Optimal Time-Consistent Monetary Policy in a Phillips-Curve World*

Thomas F. Cooley
New York University and
University of Rochester

Vincenzo Quadrini
Duke University and CEPR

October 18, 1999

Abstract

In this paper we study the optimal and time-consistent policy in a model economy that integrates the modern theory of unemployment with a liquidity model of monetary transmission. When the economy is subject to aggregate productivity shocks the optimal monetary policy is pro-cyclical—it increases the growth rate of money after a positive productivity shock and decreases the growth rate of money after a negative technology shock—and the model generates the Phillips Curve feature of a positive correlation between inflation and employment. We also study the long-run properties of the optimal policy under full commitment and compare it to the time-consistent policy. We show that, under some conditions, the optimal policy with commitment induces a long-run inflation rate that is higher than the long-run inflation rate in absence of policy commitment (time-consistent policy). This is in contrast to many studies that have argued that the inability of the monetary authority to commit induces a higher inflation rate.

*We thank Jordi Gali, Stephen LeRoy, Allan Meltzer, and Fabio Canova for very helpful comments.

> *"Despite its disrepute within important academic and policymaking circles, the Phillips Curve persists in U.S. data. Simple econometric procedures detect it."*
>
> Thomas Sargent, 1998

Introduction

On September 29, 1998 the Federal Open Market Committee met to determine the course of monetary policy. All of the economic indicators available at the time suggested that the economy was in remarkably good shape; unemployment was very low, the inflation rate was close to zero, and the economy was growing steadily although, it was thought, not as fast as in the first quarter of the year. Surprisingly, given the available data, the FOMC decided to ease monetary policy by lowering the Federal Funds rate 25 basis points. The one concrete reason cited in support of this action was that credit seemed to be getting tight, as evidenced by the fact that commercial borrowers were experiencing increasing difficulty obtaining financing.[1] In their words "...an easing policy action at this point could provide added insurance against the risk of a further worsening in financial conditions and a related curtailment in the availability of credit to many borrowers." In the period between the September meeting and the November meeting the Fed took steps to lower the Federal Funds rate again. At the November meeting they faced much the same economic background, strong growth, low inflation, and very low unemployment, and yet they decided to ease rates again. In the fourth quarter of 1998 real GDP grew at a rate of 6.1 percent. Viewed in terms of the conventional wisdom about monetary policy rules, this sequence of decisions seems somewhat surprising. In this paper we argue that the Fed was doing exactly what the optimal and time-consistent monetary policy would recommend.

Although Alan Greenspan is regarded with near mystical reverence by the financial press, no one thinks that monetary policy is decided introspectively or capriciously. The search for formal rules that describe how policy has been conducted or should be conducted has been an important area of research. Thoughtful economists like John Taylor (1993, 1998) Lars Svensson (1997b, 1997a) and Clarida, Gali, & Gertler (1999), have proposed formal rules for monetary policy that are grounded in careful empirical research. The best

[1] This shortage of liquidity was thought to be due in part to the Russian bond default and the crisis faced by Long Term Capital Management.

known of these monetary policy rules, the Taylor rule, is one in which monetary policy responds to inflation or expected inflation and the gap between actual and potential output. If output is below potential output, the implication is that monetary policy should ease. If output is above potential output monetary policy should tighten. The conventional interpretation of this rule in the face of the facts that prevailed in the autumn of 1998 would hold that inflation and expected inflation were low (essentially zero) and that output was at or above the potential level.

The notion that monetary policy should respond to the output gap is based on the view that inflationary monetary policies have expansionary effects on real activity, at least in the short-run. This view derives from a robust empirical feature of post-war U.S. data, the positive correlation between inflation and employment (or output), as documented by Sargent (1998). We refer to this empirical observation as the Phillips curve relation. The view that inflationary monetary policies have expansionary effects on the economy, is also supported by recent empirical studies, which find significant liquidity effects of monetary policy shocks.[2] The idea that the objective of the monetary authority should be to smooth employment (or output) by expanding the stock of money during periods of low employment and reducing the stock of money during periods of high employment follows from these observations.

If the optimal policy was to smooth employment and if the monetary authority acts optimally, we should observe a negative correlation between monetary aggregates and employment, at least for those aggregates that are under the control of the monetary authority. Data from the post-war period, however, show that employment is positively correlated with all monetary aggregates, including the monetary base.[3] There are important issues of causality here but the simple correlations suggest that post-war U.S. monetary policy has been pro-cyclical rather than counter-cyclical. If so it may have reinforced business fluctuations rather than smoothed them.

Our goal in this paper is to characterize the optimal time-consistent monetary policy in a general equilibrium model where there is a direct link between monetary policy and employment. We will show that the observed properties of the post-war data are not inconsistent with the optimal and

[2] See, for example, Christiano, Eichenbaum, & Evans (1996), Hamilton (1997), Leeper, Sims, & Zha (1996).

[3] See Cooley & Hansen (1995) for a documentation of the main monetary facts in the U.S. economy.

time-consistent policy of a benevolent policy-maker. To be concrete, we show that, in an economy where inflationary monetary policy can have a positive impact on the real sector of the economy and business cycle fluctuations are driven by technology shocks, the optimal time-consistent policy increases the stock of money when employment is high and reduces the stock of money when employment is low. The intuition for this result is as follows: an increase in employment and output in the economy that results from productivity shocks increases the nominal interest rate. The increase in the interest rate generates inefficiencies that a benevolent policy-maker would like to eliminate. Because changes in the stock of money have liquidity effects, the way to prevent an interest rate increase is to expand the stock of money when shocks are good. Interpreted correctly, such a policy is consistent with the Taylor rule. If the economy experiences a positive technology shock, then output will be below the potential level associated with that shock unless more liquidity is supplied to the economy. While it is consistent with the Taylor rule correctly interpreted, it is inconsistent with the conventional wisdom about what the Taylor rule would imply.[4]

A key feature of the economy we study is that changes in the supply of money change the supply of loanable funds and thus, change nominal interest rates. These liquidity effects cause changes in real activity by lowering the cost of working capital for firms. The real side of the economy is one where search and matching frictions lead to equilibrium unemployment. Monetary policy interventions are decided on a period-by-period basis, and the monetary authority cannot credibly commit to long-run plans. This implies that the type of policies we analyze are time-consistent. We restrict the analysis to policies that are Markov-stationary, that is, policy rules that only depend on the current (physical) states of the economy. Despite this restriction, finding the optimal and time-consistent policy turns out to be a non-trivial task: it requires finding a fixed point in the space of policy functions. When an analytical characterization of these policies is not possible, we find them numerically. The solution method is a variation of the method used in Krusell, Quadrini, & Ríos-Rull (1996, 1997), for the solution of voting models.

[4]In a very different environment, Peter Ireland (1996) also finds that the optimal monetary policy is pro-cyclical, when business fluctuations are driven by technology shocks. However, the mechanism that generates this result in Ireland's model is different. There monetary policy has real effects because prices are sticky.

1 The economy

We describe here a monetary economy that is specifically designed to generate the liquidity effect of monetary interventions, that is a reduction in the nominal lending rate after a monetary expansion. The reduction in the cost of borrowing, in turn, leads to an expansion in the real sector of the economy. By designing the economy so that inflationary policies have expansionary effects, we capture the main idea behind the Phillips curve relation, that is, the idea that in the short run there is a trade-off between inflation and unemployment (a Phillips curve world), and this trade-off can be used for the design of monetary policy. However, rather than taking this relation as given, we derive it from a fully specified model. We take the view that any serious treatment of optimal monetary policy issues requires an explicit theoretical underpinning. The advantage of this modeling strategy is that it allows us to define the objective of the policy maker as maximizing the welfare of the agents in the economy (benevolent policy maker), rather than defining it as an arbitrary objective over inflation and unemployment.

1.1 The monetary authority and the intermediation sector

The total amount of households' financial assets is denoted by M. Part of these assets are used for transactions and the remaining quantity is held in the form of bank deposits. The funds collected by banks are then used to make loans to firms who use the funds for transactions.

The monetary authority controls the quantity of money in the economy by making transfers to the households. As in Fuerst (1992) and Christiano & Eichenbaum (1995) monetary transfers are in the form of bank deposits. Denoting by g the growth rate of money, the government transfers are equal to gM.

For monetary interventions to have a liquidity effect, that is a fall in the nominal interest rate after a monetary expansion, some form of rigidity has to be imposed in the household's ability to readjust their stock of deposits. We are going to describe the nature of optimal policies under two different assumptions about the portfolio rigidity. We consider two cases because, in the first we can describe the results analytically. In the second (and in some respects more interesting) case, we must resort to a numerical solution of the model. The analysis of the first case also provides some useful intuition about the properties of the model that also apply under the second form of portfolio rigidity.

The first assumption is that the household chooses the stock of nominal deposits at the end of each period, after all transactions have taken place, and it must wait until the end of the next period before being able to change its portfolio. Thus, the household cannot immediately readjust its portfolio of deposits after a new aggregate shock. This is a very standard assumption in the class of "limited participation" models. We refer to this case as "one-period portfolio rigidity".

In the second case we assume that the stock of deposits owned by the households can be changed at any moment, but there is a cost for doing so. Early withdrawals are subject to a penalty imposed by the intermediary which is increasing and convex in the amount withdrawn. At the same time, new deposits earn a lower interest rate in the first period they are made, with the interest loss being increasing and convex in the amount of new deposits. These assumptions can be justified by costs that the intermediary faces when it adjusts its portfolio of loans to firms.[5] The nominal adjustment cost is denoted by $\tau(d + gM, d')$ where d is the previous nominal holding of deposits, d' the new stock, and gM the monetary transfers in the form of bank deposits. The function τ is assumed to be continuously differentiable in both arguments. Similar assumptions are made in Christiano & Eichenbaum (1992).[6] We refer to this case as "adjustment costs".

1.2 Households

There is a continuum of agents of total measure 1 that maximize the expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t (c_t - \chi_t a) \tag{1}$$

where c_t is consumption of market produced goods, a is the disutility from working and χ_t is an indicator function taking the value of one if the agent

[5] In this model deposits should not be interpreted as checking deposits but rather as less liquid deposits that earn a higher interest rate. Usually, in order to earn higher interest rates, depositors have to commit to a minimum length of their deposits. Early withdrawals are allowed but there are penalties. An example would be certificates of deposit.

[6] Christiano & Eichenbaum (1992) assume that the cost is in the form of leisure time that the household needs to give up in order to readjust its portfolio. Here, instead, we assume that the cost is in the form of forgone income.

is employed and zero if unemployed.

Agents own three types of assets: cash, nominal deposits and firms' shares. In each period, agents are subject to the cash-in-advance constraint and budget constraint. In the case of one-period rigidity the cash-in-advance and budget constraints are:

$$P(c+i) \leq m - d \tag{2}$$

$$P(c+i) + m' = m + gM + (d+gM)R + \chi Pw + P\pi n \tag{3}$$

The variable P is the nominal price, i is the household's investment in the shares of new firms, n identifies the number of firms' shares that the household owns and π the dividends paid by these firms. The wage received by an employed worker is denoted by w and it is paid at the end of the period. The determination of the wage will be specified below. The after-transfer stock of deposits is $d+gM$ (remember that transfers are in the form of bank deposits). On these deposits the household earns the nominal interest rate R.

In the case of adjustment costs the cash-in-advance and budget constraints are:

$$P(c+i) + \tau(d+gM, d') \leq m + gM - d' \tag{4}$$

$$P(c+i) + \tau(d+gM, d') + m' = m + gM + Rd' + \chi Pw + P\pi n \tag{5}$$

where τ is the nominal cost of readjusting the portfolio of deposits.[7]

1.3 Production

The production sector is characterized by a search-matching framework similar to the labor-search model of Mortensen & Pissarides (1994) with exogenous separation. The production technology displays constant returns-to-scale with respect to the number of employees. Without loss of generality, it is convenient to assume that there is a single firm for each worker. The search for a worker involves a fixed cost κ and the probability of finding a worker depends on the matching technology $\mu V^{\alpha}(1-N)^{1-\alpha}$ where V is the number

[7] We assume that the adjustment cost enter the cash-in-advance constraint, that is, they are paid in cash. As an alternative, we could assume that this cost is in the form of a lower interest rate received from the bank. This is equivalent to saying that the cost is paid at the end of the period. Because the properties of the model are not affected, we have chosen the first formulation because it makes the analytics of the model simpler.

of vacancies (number of firms searching for a worker), N is the number of employed workers ($1-N$) is the number of searching workers), and $0 < \alpha < 1$. The probability that a searching firm finds a worker is denoted by q and it is equal to $\mu V^{\alpha}(1-N)^{1-\alpha}/V$, while the probability that an unemployed worker finds a job is denoted by h and is equal to $\mu V^{\alpha}(1-N)^{1-\alpha}/(1-N)$. Job separation is exogenous and occurs with probability λ. There is no cost to searching for a worker.

If the search process is successful, the firm operates the technology $y = Ax^{\nu}$, where A is the aggregate level of technology, x is an intermediate input. Output goods and intermediate goods are perfect substitutes, and therefore, the relative price of these two goods is 1. The aggregate technology level A is equal to $\bar{A}e^{z}$ where z is an aggregate technology shock that follows a first order Markov process with transition density function $\Gamma(z, z')$.

Transactions for the purchase of the intermediate good require liquid funds. Firms get this cash by borrowing from a financial intermediary at the nominal interest rate R.[8]

The contract signed between the firm and the worker specifies the wage w so that the worker gets a share η of the surplus generated by the match. The assumption of a constant sharing fraction of the surplus is standard in these models and it is motivated theoretically by assuming a Nash bargaining process between the firm and the worker where η depends on the bargaining power of the worker relative to the firm. The wage, $w(\mathbf{s})$, depends on the states of the economy $\mathbf{s}$.

1.3.1 Firms

Firms post vacancies and implement optimal production plans to maximize the welfare of their shareholders. Denote by $J(\mathbf{s})$ the value of a match for the firm measured in terms of current consumption. This is given by:

$$J(\mathbf{s}) = \tilde{\pi}(\mathbf{s}) + \beta(1-\lambda)EJ(\mathbf{s}') \tag{6}$$

For notational convenience, we have redefined the function $\tilde{\pi}(\mathbf{s})$ which is equal to $E[\beta P(\mathbf{s})\pi(\mathbf{s})/P(\mathbf{s}')]$, where $\pi(\mathbf{s})$ are the dividends paid by the firm to the shareholders at the end of the period. The function expresses the

[8] This model is equivalent to an alternative model in which working hours are flexible and the intermediate input is replaced by the number of hours that the worker inputs in production. By assuming that the compensation for the disutility from working has to be paid in advance, we would have the same results.

current value for the shareholder of the dividend paid by the firm. Because dividends are paid at the end of the period, the shareholder needs to wait until the next period to transform monetary assets into consumption. This implies that the current value in terms of consumption of one unit of money received at the end of the period is $\beta P(\mathbf{s})/P(\mathbf{s}')$.

The dividends paid to the shareholders are equal to the output produced by the firm minus the cost for the intermediate input, $x(1+R)$, and the labor cost, w:

$$\pi = Ax^{\nu} - x(1+R) - w. \tag{7}$$

Notice that the cost for the intermediate input also includes the interest paid on the loan used to finance the payment of the input.

Given $J(\mathbf{s})$, the value of a vacancy is denoted by $Q(\mathbf{s})$ and is defined as:

$$Q(\mathbf{s}) = -\kappa + q(\mathbf{s})\beta EJ(\mathbf{s}') + (1-q(\mathbf{s}))\beta EQ(\mathbf{s}') \tag{8}$$

Because the value of a vacancy must be zero in equilibrium, that is, $Q(\mathbf{s}) = 0$, equation (8) becomes:

$$\kappa = q(\mathbf{s})\beta EJ(\mathbf{s}') \tag{9}$$

Equation (9) is the arbitrage condition for the posting of new vacancies, and accordingly, for the creation of new jobs. It simply says that, in equilibrium, the cost of posting a vacancy, κ, is equal to the discounted expected return from posting the vacancy.

Consider now the value of a match for a worker. Define $W(\mathbf{s},\varphi)$ and $U(\mathbf{s})$ to be, respectively, the value of a match and the value of being unemployed in terms of current consumption. They are defined as:

$$W(\mathbf{s}) = \tilde{w}(\mathbf{s}) - a + (1-\lambda)\beta EW(\mathbf{s}') + \beta\lambda EU(\mathbf{s}') \tag{10}$$

$$U(\mathbf{s}) = h(\mathbf{s})\beta EW(\mathbf{s}') + (1-h(\mathbf{s}))\beta EU(\mathbf{s}') \tag{11}$$

where $\tilde{w} = E[\beta P(\mathbf{s})w(\mathbf{s})/P(\mathbf{s}')]$. As with dividends, the wage $w(\mathbf{s})$ is multiplied by the term $E\beta P(\mathbf{s})/P(\mathbf{s}')$ because wages are paid at the end of the period. Adding equations (6) and (10), and subtracting equation (11), gives the total surplus generated by the match $S(\mathbf{s})$. The surplus is shared between the worker and the firm according to the fixed proportion η, that is, $W(\mathbf{s}) - U(\mathbf{s}) = \eta S(\mathbf{s})$ and $J(\mathbf{s}) = (1-\eta)S(\mathbf{s})$. Using this sharing rule and equation (9), the surplus of the match can be written as:

$$S(\mathbf{s}) = \tilde{\pi}(\mathbf{s}) + \tilde{w}(\mathbf{s}) - a + \frac{(1 - \lambda - \eta h(\mathbf{s}))\kappa}{(1 - \eta)q(\mathbf{s})} \tag{12}$$

Moreover, by equating $W(\mathbf{s}) - U(\mathbf{s})$ to $\eta S(\mathbf{s})$, and using (7), we derive the wage $w(\mathbf{s})$ which is equal to:

$$w(\mathbf{s}) = \eta(Ax^{\nu} - x(1 + R)) + \frac{(1 - \eta)a}{E\left(\frac{\beta P(\mathbf{s})}{P(\mathbf{s}')}\right)} + \frac{\eta h(\mathbf{s})\kappa}{q(\mathbf{s})E\left(\frac{\beta P(\mathbf{s})}{P(\mathbf{s}')}\right)} \tag{13}$$

The wage $w(\mathbf{s})$ as well as the surplus generated by the match depend on the intermediate input x. Because the firm and the worker are splitting the surplus, the optimal input x maximizes this surplus. Based on this, we have:

Proposition 1.1 *The optimal input x is given by:*

$$x = \left(\frac{\nu A}{1 + R}\right)^{\frac{1}{1-\nu}}$$

Proof 1.1 *By differentiating the surplus in equation (12) after substituting $\pi(s) + w(s) = Ax^{\nu} - x(1 + R)$, we get the result.*

According to proposition 1.1, the intermediate input, and therefore, firm's output, is decreasing in the nominal interest rate R. This is because the interest rate increases the marginal cost of the intermediate input. Therefore, monetary policy interventions have real effects in the economy if they affect the equilibrium interest rate.

Using equations (9) and (6) we derive:

$$\frac{\kappa}{q(\mathbf{s})} = \beta\tilde{\pi}(\mathbf{s}') + \beta E\left(\frac{(1 - \lambda)\kappa}{q(\mathbf{s}')}\right) \tag{14}$$

where as before, $\tilde{\pi}(\mathbf{s})$ is the value in terms of current consumption of dividends distributed by the firm at the end of the period. Using forward substitution and the law of iterated expectations, we then have:

$$\frac{\kappa}{q(\mathbf{s}_t)} = \beta E_t \sum_{j=1}^{\infty} [\beta(1 - \lambda)]^{j-1} \tilde{\pi}(\mathbf{s}_{t+j}) \tag{15}$$

This equation says that an increase in the expected sum of future dividends (properly discounted) must induce a reduction in the current value of q, the probability of a vacancy being filled, because κ is fixed. The fall in q requires an increase in the number of vacancies which in turn increases the next period employment.

Equation (15) provides intuition on how changes in the interest rate affect the employment rate. As long as an expected fall in the future interest rates induces an increase in the future expected dividends, they induce an increase in the employment rate. Also notice that the future inflation rates play an important role as $\tilde{\pi}_{t+j} = \beta P_{t+j}\pi_{t+j}/P_{t+j+1}$, for $j \geq 1$. On the other hand, the current dividend π_t and the next period inflation rate P_{t+1}/P_t do not enter equation (15). These observations are important for understanding the impact of monetary policy interventions on employment and the properties of the optimal monetary policy as characterized in later sections.

2 Optimal and time-consistent monetary policy

Now that the economic environment has been described, we can define the optimal monetary policy. In this section we define the optimal policy when the monetary authority chooses the growth rate of money on a period-by-period basis and cannot credibly commit to the choice of future rates. Thus, we define policies that are time-consistent. We restrict the analysis to policies that are Markov stationary, that is, policy rules that are functions of the current aggregate states of the economy. Given $\mathbf{s}$ the current states, a policy rule will be denoted by $g = \Psi(\mathbf{s})$.

The procedure we follow to derive the time-consistent policy consists of two steps. In the first step we define a recursive equilibrium where the policy maker follows an arbitrary policy function $\Psi(\mathbf{s})$. In the second step we ask what the optimal growth rate of money would be today, if the policy maker anticipates that from tomorrow on he will follow the arbitrary policy rule $\Psi(\mathbf{s})$. This allows us to derive the optimal current g as a function of the current states and the policy rule that will be followed from tomorrow on. This will be denoted by $g = \psi(\Psi; \mathbf{s})$. If the current policy rule ψ is equal to the policy rule that will be followed starting from tomorrow, that is, $\psi(\Psi; \mathbf{s}) = \Psi(\mathbf{s})$ for all $\mathbf{s}$, then Ψ is an optimal and time consistent policy rule.

We describe these two steps in detail in the next two subsections. Given the similarity of the problems in the case of one-period portfolio rigidity and in the case of adjustment costs, we will describe these problems only for the case with adjustment costs. The derivation of the optimal policy in the case of one-period portfolio rigidity requires only minor notational changes.

2.1 The household's problem given the policy function Ψ

Assume that the policy maker commits to the policy rule $g = \Psi(\mathbf{s})$. Then, using a recursive formulation, we will describe the household's problem and define a competitive equilibrium conditional on this policy rule. In order to use a recursive formulation, we normalize all nominal variables by the pre-transfer stock of money M. The aggregate states of the economy are the technology shock z, the normalized pre-transfer stock of nominal deposits D, and the number of workers, N, who are matched with a firm (employed workers) at the beginning of a period. The individual states are the occupational status χ, the normalized pre-transfer stock of money m, the normalized pre-transfer stock of nominal deposits d, and the number of firms' shares n owned by the household. We will denote the set of individual states with $\hat{\mathbf{s}} = (\chi, m, d, n)$. The household's problem is:

$$\Omega(\Psi; \mathbf{s}, \hat{\mathbf{s}}) = \max_{v,d'} \Big\{ c - \chi a + \beta\, E\, \Omega(\Psi; \mathbf{s}', \hat{\mathbf{s}}') \Big\} \tag{16}$$

subject to

$$c = \frac{m + g - (1+g)d'}{P} - \frac{\tau(d+g, d'(1+g))}{P} - v\kappa \tag{17}$$

$$m' = d'(1+R) + \frac{P(\chi w + n\pi)}{(1+g)} \tag{18}$$

$$n' = (1-\lambda)n + vq \tag{19}$$

$$\mathbf{s}' = H(\Psi; \mathbf{s}) \tag{20}$$

$$g = \Psi(\mathbf{s}) \tag{21}$$

The variable v denotes the number of new firms' shares (vacancies) purchased by the household. The next period holdings of firms' shares is determined by equation (19). In solving this problem, the household takes as given the policy rule Ψ and the law of motion for the aggregate states H defined in equation (20). To make clear that this problem is conditional on the particular policy function Ψ, this function has been included as an extra argument in the household's value function and in the aggregate law of motion.

A solution for this problem is given by the state contingent functions $v(\Psi; \mathbf{s}, \hat{\mathbf{s}})$ for the purchase of new firm shares and $d'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$ for bank deposits. As for the value function, we make explicit the dependence of these decision rules on the policy rule Ψ.

In equilibrium, households are indifferent about the allocation of cash between the purchase of consumption goods and the purchase of firms' shares, independent of their employment status. This result derives from the assumption that the utility function is linear in consumption. Consequently, we have multiple equilibria corresponding to different distributions of firms' shares among households. Because the aggregate behavior of the economy is independent of this distribution, we concentrate on a particular equilibrium. This is the equilibrium in which all agents make the same portfolio choices of deposits and shares of firms. This implies that differences in earned wages between employed and unemployed workers give rise to different consumption levels rather than differences in asset holdings. We refer to this particular equilibrium as the *symmetric* equilibrium. We then have the following definition.

Definition 2.1 (Symmetric equilibrium given Ψ) *A recursive symmetric competitive equilibrium, given the policy rule Ψ, is defined as a set of functions for (i) household decisions $v(\Psi; \mathbf{s}, \hat{\mathbf{s}})$, $d'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$, and value function $\Omega(\Psi; \mathbf{s}, \hat{\mathbf{s}})$; (ii) intermediate input $x(\Psi; \mathbf{s})$; (iii) wage $w(\Psi; \mathbf{s})$; (iv) aggregate deposits $D'(\Psi; \mathbf{s})$ and loans $L(\Psi; \mathbf{s})$; (v) interest rate $R(\Psi; \mathbf{s})$ and nominal price $P(\Psi; \mathbf{s})$; (vi) law of motion $H(\Psi; \mathbf{s})$. Such that: (i) the household's decisions are optimal solutions to the household's problem (16); (ii) the intermediate input x maximizes the surplus of the match; (iii) the wage is such that the worker obtains a fraction η of the surplus; (iv) the market for loans clears, that is $D'(\Psi; \mathbf{s}) = L(\Psi; \mathbf{s})$, and $R(\Psi; \mathbf{s})$ is the equilibrium interest rate; (v) the law of motion $H(\Psi; \mathbf{s})$ for the aggregate states is consistent with the individual decisions of households and firms; (vi) all agents choose the same holdings of deposits and firms shares (symmetry).*

Differentiating with respect to v we get:

$$\frac{\kappa}{q} = \beta\, E\left(\frac{\beta P'\pi'}{P''(1+g')}\right) + \beta\, E\left(\frac{(1-\lambda)\kappa}{q'}\right) \tag{22}$$

which is equivalent to equation (14) found before. The first order condition with respect to d' is:

$$1 = (1+R)E\left(\frac{\beta P}{P'(1+g)}\right) - \tau_2(d+g, d'(1+g)) - \beta E\tau_1(d'+g', d''(1+g')) \tag{23}$$

This equation is complicated by the presence of the adjustment cost. Without this cost the equation would reduce to $1 = \beta(1+R)E(P/P'(1+g))$ which is the Euler equation in standard dynamic models with money when agents are risk neutral.

2.2 One-shot optimal policy and the fixed point of the policy problem

In the previous section we derived the decision rules for the households $v(\Psi;\mathbf{s},\hat{\mathbf{s}})$, $d(\Psi;\mathbf{s},\hat{\mathbf{s}})$ and the value function $\Omega(\Psi;\mathbf{s},\hat{\mathbf{s}})$, for a particular policy rule Ψ. We now ask what the optimal policy is today, if the policy maker anticipates that from tomorrow on he will follow an arbitrary policy Ψ.

Defining the optimality of a particular policy requires the definition of a welfare objective. Given that in this economy there is heterogeneity due to unemployment, the definition of a social welfare function is not trivial. One possibility is to assume that the policy maker attributes equal weight to households in the economy, independently of whether they are employed or unemployed, and maximizes the sum of all households' utility. An alternative would be to assume that workers enter insurance contracts so that they enjoy the same level of utility independent of their employment status. In this framework, the monetary authority will maximize the welfare of the representative household. The social welfare function is exactly the same in the two cases, and therefore, it does not affect the type of monetary policy undertaken by the policy maker.[9] Given the equivalence between the two assumptions, in order to simplify the notation, we assume that workers

[9] Of course, this derives from the assumption that agents are risk-neutral. With risk-averse agents, the welfare function is not the same in the two cases.

insure themselves against unemployment risks and the monetary authority maximizes the welfare of the representative household.

Under the assumption of unemployment insurance, consider the following optimization problem:

$$\mathbf{V}(\Psi;\mathbf{s},\hat{\mathbf{s}},g) = \max_{v,d'} \Big\{ c - \chi a + \beta\, E\, \Omega(\Psi;\mathbf{s}',\hat{\mathbf{s}}') \Big\} \tag{24}$$

subject to

$$c = \frac{m+g-(1+g)d'}{P} - \frac{\tau(d+g, d'(1+g))}{P} - v\kappa \tag{25}$$

$$m' = d'(1+R) + \frac{P(w_\chi + n\pi)}{(1+g)} \tag{26}$$

$$n' = (1-\lambda)n + vq \tag{27}$$

$$\mathbf{s}' = \tilde{H}(\Psi;\mathbf{s},g) \tag{28}$$

where the function $\Omega(\Psi; s', \hat{s}')$ is the next period value function conditional on the policy rule Ψ as defined in the previous section. The new function $\mathbf{V}(\Psi;\mathbf{s},\hat{\mathbf{s}},g)$ is the value function for the representative household when the current growth rate of money is g and future growth rates are determined according to the policy rule Ψ.[10]

After solving for this problem and imposing the aggregate consistency condition $\hat{\mathbf{s}} = \mathbf{s}$,[11] we are able to derive the function $\mathbf{V}(\Psi;\mathbf{s},\mathbf{s},g)$. This is the welfare level reached by the representative household, when the current growth rate of money is g and the future rates are determined by the policy rule Ψ. Because the objective of the policy maker is to choose g to maximize

[10] In the budget constraint we have used the subscript χ to differentiate the wages received by employed and non-employed workers. Even though they sign insurance contracts, the income received by employed workers differ from the income received by non-employed workers because the former face a disutility from working.

[11] In writing $\hat{\mathbf{s}} = \mathbf{s}$ we make an abuse of notation as the vector $\hat{\mathbf{s}}$ includes different variables than the vector $\mathbf{s}$. What we mean in writing $\hat{\mathbf{s}} = \mathbf{s}$ is that $m = 1$, $d = D$, and $n = N$.

the welfare of the representative household, the optimal current value of g is determined by the solution of the following problem:

$$g^{OPT} \;=\; \arg\max_{g} \; \mathbf{V}(\Psi; \mathbf{s}, \mathbf{s}, g) \;=\; \psi(\Psi; \mathbf{s}) \tag{29}$$

We then have the following definition of an optimal and time-consistent monetary policy rule.

Definition 2.2 *The optimal monetary policy rule $\Psi^{OPT}(\mathbf{s})$ is the fixed point of the mapping $\psi(\Psi; \mathbf{s})$, that is:*

$$\Psi^{OPT}(\mathbf{s}) = \psi(\Psi^{OPT}; \mathbf{s})$$

The basic idea behind this definition is that, when the agents in the economy (households, firms and the monetary authority) expect that future values of g are determined according to the policy rule Ψ^{OPT}, the optimal value of g today is the one predicted by the same policy rule Ψ^{OPT} that will determine the future values. This property assures that, in the future, the policy maker will continue to use the same policy rule, so it is rational to assume that future values of g will be determined according to this rule.

3 Optimal policy with commitment

To analyze the importance of the lack of commitment mechanisms for the conduct of monetary policy, we would like to compare the properties of the time-consistent policy(s) as defined in the previous section, with the properties of the optimal policy(s) under commitment. As we will see, under particular conditions, the time-consistent policy differs substantially from the commitment policy.

With commitment, the policy maker chooses, at time zero, a sequence of money growth g as a function of the historical realization of the shocks and the initial states. The equilibrium allocation associated with this policy is usually referred to as the Ramsey equilibrium.

Let h^t be the history of shocks realized from time zero up to time t and let H^t be the collection of all possible histories. A monetary policy with commitment can then be expressed as $g_t = g(N_0, h^t)$, for all $h^t \in H^t$ and $t \geq 0$. Similarly, the realization of the interest rate induced by this policy can be expressed as a function of N_0 and h^t, that is, $R_t = R(N_0, h^t)$. The policy maker will choose $g(N_0, h^t)$ to maximize the expected discounted utility of the representative household obtained under the competitive allocation induced by the policy $g(N_0, h^t)$. If we define $C(N_0, h^t | g(N_0, h^t))$ to be the aggregate consumption induced by the policy $g(N_0, h^t)$ in the competitive equilibrium and $N(N_0, h^t | g(N_0, h^t))$ to be the employment rate also induced by the policy $g(N_0, h^t)$ in the competitive equilibrium, the optimal policy with commitment is defined as:

$$\arg \max_{\{\{g(N_0,h^t)\}_{h^t \in H^t}\}_{t\geq 0}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \Big[C(N_0, h^t | g(N_0, h^t)) - aN(N_0, h^t | g(N_0, h^t)) \Big] \tag{30}$$

In characterizing the optimal policy with commitment we follow the primal approach which consists of choosing the optimal allocation among the set of all competitive allocations that can be induced by a feasible policy $g(N_0, h^t)$. See Chari, Christiano, & Kehoe (1996) for details about the primal approach.

4 Optimal monetary policy without portfolio rigidity

Before analyzing the optimal policy in economies with portfolio rigidity, it is instructive to study the optimal monetary policy when agents are able to adjust freely their portfolio of deposits, that is, $\tau(d + g, d'(1 + g)) = 0$ for all d and d'.

Proposition 4.1 (Time-consistent policy) *Assume no adjustment costs. Then, any policy rule $g = \Psi(\mathbf{s})$ is an optimal and time-consistent policy in the class of Markov-stationary policies.*

Proof 4.1 *The proof follows the two steps used in the definition of the time consistent policy. With portfolio flexibility, the interest rate is determined by $1 = (1+R)\beta E(P/P')$. Because the current value of g only affects the current price P, but not P/P', changes in g cannot affect R and the real variables in the economy. This implies that, whatever the policy Ψ assumed for the future, the policy maker does not have incentive to deviate from it.*

The intuition underlying this result is simple. The current monetary growth rate, g, does not have any impact on the price change between today and tomorrow, and therefore, cannot affect the nominal interest rate. It only affects the current *level* of prices rather than their future changes. Accordingly, the current growth rate of money does not have any real impact on the economy. Future growth rate of money would have real effects, but the policy maker today cannot choose the future rates. Consequently, given any arbitrary policy rule $\Psi(\mathbf{s})$ adopted in the future, the policy maker does not have any incentive today to deviate from this rule. Of course, he or she may desire to change the future rule, but that would require commitment.

Consider now the case with commitment. In this situation the policy maker chooses at time zero a sequence of policies g as function of the history of the realization of the shocks, given the initial employment rate N_0. We have the following proposition.

Proposition 4.2 (Optimal policy with commitment) *Assume no adjustment costs. If $\eta \geq 1 - \alpha$, the optimal policy with commitment implies $R(N_0, h^t) = 0$ for all $h^t \in H^t$ and $t \geq 0$. If $\eta < 1 - \alpha$, the optimal policy with commitment implies $R(N_0, h^t) = 0$ for all $h^t \in H^t$ and $t = 0$, and $R(N_0, h^t) > 0$ for all $h^t \in H^t$ and $t \geq 1$.*

Proof 4.2 *See appendix.*

According to this proposition, if the sharing fraction of the worker is too small, then the optimal policy under commitment induces a sequence of non-zero interest rates, with the exception of the first period.

There is a simple intuition behind this result. In this economy there are two possible sources of inefficiency. The first inefficiency derives from the cost of financing the intermediate input x. This cost is determined by the nominal interest rate R. On this dimension, a zero nominal interest rate would be efficient. To obtain a zero interest rate, the expected inflation rate must be negative. The second source of inefficiency derives from frictions inherent in the search and matching framework. If the worker's share of the surplus is smaller than $1 - \alpha$, the high profitability of a match for the firm induces an excessive creation of vacancies. The policy maker can reduce the profitability of a match by increasing the inflation rate. However, it is important to point out that the decision to create new vacancies is not affected by either the current inflation rate or by the next period inflation rate (change in prices between today and tomorrow). What affects the return on a new vacancy is the change in prices two periods from now. This can be seen by looking at equation (15), which for simplicity we rewrite below:

$$\frac{\kappa}{q(\mathbf{s}_0)} = \beta E_0 \sum_{t=1}^{\infty} [\beta(1-\lambda)]^{t-1} \pi(\mathbf{s}_t) \frac{\beta P(\mathbf{s}_t)}{P(\mathbf{s}_{t+1})} \tag{31}$$

Notice that the infinite sum on the right hand side of this equation starts at $t = 1$ so the creation of new vacancies does not depend on the current and next period inflation rates. On the other hand, the next period inflation rate affects the interest rate, as shown in equation (23). Because the next period inflation rate does not affect the creation of vacancies but does affect the current interest rate, the optimal inflation rate in the next period will be set so that $R_0 = 0$. Future inflation rates, instead, will be set taking into consideration the possibility of correcting for the second source of inefficiency. Under the condition $\eta < 1 - \alpha$, this requires a higher inflation rate which induces a higher nominal interest rate. If $\eta > 1 - \alpha$, then it would be optimal from the planner's point of view to have an even lower inflation rate and a negative interest rate. This latter is not compatible with a competitive equilibrium, however, so the optimal interest rate is the lowest possible rate compatible with a competitive equilibrium, that is, $R = 0$.

Corollary 4.1 *If $\eta \geq 1 - \alpha$, the optimal policy with commitment is not unique. If $\eta < 1 - \alpha$, the policy $g(N_0, h^t)$ is not determined at $t = 0$, but it is unique for $t \geq 1$, independently of $g(N_0, h^0)$.*

The policy indeterminacy under the condition $\eta \geq 1 - \alpha$ derives from the fact that with a zero nominal interest rate the cash-in-advance constraints of households and firms are not binding and several sequences of money growth can induce $R = 0$. However, when $R > 0$ the cash-in-advance constraints is binding, and this allows for the uniqueness of the optimal sequences of policies $g(N_0, h^t)$. The initial indeterminacy derives from the fact that the initial growth rate of money does not affect the current interest rate as observed above.

The optimality of a zero nominal interest rate is a common feature of several monetary models. However, in this model the optimality of a zero interest rate is a general result *only* when the policy maker cannot commit to future policies. With commitment, the optimality of a zero interest rate depends on the sharing parameter η.

These results stand in contrast to previous results on the equilibrium inflation and employment rate when the monetary authority chooses the inflation rate optimally under the two regimes of commitment and non-commitment. As discussed in Sargent (1998), the inability of the central bank to commit induces an equilibrium outcome with higher inflation and lower employment. In this paper, however, if $\eta < 1 - \alpha$ we find exactly the opposite result: the ability to commit can induce an equilibrium with higher inflation and lower employment. This, however, does not imply that the equilibrium allocation with commitment is Pareto inferior. In both models, of course, the equilibrium allocation under commitment is Pareto superior.

5 Optimal monetary policy with one-period portfolio rigidity

We have argued in the previous section that, absent commitment, the optimal monetary policy and hence the sequence of nominal interest rates is indeterminant. The introduction of some form of portfolio rigidity eliminates this indeterminacy. In this section we study the economy with one-period portfolio rigidity (the standard limited participation model).

Proposition 5.1 *Assume one-period rigidity in the household's portfolio. Then, without commitment, the optimal and time-consistent policy maintains the nominal interest rate equal to zero in any state of the economy.*

Proof 5.1 *See appendix.*

The portfolio rigidity eliminates the indeterminacy in the nominal interest rate because the current growth rate of money is now able to affect the current interest rate. Because the current stock of deposits cannot be immediately changed, the current nominal interest rate is no longer dependent on the next period inflation rate. Instead, it depends on the current growth rate of money which changes the liquidity available to make loans to firms. Because the policy maker is not indifferent about the current level of the interest rate, he or she will use the current growth rate of money to set the nominal interest rate at the optimal level $R = 0$.

With policy commitment, the results obtained in the previous section extend to the case of one period portfolio rigidity. Formally:

Proposition 5.2 (Optimal policy with commitment) *Assume one period rigidity in the household's portfolio. If $\eta \geq 1-\alpha$, the optimal policy with commitment implies $R(N_0, h^t) = 0$ for all $h^t \in H^t$ and $t \geq 0$. If $\eta < 1-\alpha$, the optimal policy with commitment implies $R(N_0, h^t) = 0$ for all $h^t \in H^t$ and $t = 0$, and $R(N_0, h^t) > 0$ for all $h^t \in H^t$ and $t \geq 1$.*

Proof 5.2 *The proof follows the argument of proposition 4.2.*

The intuition for this result is similar to the intuition provided in the previous section. Also, the corollary stated in the previous section extends to the case of one period rigidity. The only change is for the optimal growth rate of money at time zero, when $\eta < 1 - \alpha$. In this case the first period optimal growth rate of money is uniquely determined by the zero interest rate target.

Although the optimal nominal interest rate is determined once we introduce the portfolio rigidity, this interest rate can be obtained with a multiplicity of time consistent policies $g = \Psi(\mathbf{s})$. In what follows we restrict the analysis to a particular policy rule. This is the policy rule under which the whole quantity of money in circulation is used for transactions. This is equivalent to imposing that the cash-in-advance constraints for households and firms are binding. This would be the unique policy if we assume that there is some form of cost associated with unused money. This particular policy rule is characterized by the following proposition.

Proposition 5.3 (Procyclical policy) *Assume one-period rigidity in the household's portfolio. The optimal and time-consistent policy compatible with full use of money implies that $g = \beta E(1 + g_Y) - 1$, where g_Y is the growth rate of aggregate final output when the interest rate is constant at $R = 0$.*

Proof 5.3 *See appendix.*

This property extends to the case of full policy commitment and $\eta \geq 1-\alpha$, but it does not apply to the case in which $\eta < 1 - \alpha$. In this case the zero interest rate is optimal only for the initial period as stated in proposition 5.2.

By following this policy, the policy maker maintains the inflation rate constant (the nominal prices grow at the constant rate). This implies that this model does not generate the positive correlation of employment with inflation (Phillips curve). As we will see in the next section, however, this is not the case for the model with adjustment costs.

The fact that the optimal growth rate of money follows the growth rate of output, implies that the persistence of g depends on the persistence of the growth rate of output. In this respect the matching frictions play an important role in the model. In a limited participation model with a neoclassical production technology, the response of output to shocks is not hump-shaped. The matching framework, instead, is able to generate a hump-shaped response of output as shown, for example, in Andolfatto (1996), Merz (1995) and DenHaan, Ramey, & Watson (1998). Consequently, in this paper, the response of the growth rate of money to an $AR(1)$ technology shock displays some persistence that is absent in the limited participation model with a neoclassical technology. With matching frictions, the optimal growth rate of money will be above the steady state level for more than one period.

Although the economy with one-period portfolio rigidity has precise predictions for the optimal nominal interest rate, the prediction of a zero interest rate and the negative average inflation rate are clearly counterfactual. Because one objective of this paper is to calibrate the model economy and consider whether this model is able to replicate some of the cyclical monetary features of the U.S. economy, it would be desirable for the model to be consistent with the long-run inflation rate of the U.S. economy. Therefore, in what follows we will introduce an extra feature that allows for the optimality of a positive nominal interest rate. An easy way to do this is to assume that there is a negative externality associated with production. We assume that each firm generates an externality of the form ξx, where x is the intermediate input used in production and ξ is constant.[12] With the introduction of this externality, propositions 5.1 and 5.3 become:

Proposition 5.4 *Assume one-period rigidity in the household's portfolio. Then without commitment, the optimal and time-consistent policy maintains*

[12] Of course, we are not claiming that this is an interesting way to explain a positive interest rate. It simply represents a modeling strategy to facilitate the calibration of the model. Given that in this section we are interested in the cyclical properties of the response of the monetary authority, rather than its long-run behavior, and this cyclical response is not affected by the presence of ξ, this is an acceptable way to proceed.

the nominal interest rate equal to ξ in any state of the economy and $g = \beta(1+\xi)(1+g_Y) - 1$.

Proof 5.4 *The proof follows the same steps of propositions 5.3 taking into account the externality $\xi x N$ in the objective of the policy maker.*

The introduction of the externality implies that the optimal interest rate is positive but it does not change the constancy of it. At the same time, the positiveness of the interest rate implies that the cash-in-advance constraints are always binding and the optimal policy rule Ψ has the same cyclical properties as the optimal rule analyzed in proposition 5.3. What is different from the case in which there is no externality, is that this policy rule is now unique.

6 Optimal monetary policy with adjustment costs

The key feature of optimal time-consistent policy in the model with one-period portfolio rigidity is the constancy of the nominal interest rate. To maintain the interest rate constant, the growth rate of money would increase following a positive shock to productivity and fall below the steady state growth rate once aggregate output stops growing.

We now consider the optimal policy when the portfolio of deposits can be changed at any moment but with some adjustment costs. We impose the following restrictions on this cost.

Assumption 6.1 *The adjustment cost satisfies $\tau(\bar{d}+\bar{g}, \bar{d}(1+\bar{g})) = \tau_1(\bar{d}+\bar{g}, \bar{d}(1+\bar{g})) = \tau_2(\bar{d}+\bar{g}, \bar{d}(1+\bar{g})) = 0$, where $\bar{d}$ and $\bar{g}$ are the steady state value of deposits and money growth.*

This assumption imposes that the adjustment cost and its partial derivatives are zero in the steady state equilibrium. This condition is imposed to guarantee that the adjustment cost τ does not affect the properties of the steady state equilibrium and the long-run interest rate.

Although the steady state properties of the economy without policy commitment do not change, the presence of the adjustment cost changes the cyclical properties of the optimal policy response and it makes the analytical characterization of this policy difficult. In the analysis that follows we apply numerical methods to solve for the optimal policy. The numerical technique follows the theoretical description of the policy problem described in section 2.2 and it is an extension of the method used in Krusell et al. (1996, 1997) to solve models with sequential voting.

As we will see from the numerical simulation, the economy with adjustment costs displays a more persistent response of output to the growth rate of money. In the economy with one-period rigidity the nominal interest rate is constant, but with adjustment costs the interest rate smoothing is not perfect, and the nominal interest rate rises after a positive technology shock. Before going into more detail we first describe the parameterization of the model economy.

6.1 Calibration

The period in the model is one quarter and we fix the discount factor at $\beta = 0.99$. The externality parameter ξ is set at 0.018. This implies that in the steady state the optimal growth rate of money is 0.008 per quarter and the quarterly nominal interest rate is 0.018.

The matching technology is characterized by two parameters: μ and α. We set $\alpha = 0.6$ which is consistent with the estimate of Blanchard & Diamond (1989). Then to calibrate the parameter μ, we follow Andolfatto (1996), and we impose the following steady state values for the labor sector of the model: (a) the fraction of the population that is employed equals 0.57;[13] (b) the probability that a vacancy is successfully filled is $q = 0.9$; and (c) the transition probability from employment to non-employment is $\lambda = 0.15$.

The production function is characterized by the parameter ν and the technology level $\bar{A}$ (in addition to the stochastic properties of the shock). The parameter $\bar{A}$ simply acts as a rescaling factor. Therefore, without loss of generality we set $A = 1$. Regarding the calibration of ν we observe that $1-D$ represents the amount of financial assets used by households for transaction

[13] This implies that the steady state fraction of searching workers is 0.43, which is obviously larger than in the data. However, this larger fraction is imposed in order to reduce the impact that changes in the number of employed workers have on the probability that an advertised vacancy is filled. When the fraction of steady state searchers is small, a small percentage increase in the number of employed workers corresponds to a large percentage fall in the number of searching workers, which in turn implies a large fall in the probability with which new vacancies are filled.

purposes (money), as a fraction of their total fixed return financial assets. This fraction is equal to $(1-\nu)+RD$. Because RD is a small number, we can approximately assume that $1-D=1-\nu$. A proxy for $1-\nu$ is then given by the stock of $M1$ used by households as a fraction of aggregate $M3$ that they own. The value chosen is $\nu=0.9$.

The workers share of the surplus, η, is set to 0.2. This value is about half the value that would guarantee an efficient creation of new vacancies. After fixing η, the disutility from working, a, is chosen so that the steady state capital income share is 25 percent.

The adjustment cost function is specified as $\tau(d+g, d'(1+g)) = \phi\cdot(d'(1+g)-d-g+\gamma)^2$, where ϕ and γ are constant. All the results reported in the paper are based on $\bar{\phi}=100$.

Finally, the technology shock z follows the first-order autoregressive process $z'=\rho_z z+\epsilon'_z$, with $\epsilon_z \sim N(0,\sigma_z^2)$. The parameter ρ_z is assigned the value 0.95, which is in the order of magnitude commonly used in the business cycle studies. The parameter σ_z, instead, is chosen so that the volatility of output is similar to what is observed in the data. We set $\sigma_z=0.0008$. Of course, the evaluation of the performance of the model will not be based on the ability to match the volatility of output.

7 Properties of the calibrated economy: the response to technology shocks

Figure 1a-1f reports the impulse responses of several variables to a one standard deviation shock to the aggregate technology for the economy with one period rigidity and the economy with adjustment costs. Two policy regimes are considered. In the first regime the monetary authority follows a passive policy of keeping the growth rate of money constant, and in the second regime the monetary authority reacts optimally to shocks, without committing to future policy (time-consistent policy).

The first point to note is that under the optimal policy regime, the growth rate of money increases after a positive technology shock in both models. Therefore, the optimal monetary policy is pro-cyclical. However, the response is much more persistent in the model with adjustment costs. A second finding to note is that the response of employment and output is more persistent under the optimal policy regime. This is a consequence of the fact that

Table 1: Business cycle properties of the economy with adjustment costs and the economy with one period rigidity under alternative policy regimes.

	Economy with one-period rigidity		**Economy with adjustment costs**		**U.S. Economy**
	Passive	Optimal	Passive	Optimal	
Standard deviation					
Employment	0.51	1.05	0.79	0.81	0.99
Output	1.34	1.81	1.64	1.62	1.67
Consumption	1.25	1.59	1.52	1.47	1.39
Price index	1.11	0.00	1.61	1.58	1.39
Inflation	0.78	0.00	0.83	0.76	0.57
Interest rate	2.38	0.00	0.90	0.24	
Money stock		1.81		0.54	1.52
Money growth		0.83		0.13	0.73
Correlation with employment					
Prices	-0.66	0.00	-0.85	-0.71	-0.30
Inflation	0.39	0.00	0.28	0.43	0.51
Money stock		0.87		0.42	0.49
Money growth		-0.26		0.81	0.33

NOTES: Statistics for the model economy are computed on HP detrended data generated by simulating the model for 240 periods and repeating the simulation 1000 times. The statistics are averages over these 1000 simulations. Statistics for the U.S. economy are computed using HP detrended data from 1959.1 through 1996.4. Consumption includes consumer expenditures in non-durable and services. The price index is the CPI index.

monetary policy interventions have expansionary effects in this economy and the optimal policy is pro-cyclical.

Table 1 reports some business cycle statistics for the two versions of the economy (one-period portfolio rigidity and adjustment costs) and for the two policy regimes. As expected from the impulse responses plotted in Figure 1, the volatility of employment is slightly greater under the optimal policy regime.

Another interesting result is that the volatility of prices and inflation is lower when the monetary authority follows an optimal policy. This confirms the optimality of monetary policy rules that are targeted to the control of inflation variability.

The lower section of the table reports the correlation of employment with some key variables. As can be seen from the table, when the monetary authority follows an optimal policy, the model with adjustment costs replicates the negative correlation of employment with prices and the positive correlation with inflation, money stock and money growth. On the other hand, the model with one period rigidity does not generate any correlation of employment with prices and inflation, and the correlation with the growth rate of money is negative. Therefore, the model with adjustment costs performs better than the model with one period rigidity. The reason the model with one period rigidity does not generate these features, is because in this model the optimal policy consists of keeping the inflation rate constant. This is obtained by expanding the stock of money when output grows, and reducing the stock of money when the growth rate of output is negative. (See proposition 5.3.) However, output growth is above the steady state only for few periods after the shock, while employment stays above the steady state for many periods. Moreover, the response of the growth rate of money reaches the pick in the first period, while the reaction of employment starts after the first period due to the matching frictions.

8 Conclusion

In this paper we have analyzed the properties of an optimal monetary policy in a world where inflationary monetary interventions have expansionary effects in the economy through the liquidity channel. We find that in this economy, if business cycle fluctuations are driven by shocks to technology, then the optimal monetary policy is pro-cyclical. This implies that the positive correlation between inflation and employment observed in the data (the Phillips curve relation) may be in part a consequence of the implementation of such a policy.

The version of the economy with adjustment costs in which the monetary authority acts optimally is able to capture many of the features of the real economy. From this we conclude that the monetary policy implemented by the Federal Reserve Bank is broadly consistent with the notion of optimality as defined in this paper.

We have also analyzed the long-run properties of an optimal and time-consistent policy and compared it to the optimal policy under commitment. The finding is that, under some conditions, the ability to commit can lead to higher inflation and lower employment. This is in contrast to many studies concluding that the lack of commitment induces more inflation and lower employment.

Finally, we should note that the optimality of a pro-cyclical policy derives from the assumption that technology shocks are the main source of business cycle fluctuations. Different conclusions may be reached if other sources of business fluctuations are considered. For example, if we assume that firms have to finance only a fraction of their input and this fraction changes stochastically (i.e. because there are shocks to the velocity of money), then the optimal policy would be counter-cyclical. This is because an increase in velocity reduces the interest rate and expands output. To prevent the fall in the nominal interest rate the monetary authority has to implement a contractionary policy. Other forms of shocks can also be considered, like demand shocks resulting from changes in government spending. The analysis of all such possibilities, however, is beyond the purpose of this paper.

A Proof of proposition 4.2

Consider the following planner's problem in the choice of V and D':[14]

$$\Omega(A,D,N) = \max_{V,D'} \Big\{ C - aN + \beta\, E\, \Omega(A',D',N') \Big\} \tag{32}$$

subject to

$$C = \frac{(1+g)(1-D')}{P} - \frac{\tau(D+g, D'(1+g))}{P} - V\kappa \tag{33}$$

$$1+g = (1+g)D'(1+R) + PN(W+\Pi) \tag{34}$$

$$(1+g)D' = PXN \tag{35}$$

$$W+\Pi = (1-\nu)AX^{\nu} \tag{36}$$

$$X = \left(\frac{\nu A}{1+R}\right)^{\frac{1}{1-\nu}} \tag{37}$$

$$N' = (1-\lambda)N + m(V, 1-N) \tag{38}$$

Equation (33) is the aggregate cash-in-advance for households; (34) is the aggregate budget constraint, (35) is the equilibrium condition in the loan market, (36) states that the sum of dividends and wages is equal to the period surplus of the firm-worker match, and (37) defines the intermediate input X as derived in proposition 1.1. Finally, (38) is the law of motion for the next period employment. For the moment the problem is stated for the general problem with adjustment costs. The first order conditions for the planner problem are:

$$-\kappa + \beta m_1 E\left[R'X' + \Pi' + W' - a + \frac{\kappa}{m_1'}(1-\lambda-m_2')\right] = 0 \tag{39}$$

$$-(1+g) + (1+g)\left[1 + \frac{R}{1-\nu}\right] - \tau_2 - \beta E\tau_1' = 0 \tag{40}$$

The first order conditions for the household in the competitive equilibrium are (22) and (23). After some substitutions the two conditions can be written as:

$$-\kappa + \beta(1-\eta)qE\left[(\Pi' + W')E\left(\frac{\beta P'}{P''(1+g')}\right) - a + \frac{\kappa}{(1-\eta)q'}(1-\lambda-\eta h')\right] = 0 \tag{41}$$

[14] All variables are denoted in capital letters as they are aggregate variables.

$$-1 + (1+R)E\left(\frac{\beta P}{P'(1+g)}\right) - \tau_2 - \beta E\tau_1' = 0 \quad (42)$$

If the adjustment cost is zero, then equation (40) implies that $R = 0$, which means that for the planner problem $R = 0$ is optimal. In order to have an equilibrium interest rate equal to zero, the term $E(\beta P/P'(1+g))$ must be equal to 1 (see equation (42)). If $R = 0$ and $\eta = 1-\alpha$, it can be verified that (22) is exactly equal to (39) and by following a policy that maintains the interest rate equal to zero the first best allocation is obtained. If $\eta > 1 - alpha$, the policy that keeps the interest rate $R = 0$ does not reach the first best allocation. The first best allocation requires $R < 0$, which however is not compatible with a competitive equilibrium. In this case the planner will keep the interest rate at the lower feasible level, which is $R = 0$. If $\eta < 1-\alpha$, a zero interest rate policy generates an excessive creation of vacancies (the value of q in equation (41) will be smaller than in equation (39). In order to change the number of vacancies, the planner must decrease the values of $E\left(\frac{\beta P_{t+j}}{P_{t+j+1}(1+g_{t+j})}\right) = 1/(1+R_{t+j})$ and the dividend π_{t+j}, for $j \geq 1$. The increase in R_{t+j} for $j \geq 1$ is equivalent to reducing the future values of $P/P'(1+g)$ and the dividends π. At time zero, however, because the interest rate does not affect the rate of new vacancies, $R = 0$ is still optimal.

B Proof of proposition 5.1

Consider again the planner problem 32 with the proper changes due to the fact that the stock of deposits can be changed only at the end of the period. The first order conditions for this problem are:

$$-\kappa + \beta m_1 E\left[R'X' + \Pi' + W' - a + \frac{\kappa}{m_1'}(1-\lambda-m_2')\right] = 0 \quad (43)$$

$$-1 + \left[1 + \frac{R}{1-\nu}\right] = 0 \quad (44)$$

The first order conditions for the household in the competitive equilibrium are:

$$-\kappa + \beta(1-\eta)qE\left[(\Pi' + W')E\left(\frac{\beta P'}{P''(1+g')}\right) - a + \frac{\kappa}{(1-\eta)q'}(1-\lambda-\eta h')\right] = 0 \quad (45)$$

$$-1 + (1+R)E\left(\frac{\beta P}{P'(1+g)}\right) = 0 \quad (46)$$

From equation (44) we have that the optimal interest rate is $R = 0$. From equation (46) the zero interest rate requires $E\beta P/P'(1+g) = 0$. Now with $R = 0$ equation (43) is not necessarily equal to (45), that is, the optimal number of new vacancies is not necessarily optimal. However, equation (45) does not depend on the current value of g, but only on the future values of the growth rate of money. The planner will be able to affect the rate of vacancy creations only if he can credibly commit to g'' today. Without commitment the value of g'' chosen today will not be optimal tomorrow. Consequently, the zero interest rate policy is an optimal and time-consistent policy. In order to show that this policy is unique it is enough to show that by choosing g the planner is able to set $R = 0$, whatever is the future policy Ψ. Fix g and the policy rule from tomorrow on Ψ. Given (g, Ψ) the equilibrium condition implies either $R = 0$ or $R > 0$ (the interest rate cannot be negative). In the first case the planner does not need to change g to obtain $R = 0$. If instead $R > 0$, then the cash-in-advance constraint is binding. Using the cash-in-advance constraint, the budget constraint and equation (37) we derive $\nu = (1+R)(D+g)$. Because in the current period D is given, an increase in g decreases the nominal interest rate. Consequently, the planner will use g to set $R = 0$.

C Proof of proposition 5.3

Under the condition that the cash-in-advance constraint is satisfied with equality, we have $PY = 1$, where $Y = Ax^{\nu}$ is aggregate gross output. This must be satisfied in each period. Therefore, $P'Y' = 1$ must also old. Using these two relations the following condition can be derived:

$$E\left(\frac{\beta P}{P'(1+g)}\right) = \frac{\beta}{1+g} E\left(\frac{Y'}{Y}\right) \tag{47}$$

Now if $R = 0$, $E(\beta P/P'(1+g))$ must be equal to 1 and the growth rate of final output $(1-\nu)Ax^{\nu} + Rx$ is equal to the growth rate of gross output $Y = Ax^{\nu}$. This implies that $g = \beta E(1+g_Y) - 1$.

(Chapter head:)*
Bibliography

Andolfatto, D. (1996). Business cycles and labor-market search. *American Economic Review*, *86*(1), 112–32.

Blanchard, O. & Diamond, P. (1989). The beveridge curve. *Brookings Papers on Economic Activity*, *89*(1), 1–76.

Chari, V. V., Christiano, L. J., & Kehoe, P. J. (1996). Optimality of the friedman rule in economies with distorting taxes. *Journal of Monetary Economics*, *37*(2), 203–223.

Christiano, L. J. & Eichenbaum, M. (1992). Liquidity effects and the monetary transmission mechanism. *American Economic Review*, *80*, 346–353.

Christiano, L. J. & Eichenbaum, M. (1995). Liquidity effects, monetary policy, and the business cycle. *Journal of Money Credit and Banking*, *27*(4), 1113–36.

Christiano, L. J., Eichenbaum, M., & Evans, C. L. (1996). The effects of monetary policy shocks: evidence from the flow of funds. *Review of Economics and Statistics*, *78*(1), 16–34.

Clarida, R., Gali, J., & Gertler, M. (1999). The science of monetary policy: a new keynesian perspective. Forthcoming in *Journal of Economic Literature*.

Cooley, T. F. & Hansen, G. D. (1995). Money and the business cycle. In Cooley, T. F. (Ed.), *Frontiers of Business Cycle Research*, chap. 7. Princeton University Press, Princeton, New Jersey.

DenHaan, W. J., Ramey, G., & Watson, J. (1998). Liquidity flows and fragility of business enterprises. Unpublished Manuscript, University of California, San Diego.

Fuerst, T. S. (1992). Liquidity, loanable funds, and real activity. *Journal of Monetary Economics*, *29*(4), 3–24.

Hamilton, J. D. (1997). Measuring the liquidity effect. *American Economic Review*, *87*(1), 80–97.

Ireland, P. N. (1996). The role of countercyclical monetary policy. *Journal of Political Economy*, *104*(4), 704–23.

Krusell, P., Quadrini, V., & Ríos-Rull, J.-V. (1996). Are consumption taxes really better than income taxes?. *Journal of Monetary Economics*, *37*(3), 475–503.

Krusell, P., Quadrini, V., & Ríos-Rull, J.-V. (1997). Politico-economic equilibrium and economic growth. *Journal of Economic Dynamics and Control*, *21*(1), 243–67.

Leeper, E. M., Sims, C. A., & Zha, T. (1996). What does monetary policy do?. *Brookings Papers on Economic Activity*, *96*(2), 1–63.

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics*, *36*, 269–300.

Mortensen, D. T. & Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies*, *61*, 397–415.

Sargent, T. J. (1998). *The Conquest of American Inflation*. Princeton University Press, Princeton, New Jersey.

Svensson, L. (1997a). Inflation forecast targeting: implementing and monitoring inflation targets. *European Economic Review*, *41*(6), 1111–46.

Svensson, L. (1997b). Optimal inflation targets, "conservative" central banks, and linear inflation contracts. *American Economic Review*, *87*(1), 98–114.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Series on Public Policy*, *39*, 195–214.

Taylor, J. B. (1998). An historical analysis of the monetary policy rules. NBER Working Paper Series, # 6768.