

# **Recommendation Technologies:**

## Survey of Current Methods and Possible Extensions

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**Abstract.** The paper presents a survey of the field of recommender systems and describes current recommendation methods that are usually classified into the following three main categories: content-based, collaborative, and hybrid recommendation approaches. The paper also describes various limitations of current recommendation methods and discusses possible extensions that can improve recommendation capabilities. These extensions include, among others, improvement of understanding of users and items, incorporation of the contextual information into the recommendation process, support for multi-criteria ratings, and provision of more flexible and less intrusive types of recommendations.

**Keywords:** recommender systems, survey, rating estimation methods, extensions to recommender systems.

### **1 Introduction**

Recommender systems became an important research area since the appearance of the first papers on collaborative filtering since the mid-1990s (Resnick et al. 1994, Hill et al. 1995, Shardanand & Maes 1995). There has been much work done both in the industry and academia on developing new approaches to recommender systems over the last decade. The interest in this area still remains high because it constitutes a problem-rich research area and because of the abundance of practical applications that help users to deal with information overload and provide personalized recommendations, content and services to them. Examples of such applications include recommending products at Amazon.com, movies at MovieLens.org (Miller et al. 2003), books at Amazon.com and Libra (Mooney 1999), CDs at CDNOW.com (Amazon.com), and

news at AdapiveInfo.com (Billsus et al. 2002). Moreover, some of the vendors incorporated recommendation engines into the latest versions of their commerce servers (Peddy & Armentrout 2003).

In this paper, we review the state-of-the-art in recommender systems and discuss three main approaches proposed in the literature, i.e., content-based, collaborative and hybrid approaches. We will also describe various limitations in the capabilities of the current generation of these technologies and also discuss some initial approaches to extending these capabilities.

## **2 Overview of Recommender Systems**

Although the roots of recommender systems can be traced back to the extensive work in the approximation (Powell 1981), information retrieval (Salton 1989), forecasting theories (Armstrong 2001), and also to the consumer choice modeling in marketing (Lilien et al. 1992), recommender systems emerged as an independent research area in the mid-1990's when researchers started focusing on recommendation problems that *explicitly* rely on the ratings structure. In its most common formulation, the recommendation problem is reduced to the problem of estimating *ratings* for the items that have not been seen by a user. This estimation is usually based on the ratings given by this user to other items and possibly on some other information as well. Once we can estimate ratings for the yet unrated items, we can recommend to the user the item(s) with the highest estimated rating(s).

More formally, the recommendation problem can be formulated as follows. Let  $C$  be the set of all users and let  $S$  be the set of all possible items that can be recommended, such as books, movies, or restaurants. The space  $S$  of possible items can be very large, ranging in hundreds of thousands or even millions of items in some applications, such as recommending books or CDs. Similarly, the user space can also be very large – millions in some cases. Let  $u$  be a utility

function that measures usefulness of item  $s$  to user  $c$ , i.e.,  $u : C \times S \rightarrow R$ , where  $R$  is a totally ordered set (e.g., non-negative integers or real numbers within a certain range). Then for each user  $c \in C$ , we want to choose such item  $s' \in S$  that maximizes the user's utility. More formally:

$$\forall c \in C, \quad s'_c = \arg \max_{s \in S} u(c, s) \quad (1)$$

In recommender systems the utility of an item is usually represented by a *rating*, which indicates how a particular user liked a particular item, e.g., John Doe gave the movie “Gladiator” the rating of 7 (out of 10). However, as indicated earlier, in general utility can be an arbitrary function, including a profit function.

Each element of the user space  $C$  can be defined with a *profile* that includes various user characteristics, such as age, gender, income, marital status, etc. In the simplest case, the profile can contain only a single (unique) element, such as User ID. Similarly, each element of the item space  $S$  is defined with a set of characteristics. For example, in a movie recommendation application, where  $S$  is a collection of movies, each movie can be represented not only by its ID, but also by its title, genre, director, year of release, leading actors, etc.

The central problem of recommender systems lies in that utility  $u$  is usually not defined on the whole  $C \times S$  space, but only on some subset of it. This means  $u$  needs to be *extrapolated* to the whole space  $C \times S$ . In recommender systems, utility is represented by ratings and is initially defined only on the items previously rated by the users. For example, in a movie recommendation application (such as the one at MovieLens.org), users initially rate some subset of movies that they have already seen. An example of a user-item rating matrix for a movie recommendation application is presented in Table 1, where ratings are specified on the scale of 1 to 5. The “ $\emptyset$ ” symbol for some of the ratings in Table 1 means that the users have not rated these movies. Therefore, the recommendation engine should be able to estimate (predict) the

ratings of the non-rated movie/user combinations and issue appropriate recommendations based on these predictions.

	K-PAX	Life of Brian	Memento	Notorious
Alice	4	3	2	4
Bob	∅	4	5	5
Cindy	2	2	4	∅
David	3	∅	5	2

**Table 1.** A fragment of a rating matrix for a movie recommender system.

Extrapolations from known to unknown ratings are usually done by (a) specifying *heuristics* that define the utility function and empirically validating its performance, and (b) *estimating* the utility function that optimizes certain performance criterion, such as the mean square error.

Once the unknown ratings are estimated, actual recommendations of an item to a user are made by selecting the highest rating among all the estimated ratings for that user, according to formula (1). Alternatively, we can recommend n best items to a user or a set of users to an item.

The new ratings of the not-yet-rated items can be estimated in many different ways, and recommender systems are usually classified according to these estimation methods. In the next section, we present a classification of recommendation approaches that were proposed in the literature and provide a survey of different types of recommender systems.

The commonly accepted formulation of the recommendation problem was first stated in (Resnick et al. 1994, Hill et al. 1995, Shardanand & Maes 1995) and this problem has been studied extensively since then. Moreover, recommender systems are usually classified into the following categories, based on how recommendations are made (Balabanovic & Shoham 1997):

- *Content-based recommendations*: the user is recommended items similar to the ones the user preferred in the past;

- *Collaborative recommendations*: the user is recommended items that people with similar tastes and preferences liked in the past;
- *Hybrid approaches*: these methods combine collaborative and content-based methods.

## 2.1 Content-based Methods

In content-based recommendation methods, the utility  $u(c, s)$  of item  $s$  for user  $c$  is estimated based on the utilities  $u(c, s_i)$  assigned by user  $c$  to items  $s_i \in S$  that are “similar” to item  $s$ . For example, in a movie recommendation application, in order to recommend movies to user  $c$ , the content-based recommender system tries to understand the commonalities among the movies user  $c$  has rated highly in the past (specific actors, directors, genres, subject matter, etc.). Then, only the movies that have a high degree of similarity to whatever user’s preferences are would get recommended.

The content-based approach to recommendation has its roots in information retrieval (Salton 1989, Baeza-Yates & Ribeiro-Neto 1999) and information filtering (Belkin & Croft 1992) research, and employs many techniques that have been extensively studied in the past. Therefore, many current content-based systems are used to recommend “information” items, such as documents, Web sites (URLs), and Usenet news messages. The improvement over the traditional information retrieval approaches comes from the use of user *profiles* that contain information about users’ tastes, preferences, and needs. The profiling information can be elicited from users explicitly, e.g., through questionnaires, or implicitly – learned from their transactional behavior over time.

Let  $Content(s)$  be a set of attributes characterizing item  $s$ . It is usually computed by extracting a set of features from item  $s$  (its content) and is used to determine appropriateness of the item for recommendation purposes. Since, as mentioned earlier, content-based systems are

designed mostly to recommend text-based items, the content in these systems is usually described with *keywords*. For example, a content-based component of the Fab system (Balabanovic & Shoham 1997), which recommends Web pages to users, represents Web page content with the 100 most important words. Similarly, the Syskill & Webert system (Pazzani & Billsus 1997) represents documents with the 128 most informative words. The “importance” (or “informativeness”) of word  $k_i$  in document  $d_j$  is determined with some *weighting* measure  $w_{ij}$  that can be defined in several different ways.

One of the best-known measures for specifying keyword weights in Information Retrieval is the *term frequency/inverse document frequency (TF-IDF)* measure (Salton 1989) that is defined as follows. Assume that  $N$  is the total number of documents that can be recommended to users and that keyword  $k_i$  appears in  $n_i$  of them. Moreover, assume that  $f_{i,j}$  is the number of times keyword  $k_i$  appears in document  $d_j$  (i.e., the simple frequency of  $k_i$  in  $d_j$ ). If  $k_i$  does not appear in  $d_j$ , then  $f_{i,j} = 0$ . Then  $TF_{i,j}$ , the term frequency (or normalized frequency) of keyword  $k_i$  in document  $d_j$ , is defined as

$$TF_{i,j} = \frac{f_{i,j}}{\max_z f_{z,j}} \quad (2)$$

where the maximum is computed over the frequencies  $f_{z,j}$  of all keywords  $k_z$  that appear in the document  $d_j$ . However, keywords that appear in many documents are not useful in distinguishing between a relevant document and a non-relevant one. Therefore, the measure of inverse document frequency ( $IDF_i$ ) is often used in combination with simple term frequency ( $TF_{i,j}$ ). The inverse document frequency for keyword  $k_i$  is usually defined as

$$IDF_i = \log \frac{N}{n_i} \quad (3)$$

Then the TF-IDF weight for keyword  $k_i$  in document  $d_j$  is defined as

$$w_{i,j} = TF_{i,j} \times IDF_i \quad (4)$$

and the content of document  $d_j$  is defined as  $Content(d_j) = (w_{1j}, \dots, w_{kj})$ .

As stated earlier, content-based systems recommend items similar to those that a user liked in the past (Lang 1995, Pazzani & Billsus 1997, Mooney 1999). In particular, various candidate items are compared with items previously rated by the user, and the best-matching item(s) are recommended. More formally, let  $ContentBasedProfile(c)$  be the profile of user  $c$  containing tastes and preferences of this user. These profiles are obtained by analyzing the content of the items previously seen and rated by the user and are usually constructed using keyword analysis techniques from information retrieval. For example,  $ContentBasedProfile(c)$  can be defined as a vector of weights  $(w_{c1}, \dots, w_{ck})$ , where each weight  $w_{ci}$  denotes the importance of keyword  $k_i$  to user  $c$  and can also be specified using the TF-IDF measure (Lang 1995, Pazzani & Billsus 1997) described above.

In the content-based systems, the utility function  $u(c,s)$  for user  $c$  and item  $s$  is usually defined as

$$u(c,s) = score(ContentBasedProfile(c), Content(s)) \quad (5)$$

Using the above-mentioned information retrieval-based paradigm of recommending Web pages, Web site URLs or Usenet news messages, both  $ContentBasedProfile(c)$  of user  $c$  and  $Content(s)$  of document  $s$  can be represented as TF-IDF vectors  $\vec{w}_c$  and  $\vec{w}_s$  of keyword weights. Moreover, the utility function  $u(c,s)$  is usually represented in information retrieval literature by some scoring heuristic defined in terms of vectors  $\vec{w}_c$  and  $\vec{w}_s$ , such as the cosine similarity measure (Salton 1989, Baeza-Yates & Ribeiro-Neto 1999):

$$u(c, s) = \cos(\vec{w}_c, \vec{w}_s) = \frac{\vec{w}_c \cdot \vec{w}_s}{\|\vec{w}_c\|_2 \times \|\vec{w}_s\|_2} = \frac{\sum_{i=1}^K w_{i,c} w_{i,s}}{\sqrt{\sum_{i=1}^K w_{i,c}^2} \sqrt{\sum_{i=1}^K w_{i,s}^2}} \quad (6)$$

where  $K$  is the total number of keywords in the system.

For example, if user  $c$  reads many online articles on the topic of bioinformatics, then content-based recommendation techniques will be able to recommend other bioinformatics articles to user  $c$ . This is the case, because these articles will have more bioinformatics-related terms (e.g., “genome”, “sequencing”, “proteomics”) than articles on other topics, and, therefore,  $ContentBasedProfile(c)$ , as defined by vector  $\vec{w}_c$ , will represent such terms  $k_i$  with high weights  $w_{ic}$ . Consequently, a recommender system using the cosine or a related similarity measure will assign higher utility  $u(c,s)$  to those articles  $s$  that have high-weighted bioinformatics terms in  $\vec{w}_s$  and lower utility to the ones where bioinformatics terms are weighted less.

Besides the traditional heuristics that are based mostly on information retrieval methods, other techniques for content-based recommendation have also been used, such as Bayesian classifiers (Pazzani & Billsus 1997, Mooney et al. 1998) and various machine learning techniques, including clustering, decision trees, and artificial neural networks (Pazzani & Billsus 1997). These techniques differ from information retrieval-based approaches in that they calculate utility predictions based not on a heuristic formula, such as a cosine similarity measure, but rather are based on a *model* learned from the underlying data using statistical learning and machine learning techniques. For example, based on a set of Web pages that were rated as “relevant” or “irrelevant” by the user, Pazzani & Billsus (1997) use the naïve Bayesian classifier (Duda et al. 2001) to classify unrated Web pages. More specifically, the naïve Bayesian classifier is used to estimate the following probability that page  $p_j$  belongs to a certain class  $C_i$  (e.g., relevant or irrelevant) given the set of keywords  $k_{1,j}, \dots, k_{n,j}$  on that page:

$$P(C_i | k_{1,j} \& \dots \& k_{n,j}) \quad (7)$$

Moreover, Pazzani & Billsus (1997) use the assumption that keywords are independent and, therefore, the above probability is proportional to

$$P(C_i) \prod_x P(k_{x,j} | C_i) \quad (8)$$

While the keyword independence assumption does not necessarily apply in many applications, experimental results demonstrate that naïve Bayesian classifiers still produce high classification accuracy (Pazzani & Billsus 1997). Furthermore, both  $P(k_{x,j} | C_i)$  and  $P(C_i)$  can be estimated from the underlying training data. Therefore, for each page  $p_j$ , the probability  $P(C_i | k_{1,j} \& \dots \& k_{n,j})$  is computed for each class  $C_i$ , and page  $p_j$  is assigned to class  $C_i$  having the highest probability (Pazzani & Billsus, 1997).

As was observed in (Shardanand & Maes 1995, Balabanovic & Shoham 1997), content-based recommender systems have several limitations that are described in the rest of this section.

**Limited content analysis.** Content-based techniques are limited by the features that are explicitly associated with the objects that these systems recommend. Therefore, in order to have a sufficient set of features, the content must either be in a form that can be parsed automatically by a computer (e.g., text), or the features should be assigned to items manually. While information retrieval techniques work well in extracting features from text documents, some other domains have an inherent problem with automatic feature extraction. For example, automatic feature extraction methods are much harder to apply to the multimedia data, e.g., graphical images, audio and video streams. Moreover, it is often not practical to assign attributes by hand due to limitations of resources (Shardanand & Maes 1995).

Another problem with limited content analysis is that if two different items are represented by the same set of features, they are indistinguishable. Therefore, since text-based documents are usually represented by their most important keywords, content-based systems cannot distinguish between a well-written article and a badly written one, if they happen to use the same terms (Shardanand & Maes 1995).

**Over-specialization.** When the system can *only* recommend items that score highly against a user's profile, the user is limited to being recommended items similar to those already rated. For example, a person with no experience with Greek cuisine would never receive a recommendation for even the greatest Greek restaurant in town. This problem, which has also been studied in other domains, is often addressed by introducing some randomness. For example, the use of genetic algorithms has been proposed as a possible solution in the context of information filtering (Sheth & Maes 1993). In addition, the problem with over-specialization is not only that the content-based systems cannot recommend items that are different from anything the user has seen before. In certain cases, items should not be recommended if they are *too similar* to something the user has already seen, such as different news article describing the same event. Therefore, some content-based recommender systems, such as DailyLearner (Billsus & Pazzani 2000), filter out items not only if they are too different from user's preferences, but also if they are too similar to something the user has seen before.

**New user problem.** The user has to rate a sufficient number of items before a content-based recommender system can really understand user's preferences and present the user with reliable recommendations. Therefore, a new user, having very few ratings, would not be able to get accurate recommendations.

To address some of these issues, *collaborative filtering* approach to recommender systems (Resnick et al. 1994, Hill et al. 1995, Shardanand & Maes 1995) has been proposed.

## 2.2 Collaborative Methods

Collaborative recommender systems try to predict the utility of items for a particular user based on the items previously rated by other users. More formally, the utility  $u(c,s)$  of item  $s$  for user  $c$  is estimated based on the utilities  $u(c_j,s)$  assigned to item  $s$  by those users  $c_j \in C$  who are “similar” to user  $c$ . For example, in a movie recommendation application, in order to recommend movies to user  $c$ , the collaborative recommender system tries to find the “peers” of user  $c$ , i.e., other users that have similar tastes in movies (rate the same movies similarly). Then, only the movies that are most liked by the “peers” of user  $c$  would get recommended.

There have been many collaborative systems developed in the academia and the industry. The Tapestry system relied on each user to identify like-minded users manually (Goldberg et al. 1992). GroupLens (Resnick et al. 1994, Konstan et al. 1997), Video Recommender (Hill et al. 1995), and Ringo (Shardanand & Maes 1995) were the first systems to use collaborative filtering algorithms to *automate* prediction. Other examples of collaborative recommender systems include the book recommendation system from Amazon.com, MovieCritic that recommends movies on the Web, the PHOAKS system that helps people find relevant information on the WWW (Terveen et al. 1997), and the Jester system that recommends jokes (Goldberg et al. 2001).

According to Breese et al. (1998), algorithms for collaborative recommendations can be grouped into two general classes: *memory-based* (or *heuristic-based*) and *model-based*.

Memory-based algorithms (Resnick et al. 1994, Shardanand & Maes 1995, Breese et al. 1998, Nakamura & Abe 1998, Delgado & Ishii 1999) essentially are heuristics that make rating

predictions based on the entire collection of previously rated items by the users. That is, the value of the unknown rating  $r_{c,s}$  for user  $c$  and item  $s$  is usually computed as an aggregate of the ratings of some other (usually the  $N$  most similar) users for the same item  $s$ :

$$r_{c,s} = \text{aggr}_{c' \in \hat{C}} r_{c',s} \quad (9)$$

where  $\hat{C}$  denotes the set of  $N$  users  $c'$  that are the most similar to user  $c$  and who have rated item  $s$  ( $N$  can range anywhere from 1 to the number of all users). In the simplest case, the aggregation can be a simple average, as defined by expression (10a).

$$(a) r_{c,s} = \frac{1}{N} \sum_{c' \in \hat{C}} r_{c',s} \quad (b) r_{c,s} = k \sum_{c' \in \hat{C}} \text{sim}(c, c') \times r_{c',s} \quad (c) r_{c,s} = \bar{r}_c + k \sum_{c' \in \hat{C}} \text{sim}(c, c') \times (r_{c',s} - \bar{r}_{c'}) \quad (10)$$

However, the most common aggregation approach is to use the weighted sum, shown in (10b). The similarity measure between the users  $c$  and  $c'$ ,  $\text{sim}(c, c')$ , is essentially a distance measure and is used as a weight, i.e., the more similar users  $c$  and  $c'$  are, the more weight rating  $r_{c',s}$  will carry in the prediction of  $r_{c,s}$ . Multiplier  $k$  serves as a normalizing factor and is usually selected as  $k = 1 / \sum_{c' \in \hat{C}} |\text{sim}(c, c')|$ . Note that  $\text{sim}(x, y)$  is a heuristic artifact that is introduced in order to be able to differentiate between levels of user similarity (i.e., to be able to find a set of “closest peers” or “nearest neighbors” for each user) and at the same time simplify the rating estimation procedure. As shown in (10b), different recommendation applications can use their own user similarity measure, as long as the calculations are normalized using the normalizing factor  $k$ , as shown above. The two most commonly used similarity measures will be described below.

One problem with using the weighted sum, as in (10b), is that it does not take into account the fact that different users may use the rating scale differently. The *adjusted* weighted sum, shown in (10c), has been widely used to address this limitation. In this approach, instead of

using the absolute values of ratings, the weighted sum uses their deviations from the average rating of the corresponding user. In (10c), the average rating of user  $c$ ,  $\bar{r}_c$ , is defined as

$$\bar{r}_c = \frac{1}{|S_c|} \sum_{s \in S_c} r_{c,s} \quad \text{where } S_c = \{s \in S \mid r_{c,s} \neq \emptyset\} \quad (11)$$

Various approaches have been used to compute the similarity  $sim(c, c')$  between users in collaborative recommender systems. In most of these approaches, the similarity between two users is based on their ratings of items that *both* of them have rated. The two most popular approaches are *correlation-based* and *cosine-based*. To present them, let  $S_{xy}$  be the set of all items co-rated by both users  $x$  and  $y$ , i.e.,  $S_{xy} = \{s \in S \mid r_{x,s} \neq \emptyset \ \& \ r_{y,s} \neq \emptyset\}$ . In collaborative recommender systems  $S_{xy}$  is used mainly as an intermediate result for calculating the “nearest neighbors” of user  $x$  and is often computed in a straightforward manner, i.e., by computing the intersection of sets  $S_x$  and  $S_y$ . However, some methods, such as the graph-theoretic approach to collaborative filtering (Aggarwal et al. 1999), can determine the nearest neighbors of  $x$  without computing  $S_{xy}$  for the whole user base. In the correlation-based approach, the Pearson correlation coefficient is used to measure the similarity between users (Resnick et al. 1994, Shardanand & Maes 1995):

$$sim(x, y) = \frac{\sum_{s \in S_{xy}} (r_{x,s} - \bar{r}_x)(r_{y,s} - \bar{r}_y)}{\sqrt{\sum_{s \in S_{xy}} (r_{x,s} - \bar{r}_x)^2 \sum_{s \in S_{xy}} (r_{y,s} - \bar{r}_y)^2}} \quad (12)$$

In the cosine-based approach (Breese et al. 1998, Sarwar et al. 2001), the two users  $x$  and  $y$  are treated as two vectors in  $m$ -dimensional space, where  $m = |S_{xy}|$ . Then, the similarity between two vectors can be measured by computing the cosine of the angle between them:

$$sim(x, y) = \cos(\vec{x}, \vec{y}) = \frac{\vec{x} \cdot \vec{y}}{\|\vec{x}\|_2 \times \|\vec{y}\|_2} = \frac{\sum_{s \in S_{xy}} r_{x,s} r_{y,s}}{\sqrt{\sum_{s \in S_{xy}} r_{x,s}^2} \sqrt{\sum_{s \in S_{xy}} r_{y,s}^2}} \quad (13)$$

where  $\vec{x} \cdot \vec{y}$  denotes the dot-product between the vectors  $\vec{x}$  and  $\vec{y}$ . Still another approach to measuring similarity between users uses the *mean squared difference* measure and is described in (Shardanand & Maes 1995). Note that different recommender systems may take different approaches in order to implement the user similarity calculations and rating estimations as efficiently as possible. One common strategy is to calculate *all* user similarities  $sim(x,y)$  (including the calculation of  $S_{xy}$ ) in advance and recalculate them only once in a while (since the network of peers usually does not change dramatically in a short time). Then, the ratings can be efficiently calculated on demand (using precomputed similarities), e.g., whenever the user asks for a recommendation.

Note, that both the content-based and the collaborative approaches use the same cosine measure from information retrieval literature. However, in content-based recommender systems it is used to measure the similarity between vectors of TF-IDF weights, whereas in collaborative systems it measures the similarity between vectors of the actual user-specified ratings.

Many performance-improving modifications, such as *default voting*, *inverse user frequency*, *case amplification* (Breese et al. 1998), and *weighted-majority prediction* (Nakamura & Abe 1998, Delgado & Ishii 1999), have been proposed as extensions to these standard correlation-based and cosine-based techniques. For example, the default voting (Breese et al. 1998) is an extension to the memory-based approaches described above. It was observed that whenever there are relatively few user-specified ratings, these methods would not work well in computing similarity between users  $x$  and  $y$  since the similarity measure is based on the intersection of the itemsets, i.e., sets of items rated by *both* users  $x$  and  $y$ . It was empirically

shown that the rating prediction accuracy could improve if we assume some default rating value for the missing ratings (Breese et al. 1998).

Also, while the above techniques traditionally have been used to compute similarities between *users*, (Sarwar et al. 2001) proposed to use the same correlation-based and cosine-based techniques to compute similarities between *items* instead and obtaining the ratings from them. Furthermore, Sarwar et al. (2001) presents empirical evidence that item-based algorithms can provide better computational performance than traditional user-based collaborative methods, while at the same time also providing better quality than the best available user-based algorithms.

In contrast to memory-based methods, model-based algorithms (Breese et al. 1998, Billsus & Pazzani 1998, Ungar & Foster 1998, Chien & George 1999, Getoor & Sahami 1999, Goldberg et al. 2001) use the collection of ratings to learn a *model*, which is then used to make rating predictions. For example, (Breese et al. 1998) proposes a probabilistic approach to collaborative filtering, where the unknown ratings are calculated as

$$r_{c,s} = E(r_{c,s}) = \sum_{i=0}^n i \times \Pr(r_{c,s} = i \mid r_{c,s'}, s' \in S_c) \quad (14)$$

and it is assumed that rating values are integers between 0 and  $n$ , and the probability expression is the probability that user  $c$  will give a particular rating to item  $s$  given that user's ratings of the previously rated items. To estimate this probability, (Breese et al. 1998) proposes two alternative probabilistic models: cluster models and Bayesian networks. In the first model, like-minded users are clustered into classes. Given the user's class membership, the user ratings are assumed to be independent, i.e., the model structure is that of a naïve Bayesian model. The number of classes and the parameters of the model are learned from the data. The second model represents each item in the domain as a node in a Bayesian network, where the states of each

node correspond to the possible rating values for each item. Both the structure of the network and the conditional probabilities are learned from the data. One limitation of this approach is that each user can be clustered into a single cluster, whereas some recommendation applications may benefit from the ability to cluster users into several categories at once. For example, in a book recommendation application, a user may be interested in one topic (e.g., programming) for work purposes and a completely different topic (e.g., fishing) for leisure.

Moreover, (Billsus & Pazzani 1998) proposed a collaborative filtering method in a machine learning framework, where various machine learning techniques (such as artificial neural networks) coupled with feature extraction techniques (such as singular value decomposition – an algebraic technique for reducing dimensionality of matrices) can be used. Both (Breese et al. 1998) and (Billsus & Pazzani 1998) compare their respective model-based approaches with standard memory-based approaches and report that in some applications model-based methods outperform memory-based approaches in terms of accuracy of recommendations. However, the comparison in both cases is purely empirical and no underlying theoretical evidence supporting this claim is provided.

There has been several other model-based collaborative recommendation approaches proposed in the literature. A statistical model for collaborative filtering was proposed in (Ungar & Foster 1998), and several different algorithms for estimating the model parameters were compared, including K-means clustering and Gibbs sampling. Other methods for collaborative filtering include a Bayesian model (Chien & George 1999), a probabilistic relational model (Getoor & Sahami 1999), and a linear regression (Sarwar et al. 2001). Furthermore, (Kumar et al. 2001) use a simple probabilistic model to demonstrate that collaborative filtering is valuable

with relatively little data on each user, and that, in certain restricted settings, simple collaborative filtering algorithms are almost as effective as the best possible algorithms in terms of utility.

As in the case of content-based techniques, the main difference between collaborative model-based techniques and heuristic-based approaches is that the model-based techniques calculate utility (rating) predictions based not on some ad-hoc heuristic rules, but rather based on a *model* learned from the underlying data using statistical learning and machine learning techniques. A method combining both memory-based and model-based approaches was proposed in (Pennock & Horvitz 1999), where it was empirically demonstrated that the use of this combined approach can provide better recommendations than pure memory-based and model-based collaborative approaches.

A different approach to improving the performance of existing collaborative filtering algorithms was taken by (Yu et al. 2002), where the input set of user-specified ratings is carefully selected using several techniques that exclude noise, redundancy, and exploit the sparsity of the ratings' data. The empirical results demonstrate the increase in accuracy and efficiency for model-based collaborative filtering algorithms. It is also suggested that the proposed input selection techniques may help the model-based algorithms to address the problem of learning from large databases (Yu et al. 2002).

The pure collaborative recommender systems do not have certain shortcomings that content-based systems have. In particular, since collaborative systems use other users' recommendations (ratings), they can deal with any kind of content and recommend any items, even the ones that are dissimilar to those seen in the past. However, collaborative systems have their own limitations (Balabanovic & Shoham 1997, Lee 2001), as described below.

**New user problem.** It is the same problem as with content-based systems. In order to make accurate recommendations, the system must first learn the user's preferences from the ratings that the user makes. Several techniques have been proposed to address this problem. Most of them use *hybrid* recommendation approach, which combines content-based and collaborative techniques. The next section describes hybrid recommender systems in more detail. An alternative approach is presented by (Rashid et al. 2002), who explore various techniques for determining the best (i.e., most informative to a recommender system) items for a new user to rate. These techniques use strategies that are based on item popularity, item entropy, user personalization, and combinations of the above (Rashid et al. 2002).

**New item problem.** New items are added regularly to recommender systems. Collaborative systems rely solely on users' preferences to make recommendations. Therefore, until the new item is rated by a substantial number of users, the recommender system would not be able to recommend it. This problem can also be addressed using hybrid recommendation approaches, described in the next section.

**Sparsity.** In any recommender system, the number of ratings already obtained is usually very small compared to the number of ratings that need to be predicted. Effective prediction of ratings from a small number of examples is important. Also, the success of the collaborative recommender system depends on the availability of a critical mass of users. For example, in the movie recommendation system there may be many movies that have been rated only by few people and these movies would be recommended very rarely, even if those few users gave high ratings to them. Also, for the user whose tastes are unusual compared to the rest of the population there will not be any other users who are particularly similar, leading to poor recommendations (Balabanovic & Shoham 1997). One way to overcome the problem of rating

sparsity is to use user profile information when calculating user similarity. That is, two users could be considered similar not only if they rated the same movies similarly, but also if they belong to the same demographic segment. For example, Pazzani (1999) uses gender, age, area code, education, and employment information of users in the restaurant recommendation application. This extension of traditional collaborative filtering techniques is sometimes called “demographic filtering” (Pazzani 1999). A different approach for dealing with sparse rating matrices was used in (Billsus & Pazzani 1998, Sarwar et al. 2000), where a dimensionality reduction technique, Singular Value Decomposition (SVD), was used to reduce dimensionality of sparse ratings matrices. SVD is a well-known method for matrix factorization that provides the best lower rank approximations of the original matrix (Sarwar et al. 2000).

### **2.3. Hybrid Methods**

Several recommendation systems use a *hybrid* approach by combining collaborative and content-based methods, which helps to avoid certain limitations of content-based and collaborative systems (Balabanovic & Shoham 1997, Basu et al. 1998, Ungar & Foster 1998, Claypool et al. 1999, Soboroff & Nicholas 1999, Pazzani 1999, Schein et al. 2002). Different ways to combine collaborative and content-based methods into a hybrid recommender system are described in this section.

Many hybrid recommender systems, including Fab (Balabanovic & Shoham 1997) and the “collaboration via content” approach, described in (Pazzani 1999), combine collaborative and content-based approaches by (1) learning and maintaining user profiles based on content analysis using various information retrieval methods and other content-based techniques, and (2) directly comparing the resulting profiles to determine similar users in order to make collaborative recommendations. This means that users can be recommended items when items either score

highly against the user's profile or are rated highly by a user with a similar profile. Moreover, (Balabanovic & Shoham 1997) observes that content-based and collaborative approaches can be considered as special cases of the hybrid approach. If the content analysis component does not extract any features (e.g., keywords) from items and just deals with a unique item identifier, then the hybrid approach reduces to pure collaborative recommendation. Moreover, if there is only a single user, the hybrid method reduces to pure content-based recommendation. (Basu et al. 1998, Melville et al. 2002) follow similar approach and proposes to use additional sources of information (e.g., the age or gender of users or the genre of movies) to aid collaborative filtering predictions, i.e., (Basu et al. 1998, Melville et al. 2002) adds some content-based elements into the collaborative filtering framework. Similarly, (Soboroff & Nicholas 1999, Schein et al. 2002) propose to implement the hybrid approach by incorporating some elements of collaborative filtering into the content-based recommendation framework using latent semantic indexing technique.

Another approach to building hybrid recommender systems is to implement separate collaborative and content-based recommender systems. Then, we can have two different scenarios. First, we can combine the outputs (ratings) obtained from individual recommender systems into one final recommendation using either a linear combination of ratings (Claypool et al. 1999) or a voting scheme (Pazzani 1999). Alternatively, we can use one of the individual recommender systems, at any given moment choosing to use the one that is "better" than others based on some recommendation "quality" metric. For example, the DailyLearner system (Billsus & Pazzani 2000) selects the recommender system that can give the recommendation with the higher level of confidence, while (Tran & Cohen 2000) chooses the one whose recommendation is more consistent with past ratings of the user.

Yet another hybrid approach to recommendations is used by (Condliff et al. 1999, Ansari et al. 2000), where instead of combining collaborative and content-based methods the authors propose to use *all* the available information in a *single* recommendation model. Both (Condliff et al. 1999) and (Ansari et al. 2000) use Bayesian mixed-effects regression models that employ Markov chain Monte Carlo methods for parameter estimation and prediction. In particular, (Ansari et al. 2000) use the profile information of users *and* items in a single statistical model that estimates unknown ratings:

$$r_{ij} = x_{ij}\mu + z_i\gamma_j + w_j\lambda_i + e_{ij}, \quad e_{ij} \sim N(0, \sigma^2), \quad \lambda_i \sim N(0, \Lambda), \quad \gamma_j \sim N(0, \Gamma). \quad (15)$$

where  $i = 1, \dots, I$  and  $j = 1, \dots, J$  represent users and items respectively, and  $e_{ij}$ ,  $\lambda_i$ , and  $\gamma_j$  are random variables taking into effect noise, unobserved sources of user heterogeneity and item heterogeneity respectively. Also,  $r_{ij}$  is a rating assigned by user  $i$  to item  $j$ ,  $x_{ij}$  is a matrix containing user and item characteristics,  $z_i$  is a vector of user characteristics, and  $w_j$  is a vector of item characteristics. The unknown parameters of this model are  $\mu$ ,  $\sigma^2$ ,  $\Lambda$ , and  $\Gamma$ , and they are estimated from the data of already known ratings using Markov chain Monte Carlo methods. In summary, (Ansari et al. 2000) uses user attributes  $\{z_i\}$  constituting a part of a user profile, item attributes  $\{w_j\}$  constituting a part of an item profile and their combinations  $\{x_{ij}\}$  to estimate the rating of an item.

Hybrid recommendation systems can also be augmented by knowledge-based techniques (Burke 2000), such as case-based reasoning, in order to improve recommendation accuracy and to address some of the limitations (e.g., new user, new item problems) of traditional recommender systems. For example, knowledge-based recommender system *Entrée* (Burke 2000) uses some domain knowledge about restaurants, cuisines, and foods (e.g., that “seafood” is

not “vegetarian”) to recommend restaurants to its users. However, the main drawback of knowledge-based systems is a need for knowledge acquisition – a well-known bottleneck for many artificial intelligence applications.

Moreover, it was empirically demonstrated in (Balabanovic & Shoham 1997, Pazzani 1999) that hybrid methods can provide more accurate recommendations than pure collaborative and content-based approaches.

To summarize, as described in Section 2, there has been much research done on recommendation technologies over the past several years that have used a broad range of statistical, machine learning, information retrieval and other techniques and that significantly advanced the state-of-art in comparison to early recommender systems that utilized collaborative- and content-based heuristics. As was discussed in this section, recommender systems can be categorized as being (a) *content-based*, *collaborative*, or *hybrid*, based on the recommendation approach used, and (b) *heuristic-based* or *model-based* based on the types of recommendation techniques used for the rating estimation. We use these two orthogonal dimensions to classify the recommender systems research in the 2×3 matrix presented in Table 2.

### **3. Extending Capabilities of Recommender Systems**

The current generation of recommendation technologies, described in Section 2, performed well in several applications, including the ones for recommending books, CDs, and news articles (Mooney 1999, Schafer et al. 2001). However, these methods need to be extended for more complex types of applications, such as recommending vacations, financial services, and certain types of movie applications (Adomavicius et al. 2003), in order to provide better recommendations. For example, (Adomavicius et al. 2003) showed that the multidimensional

approach to recommending movies outperformed simple collaborative filtering by taking into the consideration additional information, such as when the movie is seen, with whom, and where.

Recommendation Approach	Recommendation Technique	
	Heuristic-based	Model-based
Content-based	Commonly used techniques: <ul style="list-style-type: none"> <li>• TF-IDF (information retrieval)</li> <li>• Clustering</li> </ul> Representative research: <ul style="list-style-type: none"> <li>• Lang 1995</li> <li>• Balabanovic &amp; Shoham 1997</li> <li>• Pazzani &amp; Billsus 1997</li> </ul>	Commonly used techniques: <ul style="list-style-type: none"> <li>• Bayesian classifiers</li> <li>• Clustering</li> <li>• Decision trees</li> <li>• Artificial neural networks</li> </ul> Representative research: <ul style="list-style-type: none"> <li>• Pazzani &amp; Billsus 1997</li> <li>• Mooney et al. 1998</li> <li>• Mooney 1999</li> <li>• Billsus &amp; Pazzani 1999, 2000</li> </ul>
Collaborative	Commonly used techniques: <ul style="list-style-type: none"> <li>• Nearest neighbor (cosine, correlation)</li> <li>• Clustering</li> <li>• Graph theory</li> </ul> Representative research: <ul style="list-style-type: none"> <li>• Resnick et al. 1994</li> <li>• Hill et al. 1995</li> <li>• Shardanand &amp; Maes 1995</li> <li>• Breese et al. 1998</li> <li>• Nakamura &amp; Abe 1998</li> <li>• Aggarwal et al. 1999</li> <li>• Delgado &amp; Ishii 1999</li> <li>• Pennock &amp; Horwitz 1999</li> <li>• Sarwar et al. 2001</li> </ul>	Commonly used techniques: <ul style="list-style-type: none"> <li>• Bayesian networks</li> <li>• Clustering</li> <li>• Artificial neural networks</li> <li>• Linear regression</li> </ul> Representative research: <ul style="list-style-type: none"> <li>• Billsus &amp; Pazzani 1998</li> <li>• Breese et al. 1998</li> <li>• Ungar &amp; Foster 1998</li> <li>• Chien &amp; George 1999</li> <li>• Getoor &amp; Sahami 1999</li> <li>• Pennock &amp; Horwitz 1999</li> <li>• Goldberg et al. 2001</li> <li>• Kumar et al. 2001</li> </ul>
Hybrid	Combining content-based and collaborative components using: <ul style="list-style-type: none"> <li>• Linear combination</li> <li>• Various voting schemes</li> </ul> Representative research: <ul style="list-style-type: none"> <li>• Balabanovic &amp; Shoham 1997</li> <li>• Claypool et al. 1999</li> <li>• Pazzani 1999</li> <li>• Billsus &amp; Pazzani 2000</li> <li>• Tran &amp; Cohen 2000</li> </ul>	Combining content-based and collaborative components by: <ul style="list-style-type: none"> <li>• Incorporating one component as a part of the other</li> <li>• Building one unifying model</li> </ul> Representative research: <ul style="list-style-type: none"> <li>• Basu et al. 1998</li> <li>• Condliff et al. 1999</li> <li>• Soboroff &amp; Nicholas 1999</li> <li>• Ansari et al. 2000</li> <li>• Melville et al. 2002</li> <li>• Schein et al. 2002</li> </ul>

**Table 2:** Classification of recommender systems research.

Recommender systems, as described in Section 2 and summarized in Table 2, can be extended in several ways that include improving the understanding of users and items, incorporating the contextual information into the recommendation process, supporting multi-criteria ratings, and providing more flexible and less intrusive types of recommendations. Such more comprehensive models of recommender systems can provide better recommendation capabilities. In the remainder of this section we describe the proposed extensions and also identify various research opportunities for developing them.

### **3.1. Comprehensive understanding of users and items**

As was pointed out in (Balabanovic & Shoham 1997, Ungar & Foster 1998, Konstan et al. 1998, Adomavicius & Tuzhilin 2001a), most of the recommendation methods produce ratings that are based on a limited understanding of users and items as captured by user and item profiles and do not take full advantage of the information in the user's transactional histories and other available data. For example, classical collaborative filtering methods (Resnick et al. 1994, Hill et al. 1995, Shardanand & Maes 1995) do not use user and item profiles at all for the recommendation purposes and rely exclusively on the ratings information to make recommendations. Although there has been some progress made on incorporating user and item profiles into some of the methods since the earlier days of recommender systems, e.g., (Pazzani 1999, Pennock & Horwitz 1999, Billsus & Pazzani 2000), still these profiles tend to be quite simple and do not utilize some of the more advanced profiling techniques. For example, in addition to using traditional profile features, such as keywords and simple user demographics (Pazzani & Billsus 1997, Mooney 1999), more advanced profiling techniques based on data mining rules (Fawcett & Provost 1996, Adomavicius & Tuzhilin 2001a), sequences (Manilla et al. 1995), and signatures (Cortes et al.

2000) that describe user's interests can be used to build user profiles. Similar techniques can also be used to build item profiles.

Once user and item profiles are built, the most general ratings estimation function can be defined in terms of these profiles and the previously specified ratings as follows. Let profile of user  $i$  be defined as a vector of  $p$  features, i.e.,  $\vec{c}_i = (a_{i1}, \dots, a_{ip})$ . Also, let profile of item  $j$  be defined as a vector of  $r$  features, i.e.,  $\vec{s}_j = (b_{j1}, \dots, b_{jr})$ . We deliberately did not define precisely the meanings of features  $a_{ij}$  and  $b_{kl}$  because they can mean different concepts in different applications, such as numbers, categories, rules, sequences, etc. Also, let  $\vec{c}$  be a vector of all user profiles, i.e.,  $\vec{c} = (\vec{c}_1, \dots, \vec{c}_m)$ , and let  $\vec{s}$  be a vector of all item profiles, i.e.,  $\vec{s} = (\vec{s}_1, \dots, \vec{s}_n)$ .

Then the most general rating estimation procedure can be defined as

$$r'_{ij} = \begin{cases} r_{ij}, & \text{if } r_{ij} \neq \emptyset \\ u_{ij}(R, \vec{c}, \vec{s}), & \text{if } r_{ij} = \emptyset \end{cases} \quad (16)$$

that estimates each unknown rating  $r'_{ij} = u_{ij}(R, \vec{c}, \vec{s})$  in terms of known ratings  $R = \{r_{ij} \neq \emptyset\}$ , user profiles  $\vec{c}$ , and item profiles  $\vec{s}$ . We can use various methods for estimating utility function  $u_{ij}$ , including various heuristics, nearest neighbor classifiers, decision trees, spline methods, radial basis functions, regressions, and neural networks. Moreover, we would like to point out that equation (16) presents the most general model that depends on a whole range of inputs, including the characteristics of user  $i$  ( $\vec{c}_i$ ) and possibly other users  $\vec{c} = (\vec{c}_1, \dots, \vec{c}_m)$ , characteristics of item  $j$  ( $\vec{s}_j$ ), and possibly other items  $\vec{s} = (\vec{s}_1, \dots, \vec{s}_n)$ , ratings (preferences)  $R_i$  expressed by user  $i$  and ratings (preferences) expressed by all other users  $R = \{r_{ij} \neq \emptyset\}$ . Therefore, function  $u_{ij}$  clearly subsumes collaborative, content-based and hybrid methods discussed in Section 2. However, most of the existing recommender systems make function  $u_{ij}$  dependent only on a (small) subset

of the whole input space  $R$ ,  $\bar{c}$ , and  $\bar{s}$ . For example, function  $u_{ij}$  for traditional memory-based collaborative filtering methods does not depend on inputs  $\bar{c}$  and  $\bar{s}$  and restricts  $R$  only to column  $R_j$  and usually only to the set of  $N$  nearest neighbors  $r_{ij}$  for column  $R_j$ .<sup>1</sup>

An interesting research problem would be to extend the attribute-based profiles, as defined by  $\bar{c}$  and  $\bar{s}$ , to utilize more advanced profiling techniques described above, such as rule-, sequence-, and signature-based methods.

### 3.2. Extensions for Model-Based Recommendation Techniques

As discussed in Section 2, some of the model-based approaches provide rigorous rating estimation methods utilizing various statistical and machine learning techniques. However, other areas of mathematics and computer science, such as *mathematical approximation theory* (Powell 1981, Nurnberger 1989, Buhmann 2001), can also contribute to developing better rating estimation methods defined by equation (16). One example of an approximation-based approach to defining function  $u_{ij}$  in (16) constitutes *radial basis functions* (Duchon 1979, Schaback & Wendland 2001, Buhmann 2001) that are defined as follows. Given a set of points  $X = \{x_1, \dots, x_m\}$  (where  $x_i \in \mathbb{R}^N$ ) and the values of an unknown function  $f$  (e.g., the rating function) at these points, i.e.,  $f(x_1), \dots, f(x_m)$ , a radial basis function  $r_{f,X}$  estimates the values of  $f$  in the whole  $\mathbb{R}^N$ , given  $r_{f,X}(x_i) = f(x_i)$  for all  $i = 1, \dots, m$ , as

$$r_{f,X}(x) = \sum_{i=1}^m \alpha_i \phi(\|x - x_i\|) \quad (17)$$

where  $\{\alpha_1, \dots, \alpha_m\}$  are coefficients from  $\mathbb{R}$ ,  $\|x\|$  is a norm (e.g.,  $L_2$ ) and  $\phi$  is a positive definite function, i.e., a function satisfying the condition

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<sup>1</sup> Actually, the situation is a little more complicated than this because estimation of nearest neighbors may involve other values of matrix  $R$  for some of the collaborative filtering methods.

$$\sum_{i=1}^m \sum_{j=1}^m \alpha_i \alpha_j \phi(\|x_i - x_j\|) > 0 \quad (18)$$

for all distinct points  $x_1, \dots, x_m$  in  $\mathbb{R}^N$  and all the coefficients  $\alpha_1, \dots, \alpha_m$  from  $\mathbb{R}$ . Then a well-known theorem (Schaback & Wendland 2001) states that if  $\phi$  is a positive definite function then there exists a unique function  $r_{f,X}$  of the form (17) satisfying the conditions  $r_{f,X}(x_i) = f(x_i)$  for all  $i = 1, \dots, m$ . Some popular examples of positive definite functions  $\phi$  are:

1.  $\phi(r) = r^\beta$ , where  $\beta > 0$  is a positive odd number;
2.  $\phi(r) = r^k \log(r)$ , where  $k \in \mathbb{N}$  (thin-plate splines);
3.  $\phi(r) = e^{-\alpha r^2}$  where  $\alpha > 0$  (Gaussian).

One of the advantages of radial basis functions is that they have been extensively studied in the approximation theory, and their theoretical properties and utilization of radial basis functions in many practical applications have been understood very well (Schaback & Wendland 2001, Buhmann 2001). Therefore, it should be interesting to apply them for estimating unknown ratings in recommender systems.

One caveat with using radial basis functions in recommender systems, though, is that the recommendation space  $\bar{c} \times \bar{s}$  does not usually constitute an  $N$ -dimensional Euclidean space  $\mathbb{R}^N$ . Therefore, one research challenge is to extend radial basis methods from the real numbers to other domains and apply them to recommender systems problems. The applicability of other approximation methods for estimating  $u_{ij}$  in (16) constitutes another interesting research topic.

### 3.3. Multidimensionality of recommendations

Current generation of recommender systems operates in the two-dimensional *User*×*Item* space. That is, they make their recommendations based only on the user and item information and do not take into the consideration additional *contextual* information that may be crucial in some

applications. However, in many situations the utility of a certain product to a user may depend significantly on time (e.g., the time of the year, such as season or month, the day of the week, or the time of the day, such as morning or evening). It may also depend on the person(s) with whom the product will be consumed or shared and under which circumstances. In such situations it may not be sufficient to simply recommend items to users; the recommender system must take additional contextual information, such as time, place, and the company of a user, into the consideration when recommending a product. For example, when recommending a vacation package, the system should also consider the time of the year, with whom the user plans to travel, traveling conditions and restrictions at that time, and other contextual information. As another example, a user can have significantly different preferences for the types of movies she wants to see when she is going out to a movie theater with a boyfriend on a Saturday night as opposed to watching a rental movie at home with her parents on a Wednesday evening. As was argued in (Adomavicius & Tuzhilin 2001b), it is important to extend traditional two-dimensional *User×Item* recommendation methods to multi-dimensional settings. In addition, (Herlocker & Konstan 2001) argued that the inclusion of the knowledge about user’s task into the recommendation algorithm in certain applications can lead to better recommendations.

In order to take into the consideration the contextual information (e.g., the date and time, the company of other people, and the medium in case of recommending movies), Adomavicius & Tuzhilin (2001b) propose to define the utility (or ratings) function over a multidimensional space  $D_1 \times \dots \times D_n$  (as opposed to the traditional 2-dimensional *User×Item* space) as

$$u : D_1 \times \dots \times D_n \rightarrow R \quad (19)$$

Then a recommendation problem is defined by selecting certain “what” dimensions  $D_{i1}, \dots, D_{ik}$  ( $k < n$ ) and certain “for whom” dimensions  $D_{j1}, \dots, D_{jl}$  ( $l < n$ ) that do not overlap, i.e.,

$\{D_{i_1}, \dots, D_{i_k}\} \cap \{D_{j_1}, \dots, D_{j_l}\} = \emptyset$  and recommending for each tuple  $(d_{j_1}, \dots, d_{j_l}) \in D_{j_1} \times \dots \times D_{j_l}$  the tuple  $(d_{i_1}, \dots, d_{i_k}) \in D_{i_1} \times \dots \times D_{i_k}$  that maximizes the utility  $u(d_1, \dots, d_n)$ , i.e.,

$$\forall (d_{j_1}, \dots, d_{j_l}) \in D_{j_1} \times \dots \times D_{j_l}, \quad (d_{i_1}, \dots, d_{i_k}) = \underset{\substack{(d'_{i_1}, \dots, d'_{i_k}) \in D_{i_1} \times \dots \times D_{i_k} \\ (d'_{j_1}, \dots, d'_{j_l}) = (d_{j_1}, \dots, d_{j_l})}}{\arg \max} u(d'_1, \dots, d'_n) \quad (20)$$

For example, in the case of a movie recommender system one needs to consider not only characteristics of the movie  $d_1$  and of the person who wants to see the movie  $d_2$ , but also such contextual information as (a)  $d_3$ : where and how the movie will be seen (e.g., in the movie theater, at home on TV, on video or DVD – one may want to see “Harry Potter” on a big screen in a movie theater rather than on a DVD), (b)  $d_4$ : with whom the movie will be seen (e.g., alone, with girlfriend/boyfriend, friends, parents, etc.), and (c)  $d_5$ : when will the movie be seen (e.g., on weekdays or weekends, in the morning/afternoon/evening, during the opening night, etc.). As discussed earlier, each of the components  $d_1, d_2, d_3, d_4, d_5$  can be defined as a *vector* of its characteristics, and the overall utility function  $u(d_1, d_2, d_3, d_4, d_5)$  can be quite complex and take into consideration various interaction effects among vectors  $d_1, d_2, d_3, d_4, d_5$ .

As was argued in (Adomavicius & Tuzhilin 2001b), many of the two-dimensional recommendation algorithms cannot be directly extended to the multidimensional case. Furthermore, (Adomavicius et al. 2003) identify several different approaches for rating estimation in multidimensional settings: reduction-based, heuristic-based, and model-based. For example, one possible model-based approach would be the hierarchical Bayesian method presented in (Ansari et al. 2000) that can be extended from 2- to multi-dimensional case as follows. Instead of considering the two-dimensional case, as defined in (15), where user characteristics  $d_1$  are defined with vector  $z_i$  and item characteristics  $d_2$  with vector  $w_j$ , we can also add contextual dimensions  $d_3, \dots, d_n$ , where  $d_i = (d_{i_1}, \dots, d_{i_{x_i}})$  is a vector of characteristics

for dimension  $D_i$ . Then the rating function  $r = u(d_1, d_2, \dots, d_n)$  is extended from (15) to the linear combination of  $d_1, d_2, \dots, d_n$  and also includes interaction effects among these dimensions (i.e., interaction effects, as defined by matrix  $\{x_{ij}\}$  in (15), should be extended to include other dimensions). One of the research challenges is to make these extensions scalable for large values of  $n$ .

### **3.4. Multi-criteria ratings**

Most of the current recommender systems deal with single-criterion ratings, such as ratings of movies and books. However, in some applications, such as restaurant recommenders, it is crucial to incorporate multi-criteria ratings into recommendation methods. For example, many restaurant guides, such as Zagat's Guide, provide three criteria for restaurant ratings: food, decor and service.

Although multi-criteria ratings have not yet been examined in the recommender systems research literature, they have been extensively studied in the OR community over the past few decades (Statnikov & Matusov 1995, Ehrgott 2000). Typical solutions to the multi-criteria optimization problems include (a) finding Pareto optimal solutions, (b) taking a linear combination of multiple criteria and reducing the problem to the single-criterion optimization problem, (c) optimizing only one most important criterion and converting other criteria to constraints, (d) consecutively optimizing one criterion at a time, converting an optimal solution to constraint(s) and repeating the process for other criteria. A typical example of this last approach is the so-called method of successive concessions (Statnikov & Matusov 1995).

To illustrate how some of these methods can be used in recommender systems, consider the application of approach (c) to the problem of recommending restaurants  $r$  to user  $c$  based on the user's criteria of food quality  $f_c(r)$ , décor  $d_c(r)$ , and service  $s_c(r)$ . We can take food

quality  $f_c(r)$  to be the primary criterion and use others as constraints, i.e., we want to find restaurants  $r$  that maximize  $f_c(r)$ , subject to the constraints  $d_c(r) > \alpha_c$  and  $s_c(r) > \beta_c$ , where  $\alpha_c$  and  $\beta_c$  are minimal ratings for décor and service (e.g., user  $c$  will not go to any restaurant having décor and service ratings below 10, out of possible 30, regardless of the quality of food there). This problem is complicated by the fact that we usually will not have the user's décor  $d_c(r)$  and service  $s_c(r)$  ratings for all the restaurants. Then the task of a recommender system is to estimate unknown ratings  $d'_c(r)$  and  $s'_c(r)$ , e.g., using the rating estimation methods described in Section 2, and find all the restaurants  $r$  satisfying constraints  $d'_c(r) > \alpha_c$  and  $s'_c(r) > \beta_c$ . Once we find all the restaurants satisfying the constraints with these estimated ratings, we can use *those* restaurants in search for the maximum of  $f_c(r)$ . However, as was the case with décor and service ratings, we might not usually have the user's food ratings  $f_c(r)$  for all such restaurants and, thus, will need to use rating estimation procedure for  $f_c(r)$  as well, before we are able to make any recommendations.

We believe that the problem of finding Pareto-optimal solution set and the iterative method of consecutive single criterion optimizations for multi-criteria recommendation problems mentioned above should also constitute interesting and challenging problems.

### **3.5. Non-intrusiveness**

Many recommender systems are intrusive in the sense that they require explicit feedback from the user and often at a significant level of user involvement. For example, before recommending any newsgroup articles, the system needs to acquire ratings of previously read articles, and often many of them. Since it is impractical to elicit many ratings of these articles from the user, some recommender systems use non-intrusive rating determination methods where certain proxies are

used to estimate real ratings. For example, the amount of time a user spends reading a newsgroup article can serve as a proxy of the article's rating given by this user. Some non-intrusive methods of getting user feedback are presented in (Konstan et al. 1997, Caglayan et al. 1997, Oard & Kim 1998, Schein et al. 2002). However, non-intrusive ratings (such as time spent reading an article) are often inaccurate and cannot fully replace explicit ratings provided by the user. Therefore, the problem of minimizing intrusiveness while maintaining certain levels of accuracy of recommendations needs to be addressed by the researchers working on recommender systems.

One way to explore the intrusiveness problem is to determine an optimal number of ratings the system should ask from a new user. For example, before recommending any movies, MovieLens.org first asks the user to rate a predefined number of movies (e.g., 20). This request incurs certain costs on the end-user that can be modeled in various ways, the simplest model being a fixed-cost model (i.e., the cost of rating each movie is  $C$  and the cost of rating  $n$  movies is  $C \cdot n$ ). Then the intrusiveness problem can be formulated as an optimization problem that tries to find an optimal number of initial rating requests  $n$  as follows. Each additional rating supplied by the user increases the accuracy of recommendations<sup>2</sup> and, therefore, results in certain benefits for the user. One interesting intrusiveness-related research problem would be to develop formal models for defining and measuring benefit  $B(n)$  of supplying  $n$  initial ratings in terms of the increased accuracy of predictions based on these ratings. Once it is known how to measure benefits  $B(n)$  (e.g., by measuring the predictive accuracy of a recommender system), we need to determine an optimal number of initial ratings  $n$  that maximizes expression  $B(n) - C \cdot n$ . Clearly, optimal value of  $n$  is reached when marginal benefits are equal to marginal costs, i.e., when

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<sup>2</sup> ... accuracy of recommendations or any other effectiveness measure, as described in Section 3.7.

$\Delta B(n) = C$ . The optimal solution should exist under the assumption that  $B(n)$  is a monotonically increasing function in  $n$  with decreasing marginal benefits  $\Delta B(n)$  that asymptotically converge to zero.

Another interesting research opportunity lies in developing marginal cost models that are more advanced than the fixed-cost model described above and that can potentially include cost/benefit analysis of using *both* implicit and explicit ratings in a recommender system.

### 3.6. Flexibility

Most of the recommendation methods are inflexible in the sense that they are “hard-wired” into the systems by the vendors and therefore support only a predefined and fixed set of recommendations. Therefore, the end-user has limited capabilities to customize the types of recommendations according to the user’s recommendation needs in real time. This problem has been identified in (Adomavicius & Tuzhilin 2001b), where a Recommendation Query Language (RQL) and OLAP capabilities have been proposed to address it. RQL is an SQL-like language for expressing flexible user-driven recommendation requests. For example, the request “recommend to each user from New York the best three movies that are longer than two hours” can be expressed in RQL as:

```
RECOMMEND Movie TO User  
BASED ON Rating  
SHOW TOP 3  
FROM MovieRecommender  
WHERE Movie.Length > 120 AND User.City = “New York”.
```

Also, most of the recommender systems are inflexible since they do not make recommendations at different levels of granularity, i.e., they usually recommend individual items to individual users, and cannot recommend groups of items to groups of users. In some applications it is important to be able to recommend *brands* or *categories* of products to certain

*segments* of users (Adomavicius & Tuzhilin 2001b). For example, a travel-related recommender system may want to recommend vacations in Florida (category of products) to the undergraduate students from the Northeast (user segment) during the spring break. An interesting research problem would be to incorporate OLAP-based methods (Chaudhury & Dayal 1997) into recommender systems.

### **3.7. Effectiveness of recommendations**

The problem of developing good metrics to measure effectiveness of recommendations has been extensively addressed in the recommender systems literature. Some examples of this work include (Mooney 1999, Herlocker et al. 1999, Yang & Padmanabhan 2001). However, in most of the recommender systems literature, the evaluation of a particular recommendation algorithm is usually limited only to testing its performance in terms of the coverage and accuracy metrics. Coverage measures the percentage of items for which a recommender system is capable of making predictions (Herlocker et al. 1999). Accuracy measures can be either *statistical* or *decision-support* (Herlocker et al. 1999). Statistical accuracy metrics mainly compare the estimated ratings (e.g., as defined in (16)) against the actual ratings  $R$  in the  $User \times Item$  matrix, and include Mean Absolute Error (MAE), root mean squared error, and correlation between predictions and ratings. Decision-support measures determine how well a recommender system can make predictions of high-quality items (Herlocker et al. 1999). They include classical IR measures of *precision* (the percentage of ratings classified as positive that are indeed positive) and *recall* (the percentage of positive ratings classified as positive), *F-measure* (combined effect of precision and recall), and Receiver Operating Characteristic (ROC) measure demonstrating the tradeoff between *true positive* and *false positive* rates in recommender systems (Herlocker et al. 1999).

Although crucial for measuring accuracy of recommendations, these technical measures often do not capture adequately “usefulness” and “quality” of recommendations. For example, as (Yang & Padmanabhan 2001) observe for a supermarket application, recommending obvious items, such as milk or bread, that the consumer will buy anyway will produce high accuracy rates; however, it will not be very helpful to the consumer. Therefore, it is also important to develop economics-oriented measures that capture the business value of recommendations, such as return on investments (ROI) and customer lifetime value (CLTV) measures (Schmittlein et al. 1987, Dwyer 1989, Rosset et al. 2002). Developing and studying such measures constitutes an interesting research topic in recommender systems.

### **3.8. Other Extensions**

Other important research issues that have been explored in recommender systems literature include explainability (Billsus & Pazzani 1999, Herlocker et al. 2000), trustworthiness (Dellarocas 2002), scalability (Aggarwal et al. 1999, Goldberg et al. 2001, Schafer et al. 2001, Sarwar et al. 2001), and privacy (Schafer et al. 2001, Ramakrishnan et al. 2001) issues of recommender systems. However, we will not review this work and discuss research opportunities in these areas in this paper because of the space limitation.

## **4. Conclusions**

In this paper we reviewed current collaborative, content-based, and hybrid methods for recommending items to users. One of the key problems in recommender systems pertains to estimating unknown ratings in terms of the known ratings, and we reviewed how different methods compute such estimations. Moreover, we reviewed various limitations of the current recommendation methods and discussed possible extensions that can provide better recommendation capabilities. These extensions included, among others, improvement of

understanding of users and items, incorporation of the contextual information into the recommendation process, support for multi-criteria ratings, and provision of more flexible and less intrusive types of recommendations.

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