Why Has House Price Dispersion Gone Up?*

Stijn Van Nieuwerburgh[†] Pierre-Olivier Weill[‡]

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Abstract

We investigate the 30 year increase in the level and dispersion of house prices across U.S. metropolitan areas in a calibrated dynamic general equilibrium island model. The model is based on two main assumptions: households flow in and out metropolitan areas in response to local wage shocks, and the housing supply cannot adjust instantly because of regulatory constraints. In our equilibrium, house prices compensate for cross-sectional wage differences. Feeding in our model the 30 year increase in cross-sectional wage dispersion that we document based on metropolitan-level data, we generate the observed increase in house price level and dispersion. The calibration also reveals that, while a baseline level of regulation is important, a tightening of regulation by itself cannot account for the increase in house price level and dispersion: in equilibrium, workers flow out of tightly regulated towards less regulated metropolitan areas, undoing most of the price impact of additional local supply regulations. Finally, the calibration with increasing wage dispersion suggests that the welfare effects of housing supply regulation are large.

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[†]Department of Finance, Stern School of Business, New York University, and NBER, email: svnieuwe@stern.nyu.edu.

[‡]Department of Economics, University of California, Los Angeles, email: poweill@econ.ucla.edu.

1 Introduction

This paper argues that a calibrated dynamic general equilibrium island model can explain two important long-run features of cross-sectional U.S. house prices over the last 30 years: the increase in the *level* of aggregate house price, and the increase in its *dispersion* across regions. This increase in dispersion, which perhaps has not receive as much attention in the literature as the increase in level,[1] is illustrated in Figure 1. The coefficient of variation of cross-sectional house prices increased by 0.35 from1975 to 2004.[2]

Our explanation combines an increase in the dispersion of wages across metropolitan areas with fixed quantity constraints on the housing supply. The increase in wage dispersion, on the one hand, is illustrated in Figure 2. For the same samples of metropolitan areas as in Figure 1, the coefficient of variation of the real wage per job doubles between 1975 and 2004, from 0.097 to around 0.18. Constraints on the housing supply, on the other hand are mostly due to regulations such as zoning and permitting laws.[3] We show that holding regulation fixed at its 1975 level and feeding in the observed increase in wage dispersion, the model can generate the observed increases in house price level and dispersion. Several authors have argued that the tightening of housing supply regulations over time is responsible for the increase in the level and dispersion of cross-sectional house prices.[4] In our calibrated model, however, the price impact of progressively tightening regulation (while holding wage dispersion fixed) is very small. This suggests that, although an initial level of housing supply regulation is important, a tightening in regulation cannot, by itself, account for the observed

[1]Case and Shiller (1987), Himmelberg, Mayer, and Sinai (2005), Gyourko, Mayer, and Sinai (2006), Campbell, Davis, Gallin, and Martin (2006) and many others document the historical evolution of house prices at the national and local level.

[2]The coefficient of variation is the ratio of the standard deviation to the mean, and is therefore a scale neutral measure of dispersion.

[3]These can be, for instance, zoning ordinances, height- and lot-size restrictions and outright quantity restrictions, growth moratoria, preservation, and state-level development restrictions, land use planning or restrictions on the development of new towns. See Malpezzi (1996), Quigley and Rosenthal (2005) and Saks (2005).

[4]Glaeser, Gyourko, and Saks (2007) Glaeser and Gyourko (2003), Glaeser and Gyourko (2005), Glaeser, Gyourko, and Saks (2005), Quigley and Rosenthal (2005), and Quigley and Raphael (2005). Davis and Heathcote (2005) and Glaeser, Gyourko, and Saks (2007) both observe that the non-structure component of house prices has increased substantially. This component may proxy for the intensity of regulation or the value of land. It accounts for about half of the value of the U.S. housing stock, and it has risen much faster than the structure component since the 1970s. Brunnermeier and Julliard (2006) consider the effect of money illusion on house prices and generate larger effects of inflation on house prices in regions with tighter housing supply.

increases in house price level and dispersion.

Figure 1: House Price Dispersion Among U.S. Metropolitan Areas.

The figure plots the coefficient of variation of housing prices among U.S. metropolitan areas. The coefficient of variation is the ratio of the population-weighted cross-sectional standard deviation to the population-weighted cross-sectional mean. The left panel refers to a smaller sample of 70 metropolitan areas for which we have complete house price data going back to 1975 9balanced panel). The right panel refers to the largest possible sample based on all available house price data. This sample starts with the same 70 areas as in the left panel, but grows to 322 areas by 1994 (unbalanced panel). Section 3.2 describes our data set in detail.

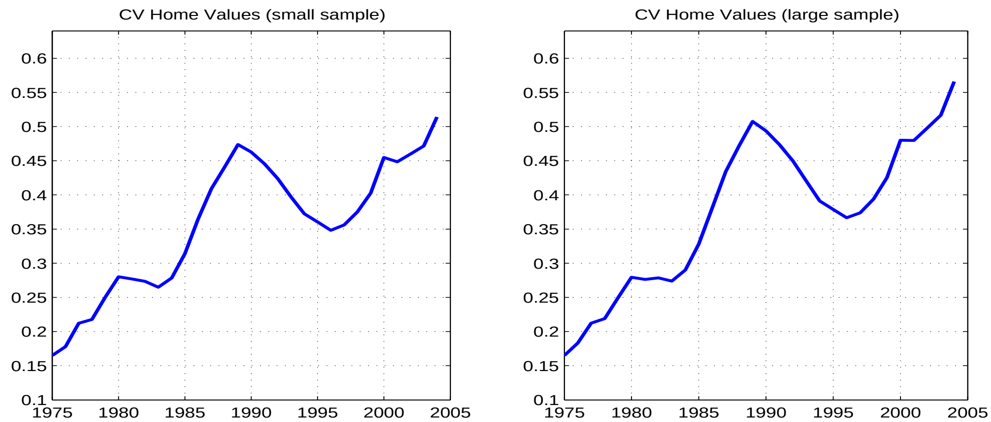
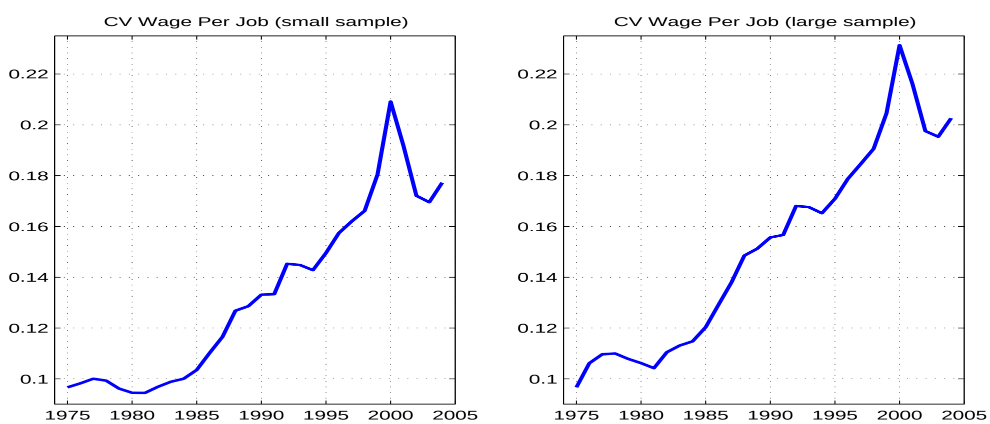


Figure 2: Wage Dispersion Among U.S. Metropolitan Areas.

The figure plots the coefficient of variation of the real wage per job among U.S. metropolitan areas. The left and the right panel refer to the same sample of metropolitan areas as in Figure 1. Section 3.1.2 describes our data in detail.



We develop a new model that is designed to study the quantitative impact of wage dispersion and housing supply regulations on cross-sectional house prices. It takes as given an exogenous process for cross-sectional wages, as well as an exogenous distribution of housing supply regulations. The equilibrium provides the endogenous joint dynamics of cross-sectional house prices, construction, and employment. More precisely, we model metropolitan areas as a collection of geographically separated islands randomly hit by idiosyncratic and persistent wage (productivity) shocks in the non-housing sector. Construction firms can build new houses in any metropolitan area, but new construction is irreversible and is subject to supply regulation, implying that the local housing supply cannot adjust instantly in response to a local wage shock. We assume that labor is mobile: households can freely move across metropolitan areas, but they are constrained to live in the same area they work. Equilibrium house prices compensate for wage differentials, to keep households indifferent between metropolitan areas: households end up living in smaller and more expensive quarters if they choose to work in a higher-wage metropolitan area. In addition, higher-wage metropolitan areas have a larger housing stock and a larger workforce.

Our main calibration exercise starts in a steady state with a baseline level of housing supply regulation, which is chosen to match the 1975 concentration of jobs in the highest quintile of the metropolitan area wage distribution. We then feed in the observed increase in the wage dispersion between 1975 and 2004, while keeping housing supply regulation constant. The increase in wage dispersion creates large flows of workers towards exceptionally high-wage metropolitan areas, driving local house prices up because of limited housing supply. Conversely, households flow away from low-wage areas, driving local house prices down. Taken together, these two effects increase house price *dispersion*: quantitatively, we find that the 0.08 increase in the coefficient of variation of wages generates a 0.35 increase in the coefficient of variation of home values, the magnitude observed in the data. Second, the same increase in wage dispersion creates an increase in the nationwide house price *level* consistent with the data. A marginal increase in the wage in an area is compensated by an equal increase in housing expenditure, which equals the marginal increase in square-foot rent multiplied by the typical house size in that area. This implies that the marginal increase in square-foot rent is inversely proportional to the house size. It increases by more in high-wage areas, where houses are smaller, than it decreases in low-wage areas, where houses are larger. This *convexity* effect increases the nationwide price level. Third, consistent with the findings in Glaeser, Gyourko, and Saks (2007) and Davis and Heathcote (2005), there is a large increase in the non-structure component of house prices which can be measured as the difference between house price and construction cost. Indeed, this difference represents the shadow value of relaxing regulation: as an island becomes more productive and attracts more households, the shadow value goes up despite the fact that the number of permit stays the same. Fourth, our mechanism creates post-1975 job flows from low-wage to high-wage metropolitan areas, consistent with the data.

We then show that tightening housing supply regulation alone is not sufficient to explain these facts. Starting from the same 1975 level of supply regulation, we gradually tighten the limits on constructions over the next thirty years while keeping the dispersion of wages constant. In order to maximize the impact of supply regulation on prices, we assume that the tightening is more pronounced in high-wage metropolitan areas. By 2004, the model does predict an increase in the level and dispersion of cross-sectional house prices, but the effects are quantitatively small. Indeed, the negative impact of regulation on local housing supply is almost completely offset by the equilibrium response of households who decide to move out of tightly regulated areas towards less regulated areas. Because this shifts the local demand down at the same time as the supply, the price impact of supply regulations ends up quantitatively small. This result is robust to the magnitude of the change in regulations, assumptions on how it is distributed across metropolitan areas and how it evolves over time, as well as to assumptions on the future path of regulation. Impediments to labor mobility, which are absent from the model, are likely to slow down this reduction in housing demand, but they are unlikely to reverse it.

Finally, we evaluate the welfare costs of increasing housing supply regulation, an exercise suggested by Arnott and MacKinnon (1977) and Glaeser, Gyourko, and Saks (2007). In our model welfare costs arise from the spatial misallocation of labor: regulation prevents households from moving towards highly productive metropolitan areas, which reduces aggregate output. The calibration shows that the net welfare cost is potentially large. The precise quantitative effects depend on whether regulation is tighter in more productive areas, on how much construction is reduced by the regulation, on whether the increase in wage dispersion comes about through an increase in idiosyncratic shocks or through an increase in the persistence of the shocks, and on the assumptions about the future path of the wage dispersion. In our benchmark case, we find that total welfare in 2004 would have been 2% higher without the increase in regulation. Comparing final steady states makes the flow welfare difference grow to 3.3%-7.8%, depending on the specification. We also calculate the welfare costs along the transition path.

The rest of the paper is organized as follows. Section 2 presents our island model. Section 3 calibrates a steady-state of the model to match features of 1975 data. Section 4 provides the quantitative impact on prices and welfare of increasing wage dispersion and regulation. Section 5 discusses extensions to the model, the related literature, and concludes.

2 An Island Economy

We design an island economy in order to study the quantitative impact of regulation and wage dispersion on cross-sectional house prices. The model takes regulation and wage dispersion as exogenous, and provides the endogenous joint dynamics of cross-sectional house prices, construction, and employment.

2.1 The Economic Environment

The first paragraph of the setup describes the stochastic environment as well as the technologies for producing housing and non-housing consumption. The second paragraph describes households.

2.1.1 Information and Technology

Time is taken to be discrete and runs for ever. The economy is made up of a measure-one continuum of homogenous metropolitan areas we call islands. At each time $t \in \{1, 2, \ldots\}$, an island's production function of non-housing consumption good is linear in labor with an idiosyncratic productivity $A_t \in \mathbb{R}_+$.[5] Together with competition, our linear specification means that the the real wage is equal to the productivity A_t.[6] We take the wage process $\{A_t\}_{t=1}^{\infty}$ to be a first-order Markov chain with a positive and possibly unbounded support $[A_{\min}, A_{\max}]$ (that is $A_{\min} \geq 0$ and $A_{\max} \leq \infty$), and with a strictly positive transition density $g_t(A_{t+1} \mid A_t)$ over $(A_{\min}, A_{\max})$. We assume that the wage process is persistent in the sense

[5]Alternatively, one can obtain such a linear specification by assuming that i) an island production function is $F(k, A_t n)$, for some constant return to scale function of capital k and effective units $A_t n$ of labor, and ii) capital is mobile across islands and can also be invested in a technology with constant return R.

[6]Although a non-linear specification would provide a more general treatment of the mapping between productivity and wage, our favorite calibration strategy would nevertheless be to pick the productivity distribution in order to match the wage distribution. Indeed, measures of the real wage per job are readily available at the metropolitan area level, whereas measures of total factor productivity are not.

that, if $A' > A$, then the density $g_t(\cdot \mid A')$ stochastically dominates the density $g_t(\cdot \mid A)$, in the first-order sense.[7]

Each island starts at time zero with some initial wage A_0 and housing stock $H_0 \in (0, H_{\max})$. Although we allow the initial housing stock of an island to be correlated with the initial wage, we assume that, conditional on A_0, it does not help predicting the future path of wage.[8] We denote by $g_0(A_0, H_0)$ the initial cross-sectional density of wage and housing stock, which we take to be strictly positive over $(A_{\min}, A_{\max}) \times (0, H_{\max})$. At each time $t \in \{1, 2, \ldots\}$, we index each island by its wage history $A^t \equiv (A_0, A_2, \ldots, A_t)$ and by its initial housing stock H_0. We denote by $g_t(A^t, H_0)$ the unconditional density of history (A^t, H_0). By the law of large numbers (Sun (2006)) this also represents the density of islands with history (A^t, H_0).

Each period, firms can purchase construction material in order to construct housing services in any island. A representative construction firm can transform Δ units of construction material into housing consumption according to the Leontief production function $\min\{\Delta, \Pi_t(A_t)\}$, where $\Pi_t(\cdot)$ is some strictly positive bounded function of the current wage A_t of the island. This function is designed to represent not only technological and physical constraints on construction (such as the amount of constructible land) but also regulatory constraints on construction. Loosely speaking, one can think of $\Pi_t(A_t)$ as the number of building permits in an island with current wage A_t. We assume that construction is irreversible and the stock of housing consumption depreciates at rate $\delta \in (0, 1)$. These assumptions are summarized by the constraints

$$\begin{aligned} \Delta_t(A^t, H_0) &\geq 0 && (1) \\ \Delta_t(A^t, H_0) &\leq \Pi_t(A_t) && (2) \\ H_t(A^t, H_0) &= (1-\delta) H_{t-1}(A^{t-1}, H_0) + \Delta_t(A^t, H_0), && (3) \end{aligned}$$

where $\Delta_t(A^t, H_0)$ denotes the construction flow and $H_t(A^t, H_0)$ denotes the housing stock in island (A^t, H_0). Inequality (1) is the irreversibility constraint, inequality (2) is imposed by the Leontief construction technology, and equation (3) is the law of motion for the housing

[7] This representation of persistent stochastic process is used, for instance, by Lucas and Prescott (1974).

[8] Formally, $g(A_t \mid A_{t-1} \ldots, A_0, H_0) = g(A_t \mid A_{t-1}, \ldots, A_0)$. This will imply that, in an dynamic equilibrium, the housing stock does not Granger (1969) cause wage.

stock. Lastly, the resource constraint for construction material is

$$\int \Delta_t(A^t, H_0) g_t(A^t, H_0)\, dA^t\, dH_0 \leq M, \tag{4}$$

where M denotes the per-period endowment of perishable construction material.

2.1.2 Preferences

The economy is populated by a measure one continuum of infinitely-lived households with discount factor $\beta \in (0, 1)$. Households have separable utility for non-durable consumption and housing services. Their flow utility for non-durable consumption is taken to be linear, while their flow utility over housing consumption is represented by some strictly increasing, strictly concave, bounded above and twice continuously differentiable function $v : (0, \infty) \to \mathbb{R}$. We assume in addition that $v(\,\cdot\,)$ is unbounded below, meaning that $v(h)$ goes to minus infinity as h goes to zero. Lastly, and without further loss of generality since $v(h)$ is bounded above, we assume that $v(h)$ goes to zero as h goes to infinity.[9,10]

We assume that, each period, a household supplies inelastically one unit of labor in the island of his choosing. Letting $n_t(A^t, H_0)$ be the number of households who choose to live in island (A^t, H_0), we have

$$\int n_t(A^t, H_0) g_t(A^t, H_0)\, dA^t\, dH_0 = 1, \tag{5}$$

since the number of households in the economy must sum to 1.

A key assumption of our model is that households are constrained to live in the same island they choose to work.[11] In other words, housing consumption $h_t(A^t, H_0)$ per household

[9] An iso-elastic utility function $v(h) = h^{1-\gamma}/(1-\gamma)$ satisfies these parametric assumptions when $\gamma > 1$. Lemma 4 of Appendix A.1 shows that these properties imply that the utility function $v(h)$ satisfies Inada (1963) conditions.

[10] The key implication of quasi-linearity is that the marginal utility of consumption is equated across islands and, in that sense, that our *ex-ante* identical households are fully insured. This result also holds with a strictly concave utility function over non-housing consumption as long as households can trade full set of claims contingent on island assignment, i.e., lotteries along the line of Rogerson (1988), Hansen (1985), Prescott and Ríos-Rull (1992), and Rocheteau, Rupert, Shell, and Wright (2006). See also the static setup of Appendix B.1. Because of full insurance, such a model would remain tractable and only differ from the present model along one dimension: the equalized marginal utility of consumption would change over time.

[11] The Bureau of Economic Analysis (Regional Economic Information System) uses this criterion to define a metropolitan area, our empirical counterpart to an island.

in island (A^t, H_0) is subject to the local resource constraint

$$n_t(A^t, H_0)h_t(A^t, H_0) \leq H_t(A^t, H_0). \tag{6}$$

An *allocation* is a collection of measurable functions specifying, for each time $t \in \{1, 2, \ldots\}$ and each island (A^t, H_0), the number $n_t(A^t, H_0)$ of households, the housing consumption $h_t(A^t, H_0)$ per household, the flow $\Delta_t(A^t, H_0)$ of construction, and the housing stock $H_t(A^t, H_0)$. An allocation is *feasible* if it satisfies the constraints (1)-(6).

2.2 Definition of a Competitive Equilibrium

Every period, competitive firms purchase construction material at price μ_t in order to produce and sell housing consumption in the islands of their choosing. The price of housing consumption in island (A^t, H_0) is denoted $p_t(A^t, H_0)$. Hence, the representative construction firm problem is to choose quantities $\Delta_t(A^t, H_0)$ of construction material in order to maximize

$$\int \left(p_t(A^t, H_0) - \mu_t\right) \Delta_t(A^t, H_0) g_t(A^t, H_0)\, dA^t\, dH_0, \tag{7}$$

subject to (1)-(2).[12]

We assume that competitive real-estate firms purchase the stock of housing consumption in all islands and rent it to households.[13] The rent in island (A^t, H_0) is denoted by $\rho_t(A^t, H_0)$. Clearly, a real-estate firm finds it optimal to supply all its housing stock as long as the rent is strictly positive.

Competition among real-estate firms implies that the current price of housing consumption is equal to the rent plus the present value of the price next period, net of depreciation:

$$p_t(A^t, H_0) = \rho_t(A^t, H_0) + \beta(1-\delta)E_t\left[p_{t+1}(A^{t+1}, H_0)\,|A^t, H_0\right].$$

[12]The assumption of a centralized market for construction material implies that construction costs are the same in every island. Although this implication of the model is obviously violated in the data, one might argue that it is a reasonable approximation for the question at hand. Indeed, very little of the cross-sectional variation in housing prices appears to be due to variation in construction costs (e.g., Davis and Palumbo (2006)).

[13]This assumption is made for expositional simplicity. As it is standard with frictionless housing markets, the same equilibrium price would arise if households were purchasing their homes instead of renting them. See Spiegel (2001) for an equilibrium model of house prices and construction with a moral hazard friction.

Under the transversality condition

$$\lim_{T\to\infty} \beta^T E_t \left[p_{t+T}\left(A^{t+T}, H_0\right) \mid A^t, H_0\right] = 0, \tag{8}$$

we obtain the Topel and Rosen (1988) result that a house price is equal to the expected present value of rents net of depreciation

$$p_t(A^t, H_0) = E_t \left[\sum_{j=0}^{\infty} \beta^j (1-\delta)^j \rho_{t+j}(A^{t+j}, H_0) \,\middle|\, A^t, H_0\right]. \tag{9}$$

Lastly, because of full mobility, the household's inter-temporal problem can be reduced to a sequence of static problems: every period, a household chooses in which island to work, and how much housing to rent in that island.[14] Consider a household who chooses to live in island (A^t, H_0). His housing consumption must solve

$$u_t(A^t, H_0) = \sup_{h\geq 0} \left\{A_t + v(h) - \rho_t(A^t, H_0)h\right\}. \tag{10}$$

Moreover, the solution of the household optimal location problem is to work and live in any island that yields the maximum value

$$U_t = \sup_{(A^t, H_0)} u_t(A^t, H_0) \tag{11}$$

of moving.[15] A *competitive equilibrium* is a price system and a feasible allocation such that: i) the price and the rent solve (9), ii) given the price $p_t(A^t, H_0)$ of housing consumption and the price μ_t of construction material, the construction flow $\Delta_t(A^t, H_0)$ solves the construction firm's problem, ii) given the rent $\rho_t(A^t, H_0)$, housing consumption $h_t(A^t, H_0)$ solves the household's problem and the allocation of households across islands is individually optimal,

[14]The inter-temporal problem in the background is as follows: every period, a household chooses the probability of being assigned to each island, his housing and non-housing consumption in each island, and trades a full set of claims conditional on his island assignment.

[15]The right-hand side of (10) contains all the terms of the full-blown inter-temporal Lagrangian that are relevant for the choice of time-t housing consumption and island location: the utility $v(h)$ of housing services, plus the wage A_t minus the rent $\rho_t(A^t, H_0)h$ times the marginal utility of consumption, which is equal to 1 because of linear utility.

that is

$$n_t(A^t, H_0) \geq 0 \qquad \text{if} \qquad u_t(A^t, H_0) = U_t \tag{12}$$

$$n_t(A^t, H_0) = 0 \quad \text{otherwise.} \tag{13}$$

Equation (12) says that, in equilibrium, utility must be equalized across populated islands.

2.3 House Price Implications of Labor Mobility

This subsection provides the mechanism through which equilibrium house prices compensate for cross-sectional wage differences. We show that a one-dollar increase in wage in some island must increase the rent of the island, and that the increase in rent is larger in a high- than in a low-wage island. This convexity property is key to the model pricing implications as it implies that a mean-preserving increase in wage dispersion increases the cross-sectional average level of rents and house prices.

2.3.1 Housing Consumption, Rent, and Price

First, in an island without population, the demand for rental housing consumption is equal to zero. Hence, in an equilibrium, a real-estate firm must be indifferent between supplying its housing stock or not, implying that the rent is $\rho_t(A^t, H_0) = 0$.[16] Plugging this back into (10) and using the fact that $v(h)$ goes to zero as h goes to infinity, we find that, if an island is not populated, then $u_t(A^t, H_0) = A_t \leq U_t$. Now consider a populated island. In that case, the rent $\rho_t(A^t, H_0)$ must be strictly positive, and (10) has an interior solution characterized by the first-order condition $v'(h_t(A^t, H_0)) = \rho_t(A^t, H_0)$. Plugging this back into the indifference condition (12) shows that

$$A_t - w\left(h_t(A^t, H_0)\right) = U_t, \tag{14}$$

where $w(h) \equiv hv'(h) - v(h)$ measures the difference between the housing expenditure and the utility from housing. This immediately implies that $h_t(A^t, H_0) = w^{-1}(A_t - U_t)$: housing consumption in island (A^t, H_0) only depends on its current wage level A_t, and the maximum value U_t of moving to some other island. Since $w(h)$ is a decreasing function, it follows that

[16]Note that the local housing supply is strictly positive in every island. Indeed, each island starts with a strictly positive housing stock, and the depreciation rate δ is strictly positive.

housing consumption is a decreasing function of the current wage A_t. This is intuitive: if a household is indifferent between a high- and a low-wage island, then this household must be enjoying more housing consumption in the low-wage island.[17]

Similarly, rents and house prices only depend on the current wage of the island. Indeed, if an island is populated, then $A_t \geq U_t$ and the rent is $v' \circ w^{-1}(A_t - U_t)$. If an island is not populated then $A_t - U_t \leq 0$, and the rent is equal to zero. In sum, $\rho_t(A^t, H_0) = R\left(\max\{A_t - U_t, 0\}\right)$, where $R(x) \equiv v' \circ w^{-1}(x)$ and $R(x)$ is an increasing function such that $R(0) = 0$.[18] Plugging the rent back into the pricing equation (9) and using the Markov property shows that

$$P_t(A_t) = E\left[\sum_{j=0}^{\infty} \beta^j (1-\delta)^j R\left(\max\left\{A_{t+j} - U_{t+j}, 0\right\}\right) \,\middle|\, A_t\right] \tag{15}$$

is a function of the current wage A_t but does not depend on other idiosyncratic characteristics of an island. Our assumption that the wage process $\{A_t\}_{t=0}^{\infty}$ is persistent also implies that $P_t(\,\cdot\,)$ is an increasing function of the current wage A_t. Intuitively, a high wage A_t not only implies that the current rent is high but also that future rents will be high on average. These properties are summarized in the following proposition:

Proposition 1. *At each time $t \in \{1, 2, \ldots\}$, housing consumption, the rent, and the price are only a function of the island's current wage, A_t, and do not depend on any other idiosyncratic characteristic of the island. In addition, housing consumption $h_t(A_t)$ is decreasing, the rent $\rho_t(A_t)$ is increasing, and the price $P_t(A_t)$ is increasing, with the island's current wage, A_t.*

2.3.2 Convexity Effect

The indifference condition (12) has the following key implication:

Proposition 2 (Convexity)**.** *At each time $t \in \{1, 2, \ldots\}$, the rent $\rho_t(A_t)$ is a convex function of the island's current wage.*

To prove this result, we rewrite the indifference condition (12) as

$$A_t + v(h_t(A_t)) - \rho_t(A_t) h_t(A_t) = U_t. \tag{16}$$

[17] The models of Rappaport (2006b, 2006a) focus on such cross-sectional differences in per capita housing consumption, or 'crowdedness'.

[18] Lemma 4 of Appendix A.1 shows that $R(x)$ is continuous at zero.

Differentiating (16) with respect to A_t and keeping in mind that the value U_t of moving is constant in the cross-section, we obtain:

$$dA + v'(h)\,dh - \rho\,dh - d\rho\,h = 0 \Leftrightarrow dA - d\rho\,h = 0 \Leftrightarrow d\rho/dA = 1/h \tag{17}$$

where the second equality follows from the fact that $v'(h) = \rho$ and time subscripts are omitted for simplicity. Equation (17) illustrates the mechanism through which equilibrium rents compensate for wage differences. A marginal increase in the wage ($dA > 0$) creates an inflow of households in the island, and reduces the equilibrium amount of housing consumption per household ($dh < 0$). This reduces the household's utility by $v'(h)\,dh$, but, holding the rent constant, it simultaneously reduces the household's housing expenditure by the same amount. An optimal housing consumption choice implies that the rent ρ is equal to the marginal utility $v'(h)$, or that $v'(h)\,dh - \rho\,dh = 0$. Hence, the net effect of the second and the third term of the indifference condition (17) is zero. Therefore, the rent has to increase by an amount $d\rho$ such that the marginal increase in housing expenditure, $d\rho\,h$, is equal to the the marginal increase in the wage, dA. This means in particular that the marginal increase in rent, $d\rho$, must be larger in an island with smaller housing consumption. Since housing consumption h decreases with wage, this immediately implies that that the rent is a convex function of the wage. Appendix B.1 shows that this convexity property holds more generally for any non-separable, concave utility function over c and h .

The house price implications of an increase in wage dispersion follow immediately from the properties of the equilibrium rent. Consider a (mean-preserving) increase in wage dispersion, Δ, holding the value U_t of moving constant, and suppose that the wage A can be either high $\bar{A} + \Delta$ or low $\bar{A} - \Delta$, with equal probability. Because ρ is an increasing function of A, the rent increases in high-wage islands and decreases in low-wage islands. Hence, the cross-sectional dispersion of rents increases. Now, convexity means that the rent increases by more in high-wage islands than it decreases in low-wage islands. This creates two level effects. First, the cross-sectional average rent goes up. Second, the house price level increases *in every island*. To understand this second effect, consider the example of an independent and identically distributed wage process. That is, every period, the wage in an island is an independent draw from the cross-sectional distribution. Our pricing equation (15) implies that the price in an island with current wage A is

$$P(A) = \rho(A) + \frac{E\left[\rho(A')\right]}{1 - \beta(1-\delta)}, \tag{18}$$

where the expectation is taken with respect to the cross-sectional distribution of wage. Convexity implies that an increase in wage dispersion increases the second term in the price equation (18).[19] In words, the house price increases because households anticipate that the rent will increase by more when the island draws a high wage than it will decrease when it draws a low wage.

2.3.3 Outside Option Effect

The previous paragraph analyzed the impact of an increase in wage dispersion on rents, under the assumption that the value of moving U_t stays constant. This is a "partial equilibrium" reasoning as U_t is an endogenous variable. Section 2.4 explains how U_t is determined in equilibrium, and our calibration illustrates how it depends on exogenous parameters. The analysis suggests that an increase in wage dispersion increases the value U_t of moving. The indifference condition then implies that the increase in U_t decreases the rent because of an outside option effect: indeed, when the value of moving to some other island goes up, households are able to bid the rent down in their island. Therefore, the outside option effect works in the opposite direction of the convexity effect. In all of our calibrated examples, however, the outside option effect is quantitatively small, and the convexity effect dominates.[20]

2.3.4 The Impact of Local Housing Supply on House Prices

An important implication of labor mobility that follows from equation (15), is the absence of a local housing supply effect on house prices.[21] Holding current wage A_t fixed, a reduction in local housing supply is offset by a simultaneous reduction in local housing demand, resulting in no price impact. Changing the *aggregate distribution* of local housing supplies, as in the calibration of Section 4.2, has an impact on the distribution of house prices through its effect

[19] If the wage process is persistent, then the same effect operates in the long run. Indeed, by ergodicity, the distribution of the wage T periods ahead converges to the cross-sectional distribution as T goes to infinity.

[20] See Appendix B.2 for an analytical argument of this dominance based on a second-order approximation.

[21] See Quigley and Raphael (2005) for time-series evidence on such effect.

on the equilibrium values of moving, $\{U_{t+j}\}_{j=0}^{\infty}$.[22]

2.4 Recursive Characterization of a Competitive Equilibrium

In this section we provide a recursive characterization a competitive equilibrium.

2.4.1 The Distribution of Households

In a populated island, the local housing market clearing condition (6) implies that $n_t(A^t, H_0) = H_t(A^t, H_0)/h_t(A^t, H_0) = H_t(A^t, H_0)/w^{-1}(A_t - U_t)$. In an island which is not populated we can write $n_t(A^t, H_0) = 0 = H_t(A^t, H_0)/w^{-1}(0)$, because $w^{-1}(x)$ goes to infinity as x goes to zero. Letting $\Phi(x) \equiv 1/w^{-1}(x)$, this can be written compactly as

$$n_t(A^t, H_0) = H_t(A^t, H_0)\Phi\left(\max\left\{A_t - U_t, 0\right\}\right). \tag{19}$$

Because there is a measure one of households, the value U_t of moving solves

$$\int H_t(A^t, H_0)\Phi\left(\max\left\{A_t - U_t, 0\right\}\right)\, dA^t\, dH_0 = 1. \tag{20}$$

Together with the distribution $H_t(A^t, H_0)$ of housing stocks, a solution U_t of this equation pins down the distribution $n_t(A^t, H_0)$ of households.

2.4.2 The Distribution of the Housing Stock

To complete our characterization of an equilibrium, we need to solve for the distribution $H_t(A^t, H_0)$ of housing stocks. To that end, we note that the linearity of the construction firm's problem implies that an optimal construction plan is simply to build $\Pi_t(A_t)$ units of housing consumption in every island such that $p_t(A^t, H_0) > \mu_t$. Since we proved that $p_t(A^t, H_0)$ only depends on the current wage A_t and is increasing, it follows that there is some wage cutoff $A_t^* \in [A_{\min}, A_{\max}]$ such that a construction firm build $\Pi_t(A_t)$ units of housing consumption if $A_t \geq A_t^*$, and does not construct otherwise. Plugging this back into

[22] One way to generate local supply effects is to change the model and assume that the production function of non-housing consumption has decreasing returns to scale. In such a model, keeping wage the same, a higher housing stock attracts more households and lowers the wage because of decreasing marginal returns. Because of the indifference condition, a lower wage translates into a lower house price.

the resource constraint (4) for construction material, we obtain

$$\int_{A_t^*}^{A_{\max}} \Pi_t(A_t) g_t(A_t)\, dA_t \leq M, \tag{21}$$

with an equality if $A_t^* > A_{\min}$, and where $g_t(A_t)$ denotes the probability density of the current wage A_t. In what follows, we make sure that equation (21) holds with equality by assuming that

$$\int_{A_{\min}}^{A_{\max}} \Pi_t(A_t) g_t(A_t)\, dA_t > M. \tag{22}$$

This means that there is a large supply of constructible land: that is, the amount of housing that could be built on all constructible land, on the left-hand side of (22), is greater than the amount of housing that can be built with the available supply M of construction material. In that case, the construction cutoff A_t^* is greater than $A_{\min}$ at each time. Moreover, at the cutoff A_t^*, the representative construction firm is indifferent between constructing and not constructing, implying that

$$\mu_t = P_t(A_t^*), \tag{23}$$

at each time $t \in \{1, 2, \ldots\}$.

2.4.3 A Recursive Characterization

The above paragraphs show that an equilibrium can be calculated recursively as follows:

1. On solves first for the sequence $\{A_t^*\}_{t=1}^{\infty}$ of construction cutoffs using (21).

2. Given the construction cutoffs, one solves for the distribution $H_t(A^t, H_0)$ of housing stocks using the first-order stochastic difference equation:

$$H_t(A^t, H_0) = (1 - \delta) H_{t-1}(A^{t-1}, H_0) + \Pi_t(A_t) \mathbb{I}_{\{A_t \geq A_t^*\}}. \tag{24}$$

3. Given the distribution of housing stocks, one solves for the sequence $\{U_t\}_{t=1}^{\infty}$ of moving values using (20), and for the distribution $n_t(A^t, H_0)$ of households using (19).

4. Given the sequence of moving values, one solves for prices using (15) and (23).

In Appendix C we describe in details a computation procedure based on these 4 steps. The main computational challenge to keep track of the joint, cross-sectional distribution $H_t(A^t, H_0)$ of wage and housing stock. It turns out that all equilibrium objects and population-weighted moments of interest can be calculated without knowledge of this joint distribution. Instead, it is sufficient to keep track of one conditional moment: the average housing stock at time $t-1$, given the current wage A_t of the island. Appendix C shows how to use this function of A_t in order to calculate cross-sectional house price moments. One benefit of this approach is that transitional dynamics can be computed without relying on any linearization technique. This turns out to be important for our results, because the price impact of wage dispersion stems from a non-linear convexity effect. We conclude this section with the following proposition:

Proposition 3 (Existence, Uniqueness, and Efficiency)**.** *Assume that (22) holds. If* $A_{\max} = \infty$*, assume that: i) at each time* $t \in \{1, 2, \ldots\}$*, for all* $U \in \mathbb{R}$*,* $\int_{A_{\min}}^{\infty} \Phi\left(\max\{A - U, 0\}\right) g_t(A)\, dA < \infty$ *and ii) the price function (15) is well defined for any constant sequence of moving values. Then there exists a unique competitive equilibrium. In addition, the equilibrium allocation is Pareto optimal, subject to the constraint embedded in the permit function* $\Pi(A_t)$*.*

Note that the technical conditions i) and ii) are immediately satisfied when $A_{\max} < \infty$. In Appendix B.3, we prove that, under natural stationarity assumption, the economy converges towards some steady state.

3 Calibration Parameters and Targets

In this section we first calibrate our parameters so that a steady state of the model matches key moments of the wage and population distribution in 1975. We then present the moments of the post-1975 house price distribution that we seek to match.

3.1 Calibration Parameters

We first explain our choice of preference and technology parameters.

3.1.1 Preferences

Because we calibrate the model at annual frequency, we follow Cooley and Prescott (1995) in taking households' discount factor to be $\beta = .95$. Households have iso-elastic utility function $v(h) = \kappa h^{1-\gamma}/(1-\gamma)$ over housing consumption, implying that the price elasticity of housing demand is equal to $-1/\gamma$. Because the micro-level evidence of Hanushek and Quigley (1980)[23] suggest an elasticity of about $-.5$, we let $\gamma = 2$. We pick κ to match the median housing expenditure to income share of 0.12 in 2000 Census data.[24]

3.1.2 Wages

Wage per job data are available for 955 U.S. metropolitan and micropolitan areas[25] for 1969-2004 from the Regional Economic Information System (REIS) compiled by the Bureau of Economic Analysis (BEA, Table CA34).[26] These 955 metropolitan areas account for 95% of all jobs in the U.S. We deflate the real wage per job by the nationwide Consumer Price Index (all items excluding shelter) with base 1983 to obtain the real wage per job.[27, 28]

Wage per job appears very persistent at the local level, and Figure 3 suggests that the 1975 cross-sectional distribution of the log real wage per job is well described by a normal.

[23] They exploit a natural experiment, the Housing Demand Allowance Experiment, where a subgroup of 586 low income renters in Phoenix and 799 households in Pittsburgh received rent subsidies ranging from 30-60%, whereas a control group received nothing. They estimate long-run elasticities of -.45 for Phoenix and -.64 for Pittsburgh, based on estimates of how fast the housing demand adjusts towards an equilibrium level in the two years of data.

[24] To that end, we consider the benchmark case in which all islands have the same wage $\overline{A}$ and the same housing stock $\overline{H} = M/\delta$. In the model income is labor income, but in the data it also includes financial income and government transfers. Therefore, total income is $\overline{A}$ divided by the labor income share, which is 0.72 in U.S. data. The housing expenditure, on the other hand, is $\overline{H}v'(\overline{H}) = \kappa/\overline{H}$. Then the housing expenditure to income share solves $.12 = \kappa/\overline{H}/(\kappa/\overline{H} + \overline{A}/.72)$. Rearranging, we obtain $\kappa = \overline{H}(\overline{A}/.72)(.12/(1-.12))$.

[25] The unit of observation is a core-based statistical area (metropolitan statistical area or MSA). Whenever possible we replace the metropolitan area by its metropolitan divisions (there are eleven such instances).

[26] Average wage per job in a region is wage and salary disbursements divided total wage and salary employment. Wage and salary disbursements consists of the monetary remuneration of employees, including the compensation of corporate officers; commissions, tips, and bonuses; and receipts in kind, or pay-in-kind, such as the meals furnished to the employees of restaurants. It reflects the amount of payments disbursed, but not necessarily earned during the year.

[27] It may be preferable to deflate wages by a regional price index, but these data are not available at this level of aggregation. The Bureau of Labor Statistics only provides regional price indices for the 26 largest metropolitan areas. We exclude shelter from the index because we need to deflate house prices by a price index that does not contain the housing component, and because we want to treat wages in the same way as house prices.

[28] Whenever we calculate population-weighted moments of the wage distribution, we use the number of jobs from the same data source as population weights.

These two observations motivate us to assume that each metropolitan area log wage per job, $a_t \equiv \log(A_t)$, follows an independent AR(1) with mean μ_a, persistence ρ_a, and innovation variance $\sigma^2_{\varepsilon t}$:

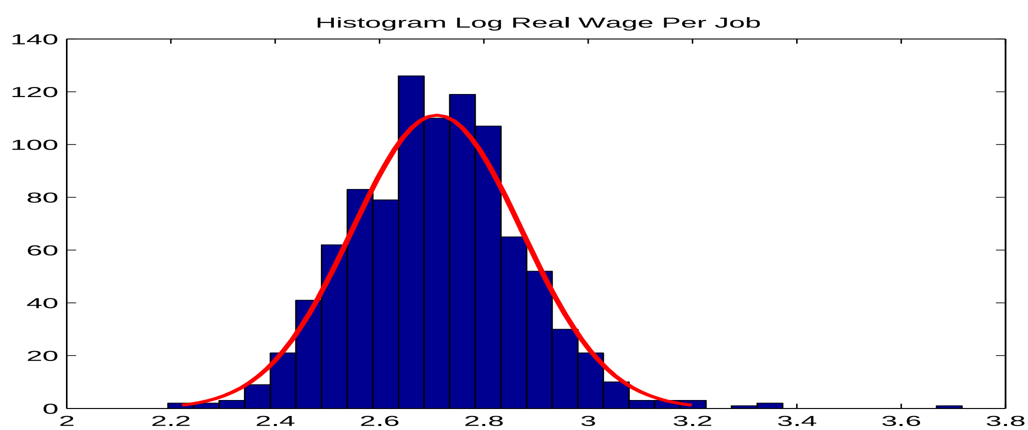
$$a_t = (1-\rho_a)\mu_a + \rho_a a_{t-1} + \sigma_{\varepsilon t}\,\varepsilon_t. \tag{25}$$

The Law of Large Numbers (Sun (2006)) equates the cross-sectional distribution of log-wage per job to its time-series counterpart: hence, log wage per job is log-normally distributed in the cross section, with a mean μ_a and a variance σ^2_{at} which can be calculated by taking variance on both sides of (25):

$$\sigma^2_{at} = \rho^2_a \sigma^2_{at-1} + \sigma^2_{\varepsilon t}. \tag{26}$$

Figure 3: The 1975 Distribution of Log Real Wage Per Job

The figure plots the distribution of the log real wage per job (horizontal axis) across 955 U.S. metropolitan and micropolitan statistical areas in 1975. The wage-per-job is deflated by the CPI excluding shelter. The bars denote the histogram; the solid line is the best fitting normal distribution. The outlier on the right is Anchorage, Alaska.



We calibrate the mean μ_a of log wage to the average real income across metropolitan areas and time. For each year, we compute the cross-regional median real wage per job and then average over time. This average wage per job in 1983 dollars is \$14,810. We find that, over the 1975-2004 period, the real wage per job only went up by 0.14% per year. Hence, a constant mean real wage per job describes the data relatively well. Since our unit of observation is a household and not a job, we multiply this average by the average number of jobs per household of 1.25 (Census data). Expressing wages in the model in thousands of dollars, this gives $\mu_a = \log(15) + \log(1.25)$.[29]

In order to pick the persistence parameter ρ_a, we estimate the AR(1) model for the log real wage per job (a) by pooled ordinary least squares. We obtain $\rho_a = .99$, with a standard error 0.0008. Consistent with our model in which all metropolitan areas have the same mean income, our regression allows no fixed effects: we impose the restriction that the regression intercept is the same for all metropolitan areas. Although a regression with fixed effects may fit the data better, this would introduce too many free parameters in our model.[30]

Finally, we set the 1975 standard deviation of the innovation in a, $\sigma_{\varepsilon 0}$, equal to 0.0173. Together with the other parameters, this value turns out to deliver a 0.0967 value for the steady-state population-weighted coefficient of variation of the real wage per job in levels (A), a level equal to the one in the 1975 data (see the 1975 value in Figure 2).[31]

3.1.3 Construction Technology

We obtain the housing depreciation rate from the ratio of depreciation at current cost and the current cost net stock of residential fixed assets from the Fixed Asset Tables provided by the BEA. After time-averaging this rate between 1969 and 2004, we find that the housing stock depreciates at a rate of 1.6% per annum ($\delta = 0.016$).

We set the yearly endowment M of construction material so that the steady-state per-capita housing consumption M/δ matches the average square footage of a single family house in the U.S. The Census provides annual data on the square foot of floor area in new one-family houses completed for the U.S. The time-average over the 1975-2004 period is 1,872 square feet.[32] Expressing housing services in thousands of square feet, and using $\delta = 0.016$, this means that $M = 0.016 \times 1.872 = 0.03$.

The last object we need to calibrate is the permit function $\Pi_t(A_t)$ which measures the

[29]Of course, Jensen's inequality implies that the log of the average wage per job is not the same as the average of the log wage. In our calibration, however, the discrepancy is quantitatively negligible.

[30]It is well known that the lack of fixed effects biases the estimate of ρ_a upwards. Indeed, an estimation with fixed effects yields a persistence estimate of $\rho_a = .95$. From the perspective of our model, a high ρ_a is a parsimonious way to capture the persistence that is implicit in the fixed-effect specification.

[31]Note that this matching exercise is non-trivial because wages are weighted by the endogenous distribution of households across metropolitan areas.

[32]More precisely, it provides the number of houses in seven square foot bins. We compute the average square footage in a given year as the weighted average of the bin midpoints. For simplicity, we abstract from growth in M over time.

maximum amount of construction per period in an island with wage A. We take this function to be:

$$\Pi_t(A_t) = \pi_a \left(\frac{A_t}{A_{\min}}\right)^{\phi}, \tag{27}$$

where π_a is some positive real number and ϕ could be either positive or negative. For the initial 1975 steady-state, we set $\phi = 0$, in order to capture that the regulation was not tighter in some metropolitan areas than in others in the 1970s. (Section 4.2 models the change in regulation after 1975 by varying ϕ.) Because the parameter π_a determines the distribution of housing across islands, it indirectly governs the distribution of households across islands. Indeed, a larger π_a allows firms to construct more housing in high-wage areas, which in turns increases the population in these areas: this observation motivates us to choose π_a in order to match the 1975 concentration of jobs in high-wage metropolitan areas, as follows.[33]

Each year, we sorts all 955 metropolitan and micropolitan areas into (equal-sized) wage quintiles and compute the fraction of jobs in each quintile. Figure 4 shows that jobs are highly concentrated in the high-wage metropolitan areas. In 1975, 64.7% of U.S. jobs were in the 20% most productive metropolitan areas. By 2004, this fraction had increased to 73.3%. Together with the other parameters, setting $\pi_a = .139$ allows us to match the 1975 value of 64.7%.

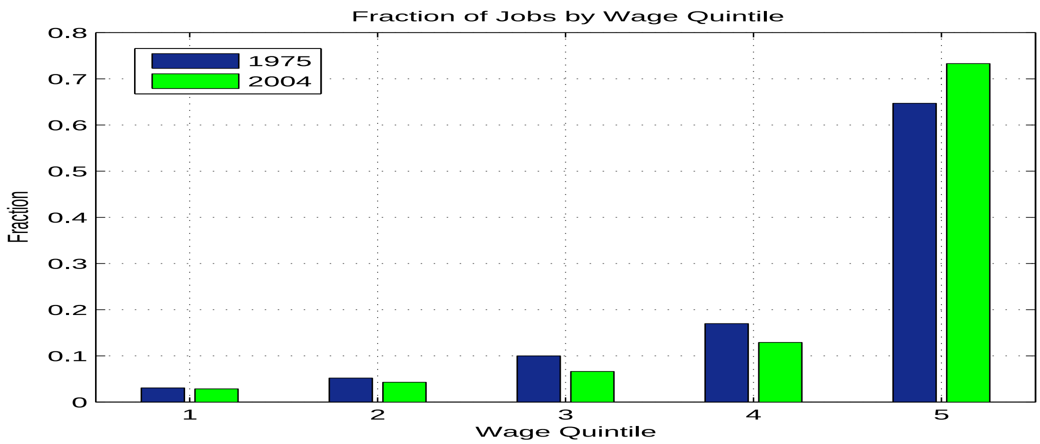
3.2 Calibration Targets

We now describes the house price moments that we seek to match: the cross-sectional population-weighted average and coefficient of variation of house prices post-1975. Figure 5 plots the average and the standard deviation, while Figure 1 showed the coefficient of variation, the ratio of the two. All moments are population-weighted, i.e. weighted by the fraction of jobs in each metropolitan areas. We construct our time-series of home prices from the 2000 Census values for the median single-family home value and the Freddie Mac Conventional Mortgage Home Price Index (CMHPI), a repeat-sale house-price index from 1975 until 2004.

[33]The strategy of calibrating $\Pi(A_t)$ directly to regulation data, instead of relying on its indirect impact on the population distribution, is no real alternative. While there certainly are indices of housing supply constraints at the metropolitan level (Malpezzi (1996) and Saks (2005), each constructed by combining several surveys), these have no time-series dimension. In addition, there is no natural mapping between such ordinal measures and our quantity constraint $\Pi(A_t)$.

Figure 4: The Distribution of Jobs by Wage Quintile

For each year between 1975 and 2004, we sort the real wage per job of 955 U.S. metropolitan and micropolitan statistical areas in five wage quintiles. Within each quintile we compute the total number of jobs in that year. We also compute the total number of of jobs in all five quintiles in that year. The figure plots the ratio of these two, the percentage of jobs in each quintile, in 1975 and in 2004.



Home values are then deflated by the CPI excluding shelter with base 1983. The left panel refers to a smaller sample of 70 metropolitan areas for which the CMHPI has complete house price data going back to 1975 (balanced panel). The right panel refers to the largest possible sample based on all available house price data from the CMHPI. It consists of the same 70 metropolitan areas in 1975, and then the sample size increases steadily until 1996 when 322 metropolitan areas are covered, and stays constant until 2004 (unbalanced panel).

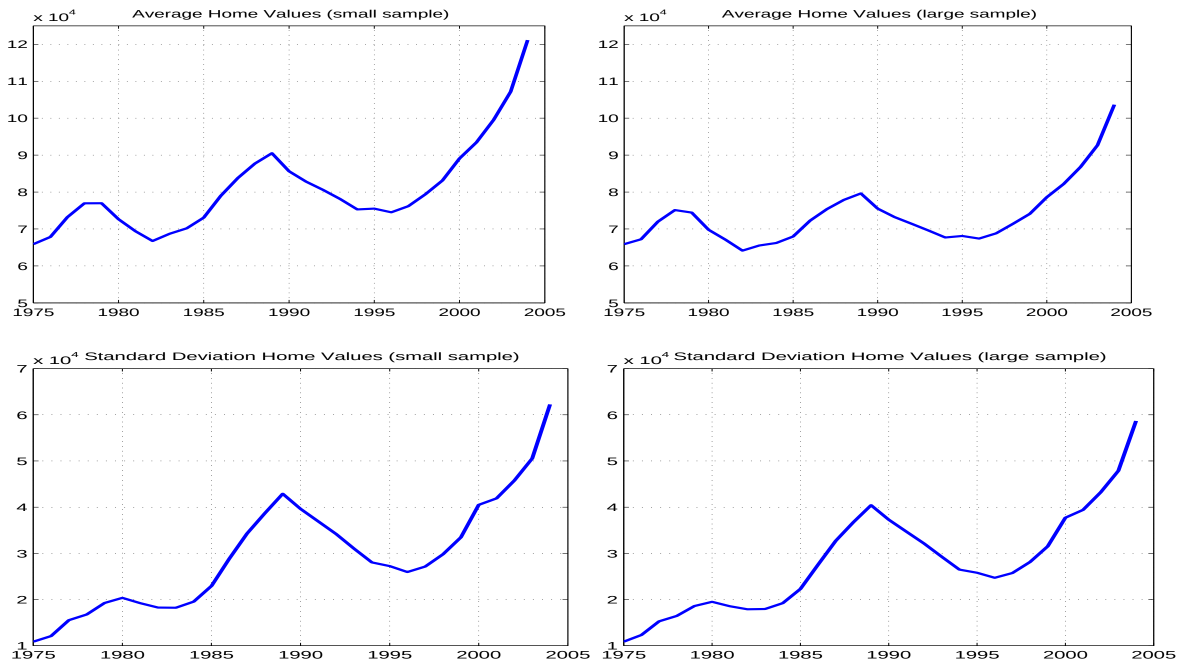
It turns out that our calibration of the 1975 steady state delivers a cross-sectional average house price of $66,000, just as in the data. We also compute year-on-year log changes in the real value of single-family home values, compute the population-weighted average in the cross-section, and average across-time. In the last 30 years, real home values increased by 1.71% per year on average in the sample of 70 MSAs with complete house price data. For all 322 MSAs with some house price data, that growth rate is 1.46%.

4 Quantitative Results

In this section we change key parameters to mimic the increase in wage dispersion (section 4.1), the increase in regulation (section 4.2), or both at the same time (section 4.4). In section 4.3, we discuss our model's implications for labor mobility. We study the economy's

Figure 5: First and Second Moments of House Prices in the Data

The top figure plots the job-weighted average of single-family home values in the data. The bottom panel plots the population-weighted standard deviation.



transition from 1975 until 2004, and ultimately towards the new steady-state. To keep the exposition concise, we focus on a a small set of price and quantity moments of interest. In the figures we present below, the red dashed line denotes the initial 1975 steady state; the green dashed line denotes the final steady state (which may be approached in much more than 30 years); and the blue solid line denotes the transition from the initial steady state to the final steady state.

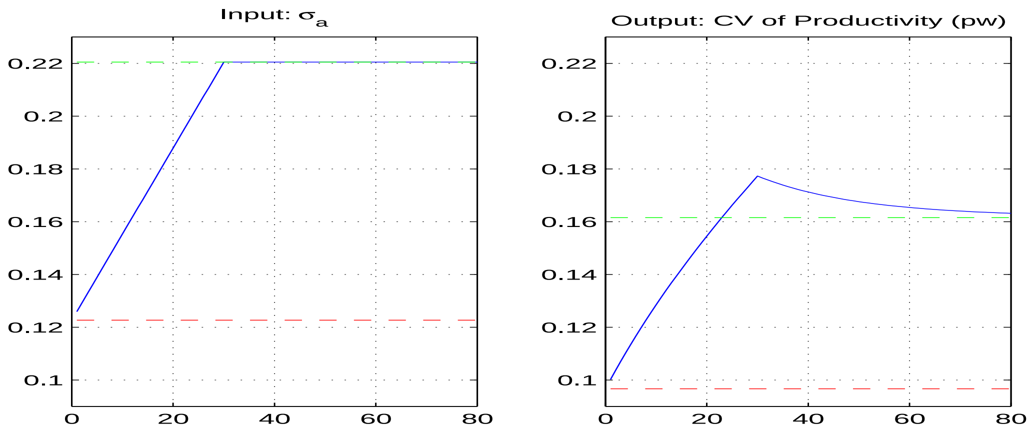
4.1 Increase in Wage Dispersion

In our main calibration exercise, we investigate the effects on an increase in the wage dispersion on house prices. We hold regulation fixed at its 1975 level ($\pi_a = .139$ and $\phi = 0$) and increase the cross-sectional dispersion of wages A.

More precisely, we match the increase in the population-weighted coefficient of variation of wages between 1975 and 2004. The right panel of Figure 6 shows that this coefficient of

Figure 6: Increasing the Wage Dispersion

The left panel plots the cross-sectional standard deviation of log wage we feed into the model (exogenous). The right panel plots the equilibrium population-weighted coefficient of variation of the level of wage that is implied by the parameters in scenario B (endogenous).



variation increased from .0967 in 1975 to 0.1772 in 2004, an increase of 0.08.[34] We match this increase as follows: we keep the persistence constant at $\rho_a = .99$ and we choose the time path of $\sigma_{\varepsilon t}$ so that the standard deviation of log wage, σ_{at}, increases linearly from 0.123 to 0.220 over 30 periods and stays constant thereafter (left panel of Figure 6).[35] The implied path of $\sigma_{\varepsilon t}$ looks as follows: σ_ε jumps from .0173 in the initial steady state (1975) to .0333 in period 1, then gradually increases to .0486 by period 30 (2004), jumps down to .0311 in period 31 and stays constant thereafter. The right panel shows that the resulting wage dispersion is assumed to contract after 2004. Hence, this benchmark case is conservative.[36]

4.1.1 Construction and Population Distribution

When wage dispersion increases, construction firms are eager to construct in the newly productive areas. The construction cutoff A_t^* gradually increases between periods 1 and 30, and stays constant from that point onwards: construction takes place in higher and higher-wage

[34]This panel repeats the left panel of Figure 2, for the small sample of 70 areas. The corresponding increase is slightly higher for the increasing sample in the right panel of Figure 2 (+0.105) and slightly lower in the full sample of 955 MSAs (+.074).

[35]The law of motion (26) suggests that one could also increase σ_{at} by increasing persistence. We discuss this alternative in Appendix B.4.

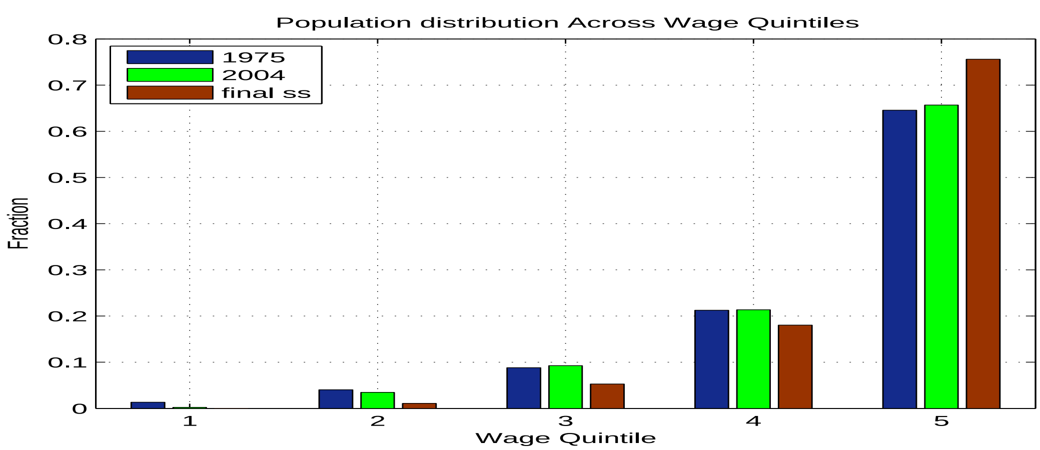
[36]Appendix B.4 also discusses the alternative where $\sigma_{\varepsilon t}$ stays constant after 2004. This leads to a further increase in the wage dispersion after 2004.

areas. These high-wage areas see an increase in their housing stock. The value U_t of moving increases and the population reallocates towards these high-wage metropolitan areas. On the other hand, low-wage islands have no construction, a declining housing stock (because of depreciation), and lose population; they may even be vacated (recall that an island is populated if and only if $A_t \geq U_t$).

Figure 7 shows that the population distribution becomes more concentrated in the top of the wage distribution. The fraction of the population in the highest-wage quintile increases from 64.6% in the initial steady state (1975) to 65.7% after 30 periods (2004). It decreases in the other wage quintiles. Even though σ_{at} is constant after 2004 and the construction threshold A_t^* has reached its steady state value, the housing distribution continues to adjust towards its steady state. The productive metropolitan areas keep growing and in the new steady state, 75.6% of jobs are in the highest-wage quintile. While a bit too slow, the increase in concentration is consistent with the data, where we found that 73.3% of the jobs were in the 20% highest-wage areas in 2004.

Figure 7: Increasing Wage Dispersion: Population Distribution

This figure plots the population distribution by wage quintile. As we did for the data in Figure 4, we now use the model with an increasing wage dispersion to generate population time-series for each MSA. We sort the MSAs into five equally sized wage bins and calculate the ratio of the number of people in each quintile to the number of people in the economy (normalized to 1). The graph shows the distribution in the initial steady state (1975), after 30 years (2004) and in the final steady state.



4.1.2 House Prices

As shown in sections 2.3.2 and 2.3.3 the increase in wage dispersion has two effects, working in opposite directions. First, because the rent is a convex function of the wage, an increase in the dispersion of A increases the expected value of future rents conditional on any current wage level. This *convexity effect* increases the house price. Second, for any current wage level A, $P_t(A)$ decreases because of an *outside option* effect: because the value of moving U_t goes up, households are able to bid their rent down.[37]

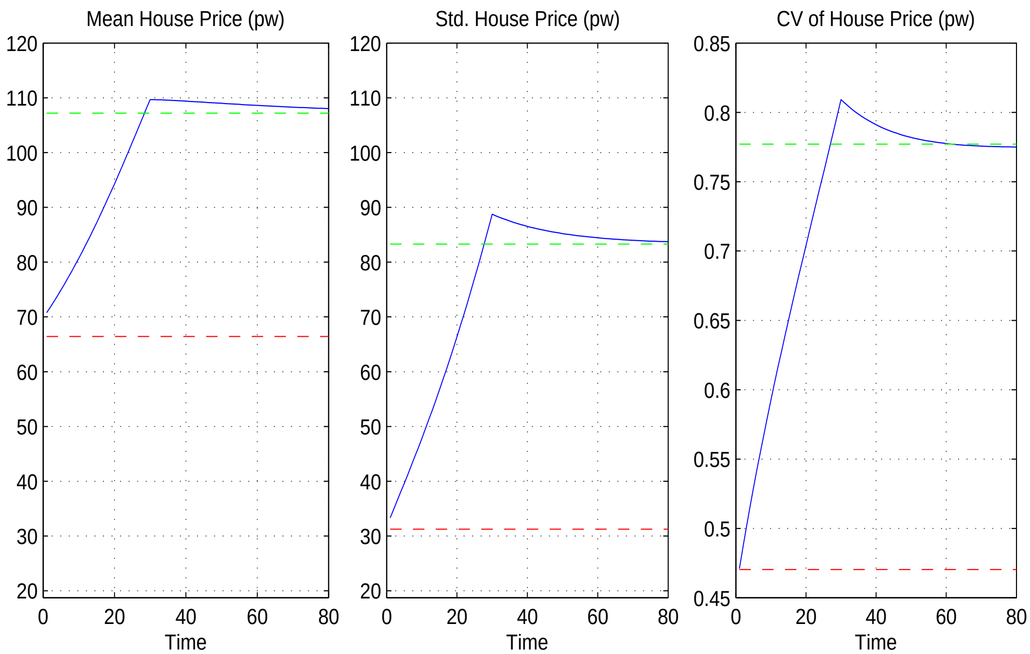
To obtain the price of a single-family house we multiply the square foot price P_t by the average square foot size of a house (1,872). The result is a value in thousands of real dollars which can be compared to the data. The left panel of Figure 8 shows that the population-weighted cross-sectional average house price increases from \$66,000 to \$110,000 over the first 30 years. This represents a cumulative increase of 50.1% or an annual increase of 1.67%, very close to the 1.71% increase in the data. Intuitively, households reallocate from low- towards high-wage areas. This change in demand decreases the price of housing in low-wage areas, and increases it in high-wage areas. However, convexity implies that the increase is larger than the decrease, and creates an increase of the average population weighted house price level. This increase is amplified by a composition effect: indeed, the reallocation of households means that our population weights also shift towards high-wage areas with higher house prices.

The model with increased wage dispersion is also able to account for the observed increase in house price dispersion. The cross-sectional standard deviation increases by 104.4% in the first 30 years and by 98.1% between steady states (middle panel of Figure 8). The standard deviation increases from \$30,000 to \$90,000, while in the data, it increases from \$10,000 to \$60,000 (see bottom panel of Figure 5). The result is a coefficient of variation of house prices that is too high in 1975 (0.47 compared to 0.17), but which has the same increase as in the data. Just as in the data, a 0.08 increase in the coefficient of variation of wages translates into a 0.35 increase in the coefficient of variation of house prices.

[37] Appendix B.2 uses a second-order approximation in order to show that, when the increase in wage dispersion is small, the convexity effect dominates. We show here numerically that this result carries over to our calibration.

Figure 8: Increasing Wage Dispersion: House Prices - First and Second Moments

The left panel plots the population-weighted cross-sectional mean of the square foot housing price multiplied by 1.875, the average housing size in the economy. The middle panel plots the corresponding population-weighted standard deviation; and the right panel plots the coefficient of variation, the ratio of standard deviation to the mean. In all panels, the red dashed line denotes the initial 1975 steady state. The green dashed line denotes the final steady state which may occur well beyond 2004. The blue solid line denotes the transition from the initial steady state to the final steady state.



4.1.3 House Price to Construction Cost Ratio

A last moment of interest is the ratio of housing price to construction cost (HP/CC). Glaeser, Gyourko, and Saks (2007) use this measure of the non-structure component of a house as an indicator of the tightness of regulation. Similarly, Davis and Heathcote (2005) and Davis and Palumbo (2006) measure the non-structure component of housing, which they label the value of land. They all show that the non-structure component has increased a lot over time and in many metropolitan areas.

Our calculations indicate that the increase in wage dispersion increases the simple average HP/CC ratio by 11% between 1975 and 2004 and by 28% between steady states. The increase in the ratio has two sources. First, house prices go up. Second, while construction costs go up initially, they come down to below their initial steady state level. Furthermore, because the population reallocates towards the high HP/CC metropolitan areas, the population-weighted average HP/CC ratio increases by much more: 35% until 2004 and 55% until the new steady state.

Note that although the quantity of permits $\Pi_t(A_t)$ remains constant between 1975 and 2004, the increase in wage dispersion increases the shadow value $P_t(A_t) - \mu_t$ of increasing the number of building permits in an area where the current wage is A_t. In other words, although constraints remain the same, the increase in wage dispersion makes them bind more intensely.

In sum, the increasing wage dispersion is quantitatively able to produce the observed increase in house price level, the observed increase in house price dispersion and to generate an increase in population concentration. It also produces an increase in the average house price to construction cost ratio.

4.2 Increase in Regulation

While the previous section fixed the housing supply constraints at their 1975 level, this section engineers a progressive tightening of housing supply regulation. It holds the wage distribution fixed at its 1975 level in order to isolate the impact of an increase in regulation, and evaluate its quantitative merits in accounting for house prices. The tightening of regulation is accomplished by decreasing ϕ in equation (27) linearly from a value of 0 in 1975 to -.5 in 2004 (see Figure 9).[38] We assume that ϕ stays constant afterwards. The negative value for ϕ means that regulation is tighter in more productive areas, and the declining value for ϕ captures that regulation becomes tighter over time. The combination of the two captures a faster tightening in more productive areas.

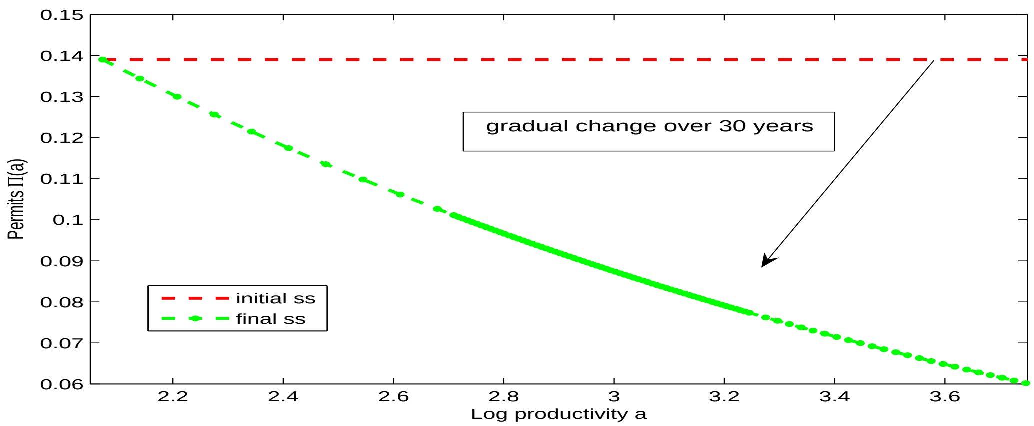
4.2.1 Construction and Population Distribution

When housing supply regulation becomes tighter, construction firms are more constrained in how many housing units they can add in high-wage productive metropolitan areas, relative to the initial steady state. Because housing units depreciate in every period, the result is not only a decline in construction in high-wage areas, but also a gradual decline in their housing stock. Note also that, because less construction takes place in high-wage areas,

[38] While a 2004 value of -.5 for ϕ is our benchmark, we study alternatives in Appendix B.4. In general, we find little change compared to the benchmark case discussed in the main text.

Figure 9: Tightening Housing Supply Regulation

This figure plots the permit function $\Pi(A) = \pi_a (A/A_{\min})^{\phi}$. The top line denotes the situation in 1975 when $\pi_a = .139$ and $\phi = 0$. The bottom line denotes the situation in 2004 and beyond when $\pi_a = .139$ and $\phi = -0.5$. In the years between 1975 and 2004, ϕ decreases linearly from 0 to -0.5, so that the permit function gradually rotates from the top line to the bottom line.



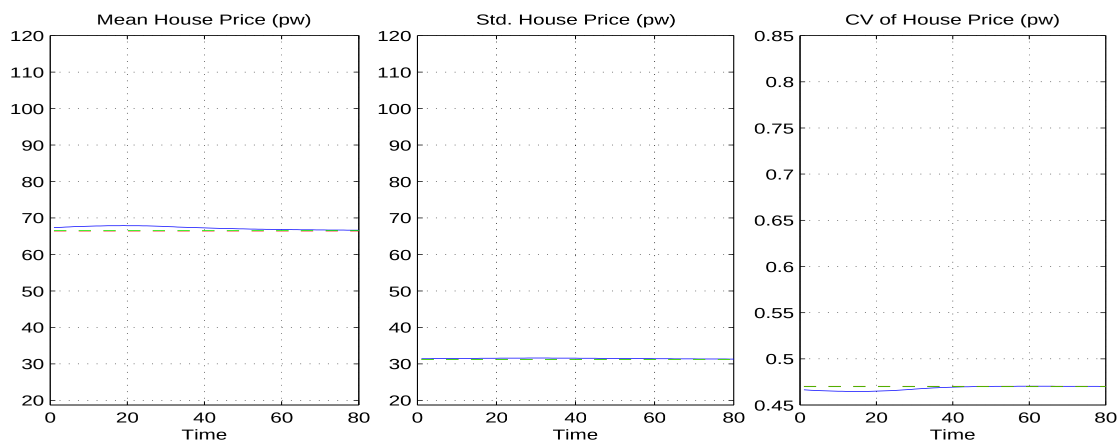
there must be more construction low-wage areas: the construction threshold A_t^* gradually declines between periods 1 and 30 (and stays constant from that point onwards). In addition households reallocate towards these areas, implying a progressive decline in the value U_t of moving. Its decline means that islands which were not populated in the initial steady state now become viable (recall that an island is populated if $A_t \geq U_t$). The population spreads out: the fraction of the population in the highest wage quintile falls from 64.6% in the initial steady state to 54.9% after 30 periods. It increases in the other wage quintiles. After 2004, the population continues to spread out and reaches 52.3% in the final steady state. This evolution of the population distribution is at odds with the data.

4.2.2 House Prices

The effect of tighter regulation is to increase unit house prices $P_t(A_t)$. Indeed, tighter supply regulation pushes some households towards low-wage areas in equilibrium, and lowers the value U_t of moving. This *outside option* effect allows landlords to bid the rent up in every metropolitan area. Housing prices increase as well, as they reflect the expected present discounted value of future rents. The *convexity effect* on house prices is not operative because, in the present scenario, the dispersion of wages is constant.

The left panel of Figure 10 shows that a tightening of regulation after 1975 increases house prices, but only by a tiny amount (the axes are the same as in Figure 8). The cumulative increase over the first 30 years is only 1.85%, which is about 30 times less than in the data. The same is true for the standard deviation of house prices, shown in the middle panel: it increases by only 1.2% in the first 30 years (0.1% between steady states). The result in the model is a coefficient of variation of 0.45 which is pretty much flat over time (right panel). The intuition for the small impact on price of regulation is simple. While tighter regulation reduces the supply of houses in high-wage metropolitan areas, the equilibrium response of labor is to move out, thereby effectively reducing the housing demand in those same areas. The net effect is a very small increase in price.

Figure 10: Tightening Housing Supply Regulation: House Prices - First and Second Moments



4.2.3 House Price to Construction Cost Ratio

An increase in regulation does have a substantial effect on the average HP/CC ratio. The 23% increase in the ratio comes from two effects working in the same direction: house prices go up in every metropolitan area and the construction cost goes down. Indeed, construction costs are set by the "marginal" metropolitan area in which some construction takes place. With tighter regulation, less construction takes place in high-wage areas, and the marginal area with construction has a lower wage (A_t^* falls) and a lower house price ($\mu_t = P_t(A_t^*)$ falls). The increase in the house price to construction cost ratio reflects an increase in the (shadow) cost $P_t(A_t) - \mu_t$ of regulation. As permits become scarcer, their value increase because it becomes more valuable to relax the permit constraint.

4.3 Labor Mobility

In real life, moving costs can limit labor mobility. However, while moving costs might substantially affect year-on-year changes, their effect at 30-year frequencies is certainly much lower. Thus, we conjecture that building such cost in the model will only slow the effects we discussed down, but not reverse them over the 30-year period we focus on.[39]

One concern is that because the model allows for frictionless reallocation, it produces excessive relocation. This turns out not to be the case. We have calculated the migration rate in the model in the scenario with increasing wage dispersion, and compared these migration rates to the data. If anything, the model is generating *too little* mobility. This may be due to the high persistence of wages.[40]

4.4 Increase in Regulation and Wage Dispersion: Implications for Output and Welfare

In the last scenario, we combine the increase in wage dispersion and the increase in regulation. Quantitatively, the results for population concentration, the average house price level, and its cross-sectional dispersion are very similar to the scenario without increasing regulation. Only for the HP/CC ratio do the two effects work in the same direction.[41]

This section focuses on the implications for aggregate output and welfare instead. Ar-

[39] One conservative gauge of such low-frequency mobility is the fraction of people residing in the state in which they were born. In the U.S. 40% of the entire population no longer live in the state in which they were born (Census 2000).

[40] More precisely, we have data for the in-migration and out-migration between 1995 and 2000 for each metropolitan area from the US Census. Together with the population, this allows us to compute a net migration rate. We have those data for the population at large, as well as for the sub-population of young (25-39), single, college-educated. That is an interesting group for us, because they are most likely to move for productive reasons. For this group, the annual out-of-county moving rate is 8.6%. We compute migration rates for groups of regions, sorted by wage-per-job. Likewise, in the model, we construct wage-per-job groups in 1995, and compute the net migration in and out each of these groups over the next five years. When we compare model to data, we find similar patterns: out-migration from low-wage areas and in-migration into high wage areas. The most productive regions see an in-migration of about 0.4% in the model and 2% in the data.

[41] The HP/CC ratio increases by 71% between 1975 and 2004 and an additional 116% afterwards, much more than in either of the previous scenarios. This is because both a decline in the number of permits and the increase in wage dispersion increase the value of relaxing constraint (2). Table 1 in the Appendix summarizes the results.

guably, the world with increasing wage dispersion is the right place to evaluate the welfare costs of housing regulation.

4.4.1 Output

On the one hand, housing supply restrictions reduce the flow of households towards higher-wage, high-productivity areas and therefore reduce aggregate output. On the other hand, an increase in wage dispersion increases the economy's output because there are now more highly productive metropolitan areas where the population can be concentrated. In the former scenario, aggregate output $\int n_t(A^t, H_0)A_t g_t(A^t, H_0)\, dA^t dH_0$ falls by .9% in the 1975-2004 and falls an additional 1.7% en route to its new steady-state. In the latter scenario, output increases by 13.3% until 2004 and an additional 3.5% afterwards. Making both changes at the same time leads to an output increase of 11.3% between 1975 and both 2004 and the final steady state.

It is instructive to isolate the effect of a tightening housing supply regulation on output. Conceptually, this is done by comparing an economy where both changes occur at the same time to an economy where housing regulation stays fixed at its 1975 level. We find that output in 2004 is 1.9% lower than what it would have been without tightening housing supply regulation. The output difference between the two economies in the final steady-state is even 5.5%. We also find that, because the housing stock adjusts slowly over time, two-thirds of the output losses come after 2004.

4.4.2 Welfare Costs of Housing Supply Restrictions

Total welfare takes into account not only the total output of the economy but also the disutility from living in a smaller house when working in a higher-wage metropolitan area. More regulation increases the output loss, but also decreases the welfare loss coming from smaller (average) per capita housing quantities. Indeed, households who flow toward low-wage, less-regulated areas end up living in larger quarters. It turns out that the latter effect is quantitatively small. Flow welfare differences are on the same order as flow output differences: total welfare in 2004 is 1.9% lower in 2004 than it would have been without the increase in regulation, and more than 5% lower between steady states.

A second welfare calculation takes the perspective of a household in 1975 and calculates the compensating variation of a policy that would keep housing regulation at its 1975 level (see equation (30)). This takes into account the transition dynamics by calculating the log difference between the inter-temporal social welfare in two economies: the economy with fixed regulation and the economy with tightening regulation. The difference is 1.06% per annum when the discounted sum runs to the final steady-states. When the discounted sums are cut off in 2004, the welfare cost is smaller, equal to 0.33% per annum. We conclude that the welfare cost of tighter regulation can be large, but because the housing stock takes time to adjust, the welfare losses are not borne until late in the transition.[42]

Our model may not fully capture other costs of living in densely populated areas that could be mitigated by regulation (e.g., congestion externalities). Our welfare calculations remain meaningful but should be interpreted with caution: they provide a lower bound on the size of the costs required for the tightening of regulation to improve welfare.

5 Discussion and Conclusion

We have argued that the observed increase in house price level and house price dispersion across metropolitan areas over the last thirty years in the US can be understood in a simple model of location choice. Faced with an increase in the wage dispersion across metropolitan areas, households chose to reallocate towards higher-wage metropolitan areas. This pushes up house prices in these locations. The observed increase in wage dispersion is sufficient to generate the observed increase in the house price level and the house price dispersion across metropolitan areas. It is also consistent with the increased concentration of the population in high-wage areas, and with the increase in the average house price to construction cost ratio. The same thirty years since 1975 also saw a tightening of housing supply regulation. We have argued that, while a baseline level of regulation is important, a tightening of regulation by itself cannot account for the increase in housing prices or their cross-sectional dispersion. This is because the equilibrium response of households is to reallocate away from tightly regulated areas. Finally, we have considered both frictions at the same time and have found substantial welfare costs of housing supply regulation in an economy with increasing wage dispersion.

[42] Appendix B.4 investigates the sensitivity of these results to alternative specifications of regulatory tightening and increasing wage dispersion.

One extension we plan to undertake is to introduce dis-utility from living in a densely populated area. Such congestion-related externalities may provide a rationale for housing supply limitations that are tighter in higher-wage areas, resembling the exogenous regulations of the present paper. Such normative approach would be complementary to existing work by Ortalo-Magné and Prat (2005) and Glaeser, Gyourko, and Saks (2007), who study the political economy of housing supply regulation. The trade-off between agglomeration effects and congestion costs also has implications for the size distribution of cities (Rossi-Hansberg and Wright (2006)).

Another extension is to consider heterogeneity within the region. If a household's income depended not only on her region of employment, but also on her type of employment (occupation or skill), then the equilibrium will feature heterogeneity within each region in terms of wages and housing consumption (Ortalo-Magne and Rady (2006)), and areas may attract a different skill mix. A simple version with two types of workers may provide a useful way to think about the "reverse causality" effect, where increasing house prices drive up wages. Consider a setting where households choose whether to work in the local manufacturing or in the local service sector upon arrival in the region, and assume that households must consume local services that must be produced locally. An increase in productivity in the manufacturing sector pushes up the housing price in the region for the reasons described in this paper. A new effect is that the wages of local service workers will need to go up to keep them indifferent between staying in this region with higher house prices and moving to a different region.

Our work is related to Gyourko, Mayer, and Sinai (2006), who also study the relationship between the U.S income distribution and cross-sectional house prices. They provide a two-location model, in which regions differ by housing supply and households differ by income and preference for a particular location. In equilibrium, households live in the low-supply location if they either have a strong preference for it or a high income. Because the model is static, it can address house price growth only through comparative statics. Our paper puts forward a different mechanism in that households move for productive rather than consumptive reason. Our focus on the quantitative analysis of the model prompts us to solve a dynamic and stochastic equilibrium model in which housing supply is endogenous. Our results suggest that these additional features have important quantitative implications.

Our model of spatial allocation shares many features with labor search models (Lucas and Prescott (1974), Alvarez and Veracierto (1999), Alvarez and Veracierto (2006)) and the spatial allocation model of Shimer (2005). We complement this literature by focusing on a different friction. In our setup, households do not incur any cost when moving between islands. Instead, the flow of households between islands is limited by the supply of housing in each island. Coen-Pirani (2006) also uses an island model to study migration patterns between US states.

Our work connects to the macroeconomics literature that documents increases in wage dispersion at the individual level (e.g., Hornstein, Krusell, and Violante (2004)) and studied its effects on risk-sharing (Krueger and Perri (2006), Storesletten, Telmer, and Yaron (2004), Heathcote, Storesletten, and Violante (2005), Lustig and Van Nieuwerburgh (2006b)) and on asset pricing (Constantinides and Duffie (1996), Cogley (2002), Lustig (2003), Storesletten, Telmer, and Yaron (2006) and Lustig and Van Nieuwerburgh (2006a)). Our model points to additional welfare costs of rising income inequality in the presence of housing supply restrictions, associated with the spatial allocation of labor.

Finally, our work is complementary to asset pricing models that focus on the role of housing as a consumption good and/or a collateral asset (Iacoviello (2005), Lustig and Van Nieuwerburgh (2005), Piazzesi, Schneider, and Tuzel (2006), Brunnermeier and Julliard (2006), and Lustig and Van Nieuwerburgh (2006a)). Their main focus is on the properties of the stochastic discount factor that prices financial assets. In our model, the discount factor is constant across dates and states. An interesting avenue for future work is to incorporate the insights from the asset pricing literature. In particular, it seems important to incorporate richer term structure dynamics and to study their role in determining house prices in conjunction with the determinants that we put forward.

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A Proofs

A.1 Preliminary Results

The following Lemma compiles technical results which are used in the following subsection.

Lemma 4. *Consider some strictly increasing strictly concave, and twice continuously differentiable function $v : (0, \infty) \to \mathbb{R}$. Suppose that $v(h)$ goes to minus infinity as h goes to zero, and that $v(h)$ goes to zero as h goes to infinity. Then*

1. *The derivative v'(h) goes to infinity as h goes to zero, and goes to zero as h goes to infinity.*
2. *The function $hv'(h)$ goes to zero as h goes to infinity.*
3. *The function $w(h) \equiv hv'(h) - v(h)$ is continuous and strictly decreasing, goes to zero as h goes to infinity, and goes to infinity as h goes to zero.*
4. *The function $\Phi(x) = 1/w^{-1}(x)$ is continuous and strictly increasing. It can be extended by continuity at zero with $\Phi(0) = 0$. It goes to infinity as x goes to infinity.*
5. *The function $R(x) \equiv v' \circ w^{-1}(h)$ is increasing, convex, continuous, goes to zero as x goes to zero and goes to infinity as x goes to infinity.*
6. *Consider any density $g(A)$ such that, for all $x \in \mathbb{R}$,*

$$G(x) = \int_{A_{\min}}^{A_{\max}} \Phi\left(\max\{A - x, 0\}\right) g(A)\, dA < \infty.$$

Then, the function $G(x)$ is continuous.

Proof.

1. For any $h_1 > h_2$, concavity implies that $v'(h_2)(h_1 - h_2) \geq v(h_1) - v(h_2)$. Therefore, $v'(h_2)h_1 \geq v'(h_2)h_2 + v(h_1) - v(h_2) \geq v(h_1) - v(h_2)$. Letting h_2 go to zero in the inequality implies that $v'(h_2)$ goes to infinity as h_2 goes to zero. Second, since $v'(h)$ is positive and decreasing, it has some positive limit $\underline{v}'$ as h goes to infinity. Since $v(h)$ is concave, then for all $h_1 > h_2$, $0 \geq v(h_1) \geq v(h_2) + v'(h_1)(h_1 - h_2) \geq v(h_2) + \underline{v}'(h_1 - h_2)$. Letting h_1 go to infinity shows that $\underline{v}' = 0$. Therefore, $v'(h)$ goes to zero as h goes to infinity.
2. Rearranging the previous inequality implies that

$$v(h_1) + h_2 v'(h_1) - v(h_2) \geq h_1 v'(h_1) \geq 0.$$

Letting h_1 go to infinity shows that $-v(h_2) \geq \limsup_{h\to\infty} hv'(h) \geq 0$ for all h_2. Letting h_2 go to infinity shows that $hv'(h)$ also goes to zero as h goes to infinity.

3. Consider the function $w(h) \equiv hv'(h) - v(h)$. The above results show that $w(h)$ goes to zero as h goes to infinity. Because $w'(h) = hv''(h) < 0$, it follows that $w(h) \geq 0$. Lastly, since $w(h) \geq -v(h)$, letting h go to zero shows that $w(h)$ goes to infinity as h goes to zero.

4. Previous paragraph implies that the function $\Phi(x) = 1/w^{-1}(x)$ is well defined. It is continuous, increasing, goes to zero as x goes to zero, and to infinity as x goes to infinity. Lastly, consider the function $R(x)$ is increasing because both $v'(x)$ and $w^{-1}(x)$ are decreasing. Point 1 and 3 of the Lemma imply that it is goes to zero as x goes to zero, and to infinity as x goes to infinity.

5. In order to prove that $R(x)$ is convex, note that

$$\begin{aligned} R'(x) &= \frac{v'' \circ w^{-1}(x)}{w' \circ w^{-1}(x)} \\ &= \frac{v'' \circ w^{-1}(x)}{w^{-1}(x) \times v'' \circ w^{-1}(x)} \\ &= \frac{1}{w^{-1}(x)}, \end{aligned}$$

where the second line follows from the fact that $w'(h) = hv''(h)$. Since $w^{-1}(x)$ is decreasing, it follows that $R'(x)$ is increasing, which establishes convexity.

6. Pick any $x \in \mathbb{R}$ and some $\eta > 0$. Then that, for all $y \in [x-\eta, x+\eta]$

$$\begin{aligned} |G(x) - G(y)| &\leq \int_{A_{\min}}^{\alpha} \big|\Phi\left(\max\{A-x,0\}\right) - \Phi\left(\max\{A-y,0\}\right)\big| g(A)\, dA \\ &\quad \int_{\alpha}^{A_{\max}} \big|\Phi\left(\max\{A-x,0\}\right) - \Phi\left(\max\{A-y,0\}\right)\big| g(A)\, dA \\ &\leq \int_{A_{\min}}^{\alpha} \big|\Phi\left(\max\{A-x,0\}\right) - \Phi\left(\max\{A-y,0\}\right)\big| g(A)\, dA \\ &\quad 2\int_{\alpha}^{A_{\max}} \Phi\left(\max\{A-x+\eta,0\}\right) g(A)\, dA \end{aligned}$$

where the second inequality follows because $\Phi(x)$ is decreasing. Now, because $G(x-\eta) < \infty$, it follows that for all $\varepsilon > 0$ there exists some $\alpha > 0$ such that the second integral on the right-hand side is less than $\varepsilon/2$. Since the function $\Phi\left(\max\{z,0\}\right)$ is uniformly continuous over the compact $[0, \alpha - x + \eta]$, there exists some $\eta' < \eta$, such that $|x-y| < \eta'$ implies that $|\Phi\left(\max\{A-x,0\}\right) - \Phi\left(\max\{A-y,0\}\right)| < \varepsilon/2$. Plugging this back into the first integral on the right-hand side shows that $|x-y| < \eta'$ implies that $|G(x) - G(y)| < \varepsilon$.

□

A.2 Proof of Proposition 3

We first prove existence and uniqueness, and then prove the First Welfare Theorem.

A.2.1 Existence and Uniqueness

Because $g_t(A_t)\Pi_t(A_t) > 0$ for all $A \in [A_{\min}, A_{\max}]$, there exists a unique sequence $\{A_t^*\}_{t=1}^{\infty}$ of construction cutoffs solving (21). Given this sequence, we can calculate the housing stock $H_t(A^t, H_0)$ in every island at each time using equation (3). We can then write equation (20) as $F_t(U) = 1$, where

$$F_t(U) \equiv \int H_t(A^t, H_0)\Phi\left(\max\{A_t - U, 0\}\right) g_t(A^t, H_0)\, dA^t dH_0$$

In order to show that, at each time, equation (20) has a unique solution, we first establish that there exists $\underline{U}$ and $\overline{U}_t$ such that $F_t(\underline{U}) > 1$ and $F_t(\overline{U}_t) < 1$. Let $\overline{H}_t \equiv \int H_t(A^t, H_0) g_t(A^t, H_0)\, dA^t\, dH_0$ be the aggregate housing stock at time $t \in \{1, 2, \ldots\}$. Since all construction material is used every period, integrating both sides of (3) shows that $\overline{H}_t = (1-\delta)\overline{H}_{t-1} + M$, which immediately implies that

$$\overline{H}_t = (1-\delta)^t \overline{H}_0 + \left(1 - (1-\delta)^t\right) \frac{M}{\delta} > k \equiv \min\left\{\frac{M}{\delta}\,,\, (1-\delta)\overline{H}_0 + M\right\} > 0,$$

for all $t \in \{1, 2, \ldots\}$. Now, for $U \le 0$, we have $\max\{A - U, 0\} \ge -U$. Since the function $\Phi(x)$ is increasing, we have

$$\begin{aligned} F_t(U) &= \int H_t(A^t, H_0)\Phi(\max\{A_t - U, 0\}) g_t(A^t, H_0)\, dA^t\, dH_0 \\ &\ge \Phi(-U) \int H_t(A^t, H_0) g_t(A^t, H_0)\, dA^t\, dH_0 = \Phi(-U)\overline{H}_t \\ &> k\,\Phi(-U). \end{aligned} \tag{28}$$

Since $\Phi(x)$ goes to infinity as x goes to infinity, it follows from (28) that there exists some $\underline{U} > -\infty$ such that $F_t(\underline{U}) > 1$.

Because H_0 has a bounded support and the function $\Pi_t(\,\cdot\,)$ is bounded above, it follows from (3) that there exists some $K > 0$ such that $H_t(A^t, H_0) \le K$ for all times and islands. Now pick some $U \in [A_{\min}, A_{\max})$:

$$\begin{aligned} F_t(U) &= \int H_t(A^t, H_0)\Phi(\max\{A_t - U, 0\}) g_t(A^t, H_0)\, dA^t\, dH_0 \\ &\le K \int_U^{A_{\max}} \Phi(A_t - U) g_t(A_t)\, dA_t \le K \int_U^{A_{\max}} \Phi(A_t - A_{\min}) g_t(A_t)\, dA_t. \end{aligned} \tag{29}$$

Because $\int_{A_{\min}}^{A_{\max}} \Phi\left(A_t - A_{\min}\right) g_t(A_t)\, dA_t < \infty$, it follows that the right-hand side of (29) goes to zero as U goes to $A_{\max}$. Therefore, there exists some $\overline{U}_t$ such that $F_t(\overline{U}_t) < 1$. Now, for $U < U'$,

$$\begin{aligned} & |F_t(U) - F_t(U')| \\ = & \int_{A_{\min}}^{A_{\max}} H_t(A^t, H_0)\big[\Phi(\max\{A_t - U, 0\}) - \Phi(\max\{A_t - U', 0\})\big] g_t(A^t, H_0)\, dA^t\, dH_0 \\ \le & \; K|G(U) - G(U')|, \end{aligned}$$

where the first equality uses the fact that $\Phi(x)$ is an increasing function, and $G(U)$ is the continuous function defined in point (v) of Lemma 4. This inequality clearly implies that $F_t(U)$ is continuous. Therefore an application of the Intermediate Value Theorem shows that there exists some $U_t \in (\underline{U}, \overline{U}_t)$ such that $F_t(U_t) = 1$. Uniqueness follows from the fact that $F_t(\,\cdot\,)$ is strictly decreasing whenever $F_t(U) > 0$. Having shown the existence of a unique sequence $\{U_t\}_{t=1}^{\infty}$ of moving values, we find house prices using equation (15), and the construction cost using equation (23). Note that $U_t \geq \underline{U}$ implies that

$$0 \leq R\left(\max\{A - U_t, 0\}\right) \leq R\left(\max\{A - \underline{U}, 0\}\right).$$

Together with our assumption that (15) is finite for any constant sequence of utilities, the above inequality implies that the price $P_t(A_t)$is well defined at each time.

A.2.2 First Welfare Theorem

A feasible allocation is said to be Pareto optimal, subject to the regulatory constraints embedded in the permit function $\Pi(A_t)$, if it cannot be Pareto improved by choosing another feasible allocation and making time-zero consumption transfers. As it is standard with quasi-linear preferences (see Chapter 16 of Mas-Colell, Whinston, and Green (1995)), it can be shown that any Pareto optimal allocation must maximize

$$\sum_{t=1}^{\infty} \beta^{t-1} \int n_t(A^t, H_0) \Big(A_t + v\left(h_t\left(A^t, H_0\right)\right) \Big) g_t(A^t, H_0)\, dA^t\, dH_0, \tag{30}$$

the equally weighted sum of households' utilities. We now show that the competitive equilibrium allocation maximizes (30) and is therefore Pareto optimal, subject to the regulatory constraints embedded in the permit function $\Pi_t(A_t)$. Our proof follows the standard optimality-verification argument for concave control problems. Let us denote by n_t^*, H_t^*, h_t^*, and Δ_t^* the elements of an equilibrium allocation, and let n_t, H_t, h_t, and Δ_t be the element of any feasible allocation. In what follows, in order to simplify the notations, we omit the dependence of these functions on the idiosyncratic history (A^t, H_0). We then write, for any $T \geq 1$,

$$\begin{aligned}
& \sum_{t=1}^{T} \beta^{t-1} \int n_t^* \left(A_t + v\left(h_t^*\right)\right) g_t\, dA^t dH_0 - \sum_{t=1}^{T} \beta^{t-1} \int n_t \left(A_t + v\left(h_t\right)\right) g_t\, dA^t dH_0 \\
= \; & \sum_{t=1}^{T} \beta^{t-1} \int \left(n_t^* - n_t\right) \left(A_t + v\left(h_t^*\right)\right) g_t\, dA^t dH_0 + \sum_{t=1}^{T} \beta^{t-1} \int n_t \left(v\left(h_t^*\right) - v\left(h_t\right)\right) g_t\, dA^t dH_0 \\
\geq \; & \sum_{t=1}^{T} \beta^{t-1} \int \left(n_t^* - n_t\right) \left(A_t + v\left(h_t^*\right)\right) g_t\, dA^t dH_0 + \sum_{t=1}^{T} \beta^{t-1} \int n_t v'\left(h_t^*\right) \left(h_t^* - h_t\right) g_t\, dA^t dH_0 \\
= \; & \sum_{t=1}^{T} \beta^{t-1} \int \left(n_t^* - n_t\right) \left(A_t + v\left(h_t^*\right) - h_t^* v'(h_t^*)\right) g_t\, dA^t dH_0 \\
& + \sum_{t=1}^{T} \beta^{t-1} \int v'\left(h_t^*\right) \left(n_t^* h_t^* - n_t h_t\right) g_t\, dA^t dH_0
\end{aligned} \tag{31}$$

where the density g_t is implicitly a function of the idiosyncratic history (A^t, H_0). The inequality on the third line follows from the concavity of the utility function $v(h)$.[43] For the next step we first note that household's optimality implies that $A_t + v(h_t^*) - h_t^* v'(h_t^*) \leq U_t$, with an equality if $n_t^* > 0$. This implies that

$$\left(n_t^* - n_t\right) \left(A_t + v(h_t^*) - h_t^* v'(h_t^*)\right) \geq \left(n_t^* - n_t\right) U_t.$$

[43]Note that it also holds if an island is not populated and $h_t^* = \infty$: in that case, one lets $v(\infty) = v'(\infty) = 0$.

Integrating both sides of the inequality against the density g_t we obtain that

$$\int (n_t^* - n_t)\left(A_t + v(h_t^*) - h_t^* v'(h_t^*)\right) g_t \, dA^t dH_0 \geq \int (n_t^* - n_t)\, U_t g_t \, dA^t dH_0 = 0 \tag{32}$$

where the last equality follows from the fact that U_t does not depend on the idiosyncratic history (A^t, H_0) and from the feasibility condition (5). The second thing we note is that

$$v'(h_t^*)\left(n_t^* h_t^* - n_t h_t\right) \geq \rho_t \left(H_t^* - H_t\right) \tag{33}$$

because: i) $v'(h_t^*) = \rho_t$, ii) $n_t h_t \leq H_t$ from (6), and iii) in an equilibrium allocation, $n_t^* h_t^* = H_t^*$ whenever $n_t^* > 0$. Lastly, if $n_t^* = 0$, then inequality (33) also holds because $h_t^* = \infty$ and therefore $v'(h_t^*) = 0$. Plugging (32) and (33) into (31), and using equation (9) we find

$$\begin{aligned}
& \sum_{t=1}^{T} \beta^{t-1} \int n_t^* \left(A_t + v\left(h_t^*\right)\right) g_t \, dA^t dH_0 - \sum_{t=1}^{T} \beta^{t-1} \int n_t \left(A_t + v\left(h_t\right)\right) g_t \, dA^t dH_0 \\
\geq \; & \sum_{t-1}^{T} \beta^{t-1} \int \rho_t \left(H_t^* - H_t\right) g_t \, dA^t dH_0 \\
= \; & \sum_{t=1}^{T} \beta^{t-1} \int \left(p_t - \beta(1-\delta) \int p_{t+1} g_{t+1}\left(A_{t+1} \,|\, A^t\right) \, dA_{t+1}\right) \left(H_t^* - H_t\right) g_t \, dA^t dH_0 \\
= \; & \sum_{t=1}^{T} \beta^{t-1} \int p_t \left(H_t^* - H_t\right) g_t \, dA^t dH_0 - \sum_{t=2}^{T+1} \beta^{t-1}(1-\delta) \int p_t \left(H_{t-1}^* - H_{t-1}\right) g_t \, dA^t dH_0 \\
= \; & \int p_1 (H_1^* - H_1) g_1 dA^1 dH_0 + \sum_{t=2}^{T} \beta^{t-1} \int p_t \left(\left[H_t^* - (1-\delta) H_{t-1}^*\right] - \left[H_t - (1-\delta) H_{t-1}\right]\right) g_t \, dA^t dH_0 \\
& - \beta^T (1-\delta) \int p_{T+1} \left(H_T^* - H_T\right) g_{T+1} \, dA^{T+1} dH_0 \\
= \; & \sum_{t=1}^{T} \beta^{t-1} \int p_t \left(\Delta_t^* - \Delta_t\right) g_t \, dA^t dH_0 - \beta^T (1-\delta) \int p_{T+1} \left(H_T^* - H_T\right) g_{T+1} \, dA^{T+1} dH_0
\end{aligned} \tag{34}$$

where the last line follows from equation (3). Now, note that

$$p_t \left(\Delta_t^* - \Delta_t\right) \;=\; \mu_t \left(\Delta_t^* - \Delta_t\right) + \left(p_t - \mu_t\right)\left(\Delta_t^* - \Delta\right) \geq \mu_t \left(\Delta_t^* - \Delta_t\right) \tag{35}$$

where inequality (35) follows from the optimality condition of construction firms. Indeed, if $p_t > \mu_t$, then $\Delta_t^* = \Pi_t(A_t) \geq \Delta_t$ from (2). If $p_t \geq \mu_t$, then $\Delta_t^* = 0 \leq \Delta_t$ from (1). Noting moreover that μ_t is independent of the idiosyncratic history (A^t, H_0), and integrating both side of (35) against g_t, we find

$$\int p_t \left(\Delta_t^* - \Delta_t\right) g_t \, dA^t dH_0 \;\geq\; \mu_t \int \left(\Delta_t^* - \Delta_t\right) g_t \, dA^t dH_0 \geq 0 \tag{36}$$

where the last inequality follows from the construction material resource constraint (4), and from equation (22) which implies that the construction material resource constraint is binding in an equilibrium. Plugging

(36) back into (34), we find

$$\sum_{t=1}^{T}\beta^{t-1}\int n_t^*\left(A_t+v\left(h_t^*\right)\right)g_t\,dA^t dH_0-\sum_{t=1}^{T}\beta^{t-1}\int n_t\left(A_t+v\left(h_t\right)\right)g_t\,dA^t dH_0$$
$$\geq\quad -\beta^T(1-\delta)\int p_{T+1}\left(H_T^*-H_T\right)g_{T+1}\,dA^{T+1}dH_0. \tag{37}$$

Now, as T goes to infinity, the right-hand side of (37) goes to zero. Indeed, the housing stock H_t in an island is bounded and, by construction of an equilibrium, the price satisfies the transversality condition (8).

B Supplementary Materials

This appendix derives additional results, provides natural extensions of the model setup, and studies the robustness our quantitative results.

B.1 Convexity of the Rent in a General Setting

In this appendix we show that the rent is a convex function of productivity, in a static island model where households have some non-separable, strictly concave utility function over non-housing and housing consumption, $u(c,h)$. As in the main text, there is a continuum of islands with wage A and initial housing supply H_0. The cross-sectional probability density of wage and housing stocks is denoted by $g(A,H_0)$. A household chooses on which island to live and work. We adopt the standard lottery assumption that a household chooses the probability density with which it wants to be assigned to island (A,H_0) (Hansen (1985), Rogerson (1988), Prescott and Ríos-Rull (1992)), chooses its non-housing and housing consumption in that island, and can simultaneously trade securities that pay contingent on his final location. The equilibrium price of one unit of good contingent on being allocated to some island is equal to the probability of being assigned to that island. Hence, a household chooses measurable functions $c(A,H_0)$, $h(A,H_0)$, and $n(A,H_0)$, in order to maximize:

$$\int u(c(A,H_0),h(A,H_0))n(A,H_0)g(A,H_0)\,dA\,dH_0 \tag{38}$$

subject to the budget constraint

$$\int n(A,H_0)\left(c(A,H_0)+\rho(A,H_0)h(A,H_0)-A\right)\,dA\,dH_0\leq B, \tag{39}$$

where $\rho(A,H_0)$ is the rent in island (A,H_0), and B is the budget of the household. In addition, the probability of being assigned to some island must be equal to one, that is

$$\int n(A,H_0)g(A,H_0)\,dA\,dH_0=1. \tag{40}$$

Denoting by λ the multiplier on the budget constraint (39), and by ν the multiplier on constraint (40), the first-order necessary conditions of the household's problem are

$$u_c(c(A, H_0), h(A, H_0)) = \lambda \tag{41}$$

$$u_h(c(A, H_0), h(A, H_0)) = \lambda \rho(A, H_0) \tag{42}$$

$$u(c(A, H_0), h(A, H_0)) - \lambda \left[c(A, H_0) + \rho(A, H_0)h(A, H_0) - A\right] \leq \nu, \tag{43}$$

with an equality if $n(A, H_0) > 0$, and where u_c and u_h denote the partial derivative of u with respect to c and h, respectively. Substituting (42) into (43), we find that, in any populated island,

$$u(c(A, H_0), h(A, H_0)) - \lambda \left[c(A, H_0) + \frac{u_h(c(A, H_0), h(A, H_0))}{\lambda} h(A, H_0) - A\right] = \nu, \tag{44}$$

which, together with (41), implies that non-housing consumption, housing consumption, and hence the rent, only depend on the current productivity of the island. Letting $u(A) \equiv u(c(A), h(A))$, and defining the expenditure function $e(u, \rho) \equiv c(A, H_0) + \rho(A, H_0)h(A, H_0)$, we can thus rewrite the indifference condition

$$u(A) - \lambda e(u(A), \rho(A)) + \lambda A = \nu,$$

Taking derivative with respect to A, one finds that

$$u'(A) - \lambda e_u(u(A), \rho(A))u'(A) + e_\rho(u(A), \rho(A))\rho'(A) + \lambda = 0.$$

Now, the partial derivative of the expenditure function are well known to be $\partial e/\partial u = 1/u_c(c(A), \rho(A)) = 1/\lambda$ and $\partial e/\partial \rho = h(A)$. Plugging these back into the above equation and rearranging, we obtain

$$\rho'(A) = \frac{1}{h(A)}.$$

This implies that the rent is *increasing*. It also implies that the rent is *convex* because housing consumption $h(A)$ decreases with productivity. To establish this last property, consider the system of first-order conditions

$$u_c(c(A), h(A)) = \lambda \tag{45}$$

$$u(c(A), h(A)) - \lambda \left[c(A) + \frac{u_h(c(A), h(A))}{\lambda} h(A) - A\right] = \nu. \tag{46}$$

A straightforward application of the Implicit Function Theorem shows that

$$h'(A) = \frac{\lambda u_{cc}(c(A), h(A))}{h(A)\left[u_{cc}(c(A), h(A))u_{hh}(c(A), h(A)) - u_{ch}(c(A), h(A))^2\right]} < 0 \tag{47}$$

The denominator is positive because $u_{cc}u_{hh} - u_{ch}^2$ is the determinant of the Hessian of u, a concave function of an even number of variables. The numerator is negative because $u(c, h)$ is concave. This proofs that the rent is convex in A.

As an aside, the above calculation does not apply to the quasi-linear utility function of the paper. In that case, both the denominator and the numerator are equal to zero.

B.2 The House Price Impact of Wage Dispersion

This appendix studies the model's main relationship, the impact of an increase in wage dispersion on housing prices, in closed form. While it relies on an approximation that applies to a small increase in wage dispersion, it has the same qualitative features as our calibration exercise. For simplicity, we now assume that the utility function over housing services is $v(h) \equiv -\kappa/h$, the same iso-elastic specification as in our calibration.

B.2.1 House Price Distribution without Wage Dispersion

We first analyze the benchmark situation without wage dispersion: in every island, the productivity is equal to $\overline{A}$ and the housing stock is equal to $\overline{H} = M/\delta$. In equilibrium, housing consumption per capita is equal to $\overline{H}$ and the unit price of housing services is equal to their marginal utility $\kappa/\overline{H}^2$. Note that the level of productivity has no impact on house prices. Lastly, the value of moving is $\overline{U} = \overline{A} - 2\kappa/\overline{H}$, so that $\overline{A} - \overline{U} > 0$.

B.2.2 House Price Distribution with Wage Dispersion

We now introduce a small amount of wage dispersion. The wage of a randomly chosen island is $A = \overline{A} + d\tilde{A}$, where the increment $d\tilde{A}$ is small, has a cross-sectional mean of zero and a cross-sectional variance $V(d\tilde{A})$. The tilde (˜) notation highlights that $d\tilde{A}$ is random in the cross section. The housing stock of a randomly chosen island is $H = \overline{H} + d\tilde{H}$, and we assume that $d\tilde{H}$ is of the same order of magnitude as $d\tilde{A}$. In equilibrium, $d\tilde{H}$ and $d\tilde{A}$ are not independent from each other. Because firms find it optimal to construct in the most productive islands, $d\tilde{H}$ and $d\tilde{A}$ tend to be positively correlated in the cross-section.

Note that Assumption (22) implies that all the material M is being used up for construction. Therefore, the cross-sectional average housing stock must be constant and equal to $\overline{H} = M/\delta$, regardless of the productivity distribution. This implies that $E(d\tilde{H}) = 0$.

With the current preference specification, the function $\Phi(x)$ of equation (20) is $\Phi(x) = x/(2\kappa)$. Therefore, the value $\bar{U} + dU$ of moving solves the equation

$$\frac{1}{2\kappa} E\left[\left(\overline{H} + d\tilde{H}\right) \max\left\{\overline{A} + d\tilde{A} - \overline{U} - dU, 0\right\}\right] = 1, \tag{48}$$

where the expectation is taken over the joint distribution of $(d\tilde{A}, d\tilde{H})$. Note that if dU is small, $\overline{A} - \overline{U} > 0$ implies that $\overline{A} + d\tilde{A} - \overline{U} - dU > 0$. Hence, all islands are populated, we can drop the maximum in equation (48), and solve for dU:

$$\begin{aligned}
& \frac{1}{2\kappa} E\left[\left(\overline{H} + d\tilde{H}\right)\left(\overline{A} + d\tilde{A} - \overline{U} - dU\right)\right] = 1 \\
\Leftrightarrow\quad & \frac{1}{2\kappa} E\left[\overline{H}\left(\overline{A} - \overline{U} + d\tilde{A} - dU\right) + d\tilde{H}\left(\overline{A} - \overline{U} + d\tilde{A} - dU\right)\right] = 1 \\
\Leftrightarrow\quad & \frac{1}{2\kappa}\left\{\overline{H}\left(\overline{A} - \overline{U}\right) - \overline{H} dU + E(d\tilde{A} d\tilde{H})\right\} = 1 \\
\Leftrightarrow\quad & dU = E\left[d\tilde{A}\frac{d\tilde{H}}{\overline{H}}\right],
\end{aligned}$$

where the last equality follows because $1/(2\kappa)\overline{H}(\overline{A} - \overline{U}) = 1$ by definition of $\overline{U}$ and because, by assumption, $E(d\tilde{A}) = E(d\tilde{H}) = 0$. This equation shows that an increase in wage dispersion increases households' value of moving, provided the housing stock and the wage are positively correlated. This arises in all of our calibrated examples because firms find it optimal to construct housing in the most productive islands. Intuitively, a positive correlation between the wage and the housing stock allows more households to work in more productive islands, and increases their utility of moving.

We now turn to the impact of wage dispersion on the rent. The rent in an island with wage $\overline{A} + d\tilde{A}$ is

$$\rho(d\tilde{A}) = \frac{1}{4\kappa}\left(\overline{A} + d\tilde{A} - \overline{U} - dU\right)^2. \tag{49}$$

Therefore, holding the wage $\overline{A} + d\tilde{A}$ fixed, an increase in wage dispersion lowers the rent. This is an *outside option* effect: because the value $\overline{U} + dU$ of moving goes up, households are able to bid down the rent in their island.

Equation (49) also reveals a *convexity effect*. Because the rent function is convex in $d\tilde{A}$, the rent increases more in high- than in low-wage islands.[44] This implies that the *average* rent across islands goes up. Convexity also has an impact on the price of housing services in each island, because the future wage of an island is random. Take the example of an independent and identically distributed wage process. That is, every period, the wage in an island is an independent draw from the cross-sectional distribution. The house price in an island with current wage $\overline{A} + d\tilde{A}$ is

$$P(d\tilde{A}) = \rho(d\tilde{A}) + \frac{E\left[\rho(d\tilde{A}')\right]}{1 - \beta(1-\delta)}, \tag{50}$$

where the expectations is taken with respect to the cross-sectional distribution of wage. The convexity effect means that an increase in wage dispersion increases the second term in the price equation (50).[45]

The outside option and the convexity effect have opposite impacts on the average rent. A second-order approximation suggests that the convexity effect dominates: a small increase in wage dispersion increases both the population-weighted average house price and its dispersion. Appendix B.2.3 combines equation (49) with the fact that

$$n(d\tilde{A}, d\tilde{H}) = \frac{1}{2\kappa}\left(\overline{H} + d\tilde{H}\right)\left(\overline{A} + d\tilde{A} - \overline{U} - dU\right),$$

to derive the following second-order approximation of the population weighted average rent across islands:

$$E\left[n(d\tilde{A}, d\tilde{H})\rho(d\tilde{A})\right] \simeq \frac{\kappa}{\overline{H}^2} + \frac{3}{4\kappa}V(d\tilde{A}). \tag{51}$$

The first term on the right-hand side is the rent when all islands are identical and the second term represents the impact on the average rent of wage dispersion. This term is positive, meaning that the convexity effect dominates the outside option effect. The more dispersed the wage, the larger the increase in the population-weighted average rent across islands.

A similar second-order approximation shows that the population weighted variance of rents is equal

[44] Lemma 4 of Appendix A.1 shows that the function $\rho(d\tilde{A})$ is convex for all utility functions satisfying the conditions of Section 2.1.

[45] If the wage process is persistent, then the same effect operates in the long run. Indeed, by ergodicity, the distribution of the wage T periods ahead converges to the cross-sectional distribution as T goes to infinity.

to $V(d\tilde{A})/\overline{H}^2$, meaning that the wage distribution lends its dispersion to the rent dispersion. Moreover, increasing the aggregate supply $\overline{H}$ of housing services reduces dispersion. It brings the marginal utility κ/h^2 in a less convex range, and reduces the impact on rents of cross-sectional variation in housing consumption.

B.2.3 Second-order Approximation

The population weighted rent solves

$$\begin{aligned} E\left[n(d\tilde{A}, d\tilde{H})\rho(d\tilde{A})\right] &= \frac{1}{8\kappa^2} E\left[\left(\overline{H} + d\tilde{H}\right)\left(\overline{A} - \overline{U} + d\tilde{A} - dU\right)^3\right] \\ &\simeq \frac{1}{8\kappa^3} E\left[\left(\overline{H} + d\tilde{H}\right)\left((\overline{A} - \overline{U})^3 + 3(\overline{A} - \overline{U})^2\left(d\tilde{A} - dU\right) + 3(\overline{A} - \overline{U})\left(d\tilde{A} - dU\right)^2\right)\right] \end{aligned}$$

where we ignore the third-order term $(d\tilde{A} - dU)^3$. Since $dU = E(d\tilde{A}d\tilde{H}/\overline{H})$ is of second order, it follows that, to a second-order approximation, $(d\tilde{A} - dU)^2 \simeq d\tilde{A}^2$. Plugging this back into the last equation, using the fact that $E(d\tilde{A}) = E(d\tilde{H}) = 0$, and neglecting all terms of order greater or equal than three, we obtain

$$\begin{aligned} E\left[n(d\tilde{A}, d\tilde{H})\rho(A)\right] &= \frac{\overline{H}}{8\kappa^2}\left\{(\overline{A} - \overline{U})^3 - 3(\overline{A} - \overline{U})^2 dU + 3(\overline{A} - \overline{U})E(d\tilde{A}^2)\right\} + \frac{3}{8\kappa^2}(\overline{A} - \overline{U})^2 E\left[d\tilde{H}d\tilde{A}\right] \\ &= \frac{\overline{H}}{8\kappa^2}\left((\overline{A} - \overline{U})^3 + 3(\overline{A} - \overline{U})V(d\tilde{A})\right) \\ &= \frac{\kappa}{\overline{H}^2} + \frac{3}{4\kappa}V(d\tilde{A}), \end{aligned}$$

where the second equality follows from the fact that $\overline{H}dU = E\left[d\tilde{A}d\tilde{H}\right]$, and the last equality follows from substituting $\overline{U} = \overline{A} - 2\kappa/\overline{H}$. In order to calculate the variance, we start with

$$\begin{aligned} E\left[n(d\tilde{A}, d\tilde{H})\rho(d\tilde{A})^2\right] &= \frac{1}{32\kappa^3} E\left[\left(\overline{H} + d\tilde{H}\right)\left(\overline{A} - \overline{U} + d\tilde{A} - dU\right)^5\right] \\ &= \frac{1}{32\kappa^3} E\left[\left(\overline{H} + d\tilde{H}\right)\left(\overline{A} - \overline{U} + d\tilde{A} - dU\right)^5\right] \\ &= \frac{1}{32\kappa^3} E\left[\left(\overline{H} + d\tilde{H}\right)\left((\overline{A} - \overline{U})^5 + 5\left(\overline{A} - \overline{U}\right)^4\left(d\tilde{A} - dU\right) + 10\left(\overline{A} - \overline{U}\right)^3 d\tilde{A}^2\right)\right] \end{aligned}$$

where we use the identity $(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$ and we neglect all terms of order greater or equal than three. Using the fact that $E(d\tilde{A}) = E(d\tilde{H}) = 0$, we find

$$\begin{aligned} E\left[n(d\tilde{A}, d\tilde{H})\rho(A)^2\right] &= \frac{1}{32\kappa^3}\left\{\overline{H}(\overline{A} - \overline{U})^5 - 5\overline{H}(\overline{A} - \overline{U})^2 dU + 10\overline{H}(\overline{A} - \overline{U})^3 V(A) + 5(\overline{A} - \overline{U})^4 E(d\tilde{A}d\tilde{H})\right\} \\ &= \frac{1}{32\kappa^3}\left\{\overline{H}(\overline{A} - \overline{U})^5 + 10\overline{H}(\overline{A} - \overline{U})^3 V(A)\right\} \\ &= \frac{\kappa^2}{\overline{H}^4} + \frac{5}{2}\frac{V(d\tilde{A})}{\overline{H}^2}, \end{aligned}$$

where we use $\overline{H}dU = E(d\tilde{A}d\tilde{H})$ for the first equality, and $\overline{A} - \overline{U} = 2\kappa/\overline{H}$ for the second equality. The second calculation is

$$\begin{aligned} E\left[n(d\tilde{A}, d\tilde{H})\rho(d\tilde{A})\right]^2 &\simeq \left[\frac{\kappa}{\overline{H}^2} + \frac{3}{4\kappa}V(d\tilde{A})\right]^2 \\ &\simeq \frac{\overline{H}^2}{\kappa^2} + \frac{3}{2}\frac{V(d\tilde{A})}{\overline{H}^2} \end{aligned}$$

neglecting again the term $V(A)^2$ which is of order greater than 2. Combining the last two results leads to

$$E\left[n(d\tilde{A}, d\tilde{H})\rho(d\tilde{A})^2\right] - E\left[n(d\tilde{A}, d\tilde{H})\rho(d\tilde{A})\right]^2 = \frac{V(d\tilde{A})}{\overline{H}^2}. \tag{52}$$

B.3 Convergence

In this Appendix we characterize the long-run behavior of the competitive equilibrium. As one might expect, when the productivity process is stationary and the permit constraint is time invariant, then the economy converges to a steady state:

Proposition 5 (Convergence)**.** *Under the assumptions of Proposition 3, and if i) the productivity process is stationary and ii) the permit function* $\Pi(A)$ *is time invariant. Then, there exists some value* U^* *of moving and some function* $m^*(A)$ *such that* U_t *converges towards* U^* *and* $m_{t-1}(A_t) \equiv E(H_{t-1} \mid A_t)$ *converges uniformly towards* $m^*(A)$*, for any initial distribution of the housing stock.*

Note that we only prove the convergence of the conditional moment function, $m_{t-1}(A)$, and not convergence of the joint distribution of productivity and housing stock. Convergence in distribution can be proved using Theorem 2 of Hopenhayn and Prescott (1992), but under the additional assumption that the productivity process is bounded, i.e. $A_{\max} < \infty$. In our calibration, however, we find it convenient to assume that the productivity process is unbounded, which requires the results of Proposition 5.

In order to prove these results, we first take expectations conditional on A_{t+1} on both sides of (3), we find that, for all $t \in \{1, 2, \ldots\}$:

$$m_t(A_{t+1}) = E\left[H_t \mid A_{t+1}\right] = E\left[(1-\delta)H_{t-1} + \Pi(A)\mathbb{I}_{\{A \geq A^*\}} \mid A_{t+1}\right]. \tag{53}$$

An application of the Law of Iterated Expectations implies that

$$E\left[H_{t-1} \mid A_{t+1}\right] = E\left[E\left[H_{t-1} \mid A_t, A_{t+1}\right] \mid A_t\right]. \tag{54}$$

Now note that H_{t-1} is a function of A^{t-1} and H_0. Let us consider the density of (A^{t-1}, H_0) conditional on (A_t, A_{t+1}):

$$\begin{aligned} g(A^{t-1}, H_0 \mid A_{t+1}, A_t) &= \frac{g(A^{t+1}, H_0)}{g(A_{t+1}, A_t)} = \frac{g(A_{t+1} \mid A^t, H_0)g(A^t H_0)}{g(A_{t+1}\, A_t)g(A_t)} \\ &= \frac{g(A_{t+1} \mid A_t)g(A^t, H_0)}{g(A_{t+1}\, A_t)g(A_t)} = \frac{g(A^t, H_0)}{g(A_t)} = g(A^{t-1}, H_0 \mid A_t), \end{aligned}$$

where the first two equalities on the first line follow from an application of Bayes' Rule, the first equality on the second line follows from the Markov property together with the assumption that, conditional on A^t, H_0 does not help predicting the future path of productivity. The last two equalities follow from Bayes' Rule. Summarizing, we have

$$g(A^{t-1}, H_0 \mid A_{t+1}, A_t) = g(A^{t-1}, H_0 \mid A_t)$$

Plugging this back into (54) this implies that $E\left[H_{t-1} \mid A_{t+1}\right] = E\left[m_{t-1}(A_t) \mid A_{t+1}\right]$ and, with (53), that

$$m_t(A_{t+1}) = (1-\delta)E\left[m_{t-1}(A_t) \mid A_{t+1}\right] + E\left[\Pi(A)\mathbb{I}_{\{A \geq A^*\}} \mid A_{t+1}\right]. \tag{55}$$

Equation (55) defines a mapping on the space of bounded measurable functions. It is straightforward to show that it satisfies the Blackwell sufficient conditions for a contraction mapping (Stokey and Lucas (1989) Theorem 3.3). Since the space of bounded measurable functions is complete (Rudin (1986) Theorem 3.11) an application of the Contraction Mapping Theorem (Stokey and Lucas (1989) Theorem 3.2) implies that it has a unique fixed point $m^*(A)$, and that the function $m_t(A)$ converges uniformly towards $m^*(A)$.

Now consider the sequence $\{U_t\}_{t=1}^{\infty}$ of maximum attainable utilities. At each time U_t is the unique solution of

$$\begin{aligned}
1 &= F_t(U) = \int H_t(A^t, H_0)\Phi\left(\max\{A_t - U, 0\}\right) dA^t\, dH_0 \\
&= E\left[H_t \Phi\left(\max\{A_t - U, 0\}\right)\right] \\
&= E\left[\left(H_{t-1} + \Pi(A_t)\mathbb{I}_{\{A_t \geq A^*\}}\right)\Phi\left(\max\{A_t - U, 0\}\right)\right] \\
&= E\left[\left(m_{t-1}(A_t) + \Pi(A_t)\mathbb{I}_{\{A_t \geq A^*\}}\right)\Phi\left(\max\{A_t - U, 0\}\right)\right].
\end{aligned}$$

where the last equality follows from conditioning with respect to A_t and applying the Law of Iterated Expectations. Define

$$F^*(U) \equiv E\left[\left(m^*(A_t) + \Pi(A_t)\mathbb{I}_{\{A_t \geq A^*\}}\right)\Phi\left(\max\{A_t - U, 0\}\right)\right]. \tag{56}$$

Because $m_t(A)$ converges uniformly towards $m^*(A)$, an application of the Dominated Convergence Theorem (Stokey and Lucas (1989) Theorem 7.10) shows that $F_t(U)$ converge point-wise towards $F^*(U)$. Moreover, by the same argument as in the Proof of Proposition 3, $F^*(U)$ is continuous.

Now, stationarity implies that $g_t(A) = g_0(A)$ which in the Proof of Proposition 3 implies that the upper bound $\overline{U}_t$ is independent of time and therefore that U_t is a bounded sequence. Therefore, the sequence $\{U_t\}_{t=1}^{\infty}$ has at least one accumulation point U^*. Letting $\{U_{t_k}\}_{k=1}^{\infty}$ be a subsequence of (U_t) converging towards U^*, we have

$$\begin{aligned}
|F^*(U^*) - F_{t_k}(U_{t_k})| &\leq |F^*(U^*) - F_{t_k}(U^*)| + |F_{t_k}(U^*) - F_{t_k}(U_{t_k})| \\
&\leq |F^*(U^*) - F_{t_k}(U^*)| + K|G(U^*) - G(U_{t_k})|,
\end{aligned}$$

where K is an upper bound on an island housing stock, and $G(U)$ is the function defined in point (v) of Lemma 4. Since U_{t_k} converges to U^* and $F_t(U)$ converges point-wise towards $F^*(U)$, it follows that $F^*(U^*) - F_{t_k}(U_{t_k})$ converges to zero. Because $F_t(U_t) = 1$, this means that $F^*(U^*) = 1$. Lastly, since $F^*(U)$ is strictly decreasing whenever $F^*(U) > 0$, there is a unique U^* such that $F^*(U^*) = 1$. This proves that sequence $\{U_t\}_{t=1}^{\infty}$ has a unique accumulation point, and is therefore convergent.

B.4 Quantitative Results: Robustness Analysis

This appendix discusses how the results change when we consider alternative ways of increasing wage dispersion and different degrees of regulatory tightening. Table 1 summarizes all results.

B.4.1 Two Alternative Ways to Increase Wage Dispersion

We revisit section 4.1 and consider two alternative ways of generating the same increase in the wage dispersion between 1975 and 2004.

First, instead of assuming that the variance of log wages σ_{at}^2 stays constant after 2004, we consider a constant innovation variance $\sigma_{\varepsilon t}^2$. That is, $\sigma_{\varepsilon t}$ stays constant after 2004 at its 2004 value of .0486. This implies that σ_a shows the exact same increase until 2004 as in the benchmark case, but keeps increasing from .220 to its new steady state level of .345. By the same token, the population-weighted c.v. of A further increases to a new steady state value of .242. For comparison, in our benchmark case, the c.v. of A declined to a steady state value of .156 instead (see Figure 6). The left column of Panel 1 in Table 1 summarizes the results. Output increases by the same amount until 2004, but it increases an additional 31% afterwards, instead of 3.5% in the benchmark. The population keeps spreading out as the construction cutoff A_t^* increases beyond 2004. This further concentrates the population in the highest productivity quintile (88.8% in the final steady state). Because of the continued increase in the wage dispersion, which is reflected in house prices, both average house prices and their standard deviation increase much more than in the benchmark: 128% versus 48% for the mean and 215% versus 98% for the standard deviation. The c.v. of house prices increases further from .79 after 2004 periods to 1.13 in the final steady state. Finally, the HP/CC also goes up by much more: 115% versus 55%. While the results are qualitatively the same, the increase in wage dispersion may be more potent for both output and house prices than the benchmark scenario let believe.

We explore a second alternative in which the increase in σ_{at} is entirely driven by an increase in persistence instead of an increase in the innovation variance. In particular, we keep $\sigma_{\varepsilon t}$ constant at .0173 and increase ρ_a from 0.99 to 0.9975. The variance of σ_{at} increases to 0.245 by period 2004 and stays constant thereafter, consistent with its evolution in the benchmark case. The population-weighted c.v. of productivity increases from .0967 in the initial steady state to .1347 in period 2004 and further to .1587 in the final steady state. The beginning and ending point are the same as in our benchmark case, but productivity dispersion has increased by much less after 30 periods. The right column of Table 1, Panel 1 shows that the total effects are larger, but the changes in the first 30 periods are muted. For example, output increases by only 6% (instead of 13%) until 2004, but increases 25% in total (instead of 16.8%). House prices increase 13% initially (50%) but 55% between steady states (48%). The final c.v. of house prices is 0.96 instead of 0.77.

B.4.2 Alternative Changes in Housing Supply Regulation

The parameter ϕ, which governs the tightening in housing supply regulation in section 4.2, is somewhat of a free parameter. We set $\phi = -.5$ in our baseline case, and explore a lower and a higher value here.

The left column of Panel 2, Table 1 reports the moments for $\phi = -.7$ and the right column of the same panel considers $\phi = -.3$. All effects are monotonic in ϕ, but even for $\phi = -.7$ we find small house price

Table 1: Summary Statistics of Model: Changes Relative to 1975 Steady State

The first panel reports the effect of an increase in productivity dispersion. The increase is achieved in one of three ways: through an increase in σ_ε and σ_a is assumed to stay constant after 2004 (middle column, *benchmark case*); through an increase in σ_ε and σ_ε is assumed to stay constant after 2004 (left column); and through an increase in persistence ρ_a (right column). The second panel reports the effect of an increase in housing supply regulation. The parameter ϕ decreases from 0 in 1975 to one of the following three values in 2004: $\phi = -.5$ (middle column, *benchmark case*); $\phi = -.7$ (left column); and $\phi = -.3$ (right column). The third and fourth panels report an increase in both regulation and productivity dispersion. In the third panel, the increase in productivity dispersion is achieved in the same way across columns: through an increase in σ_ε and σ_a is assumed to stay constant after 2004. In the fourth panel, the increase in regulation is the same across columns: ϕ declines from 0 to -.5. Each entry reports the changes between either the situation at period 30 and the initial steady state (columns "2004") or between the final steady state and the initial steady state (columns "ss"). In all panels, rows 1 and 3-6 report the log difference multiplied by 100; row 2 reports a simple difference multiplied by 100.

Panel 1 - Increasing Productivity Dispersion ($\phi = 0$)						
Experiment:	$\sigma_\varepsilon \uparrow$, $\bar{\sigma}_\varepsilon$		*Benchmark*: $\sigma_\varepsilon \uparrow$, $\bar{\sigma}_a$		$\rho_a \uparrow$	
Moment	ss	2004	ss	2004	ss	2004
Output	44.21	13.00	16.82	13.29	24.78	5.96
Pop. in Q5	24.27	0.45	11.04	1.12	23.69	-20.64
Mean HP	128.03	55.81	47.86	50.13	54.88	13.02
Std HP	215.41	107.19	98.08	104.40	126.29	68.69
Mean HP/CC	155.55	31.11	55.24	35.33	127.00	-9.65
Panel 2 - Increasing Regulation ($\sigma_a \rightarrow$)						
Experiment:	$\pi_b = -.7$		*Benchmark*: $\pi_b = -.5$		$\pi_b = -.3$	
Moment	ss	2004	ss	2004	ss	2004
Output	-3.65	-1.23	-2.56	-0.88	-1.51	-0.53
Pop. in Q5	-17.21	-7.04	-12.27	-5.13	-7.23	-3.09
Mean HP	0.52	2.84	0.22	1.85	0.05	1.01
Std HP	0.64	2.15	0.14	1.20	-0.09	0.52
Mean HP/CC	18.02	31.69	12.05	21.33	6.83	12.14
Panel 3 - Increasing Both at the Same Time ($\sigma_\varepsilon \uparrow$, $\bar{\sigma}_a$)						
Experiment:	$\pi_b = -.7$		*Benchmark*: $\pi_b = -.5$		$\pi_b = -.3$	
Moment	ss	2004	ss	2004	ss	2004
Output	9.03	10.67	11.26	11.38	13.50	12.12
Pop. in Q5	-9.49	-6.07	-3.65	-4.17	2.39	-2.10
Mean HP	45.03	52.68	45.53	51.59	46.25	50.79
Std HP	96.24	106.14	96.46	105.30	96.93	104.72
Mean HP/CC	88.54	96.78	77.34	76.63	67.77	58.79
Panel 4 - Increasing Both at the Same Time ($\pi_b = -.5$)						
Experiment:	$\sigma_\varepsilon \uparrow$, $\bar{\sigma}_\varepsilon$		*Benchmark*: $\sigma_\varepsilon \uparrow$, $\bar{\sigma}_a$		$\rho_a \uparrow$	
Moment	ss	2004	ss	2004	ss	2004
Output	33.85	11.37	11.26	11.38	17.02	3.85
Pop. in Q5	7.86	-4.19	-3.65	-4.17	3.74	-28.37
Mean HP	120.62	57.96	45.53	51.59	52.94	12.27
Std HP	207.24	108.22	96.46	105.30	124.32	57.45
Mean HP/CC	187.50	70.63	77.34	76.63	146.39	43.98

effects of regulation. We have considered even more extreme parameter values for ϕ, we have experimented with lowering the intercept π_a instead, or in combination with a decline in ϕ, and have solved the model under alternative assumptions on housing supply regulation post-2004. In all cases did we find small house price effects for the simple reason that labor reallocates away from regulated metropolitan areas and this equilibrium decline in demand largely undoes the price effects of the decline in supply brought about by tighter regulation.

In sum, tightening regulation quantitatively fails to produce the observed increase in house prices, the observed increase in house price dispersion and the increase in population concentration. It does generate a non-trivial increase in the house price to construction cost ratio, a measure of the part of house prices due to regulation.

B.4.3 Combining Both Effects

The last two panels of Table 1 consider a simultaneous increase in regulation and in wage dispersion. In Panel 3, the increase in wage dispersion is engineered in identical fashion across columns and different regulatory changes are considered, while Panel 4 compares alternative wage dispersion channels holding the regulatory change fixed across columns. By construction, the middle column in both panels is the same. This is the benchmark case discussed in the main text. We note that the deviations of house prices from the benchmark case are much more pronounced in Panel 4 than in Panel 3. Consistent with previous findings, the size of the regulatory change seems inconsequential for housing prices (Panel 3). The way in which the wage dispersion comes about, however, makes a big difference for house prices (Panel 4). Final steady state house price levels and their dispersion are higher in both the left and the right column, compared to the benchmark case.

Finally, we isolate the effects of housing supply regulation for output and welfare. We do this by subtracting the numbers in Panel 3 from the numbers in the middle column of Panel 1. Naturally, output losses depend on exactly how much tighter regulation became: the steady state output loss is 3.3% when $\phi = -.3$ (16.8-13.5) and 7.8% when $\phi = -.7$ (16.8-9.0). Fewer households work in the most productive regions and this decline in allocative efficiency leads to the output loss.

Table 2 reports the ex-ante welfare costs described in the main text. We recall that it measures the 1975 present discounted value of welfare flows lost because of tighter regulation. The discounted sum runs from 1975 until either 2004 or until the final steady-state. Its nine cases which arise from three different levels of regulatory increase ($\phi = -.7$, $\phi = -.5$, and $\phi = -.3$) and three different assumptions on how the increase in productivity dispersion came about. The welfare costs of housing supply regulation, measured until 2004 are between 0.02% and 0.48% per year. When measured until the final steady state, the annual losses are between 0.56% and 1.74%.

C Computations

In this Appendix we describes how to compute an equilibrium.

Table 2: Welfare Costs of Housing Supply Regulation

Experiment:	$\phi = -.7$		*Benchmark*: $\phi = -.5$		$\phi = -.3$	
	ss	2004	ss	2004	ss	2004
$\sigma_\varepsilon \uparrow$, $\bar{\sigma}_\varepsilon$	-1.52	-0.22	-1.03	-0.08	-0.56	-0.02
Benchmark: $\sigma_\varepsilon \uparrow$, $\bar{\sigma}_a$	-1.49	-0.48	-1.06	-0.33	-0.64	-0.19
$\rho_a \uparrow$	-1.74	-0.41	-1.24	-0.27	-0.75	-0.15

C.1 Functional and Distributional Assumptions

We assume that the idiosyncratic wage process is lognormal. Namely, $A_t = e^{a_t}$ where a_t follows the autoregression

$$a_t = (1 - \rho_t)\mu_t + \rho_t a_{t-1} + \sigma_{\varepsilon t}\varepsilon_t,$$

for some deterministic sequences $\{\mu_t, \rho_t, \sigma_{\varepsilon t}\}_{t=1}^{\infty}$, and some sequence $\{\varepsilon_t\}_{t=1}^{\infty}$ of independent standard normal variables. The initial log wage a_0 is taken to be normally distributed with mean μ_0 and variance σ_0^2. By the Law of Large Numbers, the cross-sectional distribution of log wage at any date is normally distributed with a mean $E(a_t)$ and variance $V(a_t)$ solving the first-order difference equations

$$\begin{aligned} E(a_t) &= (1 - \rho_t)\mu_t + \rho_t E(a_{t-1}) \\ V(a_t) &= \rho_t^2 V(a_{t-1}) + \sigma_{\varepsilon t}^2. \end{aligned}$$

For the rest of this appendix, we write all functions in terms of log wage a_t instead of A_t. We also assume that the utility function $v(h)$ for housing services is $v(h) = -\kappa/h$. This implies that $w(h) \equiv hv'(h) - v(h) = (2\kappa)/h$ and $\Phi(h) = h/(2\kappa)$.

C.2 The Recursive Procedure

We solve for an equilibrium recursively in 4 steps that we describe in details below.

1. On solves first for the sequence $\{a_t^*\}_{t=1}^{\infty}$ of construction cutoffs.
2. Given the construction cutoffs, one solves for the function $m_{t-1}(a_t) = E\left(H_{t-1}(a^{t-1}, H_0) \,|\, a_t\right)$.
3. Given the function $m_{t-1}(a_t)$, one solves for the sequence $\{U_t\}_{t=1}^{\infty}$ of moving values.
4. Given the sequence of moving values, one solves for prices.

C.2.1 Step 1: Construction Cutoffs

Let $f(a;\mu,\sigma^2)$ denote the probability distribution function of a normal random variable with mean μ and variance σ^2. Then, we find the construction cutoff $A_t^* \equiv e^{a_t^*}$ at time $t \in \{1,2,\ldots\}$ by solving

$$\int_{a_t^*}^{\infty} \Pi(a) f\left(a; E(a_t), V(a_t)\right) da = M.$$

C.2.2 Step 2: the Function $m_{t-1}(a_t)$

The function $m_{t-1}(a_t)$ solves the first-order stochastic difference equation:

$$\begin{aligned}
m_{t-1}(a_t) &= E\left(H_{t-1}(a^{t-1}, H_0) \,|\, a_t\right) \\
&= E\left((1-\delta)H_{t-2}(a^{t-2}, H_0) + \Pi_{t-1}(e^{a_{t-1}})\mathbb{I}_{\{a_{t-1} \geq a_{t-1}^*\}} \,|\, a_t\right) && (57) \\
&= (1-\delta)E\left(E\left[H_{t-2}(a^{t-2}, H_0) \,|\, a_t, a_{t-1}\right] \,|\, a_t\right) + E\left(\Pi_{t-1}(a_{t-1})\mathbb{I}_{\{a_{t-1} \geq a_{t-1}^*\}} \,|\, a_t\right) && (58) \\
&= E\left(\left[H_{t-2}(a^{t-2}, H_0) \,|\, a_{t-1}\right] \,|\, a_t\right) + E\left(\Pi_{t-1}(e^{a_{t-1}})\mathbb{I}_{\{a_{t-1} \geq a_{t-1}^*\}} \,|\, a_t\right) && (59) \\
&= (1-\delta)E\left(m_{t-2}(a_{t-1}) \,|\, a_t\right) + E\left(\Pi_t(e^{a_{t-1}})\mathbb{I}_{\{a_{t-1} \geq a_{t-1}^*\}} \,|\, a_t\right) && (60)
\end{aligned}$$

where the first equality (57) follows from the law of motion of $H_t(a^t, H_0)$ and the second equality (58) follows from the Law of Iterated Expectations. The third equality (59) is more delicate and follows by an application of Bayes' Rule:

$$\begin{aligned}
g\left(a^{t-2}, H_0 \,|\, a_t, a_{t-1}\right) &= \frac{g(a^t, H_0)}{g(a_t, a_{t-1})} = \frac{g(a_t \,|\, a^{t-1}, H_0) g(a^{t-1}, H_0)}{g(a_t \,|\, a_{t-1}) g(a_{t-1})} \\
&= \frac{g(a_t \,|\, a_{t-1}) g(a^{t-1}, H_0)}{g(a_t \,|\, a_{t-1}) g(a_{t-1})} && (61) \\
&= \frac{g(a^{t-1}, H_0)}{g(a_{t-1})} = g(a^{t-2}, H_0 \,|\, a_{t-1}).
\end{aligned}$$

where (61) follows from the Markov property together with the assumption that, conditional on a^t, H_0 does not help predicting future productivity.

Given an initial condition $m_{-1}(a_0)$, recursion (60) allows us to approximate the sequence $\{m_{t-1}(a_t)\}_{t=1}^{\infty}$ of functions. We use numerical integration and we approximate the functions $m_{t-1}(a_t)$ by linear interpolation (see Judd (1998) Chapter 7) with a grid of 100 points over an interval $[a_t^{\min}, a_t^{\max}]$, with

$$\begin{aligned}
a_t^{\min} &= E(a_t) - 7\sqrt{V(a_t)} \\
a_t^{\max} &= E(a_t) + 7\sqrt{V(a_t)}.
\end{aligned}$$

This means that, at each time, the wage draw falls in the interval $[a_t^{\min}, a_t^{\max}]$ with a probability of about $1 - 10^{-11}$.

C.2.3 Step 3: the Moving Values

The moving value U_t solves

$$
\begin{aligned}
& \int H_t(a^t, H_0) \max\left\{\frac{e^{a_t} - U_t}{2\kappa}, 0\right\} g(a^t, H_0)\, da^t dH_0 = 0 \\
\Leftrightarrow \quad & E\left[H_t(a^t, H_0) \max\left\{\frac{a_t - U_t}{2\kappa}, 0\right\}\right] = 0 \\
\Leftrightarrow \quad & E\left[E\left(H_t(a^t, H_0) \max\left\{\frac{e^{a_t} - U_t}{2\kappa}, 0\right\} \,\middle|\, a_t\right)\right] = 0 \\
\Leftrightarrow \quad & E\left[E\left(H_t(a^t, H_0) \,|\, a_t\right) \max\left\{\frac{a_t - U_t}{2\kappa}, 0\right\}\right] = 0 \\
\Leftrightarrow \quad & E\left[\left((1-\delta)m_{t-1}(a_t) + \Pi(a_t)\mathbb{I}_{\{a_t \geq a_t^*\}}\right) \max\left\{\frac{e^{a_t} - U_t}{2\kappa}, 0\right\}\right] = 0 \\
\Leftrightarrow \quad & \int_{-\infty}^{+\infty} \left((1-\delta)m_{t-1}(a) + \Pi(a)\mathbb{I}_{\{a \geq a_t^*\}}\right) \max\left\{\frac{e^{a} - U_t}{2\kappa}, 0\right\} f(a; E(a_t), V(a_t))\, da = 0. \qquad (62)
\end{aligned}
$$

Given our approximation of the function $m_{t-1}(a_t)$, for any U_t we can calculate the integral on the left-hand side of (62), and solve numerically this one-equation-in-one-unknown problem.

C.2.4 Calculating other equilibrium objects

The first paragraph explains how we calculate the rent, the distribution of households, and welfare. The second paragraph focuses on the calculation of house prices.

Rent, distribution of households, and welfare. The rent at time t in an island with current productivity a_t is given by equation (**??**)

$$
\rho_t(a^t, U_t) \quad \equiv \quad v' \circ w^{-1}\left(\max\{e^{a_t} - U_t, 0\}\right) = \frac{1}{4\kappa}\left(\max\{e^{a_t} - U_t, 0\}\right)^2.
$$

The population distribution is given by (19):

$$
n_t(a^t, U_t) \quad = \quad H_t(a^t, H_0)\Phi\left(\max\{e^{a_t} - U_t, 0\}\right) = \frac{H_t(a^t, H_0)}{2\kappa} \max\{e^{a_t} - U_t, 0\}.
$$

We do not calculate the distribution $H_t(a^t, H_0)$ of housing stocks but only the conditional moment $m_{t-1}(a_t) = E\left[H_{t-1}(a^{t-1}, H_0) \,|\, a_t\right]$. The function $m_{t-1}(a_t)$ is very useful to calculate population-weighted moments of any function $K(a_t)$ of the current productivity, as follows:

$$\begin{aligned}
& \int n_t(a^t, H_0) K(a_t) g_t(a^t, H_0)\, da^t dH_0 = E\left[n_t(a^t, H_0) K(a_t)\right] \\
= \;& E\left[H_t(a^t, H_0) \max\left\{\frac{e^{a_t} - U_t}{2\kappa}, 0\right\} K(a_t)\right] = E\left[E\left(H_t(a^t, H_0) \max\left\{\frac{e^{a_t} - U_t}{2\kappa}, 0\right\} K(a_t) \,\middle|\, a_t\right)\right] \\
= \;& E\left[E\left(H_t(a^t, H_0) \,\middle|\, a_t\right) \max\left\{\frac{e^{a_t} - U_t}{2\kappa}, 0\right\} K(a_t)\right] \\
= \;& E\left[\left((1-\delta) m_{t-1}(a_t) + \Pi(a_t) \mathbb{I}_{\{a_t \geq a_t^*\}}\right) \max\left\{\frac{e^{a_t} - U_t}{2\kappa}, 0\right\} K(a_t)\right] \\
= \;& \int_{-\infty}^{+\infty} \left((1-\delta) m_{t-1}(a) + \Pi(a) \mathbb{I}_{\{a \geq a_t^*\}}\right) \max\left\{\frac{e^{a} - U_t}{2\kappa}, 0\right\} K(a) f(a; E(a_t), V(a_t))\, da. \qquad (63)
\end{aligned}$$

Formula (63) allows to calculate a number of equilibrium objects:

1. The measure of households in island with current productivity less than a' at time t is calculated using the formula with

$$K(a) = \mathbb{I}_{\{a \leq a'\}}.$$

2. The population weighted average rent is calculated using the formula with

$$K(a) = 1/(4\kappa) \left(\max\left\{e^a - U_t, 0\right\}\right)^2.$$

3. The population weighted average square rent is calculated using the formula with

$$K(a) = 1/(4\kappa)^2 \left(\max\left\{e^a - U_t, 0\right\}\right)^4.$$

4. The population weighted dispersion of rents follows immediately from the above first and second population weighted moments.

5. The flow welfare, defined as the equally weighted sum of time t utilities is calculated using the formula with

$$K(a) = e^a + v(h) = e^a - 1/2 \max\{e^a - U_t, 0\}.$$

Prices. We now explain our calculation of the price $P_t(a_t)$ of housing services in an island with current productivity a_t. Given some sequence $\{U_t\}_{t=1}^{\infty}$ of moving value, the price is

$$P_t(a_t) \;=\; E\left[\sum_{j=0}^{\infty} \beta^j (1-\delta)^j \frac{1}{4\kappa} \left(\max\{e^{a_{t+j}} - U_{t+j}, 0\}\right)^2 \,\middle|\, a_t\right] = \frac{1}{4\kappa} \sum_{j \geq 0}^{\infty} \left(M_{t+j}(2) - 2U_{t+j} M_{t+j}(1) + U_{t+j}^2 M_{t+j}(0)\right).$$

where, for any $k \in \{0, 1, 2, \ldots\}$,

$$M_{t+j}(k) \equiv E\left[e^{k a_{t+j}} \mathbb{I}_{\{e^{a_{t+j}} \geq U_{t+j}\}} \,\middle|\, a_t\right] \qquad (64)$$

Since a_t is normally distributed, the moment (64) is known in closed form. Indeed,

$$\begin{aligned}
\int_{\log(A)}^{\infty} \exp(ka)\, f(a;\mu,\sigma^2)\, da &= \int_{\log(A)}^{\infty} \exp\left(ka - \frac{(a-\mu)^2}{2\sigma^2}\right) \frac{da}{\sqrt{2\pi\sigma^2}} \\
&= \int_{\log(A)}^{\infty} \exp\left(-\frac{a^2 - 2\mu a + \mu^2 - 2ka\sigma^2}{2\sigma^2}\right) \frac{da}{\sqrt{2\pi\sigma^2}} \\
&= \int_{\log(A)}^{\infty} \exp\left(-\frac{\left(a - \left(\mu + k\sigma^2\right)\right)^2 - (\mu + k\sigma^2)^2 + \mu^2}{2\sigma^2}\right) \frac{da}{\sqrt{2\pi\sigma^2}} \\
&= e^{k\mu + \frac{k^2\sigma^2}{2}} \left[1 - F\left(\frac{\log(A) - (\mu + k\sigma^2)}{\sigma}\right)\right] \\
&\equiv \mathcal{M}(k, A, \mu, \sigma^2).
\end{aligned}$$

where $F(A)$ is the cumulative distribution function of a standard normal random variable. Therefore, in order to calculate (64) we only need to calculate the conditional mean and variance of a_{t+j} given a_t. These solves the following first-order difference equations:

$$E(a_{t+j} \mid a_t) = (1 - \rho_{t+j})\mu_{t+j} + \rho_{t+j} E(a_{t+j-1} \mid a_t) \tag{65}$$

$$V(a_{t+j} \mid a_t) = \rho_{t+j}^2 V(a_{t+j-1} \mid a_t) + \sigma_{\varepsilon t+j}^2, \tag{66}$$

for $j \in \{1, 2, \ldots\}$ with the initial condition $E(a_t \mid a_t) = a_t$ and $V(a_t \mid a_t) = 0$. These calculations provide the following procedure for computing the price of housing services in an island with current productivity a_t.

1. Pick some large integer J and some initial productivity a_t.
2. Calculate the conditional means $E\left[a_{t+j} \mid a_t\right]$ and variances $V\left[a_{t+j} \mid a_t\right]$ using (65) and (66), for $j \in \{1, 2, \ldots, J\}$.
3. Calculate the price using the formula

$$\begin{aligned}
P(a_t) = {} & \frac{1}{4\kappa}\left(\max\{e^a - U_t, 0\}\right)^2 \\
& + \frac{1}{4\kappa}\sum_{j=1}^{J} \beta^j (1-\delta)^j \Big(\mathcal{M}\left(2, \log(U_t), E(a_{t+j} \mid a_t), V(a_{t+j} \mid a_t)\right) \\
& \quad - 2U_{t+j} \times \mathcal{M}\left(1, \log(U_t), E(a_{t+j} \mid a_t), V(a_{t+j} \mid a_t)\right) \\
& \quad + U_{t+j}^2 \times \mathcal{M}\left(0, \log(U_t), E(a_{t+j} \mid a_t)\right) \Big).
\end{aligned}$$

We approximate the price function $P(a_t)$ using linear interpolation. Since the price only depends on the current productivity, we can calculate population weighted moments using the usual formula.