Conditional Independence and Biscuit Conditional Questions in Dynamic Semantics

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Difference in Biscuit and Canonical conditionals

(1) Biscuit Conditional
If you are thirsty,
there's beer in the fridge.
ENTAILS
There's beer in the fridge

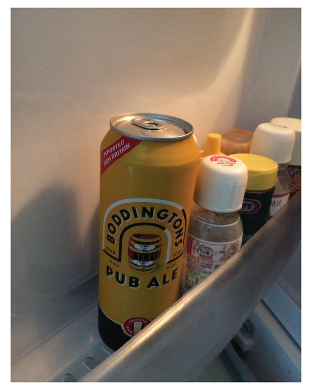
(2) Canonical Conditional
If it's raining,
the fireworks will be cancelled.
DOESN'T ENTAIL
The fireworks will be cancelled.

- the "feeling of the consequent entailment" in Biscuit Conditional
- Franke (2009): the conditional independence between the ant. and cons.

Dynamic extension and biscuit conditional questions

Question Is it possible to derive the same consequent entailment in the framework of dynamic semantics?

- Biscuit conditional questions



(3) If I'm thirsty, is there beer in the fridge?
ENTAILS Is there beer in the fridge?

Yes! ENTAILS There's beer regardless of the questioner's thirst. ☺
No! ENTAILS No beer regardless of the questioner's thirst. ☹

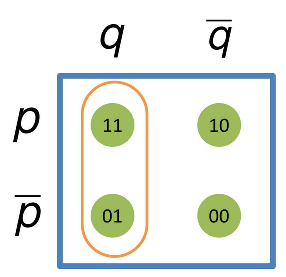
(4) If it's raining, will the fireworks be cancelled?
DOESN'T ENTAIL
Will the fireworks be cancelled?

Yes or No does not enlighten the questioner on whether the fireworks will be cancelled or not when it is not raining. ☹

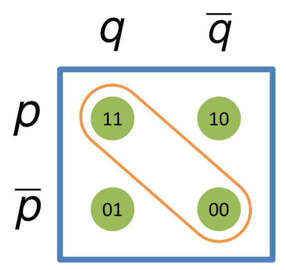
Goal Define a d-independence condition
- a dynamic and non-symmetric version of the independence condition
- correctly derives the consequent entailment in both declaratives and interrogatives

BC and independence in static semantics:

- the speaker's epistemic state $\sigma \subseteq W$
- 'if P then Q' holds in σ if $\sigma \cap P \subseteq Q$
- The speaker knows P ($\Box P$) in σ if $\sigma \subseteq P$
- P is consistent ($\Diamond P$) with σ if $\sigma \cap P \neq \emptyset$.



p and q are independent



p and q are not independent

P and Q are independent in σ if
$\Diamond X$ and $\Diamond Y$ in σ implies $\Diamond(X \cap Y)$ in σ,
for all $X \in \{P, \overline{P}\}$ and $Y \in \{Q, \overline{Q}\}$. (Franke 2009)

[Consequent Entailment] Suppose:
- P & Q are independent in σ,
- P is consistent in σ

Then
- 'if P then Q' in σ entails $\Box Q$ in σ.

- A uniform semantics for canonical and biscuit conditional can be maintained.
- Independence condition is symmetric.

Dynamic Semantics: preliminaries

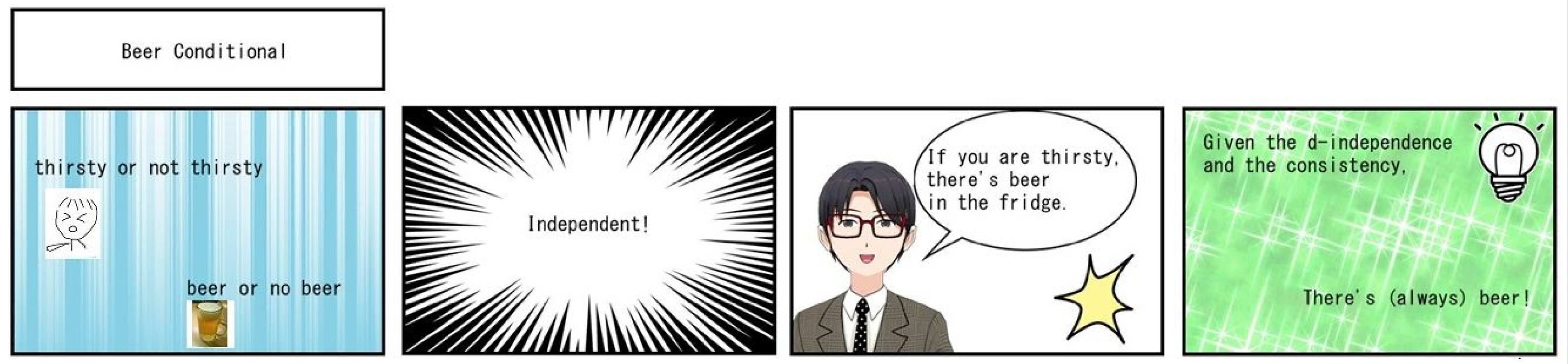
- A conditional 'if p then q': a two-step update procedure (Stalnaker 1986; Karttunen 1974; Heim 1982):

1. A temporary state is created by updating the information state with p.
2. The derived state is updated with q.

- Kaufmann's (2000) formulation of dynamic semantics:
  - Let P be a set of atomic propositions.
  - A possible world w is a mapping (truth assignment) $w : \mathrm{P} \to \{0, 1\}$
  - An information state σ is a set of possible worlds.
  - Dynamic semantics for our syntax:

$$\begin{aligned}
\sigma[p] &= \{\, w \in \sigma \mid w(p) = 1 \,\},\\
\sigma[\varphi \wedge \psi] &= \sigma[\varphi][\psi],\\
\sigma[\neg\varphi] &= \sigma \setminus \sigma[\varphi],\\
\sigma[\varphi \to \psi] &= \{\, w \in \sigma \mid w \in \sigma[\varphi] \text{ implies } w \in \sigma[\varphi][\psi] \,\},\\
\sigma[\Diamond\varphi] &= \{\, w \in \sigma \mid \sigma \cap \sigma[\varphi] \neq \emptyset \,\}.
\end{aligned}$$

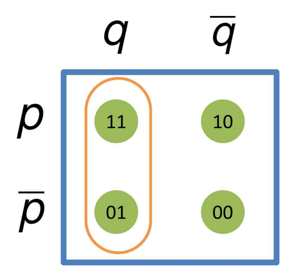
- The updates are monotone, i.e., $\sigma[\varphi] \subseteq \sigma$ for all σ and φ.
- φ is supported in σ ($\sigma \models \varphi$) if $\sigma = \sigma[\varphi]$; φ is consistent in σ if $\sigma[\varphi] \neq \emptyset$.
- $\neg\varphi$'s inconsistency in σ = φ's supportedness in σ. ($\sigma[\neg\varphi] = \emptyset$ iff $\sigma \models \varphi$)



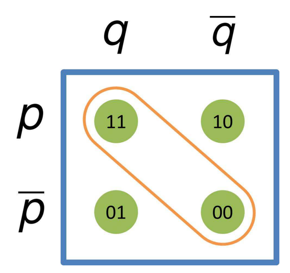
d-independence and consequent entailment

Intuition: ψ is independent of φ if updating σ with φ or $\neg\varphi$ does not affect the consistency of ψ.

- ψ is d-independent of φ in σ if, given X's consistency in σ, Y's consistency in σ is equivalent to Y's consistency in $\sigma[X]$, for all $X \in \{\varphi, \neg\varphi\}$ and all $Y \in \{\psi, \neg\psi\}$.



q is d-independent of p



q is not d-independent of p

- non-symmetric: 'ψ is d-independent of φ' does not imply the converse. (c.f. van Rooij (2007))

- [Consequent Entailment] Suppose:
  - ψ is d-independent of φ in σ and φ is consistent in σ
- Then, if σ supports $\varphi \to \psi$, it also supports ψ.

- A key observation: characterization of d-independence in terms of support:
  - Given X's consistency in σ, $\sigma \models Y$ is equivalent with $\sigma[X] \models Y$, for all $X \in \{\varphi, \neg\varphi\}$ and all $Y \in \{\psi, \neg\psi\}$.
- $\neg$ and its semantics are essential.

Extension to Biscuit Conditional Questions: Structured Contexts

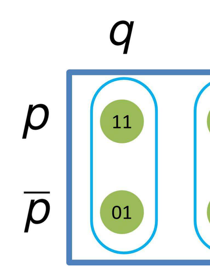
- a structured context C = an equivalence relation on $W := \{0,1\}^{\mathrm{P}}$, which gives us a partition (Groenendijk 1999, Isaacs and Rawlins 2008).
- Bool(P) := all the propositional combinations generated from P.
- the set of query-formulas: $\alpha, \beta, \gamma ::= \varphi! \mid \varphi? \mid \varphi! \to \psi! \mid \varphi! \to \psi?$ ($\varphi, \psi \in$ Bool(P))
- Updating C with a query-formula (I&R 2008):

$$C[\varphi!] = \{\, \langle w, v \rangle \in C \mid w(\varphi) = v(\varphi) = 1 \,\}, \quad C[\varphi?] = \{\, \langle w, v \rangle \in C \mid w(\varphi) = v(\varphi) \,\}.$$

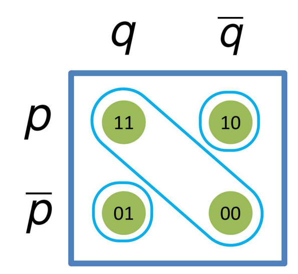
- For $\gamma \in \{\psi!, \psi?\}$, $C[\varphi! \to \gamma]$ is defined as:

$$\{\, \langle w, v \rangle \in C \mid \exists z \in W.(\langle w, z \rangle \in C[\varphi!] \text{ or } \langle z, v \rangle \in C[\varphi!]) \text{ implies } \langle w, v \rangle \in C[\varphi!][\gamma] \,\}$$

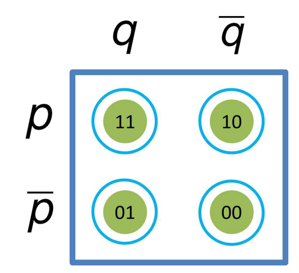
- The updates are monotone, i.e., $C[\alpha] \subseteq C$ for all C and query formulas α.
- α is supported in C ($C \models \alpha$) if $C = C[\alpha]$; α is consistent in C if $C[\alpha] \neq \emptyset$.
- $C[\neg\varphi!] = \emptyset$ is no longer equivalent to $C \models \varphi!$.



$C_1 = C_1[q?]$



C_2

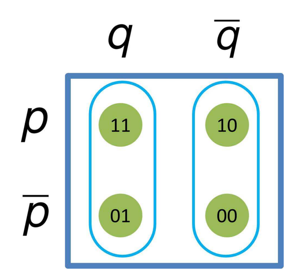


$C_2[q?]$

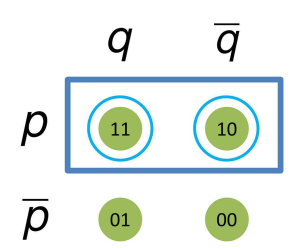
BCQ: d-independence and consequent entailment again

Intuition: a query-formula $\psi?$ is independent of $\varphi!$ in C if updating C with $\varphi!$ or $\neg\varphi!$ does not affect the supportedness of $\psi?$.

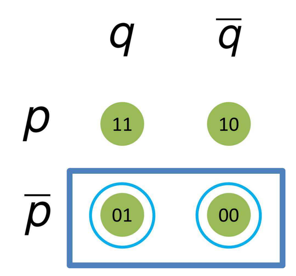
- $\psi!$ is d-independent of $\varphi!$ in C if, given α's consistency in C, $C[\beta] \neq \emptyset$ iff $C[\alpha][\beta] \neq \emptyset$, and $C \models \beta$ iff $C[\alpha] \models \beta$, for all $\alpha \in \{\varphi!, \neg\varphi!\}$ and $\beta \in \{\psi!, \neg\psi!\}$.
- $\psi?$ is d-independent of $\varphi!$ in C if, given α's consistency in C, $C \models \psi?$ iff $C[\alpha] \models \psi?$ for all $\alpha \in \{\varphi!, \neg\varphi!\}$.



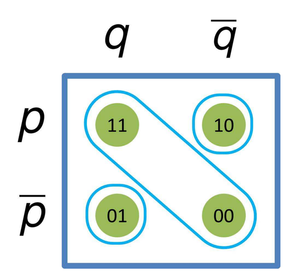
$q?$ is d-indep. of $p!$



$C_1[p!][q?]$



$C_1[\neg p!][q?]$



$q?$ is not d-indep. of $p!$

- [Consequent Entailment] Suppose:
  - $\gamma \in \{\psi!, \psi?\}$ is d-independent of φ in C and φ is consistent in C.
- Then, if C supports $\varphi \to \gamma$, it also supports γ.

- Let us take (1) and (3) and assume a non-magical situation. Given the d-independence condition, the consequent interrogative 'Is there anything in the fridge?' is supported.

Conclusion

- A dynamic and non-symmetric version of independence tailored for both information states and structured contexts.
- No need for stipulating special semantics for biscuit conditionals (Franke 2009).
- The "feeling of entailment" can be derived from the existing dynamic semantics of conditionals and our dynamic independence.

Images: http://www.comipo.com/; http://farm3.static.flickr.com/2636/3887295488_60a6bb4113_m.jpg

Supported by JSPS KAKENHI, Grant-in-Aid for Young Scientists (B) 24700146. Mail: katsuhiko.sano@gmail.com; y.hara@cityu.edu.hk