Impact of Search Engine Characteristics on Electronic Markets and Sellers' Pricing Strategies

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Center for Digital Economy Research
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Working Paper IS-98-26

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1/10/99

Abstract

Internet-based electronic markets facilitate buyer search for seller offerings and comparison of products on the basis of price and product features. Search engine capabilities such as Recall, Precision and Ranking Accuracy determine the efficiency with which buyers can search for and compare products and the resulting buyer surplus and seller profits. This research investigates the impact of each of these factors on buyer and seller strategies at equilibrium. This paper explains certain counter intuitive market phenomena where some successful electronic markets offer less choice to buyers than their competitors. The analysis is driven by a set of analytical models of an electronic market under varying conditions of sellers' market shares, buyer search strategies and search engine technology. The models developed here draw from existing theories in information economics and computer science. The results demonstrate that precision has a greater direct impact on buyer surplus than recall and that buyers will forego some choice in exchange for greater accuracy of product description, especially in products that are complex or are characterized by ambiguity in product description and terms of trade. Finally, the seller with the greatest market share stands to gain most from offering search engine services to buyers and in a market characterized by two or more sellers with significant market shares, search engine services will always be offered free of cost to buyers.

1 Introduction

The Internet has emerged as an effective forum for electronic commerce that integrates widely dispersed buyers and sellers and encourages them to transact by adopting both synchronous and asynchronous buying and selling activities. This in turn has resulted in the rapid growth of Internet based electronic markets. There are numerous products that are being bought and sold on the Internet. The following examples serve to illustrate a trend: Cisco Systems, a network equipment manufacturer that sells products from its web site amounting to $1 billion a year, General Electric buys $1 billion worth of goods from On-line suppliers, Dell Computers sells $ 1 million worth of PC's a day on the Web[Anderson 97].

Electronic markets are not restricted to a class of products such as Computers and Software or to a type of transaction such as inter-firm buying and selling. A survey published in 1997 by CommerceNet, Nielsen, a media research firm, found that 73% of Internet users had used the web for shopping in one way or the other in the past month. A survey published in March 1997 by CommerceNet/Nielsen found that while 53% of Internet users in Canada had used the Internet to reach a decision on a purchase, just 15% carried out the final transaction on the web.[Anderson 97]This trend serves to highlight another important feature of electronic markets - electronic markets offer buyers a convenience; the ability to *search for products*, research the offerings that have been identified and compare the offerings based on the *product's features, quality and price at a (comparatively) low cost.* The ability of electronic markets to enable search and comparison contributes significantly to the buyers' decision to buy a seller offering.

Electronic markets make it possible for buyers to access and evaluate product offerings from different sellers and compare the product offerings available in the market. The availability of search engines which lower the costs of searching will result in a move toward frictionless and competitive markets and lower prices.[Bakos 1987][Bakos 1991] This observation motivates an issue central to this research. This paper will examine the impact of search engine technology on market outcomes in an electronic market and examine how market welfare and buyer surplus are affected by choice of technological features.

Two factors that determine whether electronic markets will truly deliver the benefits to buyers from competition amongst sellers are: *the number of seller offerings* featured in the market and the *accuracy of product representation and description.*

Since electronic markets are an emerging phenomenon, the number of products in an electronic market is often a fraction of the number and variety of products found in traditional marketplaces. Since the extent of competition between sellers in an electronic market is determined by the number of products in the market, this factor can impact buyer welfare in an marketplace. Electronic markets reduce buyer search costs in identifying suitable products and comparing them across sellers based on price and terms of trade. In order that the benefits of reduced transactions be realized by buyers, it is necessary that the products are accurately represented in a searchable database and the search engine is able to precisely match product features with buyers' preferences. If a search results in several poorly matched products being returned, buyers will have to inspect each product in considerable detail before they can identify a candidate product that maximizes their utility from purchase. Since extensive human inspection can be a costly process, especially in the case of information rich products or products whose attributes are subject to considerable ambiguity in terms of trade and transaction settlement, buyers may well settle for sub-optimal products to minimize inspection costs[1]. Therefore, the accuracy of the search engine can be a crucial factor in determining market outcomes such as seller profits and buyer welfare in a market for differentiated products.

Both these factors search engine accuracy and market coverage (number of products in the market) have cost implications. Accuracy of the search engine depends on the granularity of the search engine's internal representation mechanism and the extent to which it has to process product descriptions in order to match them to buyer specifications. The extent of market coverage implies the number of products that are indexed and stored by the search engine in an searchable repository (database) and it is reasonable to argue that larger the repository, greater the cost of sorting, indexing and maintenance. An increase in the number of products in the market with a simultaneous increase in accuracy, can have a combined effect on cost of operation that exceeds the resulting increase in buyer welfare. While reduced

[1]Buyers will actually minimize the total costs resulting from product inspection, search, price and disutility of the candidate product.

transaction costs benefits of electronic markets accrue to buyers, the resulting operational costs of search engine have to be borne by sellers or buyers or some combination of both[2]. This motivates yet another issue that this research will address; the conditions that make it optimal for sellers to offer search engine services to buyers. In summary the three questions that are addressed by this research are as follows:

1. What are the impacts of market coverage[3] (number of products in the market) and search accuracy[4] on buyers and sellers[5]?
2. When a trade off has to be made between extent of market coverage and accuracy, which factor has greater impact on buyer welfare?
3. Under what conditions will sellers offer search services free of cost to buyers?

The remainder of this paper is organized as follows: in section 2, different aspects of search engine technology will be discussed and the terms that will be employed in the modeling and analysis exercise will be defined rigorously. In section 3 the base model is developed and in section 4, the factor of imperfect recall is added to the model. In section 5 the model further developed by considering the impact of imperfect precision. In section 6, conclusions and directions for further research are presented. The bulk of analysis and modeling are in sections 3,4 and 5 where closed form equations that represent different phenomena under analysis, are derived and discussed.

2 Impact of Search Engines on Electronic Mar-

[2]Third parties may provide search engines and charge buyers and sellers for their services.

[3]The term Recall that will be defined in section II is a precise measure of market coverage.

[4]Search Engine accuracy will be measured by "Precision", a term that will be defined in Section II.

[5]The impact on buyers and sellers will be analyzed in terms of total welfare of the market and buyer surplus wherever appropriate.

kets

The role played by search engines assumes considerable significance due to two different but not unrelated characteristics of electronic markets:

1. Electronic markets are characterized by their global reach which has resulted in a large number of geographically distributed participants transacting on a forum that lacks structure or universally defined and accepted standards for specification of the terms of trade.

2. The proliferation of a variety of products and sellers who offer these products. In electronic markets for products such as books, music, computers, cars, airline tickets etc. there is considerable variety in product offerings resulting in a high degree of *product differentiation* and several sellers for any given product.

2.0.1 An Overview of the Role of Search in Seller Location and Pricing

The study of a firm's strategic choices over price and/or location within spatial models has been studied by several researchers following the work of Hotelling in 1929 [Hotelling 1929]. Takahashi and Andre de Palma observed that the introduction of space relaxes price competition between firms because firms scattered over geographical space acquire some degree of monopoly power [Depalma 1979]. Hotelling, Greenhut, Norman and Hung have pointed out that the spatial model can be interpreted as a model of product location in characteristics space, allowing it to be extended to describe competition among firms selling products with horizontally differentiated qualities [Greenhut 1987].

The dimension of search in spatially distributed markets has been studied by different authors following Stigler's seminal work (1961) titled "The economics of Information" in which he explained the causes for price dispersion, impact of search costs on buyers and sellers and the factors that gave firms monopoly pricing power[Stigler 1961]. It has been shown by Diamond (1971) that under certain assumptions of buyer behavior and in the presence of search costs, at equilibrium, all sellers charge the same price [Diamond 1971]. This result was extended by Salop and Stiglitz (1977) to show that the equilibrium price charged by all sellers is the price that would have been charged by a monopolist[Salop 1977]. The restrictive nature of these models were analyzed by Bakos who observed that the models suffered from the following limitations[Bakos 1991]:

1. Variability of seller prices was exogenous and disappeared when the models were closed allowing profit maximizing behavior on the seller side. The equilibrium price converged to an identical price across all sellers.

2. The absence of price dispersion at equilibrium meant that buyers had no motivation to search which undermined the original premise of the models.

These models did succeed in demonstrating that even small search costs could lead to prices substantially higher than marginal costs when sellers behaved competitively. The impact of search engine technology on an electronic market has not been studied by economists since search engines are recent phenomena. This is an emerging area in the field of electronic commerce and represents the technological frontier in the retrieval of Web based distributed information.

2.1 Directed Search and Random Search

The phenomenon of search and its impact on prices, seller location and buyer behavior has been extensively studied in economics. It is necessary to point out an important difference in meanings of the term search as it has been used in economics and the concept of search as used in the context of this research. In economics, search is essentially seen as *random search* where buyers (or sellers) have to search a bounded space (usually geographically bounded) for the existence of sellers or for the existence of products that match their requirements. The buyer has no way of ascertaining if some locations are more likely to have sellers than others, nor does she have a way of selectively excluding (or including) certain products or features of products from her search space. In the case of random search, a buyer cannot begin by excluding products beyond a certain price range from her search space. In other words, *buyers do not have the ability to refine the search space by excluding certain products* based on either price or one or more product features. As opposed to this, I use the term Search to indicate **Directed**

Search using a search engine, where the buyer has the capacity to include or exclude several areas of the search space from the search by specifying features of the product to the search engine. The process of Directed Search is characterized by the following search features:

1. Buyers can specify a product space that is to be searched based on either the product features or sellers' characteristics.

2. Buyers can restrict the search space to products within a pre-determined price range.

3. Buyers can specify price-product feature combinations to the search engine that define the search space.

4. Buyers can specify a *search order*, which results in the search engine searching certain sub-spaces first before it searches others.

These factors make the role of search and search engines specially significant in electronic markets. Search Engine features such as **Recall, Precision and Ranking Accuracy** can influence the behavior of buyers and sellers and can have an impact on the price discovery mechanism as well as price dispersion in these markets.

2.2 Recall and Precision

These terms have their origins in the field of information retrieval [?]. In the field of information retrieval, recall refers to the fraction of relevant documents that are retrieved from the universe of all documents that satisfy the search conditions. In the context of electronic markets, a search engine's recall is the ratio of the number of relevant seller offerings indexed and stored in the search engine's database to the total number of relevant seller offerings in the market. Consider the following example. A search engine operating in a market for cameras has a collection of 15 cameras in its database that match a set of buyer preferences out of a total of 20 cameras in the market that satisfy the same set of specifications. The recall of this search engine is said to be 0.75 $\left(\frac{15}{20} = 0.75\right)$. More formally, recall is defined as follows:

$$\rho = \frac{\text{Number of products retrieved that meet the buyer's search criteria}}{\text{Total number of products that meet the buyer's search criteria in the market}}$$

Intuitively, it is clear that in an electronic market for a differentiated product, a search engine with a higher recall retrieves both a greater variety and a greater number of products.

Precision measures the relevance of the items that are retrieved by the search engine. Search engines that operate in the domain of document retrieval rank the results of search and provide confidence scores for each document. The extent to which the confidence score matches the relevance of the document depends on the precision of the search engine. Similarly in an electronic market, the search engine estimates the utility of each product to the buyer based on the buyer's preferences. Greater the precision of the search engine, closer is the estimate to the actual utility of the product to the buyer. From here onwards, the term disutility rather than utility will be employed in analysis and modeling. Since a buyer a specifies his ideal product to the search engine, all products other than her ideal product have disutility implications to the buyer. It is this disutility that will be sought to be minimized in the modeling that follows. Precision is defined in terms of disutility as follows:

$$\mu = 1 - \frac{|\text{Actual Disutility of the Product - Estimated Disutility of the Product}|}{\text{Actual Disutility of the Product}}$$

Poor precision in a search engine results in inaccurate estimates of products' location in product space which necessitates buyer inspection of seller offerings.

3 Model of an Electronic Market with a Search Engine

In this section analytical models will be developed to investigate the three issues discussed earlier. Each model will be followed by analysis and a brief discussion of findings. The models are defined in terms of certain factors that drive market outcomes such as buyer and seller surplus, fees that sellers can charge for use of search engines and seller profits when they offer search engines. These factors are: recall, precision, cost of technology, market size, market shares of different sellers and cost of inspecting products (incurred by buyers).

3.1 Outline of the Model

I present a model of an electronic market containing buyers and sellers and an electronic search mechanism (which will be referred to as the search engine), that allows buyers to specify their preferences and locate seller offerings. There are some basic features that are applicable to all the situations that will be discussed in the sections that follow. These are: the product space, seller offerings, buyers characteristics and the search process. A basic model with these features as the primitives of modeling will be created and specific complexities such as imperfect recall and precision will be added to augment the models in later sections.

3.2 Product Space

The market consists of a highly differentiated product[6] that has a location in two dimensional space given by two coordinates (x, y). The product (or service) may have several attributes. The attributes themselves can be classified into two mutually exclusive categories. An attribute is either characterized by a simple product description that can be unambiguously understood by a search engine or it has a complex product description and has numerous conditions associated with it[7].

To understand what these terms mean, consider the following example. Airline tickets can be said to have two kinds of attributes. The departure time, flying time and arrival time are all unambiguously described and can be understood by a search engine. The maximum number of stopovers, the minimum and maximum separation (in terms of number of days) between onward and return trips, the minimum notice period for cancellation, the cancellation fee etc. are all subject to restrictions. While a seller may list a cancellation fee as \$50, the actual amount may be greater depending on a set of rules (cancellation fee for promotional offers may be higher, the charges may be higher for certain times of the year when travel is volume is lower etc.) The minimum notice period or the cancellation fee may also depend on the conditions under which the ticket is offered or on the number of stopovers.

[6] This anlysis would hold for a differentiated service as well.

[7] Both kinds of attributes, simple and complex, used in this model are measurable. Attributes that are not measurable such as color, texture etc., will not be considered.

For *expositional simplicity, we refer to the two sets of attributes as* ***simple*** *and* ***complex*** *attributes* in the remainder of this dissertation. The resultant of all simple attributes is represented by x (which is a combination of these values according to some predefined rule). Similarly, those attributes that are complex (but measurable nevertheless) have a resultant value (calculated by combining the measured values according to some mathematical function) y. Together, the two dimensions define a product uniquely.

3.3 Seller Offerings

1. Two products that vary in even one attribute, will have two different coordinates in product space. Therefore, each seller offering is uniquely represented by a point in space.

2. There are N_0 seller offerings in the market (where N_0 is some suitably large number) all of which are priced at the same price $= P^*$.

3. There are N seller offerings which are uniformly and randomly distributed within the buyer's search domain[8].

3.4 Buyers

Figure 2 below is a graphical representation of Buyers Preferences distributed along the two dimensions of product differentiation.

1. Buyers' product preferences are heterogenous and randomly distributed along the same two dimensions (see Figure 2).

2. A Buyer's preferences define her ideal product, whose location is given by (x_0, y_0) in product space. The buyer experiences a disutility for products whose attribute values do not coincide with her ideal product's attribute values. This disutility or cost of mismatch is a function of the distances between the seller offering's attribute values and the buyer's ideal product attribute values. In this model, buyers have an

[8]The term **search domain** is the area arround the buyer's position in which the buyer will search. A buyer will not look for products outside this area. We will define this term precisely in the section that follows.

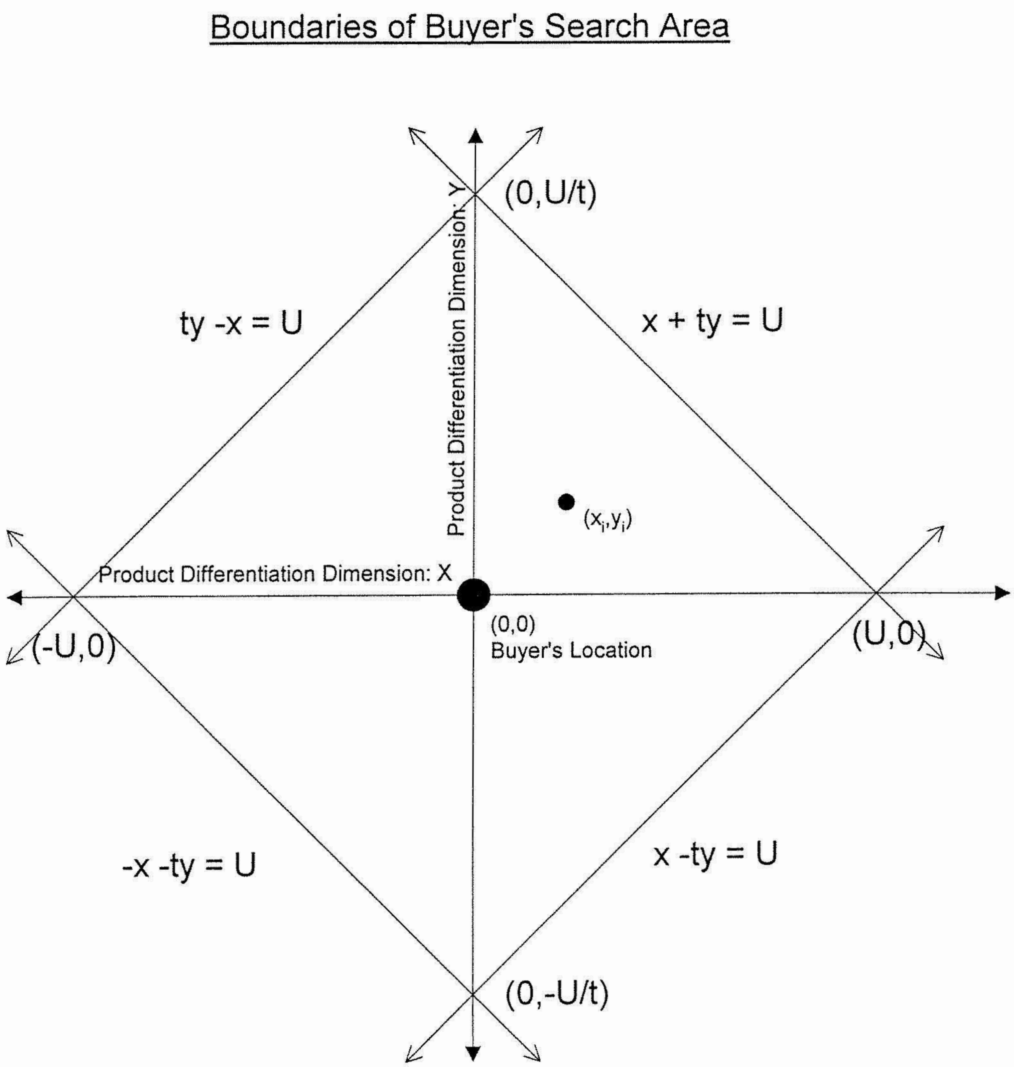

Figure 1: Boundaries of Buyer Search

identical cost of mismatch t, which is known to sellers. The value of this parameter is stored in the search engine.

A buyer who is located at $[x_{0,}y_0]$ experiences a disutility for a seller offering located at $[x_{1,}y_1]$ which is given by[9]:

$$D = |x_1 - x_0| + t \times |y_1 - y_0|$$

3. A buyer can search for products and inspect the products that are returned by the search engine. In general, the i^{th} seller offering (x_i, y_i), located at a distances of x_i and y_i from the location of the buyer (as shown in Figure 2), along the two product dimensions and whose price is given by P^*, has a disutility level associated with it, which is given by:

$$D = x_i + t \times y_i + P^* \tag{1}$$

The expression above is the sum of the two disutilities from product features and price that the buyer incurs. Since the buyer has no control over the price, she will search in such a way as to *minimize her total disutility from the product she eventually buys and the search costs that she incurs* in finding it.

4. All buyers have identical reserve utilities of U_0. Since the price of the product is P^* the maximum disutility that the buyer will suffer from product mismatch is given by: $U = U_0 - P^*$. A buyer will not search for products that lie outside the region defined by the four equations: $|x| + t\,|y| \leq U$ (refer to Figure 1).

[9] We associate a penalty of t_1 for unit distance deviation from the ideal attribute along the X coordinate dimension and t_2 for unit distance deviation from the ideal attribute along Y coordinate dimension, resulting in an equation that looks like this:

$$D_1 = t_1\,|x_1 - x| + t_2\,|y_1 - y|$$

Dividing both sides by $t_1$and substituting $D = \frac{D_1}{t_1}$ and $t = \frac{t_2}{t_1}$ we have:

$$D = |x_1 - x| + t\,|y_1 - y|$$

This gives us the comparitive importance of the dimension Y. We will derive results based on the parameter t and comment on its significance wherever appropriate.

3.5 Search

1. Each search costs the same (for each buyer) and the search cost is represented by C.

2. The search mechanism available in the electronic market allows buyers to specify a range of acceptable product characteristics. For example, a buyer can request products located in the $[0.4, 0.8]$ interval along the X dimension and in the $[0.2, 0.5]$ interval along the Y dimension. This range defines the maximum disutility that the buyer will endure before she exits the market.

3. Since the search engine is assumed to have access to both the simple and complex attributes of a product, it assigns a position in product space (based on the product features) to each seller offering. The search engine will search for all products that will provide the buyer with a disutility less than or equal to U. Initially, we will assume that the search engine can exhaustively examine every seller offering and return all sellers who meet buyers' search criteria. We will relax this assumption later and examine the impact of less than perfect recall and precision in the search technology.

4. When the buyer searches in her search domain, the search engine will return with all the products[10] in the area and rank them in an ascending order of disutility (where the product that represents the least disutility for the buyer, is top ranked). The strategy adopted by the buyer to identify the candidate product depends on whether or not the search engine's precision is perfect.

3.5.1 Perfect Recall and Perfect Precision

1. When the search engine ranks products in an ascending order of disutility, the buyer will simply select the top ranked product since it is assured to be the best choice.

[10]The recall of the search engine is assumed to be perfect. The case when the recall is less than perfect will be examined in subsequent sections.

2. Under the above dispensation, the buyer's surplus is computed as follows.

Let the disutility of a product chosen at random from the search domain be represented by the random variable X. If N products are returned by the search engine and ranked in ascending order, $Y_1, Y_2, Y_3 ... Y_n$ where $Y_1$is the best product and Y_n the worst, the vector $Y_1, Y_2, Y_3 ... Y_n$ can be treated as composed of N samples of X and the probability density function of Y_1, $f(y_1)$is given by[11] [Hogg 1995]:

$$f(y_1) = N\left[1 - \frac{y_1^2}{U^2}\right]^{N-1} \times \left(\frac{2y_1}{U^2}\right)$$

From (1), the expected disutility value of the best product, i.e., Y_1,is given by:

$$E(Y_1) = N\int_0^U \left(\left[1 - \frac{y_1^2}{U^2}\right]^{N-1} \times \left(\frac{2y_1^2}{U^2}\right)\right) dy_1 = \frac{\sqrt{\pi}U Gamma\,[N+1]}{2 Gamma\left[\frac{3}{2}+N\right]} \quad (2)$$

3.5.2 Buyer Surplus

Buyer surplus under perfect recall, can be derived from equation (2). Buyer Surplus B is given by:

$$\begin{aligned} B &= U - \left(\frac{\sqrt{\pi}U Gamma\,[N+1]}{2 Gamma\left[\frac{3}{2}+N\right]}\right) \quad (3) \\ \text{Where } U &= U_0 - P^* \end{aligned}$$

In later sections, the cost of operating the search engine will be factored into the calculation of buyer surplus. The loss of surplus has two components to it, the price of the product P^*,the and the expected disutility of the candidate product which is given by the expression in equation (2).

[11]The expression for order statistic is applied here where $Y_1 < Y_2 < ... < Y_N$ denotes the order statistics of a sample of size N.

4 Imperfect Recall

In a survey of several search engines such as Alta Vista, Excite, Lycos, and electronic markets that are currently operational, such as Expedia, [Expedia 1998], Travelocity [Travelocity 1998],SurplusAuction [SurplusAuction 1998], and structured search engines for locating products, such as CNet [?], etc. it was observed that the search engines' recall was less than perfect. When a buyer searched the product space based on a set of parameters, the search engine returned some fraction of the product space. If the buyer searched with the same parameters again, the same set of products were returned. In order to view a different set of products or expand the results of her search, the buyer had to change her search criteria. A *set of search parameters uniquely determined the results of the search for a given level of recall.*

In the model under discussion, the search engine periodically captures a fraction of products in the market, records them in a retrievable database. Buyers can search the search engine's database and buy only those products that have been captured by the search engine and indexed by it. Products that are not indexed in one period may (or may not) be captured in a subsequent period. Further, products that arrive in the market (or sellers who are new entrants) in a particular period may be captured by the search engine in subsequent periods and indexed. For the remainder of this paper, the discussion will be restricted to a *single period* and comparative statics will be employed to explain phenomena that take place within a single period. Thus, accretion or diminution of sellers over time will not impact the results.

Each attribute of each product is examined by the search engine and the product is accorded a position in product space. In this model, the products that are captured and indexed by the search engine are uniformly distributed in the buyer's search domain.

4.0.3 Disutility of the Candidate Product Under Imperfect Recall

Buyers specify their search parameters (their location in product space and the value of t) and the search engine will retrieve ρN products from the search domain. At $\rho = 1$,buyer surplus[12] is maximized and the expected disutility of the resulting best product is given by equation (3).

[12] As before, the cost of operating the search engine is ignored in calculating the buyer surplus here. It will be factored into the calculations in the sections that follow.

For imperfect recall, i.e. $0 \leq \rho < 1$, the expected disutility of the resulting best product is derived by substituting ρN for N in equation (2). Which results in:

$$E(Y_1) = \int_0^U \left(\left[1 - \frac{y_1^2}{U^2} \right]^{\rho N - 1} \times \left(\frac{2y_1^2}{U^2} \right) \right) dy_1 = \frac{\sqrt{\pi} U Gamma\left[\rho N + 1\right]}{2 Gamma\left[\frac{3}{2} + \rho N\right]} \quad (4)$$

4.0.4 Costs Associated with Recall:

There are say, N products in the market and $n_1$of these representing some fraction ρ of the market have been captured by the search engine. The search engine has to perform some operations to capture the n_1 different products and record them . We will model the cost of capturing products, F^c as a function of n_1 given by:

$$F^c(n_1) = a_1 + b_1 n_1$$

Where a_1, is a constant that represents a fixed number of preparatory operations (such as loading and closing databases etc.) that need be performed prior to and after recording the products and b_1 is a constant that represents the number of binary operations required to record a product in the database.

Once recorded, each time the buyer executes a search query, the engine will have to perform some operations to retrieve the products that meet the buyer's search criteria. The cost of retrieval, F^r, is modeled as a function of n_1 given by:

$$F^r(n_1) = a_2 + b_2 n_1$$

Where a_2 and b_2 are constants similar to a_1 and b_1.

Finally, the search engine will have to sort an unsorted list of n_1 products and rank them in ascending order of disutility. The number of operations required to achieve this is of the order[13] of $O(n_1 Log(n_1))$ .We will model this as a function of n_1 given by:

$$G(n_1) = K n_1 Log(n_1)$$

[13]From the Optimality Theorem of Sorting which states that an unsorted list of length $= N$ can be sorted at best in $O(NLog(N))$ operations.

Given that the impact of G *dominates* F^c and F^r, for larger values of n_1, $\varphi(n_1) = F^c(n_1) + F^r(n_1) + G(n_1)$, can be rewritten as:

$$\varphi(n_1) = Kn_1 Log(n_1)$$

If the cost per operation is set at some constant K_{11}, then cost of operating the search engine will amount to:

$$C_R(n_1) = K_1 n_1 Log(n_1)$$

where $K_1 = K \times K_{11}$

Since $n_1 = \rho N$, the cost of recall for a market size N is given by:

$$C_R(\rho, N) = K_1(\rho N) Log(\rho N) \tag{5}$$

4.1 Welfare Under Imperfect Recall

Since this model addresses the impact of search engine technology on buyer and seller strategies and sellers' costs are exogenous to the model and not relevant to the current discussion, the term welfare will be restricted to mean the difference of buyer surplus and the costs of offering search engine technology. Welfare in the market is given by:

Buyer Surplus at a level of Recall ρ, - costs of technology at that level of recall. This is given by:

$$W_R = U_0 - P^* - E(Y_1) - C_R(\rho, N)$$

From equations (3), (4) and (5), W_R can be rewritten as:

$$\Rightarrow W_R = U - \frac{\sqrt{\pi} U Gamma[\rho N + 1]}{2Gamma\left[\frac{3}{2} + \rho N\right]} - K_1(\rho N) Log(\rho N) \tag{6}$$

Lemma 1 *For any given Market Size, N, there exists a welfare maximizing level of Recall, ρ.*

Proof: From equation (6), W_R can be re-written as:

$$\begin{aligned} W_R &= U - D(\rho, N) - C_R(\rho, N) \\ \text{Where} &: D(\rho, N) = \frac{\sqrt{\pi} U Gamma[\rho N + 1]}{2Gamma\left[\frac{3}{2} + \rho N\right]} \end{aligned}$$

Case 1: $\frac{dW_R}{d\rho} = 0$,for some value of ρ.

Taking first order conditions for W_R w.r.t. ρ :

$$\frac{dW_R}{d\rho} = -\left[D'(\rho, N) + C'_R(\rho, N)\right]$$

Since[14] $D'(\rho, N) < 0$ and $C'_R(\rho, N) > 0$,

and further $|C'_R(\rho, N)| > |D'(\rho, N)|$ for some $\rho > \rho_0$ depending on N and K_1,

it follows that if $\frac{dW_R}{d\rho} = 0$ for some $\rho = \rho_1$,then at that value of ρ_1, W_R attains a maximum[15]. This is the welfare maximizing level of Recall (ρ_1).

Case 2: $\frac{dW_R}{d\rho} \neq 0$ for any ρ.

Since, $|C'_R(\rho, N)| > |D'(\rho, N)|$ for all $\rho > \rho_0$,

and $\frac{dW_R}{d\rho} = -\left[D'(\rho, N) + C'_R(\rho, N)\right]$,

it follows that $\frac{dW_R}{d\rho} < 0 \; \forall \; \rho \in [0, 1]$.

In this case the function attains a maximum at $\rho = 0$, and the buyer will exit the market without searching for any products.

4.1.1 Impact of Cost of Technology and Market Size on Welfare

In this section the impact of market size and cost of technology on buyer surplus will be discussed. Market size impacts buyer surplus in two conflicting ways. As the number of products increases, the expected disutility of the candidate product decreases thereby increasing the buyer surplus. The intuition behind this is that as market size increases, there are more products within the buyer's search domain and therefore, the best product (the one 'closest' to the buyer) is found closer to a buyer's location than in a sparser market.

As market size increases, the cost of indexing and retrieving products increases thereby increasing the overall cost of operating a search engine in the market. The gains that accrue to the buyer from increase in market are partly or wholly offset by the increased cost of operating the search engine.

[14]Since the derivative of $D(\rho, N)$ involves PolyGamma Functions, a formal proof is not included here. The proposition was verified using Mathematica and the results will be made available by the author on request.

[15]Since $|C'_R(\rho, N)| > |D'(\rho, N)|$, for $\rho > \rho_0$,it follows that the derivative changes sign in a small interval $Lim_{h \to 0}$ $[\rho_1 - h, \rho_1 + h]$ around ρ_1 . For all values of $\rho > \rho_1, \frac{dW_R}{d\rho} < 0$.

Intuitively, it is clear that the welfare maximizing level of recall of the search engine will reflect the trade-off between the two conflicting effects.

The level of recall that maximizes the welfare will also be impacted by the costs of technology. In equation (6), cost of technology is represented by the term K_1. As K_1 increases, it is clear that increase in market size will be met by a decrease in recall and vice versa. To examine the combined impact of cost of technology and market size on buyer surplus, three different technology cost regimes will be considered. These regimes will be called "High Cost Regime", "Moderate Cost Regime" and "Low Cost Regime". The technology cost discriminant term will vary by a multiple of 10 across the three regimes. The high cost regime will feature a technology cost that is 10 times the cost in the moderate cost regime and 100 times the cost factor of the low cost regime[16]. Similarly three market sizes will be considered, which correspond to "Large", "Moderate" and "Small" sized markets.

Figure 2 summarizes the relationship between recall, market size and welfare under the moderate technology costs regime.

INSERT FIGURE 2 HERE

From Figure 2, it can be seen that for a small market, welfare increases with recall. For moderate and large market sizes, we can see that welfare is maximized at a relatively low level of recall ($\rho = 0.2$ or so) and for larger values of recall, welfare declines rapidly.

Table 1 below Summarizes the impact of Market Size and Technology Costs on the welfare maximizing level of Recall.

Table 1: Welfare maximizing level of Recall

	Technology Cost Regime		
Market Size	Low Cost	Moderate Cost	High Cost
Small	0.863	0.220	0.056
Medium	0.173	0.044	0.011
Large	0.086	0.022	0.006

Table 2 below Summarizes the impact of Market Size and Technology Costs on total welfare.

[16]Since the focus of this discussion is on providing comparisons of trends and direction of movements of different factors at equilibrium, the actual values of constants used in setting up the simulations are not meaningful.

Table 2: Total Welfare

	Technology Cost Regime		
Market Size	Low Cost	Moderate Cost	High Cost
Small	0.867	0.746	0.552
Medium	0.867	0.746	0.552
Large	0.867	0.746	0.552

In the case of recall, as Market size N, increases, the optimal level of recall decreases. Similarly as the cost of technology increases, the optimal level of recall decreases. In the case of total welfare, the impact of increase in cost of technology dominates all other effects. For a given cost of technology, the same level of optimal welfare can be preserved by decreasing the recall suitably. However, an increase in cost of technology results in some welfare loss that cannot be compensated by either a lower level of recall or by an increase in market size. Thus the cost of technology emerges as one of the key drivers of the success of an electronic market.

5 Imperfect Precision & Imperfect Recall

In the earlier section while discussing the recall of the search engine, it was assumed that the search engine was not inaccurate in estimating the utility of the products that were returned by the search. This assumption will now be relaxed and the impact of inaccuracies in the search engine's estimation of a product's utility will be examined. For instance, in a market for airline tickets, the search engine may have to examine ticketing codes and look into a dictionary of conditions to ascertain what rules and conditions apply to each ticket. The search engine may then have to infer the degree of fit between the buyer's specifications and the seller offering, and estimate the aggregate disutility of the product. The following features describe the model.

1. The results of the search are ranked in ascending order of disutility by the search engine. This ranking is inaccurate and a lower ranked product may in effect be a better match (lower level of disutility) than a higher ranked product. The disutility of the candidate product is a function of the following parameters in this model:

μ : The base Precision[17] of the search engine. The search engine's base precision can vary from 0 to 1 (perfect precision case). $\therefore 0 \leq \mu \leq 1$

x : The number of inspections made by the buyer. The number of products a buyer inspects before she stops further inspection.

N : The number of products within the buyer's search domain.

r : The mean product rank of the x products inspected.

If U_p denotes the disutility of the best product (candidate product), in n inspections, then, $U_p = F(\mu, x, r, N)$. The exact functional form of F will be explicitly specified later.

2. If the buyer inspects, say, x products, before stopping further inspection, she will choose the best product from the x products examined until that point. Recording and retrieving the best product out of a sequence of x inspections is assumed to be costless.

3. **Mean Rank of Inspection**: As the buyer inspects products in the ascending order of disutility the mean rank of the products inspected increases from 1 (corresponding to the first inspection) to $\frac{x+1}{2}$ after x inspections. If the buyer chooses to inspect products randomly (not according to their ranks) or by ignoring their rankings, then the mean rank will simply be the sum of the ranks of the products divided by the number of inspections[18]. This definition will be used in the modeling constructs that follow.

4. In the case of search engines that operate in the domain of document retrieval, there is greater uncertainty associated with the degree of fit of documents which contain fewer search parameters (keywords) than those that contain more search parameters. The search engine employed by this model operates with an analogous logic. The precision of the search engine decays as the extent to which it has to infer the degree of match between the product's attributes and buyer preferences increases. Therefore, products that are close to buyer location in

[17]The meaning of the term "base precision" will be explained later in this section. It can be thought of as the precision of the search engine when it estimates the disutilities of products closest to the buyer's position.

[18]A buyer will not overlook the ranking scheme as it would be a sub-optimal inspection strategy.

product space have a lower uncertainty associated with their level of disutility as compared to products that are farther away from buyers. As the rank of a product decreases (the best products are ranked highest, and poorer matches are ranked lower) the degree of imprecision associated with the product increases. If the buyer inspects x ($x > 1$) products such that the mean rank of the products examined is r, then the mean precision of the search engine over the range of products inspected is given by: $\mu_r = 1 - \mu(1 - \frac{1}{r})$; where μ is the precision of the search engine for the top ranked product which will be referred to as the *base precision* of the search engine. Given that a buyer will never inspect a lower ranked product before she inspects all higher ranked products, after x inspections, the mean precision of the search engine over the range of x inspections is given by[19]:

$$\mu_x = 1 - \mu\left(1 - \frac{2}{x+1}\right) \tag{7}$$

For a given number of inspections, the disutility of the candidate product increases as the precision of the search engine decreases. Holding all other factors constant, the disutility of the best product is a decreasing function of the search engine's precision. Or:

$$\frac{\delta F}{\delta \mu} < 0 \tag{7a}$$

The base precision decreases as the number of inspections increase. Or:

$$\frac{\delta \mu_x}{\delta x} < 0 \tag{7b}$$

5. As the number of inspections increase, the disutility of the candidate product is impacted by two factors whose effects are in opposing directions. As the buyer inspects products sequentially, she may find better products which are lower ranked. However, as the buyer inspects lower ranked products, the mean precision over the inspection range decays and diminishes some of the gains of finding better products which are lower ranked.

[19] $r = \frac{\sum_{i=1}^{x} i}{x} = \frac{x+1}{2}$

If the buyer chooses to ignore the ranking of the search engine and inspects products either randomly or according to some other rule, she looses the value of information that is provided by the search engine. Holding all other factors constant, the disutility of the candidate product increases with increasing mean rank, r of inspection[20].Or:

$$\frac{\delta F}{\delta r} > 0 \tag{7c}$$

Since the buyer will always inspect the products in accordance with search engine's ranking scheme, the mean rank r of x products inspected is $r = \frac{x+1}{2}$. Therefore, when the buyer adopts an optimal inspection strategy (that minimizes her product related disutility), each additional inspection will cause the mean rank of inspected products to increase by $\frac{1}{2}$ and the mean ranking will increase only if there one or more additional inspections have been made. In this model, when optimal inspection strategy is adopted by the buyer, the impact of *additional inspection dominates the impact of decay of precision.* This implies:

$$\frac{\delta F}{\delta x} < 0 \tag{8}$$

Further, in this model of search, the *rate* of decrease in disutility increases with the number of inspections Or:

$$F''_x > 0 \tag{8a}$$

6. When a buyer has to inspect a product, she incurs an inspection cost of e. Each product inspection costs[21] e. After an inspection, the buyer knows the exact disutility of the product with certainty. In this model, the buyer has to inspect a product that she chooses to buy, for reasons exogenous to the model. This is in keeping with the functioning of currently operational electronic markets (such say, Expedia, CarPoint,

[20] A buyer could inspect products according to any rule or randomly and not necessarily according to the search engine's ranking. However, by this strategy would yield a sub-optimal solution.

[21] This is the disutility the buyer experiences from having to spend the time and effort in inspecting the product.

Auto-by-tel etc.) where buyers must inspect products and indicate that they are in agreement with terms of trade for regulatory and contract enforcement reasons.

7. If the buyer inspects x products before stopping, the buyer's total disutility is given by:

$$D(x, \mu, N) = F(x, \mu, N) + xe \tag{9}$$

Since $r = \frac{x+1}{2}$, under optimal inspection strategy, the function will be given by x, μ, e and N.

In the next section some results will be derived using the general functional form in equation (5). These results will then be interpreted by resorting to simulations by attributing numerical values to the modeling primitives.

Optimal Inspection Strategy: The buyer will inspect the product until she reaches the point of optimal disutility. Applying first order conditions, we have:

$$\frac{\delta D(x, \mu, r, N)}{\delta x} = 0 \Rightarrow F_x(x, \mu, r, N\ ) + e = 0$$
$$F_x(x, \mu, r, N) = -e \tag{10}$$

Equation (10) has a solution for a large enough value of e for a given set of parameters $\{\mu, N\}$. Let $\overline{\nu_1} = \{\mu_1, N_1, e_1\} : D_x(x, \overline{\nu_1}) = 0$. Given that $F_{xx} > 0$ and $F_x < 0$, the equation $D_x(x, \overline{\nu_1}) = 0$, has a unique solution[22].

Lemma 2 *For any market size, N level of Precision μ, and inspection cost e there is an optimal level of inspection that minimizes buyer disutility.*

Proof: Two cases will be considered which correspond to equation (6) having either no solutions or exactly one solution[23].

Case 1: Let $\overline{\nu_1} = \{\mu_1, N_1, e_1\} : D_x(x, \overline{\nu_1}) \neq 0$. If $D_x(x, \overline{\nu_1}) > 0$, then the buyer's net disutility increases with each additional inspection. The benefits

[22] $D_x(\overline{\nu_1}) = 0 \Rightarrow F_x(x, \overline{\nu_1}) = -e_1$.
Since $F_{xx} > 0$, if $F_x(x_1, \overline{\nu_1}) = F_x(x_2, \overline{\nu_1}) = -e$,then $x_1 = x_2$. Thus $D_x(\overline{\nu_1}) = 0$ has a unique solution for a given $\overline{\nu_1}$.

[23] Equation (6) cannot have more than one solution, as discussed earlier.

of finding a better product are offset by the inspection cost at the very first inspection thereby making it suboptimal for the buyer to inspect any further. The optimal inspection strategy results in the top ranked product being chosen as the candidate product, since each inspection will result in higher net disutility. If $D_x(x,\overline{\nu_1}) < 0$, the buyer will inspect all products. Since the net disutility declines with each additional inspection, the optimal strategy results in buyer inspecting all the products returned by the search engine exhaustively.

Case 2: $D_x(x,\overline{\nu_1}) = 0$, for some $x = x_1$. The lemma is proved if it is established that, at $x = x_1$, $D(x,\overline{\nu_1})$ attains a minimum value or $D(x_1,\overline{\nu_1})$ is a global minimum of the function $D(x,\overline{\nu_1})$. This will be proved by adopting proof by contradiction.

Since $D_x(x_1,\overline{\nu_1}) = 0$,if $x = x_1$ is not a minimum value of the function then it is either a point of inflection or a global maximum value[24]. If it is a point of inflection, then in an interval $[x_1 - h, x_1 + h]$, $D_x(x_1,\overline{\nu_1})$ will not change sign. But $D_x(x_1 - h,\overline{\nu_1}) = F_x(x_1 - h,\overline{\nu_1}) + e$ and $D_x(x_1 + h,\overline{\nu_1}) = F_x(x_1 + h,\overline{\nu_1}) + e$

From equation (5), it follows that:

$$\begin{aligned}
F_x(x_1 - h,\overline{\nu_1}) &> F_x(x_1,\overline{\nu_1}) > F_x(x_1 + h,\overline{\nu_1}) \\
&\Rightarrow F_x(x_1 - h,\overline{\nu_1}) + e > F_x(x_1,\overline{\nu_1}) + e > F_x(x_1 + h,\overline{\nu_1}) \\
&\Rightarrow F_x(x_1 - h,\overline{\nu_1}) + e > 0 > F_x(x_1 + h,\overline{\nu_1}) + e \\
&\therefore D_x(x_1 - h,\overline{\nu_1}) > 0 > D_x(x_1 + h,\overline{\nu_1})
\end{aligned}$$

Therefore x_1 is not a point of inflection. If $x_1$was a global maximum then $D(x_1,\overline{\nu_1}) > D(x_1 + 1,\overline{\nu_1})$ and $D(x_1 - 1,\overline{\nu_1}) < D(x_1,\overline{\nu_1})$. This implies that:

$$\begin{aligned}
F(x_1,\overline{\nu_1}) + x_1 e &> F(x_1 + 1,\overline{\nu_1}) + (x_1 + 1)e \Rightarrow F(x_1,\overline{\nu_1}) - F(x_1 + 1,\overline{\nu_1}) > e \\
&\text{And} \\
F(x_1 - 1,\overline{\nu_1}) + (x_1 - 1)e &< F(x_1,\overline{\nu_1}) + x_1 e \Rightarrow F(x_1 - 1,\overline{\nu_1}) - F(x_1,\overline{\nu_1}) < e
\end{aligned}$$

Since $F_{xx}(x,\overline{\nu_1}) > 0 \ \forall \ \overline{\nu_1}$(from equation (5), it follows that:

$$\begin{aligned}
F(x_1 - 1,\overline{\nu_1}) - F(x_1,\overline{\nu_1}) &> F(x_1,\overline{\nu_1}) - F(x_1 + 1,\overline{\nu_1}) \\
F(x_1 - 1,\overline{\nu_1}) - F(x_1,\overline{\nu_1}) &> e
\end{aligned}$$

[24]It cannot be a local minimum or a local maximum since there is *exactly one solution* to the equation, $D_x(x,\overline{\nu_1}) = 0$.

Therefore, x_1 is not a global maximum. Hence x_1 is a global minimum. Therefore, the buyer's optimal inspection level is to stop after x_1 inspections.

Candidate Product: In the previous sections several conditions the defined the behavior of the search engine under imperfect precision were discussed. Further, it was showed that the search engine's precision decayed as the mean rank of the products inspected increased. The search engine's precision for the i^{th} product was given by: $\mu_i = 1 - \mu\left(1 - \frac{2}{i+1}\right)$ (from equation (7)), where μ is the base precision of the search engine or simply, the precision of the search engine.

Disutility Differential If the search engine retrieves n products, the expected value of the disutility of the best product is given by[25]:

$$E(d_1) = \frac{\sqrt{\pi} U Gamma\left[n+1\right]}{2Gamma\left[\frac{3}{2}+n\right]}$$

The expected value of the worst product (worst fit with buyer's preferences) is given by[26]:

$$E(d_n) = \frac{2nU}{2n+1}$$

The *disutility differential* is the difference in disutility between the best and worst product in the market. This is given by:

$$\begin{aligned} \Psi(n) &= \frac{2nU}{2n+1} - \frac{\sqrt{\pi} U Gamma\left[n+1\right]}{2Gamma\left[\frac{3}{2}+n\right]} \\ \Psi(n) &= U\left[\frac{2n}{2n+1} - \frac{\sqrt{\pi} Gamma\left[n+1\right]}{2Gamma\left[\frac{3}{2}+n\right]}\right] \end{aligned} \tag{11}$$

[25]This result follows directly from equation (2).

[26]Derived by applying the n^{th}order statistic for n independent samples from a distribution.

Disutility of the Candidate Product: As the buyer examines more and more products, it is intuitively clear that the disutility of the candidate product should decrease, even as she incurs increasing costs of inspection. Further, if the buyer were to inspect most of the products retrieved and ranked (imperfectly) by the search engine, the disutility of the candidate product should be very close (in value) to the disutility of the best product in the list[27].The disutility of the candidate product after say, x inspections is given by the disutility of the best product plus some declining function of the disutility differential. This implies that as the buyer inspects more and more products, the disutility of the candidate product declines to the value of the disutility of the best product. Combining equations (7),(9) and (11), the expression that gives the value of the disutility of the candidate product after x inspections is given by:

$$F(x,\mu,N) = \frac{\sqrt{\pi}U Gamma[N+1]}{2Gamma\left[\frac{3}{2}+N\right]} + \Psi(N)\left[1-\mu\left(1-\frac{2}{x+1}\right)\right]^x$$

$$\Rightarrow F(x,\mu,N) = \frac{\sqrt{\pi}U Gamma[N+1]}{2Gamma\left[\frac{3}{2}+N\right]} +$$

$$\left(U\left[\frac{2N}{2N+1} - \frac{\sqrt{\pi}Gamma[N+1]}{2Gamma\left[\frac{3}{2}+N\right]}\right]\right)\left[1-\mu\left(1-\frac{2}{x+1}\right)\right]^x \quad (12)$$

From equations (9) and (12), the total disutility of the candidate product is given by:

$$D(x,\mu,N) = F(x,\mu,N) + xe$$

$$\therefore D(x,\mu,N) = \frac{\sqrt{\pi}U Gamma[N+1]}{2Gamma\left[\frac{3}{2}+N\right]} +$$

$$\left(U\left[\frac{2N}{2N+1} - \frac{\sqrt{\pi}Gamma[N+1]}{2Gamma\left[\frac{3}{2}+N\right]}\right]\right)\left[1-\mu\left(1-\frac{2}{x+1}\right)\right]^x + xe \quad (13)$$

[27] The best product retrieved by the search engine need not necessarily be the top ranked product. In this model, however, as the buyer inspects more and more products it is less and less likely that the best product is to be found in the remaining uninspected collection of lower ranked products.

5.0.2 Costs Associated With Precision

Since a search engine must perform a certain number of operations in order to achieve a level of precision, it is reasonable to associate a cost structure with a search engine's precision. In this paper, the cost of achieving a level of precision is modeled as a quadratic function of the level of precision achieved. A brief discussion of how the cost function was derived is provided below.

Let there be m imprecision (ambiguity) inducing parameters associated with each product. Together they can be present in 2^m different ways[28]. In this model, in order to be able to exhaustively eliminate all possibilities a search engine has to make $2^m \times 2^m$ discrete operations. When all m parameters are present (the search engine has not eliminated any parameters), let the precision of the search engine be given by[29]: $\mu_m = \frac{A}{2^m}$. When the search engine has eliminated some parameters $(m - m_1)$ and there are m_1 parameters that have not been eliminated, the precision is given by: $\mu_{m_1} = \frac{A}{2^{m_1}}$. Number of operations that are needed to eliminate m_1 parameters is given by: $2^{(m-m_1)} \times 2^{(m-m_1)}$. If the cost of performing an operation is given by K_0, then cost of a precision level μ_{m_1} is given by:

$$C\left(\mu_{m_1}\right) = K_0 2^{2(m-m_1)} = K_0 \times \frac{2^{2m}}{2^{2m_1}} = \frac{K_0}{\frac{A}{\mu_{m_1}^2}}$$

$$C\left(\mu_{m_1}\right) = K \mu_{m_1}^2$$

In general,

$$C(\mu) = K\mu^2 \tag{14}$$

5.0.3 Welfare Under Imperfect Precision

As before, Total Welfare in the market will be given by the difference of Buyer Surplus at a level of Precision μ, and the cost of operating the search

[28] The number of different combinations in which m parameters that may impact a particular product is given by:

$$\binom{m}{1} + \binom{m}{2} + \ldots \binom{m}{m} = 2^m$$

[29] The precision of the search engine is never 0. It varies between $\frac{1}{2^m}$ and 1.

engine at that level of precision. This is given by:

$$W_P = U_0 - P^* - E(Y_1) - C_P(\mu) \tag{15}$$

When the *dimension of Recall* is considered, then equations (5) and (14) can be combined to yield, the welfare under a level of Recall, ρ and Precision, μ :

$$W_{R,P} = U - D(x, \mu, N) - K_1(\rho N) \, Log(\rho N) - K_2\mu^2 \tag{16}$$

Combining equations (12), (15) and (16) results in:

$$W_{R,P} = U - (F(x, \mu, N) + xe) - K_1(\rho N) \, Log(\rho N) - K_2\mu^2$$

Or:

$$W_{R,P} = U - F(x, \mu, N) - xe - K_1(\rho N) \, Log(\rho N) - K_2\mu^2$$

$\Rightarrow$

$$W_{R,P} = U\left(1 - \left[\begin{array}{c} \frac{\sqrt{\pi}Gamma[\rho N+1]}{2Gamma\left[\frac{3}{2}+\rho N\right]} + \\ \left[\frac{2\rho N}{2\rho N+1} - \frac{\sqrt{\pi}Gamma[\rho N+1]}{2Gamma\left[\frac{3}{2}+\rho N\right]}\right] \times \left[1 - \mu\left(1 - \frac{2}{x+1}\right)\right]^x \end{array}\right]\right)$$
$$- \left[xe + K_1(\rho N) \, Log(\rho N) + K_2\mu^2\right] \tag{17}$$

Equation (17) will be used to provide comparative statics to analyze the impact of factors such as Level of Precision, Cost of Technology and Market Size on buyer surplus.

As before, three different market sizes (large, medium and small) and three different technology cost regimes, high, moderate and small will be considered. In addition, three different *Inspection Cost (e)* regimes (high, moderate and small) will also be considered to examine the effect of cost of inspection on market welfare and optimal levels of Precision and Recall. The impact of precision and recall under these conditions will be investigated by employing comparative statics.

Impact of Cost of Technology: As technology costs increases, the costs of operating a search engine increase. A larger market (greater number of products) would result in greater processing costs, but would also increase buyer surplus by featuring candidate products with lower disutility. When the two opposing trends are balanced, it is found that as market size increases, buyers are better off when the search engine's recall is lower (under any technology cost regime). This implies that the additional cost dominates the additional surplus generated by a larger market. In figure 3, below the effect of cost of technology and market size on the *welfare maximizing level of recall* (under moderate inspection cost) is displayed.

INSERT FIGURE 3 HERE

It is clear that for all three technology cost regimes, the size of the market and the optimal level of recall move in the opposite directions. Further, the decline in optimal recall level is sharper as the cost of technology increases. The optimal level of recall under the Low Technology cost regime is higher for any given market size than the corresponding level of recall under the High Technology cost regime[30]. I will now define a term that I will use in the remainder of this paper. **Effective Market Size** refers to the actual number of products that a buyer can search for in an electronic market. It is the product of recall (ρ) and the Market Size (N). If the number of products in the buyer's search domain is 1000 and the recall of the search engine is 0.2, then the effective market size is $\rho \times N = 0.2 \times 1000 = 200$. From figure 3, it is evident that as the market size increases, optimal recall declines so that the *change in Effective Market Size is relatively much smaller.* This observation will be used to explain two important phenomena in electronic markets.

Are there benefits to restricting buyer choice? When buyer choice is measured by the Effective Market Size, it appears there is enough evidence from figure 3, to support the argument that, in larger markets, higher levels of recall would result in greater Effective Market Sizes, but the resulting total

[30]There is a small portion of the graph where this is not so. This is not because of deviations from theoretical prediction as much as a 'smoothing error' created when discrete values generated by a simulation were joined into a graph using a graph plotter.

welfare of the market would be sub-optimal. At lower levels of recall, the total welfare of the market would be maximized, but the resulting effective market size will also be smaller. Intuitively this can be explained as follows. The marginal benefit (to the buyer) of an increase in Market Size N tis less than the marginal cost (incurred by the buyer and/or seller) of an increase in market size N. Table 3: below provides further evidence in support of the argument (the figures for moderate and high inspection costs are very similar and the data will be furnished by the author on request)

Table 3:Technology Costs, Optimal Recall and Total Welfare (under Low Inspection Cost)

		Market Size		
Tech Costs Regime		Small	Medium	Large
Low Tech Costs	Optimal Recall	0.86	0.172	0.086
	Effective Mkt. Size	86	86	86
	Optimal Welfare	0.85997	0.85997	0.85997
Moderate tech Costs	Optimal Recall	0.2	0.044	0.022
	Effective Mkt. Size	20	22	22
	Optimal Welfare	0.73969	0.73969	0.73969
High Tech Costs	Optimal Recall	0.1	0.011	0.006
	Effective Mkt. Size	10	6	6
	Optimal Welfare	.54455	0.54455	0.54455

From Table 3 above, the following effects can be seen:

1. The welfare maximizing effective size of the market is almost independent of the market size N and is influenced by the technology costs more than any other factor. The greater the technology costs the lower the optimal level of recall and hence the smaller the Effective Market size.

2. The welfare lost when technology costs increase cannot be compensated by a decrease in the optimal level of recall (and therefore a smaller Effective Market size).

The above observations indicate that a lower effective market size may lead to *less choice for buyers but will nevertheless, lead to optimal market welfare.* In situations where technological costs associated with operating a search engine are high, the expected gain to buyers from each additional product in the market can be less than the cost of indexing, storing and retrieving the product via an electronic search engine. Such cases typically occur when the market size is moderate or large. It can be seen from Table 3, that higher market sizes are associated with very low recall. This implies that by covering only a small fraction of the products available in the market, providers[31] of the search engine restrict buyer choice to a significant extent and maximize total welfare in the market. In the case of products that are highly information rich where the cost of processing each product and storing it in a database as well as accurate internal representation of the product for retrieval by a querying mechanism is likely to be high, the market is better served by limiting buyer choice. The alternative may be either increased operational costs or inaccurate and imprecise representation of the product resulting in poor precision. This leads us to the second question that we wish to address.

Which factor has greater impact on Total Welfare; Recall or Precision? To answer this question, it is necessary to investigate the levels of precision that maximize total welfare under different market conditions. Table 4, below shows the impact of Market size and technology cost types on Optimal levels of recall and precision under the low inspection cost regime (since the results and trends discussed here are identical for the other two inspection cost regimes, the corresponding data are not shown here and will be furnished the author on request).

Table 4:Technology Costs, Optimal Recall and Precision (under Low Inspection Cost)

[31]The question of who will provide the search engine will be addressed in the sections that follow.

		Market Size		
Tech Costs Regime		Small	Medium	Large
Low Tech Costs	Optimal Recall	0.86	0.172	0.086
	Optimal Precision	0.99	0 99	0.991
Moderate Tech Costs	Optimal Recall	0.2	0.044	0.022
	Optimal Precision	0.99	0.99	0.991
High Tech Costs	Optimal Recall	0.1	0.011	0.006
	Optimal Precision	0.99	0.99	0.991

From Table 4: above, the following conclusions can be made:

1. The welfare maximizing level of precision is independent of the market size (N) or the magnitude of technology costs and is almost equal to perfect precision.

2. In direct contrast to precision, optimal recall declines with increase in both market size and technology costs and in larger markets or under higher technology cost regimes, it is relatively small.

These observations support the argument that higher precision is far more important than recall or market size from the standpoint of maximizing market welfare. In the case of products that are information rich or those that are characterized by relatively high ambiguity in product description or terms of trade, buyers are better off with a search engine that may cover a smaller market in return for accurate representation and ranking of products retrieved by the search. In other words, *as the product complexity or uncertainty in terms of trade increases, buyers stand to gain when the search engine trades off variety or extent of market coverage for accuracy of ranking and representation.*

5.1 Incentives for Providing Search Engine Services

This section addresses incentives related issues by examining the conditions under which a seller or a coalition of sellers will provide search engine services free of cost to buyers. I will first investigate the conditions under which it is optimal for a single seller to provide search engine services to the market and then extend the results to a coalition of sellers.

5.1.1 Dominant Seller

The same model of electronic market that was used in the earlier section is used here. A new concept of 'Dominant Seller' is introduced as an addition to the model. The seller with the largest fraction of products within the buyer's search domain is called the dominant seller in this model. It is assumed that there is a single seller who offers the largest fraction of products within the buyer's search domain. The following modelling primitives will now be added to the model:

1. The fraction of products offered by the dominant seller $= f$. If there are 100 products in the buyer's Search Domain and $f = 0.25$, then the dominant seller offers 25 products within the search domain.

2. The products offered by the dominant seller are uniformly distributed within the buyer's search domain.

3. The same seller is the dominant seller for all buyers. This means that in the search domain of every buyer (not just some buyers) the dominant seller offers the largest fraction of products. An alternate way of stating this is to say that the dominant seller offers a fraction f of *all* products in the *product space* and her offerings are distributed *uniformly through the product space.*

4. Collaboration between sellers is costly due to a variety of causes ranging from regulatory constraints to coordination costs and therefore, sellers cannot cooperate in offering a search engine jointly.

5. The dominant seller offers the search engine to all buyers. He allows all sellers access to the search engine. The search engine has a recall factor that is less than unity and as before, it covers only a fraction of the products in the market. However, the search does not discriminate in *any systematic* way against any *particular* seller. It does not misrepresent the estimated disutility of any seller offering (either the dominant seller's or other sellers' offerings) when ranking the products retrieved by the search.

6. The dominant seller first selects all his products and stores them in the search engine's searchable database. If the optimal recall level for the market is ρ and the dominant seller's market share is f, the fraction of the market that is now available for other sellers $= \rho - f$. The following example illustrates the principle. If the optimal level of recall, $\rho = 0.1$ and the dominant seller offers 2% of the products in the market ($f = 0.02$), then the fraction of the market that is now available to other sellers = 8% (0.08). The probability that a given seller's good will be covered by the search engine in this situation $= \rho - f = 0.08$. The dominant seller ensures that his products are always covered by the search engine[32]. The *dominant seller improves the chances of his product being the candidate product by offering the search engine.*

5.1.2 Profits to the Dominant Seller

In this section an expression will be derived that measures the value of the search engine to the dominant seller (his additional profits from offering the search engine). The objective of the analysis here is not to comment on the actual profits to the dominant seller under specific market conditions, but to identify the conditions under which the dominant seller will find it optimal to offer search engine services to participants in an electronic market and to examine if other sellers can offer such services in competition with the dominant seller.

When the search engine is offered by third parties, the profits to the dominant seller, represented by π^{NS} are given by[33]:

$$\pi^{NS} = \sum_{i=0}^{fN} \left[\rho^i (1-\rho)^{(fN-i)} \binom{fN}{i} \left[\frac{i}{N} P^* \right] \right] \tag{18}$$

[32]This however, does not imply that the Dominant Seller's product will necessarily be selected for purchase by a buyer.

[33]This is derived as follows. If i products of the Dominant Seller are retrieved by the Search Engine, then the probability that one these products will be the candidate product is given by: $\frac{i}{\rho N}$ and the expected profit: $\frac{i}{\rho N} \times P^*$

The probability that i products out of fN products will be retrieved is given by:

$$\sum_{i=0}^{fN} \left[\rho^i (1-\rho)^{(fN-i)} \binom{fN}{i} \right]$$

This profits that would accrue to the dominant seller if he were to provide the search engine[34]:

$$\pi^S = \left(\frac{f}{\rho}\right) P^* - K_1 (\rho N)\, Log\, (\rho N) - K_2 \mu^2 \tag{19}$$

For π^S to be positive, it is necessary for the following inequality to hold:

$$P^* \geq \frac{K_1 (\rho N)\, Log\, (\rho N) + K_2 \mu^2}{\left(\frac{f}{\rho}\right)} \tag{20}$$

The gain to the dominant seller from offering Search Engine services, $G^S(f)$, is given by:

$$\begin{aligned} G^S(f) &= \pi^S - \pi^{NS} = \left(\frac{f}{\rho}\right) P^* - K_1 (\rho N)\, Log\, (\rho N) - K_2 \mu^2 - \pi^{NS} \\ &\Rightarrow G^S(f) = \left(\frac{f}{\rho}\right) P^* - K_1 (\rho N)\, Log\, (\rho N) - K_2 \mu^2 \\ &\quad - \sum_{i=0}^{fN} \left[\rho^i (1-\rho)^{(fN-i)} \binom{fN}{i} \left[\frac{i}{N} P^*\right]\right] \end{aligned} \tag{21}$$

For the gain to be positive, the following inequality should hold.

$$\begin{aligned} P^* &\geq \frac{K_1 (\rho N)\, Log\, (\rho N) + K_2 \mu^2 + \left[\sum_{i=0}^{fN} \left[\rho^i (1-\rho)^{(fN-i)} \binom{fN}{i} \left[\frac{i}{N} P^*\right]\right]\right]}{\left(\frac{f}{\rho}\right)} \\ &\Rightarrow P^* \geq \frac{K_1 (\rho N)\, Log\, (\rho N) + K_2 \mu^2}{\left(\left(\frac{f}{\rho}\right) - \left[\sum_{i=0}^{fN} \left[\rho^i (1-\rho)^{(fN-i)} \binom{fN}{i} \frac{i}{N}\right]\right]\right)} \end{aligned} \tag{22}$$

Therefore, expected profits are given by:

$$\pi^{NS} = \sum_{i=0}^{fN} \left[\rho^i (1-\rho)^{(fN-i)} \binom{fN}{i} \left[\frac{i}{N} P^*\right]\right]$$

[34]It is assumed that $\rho > f$. If not, no other seller's product would be featured by the search engine, and other sellers would turn to a third party for providing the electronic market.

This result can be restated in terms of the dominant seller's Market Share as follows:

$$f \geq \frac{\rho\left(K_1(\rho N)\,Log\,(\rho N) + K_2\mu^2 + P^*\left[\sum_{i=0}^{fN}\left[\rho^i(1-\rho)^{(fN-i)}\binom{fN}{i}\frac{i}{N}\right]\right]\right)}{P^*} \tag{23}$$

For the dominant seller to offer search engine services, it is sufficient if inequality (22) holds, since the earlier condition (inequality (21)) is implied by (22).

From equation (23), it follows that greater the market share of the dominant seller, greater is the likelihood that he will offer the search engine free of cost to buyers.

Can Other Sellers Provide Search Engine Services? Suppose there are a certain number of sellers in the market (in addition to the dominant seller) each of whom offered a certain fraction of the products in the market, $f_1, f_2, f_3, ...f_k$ such that: $f > f_1 \geq f_2 \geq f_3 \geq ...f_k$ and their corresponding gains in offering the search engine is given by: $G(f), G(f_1)...G(f_k)$.

From equation (21), it can be shown that Gain, $G^S(f)$ is increasing[35] in f for $\forall\ f \in [0,1]$. This implies that the *greater the market share of the search provider, the greater is his gain.*

Therefore, the following condition holds:

$$G(f) > G(f_1) \geq G(f_2) \geq ...G(f_k)$$

As a result, it follows that if various sellers provide search engine services, the *dominant seller's gain is the greatest.*

If the dominant seller offers discounts equal[36] to $G(f_1)$ to buyers, all buyers would use his search engine and ignore other seller sponsored search engines.

[35] The proof of this claim will be made available by the author on request.

[36] The discount would have to be greater than $G(f_1)$, but at a limiting value of $G(f_1)$, all other seller sponsored search engines would fail to attract any buyers.

6 Conclusions

The modeling and analysis provide explanatory insights into the three issues that motivated this research. Each issue will be addressed separately and the findings discussed. In analyzing the business impact of this research, some important developments in electronic commerce, such as personalization of market sites and constraints in selling information rich products will be discussed. The impact of factors such as cost of technology, market size (number of seller offerings) and the market share of dominant sellers will be discussed.

6.1 Impact of Recall, Precision and Dominant Seller's Market Share on Buyers and Sellers

The three research questions addressed by this research are directly related to the impact of search technology on the market. The questions will first be raised and answered and then the managerial insights that follow as a consequence will be discussed in detail. It is important to note that ranking accuracy is directly related to precision. Greater precision leads to higher ranking accuracy and a relatively low level of precision leads to low ranking accuracy. In order to investigate how Ranking Accuracy impacts buyers and sellers, it will suffice if the impact of precision on total welfare is analyzed.

1. **What is the impact of Recall and Precision on Buyers and Sellers?**

In a market where search engine technology has no operational or developmental costs associated with it, higher recall leads to greater total welfare in the market. Since this situation unrealistic, the optimal recall level of the search engine will be limited by the cost of technology. In larger markets, the level of optimal recall is relatively low, indicating that as market size rises, the level of optimal recall declines rapidly. It is clear from Figure-2, that as technology costs increase, the level of optimal recall falls. From Figure 3, it can be seen that even when the search engine's precision is perfect, the cost of technology makes it sub-optimal to have higher levels of recall in moderate and large sized markets. In general, as the market size increases, the optimal level of recall declines. And from Figure 3, it is clear that the decline in optimal level of recall is steeper when the cost of search technology increases. The finding with respect to precision is exactly the opposite. The size of the market, the cost of technology and the cost of human inspection etc. have almost no impact on the optimal precision level. It is clear from Table 4, that the welfare maximizing level of precision is consistently high irrespective of the size of the market or cost of technology while the optimal level of recall varies greatly depending on both factors. These observations lead us to key findings of this research.

- In large and moderate sized markets total welfare is maximized by limiting the size of recall to low levels.
- In comparatively smaller markets, welfare is maximized when the search engine has a relatively higher level of recall.
- The optimal level of Precision is independent of market size and technology costs. At a consistently high level of precision, the total welfare of the market is maximized.

2. **When cost of technology induces a trade-off between Recall and Precision, which factor has greater impact on buyer welfare?**

It is clear from Tables 3 and 4, that while precision remains unaffected under different technology cost regimes and market sizes, the level of optimal recall is highly sensitive to increase in technology costs and declines rapidly with increases in market size. The value to the buyer of an additional product in the search engine's database is less than the operational costs incurred in representing it accurately within a searchable database and retrieving it when the database is queried. The impact on the buyer of the trade-off between Recall and Precision cam be summarized as follows:

- In a small market, welfare is maximized by having relatively high levels of Precision and Recall.
- In large or medium sized markets, the total welfare is maximized when Precision is high and Recall is low.

- As the cost of technology increases, welfare is maximized by lowering Recall but providing a high level of Precision.

3. **Under What conditions will a seller (or sellers) offer search engine services free of cost to buyers?**

It is clear from that the analysis that followed equation (23), that greater the market share of the dominant seller, greater was the likelihood that he would offer free search engine services to buyers. The thrust of this research is to investigate if there are conditions under which the seller would offer free search engine services to buyers. The *dominant seller can attempt to charge fees for the usage of the search engine, only if there is no other seller in with a market share large enough to profit by offering the search engine.*

If there were one or more sellers with market shares f_i such that $G^S(f) > G^S(f_i) > 0$, then *not only will the seller have to provide the search engine for free, but also offer discounts* (or equivalent side payments) to buyers[37] which are at least equal to $G^S(f_1)$, where f_1 is the market share of the seller with the second highest market share.

A factor that was instrumental in establishing this result was that coordination and collaboration between multiple sellers is costly enough to be unviable. In the case of a market with many sellers having very small market shares, this may well be the case. However, if the collaboration between individual sellers were not prohibitively expensive and permitted under the regulatory regime, *it can be shown that the coalition of sellers that offers the search engine will have to include the dominant seller*[38].

6.1.1 Managerial Insights and Implications for Businesses

From the analysis in the proceeding section, it is clear that markets that trade-off buyer choice for enhanced accuracy are likely to deliver greater value to buyers. It can be seen in currently operational electronic markets for Airline Tickets, such as Expedia and Travelocity, that market that provides

[37] Who buy the Dominant Seller's product(s).

[38] Under the assumption that the cost of collaboration between any two sellers is identical.

better accuracy tends to attract more buyers. Expedia offers less choice than Travelocity[39] but its search interface is friendlier and the results are far more accurate. Recent analyses in the trade press [Penenberg 1998] cite Expedia's superior search engine features as having contributed to its higher market share. Similarly, Auto-by-Tel, an electronic market for used cars, offers an easy to operate search engine interface and accurate retrieval based on buyer specifications. Recent articles in the trade press[Gelsi 1997] point to these factors in explaining the success of this market.

Cybermalls, retailers of differentiated products and merchants who offer search intensive merchandise such as books, music, airline tickets and cars would benefit greatly by offering buyers accurate search features. In the case of information rich products (such as expert analyses of events and trends, analyst reports, financial news and reports etc.) and/or products characterized by complexity of description (such as SLR Cameras, hand held digital devices, PC's etc.) or ambiguity in terms of trade (airline tickets, vacation packages, investment services, legal services) search engine providers have strong reasons to trade-off market coverage for enhanced accuracy.

Customization of Electronic Markets: Customization of electronic markets has been a recent trend that has received considerable attention from the trade press and industry analysts. Many customized markets offer buyers a fraction of the products found in the market but customize their offering to precisely match buyer needs. A recent report in the trade press [Hof 1998] suggested that in search intensive markets for products such as music, information and airline tickets, customers are offered a smaller choice in exchange for precise match with customer preferences. The strategy of Portals such as 'Excite' [Hof 1998] centers on minimizing costly buyer inspection by making the search engine (in this case Excite's database of profiles) preprocess content to produce a close degree of fit between buyer preferences and products (information) retrieved. This research explains why this is a profitable strategy and predicts that increasingly, this will be the trend in markets for information rich or search intensive products.

[39] A random choice of 3 international and 3 domestic (within US) travel itineraries were made and Expedia and Travelocity were compared. While Expedia generally features fewer offerings, it is quite possible that for certain destinations, Expedia offers a greater choice of tickets. The above exercise is not be construed as a rigorous sampling effort conducted to prove propositions. There are numerous reports in the Trade press however, that make the same point.

Free Search Engine Services Most of the electronic markets that are currently operational offer buyers free search services. The findings in the preceding sections explain the conditions under which sellers stand to gain by offering buyers free search services. It is clear that when there are multiple sellers with significant market shares, no single seller will be able to charge buyers for search engine services. Further, given that in large markets buyer gain in including additional products in a search engine's repository is minimal, a dominant seller can maximize his expected revenue by strategically including a larger fraction of his products in the search engine's database or by adopting other methods to ensure that his products enjoy a higher probability of being selected as the candidate product. Such a development was in fact, observed when American Airlines introduced its SABRE airline reservation system, wherein American Airlines's products were featured ahead of competing airlines' offerings. These factors alone are enough to ensure higher seller profits and incentivize sellers to offer free search services to buyers.

6.2 Directions for Future Research

Almost all currently operational electronic markets feature considerable price dispersion across different products and for the same product across different sellers. This research is being extended to investigate the impact of search engine technology on a market characterized by price dispersion. Search engines may offer different levels of precision depending on whether buyers search for product or price information. The impact of such context dependent precision, on buyer surplus is an area that requires further research. Customization of buyer sites can result in significant benefits to buyers. It can also provide sellers with an accurate estimate of buyers reserve utility and enable them to indulge in price discrimination. The trade-offs between reducing buyers' search and inspection costs and giving sellers the power to price discriminate has welfare implications which are being investigated by my current research.

The advent of Intelligent agents and advances in collaborative filtering techniques have made it easier for sellers to search for buyers. This research can be extended to investigate the impact of intelligent agents on sellers' pricing strategies.

Appendix A

		Market Size		
Tech Costs Regime Type		**Small**	**Medium**	**Large**
Low Tech Costs Regime	Optimal Recall Effective Mkt. Size	0.86 86	0.172 86	0.086 86
Moderate tech Costs Regime	Optimal Recall Effective Mkt. Size	0.2 20	0.044 22	0.022 22
High Tech Costs Regime	Optimal Recall Effective Mkt. Size	0.1 10	0.011 6	0.006 6

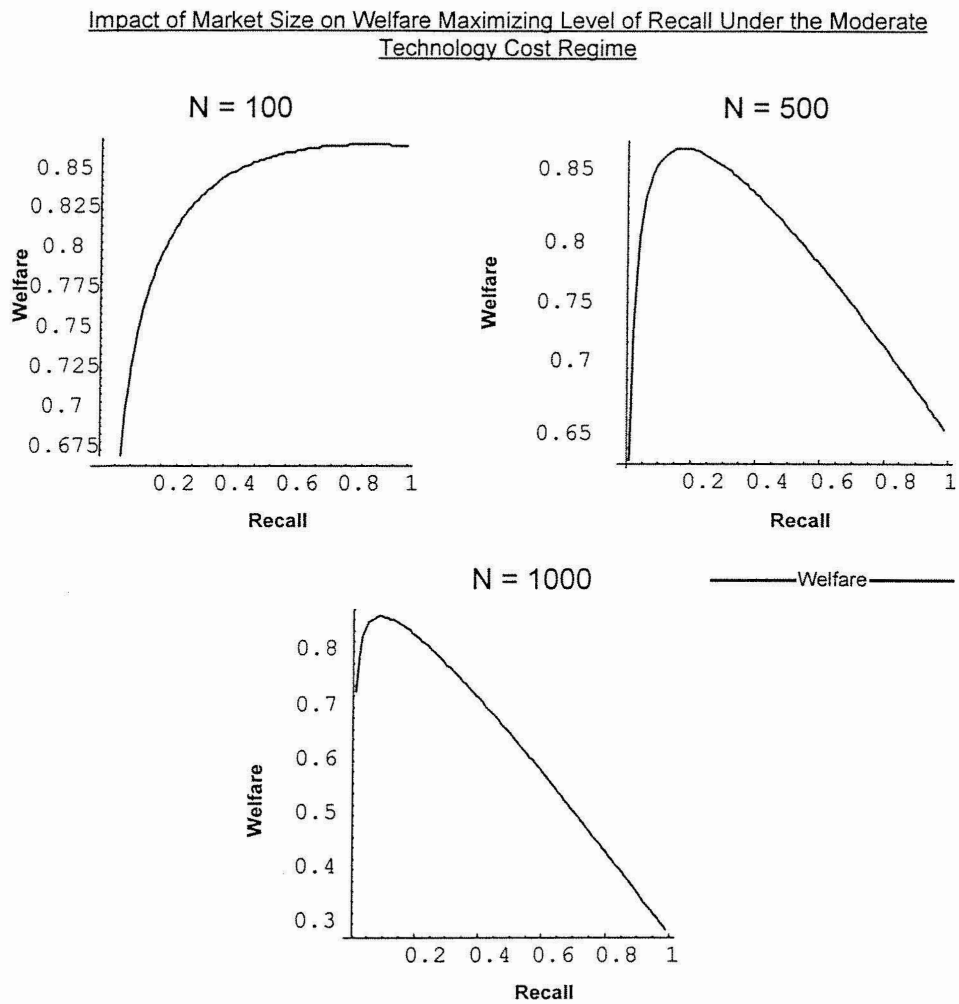

Figure 2: Market Size, Recall and Welfare

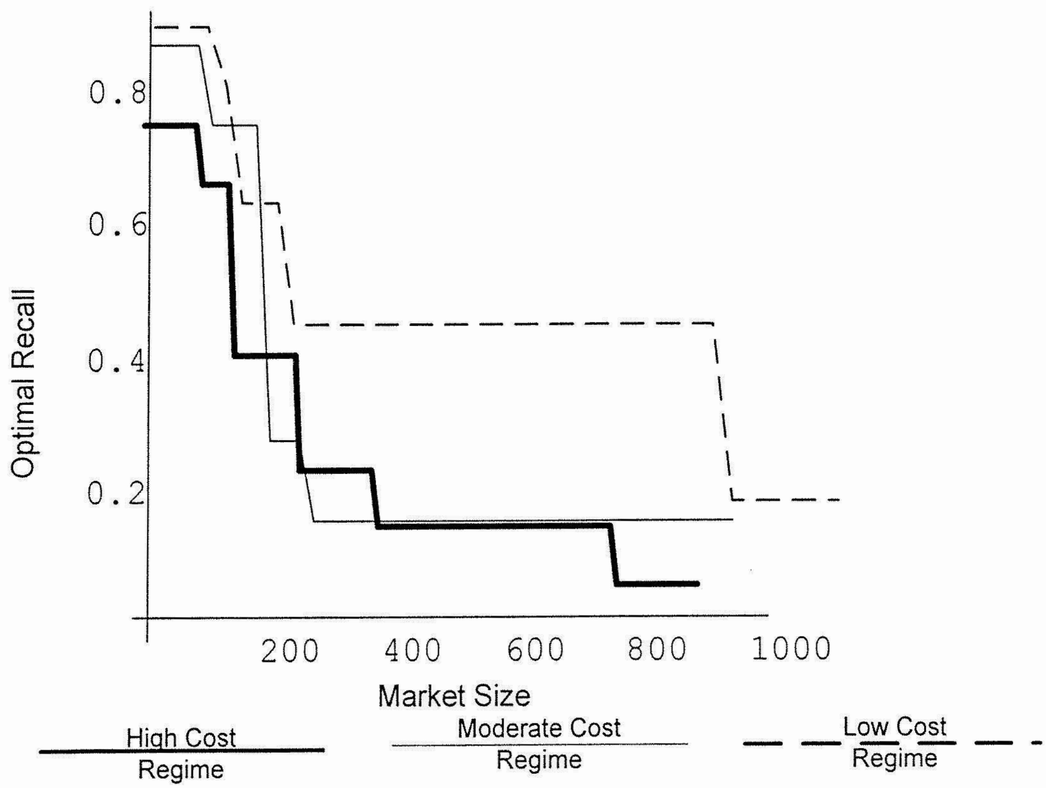

Figure 3: Welfare Maximizing Level of Recall under Different Cost Regimes

References

[Anderson 97] Anderson,C.; "In search of the perfect market"; www.economist.com/editorial/freeforall/14-9-97/ec1.html

[Bakos 1987] Bakos, J. Y.; "Interorganizational Information Systems: Strategic Opportunities for Competition and Cooperation." Ph.D. Dissertation, MIT Sloan School of Management, November 1987.

[Bakos 1991] Bakos, J. Y.; "A Strategic Analysis of Electronic Marketplaces"; MIS Quarterly, September 1991.

[Bakos 1997] Bakos, Y.; "Reducing Buyer Search Costs: Implications for Electronic Marketplaces"; Management Science, December 1997.

[Carpoint 1998] Carpoint.com; http://WWW.CarPoint.com

[CNet 1998] CNet.com; http://www.cnet.com

[Depalma 1979] Takahashi T. and de Palma, A.,; "A Unified Treatment of the Segment and the Circular Models; Does Economic Space Matter,"; pp 248-273.

[Diamond 1971] Diamond, P.A. "A model of price adjustment,"; Journal of Economic Theory(3), 1971, pp. 156-168.

[Expedia 1998] Expedia.com; www.expedia.com

[Gelsi 1997] Gelsi, S.;"Kicking Tires", Forbes, 07/23/97; http://www.forbes.com/tool/html/97/jul/0723/feat.htm

[Greenhut 1987] Greenhut, M.L., G. Norman, C.-S. Hung;"The economics of imperfect competition: A spacial approach"; Cambridge: Cambridge University press - 1987.

[Hof 1998] Hof, R.D., Green, H., Himelstein, L.;"Now it's Your Web"; Businessweek, 10/05/98; http://www.businessweek.com/1998/40/specrep.htm

[Hogg 1995] Hogg, R.V. and Craig, A.T.;"Introduction to Mathematical Statistics", Prentice Hall, Fifth Edition, pp. 198-199.

[Hotelling 1929] Hotelling, H.; "Stability in Competition"; Economic Journal, vol 39, pp 41-57.

[OnSale 1998] OnSale.com; http://www.onsale.com

[Penenberg 1998] Penenberg, A.L.;"Expedia gets rave", Forbes 02/27/98; http://www.forbes.com/tool/html/98/feb/0227/side2.htm

[Pinkerton 1994] Pinkerton, B.; "Finding What People Want: Experiences with the WebCrawler"; Paper presented at the Second International WWW Conference 1994 http://info.webcrawler.com/bp/WWW94.html

[PriceWatch 1998] PriceWatch.com; www.pricewatch.com

[Salop 1977] Salop, S. and Stiglitz, J.E.; "Bargains and Rip-offs: A Model of Monopolistically Competitive Price Dispersion"; Review of economic studies (44), October 1977, pp. 493-510.

[Stigler 1961] Stigler, G.; "The economics of Information", Journal of political economy (69), June 1961, pp. 213-225.

[Travelocity 1998] Travelocity.com; http://WWW.TraveloCity.com

[SurplusAuction 1998] SurplusAusction.com; http://WWW.surplusauction.com