Quality and Advertising in a Vertically Differentiated Market

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Abstract

In this paper, we examine firms' quality positions when consumers can only consider purchasing products that they are aware of through advertising. Consumers compare the products they are aware of, and choose the product that maximizes their utility net of price. Firms choose product quality in a first stage, their advertising strategy in a second stage, and set prices in the last stage. Two forms of advertising are studied– *blanket* and *targeted*. Under blanket advertising, firms communicate to all consumers and the probability that each consumer sees an ad depends on the level of ad expenditure. We find that when blanket advertising is relatively ineffective, i.e., it is costly to ensure high consumer awareness, both firms will choose a light level of ad spending. This allows firms to select relatively undifferentiated qualities, without concern of intense price competition. When blanket advertising is very effective, the high quality firm expends heavily on advertising, while its rival differentiates with a low quality product and expends less on advertising. Interestingly, in a mid range of advertising effectiveness, one firm chooses a high quality product, but because its rival positions close by, it selects a low ad expenditure to avoid competing fiercely in prices. Under targeted advertising, firms choose the specific segment(s) they wish to make aware of their product. We identify conditions such that both firms choose equally high quality products, but advertise to different segments. This can result in a middle pocket of unserved consumers, even though consumers with lower willingness to pay are served.

(*Product Quality, Advertising Strategy, Differentiation, Competition*)

1 Introduction

In order for consumers to consider the purchase of a product, they must first be aware of its existence.[1] Though there are a number of ways for consumers to become aware of the products available in the marketplace, firm initiated communications is a primary vehicle. In some contexts, firms can send targeted messages only to those consumers who are part of their selected target market. For example, by purchasing or compiling a list of consumers that meet certain criteria, firms can send targeted e-mails or direct mail. In other cases, firms use media outlets where they cannot directly control who sees their ad. For example, by placing a commercial on national TV or placing an ad in a general interest magazine, a firm will potentially reach a broad set of consumers. It is noteworthy that in the US alone companies spent over $268 Billion in 2005 to advertise their offerings to consumers.[2]

Consumers consider the various products they are aware of and choose the one that delivers them maximum utility. Consequently, the return to a firm on advertising will critically depend on how its offering compares to the other offerings in the marketplace *and* how those products are advertised, i.e., which products constitute consumers' consideration set. In this context, the decision of what quality product to offer, and then how heavily to promote it through advertising, become intertwined, and further depend on the rival's product positioning and advertising strategy.

In this paper, we study a market where consumers are initially unaware of which products are available for purchase. We seek to understand how the need to advertise in order to be included in the consumer's consideration set affects a firm's decision of where to position its product vis-à-vis the competition. Specifically, we develop a duopoly model in which firms choose product quality in a first stage, their advertising strategy in a second stage, and set prices in the last stage. Consumers are heterogeneous with respect to their valuation for quality allowing for vertical differentiation.

[1] Behavioral models of the consumer's decision making process (DMP) typically include the stages of awareness, consideration, preference and ultimately purchase (**?**, see, for example,).

[2] Advertising Age, October 2, 2006.

We study two forms of advertising that firms can use to generate awareness for their product. The first type– *blanket advertising*– captures such situations where firms communicate to all consumers indiscriminately (on poupular shows on broadcast television or radio). The probability that a given consumer receives a firm's communication is a function of the advertising expenditure. Specifically, a high ad spend guarantees that all consumers will receive the ad while a low ad spend results in only a likelihood that each consumer receives the ad. In general, we show that firms can be better off under low levels of advertising because this allows them to choose higher quality product positioning, yet avoid fierce price competition. Three main equilibria can emerge depending on the effectiveness of blanket advertising, which is a function of the cost differential between heavy and light advertising expenditure. When advertising is relatively ineffective, neither firm gains much from advertising heavily. One firm selects maximal quality while its rival chooses a quality level that is similar, i.e., firms are relatviely undifferentiated. When, at the other extreme, blanket advertising is very effective, one firm chooses maximal quality and expends heavily on advertising. The rival now prefers to differentiate with a lower quality product and to expend less on advertising. In a mid range advertising effectiveness, we get the intriguing result that by choosing quality appropriately, the low-quality firm induces the high-quality firm to expend only lightly on advertising. This happens because the benefit to the high-quality firm from advertising heavily is diminished by the ensuing fierce price competition.

The second type of advertising we examine–*targeted advertising*–captures situations where firms can communicate to specific segment(s) they wish to make aware of their product. We establish the uniqueness of equilibria where each firm advertises to a different segment. Because each consumer is only aware of one product, the firms do not compete in the pricing stage and hence both choose equally high quality– that is, their products are entirely undifferentiated in quality. Interestingly, due to the relatively high prices firm charge, there exist conditions such that a set of consumers with moderate valuation for quality are unserved by the firm that advertises to them, even though consumers in a different segment with lower willingness to pay are served by the rival firm. We also show that the total number of consumers served follows an inverted-U shape as a function of the size of the lower-valuation segment.

In an extension we study the role of persuasive advertising. Contrary to the two types of advertising that build awareness, persuasive advertising changes the way consumers perceive the quality of product; thereby, affecting their willingness to pay. Our main result here is that firms differentiate less in objective quality than under no advertising, but the difference in advertising levels counterbalances this effect leading to a higher degree of differentiation in perceived qualities.

The rest of the paper is organized as follows. The next section relates our work to the relevant literature and summarizes our contribution. Section 3 sets up the basics of the model in terms of demand and firm behavior. Section 4 solves the case of blanket advertising and Section 5 solves the case of targeted advertising. We consider the extensions with persuasive advertising in Section 6. Finally, Section 7 concludes and discusses limitations. To enhance readability, we postpone all proofs to the Appendix.

2 Related Literature

Our work is primarily related to the vertical differentiation literature, beginning with the widely known work of Shaked and Sutton (1982). They extend the model of price competition between firms by allowing them to choose the quality of the products they sell. Since consumers are heterogeneous with respect to their valuation of quality, in equilibrium, firms choose different qualities in order to reduce price competition in the last stage. The authors also consider entry behavior and show that no matter how many potential entrants there are, exactly two of them enters the market. Moorthy (1988) relaxes Shaked and Sutton's zero cost of production assumption by introducing a quadratic cost function for quality, resulting in an equilibrium where the firm choosing the lower quality may be better off. Moorthy further shows that both in the simultaneous-quality-choice and the sequential-quality-choice model, the equilibrium strategies are to differentiate. Assuming that firms do not cover the market, Choi and Shin (1992) show that the low quality firm will choose a quality level which is a fixed proportion of the high quality firm's choice. Wauthy (1996) on the other hand, gives a full characterization of quality choice, allowing the coverage of the market to be endogenous.

Although the previous studies show that quality differentiation is an equilibrium outcome, similar quality products are often observed in the market. Rhee (1996) offers an explanation incorporating consumer heterogeneity along unobservable attributes into the vertical differentiation model. As a result, if consumers are sufficiently heterogeneous, firms offer identical quality products in equilibrium. Banker et al. (1998) investigate the relationship between equilibrium quality levels and the intensity of competition between firms. They find that the relation depends on how firms interpret increased competition and also the parameters describing the cost and demand structure. In the recent literature, Choudhary et al. (2005) and Jing (2006) both find that the higher quality firm can be worse off in equilibrium. Choudhary et al. examine a model of vertical differentiation where personalized pricing is allowed. While personalized pricing results in a higher market coverage, it also intensifies the competition, which can hurt the high quality firm. Jing explicitly identifies the conditions under which producing the low-quality good is more profitable by examining the cost structure Notably, the papers in this stream analyze firms' quality positions under various price and cost assumptions, but assume that all consumers are fully aware of these qualities and ignore the role or need for advertising.

Another stream of literature related to our work is the vast amount of studies on advertising that we do not summarize here. We thus focus on those which examine the relationship between advertising and product quality. It is theoretically well-established in the economics and marketing literature that advertising can be a signal of quality (Nelson 1974, Milgrom and Roberts 1986), however these papers ignore the informativeness of advertising and thus its effect on the market size. On the contrary, Zhao (2000) shows that when advertising raises awareness, spending less is the correct signaling approach of the high-quality firm. Iyer et al. (2005), on the other hand, investigate how firms should target their advertising. They find that firms advertise more to consumers who have a strong preference for their product and argue that this is a way to increase differentiation in the market.

Our contribution lies in extending the first stream of literature by exploring the interaction between advertising spending and quality choice and by showing how advertising can lead to less or no differentiation.[3] As Zhao, we also employ models where advertising generates general awareness and also explore targeted and pursuasive advertising.

3 Model Setup

We assume that there are two competing firms selling their products and every consumer purchases at most one unit of the product. We index the firms by the numbers 1 and 2 or the letters i and j, always assuming that $i \neq j$. Note also, that if firms offer different quality products we denote the firm offering the lower quality product by 1. We assume that the consumers are heterogeneous with respect to their valuation of quality, denoted by ϑ. The parameter ϑ is uniformly distributed in the interval $I = [0, 1]$. A consumer with parameter ϑ gains utility $\vartheta s - p$ by purchasing a product with quality s for price p. The consumers purchase the product for which their utility is higher, however, consumers can only purchase products that they are aware of. A consumer is only aware of a product if s/he sees an advertisement of it. We study two different types of advertising mechanisms that generate awareness. In the first, which we call *blanket advertising* every consumer sees the ad of firm i with probability a_i, independently, where a_i represents the advertising effort. In the second, which we call *targeted advertising*, firms can target segments (subsets) of consumers and a consumer only sees the ad if s/he is in the targeted segment.

Timing

[3]The only exception we are aware of is Colombo and Lambertini (2003), who study persuasive advertising in a market where quality levels are determined endogenously. they find that if the relative advertising efficiencies are sufficiently different then the low quality firm earns a higher market share and profits in equilibrium. Our model differs in that...

The timing of the game is as follows. First, firms choose their qualities $s_1 \leq s_2$. Qualities are positive and have a maximum value that is normalized to 1. Second, firms make their advertising decisions, and incur the promotion costs.[4] Third, firms set prices. Finally, consumers make their purchases. We assume no fixed entry costs, hence both firms participate in the market.

Costs and profit

We assume that variable costs are constant and we normalize them to zero; hence, our model is consistent with Shaked and Sutton (1982).[5]. However, advertising costs depend on the advertising efforts in the following way. In the blanket advertising case we assume that $c(a_i)$ is an increasing function of a_i. In the targeted advertising case we assume that cost is a linear function of the size of the targeted segment. Firms' profits are therefore simply their revenues (price $\times$ sold quantity) minus advertising costs;

$$\Pi_i = p_i D_i - c_i^{adv}.$$

We examine the pure-strategy sub-game perfect equilibria of the game. We now determine the solutions in the two different cases; blanket advertising and targeted advertising.

4 Blanket advertising

In this model both firms advertise to the whole mass of consumers but the probability of the ad reaching a consumer (the effectiveness of the ad) depends on the effort the firms make. For the sake of simplicity let us assume that there are only two levels of

[4]In the blanket advertising case they set a_i and in the targeted advertising case they choose the segments

[5]Having a fixed cost of developing a product would not qualitatively affect our results. Our normalizing both variable and fixed costs of production to zero might seem an oversimplifying assumption but is done for two reasons. First, our assumptions in this respect correspond to Shaked and Sutton (1982), allowing us to compare our results to theirs. Second, it allows us to focus on the strategic incentives of firms to differentiate in qualities when there are advertising costs invovled; including production costs would complicate this analysis.

advertising $a_L \leq a_H$, with costs c_L and c_H, respectively, $c_H > c_L$. As a benchmark, let us solve the case of a monopolist. A consumer who is aware of the product, buys it if and only if $\vartheta s - p \geq 0$. A consumer who is not aware of the product will never buy it. That is, the demand of the monopolist consists of consumers that are aware of the product and for which $\vartheta \geq p/s$. Therefore,

$$D_m(p_m, a_m) = (1 - p_m/s_m)a_m,$$

where a can be equal to a_L or a_H. The monopolist chooses price to maximize its revenues, $R(p, a) = p(1 - p/s)a$. Given a_m and s_m, $p_m^* = s_m/2$. Then its profit can be written as

$$\Pi_m(a) = s^2 a/4 - c(a),$$

which is increasing in s no matter what the advertising level is. Therefore, the monopolist sets $s = 1$ and chooses a_H over a_L if and only if $\frac{a_H - a_L}{4} > c$, where $c = c_H - c_L$. That is, the extra revenue of choosing the high level is greater than the additional cost needed to advertise heavily rather than lightly.

4.1 Duopoly

After analyzing the case of a monopolist as a benchmark, let us turn to the case of a duopoly. First, we solve the last stage, the pricing game, given qualities and advertising levels. Fixing the qualities chosen by firms 1 and 2 at s_1, s_2, and the advertising levels at a_1, a_2 we can characterize the pricing equilibria. Note that a pure-strategy equilibrium does not always exist. The function $f(a_1, a_2)$ describes the critical s_1/s_2 under which a unique pure-strategy pricing equilibrium exists.

Claim 1 *There exists a function $f(a_1, a_2)$, such that $0 \leq f(a_1, a_2) \leq 1$, $f(a_1, 1) = 1$ for any a_1, $f(a, a)$ is increasing for $a \geq 1/2$, and*

1. *If $s_1 < s_2$ and $s_1 \leq f(a_1, a_2)s_2$, then the equilibrium prices are*

$$p_1^* = \frac{s_1(s_2 - s_1)(2(s_2 - s_1) - a_2 s_2 + 2s_1(a_1 + a_2 - a_1 a_2)}{4s_1^2(1 + a_1 a_2 - a_1 - a_2) + s_1 s_2(4a_1 + 4a_2 - a_1 a_2 - 8) + 4s_2^2},$$

$$p_2^* = \frac{s_2(s_2-s_1)(2(s_2-s_1)+s_1(a_1+2a_2-a_1a_2))}{4s_1^2(1+a_1a_2-a_1-a_2)+s_1s_2(4a_1+4a_2-a_1a_2-8)+4s_2^2}.$$

2. *If $s_1 < s_2$ and $s_2 > s_1 > f(a_1, a_2)s_2$, then there is no pure-strategy equilibrium.*

3. *If $s_1 = s_2$ and $a_1, a_2 > 0$, then the equilibrium prices are always zero.*

The examination of the equilibria goes on similar lines as in the original vertical differentiation model. However, the equilibrium does not always exist, because the low quality firm may have an incentive to set a high price to serve only those consumers who are aware of its product and not the competitor's. This structure may result in a game where there is no equilibrium. The details are postponed to the Appendix.

Now let us turn our attention to the stage where firms decide whether they want to advertise heavily or lightly. Let us normalize s_2 to 1, fix $0 < s_1 < 1$ and set $a_H = 1$. Let $R^i_{a_ia_j}(s_i)$ denote the revenue of firm i in the equilibrium of the pricing stage, if it exists, given the advertising level choices a_i, a_j. Also, let $G^i_{a_j}(s_1) = R^i_{a_Ha_j}(s_i) - R^i_{a_La_j}(s_i)$, denote the gains to firm i of advertising heavily instead of lightly given firm j's advertising level. The following claim describes the possible equilibria at the advertising stage, denoted by (L, L), (L, H) or (H, H), where the first letter shows the level chosen by the low quality firm, whereas the second shows the choice of the high quality firm.

Claim 2 *There exists a $0 < a^* < 1$, such that, for $a_L > a^*$, we have the following.*

1. *If $c \geq G^2_L(s_1)$ and $s_1 \leq f(a_L, a_L)$ then the advertising equilibrium is (L, L).*

2. *If $G^2_L(s_1) \geq c \geq G^1_L(s_1)$ then the advertising equilibrium is (L, H).*

3. *If $G^1_L(s_1) \geq c$ then the advertising equilibrium is (H, H).*

4. *If $c \geq G^2_L(s_1)$ and $s_1 > f(a_L, a_L)$ then there is no pure-strategy advertising equilibrium.*

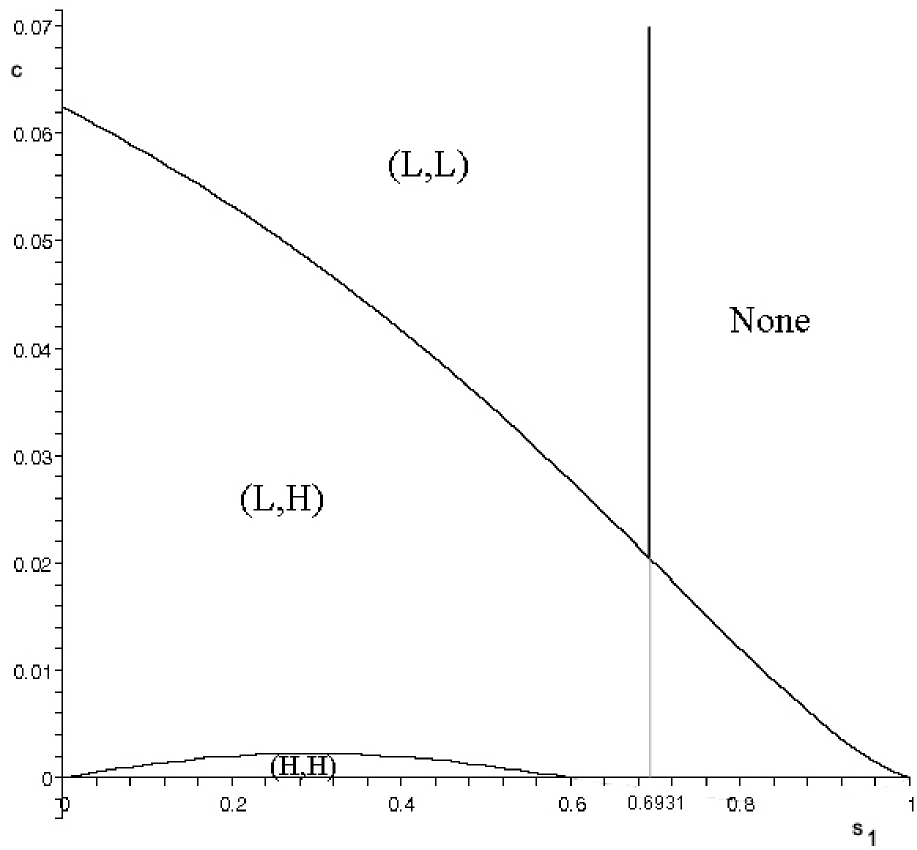

Figure 1: The different types of equilibria in the advertising stage for $a_l = 0.75$ and $a_2 = 1$.

Figure 1 shows the different types of equilibria for the different values of s_1 and c. Finally, we can solve for the qualities chosen in the (first) stage. We assume that $c < c_M = (a_H - a_L)/4$, which is the critical price for a monopolist to advertise high.

Proposition 1 *For any $a_L > a^*$, there exist $0 < \underline{c} < \overline{c} < c_M$, such that with some $0 < \underline{s} < 1, 0 < \overline{s} < 1$, we get the following equilibria*

1. *If $\overline{c} \leq c < c_M$, then $s_i = \overline{s}$,$s_j = 1$ and the advertising equilibrium is (L, L).*
2. *If $\underline{c} < c < \overline{c}$, then $s_i = (G_L^2)^{-1}(c)$,$s_j = 1$ and the advertising equilibrium is (L, L).*
3. *If $0 < c < \underline{c}$, then $s_i = \underline{s}$,$s_j = 1$ and the advertising equilibrium is (L, H).*

The results in regions 1 and 2 are intuitive, if the extra cost of heavy advertising is sufficiently high then both firms choose to advertise lightly, whereas with a lower cost difference one firm chooses to advertise heavily and the other advertises lightly. We calculate the numerical values in the following example.

Example 1 *If $a_L = 3/4$ and $a_H = 1$, the proposition describes the equilibria with $c_M = 1/16 = 0.0625$, $\overline{c} \approx 0.0238$, $\underline{c} \approx 0.0205$, $\overline{s} \approx 0.6503$ and $\underline{s} = 0.64$. See Figure 2.*

It is interesting to see, that with advertising the degree of differentiation is lower than in the original model without advertising. The intuition is that advertising reduces price competition in the last stage, because not all of the consumers are aware of both products. The degree of differentiation is generally increasing as c decreases, but there is an exception. In the range $\underline{c} < c < \overline{c}$ the differentiation decreases as c decreases. The explanation is very interesting. With a lower c, firms would move into the (L, H) advertising equilibrium but that is worse for the low quality firm than the (L, L), hence it increases its quality to attain the (L, L) equilibrium. However, the low quality firm ends up setting a quality level that is higher than optimal, given the (L, L) advertising strategies. The following corollary is about the equilibrium profits.

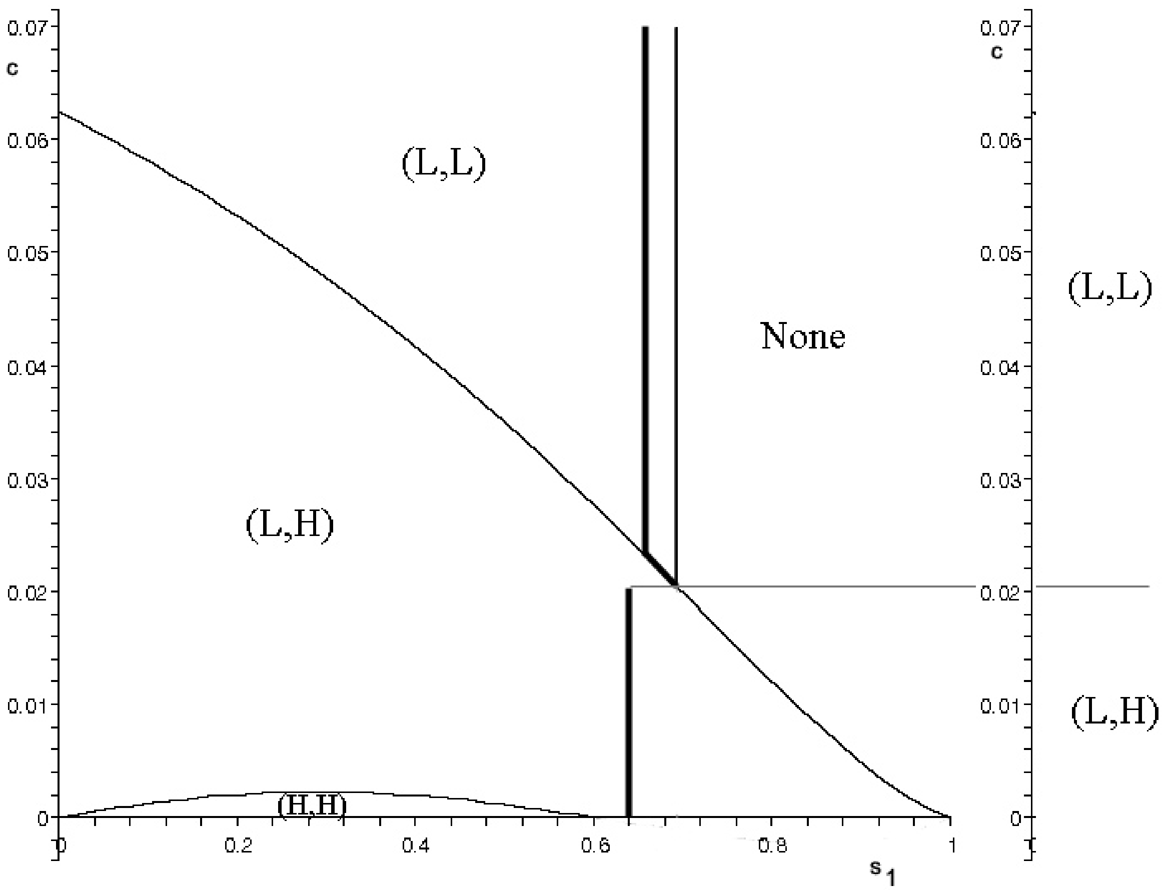

Figure 2: The bold line shows the different s_1 values and advertising equilibria as c changes for for $a_L = 0.75$ and $a_H = 1$. The right-hand size summarizes the types of advertising as c changes.

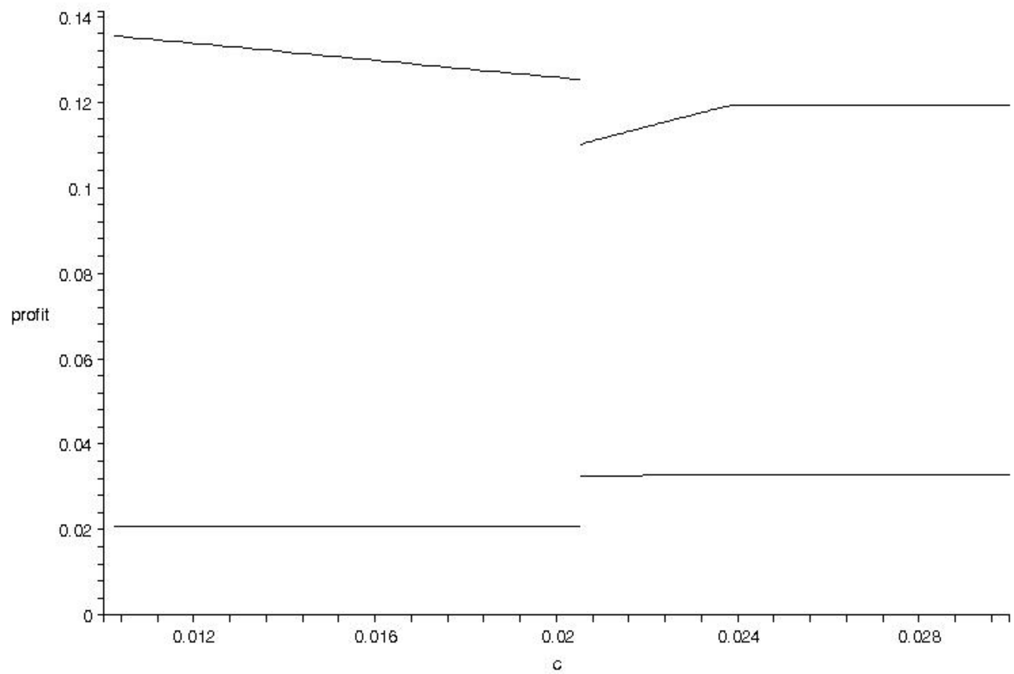

Figure 3: Profits as functions of c for for $c_L = 0$, $a_L = 0.75$ and $a_H = 1$.

Corollary 1 *π_2 is a decreasing function of c for $c \in [0, \underline{c}]$, it is increasing for $c \in [\underline{c}, \overline{c}]$ and constant for $c > \overline{c}$. Similarly π_1 is constant for $c \in [0, \underline{c}]$ and $c > \overline{c}$ and it is increasing for $c \in [\underline{c}, \overline{c}]$.*

Figure 3 shows firms' profit functions. Although the firm producing the low-quality product always makes less profits than the other one, the result reveals a surprising phenomenon in the $[\underline{c}, \overline{c}]$ interval: Firms' profits are increasing functions of c, that is, the extra cost of advertising at a high level. The intuition is the following. Although in this range neither of the firms advertises at a high level, their profits are influenced by c, since the equilibrium differentiation level changes with c. It is straightforward to see from the proof of Proposition 1, that the differentiation increases as c increases. Obviously this results in an increased profit for firm 2, but also raises firm 1's profits, as the the level of differentiation gets closer to the optimal value for firm 1. This implicit strategic effect explains the surprising impact of the cost difference.

Although in Proposition 1 we do not have an equilibrium where the advertising equilibrium is (H, H), there are certain a_L, a_H levels where we would get both firms advertising high at a low cost difference level. Let us consider the following example.

Example 2 *If* $a_L = 1/2$ *and* $a_H = 3/4$*, we get a similar structure of equilibria as in the proposition, but we have to take into account that the pricing equilibrium might not exist in every region. That is, we have to calculate* $f(3/4, 3/4) \approx 0.6930$*,* $f(1/2, 3/4) \approx 0.8250$*,* $f(1/2, 1/2) \approx 0.5755$ *and we get the following equilibria*

1. *If* $0.0414 < c < c_M = 0.0625$*, then* $s_i \approx 0.5755$*,*$s_j = 1$ *and the advertising equilibrium is* (L, L)*.*

2. *If* $0.0308 < c < 0.0414$*, then* $s_i = (G_L^2)^{-1}(c)$*,*$s_j = 1$ *and the advertising equilibrium is* (L, L)*.*

3. *If* $0.0045 < c < 0.0308$*, then* $s_i = 0.7155$*,*$s_j = 1$ *and the advertising equilibrium is* (L, H)*.*

4. *If* $0 < c < 0.0045$*, then* $s_i = 0.6503$*,*$s_j = 1$ *and the advertising equilibrium is* (H, H)*.*

5 Targeted Advertising

In this advertising model firms target segments (subsets) of consumers and a consumer can only consider buying a firm's product if s/he is in the targeted segment.[6] For the sake of simplicity, let us assume that consumers can be divided to two disjunct intervals and firms can target none, both, or either one of these segments. With

[6] Here we assume that advertising is perfectly efficient, that is, the ads reach every single consumer in the segment

the parameter $0 < t < 1$, the two intervals are defined as consumers who have ϑ less than t and consumers who have ϑ greater than t. Therefore, the size of the low-valuation interval ($\mathcal{L}$) is t, whereas the size of the high-valuation interval ($\mathcal{H}$) is $1-t$. Then the segment targeted by firm i(denoted by S_i) can be $\emptyset$, $\mathcal{L}$, $\mathcal{H}$or $\mathcal{L} \cup \mathcal{H}$. The costs associated with advertising to the four possible segments are $0, c_{\mathcal{L}}, c_{\mathcal{H}}, c_U$, respectively. We assume that costs are linear in the absolute size of the segment. That is, $c_{\mathcal{L}} = c_U t, c_{\mathcal{H}} = c_U(1-t)$,and $c_{\mathcal{L}\cup\mathcal{H}} = c_U$. The general setup and timing of the game is the same as described in Section 3. We assume that $c_U < t/4$ to ensure that both firms make positive profits in equilibrium.

It should be obvious that if t appraoches 0, i.e., the size of the low-valuation segment is negligible, the problem reduces to the blanket advertising case where firms can only advertise heavily. Hence,we focus our analysis on the case of $t \geq 1/3$ (the high-valuation segment is at most twice as large as the low-valuation segment). The following result shows that if $t \geq 1/3$, then in equilibrium firms do not differentiate. They divide up the market in the advertising stage and do not compete in prices. If $t < 1/3$, then the lower interval might be too small for firms to advertise to it and they may end up both settling in the high intervals. However, in this case, the exact description of equilibria would be too complicated.

Proposition 2 *If $t \geq 1/3$ then the only equilibria of the game are those sets of strategies where $s_2 = s_1 = 1$, $S_i = \mathcal{L}$, $S_j = \mathcal{H}$, $p_i = t/2$and $p_j = \max(t, 1/2)$.*

The intuition behind the results is that in the advertising stage firms are better off choosing disjunct segments, because then they are monopolists in their own segments and do not have to compete in prices in the final stage. In the first stage, when firms choose qualities, they separately maximize their qualities, since there is no strategic interaction, they both choose the highest possible quality. Since both firms offer the same quality, in equilibrium, either firm can end up advertising to the low-valuation segment or to the high-valuation segment. Let us denote the firm advertising to the low-valuation segment by 1 and the other one by 2.

Corollary 2 *If $1/3 < t < 1/2$, then equilibrium prices are $p*_1 = t/2$ and $p*_2 = 1/2$. Consumers with valuation in the interval t and $1/2$ are unserved, in the sense that they do not buy either product.*

It is usual in vertical differentiation models that consumers with the lowest valuation, here below $t/2$, are unserved, because prices are too high for them to purchase the products. However, note that if $1/3 < t < 1/2$, then consumers in the middle, with valuation between t and $1/2$ are also unserved. They only consider buying the product of the firm advertising to the high-valuation segment but the price is to high for them to buy the product. The following corollary summarizes how the demand changes as a function of t

Corollary 3 *The demand for product 1 is an increasing function of t. The demand for product 2 is a decreasing function of t. The total demand (consumers served) is first increasing ($t \in (1/3, 1/2]$), then decreasing ($t \in [1/2, 1]$), attaining its maximum at $t = 1/2$.*

Figure 4 shows the demand for firms 1 and 2 and the total demand. As t increases, firm 1 serves more consumers (those with a ϑ between $t/2$ and t). However, firm 2 serves all those who it advertises to except those below 1/2. The combination of the two functions yields that the total number of consumers served and is an inverted-U shaped function, with a maximum at 1/2.

6 Persuasive Advertising

In this section, we analyze an extension of the original model. Instead of focusing on the awareness building features of advertising, we examine the case of persuasive advertising. When consumers consider buying a product, its quality is an important factor in determining how much they are willing to pay for it. But the actual quality can be different from how consumers perceive it and obviously informative advertising can change the process of quality assessment. In an empirical study, Moorthy and

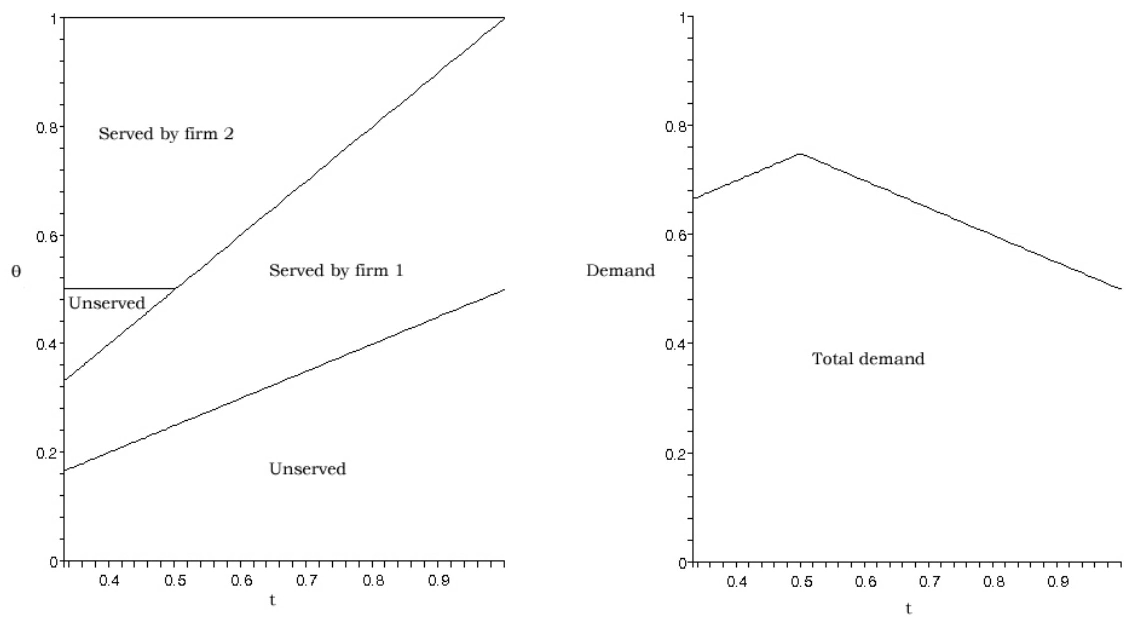

Figure 4: The left tab shows the consumers served by the firms as a function of t, whereas the right tab plots the total demand.

Zhao (2000) find (in ten product categories for more than one hundred brands) that advertising effort (spending) has a positive effect on perceived quality. Based on this phenomenon, we modify the vertical differentiation model in the following way.

Let s_i denote the real quality of the product offered by firm i. As before, let a_i denote the advertising effort of firm i. We assume that $0 \leq a_i \leq 1$ and that the perceived quality of product i is $a_i s_i$. This simple formulation captures the fact, that by advertising more, the firm can raise its product's perceived quality. However, the real quality forms an upper limit to the perceived quality.[7] Aside from the formulation of advertising, the setup of the game is equivalent to that of the blanket advertising model presented in Section 4. Firms first choose qualities simultaneously, then advertising efforts, finally, the prices.

As a first step, let us examine a very simple, but rather unrealistic case, when advertising is costless. Despite the oversimplifying assumption, this example sheds light on how firms differentiate in qualities when they can use advertising to change quality perception.

Claim 3 *The game has infinitely many sub-game perfect equilibria, where $s_j = 1$ and s_i can take any value between $4/7$ and 1. Furthermore, in every equilibrium $a_i s_i = (4/7) a_j s_j$.*

Interestingly, the degree of differentiation is less than or equal to the ration $s_1/s_2 = 4/7$ in the original vertical differentiation model (with no advertising). However, firms further differentiate in perceived qualities and reach the ratio of $4/7$.

In order to relax the assumption of costless advertising let us assume that advertising effort is a discrete variable (as in Section 4). Firms can choose between $a_i = a_L$ and $a_i = a_H$, where $0 \leq a_L < a_H \leq 1$. Furthermore, to ensure the existence of an interior solution, we assume that $7a_L > 4{a_H}$[8]. Let c_L and c_H denote the costs

[7] This formulation is equivalent to that in which the upper limit is a linear function of the real quality

[8] The assumption does not change the structure of the equilibria

associated with the two advertising levels, with c_L sufficiently low, that both firms enter the market. Again, let $c = c_H - c_L$.

Proposition 3 *There exist* $0 < \underline{c} < \bar{c}$*, such that we get the following sub-game perfect equilibria*

1. *If* $\bar{c} \leq c$*, then* $s_i = 4/7$*,*$s_j = 1$ *and the advertising equilibrium is* (L, L)*.*
2. *If* $\underline{c} \leq c < \bar{c}$*, then* $s_i = (4/7)(a_H/a_L)$*,*$s_j = 1$ *and the advertising equilibrium is* (L, H)*.*
3. *If* $0 < c < \underline{c}$*, then* $s_i = 4/7$*,*$s_j = 1$ *and the advertising equilibrium is* (H, H)*,*

The intuition behind the results is simple; if the extra cost of high-level advertising is sufficiently high then both firms choose the low level. If the cost difference is in the middle range one firm chooses to advertise high and the other advertises low, whereas if the cost difference is low, both firms can afford to advertise high (see Figure 5). Note that in the low and high regions the degree of differentiation is the same as in the model without advertising (4/7). However in the middle range we find the same phenomenon as in the simple example of Claim 3. The quality ratio s_1/s_2 is higher than 4/7, corresponding to a smaller degree of differentiation, which is compensated by the difference in advertising efforts leading to a perceived quality ratio of $4/7 = (s_1 a_1)/(s_2 a_2)$.

7 Conclusion

We studied a market where consumers are initially unaware of which products are available for purchase. We examined how the need to advertise in order to be included in the consumer's consideration set affects a firm's decision of where to position its product vis a vis the competition. We developed a model in which firms choose product quality in a first stage, their advertising strategy in a second stage, and set prices in the last stage. We studied three forms of advertising. Two of them,

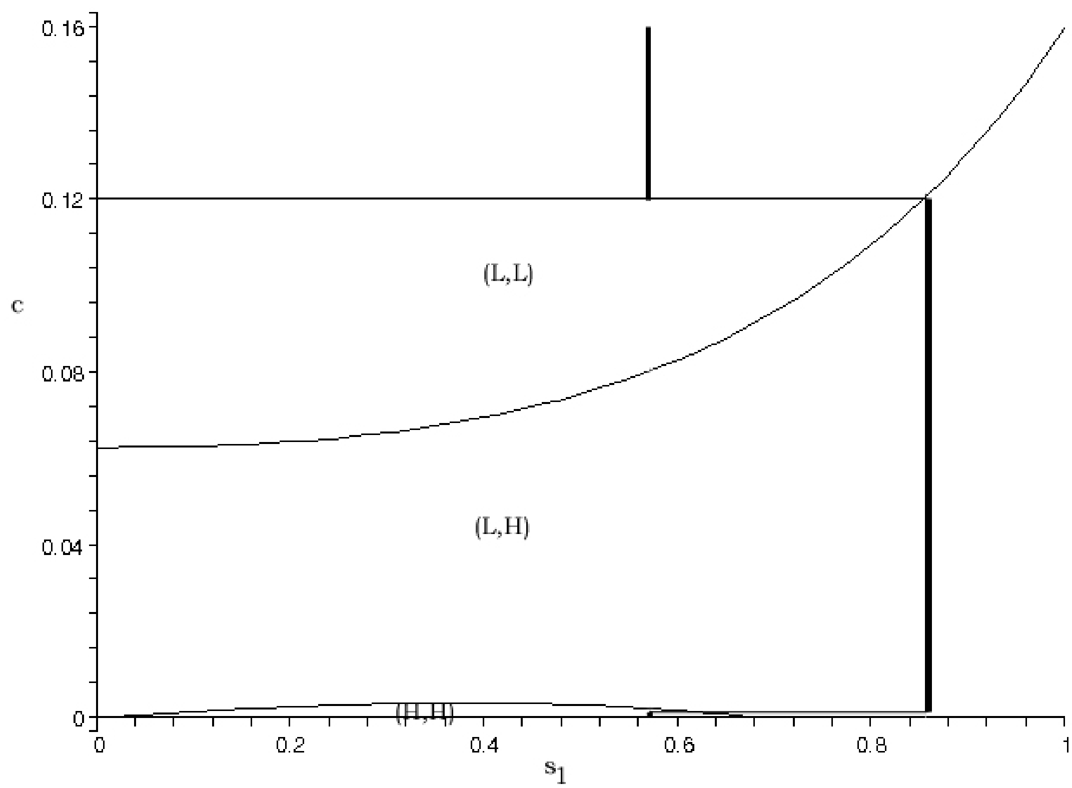

Figure 5: The bold line shows the different s_1 values and advertising equilibria as c changes for for $a_L = 0.5$ and $a_H = 0.75$.

firms can use to make consumers aware of their product – blanket advertising and targeted advertising. The third form is persuasive advertising, which serves to change consumers' perception of the product, resulting in an increase in perceived quality.

In the case of blanket advertising, we showed that firms can be better off under low levels of advertising because this allows them to choose higher quality product positioning, yet avoid fierce price competition. The equilibria of the game depend on the cost efficiency of advertising. When advertising is relatively ineffective, differentiation is relatively low and both firms advertise at a light level. When, at the other extreme, advertising is very effective, one firm chooses maximal quality and expends heavily on advertising. The rival now prefers to differentiate with a a much lower quality product and expend less on advertising. In a mid range, we obtained the intriguing result that by choosing quality appropriately, the low-quality firm induces the high-quality firm to expend only lightly on advertising. This result sheds light on how advertising and quality choice interact in a firm's strategy. It is important for managers to see that by making the two decisions separately they may be worse off.

Under targeted advertising, our main result was that each firm advertises to a different segment. Thus, firms do not compete in the pricing stage and hence both choose equally high quality– that is, their products are entirely undifferentiated in quality. Interestingly, due to the relatively high prices firm charge, there exist conditions such that a set of consumers with moderate valuation for quality are unserved by the firm that advertises to them, even though consumers in a different segment with lower willingness to pay are served by the rival firm. We also showed that the number of consumers served is an inverted-U shaped function of the lower segments size.

In an extension we studied the role of persuasive advertising. Our main result here is that firms differentiate less than under no advertising but the difference in advertising levels counterbalances this effect leading to a higher degree of differentiation in perceived qualities.

Although we thoroughly examined three different types of advertising, our study has several limitations. First, we analyze the advertising models separately. One could imagine a general model, where advertising is persuasive and also builds awareness at the same time. Also, combining the first two models, we would get a general setting in which advertising can be targeted but the reach of consumers is not perfect, resulting in two decision variables: the segment to target and the effectiveness of reaching consumers in the chosen segment. Second, we have employed very stylized assumptions on costs. In order to simplify the models, we assume that products are made at a zero variable cost and that producing a high quality product does not cost more. This way we were able to focus on the strategic forces that drive the differentiation. We leave the incorporation of variable costs to the model to future research.

Appendix: Proofs

Proof of Proposition 1

Proof of Claim 1

The demand structure is very similar to the original model, for those consumers, who aware of both firms' products. However, we have to consider those who are only aware of one of the two firms' product. Let us introduce notations for three types of indifferent consumers. The consumer, who's ϑ parameter is $t_2 = \frac{p_2 - p_1}{s_2 - s_1}$ obtains the same utility from buying firm 1's and firm 2's product. The consumer with $\vartheta = t_2' = \frac{p_2}{s_2}$ obtains 0 utility from buying firm 2's product, that is, s/he is indifferent between purchasing from firm 2 and not buying anything. Finally, $t_1 = p_1 s_1$ is the critical point of buying firm 1's product versus not buying anything.

If firm 1's relative price is lower ($\frac{p_1}{s_1} < \frac{p_2}{s_2}$), then $t_1 < t_2' < t_2$. In this case the demands are the following.

$$D_1(p_1, p_2) = a_1(t_2 - t_1) + a_1(1 - a_2)(1 - t_2).$$

$$D_2(p_1, p_2) = a_2(1 - t_2) + a_2(1 - a_1)(t_2 - t_2').$$

If, on the other hand $\frac{p_1}{s_1} \geq \frac{p_2}{s_2}$, then firm 1 only sells its product to the consumers who are not aware of firm 2. In this case

$$D_1'(p_1, p_2) = a_1(1 - a_2)(1 - t_1),$$

$$D_2'(p_1, p_2) = a_2(1 - t_2').$$

Since the variable cost is zero, the revenue is simply the price multiplied by the demand.

$$R_1(p_1, p_2) = \begin{cases} p_1 D_1(p_1, p_2), & \text{if } p_1 < \frac{p_2 s_1}{s_2}, \\ p_1 D_1'(p_1, p_2), & \text{if } p_1 \geq \frac{p_2 s_1}{s_2}. \end{cases}$$

$$R_2(p_1, p_2) = \begin{cases} p_2 D_2(p_1, p_2), & \text{if } p_2 > \frac{p_1 s_2}{s_1}, \\ p_2 D_2'(p_1, p_2), & \text{if } p_2 \leq \frac{p_1 s_2}{s_1}. \end{cases}$$

First we calculate the best response function of firm 2. Its revenue function consists of two quadratic functions, with maximums at $\frac{s_2}{2}$ and $\frac{s_2 - s_1 + p_1 a_1) s_2}{2(s_2 - s_1 + s_1 a_1)}$, respectively. Depending on the value of p_1 , the maximum is either attained in the first, the second, or the intersection. If $p_1 < \frac{s_1(s_2 - s_1)}{2(s_2 - s_1) + s_1 a_1}$, then the best response is $b_2(p_1) = \frac{s_2 - s_1 + p_1 a_1) s_2}{2(s_2 - s_1 + s_1 a_1)}$, if $\frac{s_1(s_2 - s_1)}{2(s_2 - s_1) + s_1 a_1} \leq p_1 < \frac{s_1}{2}$, then the best response is $b_2(p_1) = \frac{p_1 s_2}{s_1}$, and if $\frac{s_1}{2} \leq p_1$, then the best response is $b_2(p_1) = \frac{s_2}{2}$.

For firm 1, the best response is either $b_1(p_2) = \frac{(s_2-s_1-a_2s_2+a_2s_1+a_2p_2)s_1}{2(s_2-s_1+s_1a_1)}$ or $b_1(p_2) = \frac{s_1}{2}$, depending on where the revenue is higher. Since $b_1(s_2/2) < s_1/2$, the only possible equilibrium is

$$p_1^* = \frac{s_1(s_2-s_1)(2(s_2-s_1)-a_2s_2+2s_1(a_1+a_2-a_1a_2)}{4s_1^2(1+a_1a_2-a_1-a_2)+s_1s_2(4a_1+4a_2-a_1a_2-8)+4s_2^2},$$

$$p_2^* = \frac{s_2(s_2-s_1)(2(s_2-s_1)+s_1(a_1+2a_2-a_1a_2))}{4s_1^2(1+a_1a_2-a_1-a_2)+s_1s_2(4a_1+4a_2-a_1a_2-8)+4s_2^2}.$$

This is an equilibrium if and only if firm 1 has no incentive to deviate, that is, if

$$p_1^*D_1(p_1^*,p_2^*) \leq \frac{s_1}{2}D_1'\left(\frac{s_1}{2},p_2^*\right) = \frac{s_1a_1(1-a_2)}{4}. \tag{1}$$

In order to define f, we need the following the lemma.

Lemma 1 *$R(s_1) = p_1^*D_1(p_1^*,p_2^*)$ is a concave function of s_1 for $0 \leq s_1 \leq s_2$, $R(0) = R(s_2) = 0$ and $R'(0) > a_1(1-a_2)/4$ for $a_1, a_2 > 0$.*

Proof: We can assume without loss of generality that $s_2 = 1$. Then

$$R(s_1) = \frac{(1-s_1)(1-s_1+a_2s_1)(-2a_2s_1+2s_1-2s_1a_1+2a_1s_1a_2-2+a_2)^2a_1s_1}{(-a_1s_1a_2+4-8s_1+4s_1a_1+4s_1^2-4s_1^2a_1+4a_2s_1-4a_2s_1^2+4a_2s_1^2a_1)^2},$$

which is concave for $0 \leq s_1 \leq 1$, and

$$R'(0) = \frac{(2-a_2)^2a_1}{16} > \frac{a_1(1-a_2)}{4}$$

for $a_1, a_2 > 0$ □

Now, let $f(a_1,a_2)$ be the solution of the equation

$$p_1^*D_1(p_1^*,p_2^*) = \frac{s_1a_1(1-a_2)}{4},$$

with respect to s_1, setting s_2 to 1. Since the left hand side is a concave and the right hand size is linear function of s_1, the solution is unique in the interval $0 \leq s_1 \leq 1$, and according to the lemma $0 < f < 1$. Since s_1 only satisfies (1) if and only if $s_1 \leq f(a_1,a_2)$, we have proved parts *1* and *2*.

Then $f(a_1,1) = 1$ for any $0 < a_1 \leq 1$ also obviously follows from the lemma. In order to prove that $f(a,a)$ is increasing in a, we have to examine the function $R(s_1)$ more carefully. One can check that

$$R(s_1) - \frac{s_1a(1-a)}{4}$$

is increasing in a if $1/2 \leq a = a_1 = a_2$ for any $0 \leq s_1 \leq 1$. Since $R(s_1)$ is concave and $R(0) = 0$ this proves that $f(a, a)$ is increasing for $a \geq 1/2$.

If $s_1 = s_2$ and both advertising probabilities are positive, then there is always a positive mass of consumers, who are aware of both products. Thus, in a symmetric equilibrium, both prices have to be zero as a consequence of the Bertrand-type competition. On the other hand asymmetric equilibria do not exist, since any of the firms would be better off by setting a price of $s_1/2 = s_2/2$. $\square$

PROOF OF CLAIM 2

In order to determine a^*, we need the following observations, that can be proved by basic algebraic calculations.

Observation 1 *Let $G(q)$ denote the derivative $(G^1_H(1) - G^2_H(1))'$ as a function of q for $0 \leq q \leq 1$. Then $G(q)$ is an increasing function, and $G(q) = 0$ has a solution in the interval $0 \leq q \leq 1$.*

Let a^* be the solution of $G(q) = 0$, which is $a^* \approx 0.7032$.

Observation 2 *If $a_L > a^*$, then $G^1_L(s_1) < G^2_L(s_1)$ and $G^1_H(s_1) < G^2_H(s_1)$ for $0 < s_1 < 1$.*

Now let us examine when the different types of equilibria are possible.

- (L,L)
  If firm 1 chooses L, then firm 2's best response to this is L if and only if $c \geq G^2_L(s_1)$. Firm 1's best response to this is L if and only if $c \geq G^1_L(s_1)$. Since $G^1_L(s_1) < G^2_L(s_1)$, this type of equilibrium emerges only if $c \geq G^2_L(s_1)$. For its existence we also need that an equilibrium in the last stage exist, that is, $s_1 \leq f(a_L, a_L)$, otherwise no equilibrium exists.

- (L,H)
  If firm 1 chooses L, then firm 2's best response to this is H if and only if $c \leq G^2_L(s_1)$. Firm 1's best response to this is L if and only if $c \geq G^1_H(s_1)$. Since in this case the pricing equilibrium always exists (f(q,1)=1), this type of equilibrium emerges if and only if $G^1_H(s_1) \leq c \leq G^2_L(s_1)$.

- (H,H)
  If firm 1 chooses H, then firm 2's best response to this is H if and only if $c \leq G^2_H(s_1)$. Firm 1's best response to this is H if and only if $c \leq G^1_H(s_1)$. Since $G^1_H(s_1) < G^2_H(s_1)$ and the pricing equilibrium always exists, this type of equilibrium emerges if and only if $c \geq G^1_H(s_1)$. This completes the proof of the claim

□

In order to complete the proof we need the following observations, that can be proved by basic algebraic calculations.

Observation 3 $G_L^2(s_1)$ *is decreasing for* $0 \leq s_1 \leq 1$.

Observation 4 *We have* $R_{LH}^2(s_1) < R_{LL}^2$ *for* $0 < s_1 < 1$.

Observation 5 *We have* $\max_{\{0 \leq s_1 \leq 1\}} R_{LH}^1(s_1) = \max_{\{0 \leq s_1 \leq 1\}} R_{HH}^1(s_1)$.

Let $\overline{s} = \arg\max(R_{LL}^1(s_1))$ and $\underline{s} = \arg\max(R_{LH}^1(s_1))$. Then Let $\overline{c} = G_L^2(\overline{s})$ and $\underline{c} = G_L^2(f(a_L, a_L))$.

Now we can start determining the equilibria at the quality choice stage. Let us first examine firm 1's best response quality choice to $s_2 = 1$.

- If $c_M > c > \overline{c}$, then depending on s_1 the advertising equilibrium is either (L, H) or (L, L). Let $s' = (G_L^2)^{-1}(c)$ denote the critical value for (L, L) to realize. Therefore firm 1 maximizes R_{LH}^1 in the interval $0 \leq s_1 \leq s'$ and R_{LL}^2 in the interval $s' \leq s_1 \leq 1$. According to Observation 4, the maximum of R_{LL}^2 is greater than the maximum of R_{LH}^1. On the other hand, it follows from the definition of $\overline{c}$ that R_{LL}^2 attains its maximum in the interval $s' \leq s_1 \leq 1$, hence firm 1's best response is $\overline{s}$ and the advertising strategies are (L, L). Note that according to the definition of $\underline{c}$, the pricing equilibrium exist in this case.

- If $\overline{c} > c > \underline{c}$, then firm 1 still maximizes R_{LH}^1 in the interval $0 \leq s_1 \leq s'$ and R_{LL}^2 in the interval $s' \leq s_1 \leq 1$. However, in this case, the $R_{LL}^2()$ does not attain its maximum in the interval $s' \leq s_1 \leq 1$. On the other hand, $R_{LL}^2(s') \geq \max_{\{0 \leq s_1 \leq 1\}} R^2LH(s_1)$, therefore, the best response of firm 1 is $s' = (G_L^2)^{-1}(c)$ and the advertising strategies are (L, L). Note that according to the definition of $\underline{c}$, the advertising equilibrium still exist in this case.

- If $\underline{c} > c \geq 0$ let s'' and s''' denote the two solutions of $G_H^1(s_1) = c$, if they exist. then firm 1 maximizes R_{LH}^1 in the interval $0 \leq s_1 \leq s'$, except for the interval $s'' \leq s_1 \leq s'''$, where it maximizes $R_{HH}^1 - c$, if s'' and s''' exist, that is, if $c \leq maxG_H^1(s_1)$. Since $\max_{\{0 \leq s_1 \leq 1\}} R_{LH}^1(s_1) = \max_{\{0 \leq s_1 \leq 1\}} R_{HH}^1(s_1)$, firm 1's best response is always $\underline{s}$ and the advertising strategies are (L, H).

- If $c = 0$, then the firm 1 is indifferent between $4/7$ (where R_{HH}^1 attains its maximum) and $\underline{s}$.

In order to show that the above strategies are equilibria, we have to show that firm 2's response to firm 1's actual action is $s_2 = 1$. Note that firm 2's profit functions $R^2_{LL}(s_2)$, $R^2_{LH}(s_2)$ and $R^2_{HH}(s_2)$ are all increasing if we fix s_1 at certain level. Thus, the only incentive for firm 2 to not to choose $s_2 = 1$, if it could change the advertising equilibrium and increase its payoff trough that. This is, however not possible, as the only change that it could attain by decreasing s_2 from 1 is to change an (L, H) equilibrium to an (L, L) which is obviously not profitable. Thus, we have shown that all the above described strategies are equilibria.

We have to show that no other equilibrium exists, that is, that s_2 is always 1 in equilibrium. Let us assume that an equilibrium exists with $s_2^* < 1$. The advertising equilibria and the best response of firm 1 can be calculated the same way as for $s_2 = 1$, except that the s_1 values have to multiplied by s_2^*. As mentioned before, for a fixed advertising equilibrium and a fixed s_1, the profit of firm 2 is strictly increasing in s_2. Thus, the only way firm 2 has no incentive to increase s_2 is if that would change the advertising equilibrium and decrease profits. However, the above cases show that the only possibility to such a change would be from (L, L) to (L, H), but that does not decrease firm 2's profit. This completes the proof of the proposition. $\square$

Proof of Proposition 2

First, we examine the pricing stage of the game given the quality levels and the advertising segments.

1. If the two segments are distinct, that is, if $S_1 \cap S_2 = \emptyset$, then there is no price competition between the firms. They both maximize their income in their own segment. For the firm choosing the high interval ($S_i = \mathcal{H}$), the optimal price is $p_i^* = \max(s_i/2, s_i t)$. For the firms choosing the low interval ($S_j = \mathcal{L}$), the optimal price is $p_i^* = s_j t/2$.

2. If both firms choose both intervals, that is , if $S_1 = S_2 = \mathcal{L} \cup \mathcal{H}$, then there is full price competition and the pricing equilibria are the same as in the basic model. We have covered this case in Claim 1. Substituting $a_1 = a_2 = 1$ there yield the equilibria in this case. That is, if $s_1 = s_2$, then prices go down to zero, whereas if $s_1 < s_2$, then

$$p_1^* = \frac{s_1(s_2 - s_1)}{4s_2 - s_1},$$

$$p_2^* = \frac{2s_2(s_2 - s_1)}{4s_2 - s_1},$$

where the indifferent consumer has a parameter of $t_2^* = \frac{p_2^* - p_1^*}{s_2 - s_1} = \frac{2s_2 - s_1}{4s_2 - s_1}$.

3. If the firms choose the same intervals, for example $S_1 = S_2 = \mathcal{H}$, then in case of $s_1 = s_2$ equilibrium prices are zero. If $s_1 < s_2$, then depending on t four equilibria are possible. If $t \leq \frac{s_2 - s_1}{4s_2 - s_1}$, then

$$p_1^* = \frac{s_1(s_2 - s_1)}{4s_2 - s_1},$$

$$p_2^* = \frac{2s_2(s_2 - s_1)}{4s_2 - s_1}$$

form an equilibrium. If $\frac{s_2 - s_1}{4s_2 - s_1} \leq t \leq \frac{s2 - s1}{2s_2 + s1}$, then

$$p_1^* = ts_1,$$

$$p_2^* = \frac{s_2 - s_1 + ts_1}{2}.$$

If $1/2 > t \geq \frac{s2 - s1}{2s_2 + s1}$, then

$$p_1^* = \frac{(s_2 - s_1)(1 - 2t)}{3},$$

$$p_2^* = \frac{(s_2 - s_1)(2 - t)}{3}.$$

If $t \geq 1/2$, then

$$p_1^* = 0,$$

$$p_2^* = s_2 t/2.$$

4. If $S_1 = S_2 = \mathcal{L}$, then in case of $s_1 = s_2$ equilibrium prices are zero. If $s_1 < s_2$, then the game is equivalent to case 2, but we have to normalize the top of the interval to 1, that is,

$$p_1^* = t\frac{s_1(s_2 - s_1)}{4s_2 - s_1},$$

$$p_2^* = t\frac{2s_2(s_2 - s_1)}{4s_2 - s_1},$$

5. If $S_1 = \mathcal{L}$ and $S_2 = \mathcal{L} \cup \mathcal{H}$, then in case of $s_1 = s_2$ equilibrium prices are zero. If $s_1 < s_2$, then the price competition is limited to consumers below t. That is, firm 1 only has an incentive to decrease prices until the position of indifferent consumer $t_2 = \frac{p_2 - p_1}{s_2 - s_1}$ reaches either t or t_2^*. That is, in case of $t > t_2^*$, the equilibrium prices are the same as in case 2. On the other hand, if $t \leq t_2^*$ then $p_1^* = s_1 t/2$ and $p_2^* = \max(s_2/2, s_2 t)$.

6. If $S_1 = \mathcal{L} \cup \mathcal{H}$ and $S_2 = \mathcal{H}$, then in case of $s_1 = s_2$ equilibrium prices are zero. If $s_1 < s_2$, the price competition is limited to consumers above t. That is, firm 2 only has an incentive to decrease prices until the position of indifferent consumer $t_2 = \frac{p_2 - p_1}{s_2 - s_1}$ reaches either t or t_2^*. That is, in case of $t < t_2^*$, the equilibrium prices are the same as in case 2. On the other hand, if $t \geq t_2^*$ then $p_1^* = s_1 t/2$ and $p_2^* = \max(s_2/2, s_2 t)$.

7. If $S_1 = \mathcal{H}$ and $S_2 = \mathcal{L} \cup \mathcal{H}$, then in case of $s_1 = s_2$ equilibrium prices are zero. If $s_1 < s_2$, then two equilibria are possible. If $t \leq \frac{s_2 - s_1}{4s_2 - s_1}$, then

$$p_1^* = \frac{s_1(s_2 - s_1)}{4s_2 - s_1},$$

$$p_2^* = \frac{2s_2(s_2 - s_1)}{4s_2 - s_1}$$

form an equilibrium. If $t \geq \frac{s_2 - s_1}{4s_2 - s_1}$, then

$$p_1^* = \frac{(s_2 - s_1)(2ts_1 + s_2 - 3ts_2)}{7s_2 - 4s_1},$$

$$p_2^* = \frac{(s_2 - s_1)(t + 2)s_2}{7s_2 - 4s_1}$$

form an equilibrium.

8. If $S_1 = \mathcal{L} \cup \mathcal{H}$ and $S_2 = \mathcal{L}$, then in case of $s_1 = s_2$ equilibrium prices are zero. If $s_1 < s_2$, then if

$$t < \frac{20s_2^2 + s_1^2 - 12s_1 s_2 + (4s_2 - s_1)\sqrt{8s_2^2 + s_1^2}}{2(17s_2^2 - 9s_1 s_2 + s_1^2)}$$

there is no price competition and $p_1^* = \max(s_1/2, s_1 t), p_2^* = s_2 t/2$. Otherwise there is price competition and

$$p_1^* = \frac{s_1(s_2 - s_1)(2 - t)}{4s_2 - s_1},$$

$$p_2^* = \frac{(s_2 - s_1)(2ts_2 - ts_1 + s_1)}{4s_2 - s_1}.$$

9. If either firm chooses not to advertise then the other firm is a monopolist in its segment, setting the price to $\max(s_i x, s_i y/2)$ if the segment is the $[x, y]$ interval.

Now we study the advertising stage, that is, which segments firms choose to advertise to. Let us examine the different types of possible equilibria. A type is denoted by a pair (S_1, S_2), where the first set denotes the segment chosen by firm 1 and the second denotes the segment chosen by firm 2. First we fix $s_1 < s_2$ and determine which types of equilibria are possible if $t \geq 1/3$. Note that we do not consider cases where a firm does not advertise.

- $(\mathcal{L}, \mathcal{H})$ In this case $R_1 = s_1 t^2/4$, $R_2 = s_2/4$ if $t < 1/2$ and $R_1 = s_1 t^2/4$, $R_2 = s_2 t(1-t)$ if $t \geq 1/2$. In order to check when this constitutes and equilibrium we have to check three cases. First, if $t \geq 1/2$ firm 1 has no incentive to deviate since it cannot reach consumers above $t_2^* = \frac{2s_2 - s - 1}{4s_2 - s_1} \leq 1/2$. Firm 2 is better of by choosing $S_2 = \mathcal{L} \cup \mathcal{H}$ if and only if
$$\frac{4s_2^2(s_2 - s_1)}{(4s_2 - s_1)^2} - s_2 t(1-t) > c_L = tc_U. \tag{2}$$
If $1/2 > t > t_2^*$, then the condition is
$$\frac{4s_2^2(s_2 - s_1)}{(4s_2 - s_1)^2} - \frac{s_2}{4} > c_L = tc_U. \tag{3}$$
Finally, if $t_2^* \geq t$, then firm 2 has no incentive deviate, however firm 1 does have if and only if
$$\frac{s_1 s_2(s_2 - s_1)}{(4s_2 - s_1)^2} - s_1 t^2/4 > c_U = (1-t)c_U. \tag{4}$$
- $(\mathcal{L} \cup \mathcal{H}, \mathcal{H})$ In this case firm 2 has no incentive to deviate. However, firm 1 is better of setting $S_1 = \mathcal{L}$ if $t \geq t_2^*$. If $t < t_2^*$ firm 1 deviates if and only if (4) does not hold.
- $(\mathcal{L}, \mathcal{L} \cup \mathcal{H})$ In this case firm 1 has no incentive to deviate. However, firm 2 is better of setting $S_1 = \mathcal{H}$, if $t \leq t_2^*$. If $t > t_2^*$, firm 2 will deviate if and only if (2) and (3) do not hold in the cases $t \geq 1/2$ and $1/2 > t > t_2^*$, respectively.

In the following cases one of the firms always has an incentive to deviate, thus, they do not constitute equilibria. We assume that firms make positive profits otherwise it would be profitable for them to not advertise at all.

- $(\mathcal{L} \cup \mathcal{H}, \mathcal{L} \cup \mathcal{H})$ In this case either firm 1 or firm 2 has an incentive to deviate. If $t \leq t_2^* = \frac{2s_2 - s_1}{4s_2 - s_1}$, then consumers above below t_2^* will not buy from firm 2, therefore firm 2 is better of setting $S_2 = \mathcal{H}$. On the other hand, if $t \geq t_2^*$, then firm 1 is better off setting $S_1 = \mathcal{L}$.
- $(\mathcal{L}, \mathcal{L})$ In this case firm 2's revenues are $t^2 \frac{4s_2^2(s_2 - s_1)}{4s_2 - s_1}$. If firm 2 chooses $S_2 = \mathcal{L} \cup \mathcal{H}$ instead, then its revenues are at least $\frac{4s_2^2(s_2 - s_1)}{(4s_2 - s_1)^2}$, that is, are at least $1/t^2$ times more. Since costs are only $1/t$ times more, firm 2 has incentive to deviate.
- $(\mathcal{H}, \mathcal{H})$ If firm $t \geq 1/2$, firm 1 does not get any revenues, thus it obviously deviates. If $t \geq \frac{s_2 - s_1}{2s_2 + s_1}$, then firm 1 has an incentive to choose $S_1 = \mathcal{L}$, because then it makes

$s_1t^2/4 - tc_U$ instead of $(2t-1)^2(s_2-s_1)/9 - (1-t)c_U$ and $\frac{s_1}{s_2-s_1} \geq \frac{1-2t}{3t}$ in this region. Then one can check that if $1/5 \leq t \leq 1/2$, then

$$(1-t)\frac{1-2t}{3t}\frac{t^2}{4} - t\frac{(2t-1)^2}{9} \geq 0.$$

If 1/3
$leqt \leq \frac{s_2-s_1}{2s_2+s_1}$, then has an incentive to choose $S_1 = \mathcal{L}$, because then it makes $s_1t^2/4 - tc_U$ instead of $\frac{ts_1(s_2-s_1+ts_1-2ts_2)}{2(s_2-s_1)} - (1-t)c_U$. If $t \leq 1/3$, one can check that

$$(1-t)\frac{s_1t^2}{4} - t\frac{ts_1(s_2-s_1+ts_1-2ts_2)}{2(s_2-s_1)} \geq 0.$$

- $(\mathcal{H}, \mathcal{L})$ In this case firm 2's revenues in the pricing stage are $R_2 = s_2 * t^2/4$, thus it has an incentive to deviate to $S_2 = \mathcal{H}$ or $S_2 = \mathcal{L} \cup \mathcal{H}$.
- $(\mathcal{L} \cup \mathcal{H}, \mathcal{L})$ In this case firm 2 wants to choose $S_2 = \mathcal{H}$ or $S_2 = \mathcal{L} \cup \mathcal{H}$ depending on whether $t_2^* < t$.
- $(\mathcal{H}, \mathcal{L} \cup \mathcal{H})$ In this case firm

If $s_1 = s_2$, then in the cases where $|S_1 \cap S_2| > 0$, equilibrium prices are zero, that is, firms always have an incentive to deviate. The only possible advertising equilibria are $S_i = \mathcal{L}$ and $S_j = \mathcal{H}$. In this case firms do not have an incentive to deviate if they make positive profits.

Now we can examine the first (quality choosing) stage of the game. We start with the case $t \geq 1/2$. Note that firm 2's revenue is increasing in s_2 in any case, that is, in equilibrium $s_2 = 1$. Firm 1's profit is $\frac{s-1(1-s_1)}{(4s_2-s_1)^2}$ if (2) holds and $s_1 * t^2/4$ if not. Therefore, firm 1 is better of if (2) does not hold, therefore in equilibrium $s_1 = 1$ must hold. It is easy to check that there are equilibria with $s_1 = s_2$. Firms choose disjunct segments: $S_i = \mathcal{L}, \mathcal{S}_| = \mathcal{H}$ and the corresponding prices $p_i = t/2$, $p_j = t$.

If $1/3 \geq t < 1/2$, firm 2 again sets $s_2 = 1$ in equilibrium. Firm 1 is obviously better off if $t_2^* > t$, since t_2^* is decreasing in s_1 and firm 1's profit is at least $s_1t^2/4 - c_{\mathcal{L}}$ in this case. As s_1 approaches s_2, t_2^* goes to 1/2, hence firm 1 can attain that $t_2^* > t$. However, if $t_2^* > t$, then firm 1's profit is increasing in s_1 thus, in equilibrium, $s_1 = 1$ must hold. As in the previous case It is easy to check that there are equilibria with $s_1 = s_2$. Firms choose disjunct segments: $S_i = \mathcal{L}, \mathcal{S}_| = \mathcal{H}$ and the corresponding prices $p_i = t/2$, $p_j = 1/2$. This completes the proof of the proposition.

Proof of Claim 3

The pricing stage of the game is equivalent to that of the original vertical differentiation model with $s_1' = a_1 s_1$ and $s_2' = a_2 s_2$. Therefore, the revenues, as functions of s_1' and s_2', are

$$R_1 = \frac{s_1' s_2' (s_2' - s_1')}{(4s_2' - s_1')^2} \quad R_2 = \frac{4s_2'}{s_1'} R_1. \tag{5}$$

At the advertising stage, differentiating the two function with respect to a_1 and a_2 gives the best response functions

$$a_1 = \min\left(\frac{4s_2 a_2}{7s_1}, 1\right)$$

for the low-quality firm and $a_2 = 1$ for the high quality firm. Plugging these in the expressions in (5), we get $R_2 = 7/48s_2$. Thus, one firm always chooses $s_j = 1$. The other firm can choose anything above or equal to 4/7, that is $1 \geq s_i \geq 4/7$. A lower choice for s_i would not allow the low-quality firm to reach the optimal advertising level in the next stage. All the described strategies constitute sub-game perfect equilibria, completing the proof.

Proof of Proposition 3

The proof goes on similar lines as the proof of Proposition 1. However, the pricing stage of the game is much simpler here. In fact, it is equivalent to that of Claim 3. Thus, we can use the resulting revenue functions from (5) and turn our attention to the stage where firms decide whether they want to advertise at high or low levels. Let us normalize s_2 to 1 and fix $0 < s_1 < 1$. Let $R^i_{LL}(s_1)$ denote the revenue of firm i in the equilibrium of the pricing stage when both firm's choose $a_1 = a_2 = L$. In the other three cases let $R^i_{LH}(s_1)$, $R^i_{HL}(s_1)$, and $R^i_{HH}(s_1)$ denote the same revenue functions of firm i where the indices show the advertising level chosen by firm 1 and 2, respectively. Also, let $G^1_L(s_1) = R^1_{HL}(s_1) - R^1_{LL}(s_1)$, $G^1_H(s_1) = R^1_{HH}(s_1) - R^1_{LH}(s_1)$ $G^2_L(s_1) = R^2_{LH}(s_1) - R^2_{LL}(s_1)$, $G^2_H(s_1) = R^1_{HH}(s_1) - R^2_{HL}(s_1)$ denote the gains of setting advertising to high instead of low. One can check that $G^1_L(s_1) < G^1_H(s_1) < G^2_L(s_1) < G^2_H(s_1)$ if $1 \geq s_1 > 0$. In the following we determine the possible advertising equilibria given different c and s_1 values.

- (L,L)
  If firm 1 chooses $a_1 = L$, then firm 2 chooses $a_2 = L$ iff $c \geq G^2_L(s_1)$. However, firm 1's best response to this is L iff $c \geq G^1_L(s_1)$. That is, an (L, L) advertising equilibrium exist iff $G^2_L(s_1) \leq c$.

- (L,H)
  If firm 1 chooses $a_1 = L$, then firm 2 chooses $a_2 = H$ iff $c < G^2_L(s_1)$. However, firm 1's best response to this is L iff $c \geq G^1_H(s_1)$. That is, an (L, H) advertising equilibrium exist iff $G^1_H(s_1) \leq c < G^2_L(s_1)$.

- (H,L)
  If firm 1 chooses $a_1 = H$, then firm 2 chooses $a_2 = L$ iff $c \geq G^2_H(s_1)$. However, firm 1's best response to this is H iff $c < G^1_L(s_1)$. That is, an (H, L) advertising equilibrium never exists.

- (H,H)
  If firm 1 chooses $a_1 = H$, then firm 2 chooses $a_2 = H$ iff $c < G^2_H(s_1)$. However, firm 1's best response to this is H iff $c < G^1_H(s_1)$. That is, an (H, H) advertising equilibrium exist iff $0 < c < G^1_H(s_1)$.

Now we can start determining the equilibria at the quality choice stage. Let us first examine firm 1's best response quality choice to $s_2 = 1$. Given that the advertising equilibrium is (L, L) or (H, H), firm 1's best response is to choose $s_1 = 4/7$ as in the original model, since advertising has the same effect on both firms' perceived qualities. However, if the advertising equilibrium is (L, H), then firm 1 maximizes $R^1_{LH}(s_1)$ yielding $s_1 = 4/7(a_H/a_L)$. Note that the $\max_{s_1} R^1_{LH}(s_1) > \max_{s_1} R^1_{LL}(s_1)$ and $G^2_L(s_1)$ is increasing, thus always chooses an s_1 that leads to an (L, H) equilibrium over an (L, L) when it is possible, yielding $\overline{c} = G^2_L(4/7(a_h/a_L))$. Furthermore, firm 1 chooses an (H, H) equilibrium over and (L, H) if and only if $c < \underline{c} = R^1_{LL}(4/7) - R^1_{LH}(4/7(a_h/a_L)) > 0$. In order to show that firm 2 chooses $s_2 = 1$ in equilibrium, one can check that given any advertising equilibrium, firm 2's profit is an increasing function of s_2, thus it chooses the maximum quality of 1. $\square$

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