

# Industry Size and the Distribution of R&D Investment

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## Abstract

We analyze the link between industry size and R&D spending distribution. We consider a monopolistically competitive market in which firms can invest in cost-cutting R&D by paying a fixed cost first. For an intermediate level of fixed cost, there is a unique equilibrium in which the market segments into investing and non-investing firms. Using this equilibrium, we study how the distribution and level of R&D expenditure changes as industry size increases. In particular, we show that, as the market size increases, R&D spending can become more concentrated. Data motivating these results are drawn from the Taiwanese and Korean semiconductor industries.

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# 1 Introduction

This paper addresses the link between changes in industry size, as measured by the number of active firms, and the distribution of R&D investment in the market. We analyze a model in which a continuum of firms simultaneously invest in cost-cutting R&D and then compete in the product market. When investing in R&D involves a positive fixed cost  $K$  (e.g. establishment of an R&D department, investment in R&D capital, etc.), for an intermediate interval of values of  $K$  there is a unique equilibrium in which the market segments into two sets of firms. The firms in the first set pay the fixed cost  $K$  and invest in cost-cutting R&D. As a result, they are able to charge lower prices in the product market (because they have lower costs) and have higher product-market profits. The firms in the second set do not invest in R&D and have a higher price level and lower profits in the product-market. The relative measures of the two sets are determined in equilibrium by the fact that all firms must be ex-ante indifferent between being in the first or in the second set.

In Section 4, we use this equilibrium to analyze the comparative statics of a change in industry size on the distribution of R&D spending on the market. The distribution of R&D spending in this model is described by two variables: (i) the measure of investing firms and (ii) their level of R&D spending. Both these variables affect market price levels and consumer welfare.

We find that if the industry size increases, the measure of firms investing in R&D *always decreases* in equilibrium. The intuition behind this result is as follows: for a given industry size, suppose that we are in the asymmetric equilibrium as described earlier. If the industry size increases - that is, more firms join the market - and the set of investing firms remains the same, the profits of both types of firms decrease. However, the negative impact is greater for the investing firms than for the non-investing ones. Thus, to restore the equilibrium indifference condition between the two types of firms, we need to decrease the set of firms investing in R&D.

This implies that as industry size increases, fewer firms invest in R&D. As the firms investing in R&D charge lower prices and sell more product than the others, the decrease in the measure of firms investing in R&D results in fewer market leaders, which amounts in more market concentration.

Second, we examine the level of R&D spending of the investing firms. We show that two different effects influence the level of R&D spending. First, the increase in industry size tends to decrease the returns of R&D investment and, thus, to decrease R&D investment. We name this the “*industry size effect*.” Second, the decrease in the measure of investing firms could lead to lower market competition, higher R&D investment returns and, thus, a higher investment level. This effect is novel in the literature and is named the “*concentration effect*.” The overall effect on investment depends on the interplay between these two effects, and we provide sufficient conditions under which the R&D level of spending of the investing firms increases in equilibrium. This, together

with the decrease of investing firms, amounts to a *more concentrated R&D spending distribution* upon industry entry.

In Section 5 we derive a number of empirical implications from our results. In Section 6 we discuss the effects of industry entry on market efficiency and in particular on consumer welfare.

## 1.1 Empirical Motivation

The model explored in this paper is motivated by the striking distributional patterns in R&D expenditure observed in some data from industries experiencing significant expansions. Table 1 shows plant level data on the Taiwanese Semiconductor Industry (SIC 3211) drawn from manufacturing surveys conducted by the Ministry of Economic Affairs.<sup>1</sup> During the 1980s the Taiwanese semiconductor industry grew 648% (in terms of revenue) and 144% (in terms of establishments). As such, it is a neat example of an industry undergoing significant size changes.

Panel 1 of Table 1 shows, for each survey year, the sum of (nominal) revenue attributed to each plant, the total number of plants, the number and proportion of plants that recorded some expenditure on R&D, and the average (across all plants) and total (nominal) expenditures on R&D. The period between 1981 and 1986 is particularly interesting. The number of plants increased from 1279 to 2808 during this period. However, despite this rapid industrial expansion, the proportion of plants engaged in R&D steadily decreased from 26.5% to 18.4%.

Panels 2 and 3 of Table 1 show the percentiles of expenditures on R&D. Panel 2 shows the level of expenditure corresponding to each percentile of expenditure, so that in 1981 95% of plants had R&D expenditure equal to or less than 14,729. Panel 3 examines the proportion of total R&D expenditures conducted by plants at or below each percentile level, so that in 1981 firms with R&D expenditure equal to or less than 14,729 (i.e. corresponding to those firms at or below the 95th percentile) accounted for 18.4% of total R&D expenditure. Read together Panels 2 and 3 show a striking shift in the distribution of R&D expenditure, with R&D expenditure becoming more concentrated during the period of dramatic industry expansion occurring between 1981 and 1986. That is to say, those establishments in the far right tail of the distribution of R&D expenditures increase their R&D intensity markedly, while other establishments decrease their expenditures. It is notable that these trends become reversed in the years after 1986.<sup>2</sup>

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<sup>1</sup>The data in these surveys cover between 88% and 94% of employment in the manufacturing sector (depending on the survey year). We are grateful to Daniel Xu for his help in accessing these data. The data is available at <http://pages.stern.nyu.edu/~jasker/>

<sup>2</sup>During the early- to mid-80s the Taiwanese semiconductor industry was expanding in areas such as chip design, testign, packaging and some specialist fabrication. Significantly, the large DRAM (memory chip) market during this period was dominated by Japan and the US, with Taiwan not being a participant. A significant structural break

Similar patterns in R&D expenditure emerge in other data. Table 2 shows an abbreviated version of Table 1, reporting plant level data on the Korean semiconductor industry drawn from manufacturing surveys conducted by the Korean Government (see Xu 2008 for a description of these surveys). The same patterns seen in the Taiwanese data emerge in the Korean data during the period 1993-1996 which also corresponds to a period of dramatic industry expansion. The emergence of similar patterns in the Korean and Taiwanese data is all the more noteworthy due to the differences in government policy toward the semiconductor industry in each industry and the much higher concentration of the Korean semiconductor industry relative to the Taiwanese industry (see Matthews and Cho (2000) for a comparative history).

The model developed in this paper seeks to provide a market based explanation for these contemporaneous industry expansions and shifts in R&D expenditures. First, it captures the fact that while some establishments do engage in R&D, many do not. Second, it develops an intuition for why the distribution of R&D expenditures may change with industry size. Third, it explains why the maximum level of R&D expenditure in the industry increases (a topic of interest in earlier literature (e.g. Sutton (2001))). Lastly, it suggests some intuition behind the drop in the proportion of establishments engaged in R&D.<sup>3</sup>

It is worth stating explicitly that the model is intentionally stylized and does not purport to explain the Taiwanese and Korean experiences specifically or completely. The purpose of the model is to carefully develop a set of intuitions for how changes in industry size affects the distribution of R&D. The contribution of the model is to provide a set of comparative statics linking industry size and R&D patterns that provide a framework for applied researchers to balance the incentives provided by (say) R&D, or trade policies against the incentives provided by the market. It is designed to be broadly applicable to a large class of industrial settings. That said, the model is constructed with the patterns evident in the data firmly in mind.

The Taiwanese and Korean case studies, from which empirical motivation is drawn, contribute to several modelling decisions. First, the model focuses on cost cutting innovations rather than demand shifting innovations. Fitting any innovation into this taxonomy is always problematic, but necessary for parsimony. Since the Taiwanese and Korean semiconductor industries were never defining the frontier of technological accomplishment during the two periods of industry expansion, we have chosen to examine cost-cutting innovation. Second, we focus on exogenous changes in industry size as measured by firm numbers. An alternative may have been to model demand shifts

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occured in the industry in 1986 with the development of large scale foundry facilities by TSMC, heralding the beginning of Taiwan's entry into the DRAM fabrication (see Matthews and Cho (2000)).

<sup>3</sup>To capture the drop in the proportion but the (weak) increase in the number of firms doing R&D requires a minor extension of the basic model. As an example, a demand shift along with industry expansion can do this.

with a model of firm entry. However, since we wanted to be agnostic as to whether entry was a function demand shocks, government policy, globalization, or some other shock, we chose not to make the entry process endogenous.<sup>4</sup> Third, the data examined here is at the plant level and while the theory is at the firm level. This is of particular relevance to the Korean data, since the Korean industry is highly concentrated. Sadly, data on ownership structures are unavailable. Our concern about this aspect of the data lead us to examine both the Korean and Taiwanese data, hoping that seeing similar patterns across industries with very different structures would mitigate measurement problems arising from this data constraint. Similarly, the data is more aggregated than we would like, covering firms at several levels in the industry's vertical chain.<sup>5</sup> Lastly, we consider a model in which firms produce differentiated products. Servati and Simon (2005) describe the various dimensions on which products in the semiconductor industry are differentiated.

## 1.2 Existing Literature

Market structure was first identified by Shumpeter as one of the key determinants of R&D spending, the connection between industry size and R&D spending. This observation has generated a vast literature. In particular, Loury (1979), Lee and Wide (1980) and Reinganum (1982, 1985) first studied the impact of entry on R&D in the contest of patent races.

More recently, Sutton (2001) tackled the problem of linking industry concentration and R&D intensity. In his analysis, firms have the possibility to invest in several R&D trajectories. The scope economies across these trajectories and the effectiveness of R&D technology together determine a measure of how much a firm that outspends its rival in R&D can steal consumers away from other firms. This measure sets a lower bound to both the equilibrium market concentration and R&D intensity of the highest spender in the market. This approach is different from ours as our focus is on the entire R&D market distribution. In other words, besides studying the determinants of the highest R&D level on the market, we are also interested in studying the proportion of firms on the market that decide to spend in R&D at all, as a second key determinant of market efficiency.

An alternative modelling approach to that taken in this paper may have been to adopt the fully dynamic framework of Ericson and Pakes (1995) and Pakes and McGuire(1994). Setting aside the expositional advantages of an analytic model, the asymmetric equilibrium that is explored in this

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<sup>4</sup>Clearly, some shocks, like a demand shock, may affect other parameters beyond just the number of entering firms. A demand shock will shift the demand curve as well as attract entry. Our analysis isolates the entry effect as the comparative static that generates novel effect. The model is easily extended to accomodate a simultaneous demand shift (for instance).

<sup>5</sup>These considerations limit the extent to which the model can be meaningfully tested using these data. Testing - while possibly feasible using these, or similar, data - would require a data collection exercise beyond the scope of the current paper.

paper does not easily translate to the Markov-Perfect equilibrium concept exploited in the Pakes- and McGuire-style dynamic frameworks.

Empirical findings related to our results are in Dieter (1997). This paper is an empirical analysis on how globalization (seen as industry entry at several levels of production) leads to more concentration in the hard disk drive industry. These results are consistent with our results.

## 2 The Model

### 2.1 Firms and consumers

Consider a monopolistic competitive market populated by a fixed interval of firms  $\mathcal{N} = [0, n]$ . The preferences of the representative consumer are described by a Dixit-Stiglitz utility function<sup>6</sup>

$$u(y) = m + W \ln \left[ \int_0^n y(i)^\alpha di \right]^{1/\alpha}$$

where  $y(i)$  is the consumption of the good produced by firm  $i \in \mathcal{N}$ ,  $m$  is the numeraire,  $W > 0$  and  $\alpha \in (0, 1)$ . As is well known, utility maximization subject to the budget constraint  $m + \int_0^n p(i)y(i) di \leq E$ , yields to the demand function for good  $i$  given its price  $p(i)$  is

$$y(i) = Mp(i)^{-\frac{1}{1-\alpha}}$$

with  $M \equiv \frac{W}{\int_0^n p(j)^{-\frac{\alpha}{1-\alpha}} dj}$ .

After having paid a fixed cost  $K > 0$ , each firm  $i \in \mathcal{N}$  can invest an amount  $k(i) \in R_+$  in developing cost-cutting technology. Such investment has the effect of reducing the (constant) marginal cost of production of the firm according to the function  $c : R_+ \rightarrow (0, 1]$  defined as  $c(k) = (1 + k)^{-\rho}$  with  $\rho \in (0, 1)$ .<sup>7</sup>

To guarantee that the optimal investment problem has an interior solution and to focus the analysis on the most interesting cases, we assume that the parameters of the model satisfy the following requirements:

**Assumption 1**  $\alpha$  and  $\rho$  satisfy  $\alpha + \alpha\rho - 1 < 0$ .

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<sup>6</sup>See Dixit and Stiglitz (1977).

<sup>7</sup>Note that we do not require all the firms to develop the same technology, but we assume that two technologies developed with the same investment  $k$  cut the costs to the same level  $c(k)$ . Also, we focus on a particular functional form for  $c(\cdot)$  for the sake of simplicity. None of the results of the paper crucially depends on the specific characteristics of this functional form.

## 2.2 Timing

The timing of the game is as follows:

(1) All firms  $i \in N$  simultaneously decide whether and how much to invest in R&D by paying  $K + k(i) \in R_+$ . If a firm decides not to invest in R&D, that firm pays zero.

(2) Each firm  $i \in N$  adopts the technology  $c(i) = (1 + k(i))^{-\rho}$  and decides how much to produce by choosing  $q(i) \in R_+$ .

(3) The production is sold on the market and profits are realized.

In this paper, we adopt *Subgame Perfect Nash Equilibrium (SPNE)* as the solution concept, and focus on pure strategy SPNE.

### 2.2.1 Payoffs

The payoff of a generic firm  $i \in \mathcal{N}$  in the monopolistic competitive market is:

$$\tilde{\pi}(i) = [p(i) - c(i)] y(i) - k(i) - K 1_{\{k(i) > 0\}}$$

where  $[p(i) - c(i)] y(i)$  is the product-market profit,  $k(i)$  is the investment in R&D, and  $K$  is the fixed cost paid for R&D.<sup>8</sup> After solving for equilibrium in the final product market, because the firms' optimal mark-up rule in this model is  $p(i) = \frac{c(i)}{\alpha}$ , it is easy to check that

$$\tilde{\pi}(i) = \frac{W(1-\alpha)}{\int_{\mathcal{N}} c(j)^{\frac{-\alpha}{1-\alpha}} dj} c(i)^{-\frac{\alpha}{1-\alpha}} - k(i) - K 1_{\{k(i) > 0\}} \quad (1)$$

It is easy to see that the payoff of firm  $i$  is decreasing in the technology level of its competitors. This is because the technology level of the other firms affects the other firms' prices and, via monopolistic competition, the demand that firm  $i$  faces on the final product market.<sup>9</sup>

## 3 Equilibrium Analysis

Observe that, given that one firm decides to pay the fixed cost  $K$  and invest in R&D, the optimal investment level of firm  $i$  is given by the solution of the problem

$$\max_{k(i) \in R_+} A(1 + k(i))^{\frac{\alpha\rho}{1-\alpha}} - k(i) \quad (2)$$

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<sup>8</sup>We denote by  $1_E$  the indicator variable equal to 1 if the event  $E$  occurs and zero otherwise.

<sup>9</sup>In what follows, when describing the profit of a firm, we will use the notation  $A \equiv \frac{W(1-\alpha)}{\int_{\mathcal{N}} c(j)^{\frac{-\alpha}{1-\alpha}} dj}$ .

where  $A \equiv \frac{W(1-\alpha)}{\int_{\mathcal{N}} (1+k(j))^{\frac{\alpha\rho}{1-\alpha}} dj}$  is a function of the other firms' investment levels. By Assumption 1, such a problem has a unique solution,  $k^*(i) = \left(\frac{A\alpha\rho}{1-\alpha}\right)^{-\frac{1-\alpha}{1-\alpha-\alpha\rho}} - 1$ .

### 3.1 Symmetric Equilibria

Let us focus first on the symmetric equilibria of the model. If all firms invest, it is easy to see that the investment in equilibrium is  $k^* = \frac{W\alpha\rho}{n} - 1$ . If  $\frac{W\alpha\rho}{n} \leq 1$ , no firm invests in a symmetric equilibrium. We now focus on the case in which such investment is positive ( $\frac{W\alpha\rho}{n} > 1$ ), and, in the next result, we characterize the conditions under which all firms invest in R&D in equilibrium.

**Proposition 1** *There exists a  $\underline{K} > 0$  such that there is a equilibrium in which all firms invest in R&D if and only if  $\frac{W\alpha\rho}{n} > 1$  and  $K < \underline{K}$ . Under these conditions, such equilibrium is also unique.*

The next results provides necessary and sufficient conditions for an equilibrium in which no firm invests in R&D to exist.

**Proposition 2** *There exists  $\overline{K} > 0$  such that there is a equilibrium in which no firms invest in R&D if and only if either  $\frac{W\alpha\rho}{n} < 1$  or  $K > \overline{K}$ . Under is condition, this equilibrium is also unique.*

Since it is easy to show that  $\overline{K} > \underline{K}$ , there is no symmetric equilibrium if  $K \in (\underline{K}, \overline{K})$ .

### 3.2 Asymmetric Equilibrium

To complete the equilibrium analysis, let us look at the asymmetric equilibria of our model. In particular, in the next result, we characterize necessary and sufficient conditions for an asymmetric equilibrium to exist. We also show that, under the same conditions, this equilibrium is unique.

**Proposition 3** *If  $\frac{W\alpha\rho}{n} > 1$  and  $K \in (\underline{K}, \overline{K})$  there exists a unique equilibrium. In such equilibrium, only a positive measure  $\mu^* \in (0, n)$  of firms invest in R&D. If either  $\frac{W\alpha\rho}{n} \leq 1$  or  $K \notin (\underline{K}, \overline{K})$ , no asymmetric equilibria exist.*

The equilibrium described in Proposition 3 is built as follows. Let  $\mu$  be the measure of firms making a positive investment in R&D. Observe that, since all investing firms face the same problem (2), their R&D investment has to be the same. For any  $\mu$ , let this investment level be  $k(\mu)$ . Let us now denote by  $\pi_I(\mu)$  the equilibrium profits of a firm investing in R&D when a measure  $\mu$  of firms are investing  $k(\mu)$  in R&D. Similarly, denote by  $\pi_{NI}(\mu)$  the equilibrium profits of a firm not investing in R&D when a measure  $\mu$  of firms are investing  $k(\mu)$  in R&D. In equilibrium,  $\mu$  should



guarantee that the payoff of an investing firms is equal to the payoff of the non-investing firms - that is, the equilibrium  $\mu^*$  must satisfy

$$\pi_I(\mu^*) = \pi_{NI}(\mu^*) \quad (3)$$

In order to guarantee that such  $\mu^*$  exists in the range  $(0, n)$ , let us analyze the extremes of this interval. Suppose that  $\mu = 0$ , and observe that Proposition 2 implies that, if  $K < \overline{K}$ , then  $\pi_I(0) > \pi_{NI}(0)$ . On the other hand, suppose that  $\mu = n$ . Because of Proposition 1, we have that if  $K > \underline{K}$ , then  $\pi_I(n) < \pi_{NI}(n)$ . Thus, if  $K \in (\underline{K}, \overline{K})$ , the functions  $\pi_I(\mu)$  and  $\pi_{NI}(\mu)$  have to cross at least one  $\mu^*$  in the interval  $(0, n)$ .<sup>10</sup> The uniqueness part of the proof follows from the fact that, under our assumptions, the function  $\pi_I(\mu)$  decreases faster than  $\pi_{NI}(\mu)$  at any  $\mu$ . This implies that the two functions can cross at most once.

The result above describes the sufficient and necessary conditions for the existence of an asymmetric equilibrium in which a measure  $\mu^* > 0$  of firms invest  $k(\mu^*)$  in R&D, achieve a better technology, charge lower prices and collect higher profits on the product-market. On the other hand, a measure  $n - \mu > 0$  of firms do not invest in R&D, save in R&D fixed costs, charge higher prices and realize lower profits on the product-market.

## 4 Industry Size and the Distribution of R&D

In this section, we study the impact of a change in the size of the market (that is,  $n$ ) on the equilibria structure of the model. In particular, we are interested in looking at the distribution of R&D investment across firms after an increase in  $n$ . The R&D distribution is captured by the two endogenous variables  $\mu$ , the measure of firms investing in R&D, and  $k$ , the per-firm investment.

### 4.1 Industry Size and Concentration of R&D Investing

Note first that, by Proposition 2, when  $n > W\alpha\rho$ , the only equilibrium is a symmetric one in which no firm invests. Thus, letting  $n$  grow arbitrarily large, eventually causes firms to stop investing in R&D. Suppose now that  $n$  increases in the range  $[0, W\alpha\rho]$  The implications of an increase in the industry size  $n$  is described in the following Proposition.

**Proposition 4** *As  $n$  increases in the interval  $[0, W\alpha\rho]$ , the equilibrium measure of investing firms  $\mu^*$  decreases.*

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<sup>10</sup>We refer to the Appendix for the necessity of the conditions in Proposition 3 for the existence of an asymmetric equilibrium.

The intuition behind Proposition 4 is the following. For a given industry size  $n$ , suppose that we are in an asymmetric equilibrium as described in Proposition 3, and  $\mu^*$  is the equilibrium measure of investing firms. Let us now increase the industry size  $n$  and look at the profits of investing and not-investing firms if the measure of investing firms is still  $\mu^*$ . While the profits of both types of firms decrease, the negative impact is greater for the investing firms than for the non-investing ones. Thus, *the equilibrium indifference condition between the two types of firms must be satisfied at a  $\mu < \mu^*$ .*

The next results summarize how the equilibrium measure of investing firms changes as  $n$  varies.

**Corollary 5** *There are  $\underline{n}$  and  $\bar{n}$  such that if  $n < \underline{n}$  all firms invest in R&D, if  $n \in [\underline{n}, \bar{n}]$ , then the measure of investing firms  $\mu$  decreases as  $n$  increases, and if  $n > \bar{n}$ , then no firm invests.*

## 4.2 Industry Size and Level of R&D Investing

Let us now focus on the second variable that determines the R&D spending distribution in the industry- that is, the investment level of the investing firms  $k$ . The equilibrium investment level  $k$  is the maximum technology level reached in this economy, and, in equilibrium, it is affected by a change in the industry size  $n$ .

The impact of a change in  $n$  on the equilibrium investment  $k(n)$  is influenced by two separate effects. The first one, that we name “*industry size effect*,” measures how the increase in industry size, keeping the measure of investing firms constant, affects the investment level of the firms investing in R&D. Since an increase in industry size, via an increase in the level of market competition, decreases the returns of the R&D investment, it is always the case that the industry size effect affects the R&D spending level negatively. The second effect, that we name the “*concentration effect*,” is novel and measures how the increase in industry size affects the R&D investment level via the decrease in the measure of investing firms. Since a decrease of investing firms increases in R&D investment returns, the concentration effect affects the R&D spending level positively.

This discussion implies that the net effect of increase in industry size on R&D spending level depends on the relative size of two effects. Let  $\mu(n)$  be the function that maps the industry size  $n$  into the *equilibrium* measure of investing firms  $\mu$ . We can state now the following result.

**Proposition 6** *If  $\left| \frac{\partial \mu(n)}{\partial n} \left[ (1+k)^{\frac{\alpha \rho}{1-\alpha}} - 1 \right] \right| > 1$ , the concentration effect dominates the market effect and the level of R&D spending increases if  $n$  increases. If  $\left| \frac{\partial \mu(n)}{\partial n} \left[ (1+k)^{\frac{\alpha \rho}{1-\alpha}} - 1 \right] \right| < 1$ , the market effect dominates the concentration effect and the level of R&D spending decreases if  $n$  increases.*

Proposition 6 implies that  $\left| \frac{\partial \mu(n)}{\partial n} \left[ (1+k)^{\frac{\alpha \rho}{1-\alpha}} - 1 \right] \right| > 1$  is a sufficient condition for an increase in  $n$  to cause a more concentrated R&D resulting distribution-that is, a distribution with smaller

support and higher spending level.

## 5 Empirical Implications

Propositions 3 and 4, and Corollaries 5 and 6 allow us to derive the empirical implications of the model.

First, let us describe the implications of Proposition 3.

**Implication 1:** *In markets in which cost-cutting R&D involves a fixed cost, the market segments into two sets of firms. The firms in the first set invest in R&D, charge lower prices, sell more products and have higher product-market profits. The firms in the second set do not invest in R&D, charge higher prices, sell less product and have lower product-market profits.*

Implication 1 is reflected in the motivating empirical example from the Taiwanese semiconductor industry. On average, only around 25% of establishments are active in R&D in any year.

Second, let us turn to the impact of an increase in industry size on the measure of firms investing in R&D analyzed in Proposition 4 and Corollary 5. Because of Proposition 4,  $\left| \frac{\partial \mu(n)}{\partial n} \right|$  increases in  $(1+k)^{\frac{\alpha\rho}{1-\alpha}}$ . This implies the following:

**Implication 2:** *As the industry size increases, the set of firms investing in R&D on the market reduces. In addition:*

- (a) *This reduction is larger the higher the degree of substitution among the products ( $\alpha$ ).*
- (b) *This reduction is larger the better the R&D cost-cutting technology ( $\rho$ ).*

Implication 2 implies that, if data existed that allowed a comparison of the Korean and Taiwanese semiconductor industries on the basis of product and technology mix, we would expect to see (after appropriate econometric conditioning) that the industry with better technologies and closer substitutes experiencing a greater drop in the number of R&D active establishments during their expansionary periods.

Finally, let us turn to the empirical implications of Proposition 6. Both a higher degree of substitution among products (higher  $\alpha$ ) and a better R&D cost-cutting technology (higher  $\rho$ ) increase the concentration effect. Thus, we can derive the following:

**Implication 3:** *As entry occurs in a market characterized by a high (low) degree of substitution among products- that is, higher (lower)  $\alpha$ - the best technology level reached in the market increases (decreases).*

**Implication 4:** *As entry occurs in a market characterized by more (less) efficient R&D cost-cutting technology- that is, higher (lower)  $\rho$ - the best technology level reached in the market increases (decreases).*

Implications 3 and 4 suggest that, (again) if data existed that allowed a comparison of the Korean and Taiwanese semiconductor industries on the basis of product and technology mix, we would expect to see (after appropriate econometric conditioning) that the industry with better technologies and closer substitutes should experience a greater increase in R&D spending during their expansionary periods.

The intuition behind Implications 3 and 4 is the following. Because of Proposition 4,  $\left| \frac{\partial \mu(n)}{\partial n} \right|$  increases in  $(1+k)^{\frac{\alpha\rho}{1-\alpha}}$ . This implies that the condition  $\left| \frac{\partial \mu(n)}{\partial n} \right| \left[ (1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1 \right] > 1$  (which is necessary and sufficient for the concentration effect to dominate the market effect) is more likely to be satisfied as  $\frac{\alpha\rho}{1-\alpha}$  increases. Note that both a higher  $\alpha$  and a higher  $\rho$  increase the ratio  $\frac{\alpha\rho}{1-\alpha}$ , and, thus, they can increase the chances of  $\frac{\partial k(n)}{\partial n} > 0$  to be satisfied. Let us recall that a higher  $\alpha$  represents a higher degree of substitution among the products in the market. Thus, a high degree of substitution (hence, a higher degree of competition on the original market) leads to higher concentration after an increase in industry size and to a higher level of equilibrium investment. The same is true for a high  $\rho$ , which represents a better R&D cost-cutting technology. Thus, in markets that have better R&D cost-cutting technology, we should observe more market concentration after an increase in industry size and greater equilibrium R&D investment.

## 6 Welfare Analysis

In the previous sections we have analyzed how a change in industry size affects the R&D distribution in an industry. There may be multiple reasons why we should be concerned about R&D distribution from a market efficiency perspective. First, in an industry in which cost-cutting technologies are similar across differentiated products, a distribution in which many firms invest could imply an efficiency loss due to duplication costs. On the other hand, technology diffusion throughout the market could improve the chances of new ideas arising and incremental research to develop in the future. Third, the characteristics of the R&D distribution affect, via its effect on prices, the price distribution and the consumer welfare. In what follows, we discuss the consumer welfare implications of the results presented so far. In particular, we address the question of whether an increase in industry size is always beneficial for consumers. We show that there are conditions under which an increase in market size, via more concentrated R&D spending distribution, causes the consumers to be worse off.

First of all, note that since an increase in  $n$  results in an increase in the variety of products, there is a benefit for consumers that is not quantified in this model. Thus, here we focus on the price effects coming from the increase in industry size only. This implies that the net benefit we address should be considered a *lower bound* on the total net benefit for consumers.

Consider an increase in  $n$ . By Proposition 4, this will result in an increase in R&D concentration. By Proposition 6, if  $\left| \frac{\partial \mu(n)}{\partial n} \right| \left[ (1+k)^{\frac{\alpha \rho}{1-\alpha}} - 1 \right] < 1$ , we have that  $\frac{\partial k(n)}{\partial n} < 0$ . This would imply that the resulting two effects, an increase in market concentration and a decrease in R&D spending, are negative for consumers. Thus,  $\left| \frac{\partial \mu(n)}{\partial n} \right| \left[ (1+k)^{\frac{\alpha \rho}{1-\alpha}} - 1 \right] < 1$  is a *sufficient condition* for an increase in industry size to be negative for the consumers, and, as we already discussed in the previous section, both a higher  $\alpha$  and a higher  $\rho$  make this condition more likely to be satisfied.

However, if  $\left| \frac{\partial \mu(n)}{\partial n} \right| \left[ (1+k)^{\frac{\alpha \rho}{1-\alpha}} - 1 \right] > 1$ , we have that  $\frac{\partial k(n)}{\partial n} > 0$ . Then, for consumers' point of view, there is a tension between the increase in market concentration (which is a negative effect) and an increase in the R&D spending of the investing firms (which is a positive effect). Thus, the effects on consumer welfare could be either positive or negative depending on the relative size of these effects.

## 7 Conclusion

The model presented in this paper is novel in that it shed light on some of the determinants of the highest R&D level on the market and, particularly, studies the proportion of firms on the market that decide to spend in R&D at all and the R&D intensity of these firms. As such it examines an important sub-set of the determinants of the broader distribution of R&D activity with an industry.

The focus of the paper on how industry size affects R&D is motivated by the striking changes in R&D patterns observed in data from the Taiwanese and Korean semiconductor industries during periods of dramatic industry expansion. The model developed to explore market based intuitions for these sorts of changes captures several elements of the data including: the division of establishments into those that do and do not engage in R&D (empirical implication 1); the increase in the intensity of R&D conducted by the top percentile of firms engaged in R&D (empirical implications 3 and 4); and, gives some sense of the equilibrium forces that may have contributed to the proportion of firms engaged in R&D decreasing (building on empirical implication 2).

While the model is much more generally applicable, this motivating example provides an empirical setting that is useful to see the relevance and usefulness of the framework provided.

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## Appendix

*Proof of Proposition 1.* Let us assume that all firms invest in R&D. In this situation, the equilibrium investment of all firms is  $k^* = \frac{W\alpha\rho}{n} - 1$ . Thus, if a firm invests, the profit is  $\frac{W(1-\alpha)}{n} - \frac{W\alpha\rho}{n} + 1 - K$ , while if it doesn't the profit is  $\frac{W(1-\alpha)}{n(\frac{W\alpha\rho}{n})^{\frac{\alpha\rho}{1-\alpha}}} = \left(\frac{W}{n}\right)^{\frac{1-\alpha-\alpha\rho}{1-\alpha}} (1-\alpha)(\alpha\rho)^{\frac{-\alpha\rho}{1-\alpha}}$ . This implies that all firms investing in R&D is an equilibrium if

$$\frac{W(1-\alpha)}{n} - \frac{W\alpha\rho}{n} + 1 - K \geq \left(\frac{W}{n}\right)^{\frac{1-\alpha-\alpha\rho}{1-\alpha}} (1-\alpha)(\alpha\rho)^{\frac{-\alpha\rho}{1-\alpha}}$$

or

$$K \leq \frac{W}{n} [1 - \alpha - \alpha\rho - \left(\frac{W\alpha\rho}{n}\right)^{\frac{-\alpha\rho}{1-\alpha}} (1-\alpha)] + 1 \equiv \underline{K} \blacksquare$$

*Proof of Proposition 2.* Suppose that no firm invests in R&D. In this case, if a firm invests, the optimal investment is  $k^* = \left(\frac{W\alpha\rho}{n}\right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}} - 1$ . By Assumption 1,  $\frac{W\alpha\rho}{n} > 1$  guarantees  $k^*$  to be

positive. Thus, a firm is better off not investing if

$$\frac{W(1-\alpha)}{n} \geq \frac{W(1-\alpha)}{n} (1+k^*)^{\frac{\alpha\rho}{1-\alpha-\alpha\rho}} - k^* - K.$$

This implies that if

$$K \geq \bar{K} \equiv \left( \frac{W\alpha\rho}{n} \right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}} \left[ \frac{1-\alpha}{\alpha\rho} - 1 \right] - \frac{W(1-\alpha)}{n} + 1$$

there exist a unique equilibrium in which no firm invests in R&D ■

*Proof of Proposition 3.* First, we build an asymmetric equilibrium and we find the equilibrium measure  $\mu$  of investing firms, then, we address uniqueness. Let  $\mu$  be the measure of firms making a positive investment in R&D. Observe that, since all investing firms face the same problem (2), their investment has to be the same. In particular, the optimal investment problem of an investing firm  $i$  (2) has solution  $k_i^* = \left( \frac{A(\mu, k_{-i})\alpha\rho}{1-\alpha} \right)^{\frac{1-\alpha}{1-\alpha\rho-\alpha}} - 1$  where  $A(\mu, k_{-i}) = \frac{W(1-\alpha)}{\mu(1+k_{-i})^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu)}$  and  $k_{-i}$  represents the investment level of the other investing firms. As  $dk_i^*/dk_{-i} < 0$ , for any  $\mu$ , we have a unique  $k^*$  satisfying the equilibrium condition

$$k^* = \left( \frac{A(\mu, k^*)\alpha\rho}{1-\alpha} \right)^{\frac{1-\alpha}{1-\alpha\rho-\alpha}} - 1$$

which we denote by  $k^*(\mu)$ . Let us now denote by  $\pi_I(\mu)$  the equilibrium profits of a firm investing in R&D- that is,  $\pi_I(\mu) = \frac{W(1-\alpha)(1+k^*(\mu))^{\frac{\alpha\rho}{1-\alpha}}}{\mu(1+k^*(\mu))^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu)} - K$  - and by  $\pi_{NI}(\mu)$  the equilibrium profits of a firm non investing in R&D- that is,  $\pi_{NI}(\mu) = \frac{W(1-\alpha)}{\mu(1+k^*(\mu))^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu)}$ . In equilibrium,  $\mu$  should be such that the payoff of the investing firms should be equal to the payoff of the non-investing firms- that is,

$$\pi_I(\mu) = \pi_{NI}(\mu) \tag{4}$$

In order to guarantee that  $\mu$  is in the range  $(0, n)$ , let us analyze the extremes of this interval. Suppose that  $\mu = 0$ , and let us check that  $\pi_I(0) > \pi_{NI}(0)$ . If  $\mu = 0$ , we have  $k^* = \left( \frac{W\alpha\rho}{n} \right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}} - 1$ . Because of Assumption 1, we have that  $k^* > 0$  if and only if  $\frac{W\alpha\rho}{n} > 1$ . In this case,  $\pi_I(0) > \pi_{NI}(0)$  if  $K < \bar{K}$ . On the other hand, if a measure  $\mu = n$  of firms invest,  $\pi_I(n) < \pi_{NI}(n)$  if  $K > \underline{K}$ . Thus, condition (4) has to be satisfied for at least one  $\mu^* \in (0, n)$ . To show uniqueness of the equilibrium, follow exactly steps (a) and (b) of Proposition 9 in Baccara (2007).

For the argument we just made on the extremes of the interval  $(0, n)$ , Propositions 1 and 2 guarantee that if  $K \notin (\underline{K}, \bar{K})$ ,  $\pi_I(\mu)$  and  $\pi_{NI}(\mu)$  do not cross in the range  $(0, n)$  and no  $\mu$  satisfies (4).

Finally, let us now focus on the case  $\frac{W\alpha\rho}{n} < 1$ , and let us check if it is possible in this case to sustain asymmetric equilibria in which a measure  $\mu \in (0, n)$  of firms invest in equilibrium. Note that, if  $\mu = 0$ , we have  $k^* = \left(\frac{W\alpha\rho}{n}\right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}} - 1 < 0$  (this is guaranteed by Assumption 1, which implies that  $k^* > 0$  if and only if  $\frac{W\alpha\rho}{n} > 1$ ). Thus, since  $\frac{dk(\mu)}{d\mu} < 0$ , the R&D level for investing firms has to be zero for any  $\mu$ . However, if this is the case, non-investing firms are always better off than investing firms, and there is no asymmetric equilibrium. This guarantees that there are no asymmetric equilibria if  $\frac{W\alpha\rho}{n} < 1$  and concludes the characterization of the asymmetric equilibria. ■

*Proof of Proposition 4.* Fix  $n \in (0, W\alpha\rho)$  and  $K \in (\underline{K}, \overline{K})$ . Thus, by Proposition 3, there exists an asymmetric equilibrium where a measure  $\hat{\mu}$  of firms invests in R&D. Now, keeping  $\hat{\mu}$  constant, let  $k_i$  be the solution of firm  $i$ 's problem

$$k_i = \arg \max_{k \in \mathbb{R}_+} \frac{W(1-\alpha)}{\hat{\mu}(1+k_{-i})^{\frac{\alpha\rho}{1-\alpha}} + (n-\hat{\mu})} (1+k)^{\frac{\alpha\rho}{1-\alpha}} - k \quad (5)$$

and denote  $k(n) \equiv k_i = k_{-i}$  the investment level of the investing firms as a function of  $n$ . Let  $A(n) \equiv \frac{W(1-\alpha)}{\mu(1+k(n))^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu)}$ .

*First step:* First, let us show that  $\frac{\partial A(n)}{\partial n} < 0$ . We have

$$\frac{\partial A(n)}{\partial n} = \frac{-W(1-\alpha) \left[ \mu \frac{\alpha\rho}{1-\alpha} (1+k(n))^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} \frac{\partial k(n)}{\partial n} + 1 \right]}{\left[ \mu(1+k(n))^{\frac{\alpha\rho}{1-\alpha}} + (n-\mu) \right]^2}$$

thus,  $\frac{\partial A(n)}{\partial n} < 0$  if and only if

$$\mu \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} \frac{\partial k(n)}{\partial n} + 1 > 0 \quad (6)$$

Now, let us evaluate  $\frac{\partial k(n)}{\partial n}$ . From the first order condition of 5, we have that the function  $k(n)$  is implicitly defined by

$$\frac{W\alpha\rho(1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}}{n + \mu \left[ (1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1 \right]} - 1 = 0$$

Thus, we have

$$\frac{\partial k(n)}{\partial n} = \frac{1}{\frac{\alpha\rho-1+\alpha}{1-\alpha} (1+k)^{\frac{-1+\alpha}{1-\alpha}} \left[ n + \mu \left[ (1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1 \right] \right] - \mu \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}} \quad (7)$$

Substituting (7) into (6), we get that (6) is equivalent to



$$\frac{\mu \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}}{\frac{\alpha\rho-1+\alpha}{1-\alpha} (1+k)^{\frac{-1+\alpha}{1-\alpha}} \left[ n + \mu \left[ (1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1 \right] \right] - \mu \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}} > -1$$

or

$$\frac{\frac{\alpha\rho\mu}{1-\alpha} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}}{\frac{-\alpha\rho\mu}{1-\alpha} (1+k)^{\frac{-1+\alpha}{1-\alpha}} + \frac{n\alpha\rho}{1-\alpha} (1+k)^{\frac{-1+\alpha}{1-\alpha}} - (1+k)^{-1} \left[ n + \mu \left[ (1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1 \right] \right]} > -1$$

which is equivalent to

$$\frac{\frac{\alpha\rho\mu}{1-\alpha} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}}{\frac{n\alpha\rho}{1-\alpha} (1+k)^{-1} - \frac{\alpha\rho\mu}{1-\alpha} (1+k)^{-1} - \frac{n}{(1+k)} - \mu (1+k)^{\frac{-1+\alpha+\alpha\rho}{1-\alpha}} + \frac{\mu}{(1+k)}} > -1$$

Now, let  $D \equiv n \frac{\alpha\rho}{1-\alpha} (1+k)^{-1} - \mu \frac{\alpha\rho}{1-\alpha} (1+k)^{-1} - n (1+k)^{-1} - \mu (1+k)^{\frac{-1+\alpha+\alpha\rho}{1-\alpha}} + (1+k)^{-1} \mu$ . If  $D \geq 0$ , (6) is satisfied and the claim is true. Suppose, instead, that  $D < 0$ . In this case, (6) becomes

$$\mu \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} < -n \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{-1+\alpha}{1-\alpha}} + \mu \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{-1+\alpha}{1-\alpha}} + n (1+k)^{\frac{-1+\alpha}{1-\alpha}} + \mu (1+k)^{\frac{-1+\alpha+\alpha\rho}{1-\alpha}} - (1+k)^{\frac{-1+\alpha}{1-\alpha}} \mu \quad (8)$$

Since, by Assumption,  $\frac{\alpha\rho}{1-\alpha} < 1$ , a sufficient condition to guarantee (8) to be satisfied is

$$\mu (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} < -n \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{-1+\alpha}{1-\alpha}} + \mu \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{-1+\alpha}{1-\alpha}} + n (1+k)^{\frac{-1+\alpha}{1-\alpha}} + \mu (1+k)^{\frac{-1+\alpha+\alpha\rho}{1-\alpha}} - (1+k)^{\frac{-1+\alpha}{1-\alpha}} \mu$$

which reduces to

$$0 < (n - \mu) \frac{-\alpha\rho}{1-\alpha} + (n - \mu)$$

or  $(n - \mu) \frac{1-\alpha-\alpha\rho}{1-\alpha} > 0$ , which is true since  $\frac{1-\alpha-\alpha\rho}{1-\alpha} > 0$ .

*Second step:* To show that  $\mu$  decreases as  $n$  increases, first observe that, if we start with an equilibrium  $\hat{\mu}$  measure of investing firms such that  $\pi_{NI}(\hat{\mu}) = \pi_I(\hat{\mu})$ , and we denote by  $k(n)$  equilibrium investment level at  $\hat{\mu}$ , we have that

$$\begin{aligned} \frac{\partial \pi_I(n)}{\partial n} &= \frac{\partial A(n)}{\partial n} (1+k(n))^{\frac{\alpha\rho}{1-\alpha}} + \\ &+ \left[ A(1+k(n))^{\frac{\alpha\rho-1+\alpha}{1-\alpha}} \frac{\alpha\rho}{1-\alpha} - 1 \right] \frac{\partial k(n)}{\partial n} \\ &= \frac{\partial A(n)}{\partial n} (1+k(n))^{\frac{\alpha\rho}{1-\alpha}} \\ &< \frac{\partial A(n)}{\partial n} = \frac{\partial \pi_{NI}(n)}{\partial n} < 0 \end{aligned}$$

where  $\frac{\partial A(n)}{\partial n} (1+k(n))^{\frac{\alpha\rho}{1-\alpha}} < \frac{\partial A(n)}{\partial n}$  is guaranteed by the fact that, because of the first step of the proof,  $\frac{\partial A(n)}{\partial n} < 0$ . This implies that if  $n$  increases,  $\pi_I(n)$  decreases faster than  $\pi_{NI}(n)$ . So, we have that, if we increase  $n$ , the new payoff functions computed at  $\hat{\mu}$  are such that  $\pi_I(\hat{\mu}) < \pi_{NI}(\hat{\mu})$ , which implies that the intersection between the two curves occurs at a lower  $\mu$ . ■

*Proof of Corollary 5.* Recall that if  $K < \underline{K} = \frac{W}{n}(1-\alpha-\alpha\rho) + 1$ , in the unique equilibrium all firms invest, thus, if  $n < \frac{W}{K}(1-\alpha-\alpha\rho) + 1 \equiv \underline{n}$ , the measure of investing firms is  $n$ . If  $K > \bar{K} = \left(\frac{W\alpha\rho}{n}\right)^{\frac{1-\alpha}{1-\alpha-\alpha\rho}} \left[\frac{1-\alpha}{\alpha\rho} - 1\right] - \frac{W(1-\alpha)}{n} + 1$ , no firm invests. Let us show that this implies a threshold for  $n$ . To see this, it is sufficient to show that the  $\bar{K}$  is monotonically decreasing in  $n$ . However, it is easy to see that

$$\frac{\partial \bar{K}}{\partial n} = -\frac{1-\alpha}{1-\alpha-\alpha\rho} \left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha-\alpha\rho}} \frac{W\alpha\rho}{n^2} \left[\frac{1-\alpha}{\alpha\rho} - 1\right] + \frac{W(1-\alpha)}{n^2} < 0$$

since  $-\left(\frac{W\alpha\rho}{n}\right)^{\frac{\alpha\rho}{1-\alpha-\alpha\rho}} + 1 < 0$  because  $\frac{W\alpha\rho}{n} > 1$ .

Thus, there is  $\hat{n}$  such that, if  $n > \hat{n}$ , then in the unique equilibrium no firm invests. Recall that, by Proposition 2, if  $n > W\alpha\rho$ , nobody invests either, so let us set  $\bar{n} = \min\{W\alpha\rho, \hat{n}\}$ . The rest of the proof follows from Proposition 4. ■

*Proof of Proposition 6:* The impact of a change in  $n$  on the equilibrium investment  $k(n)$  is captured by  $\frac{\partial k(n)}{\partial n}$ .<sup>11</sup> This derivative is implicitly defined by the equilibrium condition

$$\frac{W\alpha\rho(1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}}{n + \mu \left[(1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]} - 1 = 0$$

In particular, by the Implicit Function Theorem, we get

$$\begin{aligned} \frac{\partial k(n)}{\partial n} &= \frac{1 + \frac{\partial \mu(n)}{\partial n} \left[(1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]}{\left\{n + \mu \left[(1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]\right\} \frac{\alpha\rho-1+\alpha}{1-\alpha} (1+k)^{-1} - \mu \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}} \\ &= MS + CON \end{aligned}$$

where, to separate the two effects that affect  $\frac{\partial k(n)}{\partial n}$ , we let

$$MS = \frac{1}{\left\{n + \mu \left[(1+k)^{\frac{\alpha\rho}{1-\alpha}} - 1\right]\right\} \frac{\alpha\rho-1+\alpha}{1-\alpha} (1+k)^{-1} - \mu \frac{\alpha\rho}{1-\alpha} (1+k)^{\frac{\alpha\rho-1+\alpha}{1-\alpha}}}$$

be the “*industry size effect*.” It is easy to show that it is always the case that  $MS < 0$ . On the other hand, let

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<sup>11</sup>Note that it is not the derivative defined in (7), since, in that case, we were keeping  $\mu$  constant. In this case, we let  $\mu$  move to the new equilibrium.

$$CON = \frac{\frac{\partial \mu(n)}{\partial n} \left[ (1+k)^{\frac{\alpha \rho}{1-\alpha}} - 1 \right]}{\left\{ n + \mu \left[ (1+k)^{\frac{\alpha \rho}{1-\alpha}} - 1 \right] \right\} \frac{\alpha \rho - 1 + \alpha}{1-\alpha} (1+k)^{-1} - \mu \frac{\alpha \rho}{1-\alpha} (1+k)^{\frac{\alpha \rho - 1 + \alpha}{1-\alpha}}}$$

be the “concentration effect.” Note that, since by Proposition 4  $\frac{\partial \mu(n)}{\partial n} < 0$ ,  $CON > 0$ .

This implies that the sign of  $\frac{\partial k(n)}{\partial n}$  depends on the relative size of two effects- that is,  $\frac{\partial \mu(n)}{\partial n} \left[ (1+k)^{\frac{\alpha \rho}{1-\alpha}} - 1 \right] \geq -1$ , or  $\left| \frac{\partial \mu(n)}{\partial n} \left[ (1+k)^{\frac{\alpha \rho}{1-\alpha}} - 1 \right] \right| \geq 1$  ■

Table 1: R&D in the Taiwanese Semiconductor Industry 1981-93

Panel 1: Summary Statistics						
Year	No. Of Plants	Industry Revenue	No. of R&D Active Plants	Proportion doing R&D	Avg R&D Exp.	Total R&D Exp.
81	1279	234,795,584	339	26.5%	4,072	1,380,501
82	1350	289,466,529	373	27.6%	5,353	1,996,663
83	1469	366,308,102	328	22.3%	6,691	2,194,586
84	1710	525,547,715	342	20.0%	8,437	2,885,322
86	2808	803,053,684	517	18.4%	9,507	4,914,974
87	3023	971,443,116	635	21.0%	11,262	7,151,391
88	3127	1,183,736,845	722	23.1%	12,832	9,264,517
89	3121	1,751,324,399	815	26.1%	15,436	12,580,019
91	3184	1,736,267,113	925	29.1%	21,249	19,655,308
92	3437	2,156,047,175	999	29.1%	22,573	22,550,042
93	3671	2,666,828,799	1089	29.7%	25,026	27,253,825

  

Panel 2: R&D Expenditure Percentiles (Actual Expenditures)						
Year	.75 percentile	.8 percentile	.85 percentile	.9 percentile	.95 percentile	Max
81	324	1,200	2,472	5,260	14,729	536,419
82	200	1,106	2,158	4,546	14,989	1,262,137
83	0	534	1,676	4,792	16,382	1,280,965
84	0	2	1,247	4,526	15,699	1,848,640
86	0	0	719	4,434	17,552	2,961,303
87	0	100	1,509	6,295	21,467	4,336,087
88	0	608	3,300	10,581	30,818	3,374,052
89	120	1,727	4,591	11,197	31,855	7,190,542
91	1,500	5,129	12,440	24,614	60,719	9,924,862
92	1,772	6,203	13,914	29,171	68,304	10,176,789
93	2,004	6,347	13,981	29,282	65,997	9,299,129

  

Panel 3: R&D Expenditure Percentiles (% of total R&D exp. incurred by plants in equal or lower percentiles)						
Year	.75 percentile	.8 percentile	.85 percentile	.9 percentile	.95 percentile	Max
81	0.1%	1.1%	3.2%	7.5%	18.4%	100%
82	0.1%	0.6%	2.1%	5.3%	12.6%	100%
83	0	0.1%	0.9%	3.1%	9.7%	100%
84	0	0.0%	0.4%	2.0%	6.9%	100%
86	0	0	0.1%	1.2%	6.4%	100%
87	0	0.0%	0.3%	1.9%	7.4%	100%
88	0	0.1%	0.8%	3.2%	10.5%	100%
89	0.0%	0.3%	1.2%	3.7%	10.1%	100%
91	0.1%	0.8%	2.8%	7.0%	16.3%	100%
92	0.1%	1.0%	3.2%	7.6%	17.5%	100%
93	0.2%	0.9%	2.9%	6.9%	15.9%	100%

Notes: All expenditures are nominal thousands of TW\$. Data unavailable in 1985 and 1990. In Panels 2 and 3 all 70th percentile entries = 0

Table 2: R&D in the Korean Semiconductor Industry 1993-96

Panel 1: Summary Statistics						
Year	No. Of Plants	Industry Revenue	No. of R&D Active Plants	Proportion doing R&D	Avg R&D Exp.	Total R&D Exp.
93	203	6,682,204	41	20.2%	1,906	78,153
94	229	9,646,433	42	18.3%	2,432	102,155
95	238	17,661,540	44	18.5%	3,691	162,402
96	261	17,044,308	40	15.3%	4,824	192,961
Panel 2: R&D Expenditure Percentiles (Actual Expenditures)						
Year	.75 percentile	.8 percentile	.85 percentile	.9 percentile	.95 percentile	Max
93	0	1.8	48.9	188.6	888.2	148,801
94	0	0	28	123.2	310.8	177,022
95	0	0	15	124.3	542	360,338
96	0	0	1	150	907	559,258

Notes: All expenditures are nominal Korean Won (in millions)