

# **Liquidity Risk and Competition in Banking**

Yoram Landskroner

*Stern School of Business, New York University and School of Business  
Administration, Hebrew University.  
ylandskr@stern.nyu.edu*

Jacob Paroush

*Department of Economics, Bar-Ilan University and Ashkelon Academic  
College.  
parousj@mail.biu.ac.il*

Last Revision 30 January 2008

JEL Classification: G21

Key words: Liquidity risk; liquid assets; volatile liabilities; loan commitments;  
deposit withdrawal; liquidity shortage (deficit).

### **Abstract**

Liquidity risk is one of the major risks faced by banks in addition to credit risk, market risk and operating risk. In this paper we construct a stylized model of bank management where the asset and liabilities liquidity structure are a key element in determining the bank's exposure to liquidity risk. The main results of our model are that liquidity risk increases when competition in the credit market increases while increasing competition in the deposit market will decrease the liquidity shortage. Our results are of particular importance as banks face increased liquidity risk due to the recent developments in the financial markets.

## I. Introduction

The traditional functions of banks are transformation of maturity and the provision of liquidity. Banks transform short-term liquid liabilities into long term illiquid assets. Banks provide liquidity to demand depositors and also to borrowers via lines of credit.<sup>1</sup> In doing that they are undertaking liquidity risk of their customers thus exposing themselves to this risk. In extreme cases bank liquidity problems can result in bank runs when depositors withdraw their funds on a massive scale, such problems can be contagious, spreading in the banking system. The classical role of the central bank as a lender of last resort was supposed to serve as a buffer for this risk. More important today are the government provision of deposit insurance and capital requirements that should prevent a liquidity crisis and bank runs<sup>2</sup>. Recently in the summer of 2007 following the sub-prime crisis in the U.S. we have witnessed the return of the runs however this time the target has been not banks but asset backed securities. Investment funds that were leveraged and holding these assets were hurt by the “evaporation” of liquidity in the markets.

Kashyap, Rajan and Stein (2002) show that the function of banks as liquidity providers may explain why banks tie together the activities of deposit taking and lending under the same roof based on risk management motivation. Gatev, Schuerman, and Strahan (2006) test the KRS model and find that bank risk increases with unused loan commitments but the risk is mitigated by deposits. There is an incompatibility between the two activities: the timing demand for liquidity may force fire sale of illiquid assets. Bank deposits that are subject to runs (“fragile”) create an incentive for banks to provide liquidity, on the other hand capital requirements may reduce the liquidity creation by banks but increases their ability to overcome distress, see Diamond and Rajan (2001).

Liquidity in banking is conventionally defined as the ability to fund increases in assets and meet obligations as they become due<sup>3</sup>. Liquidity involves uncertainty resulting for example from unexpected deposit withdrawals and or utilization of loan commitments.. Liquidity and liquidity risk have two related dimensions: first, funding liquidity is the ability to borrow in the market, accordingly funding risk as defined by

---

<sup>1</sup> Fama (1985) argues that liquidity production makes bank unique and private information from deposits gives banks an advantage in lending

<sup>2</sup> For a now classical model of liquidity, bank runs and deposit insurance see Diamond and Dybvig (1983)

<sup>3</sup> Basel (2000) on sound practices for managing liquidity in banking organizations.

the Federal Reserve is the possibility that the bank will face difficulties in meeting its obligations as they come due because of increasing costs of new funds or of liquidating assets. The second dimension is market liquidity, the risk involved here is the possibility that the bank will not be able to sell or unwind its asset position without adversely affecting market prices due to market illiquidity. The first risk is more important in the context of maturity transformation in the banking book while the second risk is more important in the trading book.

The bank acting as a risk bearing maturity transformer and provider of liquidity faces the usual trade-off between risk and return. Holding more liquid assets and better matching cash-flows of assets and liabilities will reduce the liquidity risk of the bank but also its profitability. Liquidity management involves finding the right balance between liquidity risk and profitability.

There has been extensive academic and regulatory discussion of the different major banking risks: credit risk, market risk and even operational risk. However relative little attention has been paid to liquidity risk that has become one of the major risks faced by banks and other financial institutions in recent years. The Basel II Accord (2004) for example sets out regulatory standards for credit risk, market risk and operational risks but says little about liquidity risk. Recent developments in domestic and international financial markets such as globalization, deregulation and financial innovation have increased competition in these markets and enhanced financial instability. These developments have contributed to the increase in relative importance of liquidity risk a risk that cannot be hedged in the market.

Liquidity is crucial to the viability of the bank and therefore managing it is among the most important activities of the bank. In this paper we present a model for the management of liquidity by the bank taking a (cash) flow approach to quantify liquidity risk. The model is stylized so that it can be solved and explicit results obtained. When considering the asset and liability structure of the bank a key issue is the distinction between liquid and illiquid assets and stable (core) vs. volatile liabilities. In the model for managing liquidity we relate liquid assets to volatile liabilities where the difference between liquid assets and volatile liabilities is the net liquidity position of the bank. The main result of our models is that: liquidity risk increases when competition in the credit market increases. This is consistent with recent increased liquidity risk faced by banks due to the developments in the markets mentioned above.

## II. The Model

Liquidity risk has two stochastic sides that is related to the likelihood of net repayment of liabilities (funding liquidity risk) and/or inability unwillingness to unwind their asset positions to meet short term obligations (asset liquidity risk). The probability of shifts in asset liquidity and liabilities structure will differ under normal business conditions, in an individual bank crisis when the bank's access to liquidity may be restricted and in a general market crisis when marketability of assets deteriorates.

Liquidity management involves primarily balancing the cost benefit trade-off between profitability and risk of illiquidity. A high level of liquidity, a bank holding a stock of high quality liquid assets ("liquidity warehouse"), indicates a capacity to meet liquidity needs and take advantage of business opportunities. However such assets are generally associated with lower returns and therefore too much liquidity in the form of cash and low-earning assets will reduce profitability. On the liability side banks raise core deposits (small demand deposits) that provide the bank a long term stable asource of funding but these may limit the growth of the bank.

Our model is based on a static balance sheet of the bank ( we do not consider loan growth and deposit growth). For liquidity measurement and management we define liquid assets and volatile liabilities<sup>4</sup>. Liquid assets, considered as a primary source of liquidity, generally include excess cash (balances over and above those needed for daily operations) and deposits with other financial institutions; money market instruments and investment securities (designated as available for sale or trading and those maturing over the short term). The main assets of the bank are illiquid term loans. The two sources of debt funding of banks are retail deposits and wholesale funding. Many banks are increasing their use of wholesale funding to replace lost retail deposits or to keep up with loan growth that cannot be sustained by deposits.<sup>5</sup>

---

<sup>4</sup> We follow the definitions of the Office of Thrift Supervision (OTS) regulatory bulletin (2003) on liquidity on the dichotomy in the balance sheet on the assets and liability sides.

<sup>5</sup> In several European countries banks had to rely more on market financing and wholesale deposits to keep up with strong loan growth, see ECB (2002) study.

Retail deposits are received from the general public, individuals and small businesses and usually are fully insured.<sup>6</sup> These “core” sources of funds are considered to be stable. Volatile liabilities on the other hand include wholesale rate- sensitive deposits and short-term liabilities (market funding) that are sensitive to credit risk and market conditions causing them to be more volatile and pose greater liquidity risk to the bank. This “hot money” includes large uninsured deposits, repurchase agreements, federal funds, lines of credit with other financial institutions brokered deposits and other short term rate sensitive borrowings.

The model is a single period model of liquidity management that relates liquid assets to volatile liabilities where the size of the bank is given and normalized to one, the initial balance sheet of the bank (at T=0) is in relative terms<sup>7</sup>:

$$\alpha + (1 - \alpha) = \beta + (1 - \beta) \quad (1)$$

Where  $(1-\alpha)$  is the proportion of liquid assets and  $(1-\beta)$  is the proportion of volatile liabilities. At time 0, the bank extends illiquid term loans that are paid off (mature) at time 1 (end of period). The interest rate on these loans is  $r_L(\alpha)$ , we assume imperfect competition in the loan market that is the bank has some market power in this market due to geographic or industry concentration and thus the bank is facing a downward sloping demand curve, thus  $\partial r_L / \partial \alpha < 0$ . In addition the bank holds liquid assets with a rate of return  $r_s$  which are assumed to be traded in a perfectly competitive market. The bank also has off-balance sheet assets in the form of loan commitments that are assumed to be a proportion  $\lambda$  of the loans  $\alpha$ . A random fraction  $z$  ( $0 \leq z \leq 1$ ) of these commitments will be exercised at some random point in time  $t$  where  $0 \leq t \leq 1$ . The bank is charging a fee  $\phi$  on the unused commitments.

The bank finances its assets by retail deposits, the bank is assumed to have some market power in this market which is assumed to be mostly a local market, and thus is facing an upward sloping supply curve. The rate paid on these deposits is  $r_0(\beta)$  with  $\partial r_0 / \partial \beta > 0$ . The second source of financing is wholesale volatile funding on which the bank is paying a rate of  $r_1$  in a national or even international market and thus is assumed a perfectly competitive market (horizontal supply curve). We assume that a

---

<sup>6</sup> In the U.S. they include demand deposits (DD), negotiable order of withdrawal accounts (NOW), Money-Market Demand Accounts (MMDA), saving accounts and certificate of deposits. These accounts usually maintain balances up to \$100,000 to be fully insured by the FDIC.

<sup>7</sup> Size may be an additional choice variable however we assume that in the short run it is fixed.

random proportion  $y$  ( $0 \leq y \leq 1$ ) of these volatile deposits will be withdrawn at time  $t$ .<sup>8</sup>

For simplicity we assume the same timing of the exercise of the loan commitments and withdrawal of deposits.<sup>9</sup>

The bank finances the commitments that are taken down and deposits withdrawn by first drawing down its liquid assets (we assume no new deposits are raised beyond time 0). We define a random variable  $x$  which is the liquidity shortage (deficit):

$$x = z\lambda\alpha + (1-\beta)y - (1-\alpha) = (\alpha-\beta) + \alpha\lambda z - (1-y)(1-\beta). \quad (2)$$

The liquidity shortage is composed of two parts, the liquidity gap (decisions of the bank)  $g = (1-\beta) - (1-\alpha) = \alpha - \beta$  that is the proportion of illiquid assets (loans) financed by volatile deposits and a random component,  $\omega$ :

$$x = g + \omega \text{ where } \omega = \alpha\lambda z - (1-y)(1-\beta). \quad (3)$$

The deficit  $x$  can be either positive or negative however we confine ourselves to the case where the bank is facing a non-negative expected liquidity deficit. This deficit will have to be financed by increasingly expensive sources of contingent funds that have an expected cost  $R(x)$ . These sources of funds may include borrowing in the money market, interbank borrowing and the usually more expensive borrowing from the central bank.<sup>10</sup> As can be seen in (3) the bank's liquidity position is determined in part by its two control variables: illiquid assets and its stable liabilities that is  $\alpha$  and  $\beta$  and in addition we introduce uncertainty explicitly via the random variables  $z$  and  $y$ . Note that the impact of the two control variables on the liquidity deficit is not symmetric:  $\partial x / \partial \alpha > 1$  and  $\partial x / \partial \beta > -1$ . That is to maintain its liquidity position the bank needs to raise more than \$1 of stable deposits for each \$1 of loans. This is because in our model loan commitments are a proportion of loans and thus are added to loans.

The end of the period balance sheet of the bank (time  $T=1$ ) is given by:

$$\alpha(1+z\lambda) = \beta + (1-\beta)(1-y) + x \quad (4)$$

Note that it includes the financing of a liquidity shortage  $x$ .

---

<sup>8</sup> Illiquidity may be interpreted as a signal of insolvency and can cause massive outflows of deposits.

<sup>9</sup> Empirical studies indicate that these variables are positively correlated. According to Gatev and Strahan (2006) these variables may have low positive or even negative correlation.

<sup>10</sup> These are the "discount window" borrowing from the Federal Reserve in the US and the standing facilities of the European Central Bank (ECB) in the EU.

The bank's objective function is maximizing expected profits w.r.t.  $\alpha$  and  $\beta$ :

$$\begin{aligned} \bar{\Pi} = & \alpha r_L(\alpha) \left(1 + \lambda \bar{z} (1 - \bar{t})\right) + \alpha (1 - \bar{z}) \lambda \varphi + (1 - \alpha) r_s \bar{t} - \\ & - \beta r_0(\beta) - (1 - \beta) r_1 \left[\bar{t} + (1 - \bar{y})(1 - \bar{t})\right] - R(x)(1 - \bar{t}) \end{aligned} \quad (5)$$

Where  $\bar{t}, \bar{z}, \bar{y}$  indicate the conditional expected values of the respective variables; the expected cost  $R$  is assumed an increasing function of  $x$ , where the function is increasing at an increasing rate, i.e.  $R'(x) > 0$  and  $R''(x) > 0$ . We assume that the bank operates within the efficiency zone so that at the optimum the expected liquidity shortage is not negative:  $\bar{x} = \alpha(1 + \bar{z}\lambda) + (1 - \beta)\bar{y} - 1 > 0$

Combining the two first order conditions of  $\bar{\Pi}$  w.r.t  $\alpha$  and  $\beta$  (See Appendix equations (A1) and (A2)) yields the equilibrium equation

$$\begin{aligned} r_L(\alpha) \left(1 - \frac{1}{\eta}\right) \left(1 + \lambda \bar{z} (1 - \bar{t})\right) + (1 - \bar{z}) \lambda \varphi = \\ = r_0(\beta) \left(1 + \frac{1}{\varepsilon}\right) + R'(x)(1 - \bar{t}) \left(1 + \lambda \bar{z} - \bar{y}\right) + r_s \bar{t} - r_1 \left[\bar{t} + (1 - \bar{y})(1 - \bar{t})\right] \end{aligned} \quad (6)$$

where  $\eta = -\frac{d\alpha}{dr_L} \frac{r_L}{\alpha}$  is demand elasticity for loans, and  $\varepsilon = \frac{d\beta}{dr_0} \frac{r_0}{\beta}$  is the elasticity of supply of retail deposits

This equation equates the marginal revenues from loans and loan commitments (first term on the LHS) and from unused loan commitments (second term on the LHS) to the marginal cost of obtaining liquidity in the deposit markets (first term on RHS) and from selling liquid assets and obtaining contingent funds. The second order conditions hold globally (See Appendix (A3)-(A5)).

### III. Comparative Statics Analysis

In this section we perform comparative statics analysis of  $\alpha$  and  $\beta$  and  $\bar{x}$  w.r.t the different parameters of the model.

Consider a change in a generic parameter  $\theta$ ; we differentiate the FOC (see Appendix (A1) and (A2)) w.r.t  $\theta$ :

$$\frac{d\alpha}{d\theta} = - \frac{\frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial \theta} \frac{\partial^2 \bar{\Pi}}{\partial \beta^2}}{\Delta} \quad \text{and} \quad \frac{d\beta}{d\theta} = - \frac{\frac{\partial^2 \bar{\Pi}}{\partial \beta \partial \theta} \frac{\partial^2 \bar{\Pi}}{\partial \alpha^2}}{\Delta} \quad (7)$$

Using the second order conditions (See Appendix (A3)-(A5)) we obtain that

$$\text{sign} \frac{d\alpha}{d\theta} = \text{sign} \frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial \theta} \quad \text{and} \quad \text{sign} \frac{d\beta}{d\theta} = \text{sign} \frac{\partial^2 \bar{\Pi}}{\partial \beta \partial \theta} \quad (8)$$

We now use (7) and (8) to analyze the effect of a change in the parameters of our model:  $\eta, \varepsilon, R, \varphi, \bar{z}, \bar{y}, \bar{t}, r_l$  on the decision variables  $\alpha, \beta$  and the expected liquidity shortage  $\bar{x}$ . First analyze the effect of the elasticity of the demand for loans  $\eta$

$$\begin{aligned} \frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial \eta} &= \left( \frac{r_L(\alpha)}{\eta^2} \right) [1 + \lambda \bar{z}(1 - \bar{t})] > 0 \Rightarrow \frac{\partial \alpha}{\partial \eta} > 0 \\ \frac{\partial^2 \bar{\Pi}}{\partial \beta \partial \eta} &= 0 \Rightarrow \frac{\partial \beta}{\partial \eta} = 0 \end{aligned} \quad (9)$$

Combining (2) and (9) yields  $\frac{\partial \bar{x}}{\partial \eta} > 0$  that is an increase in the elasticity of demand for loans, that may result from an increase in competition in the credit market, will increase the expected liquidity shortage of the bank. As can be seen an increase in the elasticity affects the loans ( $\alpha$ ) but not the deposits of the bank ( $\beta$ ): increase in the proportion of loans on the banks balance sheet and hence an increase in its liquidity needs. Recent globalization and integration of financial markets may have such an effect of increased competition in the loan market that increases liquidity risk faced by banks.

Analyze now the effect of change in elasticity of the supply of the stable deposits  $\varepsilon$  on  $\alpha$ ,  $\beta$  and the expected liquidity shortage  $\bar{x}$

$$\begin{aligned}\frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial \varepsilon} &= 0 \Rightarrow \frac{\partial \alpha}{\partial \varepsilon} = 0 \\ \frac{\partial^2 \bar{\Pi}}{\partial \beta \partial \varepsilon} &= \frac{r_0(\beta)}{\varepsilon^2} > 0 \Rightarrow \frac{\partial \beta}{\partial \varepsilon} > 0\end{aligned}\quad (10)$$

Combining (2) and (10) yields  $\frac{\partial \bar{x}}{\partial \varepsilon} < 0$

Thus, an increase in competition in the deposit market will increase the proportion of stable deposits without affecting the asset side of the balance sheet thus reducing the expected liquidity shortage of the bank. Recent deregulation especially in the US that removed geographical and product barriers on the activity of financial intermediaries has also increased competition in the markets where these firms raise funds, thus reducing the liquidity shortage in our model.

Analyze now the effect of an increase in the fee charged on unused loan commitments  $\varphi$ . Since loan commitments are proportional to the amount of loans an increase in the fee will increase  $\alpha$  without affecting  $\beta$  and thus increase the liquidity shortage:

$$\begin{aligned}\frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial \varphi} &= (1 - \bar{z})\lambda > 0 \Rightarrow \frac{\partial \alpha}{\partial \varphi} > 0 \\ \frac{\partial^2 \bar{\Pi}}{\partial \beta \partial \varphi} &= 0 \Rightarrow \frac{\partial \beta}{\partial \varphi} = 0 \\ \Rightarrow \frac{\partial \bar{x}}{\partial \varphi} &> 0\end{aligned}\quad (11)$$

We now analyze the effects of a change in the return on the liquid assets  $r_s$  and the cost of volatile deposits  $r_1$ ; interestingly both have the same effect on the liquidity position of the bank

$$\frac{\partial^2 \bar{\Pi}}{\partial \beta \partial r_1} = \bar{t} + (1 - \bar{y})(1 - \bar{t}) > 0 \Rightarrow \frac{\partial \beta}{\partial r_1} > 0$$

$$\frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial r_1} = 0 \Rightarrow \frac{\partial \alpha}{\partial r_1} = 0$$

$$\Rightarrow \frac{\partial \bar{x}}{\partial r_1} < 0$$

And

$$\frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial r_s} = -t < 0 \Rightarrow \frac{\partial \alpha}{\partial r_s} < 0 \quad \frac{\partial^2 \bar{\Pi}}{\partial \beta \partial r_s} = 0 \Rightarrow \frac{\partial \beta}{\partial r_s} = 0$$

$$\Rightarrow \frac{\partial \bar{x}}{\partial r_s} < 0$$

(12)

Thus an increase in  $r_1$  will increase  $\beta$  the proportion of stable deposits but will not affect  $\alpha$ , and an increase in  $r_s$  will reduce  $\alpha$ , the proportion of loans, but will not affect  $\beta$ , so that an increase in both rates will cause the expected liquidity needs of the bank to decline.

The comparative statics analysis yield ambiguous results with respect to the deposits withdrawals and loan commitments  $\bar{z}, \bar{y}$ . Therefore we assume for simplicity that the expected cost  $R$  is a linear function of  $\bar{x}$ , i.e.  $R(\bar{x}) = R\bar{x}$ , where  $R$  is now a constant.

From the first order conditions we obtain

$$\frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial z} = r_L \left(1 - \frac{1}{\eta}\right) \lambda (1 - \bar{t}) - \lambda \varphi - R \lambda (1 - \bar{t}) = \lambda \left[ \left( r_L \left(1 - \frac{1}{\eta}\right) - R \right) (1 - \bar{t}) - \varphi \right] \quad (13)$$

$$\frac{\partial^2 \bar{\Pi}}{\partial \beta \partial y} = R - r_1 > 0 \Rightarrow \frac{d\beta}{dy} > 0 \quad (14)$$

$$\text{and} \quad \frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial y} = \frac{\partial^2 \bar{\Pi}}{\partial \beta \partial z} = 0 \Rightarrow \frac{d\alpha}{dy} = \frac{d\beta}{dz} = 0$$

Note that  $\bar{z}$  and  $\bar{y}$  have positive direct effects on  $\bar{x}$  i.e.,

$$\frac{\partial \bar{x}}{\partial z} = \lambda \alpha > 0 \quad \text{and} \quad \frac{\partial \bar{x}}{\partial y} = 1 - \beta > 0$$

The total effects however include also indirect effects via  $\alpha$  and  $\beta$

$$\frac{d\bar{x}}{dz} = \lambda\alpha + \frac{d\alpha}{dz}(1 + \lambda\bar{z}) \text{ and } \frac{d\bar{x}}{dy} = 1 - \beta - \bar{y} \frac{d\beta}{dy} \quad (15)$$

The indirect effect of  $\bar{z}$  on  $\bar{x}$  is negative i.e.  $\frac{d\alpha}{dz} < 0$

If  $R > r_L \left(1 - \frac{1}{\eta}\right) - \frac{\varphi}{1-t}$  (See (13)).

Also since  $\frac{d\beta}{dy} > 0$  (See (14), the indirect effect of  $\bar{y}$  on  $\bar{x}$  is always negative. Thus the

total effects are positive if and only if the negative indirect effects do not offset the positive direct effects. The necessary and sufficient conditions for that is that the elasticities of  $\alpha$  w.r.t  $\bar{z}$  and  $(1-\beta)$  w.r.t  $\bar{y}$  are not too large. The specific conditions obtained from (15) are

$$\frac{d\alpha}{dz} \frac{\bar{z}}{\alpha} < \frac{\lambda\bar{z}}{1 + \lambda\bar{z}} \text{ and } \left| \frac{d(1-\beta)}{dy} \frac{\bar{y}}{(1-\beta)} \right| < 1 \text{ Note this will depend on the}$$

difference  $R - r_1$  see (14). Following the Subprime crisis of 2007 in the US the

Federal Reserve has reduced the cost of borrowing from its facility thus reducing  $R$ .

This has reduced the elasticity (in absolute terms) of  $(1-\beta)$  w.r.t  $\bar{y}$ .

Empirical studies indicate that there is a positive correlation between  $\bar{z}$  and  $\bar{y}$ ; this will increase the effects of  $\bar{z}$  and  $\bar{y}$  on  $\bar{x}$  since in this case another positive factor is added to the effects as follows

$$\frac{d\bar{x}}{dz} = \lambda\alpha + \frac{d\alpha}{dz}(1 + \lambda\bar{z}) + (1 - \beta) \frac{d\bar{y}}{dz}$$

and (16)

$$\frac{d\bar{x}}{dy} = 1 - \beta - \bar{y} \frac{d\beta}{dy} + \alpha \left(1 + \lambda \frac{d\bar{z}}{dy}\right)$$

Finally consider an increase in  $R$  the cost of obtaining contingent liquidity, this will reduce the proportion of illiquid assets (loans) and increase stable deposits thus as expected reducing the expected liquidity shortage:

$$\frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial R} = -(1 + \lambda\bar{z}) < 0 \Rightarrow \frac{\partial \alpha}{\partial R} < 0$$

$$\frac{\partial^2 \bar{\Pi}}{\partial \beta \partial R} = \bar{y} > 0 \Rightarrow \frac{\partial \beta}{\partial R} > 0$$

$$\Rightarrow \frac{\partial \bar{x}}{\partial R} < 0$$

It should be noted that in general the effect of a change a random variable on the liquidity shortage of the bank is ambiguous. It will depend, among other things, on the cost of obtaining contingent liquid funds  $R$  relative to the cost of wholesale funds and relative to the marginal revenue from loans that is determined by the interest rate on loans and the elasticity of demand for loans.

Insert Table 1

The major results are presented in Table 1. Other results concerning market interest rates and behavior of depositors and borrowers are: a change in the return on the liquid assets  $r_s$  and the cost of volatile deposits  $r_1$  will have a similar effect on the liquidity position of the bank but for different reasons. An increase in  $r_1$  will cause the bank to increase its stable deposits while an increase in  $r_s$  will reduce the proportion of loans on the balance sheet of the banks. It should be noted that the random variables ( $z$ ,  $y$  and  $t$ ) have ambiguous effects on the liquidity position of the bank due to their direct as well as indirect effects on the optimal expected liquidity shortage.

#### **IV. Summary and Concluding Remarks**

An obvious trade-off exists between the risk and return of liquidity. The bank when transforming liquid liabilities into illiquid assets earns a profit on this activity, mostly reflected in the net interest income of the bank. On the other hand the bank is exposed to the risk (cost) of not having sufficient funds to meet its obligations.

Within a stylized model that considers the cost and benefits of liquidity we show how the bank's optimal asset-liability management produces an equilibrium expected liquidity shortage.

Comparative statics analysis with respect to the bank's competitive environment yields that increased competition in the credit market will increase the optimal liquidity shortage of the bank while increasing competition in the deposit market will reduce it.

Our results are compatible with recent increased competition in capital markets due to globalization and deregulation of markets. It is of importance therefore that national and international (BIS Basel Committee) regulators pay more attention to liquidity risk.

## References

Basel Committee on Banking Supervision, 2000, Sound Practices for Managing Liquidity in Banking Organizations, BIS.

Basel Committee on Banking Supervision, 2004, Basel II: International Convergence of Capital Measurement and Capital Standards: a Revised Framework, BIS.

Diamond, Douglas W. and Philip H. Dybvig, 1983, Bank Runs, Deposit Insurance and Liquidity, *Journal of Political Economy* 91, 401-419.

Diamond, Douglas W. and Raghuram G. Rajan, 2001, Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking, *Journal of Political Economy* 109, 287-327.

European Central Bank, 2002, Developments in Bank's Liquidity Profile and Management, Frankfurt.

Gatev Evan and Philip E. Strahan, 2006, Banks Advantage in Hedging Liquidity Risk: Theory and Evidence from the Commercial Paper Market, *Journal of Finance*, 61(2), 867-92.

Gatev Evan, Til Schuermann and Philip E. Strahan, 2006, Managing Bank Liquidity Risk: How Deposit-Loan Synergies Vary With Market Conditions, Working Paper Boston College.

Kashyap Anil K., Raghuram G. Rajan and Jeremy C. Stein, 2002, "Banks as Liquidity Providers: An Explanation for the Co-Existence of Lending and Deposit-Taking" *Journal of Finance*, 57 (1), 33-73.

Malz Allan K., 2003, Liquidity Risk: Current Research and Practice, *RiskMetrics Journal* Fall, 35-72.

Matz Leonard M. 2004, Liquidity Risk Management, Thomson.

Office of Thrift Supervision 2003, Liquidity, Regulatory Bulletin RB 32-32 Thrift Activities Regulatory Handbook Update Section: 530.

Qi Jianping, 1998, Deposit Liquidity and Bank Monitoring, *Journal of Financial Intermediation* 7, 198-218.

Rajan Raghuram, 1998, Liquidity: An Overview, *Journal of Financial Intermediation* 7, 126-129.

Sharma Paul, 2004, Liquidity Risk, FSA, BBA's Annual Supervision Conference.

Wolf Wagner, 2004, The Liquidity of Bank Assets and Bank Stability, University of Cambridge Working Paper.

**Table 1: Summary Results of Comparative Statics Analysis**

Parameter/ Variable	$\alpha$	$\beta$	$g$	$\bar{x}$
Elasticity of demand for loans $\eta$	+	0	+	+
Elasticity of supply of deposits $\varepsilon$	0	+	-	-
R Cost of contingent funds	-	+	-	-
Fee on unused loan commitments $\varphi$	+	0	+	+
Interest rate on volatile deposits $r_1$	0	+	-	-
Interest rate on liquid assets $r_s$	-	0	-	-
Exercise of loan commitments $\bar{z}$ (R is high)*	-	0	-	- indirect effect + total effect
Withdrawal of deposits $\bar{y}$ *	0	+	-	- indirect effect + total effect

\*Positive correlation between  $\bar{z}$  and  $\bar{y}$  will increase the positive effect total effects of these variables on  $\bar{x}$ .

## APPENDIX

We now derive the first order conditions with respect to the two decision variables  $\alpha$  and  $\beta$

$$\frac{\partial \bar{\Pi}}{\partial \alpha} = r_L(\alpha) \left(1 - \frac{1}{\eta}\right) \left(1 + \lambda \bar{z}(1 - \bar{t})\right) + (1 - \bar{z})\lambda \varphi - r_s \bar{t} - R'(x) \left(1 + \lambda \bar{z}\right) (1 - \bar{t}) = 0 \quad (A1)$$

where  $\eta = -\frac{d\alpha}{dr_L} \frac{r_L}{\alpha}$  is demand elasticity for loans

And

$$\frac{\partial \bar{\Pi}}{\partial \beta} = -r_0(\beta) \left(1 + \frac{1}{\varepsilon}\right) + r_1 \left[\bar{t} + (1 - \bar{y})(1 - \bar{t})\right] + R'(x) \bar{y} (1 - \bar{t}) = 0 \quad (A2)$$

where  $\varepsilon = \frac{d\beta}{dr_0} \frac{r_0}{\beta}$  is the elasticity of supply of retail deposits

The second order conditions hold globally:

$$\frac{\partial^2 \bar{\Pi}}{\partial \alpha^2} = \frac{dr_L(\alpha)}{d\alpha} \left(1 - \frac{1}{\eta}\right) \left(1 + \lambda \bar{z}(1 - \bar{t})\right) - R'' \left(1 + \lambda \bar{z}\right)^2 (1 - \bar{t}) < 0 \quad (A3)$$

$$\frac{\partial^2 \bar{\Pi}}{\partial \beta^2} = -\frac{dr_0(\beta)}{d\beta} \left(1 + \frac{1}{\varepsilon}\right) - R'' \bar{y}^2 (1 - \bar{t}) < 0 \quad (A4)$$

And since

$$\left(\frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial \beta}\right) = R'' \left(1 + \lambda \bar{z}\right) (1 - \bar{t}) \bar{y}$$

We obtain that

$$\Delta = \frac{\partial^2 \bar{\Pi}}{\partial \alpha^2} \frac{\partial^2 \bar{\Pi}}{\partial \beta^2} - \left(\frac{\partial^2 \bar{\Pi}}{\partial \alpha \partial \beta}\right)^2 > 0 \quad (A5)$$