What to Put on the Table*

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August 2007

Abstract

This paper investigates under which circumstances negotiating simultaneously over multiple issues or assets helps reduce inefficiencies due to the presence of asymmetric information. We find that a simultaneous negotiation over multiple assets that are substitutes reduces inefficiencies. The effect is stronger if goods are heterogeneous, and in this case the inefficiency can be eliminated altogether. When assets are not substitutes inefficiencies always prevail. We also study cases where co-ownership is possible (partnerships), allowing for asymmetric distributions, general valuation functions and for multiple assets. We show that efficient dissolution is possible if all agents valuations at their types where gains of trade are minimal are equal. For this to hold, the agent that most likely has the highest valuation for a given asset should initially own a bigger share of that asset. We discuss implications of these findings for the design of partnerships and joint ventures. JEL classification codes: C72, D82, L14. Keywords: *efficient mechanism design, multiple units, partnerships.*

1. Introduction

Many important economic and political decisions are determined through negotiations. Negotiations determine the terms of firm acquisitions,[1] of mergers, and of labor contracts. They also play a key role for international treaties, constitutional reforms, and dispute resolutions. There are usually multiple issues at stake and often money also changes hands, as in the cases of M&A's and labor contracts. We present a general model of negotiations under incomplete information between individuals who are risk-neutral and bargain over multiple issues. We allow for private information to be multidimensional and consider the case of private values. An agent's payoff from a given issue's settlement is a function, not necessarily linear,

*We are grateful to Mariagiovanna Baccara, Heski Bar-Isaac, Phil Reny and Balazs Szentes for useful discussions and comments. We also benefited from comments of the audiences at Northwestern University, and at the University of Chicago.

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[1]Recent empirical work by Boone and Mulherin (2007) suggests that about half of company sales are performed via negotiations.

of his type. Our main objective is to identify the forces that result in the possibility (or impossibility) of designing incentive compatible mechanisms where agents are willing to participate without coercion, and that at the same time balance the budget and achieve efficient outcomes.

An important economic insight is that the presence of asymmetric information seriously hinders the ability of negotiating parties to achieve mutually beneficial agreements. For this reason, asymmetric information is viewed as a serious form of transaction costs in Coase's tradition. In a trading environment, the seminal paper by Myerson and Satterwhaite (1983) shows that when gains from trade are uncertain, no ex-post efficient negotiation between a single seller and a single buyer exists. The work by Cramton, Gibbons and Klemperer (1987) shows that in an environment where agents valuations of the good are identically distributed, and the ownership is distributed in close to equal shares among partners, a reassignment can be done efficiently, giving complete control to the partner with the highest ex-post valuation. They conclude that similar property rights are a key factor in determining whether efficiency is achievable or not. This role of property rights has also been stressed in the context of public good settings by Neeman (1999) and Schmitz (2002) among others.

In this paper we identify the exact role of initial property rights in avoiding inefficiencies. More importantly, we identify another key force that reduces the intensity of conflict among the negotiating parties: the simultaneous negotiation over different issues that are substitutes in the eyes of the agents. We show under which circumstances this can ultimately help reduce or even eliminate the inefficiencies.

There are many situations where negotiating parties have the option to put more than one issue on the table at the same time. In multilateral trade negotiations thousands of issues are discussed simultaneously, in complex mergers the ownership of many assets is on the table at the same time, etc. To fix ideas, let us look at the market for professional sports players. There the ownership of assets (players' rights) is determined via negotiations, which usually involve multiple players and cash. Each team's valuation for a particular player is private information, players are heterogeneous across multiple characteristics and, most importantly, present strong complementarities and substitutabilities with each other. All these facts make them sometimes expendable for a team, but critical for the success of another one. Then, what are the forces that determine whether they can be efficiently allocated?

Let's first consider the case of two teams negotiating over a single player, whose rights are owned by one of them. From Myerson and Sattertwaite (1983) we know that ex-post efficient trade is impossible. But what if the seller owns, for instance, 2 forwards and has to bench one of them (so the marginal utility for the second player is lower than for the first)? These log-jams are quite common. Think, for example, Barcelona in 2007 with Henry and Eto'O. Can inefficiencies be reduced if both players are negotiated simultaneously? To what extent, and under which circumstances do simultaneous negotiations help reduce inefficiencies?

We first note that negotiating simultaneously over multiple assets that exhibit substitutabilities reduces inefficiencies. The degree to which substitutabilities help depends crucially on whether assets are *homoge-*

neous or *heterogeneous.* For *homogeneous* assets the degree of substitutability reduces inefficiencies, but does not solve the problem completely.[2] However, for *heterogeneous* assets, substitutabilities may eliminate inefficiencies altogether. It is interesting to stress that in our set-up, this result *requires* the initial ownership structure to be extreme, in the sense that one agent owns all the assets. If not, it is *never* possible to achieve efficiency. This finding contradicts conventional wisdom in the context of private values (which stems from Cramton, Gibbons and Klemperer (1987)), that suggests that it is easier to achieve efficiency if property rights are "more balanced".[3] We end by pointing out that if assets are not substitutes inefficiency always prevails.

Going back to our example, our findings suggest that in the presence of a log-jam, teams should negotiate over players simultaneously instead of doing it consecutively. Putting multiple players on the table helps to achieve efficiency when they are substitutes and this effect is stronger when they are heterogeneous, which is very much the case in this market.

What happens when the ownership of players is non-exclusive? As sport aficionados know, partial ownership of players in soccer is quite common. European soccer teams often form partnerships consisting of an upper division and a lower division team in order to acquire very young third world soccer players. By doing so they avoid higher prices in the future. Co-ownership is convenient because at that point the player is too young and inexperienced for the upper division team, but still useful for the other team. While playing in the lower division he matures and becomes also useful for the other one.

Unfortunately, this strategy has often backfired. Once the player develops and it is time to dissolve the partnership and cash in, the teams often fail to reach agreement. The break-down of negotiations sometimes forces a player even to miss a months of play, which leads to an enormous loss of value. This was exactly the case of Luis Jiménez, a Chilean forward owned in equal shares by Fiorentina (first division) and Ternana (third division). Negotiations broke down for months, and only after a third team (Lazio) stepped in and offered cash in return for borrowing the player, was Ternana able to buy the remaining half shares, allowing the player to continue his career. What was the problem? If property rights were equal, shouldn't the intuition from Cramton, Gibbons and Klemperer (1987) hold and ex-post efficiency be possible? Was there any better way to structure the partnership?

We study a general partnership environment with asymmetric distributions, general valuation functions and multiple assets. In order to focus on the ownership issue, we abstract from substitutabilities and complementarities. We show that efficient dissolution is possible if all agents' valuation of an asset at their critical type[4] are equal, that is $\pi_i(v_i^*) = \pi_j(v_j^*)$. For the standard case of linear valuations, this condition reduces to $v_i^* = v_j^*$ and, for symmetric distributions (as in Cramton, Gibbons and Klemperer (1987)),

[2] In the language of Myerson-Sattertwaite, it reduces the subsidy a broker should put in order to have existence of an ex-post efficient mechanism that is incentive compatible and satisfies the voluntary participation constraints

[3] Jehiel and Panzer (2006) show that extreme ownership helps in a setting with one-sided incomplete information and interdependent values.

[4] This is the type where gains from trade are minimized.

to equal property rights. However, for asymmetric environments we show that the property rights that guarantee $\pi_i(v_i^*) = \pi_j(v_j^*)$ can be extremely unequal.[5] Moreover, we show that agents that most likely have the highest valuation, should initially own a bigger share of the asset.[6] This insight has important implications for many situations, some of which we describe below.

In Luis Jiménez' case, the problem might just have been the *equality* in ownership shares. Fiorentina was much more likely to value the player more (playing in Serie A there is more at a stake) but owned only 50% of the shares. The same could be said in general about other joint-ventures. The ones likely to benefit the most ex-post (for example, the ones with more marketing muscle) should own bigger shares of the project. If this is not true, it may be impossible to find a mechanism that transfers control efficiently to one of the participants.

Our findings can also shed light on the problem of efficient allocation of new technologies among various firms. This is of great economic relevance because, for technologies, the opportunity cost of misallocation is very high. In a patent race, initial property rights can be seen as the probability that each agent will win the race if no partnership is developed. Our previous result indicates that to have efficient trade, the firms that are better at inventing (that is the bigger "initial share" r_i) should also have higher capacity of developing applications *after* the technology is discovered (a better distribution of valuations). This is often not true, as can be seen in the case of the technology used in Blackberry mobile devices. Research in Motion (RIM), the developer of Blackberry, did not own the right for the technology, and fought costly litigation for more than 3 years with NTP. NTP owned the rights for the technology, but it is primarily a patent owning company, with no ability to directly develop products and profit from the patents.[7] Our results suggest that in order for efficiency to prevail, it must be the case that firms that are more likely to value the invention more, are also the ones more likely to develop it. This could provide a rational for the integration of research departments into big firms. Integration may help avoid lost profits due to transaction costs associated with incomplete information.

Summarizing, we show that when the initial ownership structure is exclusive, negotiating simultaneously over multiple issues is a way to reduce the conflict of interest if some issues are substitutes. It is easier to achieve efficiency because the level of necessary transfers drops, sometimes even to zero for the case for heterogeneous goods. For the case of partial ownership, we show that property rights play a crucial role but not in the way it had been initially postulated. What matters for efficiency is that agents with a higher probability of high valuations must own a big initial share of the partnership.

[5] For linear environments, the condition reduces to $v_i^* = v_j^*$, which was discovered independently by Che (2006).

[6] In relation to Schweizer (2006), we not only state that there exist property rights that guarantee the efficient dissolution of asymmetric partnership, but we also identify that the critical condition is that $\pi_i(v_i^*) = \pi_j(v_j^*)$, which allows us to infer exactly how the properties rights should be. The relation between property rights and valuations was discovered independently by Kuribko (2005) in the context of dynamic partnerships.

[7] There is extensive press coverage of this lawsuit. For a sample see "*Detractors of BlackBerry See Trouble Past Patents.*" The New York Times, March 6, 2006.

This paper relates to the literature on efficient mechanism design, where the seminal papers are the ones by Vickrey (1961), Clarke (1971) and Groves (1973). A significant fraction of the literature on efficient mechanism design, is concerned with the design of efficient *trading* mechanisms. The seminal contribution here is Myerson and Sattertwaite (1983). Important extensions, with methodological developments from which we borrow extensively, are in the papers by Makowski and Mezzetti (1993,1994), Williams (1999), Krishna and Perry (2000) and Schweizer (2006).

This paper is also related to the literature on partnerships, which stems from Cramton, Gibbons and Klemperer (1987). The analysis of interdependent valuations with one-sided incomplete information is in Jehiel and Pauzner (2004). More recent contributions are the papers by Ornelas and Turner (2007) and Ferreira, Ornelas and Turner (2007), that separate the issue of ownership from control, and Brusco, Lopomo, Robinson and Viswanathan (2007) who examine an interdependent value environment and allow for non-cash payments.

The paper proceeds as follows. In section 2 we present the general model and results. In section 3 we examine the role of simultaneous negotiations in achieving efficiency. In section 4 we study the role of property rights and its relation to the distribution of valuations. We present our conclusions in section 5.

2. The model

There are N risk-neutral agents negotiating over some issues. An outcome $z \in Z$ specifies how the issues are resolved. We take Z to be finite. Agent i's payoff from outcome z depends on his type $v_i = (v_i^1, ..., v_i^k)$ and it is denoted by $\pi_i^z(v_i)$. The vector v_i is distributed on $V_i = \times_{k \in K}[\underline{v}_i^k, \overline{v}_i^k]$ according to F_i, where $-\infty < \underline{v}_i^k \leq \overline{v}_i^k < \infty$, for all $k \in K$. We use $F(v) = \times_{i \in N} F_i(v_i)$, where $v \in V = \times_{i \in N} V_i$ and $F_{-i}(v_{-i}) = \times_{j \neq i} F_j(v_j)$ where $v_{-i} \in V_{-i} = \times_{j \neq i} V_j$. We assume throughout that the distribution F has a continuous density function that is strictly positive everywhere.

Basic Definitions

By the revelation principle we know that any outcome that can be achieved by a bargaining procedure, arises at a truth-telling equilibrium of a direct revelation game. Therefore we can without loss of generality restrict attention to incentive compatible direct revelation mechanisms. A *direct revelation mechanism* (DRM), $M = (p, x)$, consists of an *assignment rule* $p : V \longrightarrow \Delta(Z)$ and a *payment rule* $x : V \longrightarrow \mathbb{R}^N$.

The assignment rule specifies the probability of each outcome for a given vector of reports. We denote by $p^z(v)$ the probability that outcome z is implemented when the vector of reports is v. The payment rule x specifies, for each vector of reports v, a vector of expected net transfers, one for each agent.

The interim expected utility of an agent of type v_i when he participates and declares v_i' is

$$U_i(v_i, v_i'; (p, x)) = E_{v_{-i}} \left[\sum_{z \in Z} [p^z(v_i', v_{-i}) \pi_i^z(v_i)] + x_i(v_i', v_{-i}) \right].$$

If negotiations break down because of agent i's unwillingness to participate, allocation $Q_i \in \Delta(Z)$ prevails. If $Q_i \equiv Q$, we call Q the *status quo*. The payoff from non-participation is then given by

$$\underline{U}_i(v_i) = \sum_z Q_i^z \pi_i^z(v_i)$$

where Q_i^z denotes the probability assigned to outcome z by Q_i . Notice that non-participation payoffs may depend on i's type.

The timing is as follows: at stage 0 the designer chooses mechanism (p, x). At stage 1 agents decide whether to participate or not. If all participate, they report their types and the mechanism determines the outcome of the negotiations and the payments. If agent i decides not to participate, the outcome of the negotiations is Q_i. If two or more decide not to participate, some arbitrary Q is implemented.

We now provide a formal definition of what it entails for a direct revelation mechanism to be feasible.

Definition 1 *(Feasible Mechanisms) For given outside options $\{Q_i\}_{i\in N}$, we say that a mechanism (p, x) is feasible iff it satisfies:*

(IC) Incentive Constraints
$U_i(v_i, v_i; (p, x)) \geq U_i(v_i, v_i'; (p, x))$ for all $v_i, v_i' \in V_i$ and $i \in N$
(VP) Voluntary Participation Constraints
$U_i(v_i, v_i; (p, x)) \geq \underline{U}_i(v_i)$ for all $v_i \in V_i$, and $i \in N$
(RES) Resource Constraints
$\sum_{z\in Z} p^z(v) = 1$, $p^z(v) \geq 0$ for all $v \in V$

Summarizing, feasibility requires that p and x are such that (1) agents prefer to tell the truth about their valuation parameter, (2) agents choose voluntarily to participate in the mechanism and (3) p is a probability distribution over Z.

Our objective is to investigate what are the forces that enable the existence of feasible mechanisms that are ex-post efficient.

We say that an assignment rule $p : V \longrightarrow \Delta(Z)$ is *ex-post efficient* iff for all $v \in V$, $p^z(v) > 0$ implies that $z(v) \in arg \max_{z'\in Z} \sum_{i=1}^{N} \pi_i^{z'}(v)$

Simply put, an ex-post efficient assignment rule assigns positive probability only to outcomes that maximize the sum of agents' utilities. Ties can be broken arbitrarily.

We say that a payment rule $x : V \longrightarrow \mathbb{R}^N$ *balances the budget* iff $\sum_{i\in N} x_i(v) = 0$ for all $v \in V$.

Definition 2 *A mechanism (p, x) is ex-post efficient iff p is an ex-post efficient assignment rule and x balances the budget.*

We now proceed to a characterization of feasible ex-post efficient mechanisms due to Schweizer (2006) that relies on an earlier paper by Makowski and Mezzeti (1994). For that we need to introduce the concept of the *critical type* of an agent. The critical type of agent i, v_i^* is the type where his gains from trade are minimal. Formally:

Definition 3 *For a fixed mechanism (p, x) the critical type of agent i is denoted by v_i^* and is a type where the difference between participation and non-participation payoffs are minimized:*[8]

$$v_i^* \in \arg\min_{v_i} \left[U_i(v_i) - \underline{U}_i(v_i) \right]. \tag{1}$$

Schweizer's (2006) characterization is based on the observation that the interim information rent of an agent is identical for all incentive compatible and efficient mechanisms. This follows from the revenue equivalence theorem.[9] A simple way to calculate the rent is to use a particular class of mechanisms that satisfies these properties, namely the Vickrey-Clarke-Groves class (VCG).[10]

The *total social surplus* at an ex-post efficient assignment rule is given by:

$$W(v) = \sum_{i \in I} \pi_i^{z(v)}(v_i),$$

where $z(v)$ is an ex-post efficient social alternative if the vector of types is v. Defining $W(v_i^*, v_{-i}) = \pi_i^{z(v_i^*, v_{-i})}(v_i^*) + \sum_{j \neq i} \pi_j^{z(v_i^*, v_{-i})}(v_j)$, we can write the (interim) information rent of agent i in a VCG mechanism as

$$U_i(v_i) - U_i(v_i^*) = E[W(v) - W(v_i^*, v_{-i})]$$

Then, from the analysis of Schweizer (2006) we know that there exists a Bayesian incentive compatible mechanism, that satisfies interim participation constraints, ex-post efficiency and budget balance iff

$$E[\Delta(v)] = E\left[\underbrace{W(v)}_{\text{surplus}} - \underbrace{\sum_{i \in I} E[W(v) - W(v_i^*, v_{-i})]}_{\text{info rents paid to agents}} - \underbrace{\sum_{i \in I} \sum_{z \in Z} Q_i^z \pi_i^z(v_i)}_{\text{outside payoffs}} \right] \geq 0. \tag{2}$$

Whenever critical types $\{v_i^*\}_{i \in N}$ are such (2) is true pointwise (and not only in expectation), then the possibility result is strong in the following sense: for any distribution of types F that generates the critical types $\{v_i^*\}_{i \in I}$ there exists a feasible and ex-post efficient mechanism.

Now we employ this characterization to provide answers to the questions we posed in the introduction.

[8] If there is more than one type satisfying this, any of them will do.

[9] See Krishna and Perry (2000) for a general version allowing for multi-dimensional types.

[10] This observation is also in Williams (1999) and in Krishna and Perry (2000). They establish that a mechanism is incentive feasible and efficient iff and only if is a generalized Vickrey-Groves-Clarke mechanisms.

3. Multi-Asset Trading Mechanisms: Exclusive Ownership

This section studies whether including more assets in the negotiation can potentially solve the basic impossibility result pointed out by Myerson and Sattertwaite (1983). First, we analyze the case where a seller owns multiple assets that are perceived as substitutes by both the seller and the buyer and show that simultaneous negotiation helps reduce inefficiencies. The basic intuition is that the existence of substitutabilities lessens the intensity of conflict, and therefore could help to overcome the impossibility result. The effect is stronger when assets are heterogeneous, where even full efficiency is sometimes possible. At the end of the section we show that for goods that are complements (or neither complements nor substitutes) inefficiencies always prevail.

3.1 Homogeneous Assets

We first consider a slight modification of the Myerson-Sattertwaite environment, where the seller owns two[11] *homogeneous* assets. Both the buyer and the seller derive decreasing marginal utility for them, meaning that the extra utility from owning a second unit is lower than the first. In this situation there are three possible allocations: the seller keeps both assets ($z_1 = 2, 0$), each agent ends up with one asset ($z_2 = 1, 1$) and the buyer obtains both assets ($z_3 = 0, 2$). The payoffs that accrue to the seller (indexed by 1) and the buyer (indexed by 2) are given by:

$$\begin{array}{ll} \pi_1^{2,0}(v_1) = (1+\alpha)v_1 & \pi_1^{0,2}(v_2) = 0 \\ \pi_1^{1,1}(v_1) = v_1 & \pi_1^{1,1}(v_2) = v_2 \\ \pi_1^{0,2}(v_1) = 0 & \pi_2^{0,2}(v_2) = (1+\alpha)v_2 \end{array}$$

where $\alpha \in (0,1)$[12] captures the fact that these two assets are substitutes. The status quo is given by allocation $z_1 = 2, 0$ since the seller owns both assets, and v_i is distributed according to F_i on $[0, 1]$.

Our first result shows that decreasing marginal utility does not make the design of an ex-post efficient trade mechanism possible. This impossibility result is strong since it holds even if α is almost equal to 0, and therefore the conflict of interest is minimal, because the ex-post efficient allocation is $z_2 = 1, 1$ for almost all types.

Proposition 1 *If $\alpha \in (0,1)$ and the status quo is given by allocation $z_1 = 2, 0$, then there is no ex-post efficient, incentive compatible and individually rational mechanism that balances the budget irrespective of the distributions of types F_1, F_2.*

Proof. It is easy to see that $v_1^* = 1$ and $v_2^* = 0$. This is because, similar to Myerson and Satterwhaitte (1983), the slope of the seller's payoff from non-participation is $1 + \alpha$, while the slope of his payoff from an

[11] The result does not change if the seller owns more goods.

[12] Notice that $\alpha = 0$ implies that a second object gives no utility at all, and therefore the ex-post efficient allocation is constant and efficiency is possible. On the other hand $\alpha = 1$ corresponds to constant marginal utility and is equivalent to the Myerson-Sattertwaite environment.

ex-post efficient assignment is weakly less than $1+\alpha$ for all $v_1 \in [0,1]$. The slope of the buyer's payoff from non-participation is 0, and the slope of his payoff from an ex-post efficient assignment is weakly greater than 0 for all $v_2 \in [0,1]$.

Then, $\Delta(v_1, v_2)$ from (2) reduces to

$$\begin{aligned}
\Delta(v_1, v_2) &= -W(v_1, v_2) + W(1, v_2) + W(v_1, 0) - (1+\alpha) \\
&= -\max\{(1+\alpha)v_1, v_1 + v_2, (1+\alpha)v_2\} \\
&\quad + \max\{1+\alpha, 1+v_2\} + (1+\alpha)v_1 - (1+\alpha) \\
&= -\max\{(1+\alpha)v_1, v_1 + v_2, (1+\alpha)v_2\} \\
&\quad + \max\{\alpha, v_2\} + (1+\alpha)v_1 - \alpha.
\end{aligned}$$

If $\alpha > v_2$, the above expression reduces to

$$\Delta(v_1, v_2) = -\max\{(1+\alpha)v_1, v_1 + v_2, (1+\alpha)v_2\} + (1+\alpha)v_1$$

which is equal to zero, whenever $-\max\{(1+\alpha)v_1, v_1+v_2, (1+\alpha)v_2\} + (1+\alpha)v_1 = (1+\alpha)v_1$, and strictly negative otherwise.

If $\alpha \leq v_2$, we have

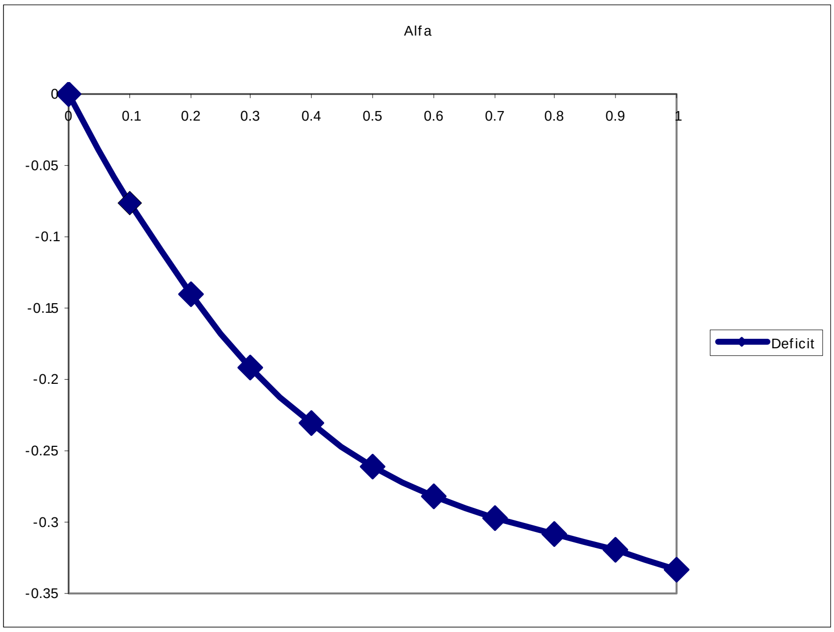
$$\begin{aligned}
\Delta(v_1, v_2) &= -\max\{v_1 + v_2, (1+\alpha)v_2\} + v_2 + (1+\alpha)v_1 - \alpha \\
&= -\max\{v_1, \alpha v_2\} + (1+\alpha)v_1 - \alpha \\
&\leq \alpha v_1 - \alpha \\
&\leq 0
\end{aligned}$$

Therefore $\Delta(v_1, v_2) \leq 0$ for all v_1, v_2, and there is a region of types with a non-empty interior where $\Delta(v_1, v_2) < 0$, establishing that it is impossible to design ex-post efficient mechanisms. ■

From Proposition 1 we conclude that irrespective of the degree of substitutability of the two assets, it is not possible to design an ex-post efficient negotiation procedure. However, we can investigate how the degree of substitutability of the two assets reduces these unavoidable inefficiencies. As a measure of the inefficiency we use the amount of transfers that a broker should bring into the system in order to make efficiency possible . The higher the outside transfers needed, the higher the degree of inefficiency. For the case where v_1 and v_2 are uniformly distributed on $[0,1]$, some straightforward calculations, yield that the expected deficit as a function of α is given by

$$E[\Delta(v_1, v_2)] = \frac{2}{3}a^2 - \frac{5}{6}a - \frac{1}{6}a^4.$$

As one can see, the deficit (the negative of the subsidy needed) is decreasing in α, but it is 0 only when $\alpha = 0$. So substitutabilities help reduce the inefficiency, but do not solve the problem unless the second asset is of *no use* once an agent already owns the first. We depict the inefficiency in the next figure.



As we can see, substitutability help but is not enough on its own to achieve efficiency. We now examine what happens when assets are heterogeneous.

3.2 Heterogeneous Assets

Suppose that two assets are at stake, A and B, and they are substitutes for the agents. The difference with the previous model is the *heterogeneity* of the assets: each asset is valued by an agent based on a different information parameter (types are multidimensional). The payoffs are given by: [13]

$$\begin{array}{ll} \pi_1^{AB,0}(v_1^A, v_1^B) = v_1^A + \alpha v_1^B & \pi_2^{AB,0}(v_2^A, v_2^B) = 0 \\ \pi_1^{A,B}(v_1^A, v_1^B) = v_1^A & \pi_2^{A,B}(v_2^A, v_2^B) = v_2^B \\ \pi_1^{B,A}(v_1^A, v_1^B) = v_1^B & \pi_2^{B,A}(v_2^A, v_2^B) = v_2^A \\ \pi_1^{0,AB}(v_1^A, v_1^B) = 0 & \pi_2^{0,AB}(v_2^A, v_2^B) = \alpha v_2^A + v_2^B \end{array}$$

Here the result is strikingly different, since the general impossibility is no longer true.

Proposition 2 *Suppose that the status quo is given by allocation $(AB, 0)$, and that $\alpha \in (0, 1)$. There exist distributions of types F_1, F_2 for which a feasible and ex-post efficient mechanism exists.*

[13] The same result is obtained if there is no ex-ante preference between the assets, and $\pi_i^{AB,0}(v_i^A, v_i^B) = \max\{v_i^A, v_i^B\} + \alpha \min\{v_i^A, v_i^B\}$.

Proof. It is easy to see that the critical types are again extreme and they are given by $(v_1^A, v_1^B) = (1, 1)$ and $(v_2^A, v_2^B) = (0, 0)$. Then

$$\begin{aligned}
\Delta(v_1^A, v_1^B, v_2^A, v_2^B) &= -W(v_1^A, v_1^B, v_2^A, v_2^B) + W(1, 1, v_2^A, v_2^B) + W(v_1^A, v_1^B, 0, 0) - (1 + \alpha) \\
&= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} + \max\{1 + \alpha, 1 + v_1^A, 1 + v_1^B\} \\
&\quad + \max\{v_1^A + \alpha v_1^B, v_1^B\} - (1 + \alpha) \\
&= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} + \max\{\alpha, v_1^A, v_1^B\} \\
&\quad + \max\{v_1^A + \alpha v_1^B, v_1^B\} - \alpha
\end{aligned}$$

We then notice that $\Delta(v_1^A, v_1^B, v_2^A, v_2^B)$ is positive at least in the region where $\max\{v_1^A, v_1^B\} > \alpha$ and $v_1^A + \alpha v_1^B > \max\{v_1^A + v, v_1^B + v_2^A, \alpha v_2^A + v_2^B\}$. Therefore, if the distribution F_1 puts enough weight on that region, we have that efficient dissolution is possible. ■

Remark 1 *Notice that the possibility result is no longer true when* $\alpha = 1$*, that is when assets stop being substitutes, since in that case* $E(\Delta(v)) < 0$*. This is addressed later in Proposition 6.*

It is interesting to note that the above result depends on the initial ownership structure being $(AB, 0)$. If the ownership structure is (A, B), efficiency is *never* possible. This is established next in Proposition 3 and it contradicts the conventional wisdom, that suggests that it is easier to achieve efficiency if property rights are "more balanced" in the sense that both agents own some part of the total endowment.

Proposition 3 *If the status quo is given by allocation* (A, B) *then for any distribution of types, there is no feasible and ex-post efficient mechanism.*

Proof. It is easy to see that the critical types for this status quo option are given by $(v_1^A, v_1^B) = (1, 0)$ and $(v_2^A, v_2^B) = (0, 1)$. Then,

$$\begin{aligned}
\Delta(v_1^A, v_1^B, v_2^A, v_2^B) &= -W(v_1^A, v_1^B, v_2^A, v_2^B) + W(1, 0, v_2^A, v) + W(v_1^A, v_1^B, 0, 1) - 2 \\
&= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} + (1 + v_2^B) + (1 + v_1^A) - 2 \\
&= -\max\{v_1^A + \alpha v_1^B, v_1^A + v_2^B, v_1^B + v_2^A, \alpha v_2^A + v_2^B\} + v_1^A + v_2^B \\
&\leq 0,
\end{aligned}$$

and there is a region of types with a non-empty interior where $\Delta(v_1, v_2) < 0$. ■

This result completes our investigation of whether negotiating over multiple assets that are substitutes helps achieve efficiency. The main conclusion is that substitutabilities help but do not completely solve the problem. If heterogeneity is added to the mix, though, efficient trading becomes possible. Moreover, we find a result that contradicts received wisdom: when ownership is more spread out in the sense that each agent owns one good, efficiency is impossible.

Our findings suggest that teams would benefit by putting many players on the table at the same time, since these assets are usually substitutes. For example, a team overloaded with two or more star forwards, knowing that 1 of them will have to be benched, should negotiating with other teams about negotiate over both players at the same time. It is important to remark that even for substitutable assets, separate negotiations will not achieve efficiency. If, for instance, asset B were to be traded in the previous environment, but with ownership of asset A fixed, we would have the impossibility result again, even if the marginal valuation of agent 1 is only αv_A^1 and for agent 2 is v_A^2.

As a conclusion to the section we briefly examine the cases when goods are not substitutes.

3.3 Independent and Complementary Assets

If goods are independent (that is the total valuation of a package is additive on its components) the results obtained for a single asset (impossibility in the case of extreme ownership and possibility for certain property rights adequately chosen) carry over to multiple assets. These conclusions follow from Proposition 6, which we state and prove in the following section.

If goods are complements, it is not difficult to see that inefficiencies increase. The strengthening of the conflict (ex-post efficiency indicates that assets are never shared in this case) just gives more force to impossibility of achieving efficiency. For the sake of completion we present the result:

Proposition 4 *Consider the heterogeneous goods version of the model presented in the previous subsection and assume $\alpha > 1$. Then if one agent owns both assets there is no incentive compatible and ex-post efficient mechanism.*

Proof. It is enough to notice that expression (2) for each v_1, v_2 becomes:

$$\begin{aligned} \Delta(v_1, v_2) &= -W(v_1^A, v_1^B, v_2^A, v_2^B) + W(1, 1, v_2^A, v_2^B) + W(v_1^A, v_1^B, 0, 0) - (1 + \alpha) \\ &= -\max\{\alpha v_1^A + v_1^B, v_2^A + \alpha v_2^B\} + 1 + \alpha + \alpha v_1^A + v_1^B - (1 + \alpha) \leq 0, \end{aligned}$$

and there is a region of types with a non-empty interior where $\Delta(v_1, v_2) < 0$ ■

This completes our investigation of whether simultaneous negotiations may reduce the inefficiencies that result from the presence of asymmetric information. This investigation has been performed assuming that the ownership of assets is exclusive. Now we move on to examine cases where co-ownership is permitted.

4. Trading Mechanisms with Co-ownership

In this section we study how property rights in partnerships affect the existence of feasible and ex-post efficient mechanisms in a general model, where payoffs and distributions of types can be asymmetric across agents. A partnership consists of a group of individuals that jointly own a number of assets. In order to focus on the ownership issue, we abstract from substitutabilities and complementarities. We first examine partnerships of one asset.

4.1 Co-ownership of One Asset

Agent $i's$ payoff from owning a fraction r of the asset is $r \cdot \pi_i(v_i)$, for $r > 0$, and it is zero otherwise. The partnership is characterized by the initial property rights, which are denoted by $Q = (r_1, ..., r_N)$. That is, there is one object owned jointly (agent i owns a proportion r_i) and each agent cares only about getting the object or not.

An ex-post efficient assignment p^* is one where for each v, the agent with the highest $\pi_i(v_i)$ is awarded exclusive ownership of the asset. Then, the expected payoff for agent i when his valuation is v_i at the ex-post efficient assignment, is

$$U_i(v_i, v_i; (p^*, x)) = \prod_{\substack{j \in N \\ j \neq i}} F_j(\pi_j^{-1}(\pi_i(v_i))) \cdot \pi_i(v_i) + E_{v_{-i}}[x_i(v_i, v_{-i})]$$

and at the status quo $Q = (r_1, ..., r_N)$, his payoff is $\underline{U}_i(v_i) = r_i \cdot \pi_i(v_i)$.

The seminal paper by Cramton, Gibbons and Klemperer (1987) examines partnerships where agents' valuations are drawn from the same distribution and their valuations are given by $\pi_i(v_i) = v_i$. In that context, they show that when the shares are close to $r_i = \frac{1}{N}$, it is possible to find a mechanism that is feasible and ex-post efficient. When compared to the result in Myerson and Sattertwaite (1983), the one of Cramton, Gibbons and Klemperer (1987) may lead to the conclusion that what makes the existence of ex-post efficient mechanisms possible, is the *equality in property rights.*

Here, we allow for asymmetric type distributions and more general payoff functions and we find that feasible and ex-post efficient mechanisms exist if all agents' valuations from the asset evaluated at the vector of critical types are equal:

Proposition 5 *If property rights* $(r_1, ..., r_N)$ *are such that*

$$\pi_i(v_i^*) = \pi_j(v_j^*), \tag{3}$$

then there exists a feasible and ex-post efficient mechanism. Moreover, suppose that for all $i \in I$, π_i *is strictly increasing in* v_i, *and they all have the same range* ($\pi_i(\underline{v}_i) = \underline{\pi}$, *and* $\pi_i(\bar{v}_i) = \bar{\pi}$), *then such property rights always exist and the result remains true for property rights in a neighborhood of* $(r_1, ..., r_N)$.

Proof. [14]From (2), we have it is possible to design an ex-post efficient mechanism iff

$$E\left[-(N-1)\max_{i\in N}\{\pi_i(v_i)\}+\sum_{i\in N}\max\{\pi_i(v_i^*),\pi_{-i}(v_{-i})\}\right]\geq\sum_{i\in N}r_i\cdot\pi_i(v_i^*). \tag{4}$$

Therefore, a sufficient condition is that (4) holds for all vectors v, that is

$$-(N-1)\max_{i\in N}\{\pi_i(v_i)\}+\sum_{i\in N}\max\{\pi_i(v_i^*),\pi_{-i}(v_{-i})\}\geq\sum_{i\in N}r_i\cdot\pi_i(v_i^*). \tag{5}$$

We just need to verify that (5) is satisfied whenever (3) is satisfied.

Suppose that for some vector of valuations v_i we have that

$$\pi_k(v_k)=\max_{i\in N}\{\pi_i(v_i)\}.$$

Then for this vector of valuations (5) reduces to

$$-(n-1)\pi_k(v_k)+(n-1)\pi_k(v_k)+\max\{\pi_k(v_k^*),\pi_{-k}(v_{-k})\}\geq\sum_{i\in N}r_i\pi_i(v_i^*)$$

which is always true. Since this holds for any v (5) always holds.

Now we move on to establish that for any distributions F_i, $i\in N$, there exists an initial ownership structure $(r_1,...,r_N)$ that guarantees that $\pi_i(v_i^*)=\pi_j(v_j^*)$ holds.

First note that at the critical types, it must be the case that

$$\prod_{\substack{j\in N\\ j\neq i}}F_j(\pi_j^{-1}(\pi_i(v_i^*)))\pi_i'(v_i^*)=r_i\cdot\pi_i'(v_i^*). \tag{6}$$

Defining $G_i(s)=\prod\limits_{\substack{j\in N\\ j\neq i}}(F_j\circ\pi_j^{-1})(s)$, and noticing that it is invertible (since it is increasing) (6) can be rewritten as

$$\pi_i(v_i^*)=G_i^{-1}(r_i). \tag{7}$$

Therefore, for $\pi_i(v_i^*)=\pi_j(v_j^*)$ to be true, we must have

$$G_i(G_j^{-1}(r_j))=r_i \tag{8}$$

which, for a given r_i determines r_j. Now, we only have to check the consistency requirement that $\sum\limits_{1=1}^{N}r_i=1$ so, for $i=1$ and $j=N$ (8) becomes

$$G_1^{-1}(r_1)-G_N^{-1}\left(1-r_1-G_2(G_1^{-1}(r_1))-...-G_{N-1}(G_1^{-1}(r_1))\right)=0 \tag{9}$$

[14] Our first proof of the result , which we present in the Appendix, was more lengthy but has the advantage that it establishes that the condition $\pi_i(v_i^*)=\pi_j(v_j^*)$ minimizes inefficiencies. The proof now is shorter thanks to Y-K Che.

Noticing that $G_1^{-1}(0) - G_N^{-1}(1) = -1$ and $G_1^{-1}(1) - G_N^{-1}(0) = 1$, continuity of G_1^{-1} implies the existence of $r_1 \in (0,1)$ such that (9) holds. ■

For the case where $\pi_i(v_i) = v_i$, (3) reduces to $v_i^* = v_j^*$. If, moreover, all types are distributed according to the same distribution F, we have that $v_i^* = v_j^*$ is satisfied if $r_i \equiv \frac{1}{N}$, exactly as in Cramton, Gibbons and Klemperer (1987). However, when distributions are asymmetric the property rights that guarantee the condition in Proposition 5 can be extremely unequal:

Example 1 *Consider a partnership with 2 players, $\pi_i(v_i) = v_i$, $F_1(v_1) = v_1^n$ and $F_2(v_2) = v_2^{\frac{1}{n}}$. Then, it is easy to see that $v_1^* = F_2^{-1}(r_1) = r_1^n$ and $v_2^* = F_1^{-1}(r_2) = r_2^{\frac{1}{n}}$. From Proposition 5 we know if $v_1^* = v_2^*$ efficient dissolution is possible. For this example this condition is equivalent to*

$$r_1^n = r_2^{\frac{1}{n}}. \tag{10}$$

Recalling that $r_1 + r_2 = 1$, (10) reduces to

$$r_1^n = (1 - r_1)^{\frac{1}{n}}.$$

For $n = 3$ we obtain $r_1 = 0.8243$ and $r_2 = 0.1757$, which give us that $v_1^* = v_2^* = 0.56009$. Moreover, a simple calculation shows that for these distributions, with property rights of $r_i = \frac{1}{2}$ there is no possibility of efficient dissolution. For $n = 99$, optimal property rights are even more extreme: $r_1 = 0.99926$ and $r_2 = 0.00074$. For this case the corresponding critical types are $v_1^* = v_2^* = 0.92933$.

We can see that, for certain distributions, very extreme property rights are needed in order to have efficient dissolution of the partnership, quite contrary to the intuition one gets from the discussion of symmetric environments. In fact, for the very extreme case of $n = 99$, property rights very close to $(1, 0)$ are needed. It is useful to contrast this result with the one in Myerson and Sattertwaite (1983), which shows that $(r_1, r_2) = (1, 0)$ will never allow an efficient dissolution. In fact, our example shows that arbitrarily close to extreme property rights can be needed for efficient dissolution, but property rights that *are* extreme will never allow it.

Moreover, this example suggests that agent 1, whose valuation is more likely to be higher, must own a higher proportion of the good and viceversa. This turns out to be a general result with interesting economic consequences:

Corollary 1 *Let's suppose that $F_1 \circ \pi_1^{-1}(\cdot) \geq F_2 \circ \pi_2^{-1}(\cdot) \geq ... \geq F_N \circ \pi_N^{-1}(\cdot)$.[15] Then the property rights that guarantee the possibility of efficient dissolution satisfy $r_1 \leq r_2 \leq ...r_N$.*

[15] This is equivalent to the distributions of valuations being ordered according to FOSD (first order stochastic dominance), since $F_i \circ \pi_i^{-1}(x) = P(\pi_i(v_i) \leq x)$.

Proof. We know that at the critical types $\prod_{j\neq i} F_j(\pi_j^{-1}(\pi_i(v_i^*))) = r_i$ and that for the property rights that guarantee dissolution we have $\pi(v_i^*) \equiv x$. Therefore we have

$$r_i = \prod_{j\neq i}[F_j \circ \pi_j^{-1}](x) = r_i,$$

from which we obtain that

$$\frac{r_i}{r_j} = \frac{[F_j \circ \pi_j^{-1}](x)}{[F_i \circ \pi_i^{-1}](x)} > 1.$$

■

Corollary 1 has important implications for many situations.

Professional Players: In the example we mentioned about Luis Jiménez, the problem lies exactly on the symmetry of the property rights. Fiorentina, which had a much higher probability of high valuations (playing in Serie A there is more at a stake) owned as many shares as Ternana, which had a much smaller probability of high valuations. An asymmetric partnership would have been advisable, with Fiorentina owning a significantly higher amount of shares.

Joint-Ventures: The same could be said in general about other joint-ventures. Even if the participation of all associates is needed, Corollary 1 suggests that the ones likely to benefit the most ex-post (for example, the ones with more marketing muscle) should own bigger shares of the project. If this fails, it may be impossible to find a mechanism that transfers the control efficiently to one of the participants.[16]

R&D and Firm Structure: It is well known that in many cases, firms that are better at generating an invention are not necessarily the best ones at developing applications for it. A famous case is the one of Research in Motion (RIM), the developer of Blackberry, who did not own the right for the technology, and fought a costly litigation during more than 3 years with NTP. NTP owned the rights for the technology but, being primarily a patent owning company, had no ability to directly develop products or profit from the patents.[17] Our result hints an explanation. To have efficient trade, firms that are better at developing applications (F_i biased toward high values of v_i) should also have high probabilities of winning the research race (higher r_i). It also provides a rational for integration of research and production under one firm. When research is conducted by independent laboratories, inventions have to bought in order to be developed. The asymmetric information present in these transactions may lead to a break-down of the negotiations and foregone profits.

We continue by noting that our findings generalize straightforwardly to the case where a partnership owns multiple assets: firms co-owning multiple plants, patents and copyrights, or a married couple that seeks a divorce and owns multiple properties.

[16] For more on this see also Kuribko (2005) that examines dynamic partnerships.

[17] There is extensive press coverage for this lawsuit. For a sample see "*Detractors of BlackBerry See Trouble Past Patents.*" The New York Times, March 6, 2006.

4.2 Co-ownership of Multiple Assets

Suppose now that there are N individuals who jointly own K assets and that agent i's valuation of asset k is given by $\pi_{ik}(v_i^k)$ and his valuation of a subset of assets $S \in K$ is additive: $\sum_{k \in S} \pi_{ik}(v_i^k)$ (so goods are neither complements nor substitutes). The ownership share of asset k by agent i is denoted by r_i^k, hence in case of disagreement his payoff is given by $\sum_{k \in K} r_i^k \cdot \pi_{ik}(v_i^k)$.

In this context, exactly as in the case of single-asset partnerships, if property rights are extreme, there is feasible and ex-post efficient mechanisms. On the other hand, there always exist property rights that guarantee the existence.

Proposition 6 *Consider a multi-asset partnership where goods are neither complements nor substitutes. Then*

- *If for every asset k there exists an agent i such that $r_{ik} = 1$ then there is no feasible and ex-post efficient mechanism.*
- *If the property rights are such that $\pi_{ik}(v_i^{k*}) = \pi_{jk}(v_j^{k*})$ for all $i, j \in N, k \in K$ then there exists a feasible and ex-post efficient mechanism.*
- *It is always possible to find property rights such that $\pi_{ik}(v_i^{k*}) = \pi_{jk}(v_{jk}^*)$ for all $i, j \in N, k \in K$.*

Proof. For the first and second points, it is enough to notice that expression (2) reduces to

$$E\left[-(N-1)\sum_{k \in K}\max_{i \in N}\{\pi_{ik}(v_{ik})\} + \sum_{k \in K}\sum_{i \in N}\max\{\pi_{ik}(v_i^{k*}), \pi_{-i,k}(v_{-i,k})\}\right] \geq \sum_{k \in K}\sum_{i \in N} r_i \cdot \pi_i(v_i^{k*})$$

which in turns can be rewritten as

$$\sum_{k \in K}\left[E\left[-(N-1)\max_{i \in N}\{\pi_{ik}(v_i^k)\} + \sum_{i \in N}\max\{\pi_{ik}(v_i^{k*}), \pi_{-i,k}(v_{-i}^k)\}\right] - \sum_{i \in N} r_i \cdot \pi_i(v_i^{k*})\right] \geq 0$$

The expression inside the first sum is pointwise non-negative for all $k \in K$ if $\pi_{ik}(v_i^{k*}) = \pi_{jk}(v_j^{k*})$ for all $i, j \in N$. This can be immediately seen from the proof of Proposition 5, establishing the second point.

Moreover, if for all $k \in K$ there exists $i \in N$ such that $r_{ik} = 1$, the expression inside the first sum reduces to

$$-(N-1)\max_{j \in N}\pi_{jk}(v_j^k) + \sum_{j \neq i}\max\{0, \pi_{-j,k}(v_{-j}^k)\} + \pi_{i,k}(\bar{v}_i^k) - \pi_{i,k}(\bar{v}_i^k)$$

which is clearly non-positive everywhere and strictly negative for the region where $\max_{i \in N}\pi_{j,k}(v_j^k) = \pi_{i,k}(v_i^k)$, establishing the first point.

Finally, the existence of optimal property rights that guarantee the existence of feasible ex-post mechanisms mirrors again the proof in Proposition 5. In fact, the condition for critical types can be rewritten as $\prod_{j\neq i} F_{jk}(\pi_{jk}^{-1}(v_i^{k*})) = r_{ik}$, where $F_{jk}(x) = \int\limits_{V_{j,-k}} \int\limits_0^x dF_j(v_j)$, and the analysis of Proposition 5 follows. ■

5. Conclusions

In this paper we investigated the forces that reduce the intensity of conflict and help alleviate the inefficiencies that arise from asymmetric information. We established under which circumstances there exists negotiation procedures that are incentive compatible, induce agents to participate without coercion and at the same time are ex-post efficient and balance the budget. We first showed that the simultaneous negotiation over multiple assets (or issues) can be helpful if these assets are perceived as substitutes by the players. This effect is stronger, and sometimes eliminates inefficiency altogether if the assets are heterogeneous.

We then investigated the role of initial property rights. We showed that in order to guarantee the existence of ex-post efficient mechanisms, property rights should be biased towards the agents with a higher probability of having a high valuation of the good. Then we used these results to understand the breakdown in negotiations where initial ownership is well balanced, and based on previous work one would have expected ex-post efficient trade to be possible. We also discussed implications for the design of joint ventures and research partnerships.

6. Appendix

We show a different proof of Proposition 5 for the linear case of $\pi_i(v_i) = v_i$ that highlights an important feature of partnerships. Property rights $(r_1, ..., r_N)$ that guarantee $v_i \equiv v^*$ not only allow for dissolution, but they also maximize the sum of expected feasible transfers, therefore are the "best" possible property rights. In order to obtain our characterization we will use Theorem 2 in Krishna and Perry (2000).

Proof. From the well known results in mechanism design (see for example Krishna and Perry 2000), we have that almost surely the derivative of the payoff from participation is given by

$$\frac{dU_i(v_i)}{dv_i} = E_{v_{-i}}[p^{z_i}(v_i, v_{-i})]$$

Using the fact that in an ex-post efficient allocation $p^{z_i}(v_i, v_{-i}) = 1$ if $v_i = \max_{j\in N} v_j$ and 0 otherwise, we get that

$$\begin{aligned}\frac{dU_i(v_i)}{dv_i} &= P(v_i = \max_{j\in N} v_j) \\ &= \prod_{j\neq i} F_j(v_i)\end{aligned}$$

From (1) we then conclude that, for the interior critical types:

$$r_i = \prod_{j\neq i} F_j(v_i^*).$$

From Krishna and Perry (2000), for a vector of critical types v^* we have

$$\sum_{i=1}^{N} x_i(v|\bar{v}^*) = [p_i(v_i^*, v_{-i})v_i^* + \sum_{j\neq i} p_j(v_i^*, v_{-i})v_j] - [\sum_{j\neq i} p_j(v_i, v_{-i})v_j] - r_i v_i^*.$$

We define $G_i(s) = \prod_{j\neq i} F_j(s)$, $v_{-i}^1 = \max_{j\neq i} v_j$, $SW(v_i^*, v_{-i}) = p_i(v_i^*, v_{-i})v_i^* + \sum_{j\neq i} p_j(v_i^*, v_{-i})v_j$ and $SW_{-i}(v) = \sum_{j\neq i} p_j(v_i, v_{-i})v_j$. The proof consists of two steps

Step 1: We prove that

$$\sum_{i=1}^{I} E(x_i(v|\bar{v}^*)) = \sum_{i\in I\setminus\{I\}} \int_{G_i^{-1}(r_i)}^{\bar{v}} sG_i^{'}(s)ds + \int_{G_I^{-1}(1-\Sigma_{i\in I\setminus\{I\}} r_i)}^{\bar{v}} sG_i^{'}(s)ds - \sum_{i=1}^{I} \int_{V_{-i}} v_{-i}^1 F_i(v_{-i}^1) dF_{-i}(v_{-i}). \tag{11}$$

In fact, simple computations show that:

$$SW(v_i^*, v_{-i}) - SW_{-i}(v) = \begin{cases} v_{-i}^1 & \text{if } v_i > v_{-i}^1 > v_i^* \\ v_i^* & \text{if } v_i > v_{-i}^1 \text{ and } v_i^* > v_{-i}^1 \\ v_i^* - v_{-i}^1 & \text{if } v_i < v_{-i}^1 < v_i^* \\ 0 & \text{otherwise} \end{cases}.$$

Then , defining $\hat{v}_{-i} = \{v_{-i} \left| v_{-i}^1 < v_i^* \right.\}$ we can write

$$
\begin{aligned}
E(x_i(v|\bar{v}^*)) &= \int_{\hat{v}_{-i}} \left[v_i^* - v_{-i}^1\right] F_i(v_{-i}^1) dF_{-i}(v_{-i}) + \int_{\hat{v}_{-i}} v_i^*[1 - F_i(v_{-i}^1)] dF_{-i}(v_{-i}) \\
&\quad + \int_{\hat{v}_{-i}^C} v_{-i}^1 [1 - F_i(v_{-i}^1)] dF_{-i}(v_{-i}) - r_i v_i^* \\
&= v_i^* \int_{\hat{v}_{-i}} dF_{-i}(v_{-i}) - \int_{\hat{v}_{-i}} v_{-i}^1 F_i(v_{-i}^1) dF_{-i}(v_{-i}) \\
&\quad + \int_{\hat{v}_{-i}^C} v_{-i}^1 [1 - F_i(v_{-i}^1)] dF_{-i}(v_{-i}) - r_i v_i^* \\
&= v_i^* \int_{\hat{v}_{-i}} dF_{-i}(v_{-i}) - \int_{v_{-i}} v_{-i}^1 F_i(v_{-i}^1) dF_{-i}(v_{-i}) \\
&\quad + \int_{\hat{v}_{-i}^C} v_{-i}^1 dF_{-i}(v_{-i}) - r_i v_i^* \\
&= v_i^* \Pi_{j \neq i} F_j(v_i^*) - \int_{v_{-i}} v_{-i}^1 F_i(v_{-i}^1) dF_{-i}(v_{-i}) + \int_{\hat{v}_{-i}^C} v_{-i}^1 dF_{-i}(v_{-i}) - r_i v_i^*.
\end{aligned}
$$

Now let's consider the term $\int_{\hat{v}_{-i}^C} v_{-i}^1 dF_{-i}(v_{-i})$. We have

$$
\begin{aligned}
\int_{\hat{v}_{-i}^C} v_{-i}^1 dF_{-i}(v_{-i}) &= E\left[v_{-i}^1 \left| v_{-i}^1 \geq v_i^* \right.\right] \cdot P(v_{-i}^1 \geq v_i^*) \\
&= \int_{v_i^*}^{\bar{v}_i} s \frac{d(P(v_{-i}^1 \leq s \left| v_{-i}^1 \geq v_i^* \right.))}{ds} \cdot P(v_{-i}^1 \geq v_i^*) \\
&= \int_{v_i^*}^{\bar{v}_i} s \frac{d\Pi_{j \neq i} F_j(s)}{ds}
\end{aligned}
$$

where the last inequality comes from

$$
\begin{aligned}
P(v_{-i}^1 \leq s \left| v_{-i}^1 \geq v_i^* \right.) &= 1 - P(v_{-i}^1 \geq s \left| v_{-i}^1 \geq v_i^* \right.) \\
&= 1 - \frac{P(v_{-i}^1 \geq s)}{P(v_{-i}^1 \geq v_i^*)} \\
&= 1 - \frac{1 - \Pi_{j \neq i} F_j(s)}{1 - \Pi_{j \neq i} F_j(v_i^*)} \\
&= \frac{\Pi_{j \neq i} F_j(s) - \Pi_{j \neq i} F_j(v_i^*)}{1 - \Pi_{j \neq i} F_j(v_i^*)}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\frac{d}{ds}(P(v_{-i}^1 \leq s \left| v_{-i}^1 \geq v_i^* \right.)) &= \frac{d\Pi_{j \neq i} F_j(s)}{ds} \frac{1}{1 - \Pi_{j \neq i} F_j(v_i^*)} \\
&= \frac{d\Pi_{j \neq i} F_j(s)}{ds} \frac{1}{P(v_{-i}^1 \geq v_i^*)}.
\end{aligned}
$$

With this, we go back to the expression of the expected payment, and we get

$$\begin{aligned} E(x_i(v)) &= v_i^* \Pi_{j\neq i} F_j(v_i^*) - \int_{v_{-i}} v_{-i}^1 F_i(v_{-i}^1) dF_{-i}(v_{-i}) \\ &\quad + \int_{v_i^*}^{\bar{v}_i} s \frac{d\Pi_{j\neq i} F_j(s)}{ds} - v_i^* \Pi_{j\neq i} F_j(v_i^*) \\ &= \int_{v_i^*}^{\bar{v}_i} s \frac{d\Pi_{j\neq i} F_j(s)}{ds} - \int_{v_{-i}} v_{-i}^1 F_i(v_{-i}^1) dF_{-i}(v_{-i}) \\ &= \int_{G_i^{-1}(r_i)}^{\bar{v}_i} s G_i'(s) ds - \int_{v_{-i}} v_{-i}^1 F_i(v_{-i}^1) dF_{-i}(v_{-i}). \end{aligned}$$

Summing over i we obtain (11).

Step 2: We maximize the surplus, taking as decision variables r_i.

Maximizing with respect to r_i we obtain:

$$-G_i^{-1}(r_i) G_i'(G_i^{-1}(r_i))(G_i^{-1})'(r_i) + G_I^{-1}(r_I) G_I'(G_I^{-1}(r_I))(G_I^{-1})'(r_I) = 0$$

Noticing that $G_i'(G_i^{-1}(r_i))(G_i^{-1})'(r_i) = 1$, we get that $G_I^{-1}(r_I) = G_i^{-1}(r_i)$, or equivalently $v_I^* = v_i^*$. Similarly differentiating with respect to the remaining $r_i's$ we obtain that

$$v_i^* \equiv v^*.$$

■

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