἀπόστη[μα
14	άπο]στήματος προσά[γ]ε[ι] πρὸς τὸ[ν"Ηλιον ὑπ]ολειπόμεν[ος
15	]ΧΗΣΤ.Σ.ΟΑΕΠΙΤΕ.[-7-].Α.ΣΠ[]ΠΟΣΤ.ΣΙΝ.[
16	]ΣΤΑΤΑΙΣ ΒΙΩΝΤΑ
17	έ]κάστην δ΄άποκατάστασιν έν ἡμέραις μικ[ρῶι ἐλάσσοσι
, 18	] Σ. ἄρχεται δέ τὴν ὑπόλειψιν Ν.Μ. Σ ἀπέχω[ν ἀπὸ τοῦ
19	] ἑσπ[ερινοῦ] στηριγμοῦ, καὶ ὑπολείπεται μέχρι τῆς ἑῶιας στά[σεως
20	] _ [ _ ] _ αι ς τμθ ν ἡμέραις σύνοδον ποιεῖται τῶι Ἡλίωι ΜΑΣΗ[
21	]αις τμθ έπὶ τὸν ἑῷιον στηριγμὸν ἀπέχων ἀπὸ τοῦ Ἡλίου ὡς[
22	] Πβ ν καὶ ἐπὶ τὴν ἐσπερινὴν παραγίνεται στάσιν ἀπέχων ἀπὸ [τοῦ
23	] δὲ ἡμέρας ἡ πάλινΕ. ὑπολείπεσθαιέν δὲ ΤΩΑ[
24	]ΝΕστάσιν. ὁ δὲ Φαέθων ἐν ν ἀποκαταστάσ[εις
25	] ἐκάστην δ' ἀποκατάστασιν ἐν ἡμέραις μικρῶι ἐλάσσ[οσι
26	] καὶ δωδεκατημόριον Ω[ ] ἄρχεται δὲ τὴν ὑπόλειψιν[
27	]_ΔΙΟΝ[_]_Ζν άπὸ τοῦ ἑσπερινοῦ στηριγμοῦ καὶ ὑπολείπ[εται
28	]ΕΤΩΝ ἕως ἑν χρόνωι ταῖς ν ρλθ ἡμέραις σύνοδ[ον
29	] ταῖς ἄλλαις ρλθ ν ἐπὶ τὸν ἑῶιον στηριγμόν, ἀπέχ[ων ἀπὸ τοῦ Ἡλίου
30	]_ μείνας ἡμέρας ν η ν προηγεῖται ἡμέρας [
31	] καὶ πάλι μείνας τὰς ἡ ἡμέρας, πάλιν ἈΡ[
32	] _ τῶν ν μδ ν ἡμέραν, γίνεται κατὰ διάμετρον [
33	άποκα]ταστάσεις έν μ[][.] <u>ψμβ</u> ν διαπορευθεὶς τὸν [
34	άпок]ατάστασιν έν [ -9- ] [ -5- ][ . ]
35	τὴ]ν ὑπόλειΨ[ι]ν Π[ -17- ]ΤΟΝ [
36	ὑπολε]ίπεται μέχρ[ι -19- ] [
37	] σ[ύ]ν[ο]δον ποιεῖτ[αι
38	στ]ηριγμὸν ἀῃ[έχ]ω[ν
39	] ἡμέρας [
40	με]ίνας ν η [
41	] κατά διάμ[ετρον
42	]_noot[
43	][

Unplaced fragment (Fragment 42)

- 1 ].[.]A[
- 2 ]ΟΓΙΣΤ[
- 3 ]ΞΔ*ν* K[

Unplaced fragment (Fragment 51)

- 1]<u>N</u>[
- 2 ]NŢK∆[
- 3 ]μείνα[ς
- 4 στηρ]ιγμο.[
- 5 ἡμ]έραις [

#### Apparatus

To indicate which letters are preserved on the various fragments of the inscribed plate (G, 26, and 27) and the accretion layer (all other fragments), the readings of each line from each fragment are reported separately below, with the fragment identified in the second column. "Gs" refers to displaced flakes of inscription adhering to G; "40R" and "43R" are letters of Fragments 40 and 43 read by Rehm in 1906 but no longer extant, and "23i" and "27i" are letters embedded inside Fragments 23 and 27.

1 G ὑπολει]πόμενος [ n: right vertical with serif, right end of horizontal along edge 2 G μεγίσ]του άποστήμ[ατος T: bottom of vertical along edge | o: lower half of loop | u: vertical, possibly a bit of the vee along edge 3 G ] ον έξ άρχῆς Π[ : serifed bottom of vertical? | o: complete but malformed, with straight right side |  $\Pi$ : left vertical, bending left at bottom, and left part of horizontal 4 G ]Σ[\_\_\_\_]ΕΙΣ[-3-] ἑπόμενα.νὸδὲΦώ[  $\Sigma^1$ : complete but blurry | E: traces at baseline, middle, and top height along edge, sigma not excluded | 1: vertical with serif at top; superimposed, an apparent narrow loop, too narrow for phi, seems to be surface damage  $|\Sigma^2$ : entire but distorted |: trace near baseline

Gs ]..[ ...: bottom of vertical, slightly sloping to right at top, possibly met near bottom by descending diagonal from its left; to the right of this, bottom of a vertical with bend (serif?) to right at baseline

along edge | v half letter |  $\omega$ : left horizontal with serif, lower left part of loop along edge

 43
 ]ΣΔΕ[

 Σ: right end of lower horizontal, sloping downwards to right, with serif

 5
 G
 ] ζωιξ[ί]ου, ε[ν] δὲ ισο[ -4- ]βL ν ἀποκαταστάσ[

 $\zeta$ ωιδι indistinct and distorted, near edge | δ: lower right corner of letter, indistinct |  $\sigma$ : serifed right end of lower horizontal and slight trace of right end of upper horizontal

Gs ]ΕΝΔ[ 43 ]ΣΟΙΣ ν Υ[ Σ: bottom half, indistinct | v half letter 43R ]\_O<\_>ΣΥΞ[  $\therefore$  bottom of steeply sloping ascending diagonal at edge Rehm | < .>: Rehm leaves no space for a letter between O and  $\boldsymbol{\Sigma}$ 6 G ]ΥΣ υξβ, ἑκάστην δ'άποκατάστασιν έν ἡμέραις φ [  $\epsilon :$  indistinct, along break  $\mid \rho :$  trace at top level along break 38 ] ἑκάστην δ'[ ε: indistinct | τ: serifed vertical | y: indistinct 43 ]ΟΚΑΤΑΣ[ 40R ]ΤΑΣ[ A: A Rehm ]T[ 40 T: serifed bottom of vertical 7 G ] ΝΑΣ. καὶ ἀπὸ μὲν [τ]ῆς πρὸς τὸν Ἡλιον συνόδου ὑπολε[ : trace near baseline | ΑΣ και: indistinct | α: apical letter | ο: trace of upper right of loop along break | ŋ: right vertical | o: top left part of loop along edge | u: bottom of vertical U: indistinct 55 ]NAΣ[ 38 ά]πὸμὲν τῆς [ ε: top and bottom horizontals | της: indistinct 43 ]TONH[ H: left half of letter 40 ] Ήλιον συν[ η: right half of letter | y: left vertical 8 G ]Ν απόστημα έν ἡμέραις σκ [ ]. προσάγει δὲ πρὸς τὸν Ἡλ[ιον N: bottom of left vertical and bottoms of diagonal and right vertical meeting 55 ]ΠΟΣΤ[ 54 ]AEN[ N: serifed bottom of left vertical 38 έ]ν ἡμέραι[ς v: serifed right vertical | pa: indistinct | 1: serifed top of vertical 43 ]Ų[ ∆: top of apical letter 43R ]∆ v ⊓[ A: descending diagonal along edge Rehm 40 ]οσαγει δε[ G 9 ] γίνεται έπὶ τὸν [ἑ]σπερινὸν στηριγμὸν, ἀπέχων ἀπὸ το[

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: blurry, perhaps a | v: indistinct |  $\epsilon$ : indistinct |  $\omega$ : left half of letter |  $\tau$ : indistinct, along break 55 ]INET[ 54 ] tòv [ 40 ]ΝΑΠΕ[ AΠE: indistinct 10 G πρ]οσάγει πρὸς τὸν [Ἡ]λιον ἐκ προηγήσε ν καὶ σύνοδον Α [ κ: blurry | : indistinct traces | v: complete but distorted | : trace at top level along edge 55 ]ΑΓΕ [ A: faint, along edge | : serifed top of vertical along edge 54 ]TONH[ H: vertical, serifed at top and bottom 56 ]ΗΣ[ H: bottom half of letter 26 ]Ω[ : trace at top level along edge 39 ]ΩΝΚΑΙΣΥ[ 11 G ] έπὶ τὸ μέγιστον ἀπόστημα ἐν ἄλλαις ἡμέραις [ o: left side of loop along break |  $\eta$ : faint |  $\mu$ : right half of letter |  $\lambda$ : faint, indistinct 26 ]Σ v ξηv [ Σ: right part of bottom horizontal | v less than half a letter | v half a letter | . : apical letter (alpha?) but instead of horizontal stroke, a gently ascending diagonal from bottom left to middle of right descending diagonal; to the right, unclear traces, possibly bottoms of two verticals 54 ]IΣ[ Σ: left half of letter 56 ]AE[ 39 ]ΛΑΙΣΗ[ στηριγ]μον. Σ προηγούμενος, ἀποστὰς δ'ἀπ[ὸ 12 G : indeterminate traces |  $\Sigma$ : top and bottom horizontals, speck in center, epsilon or xi not excluded | o: right side of loop along break | o: trace at baseline along break 26 ]οῦ Ἡλίου Μ[ ou: bottoms of letters, blurry | : confused and distorted traces, resembling messy epsilon | M: distorted 56 ]ANO[ A: trace at baseline along edge | Q: left side of loop 39 ]...∆[ ....: blurry, indeterminate traces | ∆: apical letter 13 G ] ἡμέραις μθ ὑπολειπόμενος έπὶ τὸ μέγισ[τον ημερα: blurry and distorted | μ: leftmost and rightmost strokes, sloping; middle of letter is indistinct, straddling break  $| \theta$ : large, somewhat angular loop | u: right ascending diagonal (?) along break | o: top of loop along break  $| \lambda$ : top of apical letter | o: faint loop along edge  $|\mu$ : left vertical and left descending diagonal  $|\sigma$ : trace at top level along edge

26		ἑ]ῶιον ἀπόστη[μα	
27	]YIIO[		
39	]	ΣŢ[	
: blurry, indeterminate trace	s   Σ̣̣̣̣; faint		
14 G áno]от	ήματος π[ρ]οσά [γ]ε[ι] πρὸς τὰ	<u>ې[</u> ٧	
a: left ascending diagonal and to	p of right descending diago	nal of apical letter   Ţ: hori-	
zontal   o: trace at top level			
26		ὑΠ]ολειπόμεν[ος	
27	]προσα[		
15 G ]ΧΗΣΤ.Σ	.O. ΕΠΙΤΕ [ - 7 -] Α ΣΠ[		
X: ascending and descending diag	onals clear; left half of lette	r blurry, kappa also possible	
: indistinct traces on break $  :$ to	p of vertical (?), perhaps iot	a   Ọ: top half of small loop	
$  \cdot$ : traces at top level directly abo	ve trace at baseline, and, to	right, trace of right (?) end	
of serifed descending (?) diagona	l at baseline, and further to	right, descending diagonal	
: apical letter or vertical meeting of	lescending diagonal at top lev	/el   : indistinct   A: distorted,	
doubtful   : indistinct   ∑: distorted			
26		]ΠΟΣΤΣΙΝ.[	
Π: right end of horizontal and top	of right vertical   O: top of	loop   Ț: horizontal, missing	
right end, and top of vertical  : blu	urry, indeterminate traces   : :	trace at top level along edge	
27	]AENITE[		
	ΙΣ ΒΙΩΝΤΑ		
$\Sigma^1$ : top and bottom horizontals, sp			
$  \dot{T}\dot{A} : complete but blurry   \dot{\Sigma}^2 : blur$			
eta but apparently lying low relat			
both loops might not be deliberat			
right vertical   T: left end of horiz	ontal and vertical, along bre	ak  :	
blurry and indistinct traces			
26		]τας.[	
: trace at top height along edge	1 f		
27	]IQNŢĄ[		
TA: faint and uncertain			
	ην δ΄ άποκατάστασιν έν ἡμέρα		
$\kappa$ : faint, indistinct traces   $a^1$ : blurred   $v$ : distorted   $\delta$ : top of apical letter and faint trace			
of bottom right corner			
27 18 G I I		N NA 5 4 4 4 4 4 4	
	άρχεται δέ τὴν ὑπόλειψιν		
$^{1:}$ indistinct trace near edge   $^{1:}$ indistinct traces near edge   $\delta$ : apical letter, straddling break   $_{0:}$ slight traces of tops of diagonals, straddling break   $^{2:}$ resembling distorted			
epsilon, with middle horizontal too high, followed by indistinct trace along edge $ _2$ : in-			
distinct trace $ $ M: presumed right half of letter faint and indistinct $ _{,3}$ : blurry traces $ $ $\omega$ :			
horizontal at baseline, along edge			
27	]ΕΤΑΙΔ[		
<i>L1</i>	ן בוואו שנ		

19 G ] ἑσπ[ερινοῦ] στηριγμοῦ, καὶ ὑπολείπεται μέχρι τῆς ἑῶιας στά[σεως 27 ]MOYKA[ M: right half of letter | O: traces of left and right sides of loop | Y: tops of diagonals and vertical 20 G [ ] αις τμθ ν ἡμέραις σύνοδον ποιεῖται τῶι Ἡλίωι ΜΑΣΗ[ 1: indeterminate traces along edge | . . <sup>2</sup>: bottoms of two verticals | 1<sup>1</sup>: blurry | u: faint | o: distorted  $|1^2$ : lower part of vertical  $|\lambda$ : left ascending diagonal and serif of right descending diagonal |  $1^3$ : blurry |  $\omega$ : left half of letter, faint |  $1^4$ : fat and blurry | H: faint but distinct ] v HME[ 27 : indeterminate trace at edge ]ME[ ] [ 27i : serif at baseline 37 ]H A [ HA: complete but indistinct G ] αις τμθ έπὶ τὸν ἑῷιον στηριγμὸν ἀπέχων ἀπὸ τοῦ Ἡλίου .ς....[ 21  $\overline{\theta}$ : faint and blurry |  $\underline{\tau}^1$ : horizontal and top part of vertical, near breaks |  $\epsilon\omega_1$  indistinct traces | T<sup>2</sup>: horizontal, and small trace of vertical along break | y: trace of left vertical near break | a<sup>1</sup>: indistinct | a<sup>2</sup>: faint left diagonal along break, and bottom tip of right diagonal : indistinct : faint and indistinct traces 27i ]HP[ 37 ]ΛΙΟΥΩΣ[  $\Lambda$ : faint |  $\Sigma$ : bottom left corner 22 ] πβ ν καὶ ἐπὶ τὴν ἐσπερινὴν παραγένεται στάσιν ἀπέχων ἀπὸ [ G β: small loop with a blurry extension above | 1: blurry | ηγε: blurry | 1: trace of vertical along break | 1: badly formed or blurred, appearing like a very narrow epsilon |  $\omega$ : blurry 37 ]NAREX[ 23 G ] δὲ ἡμέρας η πάλιν Ε ὑπολείπεσθαι. ἐν δὲ ΤΩ Α[ δ: apical letter | ἡμέρα: very faint | . . : faint, indeterminate traces | ŋ: faint | . . . : apical letter (?); to the right of this, apparently the left half of nu or mu, and further right, faint indeterminate traces | E: vertical and top and middle horizontals | : small trace at top height along break | u: blurry, near break | 1: blurry | n: small traces straddling break |  $\theta$ : indeterminate traces | 1: top part of vertical |  $\delta$ : apical letter |  $\Omega$ : left half of letter and faint right horizontal | ... : traces of three verticals with two faint horizontal or slightly descending diagonal strokes joining them at mid height; to the right of this, seemingly Z A: sloppy, with apparent superfluous stroke crossing end of right descending diagonal 37 ]ΝΔΕΤΩ[ 24 G ]ΝΕ... στάσιν. ο δε Φαέθων έν ν.... άποκαταστάσ[εις : indeterminate traces | 1: blurry and faint |  $\phi$ : loop along break | a: apical letter staddling break | u: blurry, straddling break | v: one letter | : faint traces near a break, suggestive of omega | . . . . : leftmost, faint traces near a break, suggestive of omega; then a coarsely damaged letter, apparently chi or sigma; then traces resembling a sloppy mu;

then a vertical with either three horizontals or two loops to its right, i.e. epsilon or beta;

then traces suggestive of mu or chi, faint towards the top | tag: faint, indeterminate traces 37 ]ΑΠΟΚΑ[ 25 G ] ἑκάστην δ'ἀποκατάστασιν ἐν ἡμέραις μικρῶι ἐλάσσ[οσι n: both vertical strokes, between which blurry traces more suggestive of nu | v: blurry | o: blurry | v: trace of top of left vertical and trace of bottom of right vertical straddling break | 1: blurry 60 ]KA[ K: bottom half of vertical with serif, descending diagonal, and trace of left end of ascending diagonal along edge | 49 ]ΣΙΝ[ 21 ]HME[ E: vertical along edge 37 ] [KPΩ[ : trace at baseline along edge | !: indistinct 41 ]ΩIE[ Ω: serifed right horizontal along baseline | !: serifed bottom of vertical 26 ]... καὶ δῷδεκατῃμόριον Ω[.] ἄρχεται δὲ τὴν ὑπόλẹ಼ψιν [ G : indeterminate traces |  $\omega^1$ : faint and indistinct |  $\Omega^2$ : faint and angular |  $\mathfrak{a}$ : traces of apex and bottoms of both diagonals straddling break |  $\varepsilon_1$ : faint |  $\psi$ : indistinct |  $\psi$ : left (?) vertical 60 ]THM[ 49 ]NΩ[ N: right vertical with serif, faint trace of diagonal  $| \Omega$ : left half, indistinct 21 ]APXET[ A : trace at baseline along edge 44 ]! [ 1: serifed bottom of vertical 1: trace at baseline along edge 37 ]N [ N: indistinct | : trace of descending diagonal at top level 41 ]ΥΠΟΛ[ ].ΔΙΟΝ[.].Ζν άπὸ τοῦ ἑσπερινοῦ στηριγμοῦ καὶ ὑπολείη[εται 27 G  $^{1}$ : indeterminate traces |  $\Delta$ : blurry |  $^{2}$ : indeterminate traces | v: one letter | ug: blurry | п: left vertical 23i ]Z v A[ v: one letter 60 ]ΥΕΣ[ 49 ]N[ 21 ] στηριγ [ γ: vertical along edge ]ΓΜΟΥΚΑ[ 44 41 ]ι ὑπολε[ |: vertical along edge, indistinct  $| \varepsilon$ : indistinct 28 G ].....ΕΤΩΝ ἔως έν χρόνωι ταῖς ν ρλθ ἡμέραις σύν[ο]δ[ον

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.....: faint and indistinct traces | Ν: blurry, straddling break | εω: blurry, straddling breaks | o: blurred | v: complete but distorted | T: vertical, faint horizontal | v: half a letter η: distorted 21 ]ΑΙΣν ρλθ [ v: half a letter 44 ]HMEPA[ A: top of ascending diagonal along edge 41 ]ΣΣΥΝΟ[ 29 G ] ταῖς ἄλλαις ρλθ ν ἐπὶ τὸν ἑῶιον στηριγμόν, ἀπέχ [ων q: indistinct |  $v^1$ : one letter |  $\theta$ : faint loop |  $v^2$ : one letter |  $\varepsilon$ : upper right corner along break | 1: distorted, indistinct | v: left vertical; remainder blurry |  $\chi$ : faint 23 ]Σ ἄλλαις [ 23i ]ΙΣ ἄλλαις [ 21 έ]ηὶ τὸν ἑῶιον στηρ[ n: right vertical, curving rightwards at bottom | 1: indistinct | 2: horizontal at baseline along edge 44 ]ILW [ M: sharp vertex at top level along edge ] NE [ 41 : top of descending diagonal at top level along edge ] μείνας ἡμέρας ν η ν προηγεῖται ἡμέρας [ 30 G : indistinct and faint |  $\varepsilon_1$ : indistinct traces near break | |  $\eta^1$ : blurry |  $\eta^2$ : blurry, traces in middle resembling nu 23 ] μείνας Η[ μεινα indistinct 23i μ]είνας ΗΜ[ a: faint | c: right end of top horizontal | M: left vertical and top of descending diagonal 21 ήμ]έρας ν η ν προηγεῖται ήμ[ ε: serifed right end of bottom horizontal along edge | ρ: serifed bottom of vertical |a: serifed letter, indistinct |c: horizontal at baseline, bending downwards towards right, with serif at right end  $|v^1$ : one letter  $|v^2$ : one letter  $|\mu$ : trace at baseline along edge 31 G ] καὶ πάλι μείνας τὰς η ἡμέρας, πάλιν ΑΡ[ . . : faint, indistinct traces |  $\kappa$ : blurry, straddling break | a: indistinct, straddling break |  $v^1$ : half a letter |  $\eta$ : blurry |  $v^2$ : half a letter | vA: faint 23 ]....καὶ πάλι Μ[ .....: indistinct | 1: indistinct | 1: right half of letter 23i ]K[ ]I[ ]IM[ K: bottom half of letter 21 μ]είνας τὰς ν η ν ἡμέρας ΠΑ[  $v^1$ : half a letter |  $v^2$ : half a letter 32 ] Ν ν ρδ ν ἡμέραν, γίνεται κατὰ δι[άμ]ετρον [ : faint traces |  $v^1$ : half a letter |  $v^2$ : one letter | o: indistinct

```
 23 ]. τῶν ν ρδ ν ἡμέρα [
 : vertical along edge | v: indistinct | α: left and right diagonals
```

23i ]N ν ΡΔ[

v: half a letter

- 21 ] γίνεται κατὰ διάμετ[ρον
- Ţ: faint

```
33 G ἀποκα]ταστάσ[ε]ις...[..].[] <u>ψμβ</u>νδιαπο[ρευ]θεὶς τὸν [
```

ra: faint | . . .: blurry, indeterminate traces | . .: indeterminate top of letter, then top of serifed vertical | u: vee, possible trace of vertical along break | v: one letter | o: left and upper parts of loop | 1: trace of top of vertical | roy: faint

23 άποκα]ταστάσεις ένμ[

 $\mu$  : vertical, sloping to right, with serif or short descending diagonal stroke meeting it at top level, along edge

23i ]ΣΤΑΣΕ[

21 ]\_B[ -6- ]PEYO[

: sharp apex at top height along edge

34 G άποκ]ατάστ[ασ]ινέν[-17-]....[

 $\mathfrak{q}$ : faint, indeterminate traces |  $\mathfrak{T}$ : trace at top level along edge | ... : indeterminate traces along edge | ... : traces at baseline and top height along edge; to the right of this, a vertical, then the bottom of an ascending diagonal | ... <sup>2</sup>: faint, indeterminate traces

29 ]ŢA[

T: trace at baseline

23 άποκ]ατάστασιν έν [

ɛ: blurry | v: vertical along edge

```
23i ]ΣΤΑ[...]E[
```

A: serifed lower part of ascending diagonal

```
35 G ]<u>NY</u>[-18-]TO<u>N</u>[
```

NY: top halves of letters | : vertical

29 ]non<sub>[</sub>

: blurry trace along edge

```
23 ]Ν ὑπολε..[
```

```
23i ]ΥΠΟΛ[..].[.]ΝΠ[
```

: serifed bottom of vertical, slightly below baseline

36 G ]..[

. : indeterminate traces of tops of letters along edge

29 ]neta[

23 ]<u>n</u>ețaim[

 $\label{eq:rescaled} \ensuremath{\mbox{IPET}}\xspace: indistinct \mid \ensuremath{\mbox{M}}\xspace: indistinct \mid$ 

23i ]INET[...]EXP[

37 29 ] σ[υ]ν[ο]δον π[

 $\sigma$ : right end of bottom horizontal | y: bottoms of both verticals |  $\delta$ : trace of right descending diagonal along edge | o: obscured by extraneous marks

23 ]δον ποιε[

o: indistinct

23i

23

]∆ON.[...]EIT[

: vertical

39

38 29 στ]ῃριγμὸν ἀҧ[έχων

 $\eta\colon$  indistinct vertical  $|\:\gamma\colon$  indistinct vertical  $|\:\mu\colon$  one sharp apex  $|\:\eta\colon$  left vertical and left end of horizontal

]ÒNÀ[

ova: doubtful traces of tops of letters

23i ]ONA[\_\_\_]Ω[

29 ] ἡμέρας [

23i ]ç[

 $\boldsymbol{\varsigma} \colon \text{serifed right end of top horizontal}$ 

40 29 με]ívaς v n [

41 29 ] κατὰ διάμ[ετρον

 $\kappa :$  traces at baseline and top level along edge |  $\mu :$  trace at baseline

42 29 ].noot[

: vertical stroke leaning slightly rightward at top (an accidental feature?), also faint trace as of an apical letter superimposed

43 29 ]...[

: indeterminate traces of tops of letters

Unplaced Fragment 42

1 : faint apparent lower part of vertical with serif, and to the right of this, a descending diagonal with large serif, possibly kappa | 2  $\Omega$ : apparent right arc and speck of lower left of small elevated loop |  $\Gamma$ : complete, but epsilon cannot be ruled out

Unplaced Fragment 51

1 N: verticals certain, blurry trace at mid height between them, possibly H | : indistinct trace 2 T: apparently complete in CT, but the PTM suggests  $\Sigma$  |  $\Delta$ : left half of the letter, with apparent horizontal stroke at baseline

3 : indistinct trace

4 μ<u>o</u>: very indistinct traces

5  $\epsilon$  : small, unidentifiable trace |  $\varsigma$  : traces of left ends of horizontals at top height and baseline

Tron	nslation
11 ai	] regressing [
2	greatest] elongation [
2	j the initial [
4	] in the following direction. Pho[sphoros
5	] zodiacal sign (?), in equal 4[6]2 years, restitutions [
6	] 462, and each restitution in 5[84] days [
7	] And after the conjunction with the Sun it regresses [
8	greatest] elongation in 224 days. It approaches the Sun [
9	] it arrives at the evening station, being distant from the [Sun
9 10	] it approaches the Sun by way of advances, and conjunction [
10	] to the greatest elongation in another 68 days [
12	morning] station advancing. Standing away from the Sun[
13	] in 49 (?) days regressing to the greatest morning elongation [
14	greatest] elongation it approaches the Sun regressing [
15	
16	][]
17	ی ا ] each restitution in a little less than [] days [
18	] It begins the regression being distant [from the Sun
19	] evening station, and it regresses as far as the morning stopping [
20	] 349 days it makes a conjunction with the Sun [
21	] 349 [days] to the morning station, being distant from the Sun [
22	] 82 and it comes to the evening stopping, being distant from [the Sun
23	]8 (?) days, again to regress. In [
24	] stopping. Phaethon [makes] restitutions [
25	] each restitution in a little less than [] days [
26	] and a twelfth part It begins the regression [
27	] []7 from the evening station and regresses [
28	] in a time interval, 139 days, conjunction [
29	] in another 139 [days] to the morning station, being distant [from the Sun
30	] after pausing for 8 days it advances for [] days [
31	] and again after pausing for the 8 days it again begins (?) [
32	] day of the 104 it comes to be diametrically opposite [
33	] restitutions in 442 having traversed the [
34	each] restitution in[
35	] the regression
36	] it regresses as far as far as [
37	] it makes a conjunction [
38	] station, being distant [from the Sun
39	] days [
40	] after pausing 8 [days
41	] diametrically opposite [
42	] [
43	] [

# 6.5 Commentary

### Synodic cycles, period relations, and terminology

A planet's synodic cycle is the periodic cycle of its apparent longitudinal motion relative to the Sun as observed from the Earth. From the point of view of ancient astronomy, we can distinguish three kinds of events, or "phases" that repeat in a fixed order in a planet's synodic cycle. Considering the planet's elongation from the Sun, the delimiting moments are the conjunctions, oppositions (only possible for the superior planets Mars, Jupiter, and Saturn), greatest elongations (only possible for the inferior planets Mercury and Venus), and first and last visibility. Secondly, the stationary points delimit the intervals of a planet's motion in the directions of increasing longitude ("direct" in modern terminology) and decreasing longitude (modern "retrograde"). The sequence of phases other than first and last visibility is as follows:

Superior planets	Inferior planets
Conjunction	Superior conjunction
Morning station	Greatest evening elongation
Opposition	Evening station
Evening station	Inferior conjunction
Conjunction	Morning station
	Greatest morning elongation
	Superior conjunction

First visibility occurs shortly after conjunction, and last visibility shortly before conjunction; however, in the case of Mercury the morning station may take place before first visibility and the evening station after last visibility.

Because the orbits of the Earth and the other planets are eccentric, neither the time intervals between successive phases nor the durations of complete synodic cycles (from any phase to the recurrence of the same phase) are constant. In both Babylonian and Greek mathematical astronomy, a common means of expressing the long-term behavior of the planets' synodic cycles was a period relation of the following form:

 $\Pi$  synodic cycles = Y years = Z revolutions of the planet around the ecliptic

which implies that after a constant period of Y years, the planet will return simultaneously to its original longitude and to its original configuration relative to the Sun.<sup>20</sup> For an inferior planet, Y and Z are equal, whereas for a superior planet  $Y = \Pi + Z$ . The mean synodic period is thus:

which can be expressed in days by multiplying the quotient by the assumed length of the year.  $^{\mathbf{21}}$ 

The Front Cover Inscription employs terminology for the synodic cycles and their phases that is mostly well known from Greek astronomical texts. In our translation we have used literal renderings rather than interpretations according to modern terminology. In particular we have respected the Greek conventions according to which longitudinal motion in the direction of the daily revolution of the heavens, i.e. westward, is characterized as forward motion and eastward motion is backward, which are the reverse of the modern nomenclature of "direct" and "retrograde." The following list sets out the technical terms, their literal meanings as given in the translation, and the modern interpretations. Those marked with an asterisk are unusual as technical terms.

Pertaining to synodic phases ἀποκατάστασις = "restitution" = synodic cycle (*literally* restitution) σύνοδος = conjunction κατὰ διάμετρον = "diametrically opposite" = opposition μέγιστον (ἐῷον/ἐσπερινὸν) ἀπόστημα = greatest (morning/evening) elongation ἑῷος/ἐσπερινὸς στηριγμός = morning/evening station ἑῷα/ἐσπερινὴ στάσις\* = "morning/evening stopping" = station

Pertaining to longitudinal motion ὑπολείπεται = "regresses" = increases in longitude ὑπολειπόμενος = "regressing" = increasing in longitude ὑπόλειψις = "regression" = direct movement

είς τὰ ἑπόμενα = "in the following direction" = eastwards

προηγεῖται= "advances" = decreases in longitude προηγούμενος = "advancing" = decreasing in longitude προήγησις= "advance" = retrograde movement

Pertaining to motion relative to the Sun  $npo\sigma \dot{\alpha}\gamma \epsilon i^* = "approaches" = decreases in elongation$  $<math>\dot{\alpha}n\dot{\epsilon}\chi\omegav^* = "being distant" = having elongation$  $\dot{\alpha}no\sigma\tau\dot{\alpha}\varsigma^* = "standing away" = having elongation$ 

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<sup>21</sup> In Babylonian and earlier Greek astronomy, no distinction was made between sidereal and tropical years. The Callippic intercalation cycle implies a year of exactly 365<sup>1</sup>/<sub>4</sub> days.

#### Models for synodic cycles

Babylonian mathematical astronomy employed arithmetical algorithms to model the intervals of time and longitudinal motion between successive phases.<sup>22</sup> These algorithms were derived from empirical data without assumption of an underlying geometrical model for the planet's motion. While Babylonian-style models were known and practiced in the Greek-speaking world, at least from the first century AD onwards,<sup>23</sup> the "main stream" of Greek planetary theory that culminated in Ptolemy assumed geometrical models based on a combination of two circular motions, one of a center (C) revolving around the Earth (0), the other of the planet (P) revolving around this moving center (Figs. 6.10-6.11). In the simplest form, such a model has the Earth at the geometrical center of the circular path of C, while C is invariably the geometrical center of the path of P, and both revolutions are performed at a uniform angular velocity relative to their centers. If the radius of P's path is less than that of C's path, then P's path, which does not enclose the Earth, is called an "epicycle", and C's path is called the "deferent." If, however, P's path encloses the Earth, it is called an "eccenter". Since the resulting motion of P relative to 0 is the sum of two uniformly revolving vectors, any simple epicyclic model is observationally equivalent to a simple eccentric model with the radii and associated rates of revolution exchanged, and vice versa.

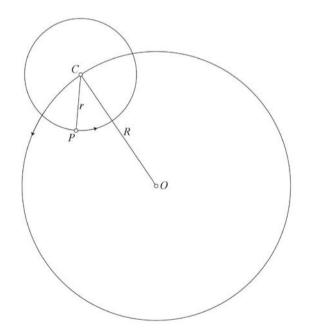


Figure 6.10: Simple epicyclic model for a planet

<sup>22</sup> Neugebauer 1955, 2.279-315.

<sup>23</sup> Jones 1998.

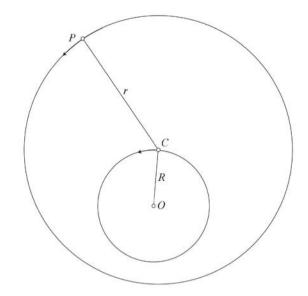


Figure 6.11: Simple eccentric model for a planet

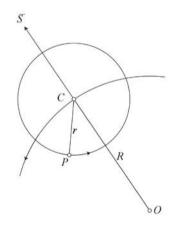


Figure 6.12: Epicyclic model for an inferior planet

In the following discussion we will employ epicyclic models. An epicyclic model for an inferior planet must satisfy the condition in the period relation that Y = Z, as well as the stronger constraint that the planet has one greatest elongation in either direction of the Sun in each synodic cycle. This requires that radius *OC* is aligned with the mean Sun ( $\overline{S}$ ) so that its period of revolution is one year, while the period of revolution of *P* around *C*, relative to the geocentric radius *OC*, is the mean synodic period *p* (Fig. 6.12). On the other hand, for a superior planet the rate of revolution of *C* around *O* is independent of the mean Sun, but the constraint that opposition always occurs between the morning and evening stations means that radius *CP* must always be parallel to the direction of the mean Sun *O* $\overline{S}$  (Fig. 6.13). Hence the period of revolution of *P* around *C*, relative to radius *OC*, or in other words the mean synodic period *p*, is:

where  $\overline{v}_s$  is the rate of revolution of the mean Sun,  $\overline{v}_c$  is the rate of rotation of  $\mathcal{OC}$ , and  $\overline{v}_p$  is the rate of rotation of  $\mathcal{CP}$  relative to  $\mathcal{OC}$ . In all epicyclic models for a planet, to obtain satisfactory representation of the retrogradations it must be assumed that the planet revolves in the same sense around its epicycle as the epicycle revolves around the Earth, so that retrogradations occur when the planet is nearest to the Earth.

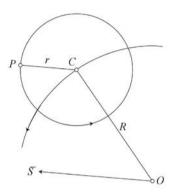


Figure 6.13: Epicyclic model for a superior planet

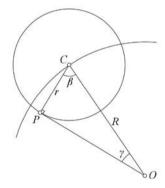


Figure 6.14: Greatest elongation of an inferior planet

According to the epicyclic model, the time interval between superior and inferior conjunction of an inferior planet or between conjunction and opposition of a superior planet is obviously p / 2. The time interval between the conjunctions and the greatest elongations of an inferior planet, as well as the actual arcs of maximum elongation, can easily be derived by trigonometry from the period relation and the assumed ratio of the epicycle's radius (r) to the deferent's radius (R). In Fig. 6.14 we have for the maximum arc of elongation,  $\Delta \lambda_{cc}$ :

 $\Delta \lambda_{\rm GF} = \gamma = \arcsin(r / R)$ 

while the time interval between inferior conjunction and greatest elongation is:

 $t_{\text{GE}} = (\beta / 360^\circ) p = (\arccos (r / R) / 360^\circ) p$ 

Conversely, these relations allow one to derive r / R from a given maximum arc or time interval.

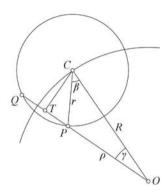


Figure 6.15: Apollonios's theorem determining the stationary points of a planet

For the stations, Ptolemy (*Almagest* 12.1) provides a theorem that he says was demonstrated by Apollonios of Perge among others; it is usually inferred that Apollonios discovered it.<sup>24</sup> In Fig. 6.15, the planet *P* is at its station; line *OP* is produced to meet the epicycle again at Q, and PQ is bisected at *T*. Apollonios's theorem states:

$$OP / PT = \overline{v}_{p} / \overline{v}_{c}$$

where, as before,  $\overline{v_{\rho}}$  is the rate of revolution of the planet around the epicycle relative to radius OC and  $\overline{v_c}$  is the rate of revolution of C around O. From this it follows that we can calculate the time interval between the planet's inferior conjunction or opposition and its station thus:

$$t_{\rm STN} = (\beta / 360^{\circ}) p$$

where:

$$\beta = \arcsin[(\rho + \overline{v}_{\rho} / \overline{v}_{c}) / R] - \arcsin[(\overline{v}_{\rho} / \overline{v}_{c})] / r)$$

$$\rho = \sqrt{\left[ \left( R^2 - r^2 \right) \left( \overline{v}_p \, / \, \overline{v}_c \right) \, / \, \left( 2 + \overline{v}_p \, / \, \overline{v}_c \right) \right]}$$

For the inferior planets, the elongation from the Sun at station,  $\Delta \lambda_{_{\rm STN'}}$  is:

$$\Delta \lambda_{\rm STN} = \gamma = 90^{\circ} - \arcsin[(\rho + \overline{v}_{\rho} / \overline{v}_{c}) / R]$$

24 Neugebauer 1975, 1.191-193.

For the superior planets, however:

$$\Delta \lambda_{\rm STN} = 180^\circ - \beta - \gamma = 90^\circ + \arcsin[(\overline{v}_{\rho} / \overline{v}_{\rho}) / r]$$

Though considerably more complicated than the calculations for greatest elongation, the derivation of  $t_{\text{STN}}$  was carried out for all the planets by Ptolemy (*Almagest* 12.2-6), so it was in principle within reach of any diligent astronomer who had the necessary trigonometrical resources. Such resources existed from Hipparchos's time if not earlier.<sup>25</sup>

For reference, we have calculated  $t_{\text{STN}}$  and  $\Delta n_{\text{STN}}$  for all planets and  $t_{\text{GE}}$  and  $\Delta n_{\text{GE}}$  for the inferior planets, assuming Ptolemy's value for r (top row) as well as a range of values surrounding Ptolemy's value; in all cases, R = 60 following Ptolemy's convention. To obtain the times between the respective phases and superior conjunction (inferior planets) or conjunction (superior planets), one subtracts the tabulated times from p / 2.

Mercury	( <i>p</i> ≈	115.88d)
---------	--------------	----------

r	t <sub>stn</sub>	$\Delta \lambda_{_{\rm STN}}$	t <sub>ge</sub>	$\Delta\lambda_{_{\rm GE}}$
22.5	11.24	17.23°	21.88	22.02°
19	9.58	12.23°	23.03	18.46°
20	10.20	13.73°	22.70	19.47°
21	10.69	15.17°	22.38	20.49°
22	11.08	16.55°	22.05	21.51°
23	11.39	17.89°	21.71	22.54°
24	11.63	19.21°	21.38	23.58°
25	11.82	20.51°	21.04	24.62°

Venus ( $p \approx 583.92d$ )

r	t <sub>stn</sub>	$\Delta \lambda_{\rm STN}$	t <sub>ge</sub>	$\Delta \lambda_{\rm GE}$
43 <sup>1</sup> / <sub>6</sub>	20.90	28.24°	71.35	46.01°
40	16.61	19.02°	78.16	41.81°
41	18.41	22.17°	76.06	43.10°
42	19.75	25.06°	73.92	44.43°
43	20.76	27.80°	71.72	45.78°
44	21.48	30.42°	69.48	47.17°
45	21.97	32.97°	67.17	48.59°
46	22.26	35.47°	64.79	50.06°

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Mars	( <i>p</i> ≈	779	.94d)
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r	t <sub>stn</sub>	$\Delta \lambda_{\rm STN}$
39.5	36.50	135.87°
36	30.34	146.83°
37	32.69	143.27°
38	34.52	140.09°
39	35.93	137.22°
40	36.98	134.58°
41	37.75	132.13°
42	38.25	129.83°
43	38.51	127.66°
44	38.56	125.59°
45	38.42	123.62°

Jupiter ( $p \approx 398.88d$ ) Saturn ( $p \approx 378.09d$ ) r t<sub>stn</sub>  $\Delta \lambda_{\rm STN}$ r t<sub>stn</sub>  $\Delta \lambda_{STN}$ 11.5 60.25 115.68° 65/6 69.86 107.24° 10 57.25 120.04° 6 68.10 109.76° 10.5 118.43° 6.25 68.70 58.41 108.93° 11 59.41 116.98° 6.5 69.24 108.17° 11.5 60.25 115.68° 6.75 69.71 107.47° 12 60.97 114.49° 7 70.13 106.81° 12.5 7.25 61.57 113.40° 70.50 106.21° 7.5 13 62.09 70.83 112.41° 105.65°

A simple epicyclic (or eccentric) model for a planet obviously generates constant and invariable synodic cycles with respect to conjunctions, oppositions, stations, and greatest elongations.<sup>26</sup> (Visibility phases are effected by the varying angle between the ecliptic and the horizon as well as meteorological conditions.) Modifying the model by displacing the Earth (now *T*) from the center *O* of the deferent (Fig. 6.16) results in varying synodic cycles while maintaining the long-term period relation. With a suitable eccentricity, such an eccenter-and-epicycle model can reproduce reasonably well the variations in the time intervals as well as the planet's total longitudinal progress from one occurrence to the next of the same phase. However, an eccenter-and-epicycle model calibrated to fit the overall durations and longitudinal progresses of the synodic periods will give a poor representation of the planet's apparent velocity when, according to the model, it is nearest to the Earth, and as a result it models the retrogradations poorly, conspicuously so in the case of Mars. This defect can be remedied quite effectively by introducing an "equant"

<sup>26</sup> For the effects of adding eccentricity and equant to an epicyclic planetary model as discussed in this paragraph see Evans 1984.

point *E*, distinct from *O* and such that *E* and *T* are equidistant from *O* in opposite directions (Fig. 6.17); the equant functions as the center of uniform revolution of *C*, i.e. the radius *EC* has a uniform rate of revolution.

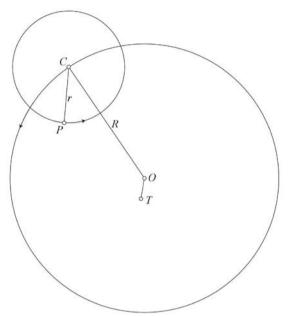


Figure 6.16: Eccenter-and-epicycle model for a planet

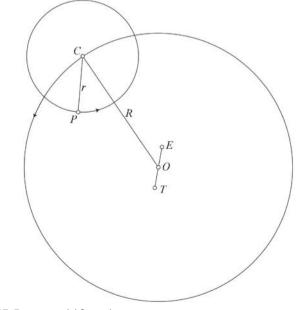


Figure 6.17: Equant model for a planet

The association of the theorem on stations with Apollonios is strong evidence that either simple epicyclic or simple eccentric modelling had been applied to the planets by the early 2<sup>nd</sup> century BC<sup>27</sup> Another passage in Ptolemy's *Almagest* (9.2) asserts that Hipparchos wrote a work in which he criticized the mathematical astronomers up to his time for working with geometrical models that did not allow for variation in the synodic cycles; this would imply that only simple models were current around the third quarter of the second century. Pliny the Elder (died AD 79) gives a confused account of planetary theory (*Hist. Nat.* 2.56-80) which contains our earliest evidence for models incorporating an eccentricity to explain synodic variations. Finally we arrive at Ptolemy, who employs equant models and is usually supposed to have introduced them, though this has been questioned.<sup>28</sup>

## Implications for the Mechanism

The idea that the Mechanism had some kind of planetary display goes back to the earliest investigations of the fragments. Various suggestions have been offered as to the nature and level of astronomical sophistication of the display:

*Display of planets' mean motion in longitude.* This would be a mechanically straightforward translation of the input drive by way of gear trains into uniform rates of longitudinal motion appropriate for each planet according to a suitable period relation; the natural place for the display would be the central front dial, with pointers standing for each of the planets along with the Sun and Moon. Aside from the period relation, a display of mean motion would not embody any specific planetary model. Rehm's unpublished reconstructions seem to be of this kind,<sup>29</sup> and it seems that Price supposed that a display of planetary longitudes, if there was one, would show mean motions only.<sup>30</sup> As Neugebauer pointed out, the mean motions of the inferior planets coincide with the mean Sun, so that it is hard to see how they could have had separate pointers.<sup>31</sup>

28 Duke 2005.

<sup>27</sup> For arguments for an early second century date for Apollonios see Toomer 1970; Evans G Carman 2014 show that the evidence could also be compatible with a late third century date. We are not persuaded by Goldstein 2009 that Ptolemy's testimony and its implications for Apollonios's knowledge of epicyclic or eccentric models should be disregarded. As Toomer (1984, 556, note 3) points out, however, Ptolemy does not assert that Apollonios operated with both kinds of model and was conversant with their interchangeability, contrary to Neugebauer 1959.

<sup>29</sup> Diagrams of hypothetical mechanism in Rehm 1906a, 92-93 and Rehm 1906b, drawings accompanying pp. 16 and 18.

<sup>30</sup> Price 1974, 59-60.

<sup>31</sup> Neugebauer 1975, 652, note 7.

*Chronological display of planets' synodic cycles.* Gear trains could also translate the input motion into displayed revolutions of synodic cycles. A single revolution of a pointer could represent a complete synodic cycle, and graduations and inscriptions around the dial could mark the dates of the synodic phases. Such a display could only represent a model according to which the synodic cycles are constant and unvarying; the subdivision of the cycle could be derived from a simple epicyclic or eccentric model, an arithmetical scheme, or unmediated empirical evidence. One of Price's vague expressions about planetary displays seem to be along these lines.<sup>32</sup> A reconstruction involving five subsidiary dials on the Mechanism's front face, one for each planet's cycle, has been offered by Evans, Carman, and Thorndike.<sup>33</sup>

Display of planets' motion in longitude according to a model assuming an invariable synodic cycle. This is the assumption underlying reconstructions of planetary displays by Wright, Edmunds and Morgan, Freeth and Jones, and Carman and Evans (in a proposal distinct from the one cited in the preceding paragraph), and apparently also Theofanidis's reconstruction.<sup>34</sup> Again, the front dial is the obvious place for a set of planetary pointers. All the reconstructions of this kind known to us employ devices involving pins mounted on gears and riding in hinged slots to effect an anomalistic motion; these are translatable into theoretical models of the simple epicyclic or eccentric type, though the kinematic equivalence is not always immediately obvious.

Display of planets' motion in longitude according to a model assuming varying synodic cycles. Wright has also suggested that a display embodying an eccenter-and-epicycle model could also be achieved within the constraints of the Mechanism's known features,<sup>35</sup> and one of his physical models incorporates a working reconstruction of the display for Mars assuming an eccentric deferent.<sup>36</sup>

It has been argued elsewhere that the Back Cover Inscription's description of the Mechanism's front face establishes beyond plausible doubt that there was in fact a display involving all five planets known in antiquity, and further, that the display consisted of a system of pointers on the central dial to indicate the planets' longitudes along the Zodiac Dial.<sup>37</sup> As we have written above, we believe that the only reasonable interpretation of the Front

<sup>32</sup> Price 1959, 65.

<sup>33</sup> Evans, Carman, & Thorndike 2010, 22-24.

<sup>34</sup> Theofanidis 1934; Edmunds & Morgan 2000; Wright 2002; Freeth & Jones 2012; Carman, Thorndike, & Evans 2012.

<sup>35</sup> Wright 2009.

<sup>36</sup> Personal communication (June 4, 2014). This is the second model referred to in Wright 2013, 9 note 4.

<sup>37</sup> See IAM 5.5, following Freeth & Jones 2012.

Cover Inscription is as a delineation of astronomical "facts" displayed by the Mechanism in action. On this basis we can rule out the notion that only mean motions of the planets were displayed, since the inscription carefully describes stages of forward and backward motion for each planet as well as Venus's varying speed relative to the Sun. On the other hand there is no indication that the synodic cycles or their constituent stages were variable in duration; specific numbers of days are allotted to each stage, and near the beginning of each planet's section was an explicit statement that each synodic cycle contained a stated number of days. A compelling case thus emerges for the third type of display in our list, one kinematically equivalent to a system of simple epicyclic (or eccentric) models.

A different point of divergence among recent discussions of the Mechanism's planetary display concerns the underlying period relations. Several proposals have favored relations equating fairly small numbers of years and synodic cycles, such as the Babylonian "Goal Year" periods which all are shorter than a century, both because short periods could be represented by simpler systems of gears having plausible tooth counts, and because the evidence for Greek knowledge of long and accurate planetary periods, such as were assumed in Babylonian mathematical astronomy, is slender before the first century AD.<sup>38</sup> By contrast, Wright has constructed his conjectural working models of the Mechanism's planetary display using very long period relations that maintain a long-term accuracy of about a degree's error in 500 years or better.<sup>39</sup> Such a period relation, to be viable as gearwork, must contain numbers of years and synodic cycles that can be reduced to factors small enough to be possible as tooth counts or factors of tooth counts; its mechanical representation then becomes a gear train involving multiple pairs of engaged gears.<sup>40</sup> Wright has given one motivation for using these accurate period relations as his desire to show the physical practicability of a planetary display representing the high end of the knowledge that can plausibly be ascribed to astronomers at the time of the Mechanism's manufacture, but he also has maintained that it is not merely possible but indeed probable that the designer would have known and sought to mechanize planetary periods comparable to those of the Babylonian mathematical models.

No complete statement of a period relation is preserved in the Front Cover Inscription, but fortunately it is sufficient to have just one of the constituent numbers in order to reconstruct the equation since the ratios of the terms are approximately known. In line 6, within the formula setting out the period relation for Venus, the number 462 is well preserved, and in line 42, within the corresponding formula for Saturn, we have the num-

<sup>38</sup> Edmunds & Morgan 2000, 6.13-15; Freeth 2002, 47-52; Freeth & Jones 2012, 3.3.1.

<sup>39</sup> Wright 2013. Evans, Carman, & Thorndike 2010, 24-31, also propose gear trains approximating Babylonian long period relations.

<sup>40</sup> Wright 2013.

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ber 442.<sup>41</sup> As we will show below, these are the numbers of years in long, accurate, and previously unattested period relations for their respective planets. The fact that the terms of both relations are suitably factorable for representation through gears adds weight to the argument that they were included in the inscription as statements of the theory built into the Mechanism, not as ideal periods that the Mechanism merely approximated. The implication that the Mechanism used compound gear trains to obtain the desired periodicities of at least two of the planets may have implications for whether specific devices for producing the anomalies would have been mechanically viable, but this is a question beyond the scope of the present paper.

A third question that the Front Cover Inscription casts light on is the relation between the design of the Mechanism and Babylonian astronomy. It has often been remarked that the lunisolar gearwork is entirely founded on two period relations that were attested in Babylonian astronomy earlier than their appearance in a Greek context: the "Metonic" equation of 19 solar years with 235 lunar months, and the "Saros" cycle equating 223 lunar months, 239 periods of lunar anomaly, and 242 periods of lunar latitude. Discussions of the assumed planetary period relations that the Mechanism might have represented either exactly or approximately have therefore tended to look to either the shorter and less precise Babylonian Goal Year periods or the long periods of Babylonian mathematical astronomy.<sup>42</sup> Now of the two period relations that can be recovered from the Front Cover Inscription, the 462-year relation for Venus could be a practicable approximation of the unfactorable Babylonian 1151-year relation, but the 442-year relation for Saturn cannot be accounted for in this way because the Babylonian 265-year relation is already practicable with gears as well as being shorter than the 442-year relation. We infer that, for the planets, the designers of the Mechanism drew on otherwise unknown research in the Greek tradition that was either independent of the Babylonians or, perhaps more likely, built on their foundations.

The descriptions of the planets' synodic cycles also tend to distance the Mechanism from Babylonian planetary theory. In Babylonian astronomy, the most prominent synodic phases are the first and last appearances, which seem not to have been mentioned at all in the Front Cover Inscription. On the other hand, the Babylonians did not include greatest elongations of the inferior planets or conjunctions of any planet among the predicted or observed phases, while sunset ("acronychal") risings of the superior planets, rather than their true oppositions, were recognized as significant phases. The Front Cover Inscription, on the contrary, takes a severely geometrical approach to the defining the key stages of the synodic cycles.

<sup>41</sup> Lines 16 and 24 contained parts of the formulas for Mars and Jupiter respectively, but we are unable to read any numerals because of the damaged condition of these lines.

<sup>42</sup> See the articles cited in notes 38 and 39 above.

## Concordance of parallel passages.

The paragraphs for all three superior planets used almost exactly the same verbal framework for describing their periodicities and synodic phenomena; a similar parallelism probably also held between the paragraphs for the two inferior planets, though our evidence is slighter since little of the paragraph for Mercury survives. The following tables facilitate comparison of the corresponding passages.

# Inferior planets

	Mercury	Venus
Planet named	_	4
Period relation	_	5-6
Synodic period in days	_	6
Superior conjunction	_	7
Greatest evening elongation	—	8
Evening station	—	9
Inferior conjunction	_	10
Greatest morning elongation	_	11
Morning station	_	12
Greatest morning elongation (again)	1-2	13-14

#### Superior planets

	Mars	Jupiter	Saturn
Planet named	?	24	?
Period relation	?	24	33
Synodic period	17	25	34
Evening station	18-19	26-27	35-36
Conjunction	20	28	37
Morning station	21	29-30	38-39
Evening station (again)	22-23	31	40
Opposition	?	32	41

# Line-by-line commentary.

# Lines 1-4: Mercury

Little can be made of these lines, which must belong to the description of the last stages of Mercury's synodic cycle. Lines 1 and 2 apparently correspond to the occurrences of ὑπολειπόμενος and [μεγίστου ἀπο]στήματοs in lines 13 and 14, which respectively describe the planet's direct motion while increasing in elongation from the Sun leading to the greatest morning elongation, and the motion, still direct but now decreasing in elongation, following that event. Lines 4-16: Venus

4-6. Name, period relation, and synodic period

Venus is identified by its descriptive name Phosphoros (line 4). The only other legible naming of a planet, in line 24, gives only the descriptive name, and it is likely that this was the practice throughout the text. By way of contrast, both descriptive and theophoric names are given in the Back Cover Inscription.<sup>43</sup>

The number 462 which appears in lines 5 and 6 identifies the period relation for Venus as:

462 years = 289 synodic periods = 462 revolutions of the ecliptic

This relation is not attested in any other known source from antiquity. The ratio 462 : 289 factorizes as  $(2 \times 3 \times 7 \times 11)$ :  $(17 \times 17)$ , so it can be represented by a gear train with reasonable tooth counts, e.g.  $(66 : 51) \times (63 : 51)$ . It is also the first continued-fraction convergent of the ratio 1151 : 720 which defines the period relation for Venus in Babylonian mathematical astronomy. Since 1151 is prime, the Babylonian period relation could not be represented by a practicable gear train. Hence it is possible that the 462 : 289 ratio was adopted for the Mechanism as a best approximation of the Babylonian ratio, without the need to presume independent empirical input.

Venus's synodic period is approximately 583.92 days; from the 462-year period relation and a 365<sup>1</sup>/4 day year one would obtain 583.89. The period as recorded in line 6 of our text was probably just 584 days; only the first digit is preserved.

#### Synodic phases

The cycle set out in the text apparently began with superior conjunction, since line 7 has the planet increasing in longitude after conjunction. The next phase reached is the greatest evening elongation (line 8). This is stated to be 224 days after superior conjunction (line 8).<sup>44</sup> If this number was obtained by accurate trigonometrical calculation from the theoretical model, it would correspond to an epicycle radius of approximately  $44^2/_3$  such that the deferent's radius is 60, which in turn could have been derived from an assumed 48° for the arc of Venus's greatest elongation, a parameter that is attested in several ancient sources.<sup>45</sup>

After the greatest elongation, the planet approaches the Sun (line 8) while continuing

<sup>43</sup> Cf. BCI lines I 19 and 23, in *IAM* 5.4.

<sup>44</sup> The traces of the numeral are also compatible with 221, but 224 appears to be the correct reading since the intervals from superior conjunction to greatest elongation and from greatest elongation to inferior conjunction should add up to half the synodic period. 45 Neugebauer 1975, 2.804.

to increase in longitude. After an interval not preserved in the text, but which ought to have been about 50 days, Venus reaches its evening station (line 9). The text appears to have specified Venus's elongation from the Sun, which should have been about 32° in the direction of increasing longitude. Following station, Venus continues to approach the Sun while now moving retrograde until inferior conjunction (line 10).

At this point the text took a step backward chronologically, stating the interval from the greatest evening elongation to the inferior conjunction, 68 days (presumably in the gap between lines 10 and 11), and the corresponding interval of 68 days from inferior conjunction to the greatest morning elongation (line 11). Then it breaks down the latter interval into a first part in which the planet moves retrograde to its morning station (line 12, again with a lost indication of the elongation from the Sun at station) and a second part, lasting 49 days if the numeral is correctly read, in which the planet moves direct to its greatest morning elongation (line 13). 46 days from morning station to greatest elongation, and thus 22 days from inferior conjunction to the station, would be in better agreement with the epicycle radius of  $44^2/_3$  obtained above.

The final stage of the cycle is the direct motion from greatest morning elongation, with the planet approaching the Sun (line 14), concluding with superior conjunction.

Lines 15-16 are in wretched condition and practically unreadable. It is not clear where the section concerning Venus ended and that concerning Mars began.

Notes on specific passages:

5. The reading  $\zeta \omega_i \delta[i] ou$  is highly uncertain, and we do not see how an allusion to a zodiacal sign would fit in here.

The point of <code>iooic</code> ("equal") is not clear, unless it anticipates the fact that the number of Venus's revolutions around the ecliptic given in line 6 is the same as the number of years.

6. The word at the beginning of the line might have been ки́клоиs, "circles" or "circuits."

10. The word following συνόδου at the line's end might have been a specification of which kind of conjunction takes place; but neither ἀπώτερον ("further") nor ἀνώτερον ("higher") would be expected for inferior conjunction.

11.  $\delta\lambda\lambda\alpha$ ; ("another") presumably because an interval of 68 days was previously specified in a lost part of the text for the time from greatest evening elongation to inferior conjunction.

# Lines 16-24: Mars

Period relation and synodic period. Mars's period relation ought to have been set out in the line preceding the statement of its synodic period (line 17), but we have not succeeded

in making sense of the traces in line 16. The synodic period is approximately 779.94 days, which the text probably expressed as "a little less than 780 days."

#### Synodic phases

The starting phase of the text's synodic cycle is not entirely clear from the surviving text, but on analogy with the paragraphs for Jupiter and Saturn, we believe it was the evening station, following opposition. Line 18 indicates a beginning of direct motion, which should mean the evening station, while line 19 refers to the entire interval of the planet's direct motion from the evening station until the morning station. Lines 20-21 breaks this interval into two equal parts of 349 days from evening station to conjunction and from conjunction to morning station. As in the case of Venus, the elongations at the stations were given, but unfortunately the numbers are lost.

In line 22, the interval of 82 days must be from morning station to evening station since:

2 x 349 days + 82 days = 780 days

82 days is in fact longer than the time of retrogradation that would be obtained from any chosen epicycle radius for Mars; the maximum possible is about 77 days, corresponding to an implausibly large radius of about 43<sup>1</sup>/<sub>2</sub>, while an accurate epicycle radius would give a retrograde time of about 73 days. It is possible that the discrepancy resulted from assuming an interval of several days of zero velocity at the stations, as the text prescribes for Jupiter and Saturn; however, in the section for Jupiter the stated duration of the retrogradation does *not* include the days of no motion.

Line 23 shows that Mars's stations were described as effectively lasting several days, like those of Jupiter (30-31) and Saturn (39-40). It seems likely that the duration of the stations were assumed to be 8 days for all three superior planets, though the reading of the numeral in the present line is not certain.

The remaining part of the section for Mars (lines 23-24) is too broken to interpret; one would expect a reference to the planet's opposition at the midpoint of its retrogradation (cf. 32 and 41).

#### Lines 24-32: Jupiter

Period relation and synodic period. The number of years of the period relation was likely written in an illegible part of line 24. The synodic period is approximately 398.88 days, so the continuation of line 25 must have given 399.

#### Synodic phases

Lines 26-29 correspond closely to lines 18-21 in the description of Mars's synodic cycle: the interval of direct motion from evening to morning station is specified, and then broken into two equal intervals of 139 days from evening station to conjunction and from

conjunction to morning station. This is, within a day, the length of the intervals between the conjunction and the stations calculated from Ptolemy's epicycle radius of  $11^{1}/_{2}$ .

According to lines 30-31, the planet stands still for 8 days at either station. The number 104 in line 32 must be the duration of the entire retrogradation, not counting the eightday pauses, since:

2 x 139 + 2 x 8 + 104 = 398

though it is not clear how the text accounted for the total's shortfall of just under a day relative to the synodic period. The opposition falls at the midpoint of the 104-day interval.

Notes on specific passages:

26. δωδεκατημόριον ("twelfth part") is likely to have the sense of "30° interval" here, and possibly refers to the amount that Jupiter progresses in longitude in one synodic period (the mean is actually a little over 33°).

28. ένχρόνω ("in a time interval") is awkward here, but no alternative reading suggests itself.

Lines 32-43: Saturn.

Period relation and synodic period. Line 33 gives the number of years in the following period relation for Saturn:

442 years = 427 synodic periods = 15 revolutions of the ecliptic

Like that for Venus, this relation is not attested in any other known ancient source. The ratio 442 : 427 factorizes as  $(2 \times 13 \times 17)$  :  $(7 \times 61)$ , so it can be expressed as a plausible gear train, e.g.  $(68 : 61) \times (52 : 56)$ . In this case, the period relation cannot be accounted for as simply an approximation of the Babylonian period relation

265 years = 256 synodic periods = 9 revolutions of the ecliptic

since the Babylonian relation is both shorter and suitably factorable for gearwork.

Line 34 is all that remains of the statement of the synodic period. Combining the ratio from the 265-year period relation with a  $365^{1}/_{4}$  day year would yield approximately 378.09 days, in agreement with the planet's actual synodic period.

# Synodic phases

Little remains of the treatment of Saturn's synodic phases. The correspondence of wording in lines 35-41 with parts of 26-32 shows that the basic pattern was the same as for Jupiter. The only numerical parameter preserved is an 8-day interval of effective immobility at

Saturn's evening station (line 40).

Lines 42-43 are too poorly preserved to make any sense of. Since line 41 corresponded to line 32, which is the last line concerning Jupiter, we suspect that the text went on to discuss material other than the planets' synodic cycles.

#### Unplaced fragment 42

Our reading of line 2 is based on identifications of each letter's traces that would be most plausible if taken in isolation, but it does not fit the known vocabulary of the inscription. If we reject the reading of the first letter, it might preserve part of  $[\mu\epsilon]\gamma(\sigma\tau[ov \dot{\alpha}n\dot{\sigma}\sigma\tau\mu\alpha]$ , "greatest elongation"; if so, the planet in question is either Mercury or Venus. If we suppose the second letter to be an epsilon, one could restore  $[\Pi up]\dot{\sigma}\epsilon_i$ , "Fiery one," i.e. Mars, in which case line 3 would be part of the statements of Mars's period relation and synodic period. Line 3 seems to give us a numeral, either 64 or a number terminating in 64. We have not succeeded in finding a plausible identification of this number among the quantities that are likely to have appeared in the inscription's text.

#### Unplaced fragment 51

Lines 3-5 appear to contain vocabulary referring to a planet's apparent pause at a station, the (following?) station, and an interval of days between stages of the synodic cycle. Line 2 might contain a numeral (324 or 224).

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