Amount Semantics

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This paper develops a new semantics for degrees under which they are nominalized quantity-uniform properties of individuals: degrees are the same sort of entity as kinds. We use English *amount* as a case study to first motivate a kind semantics for degrees, and then apply the semantics in a novel analysis of amount relatives (Grosu and Landman, 1998).

Amounts: *Amount* admits two readings: under the first, *amount* behaves like *quantity*, partitioning a predicate's denotation; (1) thus references specific apples. Under the second reading, (1) references an abstract amount of apples. We focus on the second reading. This EXISTENTIAL reading also surfaces with the noun *kind* (Carlson, 1977b; Wilkinson, 1995).

(1) John wants that amount of apples.
  a. DEFINITE: John wants those specific apples there
  b. EXISTENTIAL: John wants some apples that measure the relevant amount

(2) John wants that kind of apple.

Amount requires a substance noun (e.g., *apples*) to designate what we are referencing amounts *of*. It then relates the substance noun with a set of nominalized quantity-uniform properties, formed on the basis of a contextually specified measure μ_f (e.g., μ_{kg}, μ_{li}, etc.).

(3) $[\![\text{amount}]\!] = \lambda k \lambda d.\ \exists n[d = {}^\cap\lambda x.\ {}^\cup k(x) \wedge \mu_f(x) = n]$
where k is the kind denoted by a substance noun,
μ_f is a contextually-specified measure, and
n is some number in the range of the measure μ_f

The set of entities to which *amount* refers is a set of degrees, the individual correlates of quantity-uniform properties (cf. kind semantics, under which, e.g., the DOG kind is the individual correlate of the property of being a dog; Chierchia, 1998).

(4) $\text{DEGREE} := {}^\cap\lambda x.\ \mu(x) = n$ (for some μ, n)

Rethinking degrees: Conceiving of degrees as nominalized properties necessitates rethinking how they interact with the compositional semantics. In (1), we access the abstract amount of apples by first identifying the relevant apple individual (i.e., by establishing a pointer to it with *that*) and then picking out the degree that applies to this individual.

(5) $[\![\text{that}]\!] = \lambda P.\ \iota y[P(y) \wedge {}^\cup y(\textsc{that})]$ where $[\![\textsc{that}]\!]$ = the salient (plural) individual

In the basic case where *that* takes a set of individuals, it returns the individual that is relevant (via Partee's Id); when *that* takes a set of degrees (i.e., nominalized properties) it returns the degree that applies to the relevant individual. Assuming three apples (a, b, c) and the relevance of μ_{kg} in the context of (1), we get the following (where $\mu_{kg}(a{+}b{+}c) = n_{a+b+c}$):

(6) $[\![\text{that}]\!]([\![\text{amount of apples}]\!])$
$= [\![\text{that}]\!](\lambda d.\ \exists n[d = {}^\cap\lambda x.\ {}^\cup \text{apple}(x) \wedge \mu_{kg}(x) = n])$
$= [\lambda P.\ \iota y[P(y) \wedge {}^\cup y(\textsc{that})]](\lambda d.\ \exists n[d = {}^\cap\lambda x.\ {}^\cup \text{apple}(x) \wedge \mu_{kg}(x) = n])$
$= \iota y \in \{y = {}^\cap\lambda x.\ {}^\cup \text{apple}(x) \wedge \mu_{kg}(x) = n : n \in \mathbb{N}\}[{}^\cup y(\textsc{that})]$
$= \iota y \in \{y = {}^\cap\lambda x.\ {}^\cup \text{apple}(x) \wedge \mu_{kg}(x) = n : n \in \mathbb{N}\}[{}^\cup y(a{+}b{+}c)]$ assuming 3 apples
$= {}^\cap\lambda x.\ {}^\cup \text{apple}(x) \wedge \mu_{kg}(x) = n_{a+b+c}$

The result references a degree: a nominalized quantity-uniform set of apples. For this degree to compose with the rest of the sentence, we generalize the operation of DKP (Chierchia, 1998) to type-shift nominalized properties (kinds or degrees) for object-level argument slots.

(7) *Generalized Derived Kind Predication* (*DKP*):
If P applies to objects and y denotes a kind or degree, then $P(y) = \exists x[{}^{\cup}y(x) \wedge P(x)]$

(8) $[\![\text{I ate that amount of apples}]\!]$
$= \text{ate}([\![\text{that amount of apples}]\!])(I)$
$= \text{ate}({}^{\cap}\lambda x.\ {}^{\cup}\text{apple}(x) \wedge \mu_f(x) = n_i)(I)$
via generalized DKP
$= \exists y[{}^{\cup}({}^{\cap}\lambda x.\ {}^{\cup}\text{apple}(x) \wedge \mu_f(x) = n_i)(y) \wedge \text{ate}(y)(I)]$

Amount relatives: In relative clauses formed with *amount*, (9), we abstract over degrees at the CP level. (We use a head-external syntax for illustrative purposes only; raising or matching analyses work equally well.) Intersective modification restricts the set of degrees *amount* references to just those that apply to objects picked out by the CP. Definite *the* takes this restricted set of degrees. The set is ordered (on the basis of the measure internal to the degrees); when *the* takes an ordered set it selects the maximal element, yielding the largest apple-degree that applies to the apples that you ate. In other words, *the* returns the amount of apples that you ate in (9); generalized DKP applied at the matrix level takes this degree, asserting that I ate an apple-quantity equal in amount to the apple-quantity you ate.

(9) I ate the amount of apples λd that you ate d
$= \text{I ate the } (\lambda d.\ \exists n[d = {}^{\cap}\lambda x.\ {}^{\cup}\text{apple}(x) \wedge \mu_f(x) = n]) \cap (\lambda d.\ \exists y[\text{ate}({}^{\cup}d(y))(\text{you})])$

This sort of degree abstraction also applies in so-called "amount relatives." These relative clauses ostensibly violate the Definiteness Restriction, which precludes individuals or individual variables from occurring in the pivot of an existential construction (Heim, 1987).

(10) John ate the apples that there were on the table.
(cf. **there were the apples on the table*)

Our account of amount relatives assumes degree abstraction (e.g., Heim, 1987): the pivot of the existential contains a degree variable, not an individual. (Like degrees, kinds freely serve as pivots to existentials: *there were those kinds of apples at the store*; Wilkinson 1995). Here the RC denotes an individual: *the apples that there were on the table* references apples, not an abstract amount thereof. This object-level interpretation arises from modification of the RC head (*apples*) by a set of degrees, which yields a restricted set of individuals:

(11) $[\![P_{\langle e,t\rangle} \cap_E A_{\langle d,t\rangle}]\!] = \lambda x.\ P(x) \wedge \exists d[A(d) \wedge {}^{\cup}d(x)]$

(12) the apples λd that there were d on the table
$= \text{the } \lambda z.\ \text{apples}(z) \wedge \exists d[(\lambda d'.\ \exists n[d' = {}^{\cap}\lambda x.\ \mu_f(x) = n \wedge \text{on-table}(x)])(d) \wedge {}^{\cup}d(z)]$

(12) references the maximal apple individual that instantiates on-table degrees; in other words, it references the apples on the table. The RC is structured on the basis of the degree-internal measure; we get maximality from the semantics of *the* applied to an ordered set (cf. MAX from Grosu and Landman). We get the object-level interpretation by modifying a set of individuals by a set of degrees; Grosu and Landman stipulate an optional SUBSTANCE operator for this obligatory reading. The peculiar behavior of amount relatives (Carlson, 1977a) follow from two factors: (i) ordered/overlapping sets resist quantification, limiting the determiners that may appear; and (ii) *wh*-forms *which* and *who* preclude degree abstraction.