Why do Interest Rate Options Smile?*

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We address three questions relating to the interest rate options market: What is the shape of the smile? What are the economic determinants of the shape of the smile? Do these determinants have predictive power for the futures shape of the smile and vice versa? We investigate these issues using daily bid and ask prices of euro (€) interest rate caps/floors. We find a clear smile pattern in interest rate options. The shape of the smile varies over time and is affected in a dynamic manner by yield curve variables and the future uncertainty in the interest rate markets; it also has information about future aggregate default risk. Our findings are useful for the pricing, hedging and risk management of these derivatives.

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1. Introduction

Over-the-counter interest rate options such as caps/floors and swaptions are among the most liquid options that trade in the global financial markets, with about $28 trillion of notional principal and $570 billion in gross market value outstanding as of December 2005.[1] Given the large size of these markets, significant effort has been devoted, both in academia and in industry, to the development and testing of valuation models to accurately price and hedge these claims.[2] However, most of these studies have focused on at-the-money options, with very little attention paid to the determinants of volatility smiles/skews in interest rate options markets.[3] In this paper, we address this issue in the euro (€) interest rate options market by characterizing the smile and its time variation. We examine the economic determinants of the volatility smile patterns in the interest rate options markets. We also examine the information content of interest rate option smiles, in order to understand whether they have any power in predicting specific macro-economic variables.

Volatility smiles are an extensively documented cross-sectional feature in the equity options markets, ever since they first appeared after the October 1987 stock market crash. However, the focus of research in the equity options literature has primarily been to relax the assumptions of the Black-Scholes valuation framework to model the volatility smile patterns observed in the market. The frameworks proposed have evolved from models with deterministically varying volatility of returns to models that incorporate either stochastic volatility, or jumps in the

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[1] BIS Quarterly Review, June 2006, Bank for International Settlements, Basel, Switzerland.

[2] These include Driessen, Klaasen and Melenberg (2003), Fan, Gupta and Ritchken (2003), Longstaff, Santa-Clara and Schwartz (2001), Peterson, Stapleton and Subrahmanyam (2003), and many others.

[3] Gupta and Subrahmanyam (2005) and Jarrow, Li and Zhao (2006) do examine smile effects in interest rate options, but only from a modeling perspective.

underlying price process, or both.[4] In spite of their increasing complexity, none of these models has been successful in accurately explaining the behavior of the observed volatility smiles - the empirically observed smiles are typically more perceptible than those predicted by theory. Effort has also been devoted to explaining the volatility smile in equity options markets using liquidity effects or market frictions, with some success.[5] Very little research has been conducted on directly examining the *economic* determinants of the volatility smile patterns in the equity options markets. An exception is the paper by Pena, Rubio, and Serna (1999), who examine the determinants of the implied volatility function in the Spanish index options market.

In contrast to the literature on equity options, research on the smile in the interest rate options market has been quite sparse. The sole exception is a paper by Jarrow, Li and Zhao (2006) who examine the smile in US dollar caps and floors based on models augmented with stochastic volatility and jumps. However, they find that such augmented models do not fully capture the smile.

The conclusions from equity options markets cannot be readily extended to interest rate option markets, since these markets differ significantly from each other for several reasons. In contrast to equity options, interest rate option markets are almost entirely institutional, with hardly any retail presence. Most interest rate options, particularly the long-dated ones such as caps, floors and swaptions, are sold over-the-counter (OTC) by large market makers, typically international banks. The customers are usually on one side of the market (the ask-side), and the size of individual trades is relatively large. Many popular interest rate option products, such as caps, floors and collars are portfolios of options, from relatively short-dated to extremely long-dated ones. These

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[4] See Bakshi, Cao and Chen (1997), Dumas, Fleming and Whaley (1998), Bates (2000) and several references therein for more on this literature.

[5] See Ederington and Guan (2002), Mayhew (2002), and Pena, Rubio and Serna (1999, 2001), and Bollen and Whaley (2004), Garleanu, Pedersen and Poteshman (2006) for example.

features lead to significant issues relating to supply/demand and asymmetric information that are different from those for exchange traded equity options. Since interest rate options are traded in an OTC market, there are also important credit risk issues that may influence the pricing of these options, especially during periods of crisis. Therefore, inferences drawn from studies in the equity option markets are not directly relevant for interest rate option markets, although there may be some broad similarities.

Given the limited success of attempts to model the distribution of the underlying to explain smile, we take a different approach. We seek to directly examine the economic determinants of the smile. To give an analogy, our approach similar to finding empirical risk factors as opposed to calibrating utility-based models in order to explain the cross-section of stock returns, in the asset pricing literature. In this paper, we contribute to the literature in three distinct ways. First, we present an extensive documentation of the volatility smile patterns in the interest rate option markets for different maturities, separately for the bid and the ask sides of the market. Second, we explore the determinants of volatility smiles in these markets, in terms of macro-economic and liquidity variables. Third, we examine the bidirectional Granger-causality relationships between volatility smiles and the macro-economic and liquidity variables within a multivariate vector autoregression framework, to understand whether any of these variables have power in predicting smile patterns, or whether the volatility smiles have any information about the future values of these variables.

We find that there are clearly perceptible volatility smiles in caps and floors, across all maturities. However, the pattern of these volatility smiles varies across option maturities. Short-term caps and floors exhibit smiles that are significantly steeper than those for longer-term caps and floors. Long-term floors display more of a “smirk” than a smile. We then estimate parametric functional forms for the volatility smiles for caps and floors separately, as well as for caps and floors pooled together, and find that they display significant curvature, as well as an asymmetry in the slope of the smile.

We also find that measures of the shape of the volatility smile are significantly related to term structure variables. In particular, the curvature of the smile is positively related to the 6-month interest rate for shorter maturity options and negatively related to the slope of the term structure for longer maturity options. This suggests that away-from-the-money options, especially of shorter maturity, are significantly more expensive (compared to at-the-money options in terms of implied volatility), during higher interest rate regimes. On the other hand, the away-from-the-money options are comparatively less expensive when the term structure is relatively flat. In addition, we find that high-volatility periods are associated with flatter volatility smiles, suggesting a stochastic volatility framework with mean reversion in volatility. We also find some evidence that the curvature of the smile, especially for longer maturity options on the ask side, is positively related to the liquidity costs in this market at proxied by the bid-ask spreads. We conjecture that perhaps liquidity effects could account for a part of the smile, especially for longer maturity options. Our results for the slope of the volatility smile show that out-of-the money caps (floors) become disproportionately more expensive when interest rates go up (down). This may be a result of the existence of price pressure in this market induced by hedging demand from customers, consistent with some of the results reported in Bollen and Whaley (2004) and Garleanu, Pedersen and Poteshman (2006). Alternatively, the slope of the yield curve may capture the skew of the distribution of future interest rate and thus affect the slope of the smile.

Multivariate Granger-causality tests are used to examine if lagged values of any of the explanatory variables can predict the curvature and asymmetry of the volatility smile and vice-versa. We find that the 6-month interest rate Granger-causes the slope and the curvature of the volatility smile, while the slope of the term structure Granger-causes the curvature of the smile curve. Also, we find that a positive innovation in the 6-month interest rate results in a positive response of the curvature of smile, again indicating that away-from-the-money options become more expensive when the short term interest rate increases. On the other hand, a positive shock to the 6-month interest rate results in a reduction in slope of the smile. As before, this result is consistent with the idea that higher interest rates result in a greater demand for options that hedge against increasing interest rates, thus pushing up their prices.

In addition, we find that slope of the volatility smile curve can predict the aggregate default spread in the economy. Looking at the impulse response function, it appears that a positive shock to the slope of the smile of shorter maturity options is followed by increase in the default spread. The result is intuitive because a higher slope of the smile means higher relative prices of out-of-the-money floors (or in-the-money caps) that hedge against the risk of falling interest rates. Falling interest rates are associated with an economic downturn and higher default risk, and thus, an increase in the default spread.

The results of our paper have major implications for the modeling and risk management of interest rate derivatives, especially options. We find that even after controlling for persistence in the shape of the smile, lags of the 6-month interest rate and the slope of the yield curve have information about future shapes of the smile. Usually, while calibrating the interest rate option models, only the contemporaneous yield curve is used. Our results suggest that using lagged valued of the short term interest rate and the slope of the yield curve would improve the calibration of the models. This is intuitive if the future distribution of interest rates is not fully captured by today's yield curve, but, in addition, depends on the past values of interest rates.

We also find that the shape of the smile predicts the aggregate default spread in the economy, even after controlling for persistence in default spread and lagged valued of yield curve variables. Hence, the shape of the smile curve has information about the future state of the economy, over and above what can be explained by past values of macroeconomic variables. This result has important implications for the pricing and hedging of credit derivatives, whose payoffs directly depend on the default spread.

The structure of our paper is as follows. Section 2 describes the data set and presents summary statistics. Section 3 presents the empirical patterns of the volatility smile that we observe in the data, and the impact of several macro-economic variables on these patterns. Section 4 presents the results of the multivariate vector autoregression and the Granger-causality tests. Section 5 concludes with a summary of the main results and directions for future research**.**

2. Data

The data for this study consist of prices of euro (€) caps and floors over the 29-month period, January 1999 to May 2001, obtained from WestLB (Westdeutsche Landesbank Girozentrale) Global Derivatives and Fixed Income Group. These are daily bid and offer quotes over 591 trading days for nine maturities (2 years to 10 years, in annual increments) across twelve different strike rates ranging from 2% to 8%. This is an extensive set with price quotes for caps and floors every day, reflecting the maturity-strike combinations that elicit market interest on that day.

WestLB is one of the dealers who subscribe to the interest rate option valuation service from Totem. Totem is the leading industry source for asset valuation data and services supporting independent price verification and risk management in the global financial markets. Most leading derivative dealers subscribe to their service. As part of this service, Totem collects data for the entire range of caplets and floorlets across a series of maturities from these dealers. They aggregate this information and return the consensus values back to the dealers who contribute data to the service. The market consensus values supplied to the dealers include the underlying term structure data, caplet and floorlet prices, as well as the prices and implied volatilities of the reconstituted caps and floors across strikes and maturities. Hence, the prices quoted by dealers such as WestLB, who are a part of this service, reflect the market-wide consensus information about these products. This is especially true for plain-vanilla caps and floors, which are very high-volume products with standardized structures, that are also used by dealers to calibrate their models for pricing and hedging exotic derivatives. Therefore, it is extremely unlikely that any large dealer, especially one that uses a market data integrator such as Totem, would deviate systematically from market consensus prices for these vanilla products.[6] Our discussions with market participants confirm that the prices quoted by different dealers (especially those that subscribe to Totem) for vanilla caps and floors are generally similar.

Interest rate caps and floors are portfolios of European interest rate options on the 6-month Euribor with a 6 monthly reset frequency.[7] In addition to the options data, we also collected data on euro (€) swap rates and the daily term structure of euro interest rates curve from the same source. These are the key inputs necessary for checking cap-floor parity, as well as for conducting our subsequent empirical tests.

Table 1 provides the descriptive statistics on the midpoint of the bid and ask prices for caps and floors over our sample period. As seen in the table, the prices of these options vary significantly depending on the strike rate and maturity of the option. For a meaningful comparison, in Table 1,

[6] The euro OTC interest rate derivatives market is *extremely* competitive, especially for plain-vanilla contracts like caps and floors. The BIS estimates the Herfindahl index (sum of squares of market shares of all participants) for euro interest rate options (which includes exotic options) at about 500-600 during the period from 1999 to 2004, which is even lower than that for USD interest rate options (around 1,000). Since a lower value of this index (away from the maximum possible value of 10,000) indicates a more competitive market, it is safe to rely on option quotes from a top European derivatives dealer (reflecting the best *market consensus* information available with them) like WestLB during our sample period. Thus, any dealer-specific effects on price quotes are likely to be small and unsystematic across the over 30,000 bid and ask price quotes each that are used in this paper.

[7] Please refer to Longstaff et al (2001) for discussion of the contract structure of US dollar caps and floors. Also, please refer to Deuskar, Gupta and Subrahmanyam (2006) for a detailed discussion of the contract structure of Euro caps and floors.

the prices of options are grouped together into “moneyness buckets,” by estimating the Log Moneyness Ratio (LMR) for each cap/floor. The LMR is defined as the logarithm of the ratio of the par swap rate to the strike rate of the option. Since the relevant swap rate changes every day, the moneyness of options at the same strike rate and maturity, measured by the LMR for that maturity, also changes each day. The average price, as well as the standard deviation of these prices, in basis points, is reported in the table. It is clear from the table that cap/floor prices display a fair amount of variability over time. Since these prices are grouped together by moneyness, a large part of this variability in prices over time can be attributed to changes in volatilities over time, since term structure effects are largely accounted for by our classification.

For many of our empirical tests, we pool the data on caps and floors, since it allows us to obtain a wider range of strike rates, covering rates that are both in-the-money and out-of-the-money for both caps and floors. Before doing so, we check for put-call parity between caps floors and swaps, using both bid and ask prices. We find that, on average, put-call parity holds in our dataset, although there are deviations from parity for some individual observations. Many of these deviations may not be actual violations from parity, given the difficulty in carrying out the arbitrage using “off-market” swaps.[8] These parity computations are a consistency check, as well, to assure us about the integrity of our dataset.

3. Volatility Smiles in Interest Rate Option Markets

We use implied volatilities from the Black-BGM model throughout the analysis from here on. We do so for two reasons. First, although there may be an alternative complex model that explains at least part of the smile/skew or the term structure of volatility, it is necessary to obtain an initial sense of the empirical regularities using the standard model. In other words, we need to document

[8] Since the bid and ask prices of “off-market” swaps are not available, we cannot examine which of these observations is a *real* violation of put-call parity.

the characteristics of the smile before attempting to model it formally.[9] Furthermore, the evidence in the equity option markets suggests that even such complex models may not explain the volatility smile adequately, without considering the effect of market frictions. Second, Black-BGM implied volatilities are the common market standard for quotations between dealers for interest rate option prices.

We document volatility smiles in euro interest rate caps and floors across a range of maturities. Figure 1 presents scatter plots of the implied "flat" volatilities of caps and floors over our sample period for a set of representative maturities. Flat volatility is a volatility number common to all the caplets (floorlets) in a cap (floor), which sets the sum of their prices equal to the quoted price for the cap (floor). Thus flat volatility is a weighted average of the implied volatility of individual options included in a cap or a floor.[10] The vertical axis in the plots corresponds to the implied volatility of the mid-price (average of bid and ask price) of the cap/floor, scaled by the at-the-money volatility for the cap of the same maturity (Scaled IV). This scaling accounts for the effect of changes in the level of implied volatilities over time. The horizontal axis in the plots corresponds to the *log moneyness ratio* (LMR), our measure of the moneyness of the option.

We first examine the overall shape of the implied volatility smile. The plots are presented for three representative maturities - 2-year, 5-year, and 10-year, for the pooled cap and floor data.[11]

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[9] The use of implied volatilities from the Black-Scholes model is in line with all prior studies in the literature, including Bollen and Whaley (2004).

[10] Our implied volatility estimation is likely to have much smaller errors than those generally encountered in equity options (see, for example, Canina and Figlewski (1993)). We pool the data for caps and floors, which reduces any error due to mis-estimation of the underlying yield curve. The options we consider have much longer maturities (the shortest cap/floor is 2 year maturity), which reduces this potential error further. In addition, for most of our empirical tests, we do not include deep ITM or deep OTM options, where estimation errors are likely to be larger. Furthermore, since we consider the implied flat volatilities, the errors are further reduced due to the implicit "averaging" in this computation.

[11] The separate plots for caps and floors, and the plots for other maturities, show similar characteristics; hence, they

These plots clearly show that there is a significant smile curve in interest rate options in this market, across strike rates. The smile curve is steeper for shorter-term options, while for longer-term options, it is flatter and asymmetric around the at-the-money strike rate.

In addition, we analyze the principal components of the changes in the Black volatility surface (across strike rates and maturities) for caps and floors. If away-from-the-money option prices are just mechanical transformations of ATM option prices, we would observe that a very high proportion of the variation in these implied volatilities are explained by just one principal component. However, we find that for caps, on the ask-side, there are four significant principal components that together explain 91.7% of the daily variation in the volatility surface (the first four components explain 31.7%, 29.2%, 17.1%, and 13.9% respectively). On the bid-side, we find that two significant principal components explain 80.5% of the daily variation – the first four principal components together explain 89.9% of the daily variation in the volatility surface (42.8%, 37.7%, 5.1%, and 4.3% respectively). The structure of the principal components is similar for floors. The presence of more than one significant principal component indicates that the implied volatilities for away-from-the-money options are not just being adjusted by the dealer using a mechanical rule anchored by the at-the-money volatilities.

3.1 Functional forms for implied volatility smiles

Next, we estimate various functional forms for volatility smiles using pooled time-series and cross-sectional regressions, in order to understand the overall form of the volatility smile over our entire sample period. The most common functional forms for the volatility smile used in the literature are quadratic functions of either moneyness or the logarithm of moneyness. The scatter plots in Figure 1 also suggest a quadratic form for the smile. In order to account for the asymmetry, if any, in the smile curve, we allow the slope to differ for in-the-money and out-of-

have not been presented in the paper.

the-money options. We also estimate the linear and quadratic functional forms without the asymmetry term. In addition, we present the volatility smiles on the bid-side and the ask-side separately. Using the mid-point of the bid-ask prices may not always accurately display the true smile in the implied volatility functions, given that bid-ask spreads differ across strike rates.

The specific models that we estimate, using ordinary least squares estimation in the pooled time series and cross sectional regression, are as follows:

$$Scaled\ IV = c1 + c2 * LMR \qquad .(1)$$

$$Scaled\ IV = c1 + c2 * LMR + c3 * LMR^2 \qquad (2)$$

$$Scaled\ IV = c1 + c2 * LMR + c3 * LMR^2 + c4 * 1_{LMR<0} * LMR \qquad (3)$$

Figure 2 presents the plots of fitted implied volatility functions based on specification (3) for caps and floors separately for different maturities. These plots clearly show a smile curve for these options and display some interesting patterns. Caps always display a smile, for all maturities, although the smile flattens as the maturity of the cap increases. In-the-money caps (caps with LMR>0) have a significantly steeper smile than out-of-the-money caps, which is indicative of the asymmetric slope of the smile on either side of the at-the-money strike. More interestingly, the ask-side of the smile is steeper than the bid-side, the difference being significantly larger for in-the-money caps. Floors display somewhat similar patterns. The smile gets flatter as the maturity of the floor increases. In-the-money floors (floors with LMR<0) exhibit a significantly steeper smile, especially for short-term floors. Long-term floors display almost a “smirk”, instead of a smile. As with caps, the smile curve for floors is steeper on the ask-side, as compared to that on the bid-side.

In Table 2, we report the results for caps and floors pooled together for specification (3), the quadratic functional form with the asymmetric slope term, since it fits the observed volatility smiles the best.[12] The regression coefficients in all the specifications are highly significant. In addition, the quadratic functional form with an asymmetric slope term explains a fairly high proportion of the variability in the scaled implied volatilities. In most specifications, the asymmetry term for the slope of the smile is significant; indicating that the shape of the volatility function is different for in-the-money options compared to out-of-the-money options.[13] The coefficient of the curvature of the smile decreases with the maturity of the options, indicating that as the maturity of these options increases, the smile flattens, and eventually converts into a "smirk" when we reach the 10-year maturity. The advantage of using pooled data is that we have observations across a wider range of strike rates (moneyness), on both sides of the at-the-money strike rate. This allows us to estimate the true functional form for the smile more accurately. For all the subsequent analyses, we pool the data from caps and floors together.

3.2 Time variation in volatility smiles

In Figure 3, we present the surface plots for the implied volatilities over time, by moneyness represented by the LMR.[14] The shapes of these surface plots show similar trends – the 2-year maturity-options display a large curvature in the volatility smile, while the smile flattens out and

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[12] We also tested a specification with an asymmetric term for the curvature of the smile, but it did not add any significant explanatory power over the specification with the asymmetric term for just the slope of the smile. We got similar results when we tested a polynomial specification with higher order terms, which turned out to be statistically insignificant.

[13] We also conducted the same exercise with spot volatilities i.e. using inferred prices of individual caplets and floorlets, obtained by bootstrapping from the flat volatilities of caps and floors. Model (3) fits well there as well. Those results are not presented here to conserve space.

[14] These plots are presented for representative maturities of 2-, 5-, and 10-years, since the plots for the other maturities are similar. In addition, since 3-D plots require the data to be complete over the entire grid, we present the volatility smiles over the LMR range from -0.3 to +0.3, which is the subset of strikes over which reasonably complete data are available over a substantial number of days in our dataset.

turns into more of a skew as we move towards the longer maturity options, especially at the 10-year maturity. More importantly, both the curvature and the slope of the volatility smile show significant time-variation, sometimes even on a daily basis. The changes in the curvature and slope over time are more pronounced for the 2-year maturity options, although they are also perceptible for the longer maturity options.

Figure 3 also presents the surface plot of the euro spot interest rates for maturities from one to ten years over our sample period. Similar to the volatility surfaces, the euro term structure surface also shows significant time variation. It is clear that there is an increase in spot interest rates in the early part of our sample, followed by a flattening of the term structure due to an increase, primarily in the rates at the shorter end of the term structure, during the latter part of our sample period. Therefore, both the level of interest rates and the slope of the term structure exhibit significant time variation over our sample period.

Based on these figures, the natural question to ask is whether on a time-series basis, certain economic variables exhibit a significant relationship with the implied volatility smile patterns. In order to examine this question, we first need to define appropriate measures of the asymmetry and curvature of the smile curve each day. We can then determine empirical proxies for these attributes and estimate them using the volatility smile curve, each day. The measure of the asymmetry of the implied volatility curve, widely used by practitioners, is the "risk reversal," which is the difference in the implied volatility of the in-the-money and out-of-the-money options (roughly equally above and below the at-the-money strike rate). The measure of the curvature is the "butterfly spread," which is the difference between the average of the implied volatilities of two away-from-the-money volatilities and the at-the-money volatility.[15] The advantage of using

[15] These structures involve option-spread positions and are traded in the OTC interest rate and currency markets as explicit contracts. These prices are often used in the industry for calibrating interest rate option models. See, for example, Wystup (2003).

these empirical measures is that they explicitly capture the slope and the curvature of the smile curve. Therefore, they can be interpreted as proxies for the skewness and kurtosis of the risk-neutral distribution of interest rates.

We construct the two variables defined above, the butterfly spread and the risk reversal, to proxy for the curvature and asymmetry of the daily smile in interest rate options. We fit a quadratic function of the LMR to the scaled implied volatilities each day.[16] We then use the fitted values from this regression to construct the risk reversal and butterfly spread, defined as follows:

$$\begin{aligned} &Risk\ Reversal = Scaled\ IV_{+0.25LMR} - Scaled\ IV_{-0.25LMR} \\ &Butterfly\ Spread = \left(Scaled\ IV_{+0.25LMR} + Scaled\ IV_{-0.25LMR}\right)/\,2 - Scaled\ IV_{ATM} \end{aligned} \tag{4}$$

The butterfly spread captures the average scaled implied volatility at a 0.25 LMR away-from-the-money, on either side of 0. It is essentially a linear transformation of the curvature coefficient from the quadratic function. Hence, it is our proxy for the curvature of the smile. The risk reversal represents the difference between the implied volatility of in-the-money options and out-of-the-money options. It is a linear transformation of the slope coefficient from the quadratic function. Thus, it is a proxy for the asymmetry in the slope of the smile.

It is important to note that we estimate the risk reversal and the butterfly spread by *only* going away-from-the-money by 0.25 LMR on either side of the at-the-money strike rate. To understand the moneyness levels in terms of actual contract strikes, consider a cap with an at-the-money strike rate of 4%. In this case, a cap with an LMR of 0.25 would have a strike rate of about 3.1%, while a cap with an LMR of -0.25 would have a strike rate of about 5.1%. These strike rates are well within the range of actively traded caps in terms of moneyness.

[16] We tried fitting a quadratic function with asymmetric term, but on a day-to-day basis it leads to overfitting.

In Figure 4, we present the time-series plots of the risk reversal and the butterfly spread over our sample period, for representative maturities of 2, 5, and 10 years (the other maturities are similar). For comparison, we also present the 6-month interest rate and the at-the-money volatility on the same plots, with the 6-month interest rate plotted on the secondary vertical axis on the right. While both the slope and the curvature of the smile change almost on a daily basis, there is more variability in the slope of the volatility smile, as compared to that in the curvature. For example, for the 5-year caps and floors, the slope of the smile fluctuates around zero during the first half of our sample period. However, in the second half of our sample period, it becomes negative and more volatile. It is interesting to note that the second half of our sample period is also one where interest rates increased. These variables could potentially be linked with each other through lead/lag relationships, which is one of the central issues that we examine in this paper. The curvature of the smile also changes on a daily basis, but fluctuates within a much narrower range, especially for longer maturity options.

3.3 The determinants of the volatility smile

One of the objectives of this paper is to examine the determinants of the volatility smiles in interest rate option markets. A clear understanding of the determinants of these smile patterns can help in developing models that eventually explain the entire smile. To this end, we explore the contemporaneous relationship between the slope and curvature of the daily smiles and several economic and option variables. The economic determinants include the level of volatility of at-the-money interest rate options, the slope of the term structure (5-year rate minus the 6-month rate), the spot 6-month Euribor, the 6-month Treasury-Euribor spread (Default Spread), and the scaled ATM bid-ask spread as a proxy of liquidity costs in the market. These are time-series regressions of curvature and asymmetry measures calculated using data across all the strikes each day. The regression specifications are as follows:[17]

$$\begin{aligned} BS &= c1 + c2 * ATMVol + c3 * 6Mrate + c4 * 5yr6Mslope + c5 * DefSpread + c6 * atmBAS \\ RR &= d1 + d2 * ATMVol + d3 * 6Mrate + d4 * 5yr6Mslope + d5 * DefSpread + d6 * atmBAS \end{aligned} \quad (5)$$

The intuition for examining these variables is as follows. First, the at-the-money volatility variable is added to examine whether the patterns of the smile vary significantly with the level of uncertainty in the market. During uncertain times, reflected by higher volatility, information asymmetry issues are likely to be more important than during periods of lower volatility. If there is significantly greater information asymmetry, market makers may charge higher than normal asking prices for away-from-the-money options, since they may be more averse to taking short position at these strike rates. This would lead to a steeper smile, especially on the ask side of the smile curve. Also, during times of greater uncertainty, a risk-averse market maker may demand higher compensation for providing liquidity to the market, which would affect the shape of the smile. Since we have already divided the volatility of each option by the volatility of the corresponding ATM cap to obtain the scaled IV, we use the ATM swaption volatility, a general measure of the future interest rate volatility, as an explanatory variable here, in order to avoid having the same variable on both sides of the regression equation.[18]

[17] This time series regression is estimated by including AR(2) error terms to correct for serial correlation. We find no serial correlation in the residuals after this correction. In addition, for all maturities, the Durbin-Watson statistic is insignificantly different from 2. Therefore, the inclusion of the AR(2) error terms, indeed, takes care of any serial correlation in the regression model.

[18] Although swaption implied volatilities are not exactly the same as the cap/floor implied volatility, they both tend to move together. Hence, swaption implied volatilities are a valid proxy for the perceived uncertainty in the future interest rates. The data on the ATM swaption volatility in the Euro market was obtained from DataStream.

Second, we include the spot 6-month Euribor as another explanatory variable. The level of interest rates is indicative of general economic conditions, as well as the direction of interest rate changes in the future - for example, if interest rates are mean-reverting, very low interest rates are likely to be followed by rate increases. This would manifest itself in a higher demand for out-of-the-money caps in the market, thus affecting the prices of these options, and possibly the shape of the implied volatility smile itself.

Third, the slope of the yield curve is added as an explanatory variable, as it is widely believed to proxy for general economic conditions, in particular the stage of the business cycle. The slope of the yield curve is also an indicator of future interest rates, which affects the demand for away-from-the-money options: if interest rates are expected to increase steeply, there will be a high demand for out-of-the-money caps, resulting in a steepening of the smile curve.

The ATM volatility and the term structure variables act as approximate controls for a model of interest rates that displays skewness and excess kurtosis. Typically, in such models the future distribution of interest rates depends on today's volatility and the level of interest rates. Hence, we examine the relationship of the volatility smile to the 6-month Treasury-Euribor spread. This variable is often used as a measure of aggregate liquidity as well as the default risk of the constituent banks in the Euribor fixing. A wider spread indicates a higher default risk for the constituent banks, and possibly also higher risk of default of interest rate option dealers. It could affect the prices of away-from-the-money options more than the prices of ATM options, thus affecting the shape of the smile.

Fifth, we include a measure of the at-the-money relative bid-ask spreads of these options. The objective of including this variable is to directly control for the explicit liquidity of these options, while examining the relationship of the other economic variables to the volatility smile. The relative bid-ask spreads of ATM options capture the general level of liquidity in the market.

The results from this regression analysis are presented in Table 3. They suggest that the degree of curvature of the smile is positively and significantly related to the 6-month interest rate, with the effect being insignificant for long maturity options. When interest rates are high, the away-from-the-money options, especially the ones with shorter maturities, are priced relatively higher than during times when interest rates are lower. Shorter maturity caps and floors appear to be most sensitive to this effect, which explains the declining significance of this result as we move towards longer maturity options. On the other hand, the curvature is negatively related to the slope of the term structure; interestingly, this effect is significant only for the longer maturity options. It appears that the volatility smiles in this market have more curvature when the term structure is relatively flat. When the term structure is upward sloping, the prices of the in-the-money and out-of-the money options are affected differently as we find below while examining the relationship between the risk reversal and the slope of the yield curve. Perhaps these effects, on average, result in a flatter smile when the term structure is upward sloping. These results are consistent for the bid- as well as the ask-side quotations, as well.

The results also show that the degree of curvature is negatively related to the volatility of at-the-money options, although this effect is significant only for short/medium maturity options. Therefore, highly volatile periods tend to be associated with a lower curvature of the smile, which is consistent with the evidence in the equity options literature (Pena, Rubio, and Serna (1999)). These results suggest a stochastic volatility framework with the volatility itself exhibiting mean reversion. In such a model, high volatility periods are likely to be followed by lower volatility periods, which would result in a shallow smile when volatility is high. We also find weak evidence of the curvature of the smile being positively related to the liquidity costs in the market, but this effect is significant only for long maturity options on the ask-side. This is understandable, since an increase in bid-ask spreads would increase the curvature of the smile on the ask side. Perhaps liquidity effects do not play an important role in explaining the shape of the volatility smile for shorter maturity caps and floors, but for longer maturity options, it may be important to account for liquidity effects while modeling the volatility smile.

The slope of the volatility smile exhibits somewhat different relationships to the contemporaneous determinants examined in this section. When the short-term interest rate is high, the slope of the smile appears to be more negative. Since the slope of the smile is defined as the difference in ScaledIV between +0.25 LMR and -0.25 LMR, it is important to understand the effects separately for caps and floors. Since a negative LMR refers to out-of-the-money caps and a positive LMR refers to out-of-the-money floors, this implies that when interest rates increase, out-of-the-money caps become even more expensive. Conversely, when interest rates decline, out-of-the-money floors become disproportionately expensive. These results are quite intuitive. It is possible that the demand for out-of-the-money caps (floors) is higher when interest rates go up (down). Then, consistent with the findings of Bollen and Whaley (2004) and Garleanu, Pedersen and Poteshman (2006), demand pressure may affect the prices of interest rate options at some strikes, thereby affecting the shape of the volatility smile. Similarly, when the term structure becomes more steeply upward sloping, the smile becomes more negative, which is consistent with the previous result. With mean reversion in interest rates, an upward-sloping yield curve is a signal that interest rates will increase in the future, thereby leading to higher demand for out-of-the-money caps, which would make the volatility smile more negative. An alternate way of thinking about this effect is that the slope of the yield curve captures the skew of the distribution of future interest rates, thus affecting the slope of the smile.

Finally, we find some evidence that the slope of the smile curve is related to the default spread. However, this relationship is not consistent across all maturities. Perhaps the relationships between the parameters of the volatility smile and the economic variables exhibit a lead/lag relationship, which we explore further in the next section.

4. Multivariate Vector Autoregression

In the previous sub-section, we show that economic variables are significantly related to the shape of the *contemporaneous* smile. In this section, we examine the relationship between the *lagged* values of economic variables and the shape of the smile, and vice-versa. We use a multivariate vector autoregression for this analysis, since it can provide useful information on whether knowledge of the past values of a variable improves the short-run forecasts of the current and future values of another variable. Although this analysis does not explain causality *per se*, it may throw light on the linkages between the economic variables and the volatility smile in a dynamic, predictive sense.

We estimate a six-equation, multivariate, vector autoregression separately for the butterfly spread and the risk reversal, each of which includes the five economic and liquidity variables (at-the-money volatility, 6-month rate, the slope of the term structure, the default spread, and the at-the-money bid-ask spreads).[19] Consider the following system:

$$X_t = A(L) X_{t-1} + u_t \tag{6}$$

where, X is a vector containing six variables – five described above and BS or RR. A(L) is a polynomial of order K in the lag operator, L. If K=2, the system on the RHS includes two lags of each of the six variables. We estimate the system separately for each option maturity, and for the ask- and the bid-side. We choose K, the appropriate number of lags for the multivariate VAR estimation in each case, using the Akaike information criterion (AIC). For most option maturities, this estimation results in two or three lags, with the maximum number of lags in any system being five. These 36 VAR models (9 option maturities each, for the bid and ask sides, separately for the

[19] We thank Rob Engle for insightful discussions on the econometric procedures used in this section.

butterfly spread and risk reversal) provide a comprehensive description of the time-series movements in the shape of the smile and the economic and liquidity variables.

We first examine the cross-correlations of the innovations obtained from the VAR system. Unexpected shocks to any of the economic variables may be related to the unexpected fluctuations in the shape of the volatility smile. These correlations are presented in Table 4. The most striking relationship noticed from the table is the negative correlation between the shocks to the slope of the term structure and the shocks to the curvature and slope of the volatility smile. These correlations are significantly different from zero for all maturities for the risk reversal, but significant only for the longer maturity options for the butterfly spread. The negative sign is consistent with our results in the previous section. It appears that unexpected twists in the term structure, which may be proxies for unexpected changes in the higher moments of the risk neutral distribution of interest rates, could cause unexpected changes in the shape of the volatility smile curve. To a lesser degree, we find that the shocks to the 6-month interest rate are positively correlated with the shocks to the shape of the smile, especially to the butterfly spread. When interest rates unexpectedly go up, this appears to coincide with greater expectations of extreme moves in interest rates in the future, which would cause the butterfly spread to increase. Similarly, we find some relationship between shocks to the default spread and shocks to the shape of the volatility smile, although these correlations are not consistent across all maturities. In addition, the shocks to the liquidity of at-the-money options appear to be positively related to the shocks to the butterfly spread, especially for longer maturities, consistent with our results in the previous section. This indicates that when liquidity suddenly dries up, the away-from-the-money options become disproportionately more expensive, as reflected in the increase in the curvature of the smile, with the effects being stronger for longer maturity options.

4.1 The predictors of the volatility smile

In Table 5, we present the pair-wise Granger causality tests between the butterfly spread or risk reversal and the five economic variables. The results are presented separately for the bid- and ask-side, and for each maturity. Panel A of the table presents the p-values for rejecting the null hypothesis that variable *i* Granger-causes the shape of the smile (butterfly spread or risk reversal), by testing whether the lag coefficients of variable *i* are jointly zero when the dependent variable in the vector autoregression is BS or RR. We find evidence that for most option maturities, the 6-month interest rate and the slope of the term structure Granger-cause the butterfly spread, on the ask- or the bid-side or both. This indicates that past values of the short term interest rate and the slope of the term structure have some power in predicting the curvature of the volatility smile in this market. Similarly, we find some evidence that the 6-month interest rate Granger-causes the risk reversal. These results show that past realizations of the term structure have some information about the shape of the volatility smiles observed in this market. We also find some information in past values of the at-the-money volatility, and the at-the-money liquidity costs, in predicting the curvature of the volatility smile, but these effects are weaker and less consistent across different option maturities.

Another way of examining the dynamic relationship between these variables is to look at impulse response functions. We calculate impulse responses based on the multivariate VAR standardized by Cholesky decomposition. For the sake of brevity, we only show those cases where we do find Granger causality. Figure 5 presents the response of the butterfly spread to a one Cholesky standard deviation shock to the 6 month rate. The ordering of the VAR for this purpose is the 6-month rate, the 5 yr rate - 6 m rate differential, the default spread (6-month), the ATM BA Spread, BS, and ATM Vol.[20] We show the response for a shock of + / - two standard errors to

[20] Usually the Cholesky decomposition is sensitive to the ordering of the VAR. We order the VAR from the most exogenous variable to the most endogenous variable, based on the results of Granger causality tests. However, we vary

gauge the significance. Only three representative maturities are shown. On the ask side, except for the 2-year cap, a positive shock to the short term interest rate results in an increase in the butterfly spread. The effect is significant initially, and remains so for 5-year and shorter maturities. For longer maturities as represented by 10-year, the effect becomes insignificant as the horizon progresses. On the bid-side the results are qualitatively similar.[21]

Figure 7 shows the response of the risk reversal to one Cholesky standard deviation shock to the 6 month interest rate. The ordering of the VAR in this case is the 6-month rate, the RR, the 5 yr rate – 6 m rate differential, the default Spread (6-month), the ATM Vol, and the ATM BA Spread. On the ask-side, except for the short term maturities like the 2-year, there is a decrease in the risk-reversal following a positive shock to the short term interest rate. Although the statistical significance is mixed, the results are consistent with the idea that an increase in the short term interest rate is followed by increase in the price of the out-of-the-money caps. Recall that the risk reversal stands for the difference in scaled IV of +0.25 LMR and –0.25 LMR. Hence, a decrease in the risk reversal corresponds to the increase in the relative scaled IV on the negative LMR side, or the side corresponding to out-the-money caps. As the short rate increases, the investors are more concerned about hedging the risk of rising interest rates. So the prices of out-the-money caps relative to in-the-money caps increases. An alternate way of thinking about this result is that investors are less concerned about hedging the risk of decreasing interest rates. So, the prices of out-of-the-money floors (corresponding to a positive LMR) relative to in-the-money floors (corresponding to a negative LMR) decrease. The results on the bid-side are similar.

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the ordering to see if the results are sensitive to the change. They appear mostly unchanged by the ordering.

[21] We also examined the response of the butterfly spread to the slope of the yield curve computed in the manner explained above. Although Granger-causality points to the slope of the yield curve having information about the butterfly spread, the impulse responses do not show any clear pattern, since most of them are insignificant.

Tables 6 and 7 present variance decompositions of the butterfly spread and risk reversal. They show how much each of the variables contributes towards the variance of the error in forecasting the shape of the smile. As seen from the tables, the bulk of the variance of the forecast error in the butterfly spread or risk reversal is attributable to the innovations in that variable itself. For butterfly spreads at shorter maturity, the 6-month interest rate contributes around two percent towards the forecast error variance at the horizon of one day. This contribution increases to around six percent at the 10-day horizon. The contributions are smaller for higher maturities. At-the-money volatility is another variable that contributes towards the forecast error variance of butterfly spread.

As seen from Table 7, for the risk reversal as well, innovations to the 6-month rate are the next contributing factor, after innovations to the risk reversal itself. Excluding the 2-year maturity, the contribution of innovations to the short rate start at around one percent at a 1-day horizon and go up to 4-5 percent at the 10-day horizon.

4.2 Information in the volatility smile

Panel B of Table 5 presents *p*-values for the null hypothesis that the shape of the smile (measured by the BS or RR) does not Granger-cause any of the other variables of interest. Here, the idea is to examine if the shape of the smile has information about future values of these economic and liquidity variables. We find that the shape of the volatility smile plays a role in predicting some of the economic variables. In particular, the risk reversal Granger-causes the 6-month default spread, implying that the asymmetry in the volatility smile curves is useful for predicting the default spread in the Euribor market. This is intuitive since the option prices are forward looking. More importantly, our results suggest that the asymmetry in the prices of out-the-money options as compared to those for in-the-money options (which is the cause of the asymmetry in the volatility smile) have information about the future economic outlook, since the default spread is a reflection of the expectations for aggregate default risk in the economy. In particular, if market participants expect the general economic conditions to deteriorate, and interest rates to move down as a result, there would be enhanced interest in buying out-of-the-money caps or floors. The impulse response of the default spread to a shock in the risk reversal sheds further light on this. We discuss the impulse responses next.

Figure 7 presents the response of the default spread to a one Cholesky standard deviation shock to risk reversal computed in a manner similar to earlier responses. The ordering of the VAR in this case is 6-month rate, RR, the 5 yr rate - 6 m rate spread, the default spread (6-month), the ATM vol, and the ATM BA Spread. A positive shock to the risk reversal for shorter maturities (up to 6-year) is followed by a significant increase in the default spread. The results are insignificant for higher maturities. The results are consistent with a positive correlation, at short maturities, between unexpected shocks to risk reversal and default spread. An increase in the risk reversal occurs during the period when investors are more concerned about falling interest rates. Usually, falling interest rates coincide with an economic downturn and a consequent increase in default risk. Hence, the positive response of the 6-month default spread to a shock in the risk reversal of shorter maturity options in consistent with this intuition.

Table 7 presents the decomposition of the forecast error variance of default spread computed from the VAR involving risk reversal. Similar to previous cases, own innovations contribute the most towards forecast error variance of default spread. However, it is interesting to note that shocks to the risk reversal contribute up to 8 percent to the variance of the forecast error. This is a result consistent with what we find using Granger-causality: risk reversal has information about the default spread.

5. Concluding Remarks

We examine the patterns of implied volatility in the euro interest rate option markets, using data on bid and ask prices of interest rate caps and floors across strike rates. We document the pattern of implied volatility across strike rates for these options, separately on the bid-side and the ask-side, and find that the volatility smile curve is clearly evident in the euro interest rate cap and floor market.

We further examine the impact of bid-ask spreads along with other economic variables on the volatility smile curves. We include the level of volatility and interest rates to control for the effects arising out of a more elaborate model of interest rates. We find that these term structure variables have significant explanatory ability for the time-variation in the shape of the smile. During a high-interest-rate regime, the smile appears to be steeper and more skewed. When the yield curve is sloping upward more steeply, the smile in the interest rate options is flatter but more skewed. In addition, when the level of volatility in the interest rate markets is high, the smile is flatter consistent with mean-reverting stochastic volatility.

We investigate the behavior of the relationship between the yield curve variables and the shape of the smile over time and find that it is not static but dynamic. The yield curve variables have information about the future shape of the smile in the interest rate options market. Thus, past values of yield curve variables can be used to formulate and implement hedging and risk management strategies for the interest rate options. We also find that the shape of the smile has information about future default spreads. Thus, past prices of interest rate options can be useful for valuing and hedging credit derivatives. Many of the dealers of interest rate options are also likely to have positions in the credit derivatives. This link between interest rate options and default spread can be useful for the risk management at the firm level.

Our results suggest that understanding the dynamic relationship between the economic variables and the shape of the smile is important for developing pricing and hedging models for interest rate options. In future research, these results should be extended to other time periods and currencies.

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Table 1

Descriptive statistics for cap and floor prices

This table presents descriptive statistics on euro interest rate cap and floor prices across maturities and strike rates, over the sample period Jan 99 - May 01, obtained from WestLB Global Derivatives and Fixed Income Group. The caps and floors are grouped together by moneyness into five categories. The moneyness for these options is expressed in terms of the Log Moneyness Ratio (LMR), defined as the log of the ratio of the par swap rate to the strike rate of the cap/floor. All prices are averages, reported in basis points, with the standard deviations of these prices in parenthesis.

Maturity	Caps					Floors				
	Deep OTM	OTM	ATM	ITM	Deep ITM	Deep ITM	ITM	ATM	OTM	Deep OTM
	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3
2-year	2.1	11.1	43.2	107.7	250.5	250.5	153.7	55.5	13.6	3.6
	(0.5)	(5.8)	(19.8)	(30.9)	(58.8)	(48.1)	(50.7)	(25.4)	(7.9)	(2.0)
3-year	10.7	37.7	91.9	209.6	481.3	529.1	285.3	111.3	32.7	6.9
	(10.0)	(20.0)	(33.8)	(52.3)	(133.4)	(114.2)	(74.7)	(44.6)	(18.0)	(4.6)
4-year	22.3	72.6	152.7	311.3	674.4	728.3	406.4	176.1	62.1	12.0
	(12.5)	(32.2)	(49.7)	(78.3)	(193.1)	(138.7)	(98.9)	(64.8)	(27.8)	(7.9)
5-year	42.7	119.4	221.7	409.1	872.3	910.8	519.5	244.7	94.3	19.2
	(16.3)	(48.6)	(67.2)	(95.4)	(252.2)	(161.2)	(122.5)	(84.5)	(35.2)	(13.9)
6-year	66.9	163.7	286.6	507.9	1,006.6	1,093.1	663.8	323.7	128.6	27.2
	(20.2)	(64.4)	(84.6)	(109.5)	(257.4)	(173.2)	(133.1)	(101.9)	(43.5)	(18.7)
7-year	93.7	210.9	355.8	610.8	1206.4	1,239.0	809.3	393.3	164.1	36.9
	(25.4)	(82.2)	(99.3)	(125.3)	(275.5)	(147.0)	(127.5)	(115.2)	(51.9)	(33.0)
8-year	123.9	264.2	433.2	706.8	1,248.2	1,284.7	924.7	425.2	199.2	46.8
	(31.4)	(98.1)	(115.9)	(162.8)	(253.4)	(120.8)	(139.3)	(108.3)	(59.6)	(32.8)
9-year	152.1	309.6	509.9	811.8	1,310.3	NA	997.1	482.3	235.0	58.9
	(35.6)	(103.2)	(128.7)	(172.2)	(205.3)		(150.2)	(120.9)	(69.6)	(41.5)
10-year	179.6	347.8	598.0	881.3	1,493.4	NA	815.5	541.7	242.9	71.3
	(39.8)	(106.7)	(140.0)	(153.4)	(275.3)		(31.1)	(139.6)	(61.9)	(50.1)

Table 2

Functional forms for implied volatility smiles

This table presents regression results when the scaled implied flat volatility for euro interest rate caps and floors, for various maturities, is regressed on a quadratic function of the Log Moneyness Ratio (LMR) with an asymmetric slope term, as follows:

$$Scaled\ IV = c1 + c2 * LMR + c3 * LMR^2 + c4 * 1_{LMR<0} * LMR$$

The statistics are presented for the period, Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group. The coefficient and regression statistics are presented for caps and floors pooled together, separately for bid and ask prices, for all maturities. An asterisk implies significance at the 5% level.

Maturity	c1	c2	c3	c4	Adj R^2
Ask					
2-year	1.15*	-1.43*	4.92*	1.55*	0.65
3-year	1.15*	-0.67*	2.45*	0.98*	0.59
4-year	1.13*	-0.41*	1.78*	0.67*	0.63
5-year	1.08*	0.25*	0.68*	-0.64*	0.33
6-year	1.04*	0.62*	-0.06*	-1.05*	0.46
7-year	1.05*	0.73*	-0.19*	-1.10*	0.27
8-year	1.04*	0.44*	-0.14*	-0.53*	0.49
9-year	1.04*	0.36*	-0.07*	-0.40*	0.53
10-year	1.11*	0.37*	-0.04	-0.26*	0.59
Bid					
2-year	1.00*	-1.32*	3.55*	1.18*	0.53
3-year	0.99*	-0.35*	0.92*	0.08	0.30
4-year	1.01*	-0.42*	0.98*	0.47*	0.34
5-year	1.00*	-0.18*	0.61*	0.13*	0.40
6-year	0.97*	0.12*	0.21*	-0.27*	0.40
7-year	0.98*	0.38*	-0.04*	-0.55*	0.55
8-year	0.96*	0.31*	-0.09*	-0.35*	0.56
9-year	0.95*	0.28*	-0.06*	-0.33*	0.61
10-year	1.01*	0.31*	-0.05*	-0.27*	0.66

Table 3

Effects of economic variables on volatility smiles

This table presents regression results for the impact of economic and liquidity variables on the curvature of the volatility smile (as proxied by the butterfly spread, BS) and asymmetry in the volatility smile (as proxied by risk reversal, RR):

$$BS = c1 + c2 * ATMVol + c3 * 6Mrate + c4 * 5yr6Mslope + c5 * DefSpread + c6 * atmBAS$$
$$RR = d1 + d2 * ATMVol + d3 * 6Mrate + d4 * 5yr6Mslope + d5 * DefSpread + d6 * atmBAS$$

The statistics are presented for the period, Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group and DataStream. The coefficients and regression statistics are presented for the pooled sample of caps and floors, separately for bid and ask prices, for all maturities. Lagged error terms are included in the regression equation to correct for serial correlation. ** and * indicate statistical significance at the 5% and 10% level respectively.

Panel A: BS

Maturity	c1	c2	c3	c4	c5	c6	Adj R^2
Ask							
2-year	0.44**	-1.99**	8.23**	-1.61	-1.38**	-0.01	0.92
3-year	0.15**	-0.79**	3.05**	1.09	0.29*	-0.09	0.82
4-year	0.07**	-0.35**	2.95**	-0.35	0.34**	-0.12	0.92
5-year	-0.01*	-0.14*	2.76**	0.19	0.02*	0.14	0.97
6-year	0.01	-0.12	1.83*	-0.64	-0.07	0.36**	0.99
7-year	0.04	-0.14	0.17	-0.16	0.03	0.40**	0.94
8-year	0.02	-0.03	0.59	-1.69**	0.03	0.16**	0.98
9-year	0.01	-0.02	0.50	-1.24*	0.05	0.19**	0.97
10-year	0.02	-0.12	0.78	-1.05*	0.04	0.04**	0.95
Bid							
2-year	0.46**	-1.90**	3.45**	-1.83	-1.87**	0.21	0.86
3-year	0.20**	-1.01**	1.86**	-0.69	-0.05*	-0.22*	0.7
4-year	0.02*	-0.24**	1.90**	0.04	-0.01	-0.03	0.78
5-year	0.10**	-0.61**	0.76**	-0.49	-0.46**	0.09	0.58
6-year	0.02*	-0.09*	0.73*	-0.89	0.04*	0.09	0.82
7-year	-0.01	0.08	0.44*	-0.13*	0.04	0.18	0.87
8-year	0.03	-0.12	0.14	-1.45**	0.07	0.00	0.88
9-year	0.02*	-0.03	0.18	-1.19**	0.06	0.01	0.88
10-year	0.09**	-0.51**	-0.03	-1.03**	0.01	-0.09**	0.79

Panel B: RR

Maturity	d1	d2	d3	d4	d5	d6	Adj R^2
Ask							
2-year	-2.42**	2.42**	41.57	-11.32	2.05	-0.25	0.86
3-year	0.15*	0.49*	-7.40**	-9.98**	1.17**	0.16	0.83
4-year	0.42	0.75**	-13.77**	-8.50**	0.57*	0.09	0.92
5-year	0.69**	-0.33*	-12.62**	-14.91**	0.58**	0.02	0.73
6-year	0.65**	-1.04**	-8.13**	-8.57**	0.59**	-0.13	0.96
7-year	0.49**	-0.22*	-4.73**	-7.79**	0.47*	-0.90	0.89
8-year	0.12*	0.76	-1.77*	-1.85*	-0.06*	-0.25	0.88
9-year	0.26**	-0.03	-1.39	-1.64	-0.31	-0.98	0.85
10-year	0.41*	-0.07	-5.53**	-9.53**	-0.37*	-0.13*	0.89
Bid							
2-year	-0.75	2.87**	-5.90	-10.60	3.64**	0.01	0.94
3-year	-0.20	1.47**	-6.42**	0.73	1.50**	0.08	0.88
4-year	-0.06	0.49*	-5.74**	2.91	0.56*	0.49	0.82
5-year	0.44**	-0.20	-10.89**	-5.30**	0.30	0.12	0.9
6-year	0.42**	-0.81	-5.57*	-6.03**	0.61**	-0.01	0.93
7-year	0.51**	-0.46*	-6.22**	-6.24**	0.34*	-0.65	0.89
8-year	0.11*	0.28	-0.13*	-1.33*	-0.08	-0.19	0.89
9-year	0.10	-0.22	0.66	0.00	-0.19	0.09	0.91
10-year	0.17	0.24	-3.01**	-2.88	-0.52**	0.66**	0.89

Table 4

Correlations in VAR innovations

This table presents the correlations between innovations from the multivariate vector autoregression for six variables - the level of volatility of at-the-money interest rate options (ATM Vol), the spot 6-month Euribor (6 m rate), the slope of the term structure (5 yr rate - 6 m rate), the 6-month Treasury-Euribor spread (Default Spread), the scaled ATM bid-ask spreads (ATM BA spread) and butterfly spread (BS) or risk reversal (RR) for all maturities separately for ask and bid sides for the period Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group and DataStream. The correlations between innovations of the smile variables (BS / RR) and innovations and other variables are presented below. ** and * represent p-values less than or equal to 5% and 10% respectively.

	Ask					Bid				
	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread
BS										
2-year	-0.06	-0.15**	0.03	-0.13**	0.00	-0.01	-0.29**	0.02	-0.22**	0.06
3-year	-0.19**	0.14**	0.01	0.09**	-0.15**	-0.06	0.04	0.04	0.02	-0.09**
4-year	-0.16**	0.29**	-0.07	0.24**	-0.04	-0.09**	0.01	0.02	0.02	-0.03
5-year	-0.06	0.15**	-0.01	0.04	0.16**	-0.07	-0.04	0.10**	-0.10**	0.02
6-year	-0.03	0.09*	-0.11**	-0.07	0.30**	0.04	0.05	-0.07	0.02	0.13**
7-year	-0.02	0.04	-0.02	0.03	0.34**	0.04	0.13**	-0.01	0.04	0.24**
8-year	0.01	0.18**	-0.24**	0.09*	0.13**	-0.06	0.26**	-0.18**	0.13**	0.09*
9-year	0.00	0.14**	-0.16**	0.10**	0.12**	0.02	0.24**	-0.21**	0.15**	0.07
10-year	-0.05	0.14**	-0.14**	0.08*	0.13**	-0.10**	0.12**	-0.08	0.07	-0.15**
RR										
2-year	0.08	0.26**	-0.18**	0.19**	-0.06	0.02	0.08	-0.16**	0.24**	0.00
3-year	0.05	0.09**	-0.26**	0.17**	0.07	0.08*	0.08*	-0.04	0.19**	0.04
4-year	0.12**	-0.11**	-0.24**	0.07	0.04	-0.02	0.05	-0.03	0.06	0.11**
5-year	-0.02	-0.04	-0.46**	0.13**	0.06	0.00	-0.09**	-0.21**	0.05	0.04
6-year	-0.07	0.01	-0.24**	0.12**	-0.01	-0.08*	0.07	-0.20**	0.15**	0.00
7-year	-0.02	0.03	-0.17**	0.08*	-0.08*	-0.05	0.00	-0.19**	0.06	-0.10**
8-year	0.10**	0.03	-0.11**	-0.02	-0.07	0.04	0.07	-0.10**	-0.01	-0.05
9-year	0.05	0.02	-0.08*	-0.09*	-0.18**	-0.01	0.08*	-0.03	-0.04	-0.01
10-year	0.04	-0.13**	-0.14**	-0.09*	-0.07	0.08*	-0.07	-0.03	-0.16**	0.15**

Table 5

Granger Causality Tests

This table presents results for the Granger causality tests based on the multivariate vector autoregression for six variables - the level of volatility of at-the-money interest rate options (ATM Vol), the spot 6-month Euribor (6 m rate), the slope of the term structure (5 yr rate - 6 m rate), the 6-month Treasury-Euribor spread (Default Spread), the scaled ATM bid-ask spreads (ATM BA spread) and butterfly spread (BS) or risk reversal (RR) for all maturities separately for ask and bid sides for the period Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group and DataStream . The p-values for rejecting the null hypothesis of "No Granger Causality" are given below. ** and * represent p-values less than or equal to 5% and 10% respectively.

Panel A: Null Hypothesis – presented variables do not individually Granger cause the butterfly spread (BS) / risk reversal (RR) on the ask / bid side

	Ask					Bid				
	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread
BS										
2-year	0.61	0.05**	0.98	0.73	0.65	0.43	0.05**	0.09*	0.44	0.97
3-year	0.05*	0.00**	0.27	0.65	0.00**	0.01**	0.00**	0.30	0.55	0.09*
4-year	0.00**	0.03**	0.97	0.38	0.36	0.08*	0.03**	0.98	0.56	0.93
5-year	0.01**	0.00**	0.00**	0.57	0.04**	0.38	0.22	0.06*	0.07*	0.74
6-year	0.47	0.24	0.00**	0.52	0.02**	0.00**	0.44	0.00**	0.16	0.04**
7-year	0.11	0.31	0.09*	0.88	0.00**	0.02**	0.53	0.12	0.31	0.00**
8-year	0.15	0.03**	0.07*	0.21	0.09*	0.08*	0.02**	0.00**	0.13	0.05*
9-year	0.11	0.03**	0.03**	0.06*	0.32	0.19	0.45	0.00**	0.92	0.92
10-year	0.29	0.33	0.12	0.04**	0.71	0.00**	0.70	0.00**	0.40	0.75
RR										
2-year	0.58	0.92	0.74	0.81	0.16	0.89	0.18	0.81	0.46	0.11
3-year	0.39	0.06*	0.27	0.23	0.04**	0.22	0.02**	0.14	0.52	0.57
4-year	0.40	0.01**	0.87	0.17	0.83	0.87	0.01**	0.66	0.05*	0.86
5-year	0.00**	0.00**	0.30	0.47	0.22	0.26	0.00**	0.00**	0.06*	0.73
6-year	0.25	0.25	0.01**	0.84	0.38	0.09*	0.27	0.01**	0.66	0.18
7-year	0.39	0.53	0.86	0.82	0.74	0.30	0.22	0.88	0.66	0.52
8-year	0.73	0.09*	0.71	0.13	0.40	0.33	0.05**	0.92	0.11	0.87
9-year	0.46	0.02**	0.44	0.17	0.38	0.14	0.29	0.29	0.04**	0.30
10-year	0.01**	0.09*	0.00**	0.69	0.42	0.85	0.13	0.00**	0.50	0.00**

Panel B: Null Hypothesis – Butterfly spread (BS) / risk reversal (RR) on the ask / bid side do not Granger cause each of the presented variables

	Ask						Bid				
	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread		ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread
BS											
2-year	0.58	0.28	0.69	0.15	0.80		0.92	0.12	0.34	0.27	0.72
3-year	0.42	0.01**	0.46	0.31	0.23		0.94	0.95	0.47	0.19	0.05**
4-year	0.45	0.14	0.47	0.66	0.32		0.19	0.17	0.78	0.33	0.77
5-year	0.58	0.28	0.01**	0.16	0.60		0.95	0.47	0.81	0.72	0.60
6-year	0.72	0.13	0.03**	0.09*	0.71		0.56	0.20	0.22	0.77	0.00**
7-year	0.03**	0.27	0.17	0.80	0.64		0.00**	0.06*	0.57	0.19	0.36
8-year	0.00**	0.37	0.93	0.00**	0.05**		0.79	0.03**	0.50	0.90	0.00**
9-year	0.06*	0.81	0.71	0.01**	0.71		0.44	0.28	0.82	0.54	0.61
10-year	0.00**	0.77	0.92	0.01**	0.21		0.31	0.15	0.38	0.84	0.00**
RR											
2-year	0.83	0.78	0.45	0.47	0.33		0.39	0.56	0.79	0.02**	0.13
3-year	0.70	0.51	0.55	0.08*	0.02**		0.39	0.53	0.84	0.01**	0.01**
4-year	0.25	0.44	0.59	0.02**	0.41		0.61	0.95	0.88	0.49	0.03**
5-year	0.14	0.39	0.02**	0.28	0.00**		0.67	0.04**	0.19	0.10	0.08*
6-year	0.07*	0.03**	0.07*	0.39	0.04**		0.23	0.21	0.09*	0.25	0.01**
7-year	0.00**	0.41	0.41	0.06*	0.48		0.00**	0.51	0.25	0.04**	0.25
8-year	0.54	0.23	0.13	0.12	0.56		0.06*	0.16	0.71	0.03**	0.83
9-year	0.11	0.64	0.08*	0.19	0.27		0.01**	0.19	0.16	0.04**	0.88
10-year	0.81	0.22	0.16	0.00**	0.00**		0.19	0.97	0.37	0.02**	0.00**

Table 6

Variance decomposition of the butterfly spread

This table presents the variance decomposition (%) of butterfly spread (BS) computed from the multivariate vector autoregression for six variables - the level of volatility of at-the-money interest rate options (ATM Vol), the spot 6-month Euribor (6 m rate), the slope of the term structure (5 yr rate - 6 m rate), the 6-month Treasury-Euribor spread (Default Spread), the scaled ATM bid-ask spreads (ATM BA spread) and butterfly spread (BS) separately for ask and bid sides for the period Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group and DataStream . The VAR is ordered as follows: 6 m Rate, 5 yr rate - 6 m Rate, Default Spread (6m), ATM BA Spread, BS, ATM Vol. The results are presented for three representative maturities - 2-year, 5-year, and 10-year.

Maturity	Forecast Horizon (Days)	Ask							Bid						
		Forecast Standard Error	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread	BS	Forecast Standard Error	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread	BS
2-year	1	0.42	0.00	2.13	0.03	0.65	0.00	97.19	0.43	0.00	8.24	0.00	1.18	0.32	90.26
	5	0.88	0.39	4.38	0.03	0.43	0.27	94.51	0.83	0.33	11.73	2.74	3.07	0.19	81.94
	10	1.13	1.69	4.37	0.07	0.34	0.23	93.30	1.08	1.16	11.83	6.19	3.93	0.19	76.70
5-year	1	0.21	0.00	2.27	0.04	0.03	2.19	95.46	0.22	0.00	0.18	0.92	0.77	0.28	97.86
	5	0.47	2.64	6.34	0.33	0.97	1.29	88.43	0.48	0.39	0.39	1.28	2.92	0.30	94.71
	10	0.62	3.45	7.59	2.98	2.25	1.93	81.80	0.62	0.73	0.49	1.22	5.49	0.37	91.71
10-year	1	0.12	0.00	2.03	0.88	0.06	1.64	95.40	0.13	0.00	1.53	0.13	0.07	2.19	96.07
	5	0.24	2.23	1.65	0.32	2.14	1.17	92.48	0.26	3.79	1.80	0.36	0.22	2.12	91.72
	10	0.31	4.49	1.14	0.20	5.64	0.78	87.76	0.35	11.88	1.54	1.12	0.41	2.19	82.85

Table 7

Variance decomposition of risk reversal

This table presents the variance decomposition (%) of risk reversal (RR) computed from the multivariate vector autoregression for six variables - the level of volatility of at-the-money interest rate options (ATM Vol), the spot 6-month Euribor (6 m rate), the slope of the term structure (5 yr rate - 6 m rate), the 6-month Treasury-Euribor spread (Default Spread), the scaled ATM bid-ask spreads (ATM BA spread) and risk reversal (RR) separately for ask and bid sides for the period Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group and DataStream . VAR is ordered as follows: 6 m Rate, RR, 5 yr rate - 6 m Rate, Default Spread (6m), ATM Vol, and ATM BA Spread. The results are presented for three representative maturities - 2-year, 5-year, and 10-year.

		Ask							Bid						
Maturity	Forecast Horizon (Days)	Forecast Standard Error	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread	RR	Forecast Standard Error	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread	RR
2-year	1	0.43	0.00	6.81	0.00	0.00	0.00	93.19	0.43	0.00	0.62	0.00	0.00	0.00	99.38
	5	0.91	0.04	7.18	0.19	0.13	1.44	91.02	0.84	0.02	3.38	0.08	1.01	1.58	93.94
	10	1.21	0.04	6.30	0.28	0.48	4.03	88.88	1.10	0.02	3.32	0.26	0.93	4.04	91.43
5-year	1	0.21	0.00	0.19	0.00	0.00	0.00	99.81	0.21	0.00	0.86	0.00	0.00	0.00	99.14
	5	0.47	0.35	4.77	0.17	0.52	0.51	93.69	0.48	0.20	4.62	0.28	1.65	0.07	93.17
	10	0.61	2.31	5.81	0.62	0.50	1.05	89.70	0.62	1.02	5.86	1.65	3.15	0.11	88.21
10-year	1	0.16	0.00	1.63	0.00	0.00	0.00	98.37	0.16	0.00	0.51	0.00	0.00	0.00	99.49
	5	0.33	1.17	2.66	0.60	0.24	0.64	94.68	0.33	0.18	2.81	0.62	0.11	5.53	90.75
	10	0.43	2.50	3.29	2.62	0.40	1.70	89.50	0.43	0.55	3.42	1.54	0.11	8.17	86.22

Table 8

Variance decomposition of default spread

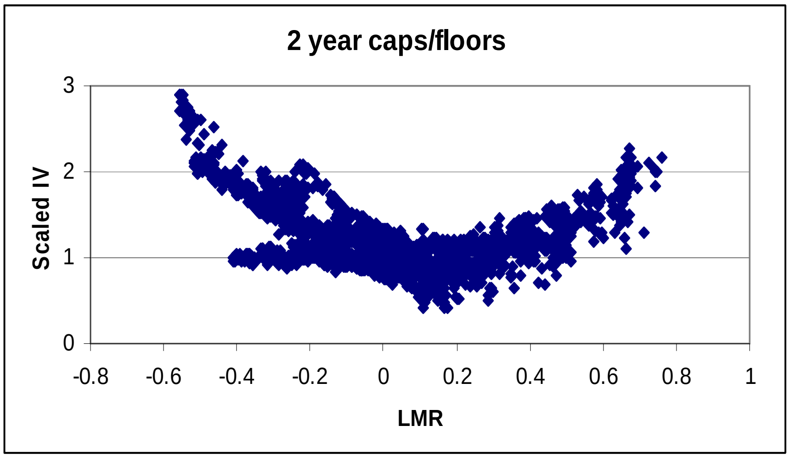
This table presents the variance decomposition (%) of default spread computed the multivariate vector autoregression for six variables - the level of volatility of at-the-money interest rate options (ATM Vol), the spot 6-month Euribor (6 m rate), the slope of the term structure (5 yr rate - 6 m rate), the 6-month Treasury-Euribor spread (Default Spread), the scaled ATM bid-ask spreads (ATM BA spread), and risk reversal (RR) for ask and bid sides for the period Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group and DataStream . VAR is ordered as follows: 6 m Rate, RR, 5 yr rate - 6 m Rate, Default Spread (6m), ATM Vol, and ATM BA Spread. The results are presented for three representative maturities - 2-year, 5-year, and 10-year.

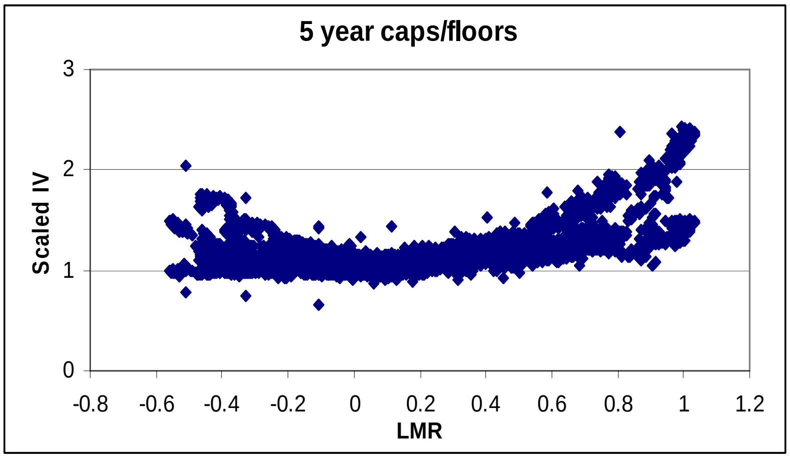
		Ask							Bid						
Maturity	Forecast Horizon (Days)	Forecast Standard Error	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread	RR	Forecast Standard Error	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	Default Spread (6m)	ATM BA Spread	RR
2-year	1	0.43	0.00	9.62	0.16	88.87	0.00	1.35	0.43	0.00	16.45	0.03	78.97	0.00	4.55
	5	0.91	0.78	16.75	0.72	78.48	0.63	2.64	0.84	0.46	26.86	0.29	62.58	0.91	8.90
	10	1.21	2.59	17.65	2.14	74.35	0.56	2.71	1.10	1.02	27.67	0.29	57.30	1.22	12.50
5-year	1	0.21	0.00	12.36	0.26	85.39	0.00	1.99	0.21	0.00	12.59	0.00	86.77	0.00	0.63
	5	0.47	0.93	18.01	0.34	76.36	0.26	4.10	0.48	1.00	18.79	0.46	77.25	0.32	2.18
	10	0.61	1.37	19.68	0.77	73.45	0.25	4.48	0.62	1.52	20.01	0.40	73.04	0.30	4.72
10-year	1	0.16	0.00	12.55	0.03	87.25	0.00	0.17	0.16	0.00	13.02	0.06	85.24	0.00	1.68
	5	0.33	0.20	16.85	0.49	76.62	0.63	5.20	0.33	0.23	17.72	0.38	75.98	1.56	4.13
	10	0.43	0.31	17.50	0.50	73.54	1.48	6.67	0.43	0.35	17.71	0.57	68.79	4.52	8.06

Figure 1

Implied volatility smiles in interest rate caps and floors

This figure presents scatter plots of the implied flat volatilities of euro interest rate caps and floors over our sample period. The vertical axis in the plots corresponds to the implied volatility of the mid-price (average of bid and ask price) of the option, scaled by the at-the-money volatility for the option of similar maturity. The horizontal axis in the plots corresponds to the logarithm of the moneyness ratio, defined as the ratio of the par swap rate to the strike rate of the option. The plots are for three representative maturities - 2-year, 5-year, and 10-year for the period, Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group.





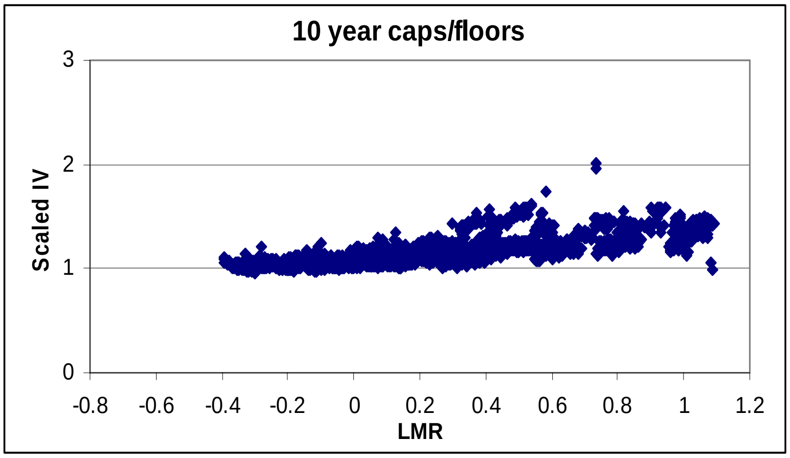


Figure 2

Functional forms of implied volatility smiles in interest rate caps and floors

This figure presents the fitted smile functions for the bid and ask implied flat volatilities of euro interest rate caps and floors separately, across different maturities. The vertical axis in the plots corresponds to the implied flat volatility of the bid and ask prices of the option, scaled by the at-the-money volatility for the option of similar maturity (Scaled IV) calculated using the regression model in Table VI. The horizontal axis in the plots corresponds to the logarithm of the moneyness ratio (LMR), defined as the ratio of the par swap rate to the strike rate of the option. The plots are three representative maturities - 2-year, 5-year, and 10-year for the period, Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group.

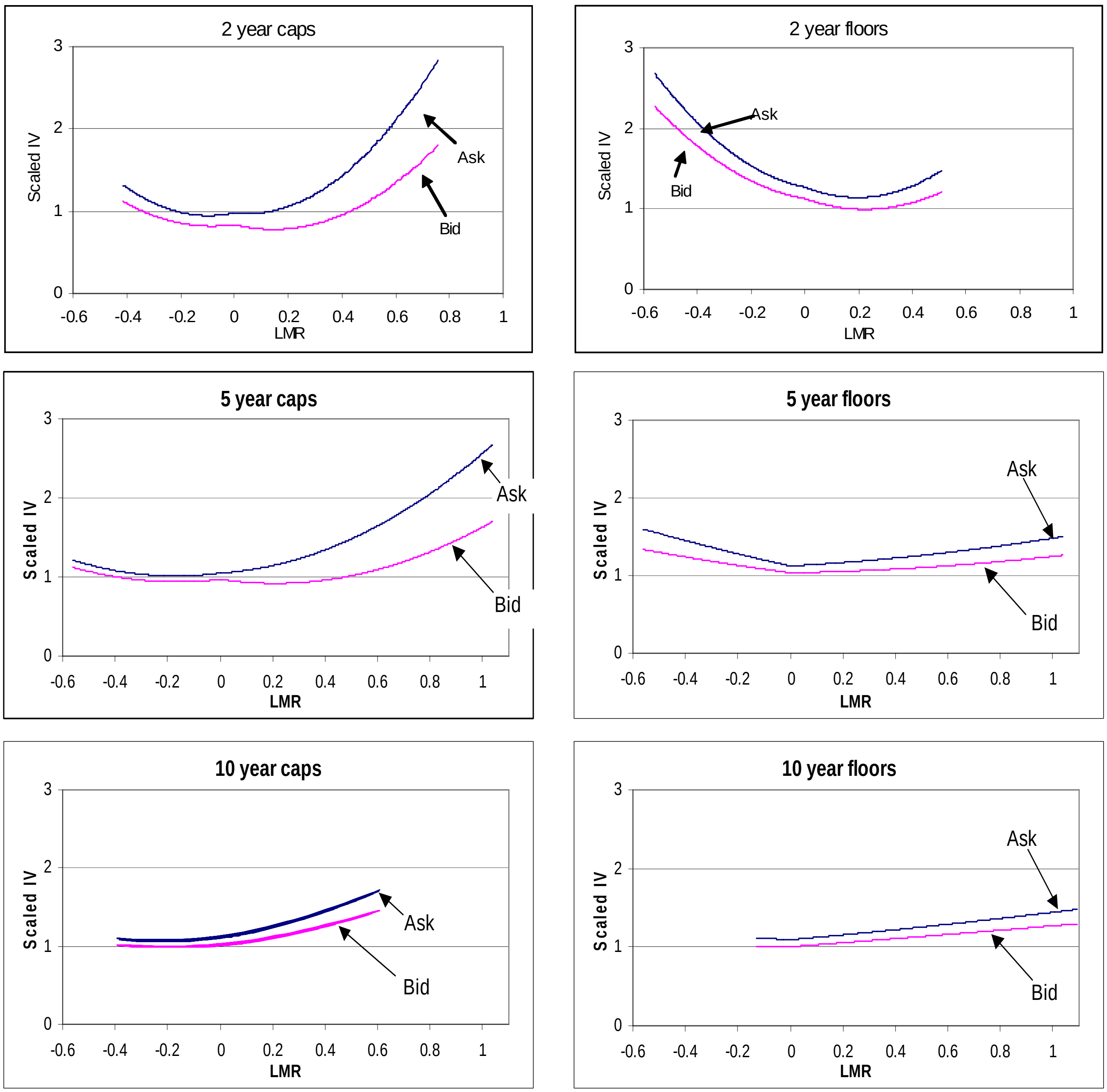
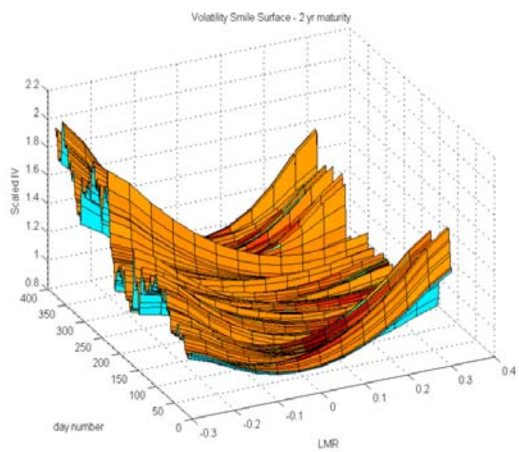


Figure 3

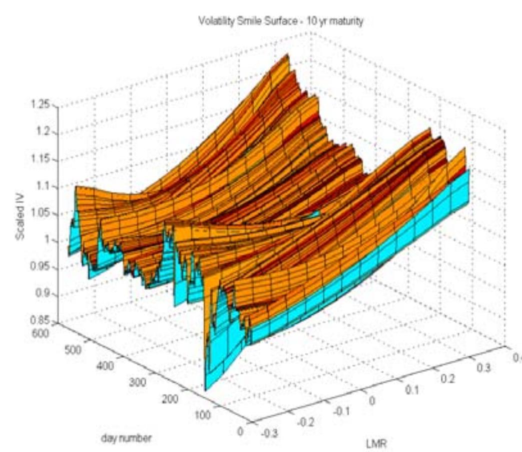
Time variation in volatility smiles and the Euro term structure

This figure presents surface plots showing the time variation in the implied flat volatilities of euro interest rate caps and floors as well as the Euro term structure over the period Jan 99 - May 01. In figures 3A, 3B and 3C show the time variation in the implied flat volatilities of euro interest rate caps and floors for three representative maturities - 2-year, 5-year, and 10-year. The vertical axis corresponds to the implied volatility of the mid-price (average of bid and ask price) of the option, scaled by the at-the-money volatility for the option of similar maturity. The horizontal axes in these plots correspond to the logarithm of the moneyness ratio (defined as the ratio of the par swap rate to the strike rate of the option), and time. Figure 3D depicts the Euro spot rate surface by maturity (in years) over time (daily). The vertical axis corresponds to the spot rates. The horizontal axes correspond to the maturity of the spot rate and time, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group.

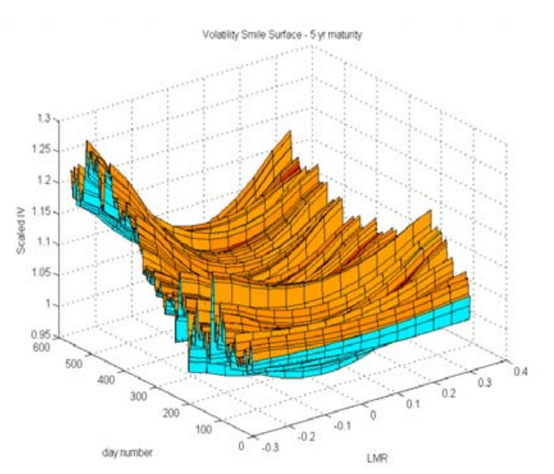
3A



3B



3C



3D

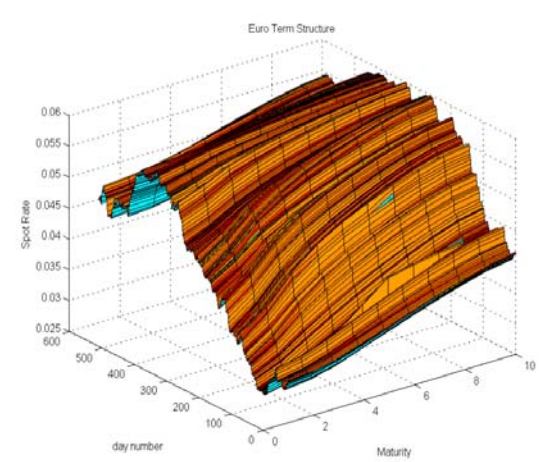
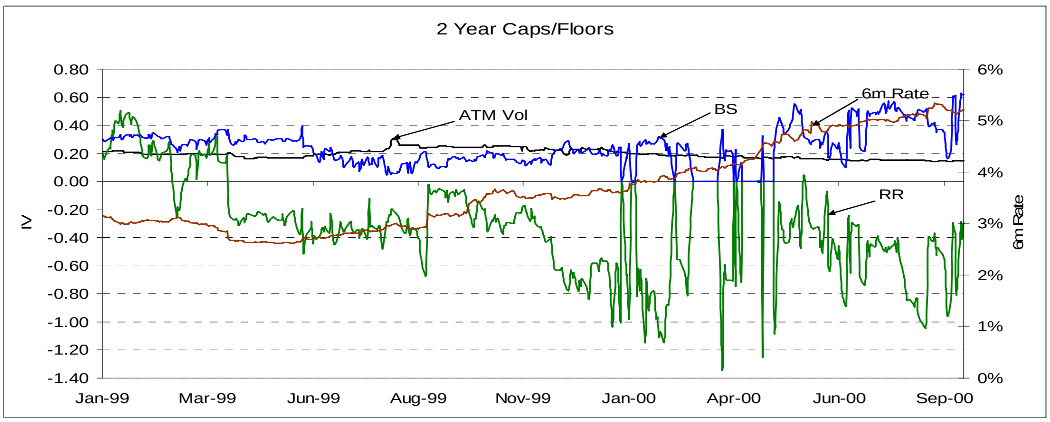
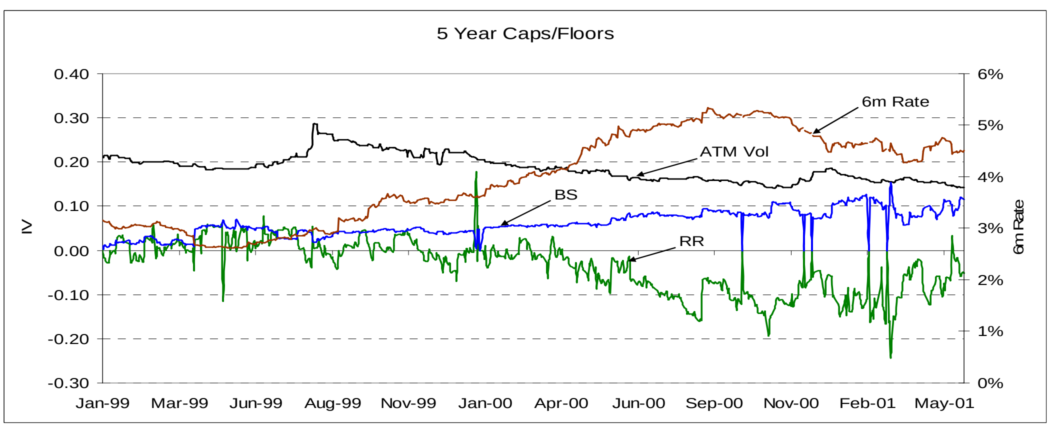


Figure 4

Time variation in butterfly spread and risk reversal

This figure shows time variation in butterfly spread (BS) and risk reversal (RR) for the mid prices along with at-the-money volatility (ATM Vol) of swaptions and 6 month interest rate over the period Jan 99 - May 01 for three representative maturities - 2-year, 5-year, and 10-year - based on data obtained from WestLB Global Derivatives and Fixed Income Group and DataStream. The horizontal axis shows time; the left vertical axis shows scaled implied volatility whereas the right vertical axis shows percentage 6 m rate and ATM volatility.





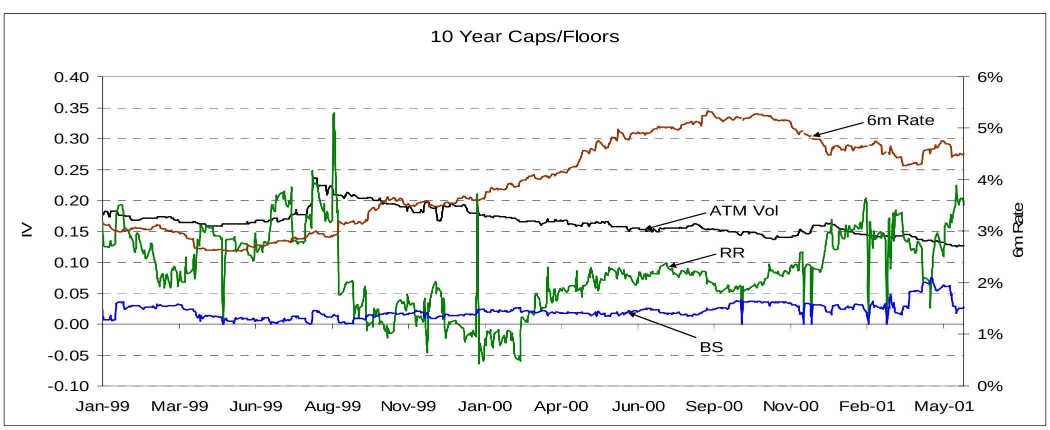
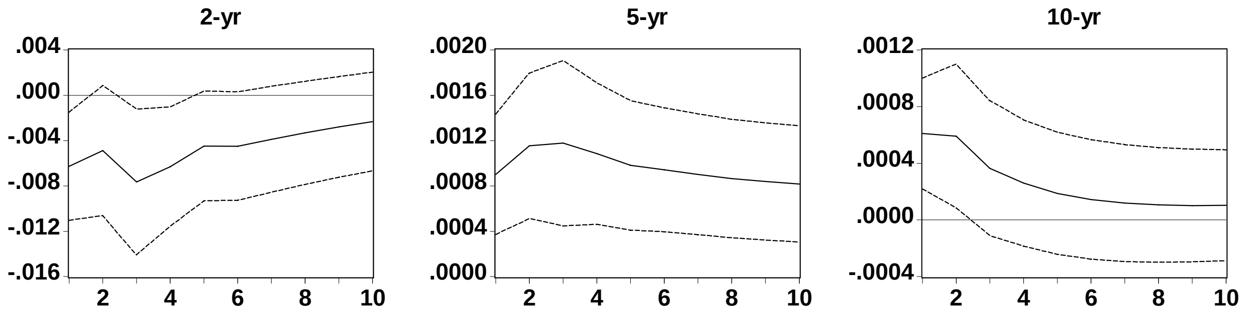


Figure 5

Response of the butterfly spread to the 6-month interest rate

This figure presents impulse response of butterfly spread (BS) to the 6-month rate computed from the multivariate vector autoregression for six variables - the level of volatility of at-the-money interest rate options (ATM Vol), the spot 6-month Euribor (6 m rate), the slope of the term structure (5 yr rate - 6 m rate), the 6-month Treasury-Euribor spread (Default Spread), the scaled ATM bid-ask spreads (ATM BA spread) and butterfly spread (BS) separately for ask and bid sides for the period Jan 99 - May 01, based on data obtained from WestLB Global Derivatives and Fixed Income Group and DataStream. The figure shows response to Cholesky one s.d. innovations +/- 2 standard errors for three representative maturities - 2-year, 5-year, and 10-year. VAR is ordered as follows: 6 m Rate, 5 yr rate - 6 m Rate, Default Spread (6m), ATM BA Spread, BS, ATM Vol.

Panel A: Ask



Panel B: Bid

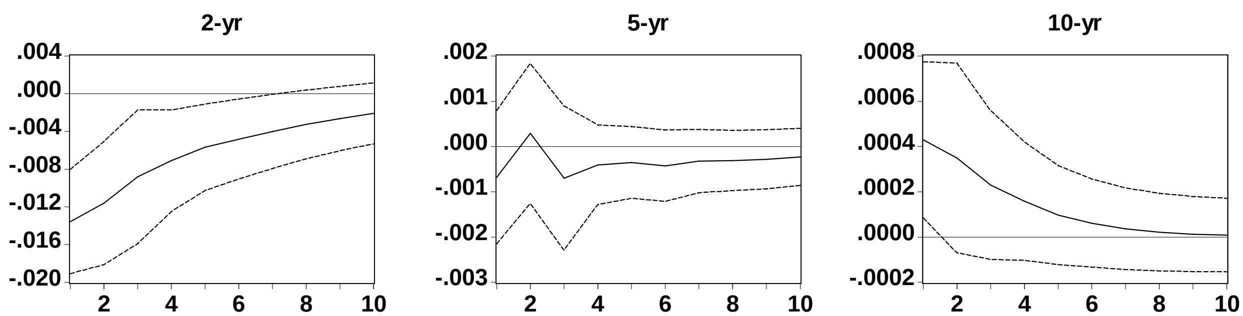
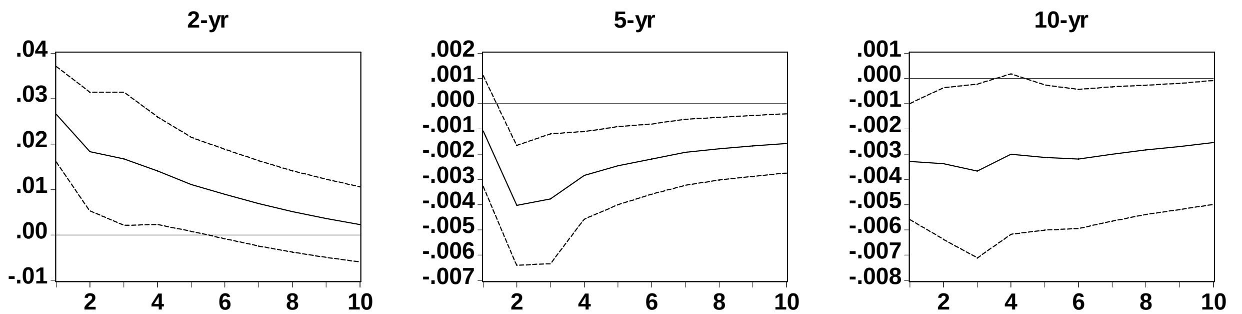


Figure 6
Response of the risk reversal to the 6-month interest rate

This figure presents impulse response of risk reversal (RR) to the 6-month rate computed from the multivariate vector autoregression for six variables - the level of volatility of at-the-money interest rate options (ATM Vol), the spot 6-month Euribor (6 m rate), the slope of the term structure (5 yr rate - 6 m rate), the 6-month Treasury-Euribor spread (Default Spread), the scaled ATM bid-ask spreads (ATM BA spread) and risk reversal (RR) separately for ask and bid sides for the period Jan 99 - May 01, based on data obtained from WestLB Global Derivatives and Fixed Income Group and DataStream. The figure shows response to Cholesky one s.d. innovations +/- 2 standard errors for three representative maturities - 2-year, 5-year, and 10-year. VAR is ordered as follows: 6 m Rate, RR, 5 yr rate - 6 m Rate, Default Spread (6m), ATM Vol, and ATM BA Spread.

Panel A: Ask



Panel B: Bid

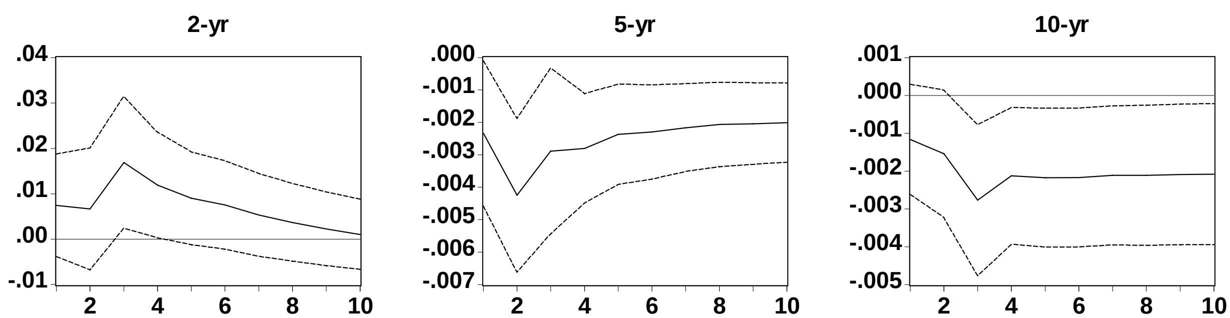
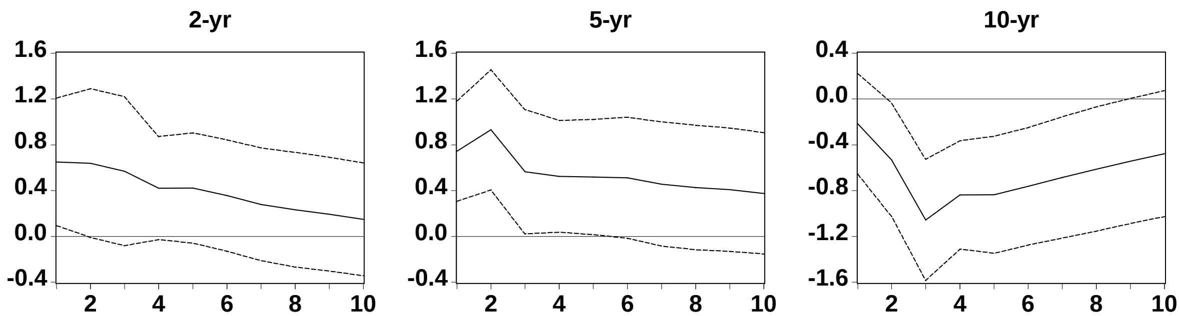


Figure 7
Response of the default spread to the risk reversal

This figure presents impulse response of default spread to risk reversal (RR) computed the multivariate vector autoregression for six variables - the level of volatility of at-the-money interest rate options (ATM Vol), the spot 6-month Euribor (6 m rate), the slope of the term structure (5 yr rate - 6 m rate), the 6-month Treasury-Euribor spread (Default Spread), the scaled ATM bid-ask spreads (ATM BA spread) and risk reversal (RR) separately for ask and bid sides for the period Jan 99 - May 01, based on data obtained from WestLB Global Derivatives and Fixed Income Group and DataStream. The figure shows response to Cholesky one s.d. innovations +/- 2 standard errors for three representative maturities - 2-year, 5-year, and 10-year. VAR is ordered as follows: 6 m Rate, RR, 5 yr rate - 6 m Rate, Default Spread (6m), ATM Vol, and ATM BA Spread.

Panel A: Ask



Panel B: Bid