Conditional Independence and Biscuit Conditional Questions in Dynamic Semantics

Katsuhiko Sano (Japan Advanced Institute of Science and Technology)
Yurie Hara (City University of Hong Kong)

Biscuit conditionals such as (1) are felt different from canonical conditionals (2) in that the consequent seems to be entailed regardless of the truth/falsity of the antecedent.

(1) If you are thirsty, there's beer in the fridge. (2) If it's raining, the fireworks will be cancelled.

BC AND INDEPENDENCE IN STATIC SEMANTICS: Franke (2009) argues that the "feeling of the consequent entailment" in biscuit conditionals is due to the conditional independence between the antecedent and consequent; thus a uniform semantics (i.e., a strict implication, $\sigma \cap A \subseteq C$, where a set σ is the speaker's epistemic state) for canonical and biscuit conditionals can be maintained. Let us define the speaker knows A ($\Box A$, in short) σ if $\sigma \subseteq A$, and A is consistent with ($\Diamond A$, in short) σ if $\sigma \cap A \neq \emptyset$. Briefly, if the antecedent of a conditional is presupposed to be possible ($\Diamond A$ in σ) and the speaker has a prior knowledge that the antecedent A and the consequent C are conditionally independent (3), it follows from $\sigma \cap A \subseteq C$ that the speaker knows C, hence the entailment of the consequent obtains.

(3) A and C are conditionally independent in σ
if $\forall X \in \{A, \overline{A}\}. \forall Y \in \{C, \overline{C}\}$: if $\Diamond X$ and $\Diamond Y$ in σ, then $\Diamond(X \cap Y)$ in σ. (Franke, 2009)

To see this, suppose for contradiction that the speaker does not know C in σ, i.e., $\Diamond\overline{C}$ hold in σ. Then, by assumption, (3) gives us $\Diamond(A \cap \overline{C})$ in σ. This contradicts with the speaker's assertion $\sigma \cap A \subseteq C$; $(\sigma \cap A) \cap \overline{C} \subseteq C \cap \overline{C}$; $(A \cap \overline{C}) \cap \sigma \subseteq \emptyset$. Therefore, $\sigma \subseteq C$, as desired.

Now, the next question pertains to whether it is possible to derive the same consequent entailment in the framework of dynamic semantics. Furthermore, there are some instances of biscuit conditional questions, as in (4). Intuitively, a BC question does give rise to a consequent entailment. In (4), answering 'yes' entails that there is something in the fridge and answering 'no' entails the opposite regardless of the state of the speaker's thirst. Put another way, if the speaker asks the unconditionalized counterpart right after the conditionalized one, it would be a superfluous question. In contrast, canonical conditional questions do not. I.e., answering 'yes' to (5) does not enlighten the questioner on whether the fireworks will be cancelled or not when it is not raining.

(4) If I'm thirsty, is there anything in the fridge? (5) If it's raining, will the fireworks be cancelled?

This paper provides a dynamic and nonsymmetric version of the independence condition, a *d-independence* condition which correctly derives the consequent entailment in both declaratives and interrogatives.

INDEPENDENCE AND BCQ IN DYNAMIC SEMANTICS: Within the dynamic view, conditionals are characterized as a two-step update procedure (Stalnaker 1986; Karttunen 1974; Heim 1982): 1. A temporary state is created by updating the information state with the antecedent of the conditional. 2. The derived state is updated with the consequent. In the current paper, we follow Kaufmann's (2000) formulation of dynamic semantics. First, we regard a *possible world* as a mapping from the set P of proposition letters to $\{0, 1\}$ and define an *information state* σ as a set of possible worlds and define $W := \{0, 1\}^{\mathsf{P}}$. We assume that our syntax $\mathcal{ML}$ consists of the negation $\neg$, the conjunction $\wedge$, the implication $\rightarrow$, and the diamond operator $\Diamond$, as well as P. Then, we define the result of updating σ with the sentence $\varphi \in \mathcal{ML}$ as follows:

$$\sigma[p] = \{ w \in \sigma \mid w(p) = 1 \}, \qquad \sigma[\varphi \wedge \psi] = \sigma[\varphi][\psi], \quad \sigma[\neg\varphi] = \sigma \setminus \sigma[\varphi],$$
$$\sigma[\varphi \rightarrow \psi] = \{ w \in \sigma \mid w \in \sigma[\varphi] \text{ implies } w \in \sigma[\varphi][\psi] \}, \quad \sigma[\Diamond\varphi] = \{ w \in \sigma \mid \sigma \cap \sigma[\varphi] \neq \emptyset \}.$$

In characterizing the intuition of "entailment", we use the notion of *support* (*acceptance* in Veltman (1996)): φ is *supported* in σ (notation: $\sigma \models \varphi$) if $\sigma[\varphi] = \sigma$. We also say that φ is *consistent* in σ if $\sigma[\varphi] \neq \emptyset$. In Kaufmann (2000), remark that we obtain the monotonicity of the updates, i.e., $\sigma[\varphi] \subseteq \sigma$ for all σ and φ. We define the nonsymmetric *d-independent* condition as in Definition 1. Intuitively speaking, ψ is independent of φ if updating σ with φ or $\neg\varphi$ does not affect the consistency of ψ.

Definition 1. We say that ψ is *d-independent* of φ in σ if, for all $X \in \{\varphi, \neg\varphi\}$ and all $Y \in \{\psi, \neg\psi\}$, $\sigma[X] \neq \emptyset$ implies that $\sigma[Y] \neq \emptyset$ is equivalent with $\sigma[X][Y] \neq \emptyset$.

Note that our condition is nonsymmetric, i.e., only defines the consequent's independence from the antecedent, since in the current analysis, a conditional is treated as a two-step update. This non-symmetry is particularly suitable for biscuit conditional questions discussed below, as the antecedent assertion sets up a context on which the consequent question operate. Van Rooij (2007) also offers a notion of independence in context in a dynamic setting to account for the strengthening of conditional presuppositions, but it is symmetrically defined. Now, by $\sigma[\neg\varphi] = \emptyset$ iff $\sigma[\varphi] = \sigma$, we can rewrite the d-independence in terms of the notion of support.

Proposition 2. ψ is d-independent of φ in σ iff, $\sigma[X] \neq \emptyset$ implies that $\sigma \models Y$ is equivalent with $\sigma[X] \models Y$, for all $X \in \{\varphi, \neg\varphi\}$ and all $Y \in \{\psi, \neg\psi\}$.
Theorem 1. Let ψ be d-independent of φ in σ. If $\sigma[\varphi] \neq \emptyset$ and $\sigma \models \varphi \to \psi$, then $\sigma \models \psi$.
Proof. Assume $\sigma[\varphi] \neq \emptyset$ and $\sigma \models \varphi \to \psi$. By Proposition 2, it suffices to show $\sigma[\varphi][\psi] = \sigma[\varphi]$, i.e., $\sigma[\varphi] \subseteq \sigma[\varphi][\psi]$ by monotonicity. Fix any $w \in \sigma[\varphi]$. Since $\sigma[\varphi \to \psi] = \sigma$, $w \in \sigma[\varphi \to \psi]$. By $w \in \sigma[\varphi]$, $w \in \sigma[\varphi][\psi]$, as desired. $\square$

Let us take (1) as an example. Assume a normal (i.e., non-magical) situation where acquiring the knowledge that the addressee is thirsty does not determine whether there is beer in the fridge or not. Thus, the proposition 'there's beer in the fridge' is independent of 'you are thirsty'. Now, the speaker uttered the sentence (1). Given the d-independence condition and Theorem 1, the consequent proposition 'there's beer in the fridge' is supported. Thus, our condition derives the consequent entailment in the dynamic framework.

BCQ: EXTENSION TO STRUCTURED CONTEXTS We extend our dynamic independence to structured contexts to handle biscuit conditional questions. As before, we stipulate $W := \{0, 1\}^{\mathsf{P}}$, where P is the set of proposition letters. In dealing with statements and questions, we now introduce a *structured context* C as an equivalence relation on some set of possible worlds (Groenendijk 1999, Isaacs and Rawlins 2008). We define the set $\mathsf{Bool}(\mathsf{P})$ as all the propositional combinations generated from P. Note that we can calculate the truth value of $w(\varphi)$ for a $w \in W$ and $\varphi \in \mathsf{Bool}(\mathsf{P})$. Now, we define the set $\mathcal{QL}$ of query-formulas by the following rule: if φ, $\psi \in \mathsf{Bool}(\mathsf{P})$ then $\varphi!$, $\varphi?$, $\varphi! \to \psi!$, $\varphi! \to \psi?$ are in $\mathcal{QL}$. We denote query-formulas of $\mathcal{QL}$ by α, β, γ, etc. Then, we define the result of updating C with a query-formula of $\mathcal{QL}$ as follows (Isaacs and Rawlins 2008):

$$C[\varphi!] = \{\langle w, v\rangle \in C \mid w(\varphi) = v(\varphi) = 1\}, \quad C[\varphi?] = \{\langle w, v\rangle \in C \mid w(\varphi) = v(\varphi)\},$$
$$C[\varphi! \to \gamma] = \{\langle w, v\rangle \in C \mid \exists z \in W.(\langle w, z\rangle \in C[\varphi!] \text{ or } \langle z, v\rangle \in C[\varphi!]) \text{ implies } \langle w, v\rangle \in C[\varphi!][\gamma]\},$$

where $\gamma \in \{\psi!, \psi?\}$. Note that $C[\varphi! \to \psi!]$ and $C[\varphi! \to \psi?]$ are also structured contexts. Let us say that C *supports* α (written: $C \models \alpha$) if $C = C[\alpha]$. We also say that α is *consistent* in C if $C[\alpha] \neq \emptyset$. As in Kaufmann (2000), we also obtain that $C[\alpha] \subseteq C$ for all C and $\alpha \in \mathcal{QL}$. What is the *d-independent* condition in this setting? Just replacing σ in Definition 1 with C is insufficient, because, for instance, $C[\neg\varphi!] = \emptyset$ is no longer equivalent to $C[\varphi!] = C$. That is, now that we have structured contexts, the condition can*not* be rewritten in terms of the notion of support. However, we can still preserve our previous intuition of independence in dynamic semantics: a query-formula $\psi!$ (or $\psi?$) is independent of $\varphi!$ in C if updating C with $\varphi!$ or $\neg\varphi!$ does not affect the supportedness and the consistency of $\psi!$ (or $\psi?$). Thus, we provide Definition 3 as the notion of independence for structured contexts. From the condition 2) of Definition 3, we can obtain the desired consequent entailment, as is the case with Theorem 1.

Definition 3. $\gamma \in \{\psi!, \psi?\}$ is *d-independent* of $\varphi!$ in C if, for all $\alpha \in \{\varphi!, \neg\varphi!\}$ and all $\beta \in \{\psi!, \neg\psi!\}$ (or $\beta = \psi?$ if γ is $\psi?$), $C[\alpha] \neq \emptyset$ implies the following: 1) $C[\beta] \neq \emptyset$ iff $C[\alpha][\beta] \neq \emptyset$ and 2) $C \models \beta$ iff $C[\alpha] \models \beta$.
Theorem 2. Let $\gamma \in \{\psi!, \psi?\}$ be d-independent of $\varphi!$ in C. If $C[\varphi!] \neq \emptyset$, then $C \models \varphi! \to \gamma$ implies $C \models \gamma$.

Let us take (1) and (4) as examples. Assume a similar non-magical situation where the speaker being thirsty does not determine the presence/absence of drinks in the fridge. Given the d-independence condition and Theorem 2, both the consequent declarative 'there's beer in the fridge' and the consequent interrogative 'Is there anything in the fridge?' are supported. Thus, our condition derives the consequent entailment for both biscuit conditional statements and questions in the dynamic framework.

CONCLUSION: We develop a dynamic and nonsymmetric version of independence tailored for both information states and structured contexts. Franke's proposal is further supported in that there is no need for stipulating special semantics for biscuit conditionals, since the "feeling of entailment" of biscuit conditional questions as well as statements can be derived from the existing dynamic semantics of conditionals and our dynamic independence.

SELECTED REFERENCES: **Franke** (2009) *Signal to Act*. **Groenendijk** (1999) 'The logic of interrogation.' **Heim** (1982) *The Semantics of Definite and Indefinite Noun Phrases.* **Isaacs and Rawlins** (2008) 'Conditional questions.' **Kaufmann** (2000) 'Dynamic context management.' **van Rooij** (2007) 'Strengthening Conditional Presuppositions' **Veltman** (1996) 'Defaults in update semantics.'