Something to Prove: Reputation in teams and hiring to introduce uncertainty

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Abstract

Agents work for their own reputations when young but for their firms' when old. An individual with an established reputation cannot credibly commit to exerting effort when working alone. However, by hiring and working with juniors of uncertain reputation, seniors will have incentives to exert effort. Incentives for young agents arise from a concern for their own reputation (and the opportunity to take over the firm) but older agents work for the reputation of their firms (and the opportunity to sell out to juniors). An important theoretical contribution is an example of a mechanism that endogenously introduces type uncertainty.

Key words: Reputation incentive, teams, overlapping generations, up-or-out

JEL: C73 (Stochastic and Dynamic Games), D82 (Asymmetric and Private Information), L14 (Transactional Relationships: Contracts and Reputation), M12 (Personnel Economics)

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1 Introduction

In numerous applications, reputation concerns play an important role in ensuring effort. For example, in professional services industries, such as law and consulting, products cannot be inspected prior to sale and it is impossible to make fully contingent descriptions of the product.[1] Developing and maintaining reputation both at the level of the firm and the individual are crucial. Indeed, previous literature has suggested that the very existence of a firm might arise as a means to manage reputation and that careers are designed to take into account reputational considerations.[2]

We outline a framework in which a young agent is motivated by concerns for her own reputations. However, once she is established, her own reputation is not at risk and cannot act as a motivation. She, therefore, chooses to hire and work with a junior whose ability is uncertain and, since only combined outcomes are observed, her actions will affect the reputation of her junior. She cares about the reputation of her junior since she controls the client list that the junior needs. She is able to provide incentives for the junior by committing in advance to a price at which she will allow the junior access to the client list (or equivalently the wage which she will pay to the junior if she retains him). Thus, in particular, young agents are motivated by concerns for their own reputations, and old, successful agents are motivated by the reputations of the firms which they own (or more specifically by concern for the reputation of their employees).

This modeling approach and the results relate and contribute to a number of different literatures.

First, there is a contribution to the literature on reputation. Any reputational concern relies almost tautologically on current actions affecting future beliefs.[3] In particular, this implies that there must be uncertainty for reputational concerns to arise since if beliefs

[1] It is relatively difficult for a consultant to show a client the report that she would write if hired or for a client after the event to complain that he did not receive the report that he was expecting.

[2] For examples, see David M. Kreps (1990) on the first point and Robert Gibbons and Kevin J. Murphy (1992) on the second.

[3] An earlier literature, including for example Benjamin Klein and Keith B. Leffler (1981) or more closely related to this paper Jacques Cremer (1986) has been built on the notion that current actions can affect future continuation values not through beliefs on fundamentals but rather through coordinating on equilibrium strategies along different subgames of infinite games. Such repeated game notions are now seldom referred to as reputation—the term in the literature now reserved for type-based models. (The interested reader is referred to the opening chapter of Heski Bar-Isaac (2004) for a discussion of notions of reputation.) Moreover, these repeated game models have been criticized on the grounds that the results rely on an infinite horizon and the equilibrium strategies, that the results rely on, are not renegotiation-proof. The results described below are robust to both such criticisms.

are sure then little can be done to change them.[4] Loosely for an agent to be motivated by reputation she needs to have something to prove. Previous literature has suggested the possibility that this might lead principals to slow down the release of information to sustain reputational concerns (Seonghoon Jeon (1996) for example) or the important role that exogenous replenishment of type uncertainty might play (Bengt Holmstrom (1999), Cripps, Mailath and Samuelson (2004) and the references therein).[5] This is the first paper, however, to suggest an approach to strategically introduce new type uncertainty and allow an agent, thereby, to credibly commit to exerting effort.

Second, the focus on small teams departs from other reputation, which seeks to explore the issue of the interaction of individual and collective reputations. Such papers have either been unable to separate the notions of individual and collective reputations (Steven Tadelis (2002) and Mailath and Samuelson (2001))[6] or have assumed that groups are very large

[4] Much of the foundational literature on reputation has, in some sense, never allowed an agent an opportunity to establish herself or equivalently as time passes no uncertainty is resolved. In particular, consider the pioneering work of Kreps, Paul Milgrom, John Roberts and Robert Wilson in a series of papers published in 1982 and developed by Drew Fudenberg and David K. Levine (1989) and (1992). In these models observers cannot distinguish a strategic type behaving well and a "crazy" type who behaves in a way in which the strategic would like to be able to commit (Fudenberg and Levine (1989) therefore term this a "Stackelberg" type). That is, since the strategic type has an incentive to convince the public that she is a Stackelberg type, in equilibrium there is uncertainty about an agent who behaves in the way that a Stackelberg type would behave, as this might be a strategic type mimicking. Thus the uncertainty necessary to maintain reputational incentives arises naturally and is maintained over time.

Martin W. Cripps, George J. Mailath and Larry Samuelson (2004) show that even with such a Stackelberg type if there is imperfect monitoring of actions, then in the very long run customers would learn an agent's true type and reputation effects would disappear. They view their results:

> as suggesting that a model of long-run reputations should incorporate some mechanism by which the uncertainty about types is continually replenished.

An important contribution of this paper is to suggest an endogenous mechanism for achieving this.

[5] In the organizational economics literature, papers that note that uncertainty might be helpful include Margaret A. Meyer, Trond E. Olsen and Gaute Torsvik (1996), Meyer and John Vickers (1997) and Cremer (1995). However, these papers are based on period-by-period *output-contingent* contracting and introduce no means to introduce *new* type uncertainty.

[6] Indeed, central to these papers is the idea that one cannot identify the current holder of a firm's name. It is further worth noting explicitly that this work and other literature based on name trading based on name-trading (Tadelis (1999) and (2003)) relies heavily on the intertemporal non-observability of name transfers. In contrast, this paper relies on the contemporaneous non-observability of joint production and the focus is on the decision of the senior agent's decision of whether or not to hire, rather than the junior agent's decision to join one firm rather than another or buy one name rather than another.

On this last aspect, in this paper it is assumed that the senior cannot screen juniors and that all juniors are ex-ante identical, so that the question of who to hire does not arise. This latter question has been considered elsewhere, in particular Björn Segendorff (2000) and Amihai Glazer and Segendorff (2001) consider how reputational concerns might, for example, lead good seniors to hire bad juniors when the relationship

(Jean Tirole (1996) and Jonathan Levin (2001)) so that a single individual's actions can have no effect on the collective reputation.[7] In particular, this focus on small teams, admits the possibility hitherto absent from the literature, that an agent might have concerns both for her own and for her group's reputation.

With respect to the application of organizational design for professional services firms, a number of other interesting features arise.[8] In particular, it is shown that teamwork can create rather than dampen incentives since mixed teams of partners and juniors can provide incentives for partners.[9] Moreover, in this framework for an up-or-out mechanism to be effective then promotion must be to partnership–an empirical feature which previous literature has not much addressed, but which arises very naturally in the context of the overlapping generations framework of the central model.[10],[11]

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between rewards and reputation are non-linear

[7] Michèle Breton, Pascal St-Amour and Désiré Vencatachellum (2002, 2003) and Axel Anderson and Lones Smith (2002) also consider reputation in small groups but focus on what types of agents work together. In these papers agents do not have any effort decision to make and so these papers cannot address reputational incentives to exert effort—the focus of this paper.

[8] See, for example Canice Prendergast (1999), for a review and further references on the broad question of organizational design and incentives.

[9] Meyer, Olsen and Torsvik (1996) also show that teamwork–precisely because it clouds the inference that can be drawn on a particular agent–might be preferred to solo production. However, in their model, this is to dampen the ratchet effect in a model of explicit outcome-contingent contracting with limited ability to commit to long-term contracts. This is quite different to this paper's model, in which agents are risk neutral and it is not possible to write explicit contracts, and where reputational considerations rather than explicit outcome-contingent contracts are the driving force behind effort. Yeon-Koo Che and Seung-Weon Yoo (2001) consider that agents within a team can monitor each other's actions better than a principal and over a long period dynamic strategies can allow team members to enforce good behavior.

[10] For an exception and elaboration on this criticism of previous literature, see James B. Rebitzer and Lowell J. Taylor (2001) who focus on employees threatening to "grab and leave" with an important client, abstracting from other incentive problems, and suggest that the up-or-out system has evolved as a resolution to this problem.

There are a number of interesting papers, relating to the phenomenon of up-or-out; most relevant to this paper are Charles Kahn and Gur Huberman (1988) and Prendergast (1993).

[11] A similar feature arises in Alan D. Morrison and William J. Wilhelm Jr. who consider the role of partnerships in ensuring that seniors mentor their juniors. In their paper a partner has an incentive to mentor a junior as only a good junior who has been mentored would be willing to buy a partnership share in the firm, as only in this case will the junior be able to maintain the firm's collective reputation and the value of her partnership stake. Thus the paper echoes a number of the themes highlighted here, in particular, a senior is motivated to work since this affects her ability to sell the firm to the junior. However, the issue at the heart of Morrison and Wilhelm (2003) is the incentive for partners to mentor juniors–an incentive which naturally leads seniors and juniors to work together–and they consider no incentive issues for the juniors. Here instead, seniors and juniors are identical and face similar decisions–it is not the case that the senior affects the productive capability of the junior, as in Morrison and Wilhelm (2003), but rather can affect how this is perceived; moreover we show that the promotion structure described can be successful in providing incentives for the junior as well as for the senior. Further their model has no role

Finally, whereas some have argued that law firm partnerships might exist to diversify risks for individuals, in practice one sees groups of lawyers working in the same or related fields. An established argument for this phenomenon is that it allows for mutual monitoring among the partners in a law firm (Armen A. Alchian and Harold Demsetz (1972)), note in addition that a more homogeneous firm makes it more difficult for clients to identify individual contributions of seniors and juniors and so enables the reputational mechanism highlighted in this paper to operate and seniors to credibly commit to exerting effort.

The central model is somewhat involved and so to gain intuition, a simpler model is introduced which highlights a number of important features, in particular including the central role of a joint production function. The central model itself makes a number of stark assumptions, in many cases (as discussed below) these are made primarily for expositional purposes and can be relaxed.[12] The remaining, crucial assumptions and their empirical plausibility are discussed in a section following the presentation of the central results. A final section concludes.

2 A motivating example

In this section, we introduce a simple model to demonstrate that a senior, an agent with an established reputation, can exploit uncertainty about something else, when there is a joint production process which does not allow individual contributions to be observed.

Specifically, consider an agent, the senior, who lives for two periods. There are many customers Bertrand competing for the good that the senior produces which may be of high or low quality depending on her effort decision, as discussed below, and for the good produced by a machine of uncertain quality. Timing is as follows.

Period 0: There is a machine which either always produces high quality or always low quality products, but can only produce one unit in each period and does so at no cost. The machine owner together with the senior and customers believes that with probability λ the machine is the type that always produces high quality. Thus, supposing no discounting between periods, a risk neutral machine owner who worked the machine independently would have an expected present value of 2λ.

The senior can make a take-it-or-leave-it offer to the current owner.[13] Thus she can

for individual reputations.

[12] For the skeptical reader, we prove similar results in a number of related models which relax many of these assumptions have been relaxed. These results are available in the appendix.

[13] Note that the assumption of a take-it-or-leave-it offer on the part of the senior is not crucial, similar qualitative results could be generated if the machine owner had the ability to make a take-it-or-leave-it

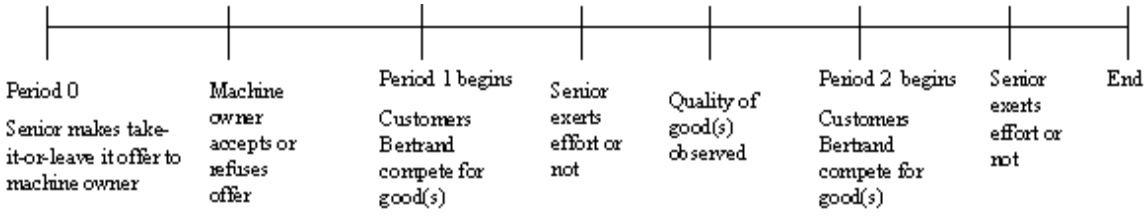

Figure 1: Timeline for example

offer 2λ to buy the machine for the 2 periods, collecting the revenue for the machine's output.

Period 1: Customers Bertrand compete for the output produced by the senior and the machine. They assign the value 1 to a high quality product and 0 to low quality and compete to buy the unit so that the sale price is exactly the customers' belief that the product will be successful, since sale occurs before the quality is realized and the price cannot be made contingent on the outcome.

The senior decides whether or not to exert effort at a cost c. If she exerts effort, she will produce a single unit which will be a high quality product with probability $g \leq 1$ and otherwise low quality. If she exerts no effort then the product will be low quality for sure. Quality is non-verifiable and not observed prior to purchase so that the price cannot be contingent on quality but will depend on customers' expectations.

Finally suppose that $g > c$ so that the costly effort is efficient. In particular, if effort were contractible or the senior could credibly commit to it and be compensated for it, then she would exert effort.

Period 2: The quality of the goods produced in period 1 is observed. If the senior bought and worked with the machine then the output cannot be directly attributed, that is if a one high quality good and one low quality good are observed, customers do not know which was produced by the senior and which by the machine. Again customers Bertrand compete for the output produced.

This timing is summarized in Figure 3.1.

First suppose that the senior has no opportunity to buy and work with the machine,

offer or under any intermediate bargaining power assumption and bargaining game. The key point here is that the total value to the senior from working with the machine is greater than the value of them working separately, and that senior is the residual claimant of at least some of the machine's reputation in the second period.

then the unique subgame perfect equilibrium of this game is clear—the senior exerts no effort in either period. This is her optimal action in the final period, and so in the penultimate period as well.

However, when the senior can buy and work with the machine, she can commit to exert effort in Period 1 (though not in Period 2). First note that the total output when the senior and the machine work together is simply the sum of their output. Thus the expected value of the output when the senior puts in effort and works with the machine is 0 with probability $(1-g)(1-\lambda)$, it is 1 with probability $g(1-\lambda)+\lambda(1-g)$ and it is 2 with the residual probability $g\lambda$. If the senior does not exert effort then the expected output is 0 with probability $(1-\lambda)$ and 1 with probability λ, and there is no chance that the output might be 2. When the senior works with the machine, however, it is assumed that the production process is unobservable in the sense that when a single unit is produced, customers cannot tell whether it was produced by the senior or the machine. In particular, this implies that if customers expect the senior to exert effort in period 1 and she does not, then the machine's reputation will in expectation be lower than it ought to be—this could induce the senior to exert effort and so allow an equilibrium in which the senior credibly commits to exerting effort. This is the intuition underlying the following result.

Proposition 1 *If* $\frac{g(1-\lambda)\lambda}{(1-g)\lambda+g(1-\lambda)} > c$ *then there is a pure strategy subgame perfect equilibrium in which the senior exerts effort in period 1. If in addition* $g(1-\lambda) > c$*, this is the unique pure strategy subgame perfect equilibrium.*

Proof. Consider the following strategy: The senior buys the machine for 2λ. In the first period the senior works with the machine and exerts effort. In the second period (the senior's "retirement" period) the senior works alone and exerts no effort and puts the machine to work alone, collecting the fee for its output. Buyers believe that the senior behaves in this way and bid for the good produced accordingly, revising their beliefs on the quality of the machine according to Bayes rule after observing the quality of the Period 1 goods at the beginning of Period 2.

It is clear that the price of the machine is an equilibrium price and that the machine owner's behaviour in equilibrium is optimal. For the senior the strategies describe optimal behaviour so long as she could do no better either by working on her own from the first period (the value of which as described above is 0) or by not exerting effort in the first

period.[14] Her value from sticking to the strategy is

$$g+\lambda-c-2\lambda+g\lambda*1+((1-g)\lambda+(1-\lambda)g)\frac{(1-g)\lambda}{(1-g)\lambda+g(1-\lambda)}+(1-g)(1-\lambda)*0 \quad (1)$$

This expression can be explained as follows. Given the anticipated equilibrium strategies the first period revenue is $g+\lambda$ but the senior must incur the cost c of effort and buy the machine for 2λ. In the second period, when the output is 2 (which will happen in equilibrium with probability λg) the public is certain that the machine is good and will pay 1 in the second period; when the output is 0 (which happens with probability $(1-g)(1-\lambda)$) the public is certain the machine is bad and is prepared to pay 0 in the second period; and, when the output is 1 the belief that the machine is good (and so also the second period revenue) is $\frac{(1-g)\lambda}{(1-g)\lambda+g(1-\lambda)}$. The expression in (1) can be simplified to $g-c$; it is simply the value of the senior committing to exert effort in the first period and buying the machine for exactly its expected output.

The value when the senior defects and exerts no effort is given by:

$$g+\lambda-2\lambda+\lambda\frac{(1-g)\lambda}{(1-g)\lambda+g(1-\lambda)}. \quad (2)$$

This follows since she receives $g+\lambda$ in the first period as customers expect her to exert effort but does not exert effort or incur its cost, but now there is zero probability of the realized output in the first period being 2 and the probability of it being 1 is λ. So the total value of deviating and not exerting effort when it is anticipated is as appears in (2). The value when she defects by not hiring is 0.[15]

So the strategies do indeed describe an equilibrium so long as $g-c>0$ (following (1)) which is assumed to be true—otherwise effort would be inefficient and so long as the senior prefers to exert effort in the first period so that (1)>(2):

$$g-c>g-\lambda+\lambda\frac{(1-g)\lambda}{(1-g)\lambda+g(1-\lambda)} \quad (3)$$

or equivalently

$$\frac{g(1-\lambda)\lambda}{(1-g)\lambda+g(1-\lambda)}>c. \quad (4)$$

[14] It is clear that there is there is no second period strategy that dominates the one described.

[15] We suppose that on this off-equilibrium path customers believe that the senior will not exert effort, this would have to be the case, for example, if imposing a trembling-hand refinement.

Uniqueness in pure strategies:[16] Now suppose that it is believed that the senior exerts no effort even when buying the machine. Suppose that the senior buys the machine and exerts no effort, the expected value of this strategy is 0. Suppose that the senior deviates by exerting effort, this yields the value:

$$-2\lambda + \lambda - c + g(1-\lambda) + \lambda(1-g) + g\lambda, \tag{5}$$

where this expression can be explained as follows. The senior buys the machine at a cost 2λ and since the machine is expected to produce high quality with probability λ and she is not expected to exert effort, the first period revenue is λ, in addition she incurs the cost of effort. In the second period, whether they see one or two high quality products, customers will believe that the machine always produce high quality. The above expression can be re-written as $g(1-\lambda)-c$ and so the senior deviates and exerts effort when $g(1-\lambda) > c$. ■

It is clear that parameter values exist for which condition (4) can be satisfied, and that it is more likely to be satisfied the smaller is c and the larger is g—this follows naturally, since the larger g or the smaller c the greater the gain from taking the costly action. However the comparative statics with respect to λ are not monotone. The intuition underlying this non-monotonicity reflects the discussion in the introduction of the need for "sufficient" uncertainty, or having something to prove. At extremal values of λ whether or not the senior exerts effort would not change the public's belief about the machine by much.[17] Note in particular that for λ close enough to 0 or 1 then the left hand side of inequality (4) is close to 0 so that the condition fails. This is illustrated in Figure 2, which illustrates combinations λ and g for the case $c = 0.1$: an equilibrium in which the senior exerts effort exists to the left of the black line line (this is the unique equilibrium in pure strategies if in addition the (λ, g) pair is below the red line).

Note that the senior's profits in an effort-inducing equilibrium are independent of λ—so long as it is in the range that supports the equilibrium—the senior's profits over the two periods are given by $g-c$ which is simply the value of the senior committing to exert effort in the first period, (the other equilibrium revenues correspond to the senior buying the machine for exactly the value of its expected output which she sells).

As a final observation in this section, note that non-observability in joint production

[16] It can be shown that there is a mixed strategy equilibrium where the senior buys the machine and exerts effort with probability $q = \lambda\frac{c+\lambda g-g}{cg(2\lambda-1)}$ so long as $q \in (0,1)$ and $(2-\lambda)cg - g^2(1-\lambda) - c^2 \geq 0$.

[17] Specifically, note that in the case following an observation that the output is 1 the public belief about the machine in the equilibrium is $\frac{(1-g)\lambda}{(1-g)\lambda+g(1-\lambda)}$; for λ close to 0 or 1 this is close to λ.

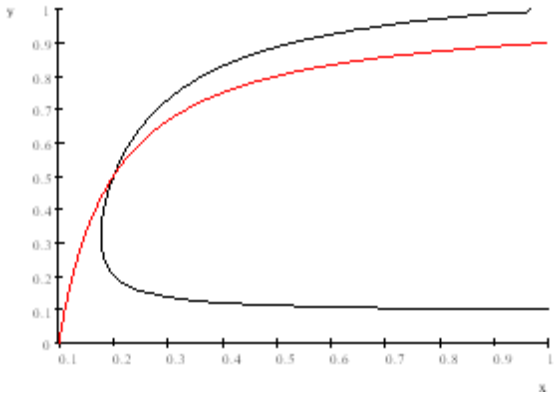

Figure 2: A Plot of lambda against g: Existence and Uniquencess at c=0.1

is crucial. Here non-observability arises in the sense that when the joint output is 1, the public does not know whether the single unit was produced by the machine or the senior. If instead this is observed, then the equilibrium described above breaks down—the senior's action would have no influence on the reputation of the machine and on the price for which its product is sold in the second period. In particular, if the senior exerts no effort and it is fully observed that the machine produces a single unit in the first period, then the machine's output can still be sold for 1 in the second period so that the senior loses the incentive to exert effort in the first period.

3 A richer model with OLG agents

The above example demonstrates that a senior can exploit the uncertainty about a machine in order to commit to an efficient, though costly action through concern about the reputation of the machine. However, the above example, though suggestive, is inadequate for many applications. In particular, in trying to apply this reputational mechanism to professional services, it is of more interest to think about introducing uncertainty through working with other agents who also need incentives to exert effort rather than machines with no such need. In this section, we consider a richer framework which treats seniors and juniors in a symmetric and consistent way.

We introduce a framework with overlapping generations of agents with a two period working life then we characterize an equilibrium, which is the focus of this paper, where agents exert costly and efficient effort in the first period of life and in the second period

only if their "firm" is successful in the first period.

3.1 Model set-up

In an infinite period framework, agents work for two periods, and can consume in a third "retirement" period. A new generation of equal size is born in each period. An agent may be either competent or inept.

Production Inept agents, whether exerting effort or not, produce low quality products for sure as do competent agents who exert no effort. A competent agent who exerts effort at a cost c produces a high quality product for sure.[18]

An agent can either work on her own, producing as above, or alternatively, an agent can work in a team with another agent, in which case the total output is the sum of the output of each agent.[19] However, in the case of joint production, if a single unit is produced then the public cannot tell which of the agents produced it—this assumption ensures that when working together the senior's choice of effort can affect the reputation of the junior.[20]

All customers are willing to pay 1 for a high quality product and 0 for a low quality product. All agents are risk neutral, maximise their expected lifetime earnings and have a discount factor of 1.[21]

Information At birth, an agent does not know her own type and believes that there is a probability λ that she is competent. Potential employers and customers also believe that there is a probability λ that any new born agent is competent. It is assumed that $\lambda > c$ so that inducing effort from new born agents is efficient.[22]

There are very many *ex-ante* identical locations; in particular, a new born agent can always find a location where there is no existing agent at which to found a new firm. At each location there are many identical customers who Bertrand compete for the product

[18] Assuming that inepts and competents who exert no effort produce high quality with probability $b > 0$ and competents exerting effort produce high quality with probability $g < 1$ leads to qualitatively similar results. This model is analyzed in the appendix.

[19] Allowing agents to work alone as well as in teams is a convenient modelling assumption. Without it, in the case where a competent agent exerting effort succeeded with a probability less than 1, the number of successful agents (that is in teams with two successes) in each generation would fall. This assumption, thus, allows us to consider a steady state with a positive fraction of successful agents.

[20] Note that this assumption can be weakened, in particular partial non-observability would do. For example, the case that the senior's output could be seen independently of the junior's but a junior only gets chance to prove herself if the senior exerts effort would lead to similar results.

[21] The assumptions that agents are risk neutral and value the present and future equally are made for ease of exposition and are not important for the qualitative results.

[22] In the appendix, we consider the case where the junior knows her own type from birth and derive qualitatively similar results.

of the agent or agents at that location, so that the revenue that an agent generates when working alone, for example, is equal to the customers' belief that the agent will produce high quality.[23] An agent's productive history (that is the sequence of high or low quality produced by the agent working alone or by the agent and her co-worker when working in a team) is assumed to be non-verifiable but observable, though only at the location where it is produced. The assumption that the quality of the agent's output is non-verifiable prevents the use of contracts contingent on this.

The assumption that the agent's history is observable only at the location where she has worked has two implications. First, when one agent has control over a location and can exclude the other agent from its use, she can prevent the other agent from simply moving to another location with no ill consequences, since in moving she would also lose any positive reputation built up through previous high quality output. Thus a junior, in order to keep her reputation, must buy the location (the firm) from the senior. Note, however, that since there are infinitely many such locations, this control is valuable only after an agent has worked there and built up a reputation, but not before. Second, an agent who has failed and consequently has a worse reputation than a new-born agent can move to another location. Then an agent who failed in the first period can pretend to be a new-born agent. This implies that an agent cannot be severely punished for past actions and so loosely speaking this assumption acts as a sort of renegotiation proof assumption and highlights that the mechanism is doing much more than could be achieved by simple repeated game considerations.[24]

Contracting As mentioned above, outcomes are observable at locations, so that customers' beliefs can change over time, but these outcomes are not verifiable so there is no outcome-contingent contracting. Similarly, a senior cannot write an outcome contingent contract with a junior. Employment contracts will be of the form (w, P) where w specifies the wage that the junior is paid and P specifies the price at which the junior will be able to buy the firm (or equivalently control of the location) at the end of the period.[25]

[23] This assumption can be relaxed, similar qualitative results would hold so long as the price for output is an increasing function in this belief.

[24] The case where an agent's age is observable appears in the appendix. In that framework and in an equilibrium characterized, customers would hold the belief that only agents who had failed would move and so observing a second period agent in a new location effectively reveals her history. In that case too, an equilibrium in which agents exert effort in the first period of life and successful agents exert effort in the second period of life can be sustained.

[25] Similar results can be obtained if instead the senior offers an employment contract of the form (w_1, w_2) where w_2 denotes a second period wage for the junior if retained and the senior keeps all revenues generated

An agent thus has a rather complicated strategy. Specifically, her strategy consists of:

- choice of mode of work in period 1 of career: work alone, work as an employee (if an appropriate position is offered), hire an employee;
- choice of mode of work in period 2 of career: move location or stay, hire an employee (in which case the strategy will include a decision of the wage contract offered), buy the firm (if previously an employee), work as an employee (if an appropriate position is offered);
- effort decision (in both periods of life).

An agent's strategy will of course depend on outcomes and decisions in previous periods.

In each period, customers Bertrand compete for the goods that are produced before observing quality. The quality is observed between periods and customers revise beliefs. An agent's strategy (such as whether to hire, buy the firm, exert effort, relocate) for period 2 of course depends on the observable outcomes in previous periods.

Note that in the case where agents can only work alone and there is no hiring, the unique perfect Bayesian equilibrium outcome is that no agent exerts effort. With no reputational incentives in the final period of her career whatever the belief about the agent at that time and no mechanism for contingent payments, there are no incentives for effort in the final period of the career. It follows by backward induction there are no such incentives in the first period either. Moreover, even when allowing for joint production, there is always an equilibrium where no agent exerts effort. Suppose that the public believe that no agent ever exerts effort and would continue to believe this even after observing an agent producing high quality, then this belief would be upheld in equilibrium since no agent will exert effort at a cost c when whether or not they do so, they sell their service at a price 0.

in that period. In this case, a constraint on equilibrium strategies would be that the senior only promotes a junior who succeeded, thus providing incentives for the junior to work, in the spirit of the up-or-out models of Kahn and Huberman (1988) and Prendergast (1993). In this case, for the promoted junior to have incentives to exert effort, she must gain from the future reputation of her own junior—this can naturally be interpreted as promotion to partnership. With the (w, P) contract the senior always would want to sell the firm, but it is worth more to a successful junior than an unsuccessful one, providing incentives for the junior to work. Since in equilibrium only a successful junior would find it worthwhile to buy the firm, the senior has incentives in the first period of the junior's life to ensure that the junior has the opportunity to succeed. This is similar to the motivation for a partner to mentor in Morrison and Wilhelm (2003).

3.2 An equilibrium with effort

Despite the result that an equilibrium always exists where no agents exert effort, allowing for joint production can allow other equilibria which do induce effort.

Proposition 2 *If* $1-\lambda-c+\frac{c\lambda}{2}-\frac{3c(2-\lambda)}{2\lambda} \geq 0$ *then there is a Perfect Bayesian Equilibrium in which all agents exert effort in the first period of their careers, and competent agents exert effort in the second period.*

Proof. We prove this result by construction, outlining equilibrium strategies and verifying that these strategies do indeed characterize an equilibrium.

Specifically, the strategies are as follows:

In the first period of life, either an agent founds her own firm, working on her own in an unoccupied location or accepts a position as an employee if offered one at sufficiently attractive terms; in either case she exerts effort in the first period of her own life. If she founds her own firm and fails in the first period, then she poses as a new-born agent. If she succeeds in the first period of life after founding her own firm, then in the second period she hires a junior (offering a wage contract that pays w and offers the firm at P) and they work together with the founder exerting effort.

Alternatively an agent might begin life as an employee in an established firm with a (w, P) contract, so long as such a contract offers her in equilibrium at least as much lifetime earnings as founding her own firm. She exerts effort in the first period of life. If the firm as a whole produced one or no high quality outputs, she chooses not to buy the firm but instead poses as a new born agent. If the firm produced two high quality outputs then she buys the firm at the specified price P, hires her own junior offering her junior the contract (w, P) and she works together with her junior and exerts effort.

A second period agent posing as a newborn either works alone or works as an employee if offered a position and if w is at least as great as the wage she could earn when working alone.

In the equilibrium described below, all agents exert effort in the first period of life and so all competents succeed in the first period of life. Thus the population of those who appear to be new borns consists of a measure 1 of true new borns and a measure $1-\lambda$ of second period inepts posing as new born, thus the probability that an agent posing as a new born is competent is $\mu = \frac{\lambda}{2-\lambda}$.[26]

[26] In the case where a competent agent exerting effort succeeds with probability $g < 1$, and an inept or competent exerting no effort with probability $b > 0$, things are a little different. An issue which arises in

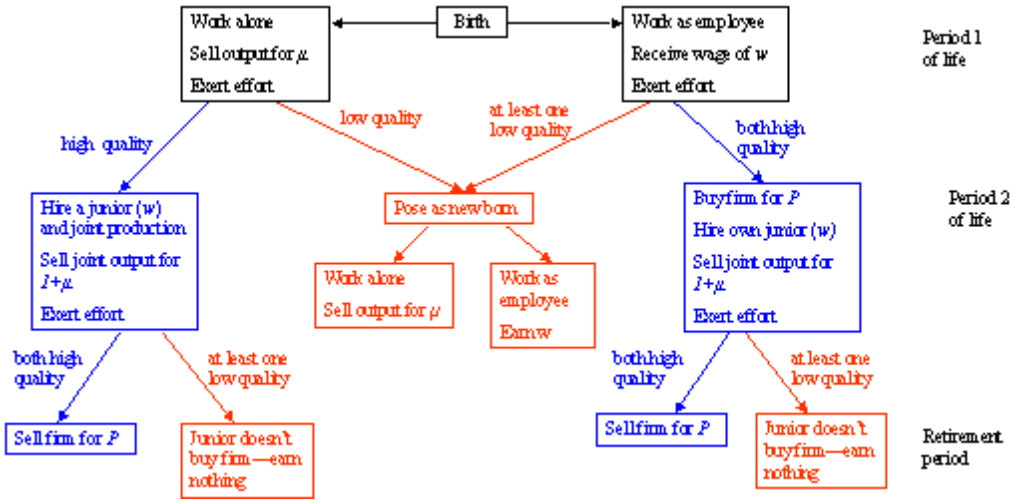

Figure 3: Equilibrium behaviour

It is common knowledge among the public and all agents that these are the equilibrium strategies and prices are set appropriately, with the industry capturing the full consumer surplus. Thus the price of the output of a new-born agent (or of an agent pretending to be new born) working alone is μ since there is a probability $\frac{1}{2-\lambda}$ that she truly is new-born, in this case she exerts effort in equilibrium and so generates high quality output with probability λ (the probability that the new born is competent). When an agent has succeeded then it is known that she is competent and will produce high quality products with certainty in the following period when exerting effort. The on-equilibrium path behaviour and strategies are summarized in Figure 3.

To complete the characterization of the equilibrium, one must also consider off-equilibrium beliefs:

- if an agent does not offer the contract (w, P) then any potential juniors who will not have seen her history assume that she has previously failed;[27] and finally,

this case, is that the probability of someone who appears to be new-born truly is new-born will depend on the fraction of second-period agents who had previously been working alone rather than as employees. Solving for a steady state, where the proportion of those starting their working lives as employees closes the model, which appears in the appendix.

[27] Note that although this does not imply that an agent will reject any offer (in particular, she would accept any offer with very high w and/or low P); however she would reject any offer that an inept senior could make to her and which would allow such a senior to make a non-negative profit by hiring her. Further

- when an apparently new born agent hires, in this off-equilibrium action, customers suppose that the agent must be a second-period inept agent.

The value of a new born who begins her career by founding her own firm is given by:

$$V_f = \mu - c + \lambda(1 + \mu - c - w + \mu P) + (1 - \lambda)(\mu w + (1 - \mu)\mu). \tag{6}$$

This expression is built up as follows. On observing an agent who appears to be new-born working alone, customers believe that she is a competent new born agent with probability μ. The new born agent, who exerts effort and knows that she is new born expects success with probability λ and in this case she is revealed as competent, she hires another agent as a junior whose product is expected to be worth μ and so can charge $1+\mu$ for the joint service (hiring the junior costs her w, further she exerts effort at a cost c) and in case of two successes, which occurs with probability μ, she sells the firm earning P. Following a failure in the first period, the agent can pose as a new-born agent and so receive the revenue w if hired, which occurs with probability μ since in equilibrium there is a measure λ of hiring agents and $1 + 1 - \lambda$ is the measure of agents claiming to be new born; if not hired then the agent works alone and earns μ.

Similarly, the value for a new born agent who works as an employee is given by the following expression:

$$V_e = w - c + \lambda(1 + \mu - c - w - P + \mu P) + (1 - \lambda)(\mu w + (1 - \mu)\mu). \tag{7}$$

In equilibrium, a new born agent is willing to become a junior or equivalently $V_e \geq V_f$. In particular, this implies that:

$$w - \mu - \lambda P \geq 0. \tag{8}$$

In addition, a second period agent posing as a new born should be willing to be hired as a junior.[28] Specifically, this condition is given by:

$$w \geq \mu \tag{9}$$

note that as discussed below, by offering (w, P) a competent senior in effect offers the junior a contract which compensates her for her outside option (working alone); the senior can therefore offer no less in expected terms.

[28] It can easily be shown that no equilibrium exists in which only new-borns are willing to work as employees and only the population of those working alone consist of true new-borns and posers.

Given that there is a scarcity of junior slots available, hiring seniors will drive down the value that the contracts deliver to employees to the point where they are indifferent. In particular this implies:

$$w = \mu + \lambda P. \tag{10}$$

It will come as no surprise that a later condition will ensure that $P > 0$ and so (10) implies (9).

Conditions (9) and (10) can be thought of as individual rationality conditions. The remaining deviations can be categorized into a number of separate groups (i) exert effort (incentive compatibility) (ii) hiring policy and (iii) buying the firm. Each group is considered in turn.

(i) exert effort (incentive compatibility)

For a second period agent who had been successful there is an incentive compatibility constraint—which is identical in both the case that she worked alone in the first period and the case when she buys the firm from her employer—specifically, this constraint is as follows:

$$\mu P \geq c \tag{11}$$

In the first period in both cases, that the agent works alone or as a junior, it must be worthwhile to exert effort. The corresponding incentive compatibility conditions are the following:

$$1 + \mu - c - w + \mu P - (\mu w + (1 - \mu)\mu) \geq \frac{c}{\lambda} \tag{12}$$

and

$$1 + \mu - c - w - P + \mu P - (\mu w + (1 - \mu)\mu) \geq \frac{c}{\lambda} \tag{13}$$

Note that by (11), it follows that (13) implies (12).

(ii) hiring policy

Without hiring in the second period of life, the agent could not commit to effort and so the best that she could do is pose as a new-born and earn $\mu w + (1 - \mu)\mu$ and so the conditions that a second period agent who had success when working alone in the first period does indeed prefer to hire a junior is given by:

$$1 + \mu - c - w + \mu P \geq \mu w + (1 - \mu)\mu \tag{14}$$

which is implied by (12). The same condition ensures that a second period agent who had been an employee and bought the firm, prefers to hire.

Suppose that an agent failed in the first period of life, then she must be inept and a single success would prove that her junior was competent and so a sufficient condition that would ensure that she does not hire is given by:

$$\mu w + (1-\mu)\mu \geq \mu - w + \mu P. \tag{15}$$

Finally, since customers hold the off equilibrium belief that any apparent new born attempting to hire must be a failed second period agent, the condition that ensures no second period agent posing as a new born hires is also given by (15). A true new born agent would not hire when $V_f = V_e \geq \mu - w + \mu P + \mu w + (1-\mu)\mu$ (note in the first period of life, customers would hold the belief that she must be a failed second period agent). Note that $V_f \geq w + \mu w + (1-\mu)\mu$ by (13) and so (10) and $\mu = \frac{\lambda}{2-\lambda} < \lambda$ ensures that a true new born would not hire.

(iii) Buying the firm

An employee who had succeeded would indeed buy the firm so long as this generates more value than her alternative—posing as a new born. This is the case when:

$$1 + \mu - c - w - P + \mu P \geq \mu w + (1-\mu)\mu \tag{16}$$

This is implied by (13).

An employee who had failed is revealed as inept, and so rather than spending $P > 0$ to remain in this location, she would rather costlessly move to another location and pose as a new born agent where she would have a higher reputation.

Thus sufficient conditions are (10), (11), (13), and (15). Substituting for $\mu = \frac{\lambda}{2-\lambda}$ and for w from (10), these conditions can be reduced to $1 - \lambda - c - \frac{(2-\lambda)c}{2\lambda} + \frac{c\lambda}{2} \geq P \geq \frac{c(2-\lambda)}{\lambda}$, or equivalently:

$$1 - \lambda - c + \frac{c\lambda}{2} - \frac{3c(2-\lambda)}{2\lambda} \geq 0 \tag{17}$$

This concludes the proof. ■

The characterized equilibrium relies on the senior's ability to commit in advance to the price at which she will sell the firm.[29] The junior either buys the firm (and goes on to her

[29] In general, many ex-post bargaining schemes which share the benefits of the junior's accumulated reputation between the senior and junior would lead to qualitatively similar results.

own junior) or else leaves. Since the senior can commit to a price for the firm, the junior's incentive problem is resolved since there are rewards to working hard, being revealed as competent and buying the firm. The senior's incentive problem is resolved since her action affects the junior's reputation, and only a junior with a good reputation would be willing to buy the firm.

A junior with accumulated reputation buys the firm since locations are assumed to be informationally separate and the senior controls access to the existing location, so that a junior who leaves would have to forego her accumulated reputation.

Note that the result that all competent agents exert effort in the second period of their careers relies on the simplifying assumptions that a competent agent who exerts effort produces high quality for sure and that an inept produces low quality for sure. Thus in the equilibrium described, a competent is always recognized as such. Relaxing these assumptions, would suggest that mistakes are possible (that is a competent agent may not be recognized as such by the end of the first period), but a qualitatively similar result holds whereby under parameter restrictions all agents exert effort in the first period of their careers and successful agents in the second.

Further note, that as in the illustrative example of Section 2, uncertainty plays an important role. The equilibrium condition (17) fails for λ close to 0 or 1, as illustrated in Figure 4, which plots c against λ where the condition is satisfied below the line.

$1 - \lambda - c + \frac{c\lambda}{2} - \frac{3c(2-\lambda)}{2\lambda} = 0$

4 Discussion

The model builds on a number of assumptions which are worthy of further discussion.

First, we assumed that there was no static advantage or disadvantage in having agents work in teams rather than as individuals. A team's production was assumed to be simply given by the sum of individual members' contributions. Although, it is perhaps more reasonable to suppose that there may be important complementarities in team production, perhaps that a team's output is determined by its weakest member, the model deliberately ignored such considerations. First, it can be shown that similar qualitative results (that seniors hire juniors to create a reputational concern to overcome a commitment problem) can be obtained with other joint production functions. More importantly, focusing on the additive case is intended to demonstrate that the effects identified in this paper are informational and not the result of a joint production technology which in itself is more or less efficient than solo production.

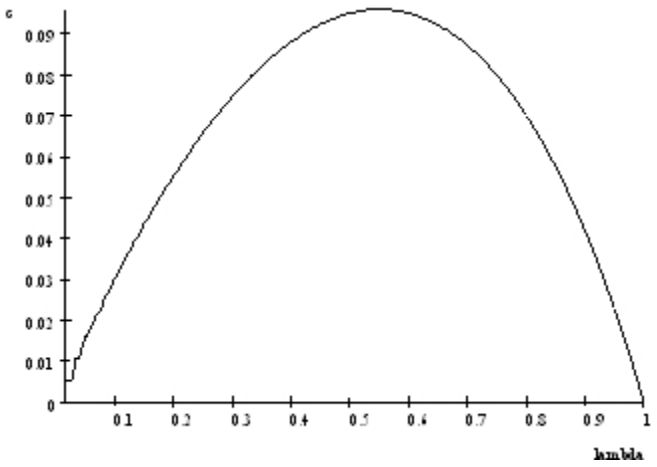

Figure 4: Equilibrium condition: c against lambda

This assumption that agents are either competent or inept, rather than competent or a Stackelberg type, means that over time uncertainty is resolved and an agent's type is learned. In the stark models considered here, this implies that an agent would have no reputational incentives.[30] Hiring juniors as a mechanism to introduce uncertainty and allow seniors to overcome this lack of incentives relies on a couple of assumptions. Specifically, that the senior's choice of actions can affect customers' information about the junior and that the senior cares about how customers will perceive the junior or equivalently that the senior is the residual claimant on at least some fraction of her co-worker's reputation in the next period. The former is implied by the much-noted and intuitive property of team production that it is difficult to attribute the specific contributions of individuals within teams (see particularly Holmstrom (1982)). The latter is addressed in this paper by an assumption that a junior cannot leave the employment of the senior with her reputation intact.

There are a number of reasons which can explain why the senior might be able to profit from a co-worker's future reputation. In the central model, we assumed that the senior controls access to the location where they work (for example in the context of professional

[30] In Holmstrom (1999) a "good" agent performs better even when exerting no effort and so a young agent may have some incentives to work but such incentives are diminished or disappear as the principal or customers learn the agent's type. We abstract from this effect by assuming that effort and ability are complimentary.

services it seems natural to think of this as access to clients or a non-compete clause in the junior's contract) and that agents build reputation only at the location where they work. Then if the co-worker leaves, she cannot take her reputation with her. This seems a reasonable assumption for law-firm associates for example and given the use of non-compete clauses is likely to apply to junior partners. Thus, although there are many locations where an agent might choose to work at the beginning of her career which are *ex-ante* identical, control of the location acts as a valuable asset *ex-post*.

Levin and Tadelis (2002) cite evidence on the prevalence of non-compete clauses in the US. In the UK Richard Turnor (2001), for example, states that most professional service firms employ restrictive covenants binding outgoing members and describes that:

> "Typical restrictive covenants include, for example:
>
> (a) a clause preventing the outgoing partner from acting for those who have been clients of the firm, or of the particular partner in a specified period (say two years) before his retirement, such restriction applying for a period of two, three or even five years after retirement;
>
> (b) a clause preventing the outgoing partner from practising at all for a specified period in a specified geographical area;
>
> (c) clauses preventing any activity within a specified period which involve the provision of services in a way which competes with the business of the firm"

However, the effectiveness of such restrictive covenants is imperfect, as discussed for example in Rebitzer and Taylor (2001). The qualitative results of this paper would apply even if such covenants were imperfect as long as they had some sufficient effect.

In this paper, the senior can act as the residual claimant on all of the junior's reputational gains and so the junior might have little incentive to exert effort for the sake of her reputation unless the senior can reward her for such effort. The senior cannot contract with a junior directly on the basis of the output produced as this is assumed to be non-verifiable (or else there would be no need for reputational incentives as explicit contractual incentives would suffice). To overcome this problem, we suppose that the senior can commit to a price at which she would sell the firm to the junior. The price is set in equilibrium at a level which would allow some returns to a successful junior providing the junior with incentives and at which only a successful junior would be willing to buy, ensuring that the senior has an incentive to exert effort and so give the junior an opportunity of succeeding. An equivalent formulation is to suppose that contracts can be made contingent on the task

to which the junior is assigned in the second period—that is a contract could allow for a different second period wage (a severance payoff) for a junior who is fired to one who is retained. As discussed by Kahn and Huberman (1988) and Prendergast (1993), promotion or task assignment can be effective in providing incentives when task assignment is at the discretion of the employer only when the expected marginal productivity of the employee differs in the different jobs.[31] In particular when this property holds, then task assignment can act as a quasi-contingent contract—an aspect which is important for the equilibrium characterized.[32]

5 Conclusions

This paper highlights the pivotal role of uncertainty for robust reputational incentives and that an agent can strategically introduce such uncertainty without damaging her existing reputation. We suggest that hiring and working with a junior with uncertain reputation as a strategic choice can endogenously introduce the uncertainty required for reputational incentives and in this way allow an agent to commit to exerting efficient though costly effort. In particular, it is perhaps worth re-iterating that whereas typically teamwork is thought to reduce individuals' incentives and lead to a free-rider problem, here if only solo production were possible (and even if this were more efficient in a complete contracting world) there would be no incentives for effort. The crucial aspects in establishing this result are that a senior and junior can work together in a non-attributable production process and that the senior can gain from the future reputation of the junior. Incentives for the junior are provided by committing to a price at which the junior can buy the firm or equivalently to an up-or-out promotion scheme in which for the promoted agent to have incentives to exert effort, promotion must be to a position where the promoted employee lays claim to future revenue generated by her own junior. This has a natural interpretation as promotion to partnership.

In the context of professional services, the inspiration for this paper, the conclusion that agents are motivated by a concern for their own reputations when young, and when old (and successful) for the reputation of the firm that they own does not seem unreasonable.

[31] This would be the case here—a successful agent is worth more to the senior in the promoted position and by committing to the wage in this position, the senior can commit to reward the junior for success.

[32] Alternative treatments of up-or-out promotion rules include Milton Harris and Yoram Weiss (1984), which generate up-or-out rules based on learning with finite lifetime and risk aversion, H. Lorne Carmichael (1988) which introduces a model of tenure built on the insight that current workers are best informed to select new workers, and Brendan O'Flaherty and Aloysius Siow (1992) and (1995), which are based on the scarcity of junior slots used to assess suitability for a senior slot.

An important aspect in this paper is that firms or teams are small—often a reasonable assumption in such industries—and so other models which have considered the interaction of individual and collective reputations and assumed that firms either consist of a single individual or very many individuals, are unable to consider such as a result.[33] Other instances where agents may be choosing to endogenously introduce uncertainty or choose to allow the reputations of a small number of agents or good to interact might include a firm's turnover policy (hiring and firing at a more aggregate level) and some instances of product bundling.

This paper has suggested direct ownership (in Section 3.3) or ex-post control of location, perhaps through restrictive covenants (in Section 3.4), as a means by which a senior can gain from the reputation of the machine or a junior.[34] Another alternative might be to suppose that the agent is exploiting uncertainty about another dimension of her own capability; as a specific example suppose that there is certainty that the agent is competent but uncertainty about her ability to select good candidates, then without recourse to directly gaining a benefit from the revenues that the good reputation of a junior can generate, the senior would have a reputational concern but over a different dimension of her ability.

Other aspects of the model which might be developed or extended are the restrictions to a binary effort choice and two types of agent. Relaxing these assumptions may yield interesting results. In particular it is not hard to imagine that in a more sophisticated model seniors would differ according to the history of their previous employer (and their employer's previous employer and so on). This would suggest that a firm's age is important and further, that working for a different type of senior (or a firm of different age) would

[33] Note that even within large law firms, individual departments or groups which are typically small have their own reputations and compensation even of partners is often tied to the performance of the individual departments.

As suggestive evidence on the first of these point, it is worth noting that law firm directories will rank a firm with respect to different specialization (and a typical specialization in a law firm will have a fairly small number of partners). In another industry, the observation that in investment banking often one sees small teams moving as teams is also revealing.

Further on compensation being determined at a fairly decentralized level. James D. Cotterman (2001), for example, discussing law firms states that:

> Historically a prominent compensation method, the lockstep approach is now the least preferred way of allocating compensation.

Cotterman argues that performance-based measures and to some extent compensation based on the revenue that a partner has generated ("eat what you kill") are important features of partners' compensation.

[34] Another possibility is explicit bonding—a junior buys a share in a partnership which is returned only on retirement.

entail a different employment contract. Another aspect which might be profitably relaxed is the form of the joint production function, the choice of an additive production function in which joint production is the sum of individual contributions is useful in highlighting the purely informational role in obscuring those individual contributions, but introducing some complementarity in joint production would perhaps be more realistic and could enrich the model.

Appendices

A Agents know their own types at birth

In this section, we adapt the model presented in Section 3 and suppose that agents know their own types at birth. Analogous to the equilibrium described in there, we construct an equilibrium in which competent agents exert effort. Specifically, we distinguish between a competent and an inept agent. In equilibrium the following occurs:

In the first period of life, a competent agent either founds her own firm, working on her own in an unoccupied location or accepts a position as an employee if offered one at sufficiently attractive terms; in either case she exerts effort in the first period of her own life. In equilibrium in either case she will succeed (since it is assumed that a competent agent who exerts effort succeeds). If she had founded her own firm, then in the second period she hires a junior (offering a wage contract that pays w and offers the firm at P) and they work together with the founder exerting effort.

Alternatively she might begin life as an employee in an established firm with a (w, P) contract, so long as such a contract offers her in equilibrium at least as much lifetime earnings as founding her own firm. She exerts effort in the first period of life. In equilibrium, the firm produces two high quality outputs and she buys the firm at the specified price P, hires her own junior offering her junior the contract (w, P) and she works together with her junior and exerts effort.

An inept agent might begin life either as an employee or founding her own firm. In either case she exerts no effort, fails and poses as a new-born in the following period, accepting employment as an employee or founding her own firm.

It is clear that an inept agent, knowing that she is inept, would never exert effort (it is costly but otherwise does not affect outcomes) and would be willing to work as an employee so long as:

$$w \geq \mu \tag{18}$$

For the competent agent, following the strategy outlined above yields:

$$CV_f = \mu - c + (1 + \mu - c - w + \mu P). \tag{19}$$

when starting life by founding her own firm, and

$$CV_e = w - c + (1 + \mu - c - w - P + \mu P). \tag{20}$$

when starting as an employee. Note that in contrast to the expressions in (6) and (7) the competent agent, knowing that she is competent knows that she will succeed.

In equilibrium, a new born agent is willing to become a junior or equivalently $CV_e \geq CV_f$. In particular, this implies that:

$$w - \mu - P \geq 0. \tag{21}$$

Given that there is a scarcity of junior slots available, hiring seniors will drive down the value that the contracts deliver to employees to the point where they are indifferent, in particular this implies:

$$w = \mu + P. \tag{22}$$

The remaining deviations can be categorized into a number of separate groups (i) exert effort (incentive compatibility) (ii) hiring policy and (iii) buying the firm. Each group is considered in turn.

(i) exert effort (incentive compatibility)

For a second period agent who had been successful there is an incentive compatibility constraint—which is identical in both the case that she worked alone in the first period and the case when she is now a manager—specifically, this constraint is:

$$\mu P \geq c. \tag{23}$$

In the first period in both cases, that the agent works alone or as a junior, it must be worthwhile to exert effort. The corresponding incentive compatibility conditions are the following:

$$1 + \mu - c - w + \mu P \geq c \tag{24}$$

and

$$1 + \mu - c - w - P + \mu P \geq c \tag{25}$$

Note that by (23), it follows that $P \geq 0$ and so (24) implies (25).

Trivially an inept agent does not exert effort

(ii) hiring policy

Without hiring in the second period of life, the agent could not commit to effort and so the best that she could do is pose as a new-born (and earn $\mu w + (1-\mu)\mu$) and so the conditions that a second period agent who had success when working alone in the first period does indeed prefer to hire a junior is given by:

$$\mu + 1 - c - w + \mu P \geq \mu w + (1-\mu)\mu \tag{26}$$

The same condition ensures that a second period agent who had been an employee and bought the firm, prefers to hire.

Suppose that an agent failed in the first period of life, then she must be inept and a single success would prove that her junior was competent and so a sufficient condition that would ensure that she does not hire is given by:

$$\mu w + (1-\mu)\mu \geq -w + \mu P. \tag{27}$$

Finally, since customers hold the off equilibrium belief that any apparent new born attempting to hire must be a failed second period agent. The conditions that ensure no second period agent posing as a new born hires is also given by (27). A true new born agent, whether competent or inept, would not hire when this condition holds.

(iii) Buying the firm

An employee who had succeeded would indeed buy the firm so long as this generates more value than her alternative—posing as a new born. This is the case when:

$$1 + \mu - c - w - P + \mu P \geq \mu w + (1-\mu)\mu \tag{28}$$

An employee who had failed is revealed as inept, and so rather than spending $P > 0$ to remain in this location, she would rather costlessly move to another location and pose as a new born agent where she would have a higher reputation. Note that (28) and (23) imply (26).

Thus sufficient conditions for the strategies to form an equilibrium are (22), (23), (24), (27), and (28). Substituting for $w = \mu + P$ from (22), and $\mu = \frac{\lambda}{2-\lambda}$, these inequalities reduce to $\min\{\frac{(2-\lambda)(1-c)-\lambda}{2(2-\lambda)}, \frac{(1-2c)(2-\lambda)}{2(1-\lambda)}\} \geq P \geq \frac{c(2-\lambda)}{\lambda}$, with $(c,\lambda) \in [0,1] \times [0,1]$. A sufficient condition is:

$$(2-\lambda)((1+c)\lambda - 4c) - \lambda^2 \geq 0 \tag{29}$$

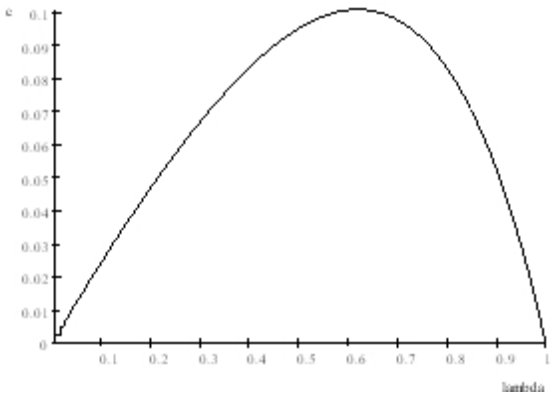

Figure 5: Sufficient condition for equilibrium (know own type): c against lambda

This proves the following Proposition:

Proposition 3 *If (29) holds then there is a Perfect Bayesian Equilibrium in which competent agents exert effort in both periods of their careers.*

Condition (29) is illustrated in the Figure 5 , where combinations of (c, λ) below the line satisfy the condition.

B Case with $g < 1$, $b > 0$ and age observable

In this section, we alter the framework and suppose that a competent agent exerting effort succeeds with a probability $g < 1$ and that a competent agent who exerts no effort succeeds with probability $b > 0$, this is also the probability of success for an inept agent. We assume $g > b$. Further, it is convenient to assume that in addition to the options of working alone or working in a team, agents have the option of leaving the industry and claiming a (per-period) outside option of R.

Proposition 4 *There are parameter values and strategies which allow a Perfect Bayesian Equilibrium in which all agents exert effort in the first period of life, and agents who produce high quality when working alone in the first period or work in a firm, that produces two high quality products, as a junior exert effort in period 2 of their lives.*

In the first period of life an agent either founds her own firm, working on her own in an unoccupied location or as a junior for an agent currently in the second period of life; in

either case she exerts effort in the first period of her own life. If she founds her own firm and fails in the first period, then she leaves the industry and claims her outside option R. If she succeeds in the first period of life after founding her own firm, then in the second period she hires a junior (offering a wage contract that pays w_1 and w_2^m following a promotion) and they work together with the founder exerting effort. In the final ("retirement") period, she promotes a junior to manager (and pays the manager's fee w_2^m) if the firm as a whole produced two high quality units in the previous period and otherwise fires the junior.

Alternatively an agent might begin life as an employee in an established firm with a (w_1, w_2^m) contract. She exerts effort in the first period of life. If the firm as a whole produced one or no high quality outputs, she is fired and leaves the industry, earning her outside option R. If the firm produced two high quality outputs then she will be retained as manager and paid w_2^m, in which case she must hire her own junior offering her junior the contract (w_1, w_2^m); in this case she works together with her junior and exerts effort and at the end of the period gains control of the firm. In the final period, she promotes her junior to manager if the firm as a whole produced to high quality units in the pervious period and otherwise fires the junior.

The public and all agents believe that these are the equilibrium strategies and prices are set appropriately, with the industry capturing the full consumer surplus. Thus the price of the output of a new-born agent working alone is $p(\lambda) := \lambda g + (1-\lambda)b$ since there is a probability λ that she is competent (and in equilibrium she is expected to exert effort and generate high quality output with probability g) and otherwise inept. To simplify notation, let the expected probability of success from a new-born agent when exerting effort be $n = p(\lambda)$; let $\lambda^s = \frac{\lambda g}{p}$ be the belief that an agent seen to have had a success is competent and let $s = p(\lambda^s)$ be the probability that an agent who had previously had a success has another success when exerting effort. With this notation, the behaviour along possible equilibrium paths, as described in the paragraph above, is illustrated in the figure below.

To complete the characterization of the equilibrium, one must also specify beliefs concerning off-equilibrium actions:

- if a senior and junior were both successful in the previous period and the senior did not promote the junior to manager but instead sought to retain her as a lone worker, then the un-promoted junior would have no concern for the future and so would not

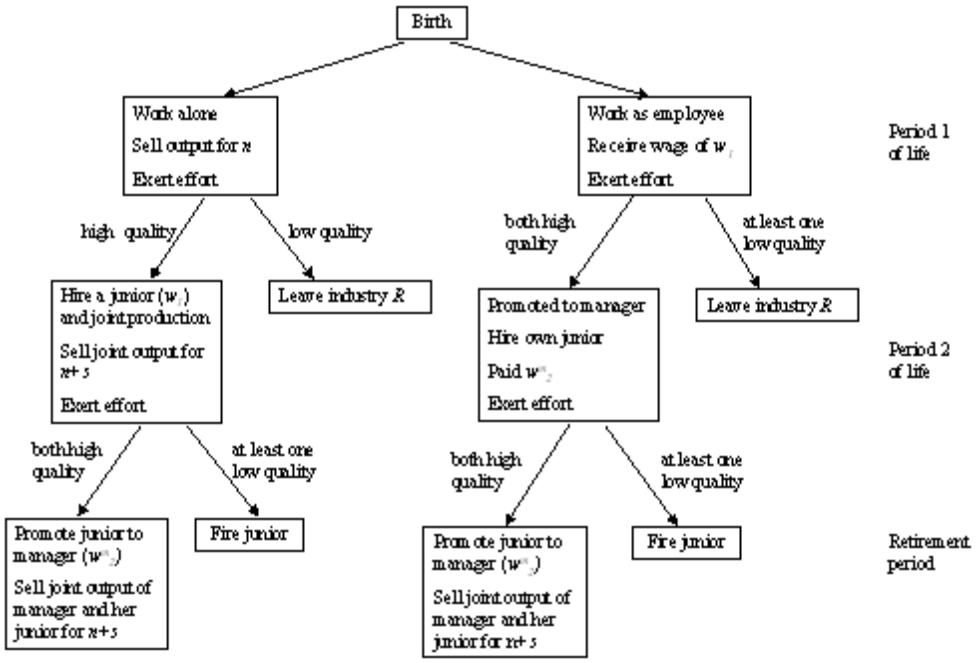

Figure 6: Behaviour on possible equilibrium paths

exert effort and the revenue generated would be b. We impose that in this case the senior would have to pay the un-promoted junior at least her outside option R and characterize this condition explicitly below, calling it the "honest senior" condition (43);

- if the senior hires a junior as manager following failure, we assume in this section that the newly promoted manager will not exert effort;
- when an agent who had been successful does not hire, again there would be no possibility of retirement period revenues or wages and so the agent would not exert effort in the current period, as there are no future revenues for such effort to affect; thus this deviation is equivalent to the second period individual rationality conditions discussed below;
- when an agent who failed does hire, then we assume that the off-equilibrium beliefs are that this senior will not exert effort;
- if an agent does not offer the contract (w_1, w_2^m) then any potential juniors who will

not have seen her history assume that she has previously failed; and finally,

- if an agent who is not expected to hire does so, she must offer the contract (w_1, w_2^m) but she is not expected to exert effort by customers and so does not raise the same revenues that a successful agent would; however, in this case customers' updating on the basis of output will be different and specifically one high quality output may be sufficient to win the junior a promotion—such a deviation is considered explicitly below in Equation (41) and a new born agent deviating by seeking to hire in the first period is addressed by (42).

Note that the expected equilibrium value of beginning life by founding a firm (starting solo) is:

$$V_s = n - c + n[s + n - c - w_1 + sn(s + n - w_2^m)] + (1 - n)R \tag{30}$$

This expression is derived as follows. In the first period the agent sells her service at a price n and exerts effort; if she fails, which occurs with probability $(1 - n)$, then in the second period she leaves the industry and so in the second period earns R. If she succeeds in the first period (which occurs with probability n) then it is known that she must be competent with probability λ^s, she hires a junior and sells their joint output at its expected value $s + n$, given that she exerts effort as does a new born agent, and she must also pay the junior w_1. Finally in the third period, if a senior and her junior succeeded in the second period, which occurs with probability sn, then again it must be that the junior is competent with probability λ^s and so when promoted to manager generates a revenue of $s + n$ (which is the expected value of the manager and her junior's output) but must be paid w_2^m. (We assume that the manager hires and pays her own junior but her wage is contingent on doing so). If a senior and junior fail in the second period, then in the third period the junior is fired, and the senior gains no revenue.

Similarly, the expected value for a new born agent who begins by working as a junior in joint production is given by the following expression.

$$V_j = w_1 - c + sn[w_2^m - c - w_1 + sn(s + n - w_2^m)] + (1 - sn)R \tag{31}$$

The proof proceeds by construction; we characterize sufficient conditions for the parameters which ensure that the strategies described are indeed equilibrium strategies and verify that there are parameter values which satisfy all the relevant constraints.

First, we suppose that a junior refuses any contract other than (w_1, w_2^m). In addition it must be that the a new agent is willing to become a junior or equivalently $V_j \geq V_f$ and given the scarcity of such opportunities, the employer would be in a position to ensure that $V_j = V_s$

$$V_j = V_s \ . \tag{32}$$

The remaining constraints fall into a number of categories. Specifically, these categories can be summarized under the headings (i) exit or remain in industry (individual rationality) (ii) exert effort (incentive compatibility) (iii) hiring policy and (iv) promotion policy.

The individual rationality constraints ensure that an agent prefers to stick to the strategy described to leaving the industry, either at the beginning of life or later when the strategy requires staying (for example following a high quality output when working alone in the first period). Individual rationality also requires that when the strategy requires that an agent leaves the industry, she prefers to do so than to remain within it. The first and second period incentive compatibility conditions ensure that an agent prefers to exert effort when the strategy requires her to do so.

The conditions on hiring policy are first that an agent hires in period 2 when in period 1 she had been in a "successful" firm, that is one that produced two high quality goods if she was there as a junior or where she produced one high quality good in a firm that she had founded on her own. In addition, we characterize sufficient conditions that ensure that an agent who had not been in a successful firm in period 1 does not hire in period 2, and that no agent hires in period 1.

Finally, on promotion policy the value of promoting a junior to manager in a successful firm must be greater than either the value of retaining the junior or firing her and a manager must prefer to fire a junior in an unsuccessful than retain or promote her.

Proof. The remaining deviations are categorized into the following groups (i) exit or remain in industry (individual rationality) (ii) exert effort (incentive compatibility) (iii) hiring policy and (iv) promotion policy. Each group is considered in turn and then we verify that there are indeed parameters at which all the requisite constraints are satisfied.

(i) exit or remain in industry (individual rationality)

It must be the case that pursuing these strategies from birth is preferred to exercising the outside option:

$$V_s \geq 2R \tag{33}$$

and $V_j \geq 2R$ or equivalently

$$w_1 - c + sn[w_2^m - c - w_1 + sn(s + n - w_2^m) - R] - R \geq 0 \ . \tag{34}$$

Note that (32) makes one of (34) and (33) redundant.

For a second period agent, the corresponding individual rationality constraints (the constraint that she wishes to remain in the industry and stick with the equilibrium strategies) depend on whether the agent worked alone in a firm she founded in her first period of life or was successful as a junior in a team that produced two high quality outputs. The conditions are given respectively by:

$$s + n - c - w_1 + sn(s + n - w_2^m) \geq R \ , \tag{35}$$

$$w_2^m - c - w_1 + sn(s + n - w_2^m) \geq R \ . \tag{36}$$

Note that in (36) it is crucial that the agent cannot leave with her reputation intact—this follows from the assumption of informationally separate locations and that control of locations is determined by seniority.[35]

For an agent who failed when working alone or as a junior, she prefers to leave the industry so long as

$$R \geq b. \tag{37}$$

Note that an agent who failed would be unable to hire a junior (see (41) below) and so with only one period of working life remaining and no reputational concerns would be unable to commit to exerting effort and so would generate only b in revenue if she remained in the industry.

(ii) exert effort (incentive compatibility)

For a second period agent who had been successful there is an incentive compatibility constraint—which is identical in both the case that she worked alone in the first period and the case when she is now a manager—specifically, this constraint is as follows:

$$s + n - w_2^m - \frac{c}{sn} \geq 0 \ . \tag{38}$$

[35] In the context of partnerships, restrictive covenants or bonding might be more plausible constraints which would allow a senior to claim at least some of the revenue attributable to a junior who succeeded, such mechanisms would operate in an informationally unified market.

In the first period in both cases, that the agent work as a founder or as a junior, it must be worthwhile to exert effort. The corresponding incentive compatibility conditions are the following:

$$s + n - c - w_1 + sn(s + n - w_2^m) - R - \frac{c}{n} \geq 0 \ , \tag{39}$$

and

$$w_2^m - c - w_1 + sn(s + n - w_2^m) - R - \frac{c}{sn} \geq 0 \ . \tag{40}$$

Note that these two equations imply (35) and (36) respectively and since $-\frac{c}{n} \geq -\frac{c}{sn}$ and by (38) $s + n \geq w_2^m$ it follows that (40) and (38) imply (39).

(iii) hiring policy

Without hiring in the second period of life, the agent could not commit to effort and so the conditions that a second period agent who had success when working alone in the first period does indeed prefer to hire a junior is given by:

$$s + n - c - w_1 + sn(s + n - w_2^m) \geq R$$

this is precisely (35) and by (37) which states $R > b$, this is the most profitable deviation (that is it is more profitable to leave the industry if not hiring rather than remaining in the industry and earning b). Similarly the corresponding condition that a manager prefers to hire is implied by (36) and (37).

Suppose that an agent failed in the first period of life, imposing the off-equilibrium belief that if she were to hire she would exert no effort, then a sufficient condition that would ensure that she does not hire is given by:

$$R \geq b + n - w_1 + (b + n)(s + n - w_2^m). \tag{41}$$

This condition is sufficient to ensure that such an agent does not hire but is not necessary. Given that the senior does not exert effort the first period revenue is $b + n - w_1$; if two successes ensue then the public believe that the junior is competent with probability λ^s and if observing one success this belief would be

$$\frac{\lambda g(1-b) + \lambda(1-g)b}{\lambda g(1-b) + \lambda(1-g)b + 2(1-\lambda)b(1-b)} < \lambda^s.$$

Consider a the value earned by an agent (who had previously failed) and hires in the second period of her life, when customers are over-optimistic in the sense that they believed that a junior who worked for such a senior was competent with probability λ^s when between them the senior and junior managed at least one success. This is the right hand side of (41) and since the condition states that even under these unrealistically favorable conditions an agent who had previously failed would still not want hire, this is a sufficient condition to ensure that such an agent does not hire. Note that in (41) implicitly it is assumed that customers in this off-equilibrium path believe that the senior would exert no effort. We have discussed this this off-equilibrium belief above.

Finally, the condition that a new born agent prefers not to hire is given by:

$$V_{joint} = V_{solo} \geq 2n - w_1 - c + n^2[s + n - c - w_1 + s + n - w_2^m + sn(s + n - w_2^m)] + (1 - n^2)R \ . \quad (42)$$

In this last expression we suppose that the new born agent exerts effort and that following one success the updated belief $\frac{1}{2}(\lambda^s + \frac{\lambda(1-g)}{\lambda(1-g)+(1-\lambda)(1-b)})$ is relatively small so that an agent with such a reputation would prefer to exercise her outside option.

(iv) Promotion policy

First, a senior must be honest, that is she must prefer to promote to manager a junior who had been successful. This conditions is given by:[36]

$$s + n - w_2^m \geq R \ . \quad (43)$$

In addition for the equilibrium characterized a senior in a firm with either one or two failures should not want to promote. This might be the case for example if the off-equilibrium beliefs in this case were that the promoted senior would exert no effort—then the relevant condition no promotion condition would be:

$$0 \geq -w_2^m + b + n \quad (44)$$

Verification

Thus sufficient conditions for an equilibrium are (32), (33), (37), (38), (40), (41), (42), (43) and (44). It can be easily verified that there are indeed parameter values and corre-

[36] Note that if the senior attempted to retain the junior without promoting her, then the retained junior would raise no more than b in revenue which is less than her outside option by (37), which is the very least that she must be paid. Thus the senior would prefer to fire the junior rather than retain her without promotion.

sponding wages (w_1, w_2^m) for which these conditions are satisfied. For example they are satisfied when $\lambda = 0.45$, $g = 0.9$, $b = R = 0.1$, $c = 0.05$ and $w_1 = 0.565$ and $w_2^m = 1.124$. ■

C A finite horizon up-or-out effort inducing equilibrium

First a three period career concern model is introduced in which reputational considerations lead to effort in the first period but not in the second of life for an agent working alone. We characterize restrictions which ensure that this is the case. Next it is shown that an agent who had been successful in the first period of life can commit to exerting effort in the second period of her life by hiring a new born agent of uncertain quality, even though that agent will have no opportunity to hire her own agent and will only exert effort in the first period of her own life.

It is supposed that there are two types of agent. A good agent and a bad agent. A good agent differs from a bad agent in two ways. First she is more likely to be successful than a bad agent even when she exerts no effort and second, effort has a greater impact on her chances of generating success. Specifically suppose that a bad agent always fails whether exerting effort or not and a good agent succeeds with probability g with no effort and with probability $g + e$ when exerting effort. Exerting effort costs c. Initially the agent and labour market share the prior that the agent is good with probability λ.

Note that following any successes, all agents know that the agent must be a good type. Inferences following failure depend on whether or not it is believed that the agent exerted effort or not. Specifically let $\lambda^f(n)$ denote the belief that the agent is good given one failure when it is believed that the agent exerted no effort; $\lambda^{ff}(e, n)$ for the belief that the agent is good given two observed failures when it is believed that the agent exerted effort in the first period but not in the second; other beliefs are denoted similarly and can be written explicitly as follows

$\lambda^f(n) = \frac{\lambda(1-g)}{\lambda(1-g)+1-\lambda} = \frac{\lambda(1-g)}{1-\lambda g}$
$\lambda^f(e) = \frac{\lambda(1-g-e)}{1-\lambda g-\lambda e}$
$\lambda^{ff}(n,n) = \frac{\lambda(1-g)^2}{\lambda(1-g)^2+1-\lambda}$
$\lambda^{ff}(e,n) = \lambda^{ff}(n,e) = \frac{\lambda(1-g)(1-g-e)}{\lambda(1-g)(1-g-e)+1-\lambda}$
$\lambda^{ff}(e,e) = \frac{\lambda(1-g-e)^2}{\lambda(1-g-e)^2+1-\lambda}$

In addition, suppose there is an outside option R which can be invoked in any period.

First, we characterize conditions which ensure that when working alone an agent would

exert effort in the first period but not in the second and would stay in the industry following a first period success but leave it otherwise.

First the condition that ensures that she would stay following success in the first period is:

$$g \geq R. \tag{45}$$

A condition which ensures that the she would leave the industry following two failures is that:

$$R \geq \lambda^{ff}(n,n)g. \tag{46}$$

Note that $\lambda^{ff}(n,n) > \lambda^{ff}(n,e) = \lambda^{ff}(e,n) \geq \lambda^{ff}(e,e)$ and so (46) is sufficient to ensure that the agent leaves the industry following two failures whatever the beliefs about her having exerted effort in the past.

Conditions which ensure that the agent leaves the industry following one failure and claims the outside option whether exerting effort or not are given respectively by:[37]

$$2R \geq \lambda^{f}(n)(g+e)(1+g) - c + (1 - \lambda^{f}(n)(g+e))R, \text{ and} \tag{47}$$

$$2R \geq \lambda^{f}(n)g + \lambda^{f}(n)g^2 + (1 - \lambda^{f}(n)g)R. \tag{48}$$

The condition which ensures that the agent prefers to stay in the industry and exert effort in the first period to leaving the industry immediately is

$$\lambda(g+e)(1+2g) - c + (1 - \lambda(g+e))2R \geq 3R. \tag{49}$$

Finally the condition that states that she prefers to stay in the industry and exert effort in the first period to deviating and not exerting effort (even though effort is anticipated) is as follows:

$$\lambda(g+e)(1+2g) - c + (1 - \lambda(g+e))2R > \lambda(g+e) + 2\lambda g^2 + (1 - \lambda g)2R \tag{50}$$

Thus inequalities (46)-(50) characterize parameters for which it is a PBE that an agent

[37] Note that since $\lambda^{f}(n) > \lambda^{f}(e)$ and $g > R$ these conditions (which state that following one failure the agent leaves the industry given that she knows that she exerted no effort in the first period), it follows that the conditions also ensure that the agent leaves the industry following a failure when it she exerted effort in the first period.

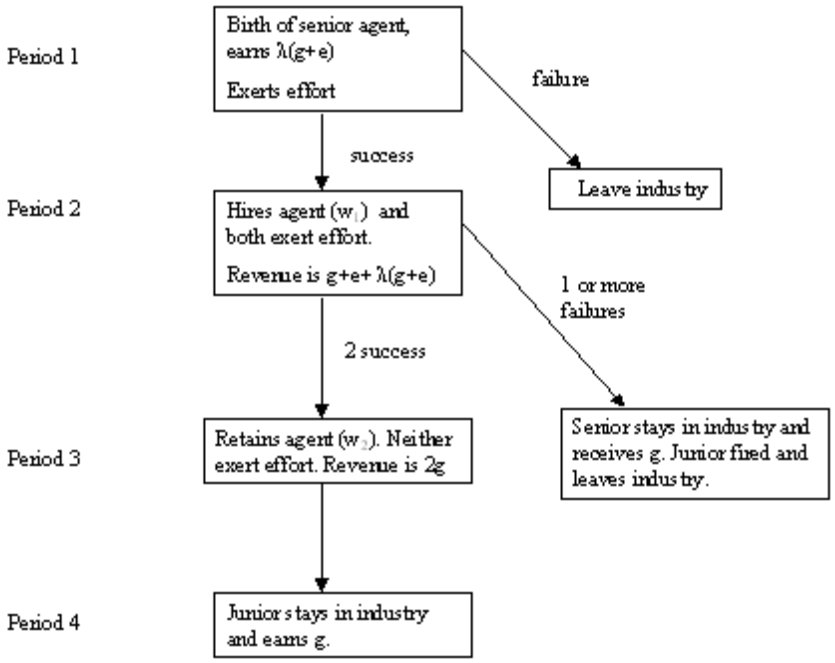

Figure 7: Finite model: equilibrium path behaviour

exerts effort in the first period of life and then stays in the industry following a success and leaves the industry following a failure.

Now suppose that the agent in the second period of her life has the opportunity to hire a new born agent though this agent cannot then go on to hire her own agent and show that this can be an equilibrium. The possible outcomes in this situation are summarized in the figure below.

An agent who works alone throughout life, supposing inequalities (46)-(50) hold, would earn

$$S = \lambda(g+e)(1+2g) - c + (1-\lambda(g+e))2R. \tag{51}$$

Following a first period success the continuation value when working alone is $2g$. The continuation value if hiring with the contract (w_1, w_2) is instead

$H = 2g + e - c + \lambda(g+e) - w_1 + \lambda(g+e)^2(g+e-w_2)$. Thus the senior will hire in her second period following success so long as

$$2g + e - c + \lambda(g + e) - w_1 + \lambda(g + e)^2(g + e - w_2) \geq 2g \tag{52}$$

She will not deviate by not exerting effort so long as the continuation value by hiring and exerting effort H is greater than the value by deviating and not exerting effort which is $2g + e + \lambda(g + e) - w_1 + \lambda(g + e)g(g + e - w_2)$. Thus the senior will not deviate by not exerting effort so long as this value is less than or equal to H, or equivalently:

$$e(g + e - w_2)\lambda(g + e) \geq c. \tag{53}$$

She behaves honestly in retaining a junior following two success so long as:

$$g \geq w_2. \tag{54}$$

Finally the junior agent must be willing to be hired and to exert effort, and stay following a success. This leads to three further conditions, specifically the value of being hired should be no less than the value to working alone:

$$w_1 - c + \lambda(g + e)^2(w_2 + g) + (1 - \lambda(g + e)^2)R \geq S, \tag{55}$$

and the junior should be willing to exert effort in the first period of her life (corresponding to period 2 of the time path) so that the left hand side of (55) must be greater than or equal to $w_1 + \lambda(g + e)g(w_2 + g) + (1 - \lambda(g + e)g)R$, or equivalently

$$\lambda(g + e)e(w_2 + g - R) \geq c. \tag{56}$$

The last condition is that following a success, the junior should prefer to stay in the firm to leaving the firm (and so also the industry) that is:

$$w_2 + g \geq 2R \tag{57}$$

Finally, having characterized the relevant inequalities, it remains to show that there are indeed parameter values for which all the relevant inequalities hold. It can be easily verified that (46)-(50) and (52)-(57) are all satisfied, for example, when $R = 0.6$, $g = 0.5$, $e = 0.3$, $c = 0.01$, $\lambda = 0.8$, $w_1 = 1$, and $w_2 = 0.55$.

D $g < 1$ and $b > 0$ age non observable

Again, as in Section 3, we suppose that agents do not know their own types at birth but, in contrast to the framework considered in Section 3, it is supposed that $g < 1$ and $b > 0$. We characterize conditions for an up-or-out effort inducing equilibrium in which both new born agents and successful agents in the second period of life exert effort. The equilibrium follows the structure characterized in Section 3 but in which agents who fail (either on their own or as part of a firm that suffered at least one failure) pose as new born agents, are willing to work as juniors when given the opportunity and exert no effort. For convenience, take $b = 0$.

The value of a new born who starts life working alone is given by:

$$HV_{solo} = \mu g - c + \lambda g(\mu g + g - c - w_1 + \mu g^2(g + \mu g - w_2^m)) + (1 - \lambda g)\mu g. \tag{58}$$

Note that this expression is built up as follows. On observing an agent who appears to be new-born working alone, customers believe that she is a competent new born agent with probability μ, we characterize μ below but note that $\mu < \lambda$ since among the pool of those agents who appear to be new born there will be some second-period agents who are hiding their histories. The new born agent, who exerts effort and knows that she is new born expects success with probability λg and in this case she hires a another agent as a junior (note that this may be a failed agent pretending to be new born and so the probability that junior will be new born and competent is μg) and so can charge $\mu g + g$ for the joint service and in case of further success she promotes the junior, paying her the manager's fee and receiving the appropriate revenue.[38] Following a failure in the first period, the agent can pose as a new-born agent and so receive the revenue μg.

The value for a new born agent who works as an employee is given by the following expression:

$$HV_{joint} = w_1 - c + \lambda g^2(w_2^m - c - w_1 + \mu g^2(g + \mu g - w_2^m)) + (1 - \lambda g^2)\mu g. \tag{59}$$

As before we impose:

[38] If there are two successes and the junior was a failed agent, she can not claim the promotion, but retires.

$$HV_{solo} = HV_{joint} \tag{60}$$

In addition to the conditions which are similar to those considered in Proposition 2. There will be new conditions to sustain an equilibrium, in particular, that a second period agent who is hiding her history prefers to work alone rather than be hired as a junior. Specifically, this condition is given by:

$$w_1 \geq \mu g \tag{61}$$

Say that there is a stable proportion of agents α in each generation who succeed in the first period of life hire new-borns is given by α. Then $\mu = \frac{\lambda}{2-\alpha}$ and $\alpha = \mu g(2-2\alpha) + \alpha\mu g^2$, so $\alpha = \frac{\lambda}{2-\alpha} g(2-2\alpha) + \alpha \frac{\lambda}{2-\alpha} g^2$. First note that

$$\left(\lambda^2 g^4 - 4\lambda^2 g^3 - 4\lambda g^2 + 4\lambda^2 g^2 + 4\right) = 4(1-\lambda g^2) + 4\lambda^2 g^2 (1-g) + \lambda^2 g^4 > 0$$

Secondly note that there is a sensible solution only when $0 \leq \alpha \leq 1$.

Write $f(\alpha) = \alpha\lambda g^2 + (2-2\alpha)\lambda g - \alpha(2-\alpha)$. Then $f(0) = 2\lambda g > 0$ and $f(1) = \lambda g^2 - 1 < 0$

so there must be at least one solution for the quadratic equation $f(\alpha) = 0$ in the range, the question is, is there exactly one and which one?

Well consider

$\lambda g(1 - \frac{g}{2}) + 1 + \frac{1}{2}\sqrt{\left(\lambda^2 g^4 - 4\lambda^2 g^3 - 4\lambda g^2 + 4\lambda^2 g^2 + 4\right)}$

This is > 1 so the root in the range $(0, 1)$ must be the negative root. That is the equation has exactly one root in the range $[0, 1]$ and so

$$\alpha = 1 + \lambda g - \frac{1}{2}\lambda g^2 - \frac{1}{2}\sqrt{\left(4 - 4\lambda g^2 + 4\lambda^2 g^2 - 4\lambda^2 g^3 + \lambda^2 g^4\right)}.$$

Corresponding to Proposition 11, the remaining constraints can be categorized in groups as follows.

(i) work alone

Now there is no opportunity to leave the industry, but an agent can always choose to work alone in both periods. It must be the case that pursuing the hypothesized up-or-out equilibrium strategies from birth is preferred to working alone:

$$V_{solo} \geq 2\mu g \tag{62}$$

and $V_{joint} \geq 2\mu g$. Note that (60) makes this latter condition redundant.

(ii) exert effort (incentive compatibility)

For a second period agent who had been successful there is an incentive compatibility constraint—which is identical in both the case that she worked alone in the first period and the case when she is now a manager—specifically, this constraint is as follows:

$$g + \mu g - w_2^m - \frac{c}{\mu g^2} \geq 0 \ . \tag{63}$$

In the first period in both cases, that the agent works alone or as a junior, it must be worthwhile to exert effort. The corresponding incentive compatibility conditions are the following:

$$g - c - w_1 + \mu g^2(g + \mu g - w_2^m) - \frac{c}{\lambda g} \geq 0, \tag{64}$$

and

$$w_2^m - c - w_1 + \mu g^2(g + \mu g - w_2^m) - \mu g - \frac{c}{\lambda g^2} \geq 0 \ . \tag{65}$$

(iii) hiring policy

Without hiring in the second period of life, the agent could not commit to effort and so the conditions that a second period agent who had success when working alone in the first period does indeed prefer to hire a junior is given by:

$$g - c - w_1 + \mu g^2(g + \mu g - w_2^m) \geq 0$$

which is implied by (64). Similarly the corresponding condition that a manager prefers to hire is implied by (65).

Suppose that an agent failed in the first period of life, imposing the off-equilibrium belief that if she were to hire she would exert no effort, then a sufficient condition that would ensure that she does not hire is given by:

$$0 \geq -w_1 + \mu g(g + \mu g - w_2^m). \tag{66}$$

Note that in this case, since $b = 0$, a single success demonstrates to customers that the junior must be competent.

Finally, the condition that a new born agent prefers not to hire is given by:

$$HV_{solo} \geq \lambda g+\mu g-w_1-c+\lambda\mu g^2[\mu g+g-c-w_1+\mu g+g-w_2^m+\mu g^2(g+\mu g-w_2^m)]+(1-\lambda\mu g^2)\mu g \ . \tag{67}$$

In this last expression we suppose that the new born agent exerts effort and that following one success the updated belief is relatively small so that an agent with such a reputation would prefer to hide her history and pose as a new born.

(iv) Promotion policy

First, a senior honest, that is she must prefer to promote to manager a junior who had been successful. This conditions is given by:

$$g + \mu g - w_2^m \geq 0 \ . \tag{68}$$

In addition for the equilibrium characterized a senior in a firm with either one or two failures should not want to promote. This might be the case for example if the off-equilibrium beliefs in this case were that the promoted senior would exert no effort—then the relevant condition no promotion condition would be:

$$0 \geq -w_2^m + \mu g \tag{69}$$

Verification

It can readily be verified that all the relevant conditions are satisfied when $\lambda = g = 0.9$, $c = 0.05$, $w_1 = 0.74$ and $w_2^m = 1.454$.

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