The y-Theory of Investment*

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Abstract

I propose a new implementation of the q-theory of investment using corporate bond yields instead of equity prices. In q-theory, the optimal investment rate is a function of risk-adjusted discount rates and of future marginal profitability. Corporate bond prices also depend on these variables. I show that, when aggregate shocks are small, aggregate q is a linear combination of risk free rates and average yields on risky corporate debt. The yield-theory of investment, unlike its equity-based counter part, is empirically successful: it can account for more than half of the volatility of investment in post-war US data, it drives out cash flows from the investment equation, and it delivers sensible estimates for the parameters of the adjustment cost function.

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I propose a new implementation of the q-theory of investment using corporate bond yields instead of equity prices. According to q-theory, investment should be an increasing function of marginal q, defined as the net present value of the future marginal product of capital. Hayashi (1982) shows that, with constant returns to scale, marginal and average q are the same. Therefore, the ratio of the market value of unlevered equity over the existing capital stock should be a sufficient statistic for the investment rate. Unfortunately, the implementation of the theory using equity has not been successful (see Caballero (1999) for a recent survey).

Just like Hayashi (1982), I use a model with constant returns to scale and convex costs of adjustment. Thus, in my setup, marginal and average q are the same. The innovation is that I use the corporate bond market, as opposed to the equity market, to construct my proxy for q. Just like equity prices, corporate bond prices react to changes in the risk-adjusted discount rate and to news about future profitability. When aggregate shocks are small and there is a continuous distribution of firm level shocks, I show that bond prices are in fact proportional to q. After deriving the yield-based equation, I find that, unlike its equity counter-part, this investment equation is empirically successful: it can account for more than half of the volatility of investment in post-war US data, it drives out cash flow variables, and it delivers quantitatively sensible estimates for the parameters of the adjustment cost function.

The yield-theory is successful because yield spreads of corporate bonds over treasuries forecast investment. It is well-known (Bernanke (1983) and Stock and Watson (1989)) that credit spreads have forecasting power for output, but there has been much disagreement about how one should interpret this fact. My work suggests a straightforward interpretation: this is just q-theory. In particular, it does not require that firms be credit constraint, or that financial markets be imperfect. As an aside, I also find that corporate credit spreads forecast output through corporate investment, not consumption or residential investment.

Beyond the macroeconomic literature already mentioned, this paper is related to Cochrane (1996) who studies the joint dynamics of physical and financial returns, Lettau and Ludvigson (2002) who emphasize the role of time varying risk premia, and Zhang (2005) who shows how irreversibility and counter-cyclical risk premia could explain the value premium. Finally, I build on the credit risk literature pioneered by Merton (1974). Berndt, Douglas, Duffie, Ferguson, and Schranz (2005) and Pan and Singleton (2005) estimate credit risk premia from default swap rates, respectively for US corporate and sovereign bonds, and find large and volatile risk premia. Almeida and Philippon (2005) show that risk premia are of first order importance for capital structure decisions.

In section 1, I present the setup of the model. In section 2, I derive the approximations to the value function and to the investment policy function that are needed to establish the main result. In section 3, I show how one can use corporate bond spreads to build a sufficient statistic for aggregate investment. Section 4 contains empirical evidence on the relevance of the theory and on the implied adjustment costs. Section 5 contains robustness checks, including numerical simulations in section 5.4. In the last section, I summarize the results in the context of existing theories of investment, and I discuss some remaining puzzles.

1 Model

I present a model of investment with aggregate shocks to discount rates, risk premia and average profitability, as well as idiosyncratic profitability shocks. This is a partial equilibrium model and the pricing kernel is specified exogenously.

1.1 Technology and Pricing Kernel

I now describe the technology of the firms, and the stochastic processes for aggregate and idiosyncratic shocks.

Cash Flows

The state of the aggregate economy is an integer $s_t \in [1, 2.., S]$ which evolves according to a stationary Markov chain with transition matrix P. Abusing notations, I will also use S to denote the aggregate state space. Firms are subject to aggregate and idiosyncratic shocks. The profits of a firm with idiosyncratic shock η in state s are

$$(a(s) + \eta - \Gamma(x))\, k, \tag{1}$$

where k is the firm's capital stock and $x := \frac{i}{k} - \delta$ is the investment rate, i is investment and δ is the depreciation rate. The function $\Gamma(.)$ is strictly convex, and the aggregate profit rate $a(.)$ is normalized so that $\Gamma(0) = 0$.[1] Capital accumulates according to

$$k_{t+1} = (1 + x_t)k_t. \tag{2}$$

Risk Neutral Probabilities

The state s characterizes the aggregate economy, and the pricing kernel between state s and s' is $m_{s,s'}$. Let $p_{s_t,s_{t+1}}$ be the objective probability of s_{t+1} given s_t. Let $\bar{m}_s$ be the price of a one period risk free bond in state s and let r_s be the corresponding risk free rate

$$\frac{1}{1 + r_s} := \bar{m}_s = \sum_{s' \in S} p_{s,s'} m_{s,s'}.$$

The risk neutral probability $\pi_{s,s'}$ is defined by

$$\pi_{s,s'} := \frac{m_{s,s'}}{\bar{m}_s} p_{s,s'},$$

and the risk-adjusted transition matrix is $\Pi = \left[\pi_{s,s'}\right]_{s \in S, s' \in S}$. I assume that Π is stationary with a unique invariant distribution, $\bar{\pi}_s$, which solves $\bar{\pi}_{s'} = \sum_{s \in S} \bar{\pi}_s \pi_{s,s'}$.

Idiosyncratic Shocks

The idiosyncratic component of profits follows a continuous Markov process over a compact interval $\Omega \subset \mathbb{R}$ with a transition density $\zeta\left(\eta_t, \eta_{t+1}\right)$ which is independent of the state of the economy. The economy has reached its ergodic steady state for idiosyncratic shocks, and the ergodic distribution of η is $\bar{\zeta}(.)$. By definition, the ergodic distribution is such that $\bar{\zeta}(\eta') = \int_{\Omega} \zeta(\eta, \eta') \bar{\zeta}(\eta) d\eta$. Finally, I normalize the mean of η to zero: $\int_{\Omega} \eta \bar{\zeta}(\eta) d\eta = 0$.

1.2 Program of the Firm

The value of the firm at time 0 is

$$V_0 = E \sum_{t=0}^{\infty} m(t) \left(a(s_t) + \eta_t - \Gamma(x_t)\right) k_t \tag{3}$$

where

$$m(0) = 1 \text{ and } m(t) = m_{s_0,s_1} \times .. m_{s_{t-1},s_t}$$

[1] This functional form is the general case when the production function has constant returns to scale and there is no adjustment cost for labor. Start from the traditional formulation of cash flows $(\tilde{a} + \eta) k - i - k\tilde{\Gamma}\left(\frac{i}{k}\right)$, where $\tilde{a} + \eta$ is the profit rate net of wages, and $\tilde{\Gamma}\left(\frac{i}{k}\right)$ the adjustment cost per-unit of capital. Using x, I then define $a = \tilde{a} - \delta - \tilde{\Gamma}(\delta)$ and $\Gamma(x) = x + \tilde{\Gamma}(x + \delta) - \tilde{\Gamma}(\delta)$ and I obtain (1).

The program of the firm is to maximize (3) with respect to the sequence $\{x_t\}_{t\geq 0}$ and subject to the capital accumulation constraint (2) and the initial condition (k_0, s_0, η_0). I assume that the solution to this program is finite and strictly positive for all possible initial conditions $(k_0, s_0, \eta_0) \in \mathbb{R}_{++} \times S \times \Omega$. Since the technology exhibits constant returns to scale, it is useful to consider the scaled value function

$$v_t := \frac{V_t}{k_t}.$$

The function $v(.)$ solves

$$v(s,\eta) = \max_{x>-1} \left\{ a(s) + \eta - \Gamma(x) + (1+x)\bar{m}_s E^{\pi}\left[v\left(s',\eta'\right)|s,\eta \right] \right\}, \tag{4}$$

and the transversality condition is

$$\lim_{\tau\to\infty} E\left[m_{t,\tau}(1+x_t)..\times..(1+x_\tau)\, v_\tau | s_t, \eta_t \right] = 0. \tag{5}$$

1.3 Investment

The first order condition for investment is

$$\gamma(x(s,\eta)) = \bar{m}_s E^{\pi}\left[v\left(s',\eta'\right)|s,\eta \right], \tag{6}$$

where $\int_0^x \gamma(t)\,dt := \Gamma(x)$. I assume that $\lim_{x\to -1}\gamma(x) = -\infty$ and $\lim_{x\to\infty}\gamma(x) = +\infty$, so that the solution is always interior. Equation (6) says that investment is equal to the expected discounted marginal product of capital, usually called marginal q. With constant returns to scale, the marginal product of capital is equal to the average product of capital. One way to test this theory is to use the value of unlevered equity over physical assets as a measure of q, but it has not worked well in practice (see Caballero (1999) for a recent survey). I discuss potential explanations for this lack of success in the last section of the paper. Instead of using equity, I argue that one can use corporate bond spreads to estimate equation (6).

1.4 Debt Policy

I assume that the Modigliani-Miller assumptions hold. Default can happen, but default does not entail any deadweight loss. In case of default, the creditors take over the assets, but the firm continues to operate. In the Modigliani-Miller world, debt policy does not affect firm value. It does, however, affect bond spreads, so I must specify debt dynamics before I can use bond prices to estimate the net present value of the marginal product of capital. I consider two polar cases: short term debt and perpetuity.

Short Term Debt

The firm issues at time t a one period bond with face value $d_t k_{t+1}$, where the book leverage d_t might be a function of the state of the firm (s_t, η_t). The price of the bond is then $B_t = b_t k_{t+1}$, where the pricing function solves

$$b(s,\eta) = \bar{m}_s E^{\pi}\left[\min\left(v\left(s',\eta'\right), d(s,\eta)\right)|s,\eta\right] \tag{7}$$

Perpetuity

Empirically, firms continuously issue and retire long term debt in order to keep their leverage ratio within reasonable bounds. To capture this in a simple way, I assume that firms keep the total coupon payments on long term debt a constant fraction c of their capital stock k. In case of default, each creditor therefore recovers v, and the value of each perpetuity solves

$$b^{\infty}(s,\eta) = \bar{m}_s E^{\pi}\left[\min\left(v\left(s',\eta'\right), c + b^{\infty}\left(s',\eta'\right)\right)|s,\eta\right] \tag{8}$$

2 First Order Approximations

In this section, I present approximations to the value and policy functions that are needed to show how one can use (7) or (8) to estimate the right hand side of (6). To make the statements more precise, define the vectors

$$A = \begin{bmatrix} a(1) \\ .. \\ a(S) \end{bmatrix}; \ \bar{M} = \begin{bmatrix} \bar{m}(1) \\ .. \\ \bar{m}(S) \end{bmatrix},$$

and the norm over vectors and functions of s as

$$\|A\| := \sup_{s \in S} |a(s)| \ .$$

Similarly, for a function that depends on both η and s

$$\|v(.,.)\| := \sup_{s \in S, \eta \in \Omega} |v(s,\eta)| \ .$$

Note that Ω is compact so the *sup* is a *max* for continuous functions. I start by considering two polar cases: an economy with one representative firm, and then an economy with no aggregate risk.

2.1 No Idiosyncratic Shocks

In this section, I assume that the idiosyncratic shock η is identically equal to zero, i.e. $\|\Omega\| = 0$, and I focus on aggregate shocks. Consider first the pseudo-steady state defined as the steady state of an economy where the profit rate is constant and equal to $\bar{a} := E^{\pi}[a(s)]$, and the discount factor is $\bar{m} := E^{\pi}[\bar{m}_s]$. Combining equation (4) evaluated at the pseudo-steady state, $\bar{v} = \bar{a} - \Gamma(\bar{x}) + (1+\bar{x})\bar{m}\bar{v}$, with the first order condition for investment $\gamma(\bar{x}) = \bar{m}\bar{v}$, I find that the pseudo-steady state is characterized by

$$\Gamma(\bar{x}) + (r - \bar{x})\gamma(\bar{x}) - \bar{a} = 0, \tag{9}$$

where $r := \frac{1-\bar{m}}{\bar{m}}$ is the steady state interest rate.

Lemma 1 *Equation (9) has at most one solution over $(-1, r]$ and one over $[r, \infty)$. If*

$$r\gamma(0) < \bar{a} < \Gamma(r), \tag{10}$$

then there exists a unique solution $\bar{x} \in (0, r)$ which does not violate the transversality condition (5).

Proof. See appendix ■

With no uncertainty, condition (10) is necessary and sufficient for balanced growth with finite firm value. It is also necessary and sufficient when uncertainty is small enough.[2] From now on, I assume that condition (10) is satisfied. The unique solution of (4) that satisfies (5) is also the solution to the original problem of the firm. Let $v^a(.)$ be the solution for the particular case of no idiosyncratic shocks, and let $x^a(.)$ be the corresponding policy function. Clearly, $v^a(.)$ converges to $\bar{v}$ when $\|A - \bar{a}\| + \left\|\bar{M} - \bar{m}\right\| \to 0$. To approximate $v^a(.) - \bar{v}$, I define the stochastic process $\hat{v}(.)$ as the solution to

$$\hat{v}(s) := a_s - \bar{a} + (1+\bar{x})\bar{v}(\bar{m}_s - \bar{m}) + (1+\bar{x})\bar{m}E^{\pi}\left[\hat{v}(s')\,|s\right], \tag{11}$$

and the stochastic process $\hat{x}(s)$ as

$$\hat{x}(s) := \frac{\bar{v}(\bar{m}_s - \bar{m}) + \bar{m}E^{\pi}[\hat{v}(s')]}{\gamma_1}, \tag{12}$$

[2] For instance, by imposing that $r\gamma(0) < \min(a)$ and $\max(a) < \Gamma(r)$.

where the curvature parameter is $\gamma_1 := \frac{\partial \gamma}{\partial x}(\bar{x})$. I now show that $\bar{v} + \hat{v}(.)$ is the correct first order approximation of $v^a(.)$ when aggregate shocks are small. The next Lemma says that the convergence of $v^a(.)$ to $\bar{v}$ is at least linear in $\|A - \bar{a}\| + \left\|\bar{M} - \bar{m}\right\|$

Lemma 2 *As shocks become small, $v^a(.)$ converges to $\bar{v}$ and*

$$\lim_{\|A-\bar{a}\|+\left\|\bar{M}-\bar{m}\right\|\to 0} \frac{\|v^a(.) - \bar{v}\|}{\|A - \bar{a}\| + \left\|\bar{M} - \bar{m}\right\|} < \infty$$

Proof. See appendix ■

The following Lemma says that the economy with small aggregate shocks and no idiosyncratic shocks is well approximated by the functions $\hat{x}(.)$ and $\hat{v}(.)$.

Lemma 3 *As shocks become small, $v^a(.) - \bar{v}$ converges to $\hat{v}(.)$, in the sense that*

$$\lim_{\|A-\bar{a}\|+\left\|\bar{M}-\bar{m}\right\|\to 0} \frac{\|v^a(.) - \bar{v} - \hat{v}(.)\|}{\|A - \bar{a}\| + \left\|\bar{M} - \bar{m}\right\|} = 0$$

and $x^a(.) - \bar{x}$ converges to $\hat{x}(.)$

$$\lim_{\|A-\bar{a}\|+\left\|\bar{M}-\bar{m}\right\|\to 0} \frac{\|x^a(.) - \bar{x} - \hat{x}(.)\|}{\|A - \bar{a}\| + \left\|\bar{M} - \bar{m}\right\|} = 0$$

Proof. See appendix ■

2.2 No Aggregate Shocks

In this section, I assume that $a(s) = \bar{a}$ and $\bar{m}(s) = \bar{m}$ for all s, and I focus on idiosyncratic shocks. Define $\tilde{v}(.)$ and $\tilde{x}(.)$ as the solutions to

$$\tilde{v}(\eta) := \bar{a} + \eta - \Gamma(\tilde{x}_\eta) + (1 + \tilde{x}_\eta)\,\bar{m}E\left[\tilde{v}\left(\eta'\right)|\eta\right] \tag{13}$$

and

$$\gamma(\tilde{x}_\eta) = \bar{m}E\left[\tilde{v}\left(\eta'\right)|\eta\right] \tag{14}$$

The average investment rate in this economy is given by $\int \gamma^{-1}(\bar{m}E[\tilde{v}(\eta')|\eta])\,\bar{\zeta}(\eta)\,d\eta$. This average rate converges to $\bar{x}$ when $\|\Omega\| \to 0$, but is in general different from $\bar{x}$. In this economy, because of constant returns to scale, there is no tendency of firm size to mean revert, so one must be careful when studying limiting distributions. Suppose, without loss of generality, that all firms start with one unit of capital at time $t = 0$, i.e., $k_{j,0} = 1$ for all j. The capital stock of a firm j at time T is then

$$\log(k_{j,T}) = \sum_{t=0}^{T-1} \log(1 + \tilde{x}_{j,t})$$

By the law of large numbers, we know that $\frac{\log(k_{j,T})}{T}$ converges in mean squares to $E^{\bar{\zeta}}[\log(1 + \tilde{x}(\eta))]$, which I assume to be strictly positive.[3] Clearly, there is no ergodic distribution for $k_{j,t}$. Let $\tilde{f}(k_t, \eta_t, t)$ be the joint distribution of k and η at time t. It evolves according to the law of motion

$$\tilde{f}(k_{t+1}, \eta_{t+1}, t+1) = \int_{\eta_t \in \Omega} \left(\int_{k_t = \frac{k_{t+1}}{1+\tilde{x}(\eta_t)}} \tilde{f}(k_t, \eta_t, t)\, dk_t \right) \zeta(\eta_t, \eta_{t+1})\, d\eta_t.$$

Define the aggregate stock of capital at time t by $\bar{k}_t := \int\int k_t \tilde{f}(k_t, \eta_t, t)\, dk_t d\eta_t$. Since $\frac{k_{jt}}{\exp\left(1+tE^{\bar{\zeta}}[\log(1+\tilde{x}(\eta))]\right)} \xrightarrow{p} 1$, we see that the scaled aggregate capital stock converges in probability

$$\frac{\bar{k}_t}{\exp\left(1 + tE^{\bar{\zeta}}[\log(1 + \tilde{x}(\eta))]\right)} \xrightarrow{p} 1.$$

Similarly, defining aggregate investment by $\bar{\imath}_t := \int\int (\tilde{x}(\eta_t) + \delta)\, k_t \tilde{f}(k_t, \eta_t, t)\, dk_t d\eta_t$, we see that the aggregate investment rate $\frac{\bar{\imath}_t}{\bar{k}_t}$ converges in probability

$$\frac{\bar{\imath}_t}{\bar{k}_t} \xrightarrow{p} E^{\bar{\zeta}}[\tilde{x}(\eta)] + \delta.$$

After a long time, the aggregate investment rate is constant and equal to the unconditional mean of the firm level investment rate. Note that the key for this result is that $E^{\bar{\zeta}}[\log(1 + \tilde{x}(\eta))] > 0$: as the economy grows, the aggregate capital stock becomes very large, and differences in k across firms, even though they are infinitely large in absolute terms, become small relative to the aggregate capital stock. To study $\frac{\bar{\imath}_t}{\bar{k}_t}$, we can thus safely assume that all firms have the same capital stock, and that the heterogeneity is simply due to η.

[3] In fact, since $\zeta(.,.)$ is ergodic and Ω is bounded, a central limit theorem holds, and $\sqrt{T}\left(\frac{\log(k_{jT})}{T} - E^{\bar{\zeta}}[\log(1 + x(\eta))]\right) \to N(0, \tilde{\sigma})$ where the variance is given by $\tilde{\sigma}^2 = var^{\bar{\zeta}} \log(1 + \tilde{x}_\eta) + 2\sum_{\tau=1}^{\infty} cov^{\bar{\zeta}}(\log(1 + \tilde{x}_t), \log(1 + \tilde{x}_{t+\tau}))$.

2.3 General Case

In the general case, there are both aggregate and firm level shocks. The following Lemma shows that, when aggregate and idiosyncratic shocks are small, the value function is well approximated by the sum of an idiosyncratic and an aggregate component. The same is true for the policy function and for the aggregate investment rate.

Lemma 4 *$\tilde{v}(\eta) + \hat{v}(s)$ is a first order approximation to $v(s,\eta)$, in the sense that*

$$\lim_{\|A-\bar{a}\|+\|\bar{M}-\bar{m}\|+\|\Omega\|\to 0} \frac{\sup_{s,\eta}\left(v(s,\eta) - \tilde{v}(\eta) - \hat{v}(s)\right)}{\|A-\bar{a}\| + \left\|\bar{M}-\bar{m}\right\| + \|\Omega\|} = 0.$$

Similarly, $\tilde{x}(\eta) - \hat{x}(s)$ is a first order approximation to the policy function

$$\lim_{\|A-\bar{a}\|+\|\bar{M}-\bar{m}\|+\|\Omega\|\to 0} \frac{\sup_{s,\eta}\left(x(s,\eta) - \tilde{x}(\eta) - \hat{x}(s)\right)}{\|A-\bar{a}\| + \left\|\bar{M}-\bar{m}\right\| + \|\Omega\|} = 0.$$

The aggregate investment rate converges to

$$\lim_{\|A-\bar{a}\|+\|\bar{M}-\bar{m}\|+\|\Omega\|\to 0} \frac{\left|\frac{\bar{\imath}_t}{\bar{k}_t} - E^{\bar{\zeta}}\left[\tilde{x}(\eta)\right] - \delta - \hat{x}(s_t)\right|}{\|A-\bar{a}\| + \left\|\bar{M}-\bar{m}\right\| + \|\Omega\|} = 0, \text{ for all } t.$$

The proof of Lemma 4 is a straightforward extension of the proof of Lemma 3 and is omitted. It is important to emphasize that both the aggregate and the idiosyncratic shocks must be small, because the convexity of $\Gamma(.)$ creates interaction effects between aggregate and idiosyncratic shocks. Therefore, it is not enough for aggregate shocks to be small in order to get additive separability of the value function into $\hat{v}$ and $\tilde{v}$. It is legitimate to wonder what happens when idiosyncratic shocks are only second-order small, and I address this issue in section 5.

3 Corporate Yields and Aggregate Investment

I now derive the relationship between the average yield on corporate bonds and the aggregate investment rate. In this section, I restrict my attention to short term corporate debt. I extend the analysis to long term debt in section 5.

3.1 Short Term Bond Prices

Equation (7) gives the value of the one-period bond issued by a firm in the state (s,η). It can be written as

$$b(s,\eta)=\bar{m}_s d(s,\eta)-\bar{m}_s\sum_{s'\in S}\pi_{s,s'}\int_{v(s',\eta')<d(s,\eta)}\left(d(s,\eta)-v\left(s',\eta'\right)\right)\zeta\left(\eta,\eta'\right)d\eta'$$

The first term is the discounted face value. The second term is the discounted value of credit losses. Integrating over the ergodic distribution of η, I define the aggregate bond price as $b(s):=\int_\Omega b(s,\eta)\bar{\zeta}(\eta)\,d\eta$, and the aggregate default loss in state s' as

$$L\left(s,s'\right):=\int_{\eta\in\Omega}\bar{\zeta}(\eta)\left[\int_{v(s',\eta')<d(s,\eta)}\left(d(s,\eta)-v\left(s',\eta'\right)\right)\zeta\left(\eta,\eta'\right)d\eta'\right]d\eta. \qquad (15)$$

so that, by definition,

$$b(s)=\bar{m}_s\int_{\eta\in\Omega}d(s,\eta)\bar{\zeta}(\eta)\,d\eta-\bar{m}_s\sum_{s'\in S}\pi_{s,s'}L\left(s,s'\right).$$

I specify the dynamics of $d(s,\eta)$ as

$$d(s,\eta)=\hat{d}(s)+\tilde{d}(\eta),$$

normalized so that $\hat{d}(\bar{s})=0$, and I define $\bar{d}$ as the mean of $\tilde{d}$. Then, using the approximations of Lemma 4 that $v(s,\eta)\approx\tilde{v}(\eta)+\hat{v}(s)$, I find that[4]

Lemma 5 *The first-order approximation of the credit loss function $L(.)$ is*

$$L\left(s,s'\right)\approx\bar{\theta}\left(\bar{l}\bar{d}+\hat{d}(s)-\hat{v}\left(s'\right)\right),$$

where $\bar{l}$ is the average loss-rate given default and $\bar{\theta}$ is the average default rate

$$\bar{\theta}:=\int_{\tilde{v}(\eta')<\bar{d}}\bar{\zeta}\left(\eta'\right)d\eta'.$$

Proof. See appendix ■

For small aggregate shocks, we therefore find that the average bond price is, to a first order, given by

$$b(s)\approx\bar{m}_s\bar{d}\left(1-\bar{\theta}\bar{l}\right)+\bar{m}\left(1-\bar{\theta}\right)\hat{d}(s)+\bar{\theta}\bar{m}\sum_{s'\in S}\pi_{s,s'}\hat{v}\left(s'\right). \qquad (16)$$

[4] To lighten the notations, I use the symbol $\approx$ to denote first order approximation.

This equation has a very natural interpretation. The market value of debt depends on three factors. The first term is the discounted value of the average recovery rate. The second term captures time variation in debt issuance: the higher the face value, all else equal, the higher the market value. The last term captures the effect of the future macroeconomic conditions on the average recovery rate. In this last term, the future marginal product of capital is scaled by $\bar{\theta}$, the likelihood of default evaluated at the risk neutral steady state.

3.2 The y-Theory of Investment

Consider now the first order approximation to the investment equation (6): $\gamma(x(s,\eta)) \approx \gamma_0 + \gamma_1 (\tilde{x}(\eta) + \hat{x}(s))$. Integrating over η, we get

$$\gamma_1 \hat{x}(s) \approx (\bar{m}_s - \bar{m}) \bar{v} + \bar{m}_s E^{\pi} \left[\hat{v}\left(s'\right) | s \right]$$

where $\bar{v} := E^{\bar{\zeta}} [\tilde{v}(\eta)]$, $\bar{x} := E^{\bar{\zeta}} [\tilde{x}(\eta)]$ and $\gamma_0 + \gamma_1 \bar{x} = \bar{m}\bar{v}$. Therefore, using equation (16) and defining the average risky yield $y(s) := \frac{d}{b(s)} - 1$, we obtain

$$\hat{x}(s) \approx \frac{r - r(s)}{(1+r)(1+r(s))} \frac{\bar{v}}{\gamma_1} + \frac{\bar{d}}{\gamma_1 \bar{\theta} (1+r(s))} \left(\frac{1+r(s)}{1+y(s)} - \frac{1+r}{1+\bar{y}} \right) + \frac{\hat{d}(s)}{\gamma_1} \frac{1-\bar{l}}{1+r} \tag{17}$$

where $r(s)$ is the one period real risk free rate.

We can summarize our finding in the following theorem

Theorem 1 *When aggregate shocks and firm level shocks are small, there exists a linear combination of risk free yields, risky corporate yields and book leverage that is a sufficient statistic for aggregate investment.*

A practical issue that I have not discussed so far is inflation. Modern research in asset pricing suggests that real risk free rates do not vary much over time. Suppose that real rates are exactly constant, and let $y^{\$}(s)$ and $r^{\$}(s)$ denote nominal yields. Then, we have the following corollary:

Corollary 1 *If the real risk free rate is constant, then*

$$\hat{x}(s) \approx \frac{\bar{d}}{\gamma_1 \bar{\theta} (1+r)} \left(\frac{1+r^{\$}(s)}{1+y^{\$}(s)} - \frac{1+\bar{r}^{\$}}{1+\bar{y}^{\$}} \right) + \frac{\hat{d}(s)}{\gamma_1} \frac{1-\bar{l}}{1+r} \tag{18}$$

4 Empirical Estimates

4.1 Data Description

In equation (18), the key variable is $\frac{1+r^{\$}(s)}{1+y^{\$}(s)}$, the price of corporate bonds relative to treasuries. As shown in **Figure 1**, the spreads of corporate bonds over treasuries have increased over the post-war period. Campbell and Taksler (2003) argue that this is due to a secular increase in the volatility of publicly traded companies.[5] I use Moody's Baa index (y_t^{Baa}) as my main measure of the yield on risky corporate debt. Moody's index is the equal weighted average of yields on Baa-rated bonds issued by large non financial corporations. To be included in the index, a bond must have a face value of at least 100 million, an *initial* maturity of at least 20 years, and most importantly, a liquid secondary market. Beyond these characteristics, Moody's has some discretion on the selection of the bonds. The number of bonds included in the index varies from 75 and 100 in any given year. The main advantages of Moody's measure are that it is available since 1919, and that it is broadly representative of the U.S. non financial sector, since Baa is close to the median among rated companies. On the other hand, the main issue with this measure is maturity. The theory presented above is based on the yield on short term bonds, but y_t^{Baa} is an average among outstanding bonds of different maturities. This creates two problems. First, it is not obvious which treasury yield one should use as a benchmark. I will follow the literature and use the 10-year treasury yield.[6] Second, if the term structure of default spreads is not flat, using a long yield to estimate the price of a one-year bond might introduce measurement errors. In section 5, I consider the case of infinite horizon debt, and I use numerical simulations to show the robustness of the results. For now, however, I focus on $\frac{1+r_t^{10}}{1+y_t^{Baa}}$ as my main proxy for $\frac{1+r^{\$}(s)}{1+y^{\$}(s)}$, where r_t^{10} is the yield on a 10-year constant maturity treasury and y_t^{baa} is Moody's yield for an index of Baa bonds. Both variables are obtained from FRED®.[7] The variables are described in **Table 1a**.

[5] See Comin and Philippon (2005) for a discussion, and Davis, Haltiwanger, Jarmin, and Miranda (2006) for evidence on privately held companies.

[6] All the results presented below are robust to using a 20-year risk free yield (which is not available on as long a sample, however).

[7] Federal Reserve Economic Data: http://research.stlouisfed.org/fred2/

4.2 Regression Results

Constant real rates and constant book leverage

Suppose that the real rate and book leverage are constant, i.e. $\bar{m}_s \equiv \frac{1}{1+r}$ and $\hat{d}(s) \equiv 0$. Then equation (18) becomes

$$\hat{x}(s) \approx \frac{\bar{d}}{\gamma_1 \bar{\theta}(1+r)} \frac{1+r^{\$}(s)}{1+y^{\$}(s)} + cte, \tag{19}$$

where $r^{\$}(s)$ is the nominal risk free rate, and $y^{\$}(s)$ is the nominal yield on corporate bonds. Because of the persistence in the investment rate and the other variables, as reported in **Table 1a**, I estimate equation (19) in changes, rather than in levels.[8] The dependent variable is thus the year-to-year change in the annual investment rate $\Delta\left(\frac{i}{k}\right)$ and the first estimation equation is simply

$$\Delta\left(\frac{i_t}{k_t}\right) = \alpha + \beta\Delta\left(\frac{1+r_t^{10}}{1+y_t^{Baa}}\right), \tag{20}$$

where i_t is real aggregate non-residential fixed investment, and k_t is the capital stock constructed using the investment series and a depreciation rate of $\delta = 12\%$. **Table 2** reports the results. Each column corresponds to a different specification. Column (i) presents the results from estimating equation (20). The regression of annual change in $\frac{i}{k}$ on annual change in the relative price of defaultable bonds has an adjusted R^2 of 42%. We can interpret column (i) as saying that

$$\frac{\bar{d}}{\gamma_1 \bar{\theta}(1+r)} \approx 1.9$$

The mean book leverage is around 5% for short term debt and 20% for long term debt, and it is customary in the empirical credit risk literature to use short term debt plus half of long term debt as a proxy for $\bar{d}$ (see Moody's KMV methodology, as explained in Crosbie and Bohn (2003)). So we can use $\bar{d} = 15\%$ as a benchmark. What is $\bar{\theta}$? It is the default rate in the average *risk-neutral* state. **Table 1b** shows that the average historical default rate for Baa bonds (annualized from the 10-year cumulative rate) is around 0.5% over 1970-2001 and 0.8% over 1920-1999. The standard deviation of this rate is 0.25% over 1970-2001 and

[8] Another reason is that there is a slow-moving upward trend in the default probability, mostly reflecting the increase in idiosyncratic risk, as explained in Campbell and Taksler (2003) and Comin and Philippon (2005).

0.9% over 1920-1999. Thus, the mean plus one standard deviation is between 0.75% and 1.7%. On the other hand, it is easy to see that $\bar{\theta}$ must be more than the average spread. Assuming a recovery rate of 20%, we would get $\bar{\theta}$ between 2% and 3%. Let us take 2%. Then we would get

$$\gamma_1 \approx \frac{15}{1.03 \times 1.9 \times 2} \approx 3.832.$$

Shapiro (1986) estimates γ between 8 and 9 using quarterly data, which corresponds to 2 to 2.2 at annual frequencies. Hall (2004) finds even smaller adjustment costs. Thus, the y-theory brings a substantial improvement compared to estimates of more than 20 that one would get from the equity based theory. Moreover, simple extensions to take into account non-convexities at the firm level would likely decrease the implied γ_1 further (see Caballero and Engle (2005)).[9]

Another important fact to keep in mind is that the Baa spread is not a pure measure of credit risk. There is an ongoing debate in the literature about the role of default risk in explaining yield spreads. Because treasuries are more liquid than corporate bonds, part of the spread should reflect a liquidity premium. Also, treasuries have a tax-advantage over corporate bonds because they are not subject to state and local taxes. These arguments suggest that we cannot attribute the entire spreads to default risk. Almeida and Philippon (2005) discuss in details the various ways one can use to estimate the default component of bond spreads, and find that roughly 3/4 of the spread is due to credit risk. Taken literally, this means that we should multiply of estimated γ_1 by 3/4 in order to obtain the correct estimate: it would be 2.875.

Other Regressions

Column (ii) adds the 10-year treasury yield and the mean current liabilities over assets of Baa firms as a measure of leverage. Column (ii) shows that it is really the spread that matters, since the coefficient on the 10-year treasury is zero. The change in book leverage has the predicted sign, and it is significant. Column (iii) introduces a cash flow variable, earnings before interest and taxes over capital, as well as the market value of equity

[9] To keep things simple, I have used a purely convex model. But it is not a good approximation at the firm level because of fixed costs and time-to-build. These issues will not disappear in the aggregate. If one approximates lumpiness at the firm level in the Calvo way, it is straightforward to show that one should scale the coefficient on the spread by the fraction of active companies to obtain the correct estimates. If only half of the firms actively change their investment plans in any given period, then we would get γ_1 around 1.9.

over capital. Quite remarkably, the cash flow variable is not significant in the investment equation, and neither is the market value of equity. So far, it seems that a model with constant real risk-free interest rates, constant book leverage, and the bond spread as a sufficient statistic, is not rejected by the data. The next three columns allow for different coefficients on the current spread and its lag. Column (iv) shows a significant increase in the R^2, and the restriction that the sum of the coefficients is zero is rejected at the 1% level. This suggests that the adjustment cost function might be misspecified, probably because of non-convexities at the firm level, as in Caballero and Engle (1999). Column (v) and (vi) suggest that controlling for book leverage can marginally improve the fit, while equity does not seem to bring in a significant amount of new information. I have also run regressions controlling for long term book leverage, which is not significant and leaves the other coefficients unchanged.

5 Extensions and Robustness

5.1 Long Term Yields

For long term bonds, the pricing equation (8) can be integrated over η with the ergodic distribution $\bar{\zeta}(.)$. This defines the aggregate long term bond price as $b^{\infty}(s) := \int_{\eta' \in \Omega} b^{\infty}(s,\eta)\,\bar{\zeta}(\eta)\,d\eta$, and (8) becomes

$$b^{\infty}(s) = \bar{m}_s c + \bar{m}_s \sum_{s' \in S} \pi_{s,s'} b^{\infty}\left(s'\right) - \bar{m}_s \sum_{s' \in S} \pi_{s,s'} L^{\infty}\left(s'\right), \tag{21}$$

where

$$L^{\infty}\left(s'\right) := \int_{v(s',\eta') < c + b^{\infty}(s',\eta')} \left(c + b^{\infty}\left(s',\eta'\right) - v\left(s',\eta'\right)\right) \bar{\zeta}\left(\eta'\right) d\eta'. \tag{22}$$

Formula (21) and (22) are exact, but cannot be used directly in the investment equation. At this point, and just like with short term debt, I make a first order approximation to the function $b^{\infty}(s',\eta')$ in equation (8) as $b^{\infty}(s',\eta') \approx \hat{b}^{\infty}(s') + \tilde{b}^{\infty}(\eta')$, where $\tilde{b}^{\infty}(\eta')$ is the price that would prevail in the idiosyncratic economy. To a first order, equation (22) then becomes

$$L^{\infty}\left(s'\right) \approx \bar{L}^{\infty} + \bar{\theta}^{\infty}\left(\hat{b}^{\infty}\left(s'\right) - \hat{v}\left(s'\right)\right), \tag{23}$$

where the average default rate is defined by $\bar{\theta}^{\infty} := \int_{\tilde{v}(\eta) < c + \tilde{b}^{\infty}(\eta)} \bar{\zeta}(\eta)\,d\eta$ and the average loss rate by $\bar{L}^{\infty} := \int_{\tilde{v}(\eta) < c + \tilde{b}^{\infty}(\eta)} \left(c + \tilde{b}^{\infty}(\eta) - \tilde{v}(\eta)\right) \bar{\zeta}(\eta)\,d\eta$. Substituting (23) into (21), and defining the average bond price $\bar{b}^{\infty} := \int \tilde{b}^{\infty}(\eta)\,\bar{\zeta}(\eta)\,d\eta$, and noting that $\bar{b}^{\infty} = \frac{c-\bar{L}^{\infty}}{r}$, I find that

$$\hat{b}^{\infty}(s) \approx (\bar{m}_s - \bar{m})\,\bar{b}^{\infty} + \left(1 - \bar{\theta}^{\infty}\right)\bar{m}E^{\pi}\left[\hat{b}^{\infty}(s')\,|s\right] + \bar{\theta}^{\infty}\bar{m}E^{\pi}\left[\hat{v}(s')\,|s\right],$$

while the investment equation is $\gamma_1\hat{x}(s) \approx (\bar{m}_s - \bar{m})\,\bar{v} + \bar{m}E^{\pi}\left[\hat{v}(s')\,|s\right]$. In matrix notations, we get

$$\begin{aligned}\left(I - \frac{1-\bar{\theta}^{\infty}}{1+r}\Pi\right)\hat{B}^{\infty} &= \bar{b}^{\infty}\hat{M} + \bar{\theta}^{\infty}\bar{m}\Pi\hat{V},\\ \gamma_1\hat{X} &= \bar{v}\hat{M} + \bar{m}\Pi\hat{V}.\end{aligned}$$

Therefore

$$\gamma_1\hat{X} = \left(\bar{v} - \frac{\bar{b}^{\infty}}{\bar{\theta}^{\infty}}\right)\hat{M} + \left(I - \frac{1-\bar{\theta}^{\infty}}{1+r}\Pi\right)\frac{\hat{B}^{\infty}}{\bar{\theta}^{\infty}}.$$

The bond price is not a sufficient statistic for investment because of the off-diagonal elements in the matrix Π. In the case of long term bonds, we must therefore extend our first order approximation to the matrix Π by assuming that Π is close to being diagonal. This is a new approximation, one that we did not need to make before. It extends the small shock approximation that we have used so far, but it applies to the transition matrix. It says that the system is not likely to jump from one extreme to the other. If Π is close to being diagonal, then the response of $\hat{x}$ to $\hat{b}$ is $\frac{r+\bar{\theta}^{\infty}}{(1+r)\gamma_1\bar{\theta}^{\infty}}$, and with $b^{\infty}(s) = \frac{c}{y(s)}$ we get $\hat{b}^{\infty}(s) = -\frac{c}{\bar{y}}\frac{\hat{y}(s)}{\bar{y}} = \bar{b}^{\infty}\frac{\hat{y}(s)}{\bar{y}}$ and therefore

$$x_t \propto -\frac{\left(r + \bar{\theta}^{\infty}\right)\bar{b}^{\infty}}{(1+r)\,\gamma_1\bar{\theta}^{\infty}}\frac{\hat{y}(s)}{\bar{y}}. \tag{24}$$

If we implement this regression using the same data as in table 1, we get $\frac{\left(r+\bar{\theta}^{\infty}\right)\bar{b}^{\infty}}{(1+r)\gamma_1\bar{\theta}^{\infty}} \approx 0.12$. Assuming that long term debt is on average $\bar{b}^{\infty} = 20\%$ of book assets, we get, with $r = 3\%$ and $\bar{\theta}^{\infty} = 2\%$

$$\gamma_1 \approx \frac{5 \times 0.2}{1.03 \times 0.12 \times 2} = 4.05.$$

I return to the issue of long term debt in section 5.4 where I perform numerical simulations.

5.2 Second Order Approximations

The appendix shows the approximation $v(s,\eta) = \tilde{v}(\eta) + \hat{v}(s) + \eta h(s)$ and characterizes the function $h(.)$. Consider the case of short term debt with constant book leverage. Note that

$$\int v\left(s',\eta'\right)\bar{\zeta}\left(\eta'\right)d\eta' = \hat{v}\left(s'\right) + \int \tilde{v}\left(\eta'\right)\bar{\zeta}\left(\eta'\right)d\eta',$$

so that the RHS of the investment equation does not change. On the other hand, the loss function becomes

$$\begin{aligned} L\left(s,s'\right) &= \int_{\tilde{v}(\eta')+\hat{v}(s')+\eta' h(s')<\bar{d}} \left(\bar{d}-\tilde{v}\left(\eta'\right)-\hat{v}\left(s'\right)-\eta' h\left(s'\right)\right)\bar{\zeta}\left(\eta'\right)d\eta', \\ &= \bar{L}-\hat{v}\left(s'\right)\bar{\theta}-h\left(s'\right)\int_{\tilde{v}(\eta')<\bar{d}} \eta'\bar{\zeta}\left(\eta'\right)d\eta'. \end{aligned}$$

so $h(s')$ creates a measurement issue. The above equation shows the effect of an increase in h: The integral is negative, so a higher value of h increases the credit losses. On the other hand, as on can see in the appendix, $h(s)$ increase with $E^{\pi}[\hat{v}(s')]$. Suppose good news arrive: Investment increases, h increases, which indirectly increases credit losses. As a result, bond prices do not rise as much as one would expect from the first order approximation. This could lead to a bias in the estimation of γ_1. To investigate this issue further, I use numerical simulations in section 5.4.

5.3 Time Varying Idiosyncratic Risk

To study this issue, let us assume that idiosyncratic shocks are *iid*. Otherwise, keeping track of the distribution of idiosyncratic shocks would be too complicated. In this case, all the firms have the same bond price, the same investment rate and η simply affects the level of the value function

$$v(s,\eta)=\bar{v}+\eta+\hat{v}(s).$$

Suppose that the conditional distribution of η' is $\bar{\zeta}(s,\eta')$

$$L\left(s,s'\right)=\int_{\eta'<\bar{d}-\bar{v}-\hat{v}(s')} \left(\bar{d}-\bar{v}-\hat{v}\left(s'\right)-\eta'\right)\bar{\zeta}\left(s,\eta'\right)d\eta'.$$

The first order approximation is

$$L\left(s,s'\right)\approx\bar{L}(s)-\hat{v}\left(s'\right)\bar{\theta}(s),$$

where $\bar{\theta}(s):=\int_{\eta'<\bar{d}-\bar{v}}\bar{\zeta}(s,\eta')\,d\eta'$ and $\bar{L}(s):=\int_{\eta'<\bar{d}-\bar{v}}\left(\bar{d}-\bar{v}-\eta'\right)\bar{\zeta}(s,\eta')\,d\eta'$. The bond price is then

$$b(s)\approx\bar{m}_s\left(\bar{d}-\bar{L}(s)\right)+\bar{\theta}(s)\,\bar{m}_s E^{\pi}\left[\hat{v}\left(s'\right)|s\right],$$

and the investment equation becomes

$$\gamma_1\hat{x}(s)\approx\left(\bar{m}_s-\bar{m}\right)\bar{v}+\frac{b(s)-\bar{m}_s\left(\bar{d}-\bar{L}(s)\right)}{\bar{\theta}(s)}.$$

The effect of time varying idiosyncratic risk is theoretically straightforward. In practice, one can simply control for changes in idiosyncratic volatility. In column (vi) of **Table 2**, I add the (change in) average idiosyncratic asset volatility of Baa-rated companies.[10] The change in mean asset volatility has the predicted sign and it is significant.

5.4 Numerical Simulations

In this section, I try to answer the following questions:

- how good is the first order approximation for estimating γ with aggregate data?
- how well would the y-theory perform on firm level data?

To do so, I first need to approximate the aggregate dynamics $a(s)$, and the firm level dynamics η. I assume that a and η can be approximated as AR(1) processes, and I estimate the persistence of the processes using aggregate and firm level data

$$\begin{aligned} a_t &= \rho^a a_{t-1} + \varepsilon_t^a, \\ \eta_t &= \rho^\eta \eta_{t-1} + \varepsilon_t^\eta. \end{aligned}$$

For each firm, I define $a + \eta$ as operating income before depreciation divided by the book value of assets. I define a as the mean across all the firms rated and η as the residual. I allow for a deterministic time trend in order to capture changes in the composition of the sample and other non-stationarities, as discussed in Comin and Philippon (2005) and Davis, Haltiwanger, Jarmin, and Miranda (2006). I find similar persistence for a for η of 0.75. I then choose the volatility of the distribution of shocks to match the mean and standard deviations of $\frac{1+r_t^{10}}{1+y_t^{Baa}}$ reported in **Table 1a**. Roughly speaking, the dispersion of η pins down the average spread, and the dispersion of $a(s)$ pins down how it changes over time. The rest of the parameters are standard: r=4%, γ=4. The parameters needed to match the data imply risk neutral volatilities of 0.012154 for $a(s)$, and 0.12103 for η. I simulate 1000 observations, and I run equation (20) and (24) in the simulated data, at the firm level

[10] I have used a simple Merton model for each Baa rated company and each year in order to extract asset volatility. Equity volatility was estimated on the 12 month of each year, and the face value of debt was computed as short term debt + one half of long term debt (as in the simplest version of KMV). Details are available on request.

and in the aggregate. The next table summarizes the results. For each regression, I report the R^2 and the ratio of the implied $\hat{\gamma}$ over the true γ, which is equal to 4.

Equation	(20)	(20)	(24)	(24)
	$\hat{\gamma}/\gamma$	R^2	$\hat{\gamma}/\gamma$	R^2
Aggregate	0.70327	0.97413	1.3751	0.99033
Firm Level	3.8673	0.13247	2.9813	0.38957

Several findings emerge. Firstly, the aggregate regression deliver very high R^2. Secondly, despite the high R^2, there can be a significant bias in the estimation of γ. The bias is downward when one uses short term debt, and upward when one uses long term debt. Thirdly, the performance of the model deteriorates substantially when we move from aggregate to firm level data. This underlines the importance of averaging across firms. Note also that the direction of the bias in the short debt equation is reversed. Finally, at the firm level, the R^2 of the long term debt equation is substantially higher than the one of the short term equation. Estimating equations (20) and (24) on firm level data is an important task for future research.[11]

6 Discussion

I show theoretically that, in an economy where aggregate shocks are small and there is a continuous distribution of firms, aggregate q is a simple function of the risk free rate and of the average yield on risky corporate debt. Empirically, I find that y-theory explains aggregate investment reasonably well. Numerical simulations suggest that y-theory might also be useful at the firm level. I now discuss the implications of this finding for the existing research on corporate investment, and I mention the remaining puzzles that need to be addressed.

On the failure of q-theory

One could summarize my results by saying that aggregate q-theory seems to work fine, provided one uses the bond market instead of the equity market to construct one's measure of q. Keeping this fact in mind, let us revisit the various explanations that have been proposed in the literature for the failure of q-theory. The q-theory relies on a first order condition linking a firm's investment rate I/K to the firm's Tobin's Q. The theory could

[11] The practical issue is to obtain reliable bond prices at the firm level.

therefore fail if the first order condition does not hold at the firm level, because of credit constraints (Fazzari, Hubbard, and Petersen (1988), Bernanke and Gertler (1989)) or non-convexities (Caballero and Engle (1999)) for instance. If these issues caused the q-theory to fail, however, they would cause the y-theory to fail as well, since the underlying equation is the same.

The theory could also fail because of measurement errors (Erickson and Whited (2000)). It is important to separate measurement errors in I or K from issues involving market prices. Measurement errors in I or in K – due to improper accounting, incorrect depreciation rates, aggregation across heterogenous types of capital, failure to take into account intangibles and human capital – would affect my investment equations as well.

Thus, it seems unlikely that credit constraints, fixed costs and measurement errors in I or in K are responsible for the first order failures of q-theory in the aggregate, and it seems more likely that the issue lies in the behavior of the market value of equity.[12]

Equity prices versus bond prices

Why is the equity-based investment equation less successful than the bond-spread investment equation? It is important to understand that there are two separate issues here. The first is why the correlation between equity and bond prices is so low in the data, given that, with small aggregate shocks, we would expect a very high correlation. The presence of bubbles in equity prices but not in bond prices is a potential explanation for the observed low correlation. Another is the presence of large growth options: a small probability of a large increase in the present value of assets would have almost no impact on bond prices, and a large impact on equity prices.

The second issue is why investment does not respond more to equity prices. One explanation is that firms choose not to respond, either because they perceive the equity prices to be disconnected from fundamentals, or because of adverse selection in the equity issuance market. Another explanation, suggested by Abel and Eberly (2005), is that firms cannot invest before the growth options are realized. It is as if firms faced two types of adjustment costs, the ones for normal investment, well captured by convex costs, and the ones

[12] Erickson and Whited (2006) reach a similar conclusion using an econometric approach to deal with the measurement errors. They find that most of the measurement errors responsible for the failure of q-theory are in the market value V, not in K. In their terminology, measurement errors in V include bubbles and other deviations of market values from fundamentals.

for growth options, highly non-convex. This would seem to be consistent with the fact that IPOs are well explained by equity market valuations, while the bulk of capital expenditures is well explained by bond market valuations.

On the importance of risk premia in macroeconomics

It is well known in the credit risk literature that risk premia explain a large fraction of credit spreads. Given that these spreads account for most of investment dynamics, one should conclude that risk premia are important for macroeconomic dynamics. Yet, risk premia play essentially no role in the standard business cycle model of Kydland and Prescott (1982).[13]

Much of the investment literature abstracts from risk premia. On the empirical side, Abel and Blanchard (1986) construct a time series for marginal q that takes into account changes in the risk free rate, while abstracting from changes in risk premia. In this setup, changes in aggregate investment are driven either by changes in the (real) risk free rate, or by news about future aggregate profitability. The ex-ante real rate is neither volatile nor very correlated with aggregate investment, however, and aggregate cash flows news explain little of the variation in aggregate valuations, as shown by Shiller (1981). Compared to the approach in Abel and Blanchard (1986), the yield theory has the advantage of taking into account changes in risk premia over time.

On the theoretical side, the leading papers focusing on credit constraints, such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), assume that investors are risk neutral. In this case, it is the objective probability of financial distress that matters for investment. This objective probability, however, is much smaller that the risk adjusted one, as shown in Berndt, Douglas, Duffie, Ferguson, and Schranz (2005). Thus, irrespective of whether firms are indeed constrained or not, one should not neglect the effects of investor risk aversion.

Similarly, models of non-convex adjustment costs in the tradition of Caballero and Engle (1999) also rely on risk neutral investors for tractability. To the extent that one is concerned with explaining firm level investment dynamics, the assumption of risk neutral

[13] Obtaining high risk premia in general equilibrium models is not straightforward. Tallarini (2000) builds a real business cycle model where consumers's preferences are modelled as in Epstein and Zin (1989), and studies the consequences of high risk aversion for asset prices and macroeconomic dynamics.

investors is probably fine, because much of the volatility at the firm level reflects cash flow news, not discount rate shocks, as emphasized for instance in Vuolteenaho (2002). In the aggregate, however, discount rate shocks are much more important. My results suggest that understanding the nature of these macroeconomic shocks could be necessary and sufficient to obtain a successful theory of aggregate investment.

References

Abel, A. B., and O. J. Blanchard (1986): "The Present Value of Profits and Cyclical Movements in Investment," *Econometrica*, 54(2), 249–73.

Abel, A. B., and J. C. Eberly (2005): "Investment, Valuation, and Growth Options," Working Paper, Wharton.

Almeida, H., and T. Philippon (2005): "The Risk-Adjusted Cost of Financial Distress," Working Paper, New York University.

Bernanke, B., and M. Gertler (1989): "Agency Costs, Net Worth and Business Fluctuations," *American Economic Review*, 79, 14–31.

Bernanke, B. S. (1983): "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression," *American Economic Review*, 73(3), 257–276.

Berndt, A., R. Douglas, D. Duffie, M. Ferguson, and D. Schranz (2005): "Measuring Default-Risk Premia from Default Swap Rates and EDFs," Working Paper Stanford GSB.

Caballero, R. (1999): "Aggregate Investment," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford, vol. 1B. Elsevier Science, North Holland.

Caballero, R., and E. Engle (1999): "Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S,s) Approach," *Econometrica*, 67, 783–826.

——— (2005): "Adjustment is Much Slower Than You Think," Working Paper, MIT.

Campbell, J. Y., and G. B. Taksler (2003): "Equity Volatility and Corporate Bond Yields," *Journal of Finance*, LVIII (6), 2321–2349.

Cochrane, J. H. (1996): "A Cross-Sectional Test of an Investment-Based Asset Pricing Model," *Journal of Political Economy*, 104(3), 572–621.

Comin, D., and T. Philippon (2005): "The Rise in Firm-Level Volatility: Causes and Consequences," in *Macroeconomics Annual*, ed. by M. Gertler, and K. Rogoff. NBER.

Crosbie, P., and J. Bohn (2003): "Modeling Default Risk," Discussion paper, Moody's KMV.

Davis, S., J. Haltiwanger, R. Jarmin, and J. Miranda (2006): "Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms," in *Macroeconomics Annual*, ed. by D. Acemoglu, K. Rogoff, and M. Woodford. NBER.

Epstein, L. G., and S. E. Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–969.

Erickson, T., and T. M. Whited (2000): "Measurement Error and the Relationship Between Investment and Q," *Journal of Political Economy*, 108(5), 1027–1057.

——— (2006): "On the Accuracy of Different Measures of Q," *Financial Management*, p. forthcoming.

Fazzari, S. M., R. G. Hubbard, and B. C. Petersen (1988): "Financing Constraints and Corporate Investment," *Brookings Papers on Economic Activity*, 1, 141–195.

Hall, R. E. (2004): "Measuring Factor Adjustment Costs," *Quarterly Journal of Economics*, 119(3), 899–927.

Hayashi, F. (1982): "Tobin's Marginal q and Average q: A Neoclassical Interpretation," *Econometrica*, 50(1), 213–24.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–248.

Kydland, F. E., and E. C. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50, 1345–1371.

Lettau, M., and S. Ludvigson (2002): "Time-Varying Risk Premia and the Cost of Capital: An Alternative Implication of the Q Theory of Investment," *Journal of Monetary Economics*, 49, 31–66.

Merton, R. C. (1974): "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449–470.

Pan, J., and K. J. Singleton (2005): "Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads," .

Shapiro, M. D. (1986): "The Dynamic Demand for Capital and Labor," *Quarterly Journal of Economics*, 101, 513–542.

Shiller, R. J. (1981): "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?," *American Economic Review*, 71(3), 421–436.

Stock, J. H., and M. W. Watson (1989): "New Indexes of Coincident and Leading Economic Indicators," in *NBER Macroeconomics Annual 1989*, ed. by O. J. Blanchard, and S. Fisher, pp. 352–394.

Tallarini, T. D. (2000): "Risk-sensitive real business cycles," *Journal of Monetary Economics*, 45(3), 507–532.

Vuolteenaho, T. (2002): "What Drives Firm-Level Stock Returns?," *Journal of Finance*, 57(1), 233–264.

Zhang, L. (2005): "The Value Premium," *Journal of Finance*, 60(1), 67–103.

A Proofs

This appendix contains the proofs of the Lemmas mentioned in the main text.

Proof of Lemma 1
Define

$$\varpi(x) := (r - x)\gamma(x) + \Gamma(x) - \bar{a}$$

The function $\varpi(.)$ is continuous over $(-1, \infty)$. Moreover

$$\frac{\partial \varpi}{\partial x}(x) = (r - x)\frac{\partial \gamma}{\partial x}(x)$$

so ϖ is increasing over $(-1, r]$ and decreasing over $[r, \infty)$. If $r\gamma(0) < \bar{a} < \Gamma(r)$, then $\varpi(0) < 0$ and $\varpi(r) > 0$, so there is a unique solution over $(0, r)$. There might be another over (r, ∞), but it violates the transversality condition.

Proof of Lemma 2
First of all, it is clear that when $\|A - \bar{a}\|$ and $\|\bar{M} - \bar{m}\|$ are small enough, we can use Lemma 1 to argue that $v^a(.)$ exists and solves

$$v^a(s) = \max_{x > -1}\left\{a(s) - \Gamma(x) + (1 + x)\bar{m}_s E^\pi\left[v^a\left(s'\right)|s\right]\right\}.$$

Then, we can define $o^v(s) := v^a(s) - \bar{v}$ and $o^x(s) := x^a(s) - \bar{x}$, and it is also clear that both $\|o^v\|$ and $\|o^v\|$ go to 0 when $\|A - \bar{a}\| + \|\bar{M} - \bar{m}\| \to 0$. To show the stronger result in the Lemma, let us assume that $\|A - \bar{a}\|$ and $\|\bar{M} - \bar{m}\|$ are small enough that there exist a neighborhood Ξ of $\bar{x}$ such that $x^a(s) \in \Xi$ for all s and a neighborhood ϱ of r such that $r(s) \in \varrho$ for all s and

$$x_{\max} = \sup \Xi < r_{\min} = \inf \varrho$$

Note that this is possible under assumption (10). Then notice that

$$\bar{m}_s E^\pi\left[v^a\left(s'\right)|s\right] \leq \frac{a_{\max} - \Gamma(x_{\max})}{r_{\min} - x_{\max}} \text{ for all } s \in S$$

Using the investment equation, this implies that

$$\chi(x_{\max}) \leq a_{\max},$$

where the function $\chi(.)$ is defined by

$$\chi(x) := (r_{\min} - x)\gamma(x) + \Gamma(x).$$

The function $\chi(.)$ is strictly increasing over Ξ, and its derivative can be bounded below by some strictly positive number ϵ

$$\frac{\partial \chi(x)}{\partial x} = (r_{\min} - x)\frac{\partial \gamma(x)}{\partial x} > \epsilon > 0 \text{ for all } x \in \Xi,$$

which implies that $\chi(x_{\max}) > \epsilon(x_{\max} - \bar{x}) + \chi(\bar{x})$. Now, given the definition of $\bar{x}$, we see that $\chi(\bar{x}) = (r_{\min} - r)\gamma(\bar{x}) + \bar{a}$ and therefore

$$x_{\max} - \bar{x} < \frac{a_{\max} - \bar{a} + (r - r_{\min})\gamma(\bar{x})}{\epsilon}$$

This last inequality then implies that

$$\lim_{\|A-\bar{a}\|+\left\|\bar{M}-\bar{m}\right\|\to 0} \frac{x_{\max}-\bar{x}}{\|A-\bar{a}\|+\left\|\bar{M}-\bar{m}\right\|} < \infty$$

The same strategy applies to the lower bound $x_{\min}$, and it is then straightforward to extend to the value function to prove the Lemma.

Proof of Lemma 3
By the mean value theorem, there exist $\tilde{x}_s \in [\bar{x}, x_s]$ such that

$$\Gamma(x_s) = \Gamma(\bar{x}) + o^x(s)\,\gamma(\tilde{x}_s)$$

and thus

$$\|a(.) - \bar{a} - \Gamma(.) + \Gamma(\bar{x})\| \leq \|a(.) - \bar{a}\| + \max_s \{\gamma(\tilde{x}_s)\} \|o^x_s\|$$

The function $o^v(s)$ satisfies

$$\bar{v} + o^v(s) = a(s) - \Gamma(x_s) + (1 + \bar{x} + o^x(s))\left(\bar{m}_s\bar{v} + \bar{m}_s E^\pi\left[o^v\left(s'\right)|s\right]\right)$$

Using the investment equation and the definitions of $\bar{v}$ and $\hat{v}(s)$, we get

$$o^v(s) - \hat{v}(s) = o^x_s\left(\bar{m}_s E^\pi\left[v^a\left(s'\right)|s\right] - \gamma(\tilde{x}_s)\right) + (1+\bar{x})\left((\bar{m}_s - \bar{m}) E^\pi\left[o^v\left(s'\right)|s\right] + \bar{m}E^\pi\left[o^v\left(s'\right) - \hat{v}\left(s'\right)|s\right]\right)$$

Therefore

$$\|o^v_s - \hat{v}(s)\| \leq \frac{\|o^x_s\|\,\|\bar{m}_s E^\pi[v^a(s')|s] - \gamma(\tilde{x}_s)\| + (1+\bar{x})\left\|\bar{M} - \bar{m}\right\|\|o^v_s\|}{1 - (1+\bar{x})\,\bar{m}}$$

By continuity of $\gamma(.)$, and using the first order condition for investment in the pseudo-steady state, we know that

$$\lim_{\|A-\bar{a}\|+\left\|\bar{M}-\bar{m}\right\|\to 0} \left\|\bar{m}_s E^\pi\left[v^a\left(s'\right)|s\right] - \gamma(\tilde{x}_s)\right\| = 0,$$

and therefore, using Lemma 2, we get

$$\lim_{\|A-\bar{a}\|+\left\|\bar{M}-\bar{m}\right\|\to 0} \frac{\|o^v_s - \hat{v}(s)\|}{\|A-\bar{a}\|+\left\|\bar{M}-\bar{m}\right\|} = 0.$$

Proof of Lemma 5
Let us consider a small change in d

$$d = \bar{d} + \varepsilon$$

Then, using Leibniz's formula

$$\begin{aligned} & \int_{v(s',\eta')<\bar{d}+\varepsilon} \left(\bar{d} + \varepsilon - v\left(s',\eta'\right)\right)\zeta\left(\eta,\eta'\right)d\eta' \\ \approx \; & \int_{v(s',\eta')<\bar{d}} \left(\bar{d} - v\left(s',\eta'\right)\right)\zeta\left(\eta,\eta'\right)d\eta' + \varepsilon\int_{v(s',\eta')<\bar{d}} \zeta\left(\eta,\eta'\right)d\eta' \end{aligned}$$

Applying to $\varepsilon = d_1(s) + d_2(\eta)$, we get

$$\begin{aligned} L(s,s') &\approx \int_{v(s',\eta')<\bar{d}} \left(\bar{d} - v(s',\eta')\right)\bar{\zeta}(\eta')\,d\eta' \\ &\quad + \int_{\eta\in\Omega} \left(d_1(s) + d_2(\eta)\right)\bar{\zeta}(\eta)\left(\int_{v(s',\eta')<\bar{d}} \zeta(\eta,\eta')\,d\eta'\right)d\eta \end{aligned}$$

Using Leibniz's formula to small changes in v, and neglecting second order terms

$$\begin{aligned} L(s,s') &\approx \int_{\tilde{v}(\eta')<\bar{d}} \left(\bar{d} - v(s',\eta')\right)\bar{\zeta}(\eta')\,d\eta' + d_1(s)\int_{\tilde{v}(\eta')<\bar{d}} \bar{\zeta}(\eta')\,d\eta' \\ &\quad + \int_{\eta\in\Omega}\left(\int_{\tilde{v}(\eta')<\bar{d}} \zeta(\eta,\eta')\,d\eta'\right) d_2(\eta)\,\bar{\zeta}(\eta)\,d\eta \end{aligned}$$

Define

$$\bar{L} := \int_{\tilde{v}(\eta')<\bar{d}} \left(\bar{d} - \tilde{v}(\eta')\right)\bar{\zeta}(\eta')\,d\eta' + \int_{\eta\in\Omega}\left(\int_{\tilde{v}(\eta')<\bar{d}} \zeta(\eta,\eta')\,d\eta'\right) d_2(\eta)\,\bar{\zeta}(\eta)\,d\eta$$

and

$$\bar{\theta} := \int_{\tilde{v}(\eta')<\bar{d}} \bar{\zeta}(\eta')\,d\eta'$$

to get

$$L(s,s') = \bar{L} + \bar{\theta}\left(d_1(s) - \hat{v}(s')\right)$$

Second Order Approximation

Suppose that aggregate shocks are first order small, and firm level shocks are second order small. Then, write

$$v(s,\eta) \approx \tilde{v}(\eta) + \hat{v}(s) + \eta h(s)$$

The value function satisfies

$$v(s,\eta) = \max_{x>-1}\left\{a(s) + \eta - \Gamma(x) + (1+x)\,\bar{m}_s E^{\pi}\left[v(s',\eta')\,|s,\eta\right]\right\}$$

Differentiating the value function with respect to aggregate shocks, we get

$$\Delta v(s,\eta) = \Delta a(s) + (1+x)\left(\Delta\bar{m}(s)\,E^{\pi}[\tilde{v}(\eta)] + \bar{m}E^{\pi}\left[\hat{v}(s') + \eta' h(s')\,|s,\eta\right]\right)$$

and then with respect to idiosyncratic shocks

$$\begin{aligned} \frac{\partial \Delta v(s,\eta)}{\partial \eta} &= \frac{\partial \tilde{x}}{\partial \eta} \times \left(\Delta\bar{m}(s)\,E^{\pi}[\tilde{v}(\eta)] + \bar{m}E^{\pi}\left[\hat{v}(s') + \eta' h(s')\,|s,\eta\right]\right) \\ &\quad + (1+\bar{x}) \times \frac{\partial}{\partial \eta}\left(\Delta\bar{m}(s)\,E^{\pi}[\tilde{v}(\eta)] + \bar{m}E^{\pi}\left[\hat{v}(s') + \eta' h(s')\,|s,\eta\right]\right) \end{aligned}$$

A second order expansion of the idiosyncratic component gives

$$\begin{aligned} \tilde{v}(\eta) &= \bar{v} + \tilde{v}_1\eta + \tilde{v}_2\eta^2 \\ \tilde{x}(\eta) &= \bar{x} + \tilde{x}_1\eta + \tilde{x}_2\eta^2 \end{aligned}$$

and neglecting the small terms, we get

$$\begin{aligned} h(s) &= \left[\frac{\partial \tilde{x}}{\partial \eta}\right]_{\eta=0} \left(\Delta \bar{m}(s)\bar{v} + \bar{m}E^{\pi}\left[\hat{v}\left(s'\right)|s\right]\right) \\ &+ (1+\bar{x})\left[\frac{\partial E\left[\eta'|\eta\right]}{\partial \eta}\right]_{\eta=0} \left(\Delta \bar{m}(s)\left[\frac{\partial \tilde{v}}{\partial \eta}\right]_{\eta=0} + \bar{m}E^{\pi}\left[h\left(s'\right)|s\right]\right) \end{aligned}$$

The driving forces are $\bar{m}(s) - \bar{m}$ and $E^{\pi}\left[\hat{v}\left(s'\right)|s\right]$. From the investment equation, we see that

$$\left[\frac{\partial \tilde{x}}{\partial \eta}\right] = \frac{\bar{m}\chi}{\gamma_1}\left[\frac{\partial \tilde{v}}{\partial \eta}\right]_{\eta=0}$$

where $\chi = \left[\frac{\partial E[\eta'|\eta]}{\partial \eta}\right]_{\eta=0}$, and

$$\begin{aligned} h(s) &= \frac{\bar{m}\chi}{\gamma_1}\left[\frac{\partial \tilde{v}}{\partial \eta}\right]_{\eta=0} \bar{m}E^{\pi}\left[\hat{v}\left(s'\right)|s\right] \\ &+ (1+\bar{x})\chi\bar{m}E^{\pi}\left[h\left(s'\right)|s\right] \\ &+ \chi\Delta \bar{m}(s)\left[\frac{\partial \tilde{v}}{\partial \eta}\right]_{\eta=0}\left(1+\bar{x}+\frac{\bar{m}}{\gamma_1}\bar{v}\right) \end{aligned}$$

$$v = \bar{a} - \Gamma(x) + (1+x)\bar{m}v$$

and from the value function equation

$$\begin{aligned} \tilde{v}(\eta) &= \bar{a} + \eta - \Gamma(\tilde{x}_\eta) + (1+\tilde{x}_\eta)\bar{m}E\left[\tilde{v}\left(\eta'\right)|\eta\right] \\ \left[\frac{\partial \tilde{v}}{\partial \eta}\right]_{\eta=0} &= \frac{1}{1-(1+\bar{x})\bar{m}\left[\frac{\partial E[\eta'|\eta]}{\partial \eta}\right]_{\eta=0}} \end{aligned}$$

So

$$\left[\frac{\partial \tilde{x}}{\partial \eta}\right] = \frac{1}{\gamma_1}\frac{\bar{m}\chi}{1-(1+\bar{x})\bar{m}\chi}$$

So

$$\begin{aligned} h(s) &= \frac{1}{\gamma_1}\frac{\bar{m}\chi}{1-(1+\bar{x})\bar{m}\chi}\left(\Delta \bar{m}(s)\bar{v} + \bar{m}E^{\pi}\left[\hat{v}\left(s'\right)|s\right]\right) \\ &+ (1+\bar{x})\chi\left(\Delta \bar{m}(s)\left[\frac{\partial \tilde{v}}{\partial \eta}\right]_{\eta=0} + \bar{m}E^{\pi}\left[h\left(s'\right)|s\right]\right) \end{aligned}$$

Table 1a: Summary Statistics

Variables in Levels. 1953-2004, 52 annual observations						
	Data Source	Mean	St. Dev.	Min	Max	Auto-correlation
I / K	NIPA	0.165	0.017	0.134	0.200	0.758
$(1+r^{10}) / (1+y^{Baa})$	FRED	0.988	0.005	0.978	0.996	0.766
$(1+r^{10}) / (1+y^{Baa})$ post 1980	FRED	0.985	0.003	0.978	0.989	0.598
Mean (Current Liabilities / Book Assets) of Baa firms	COMPUSTAT	0.199	0.021	0.156	0.263	0.582
EBIT / K	NIPA	0.285	0.049	0.188	0.405	0.869
Total Equity Value / K	CRSP & NIPA	1.478	0.533	0.766	3.048	0.843
Variables in First Difference, 51 annual observations						
I / K		0.000	0.012	-0.026	0.021	0.2472
$(1+r^{10}) / (1+y^{baa})$		0.000	0.003	-0.006	0.005	0.0146
Mean (Current Liabilities / Book Assets) of BBB firms		-0.001	0.019	-0.047	0.037	0.0496
EBIT / K		-0.002	0.022	-0.047	0.060	0.0162
Equity / K		0.025	0.272	-0.575	0.449	0.1054

Table 1b: Descriptive Statistics on Bond Default Rates

	1970-2001							1920-1999					
	Mean Default Rates (%, annualized from cumulative)				Std. Dev. of Default Rate			Mean Default Rates				Std Dev	
Years	1	5	10	20	1	5	10	1	5	10	20	1	10
Aaa	0.00	0.03	0.08	0.10	0.00			0.00	0.04	0.11	0.12	0.00	0.15
Aa	0.02	0.06	0.09	0.15				0.08	0.19	0.31	0.35	0.20	0.34
A	0.02	0.10	0.16	0.28				0.08	0.28	0.37	0.39	0.30	0.55
Baa	0.15	0.39	0.52	0.66	0.28	0.29	0.25	0.30	0.71	0.82	0.75	0.50	0.91
Ba	1.27	2.40	2.36	2.36				1.43	2.09	2.09	1.83	1.70	1.38
B	6.66	7.15	6.26	4.15	4.66			4.48	4.58	3.77	2.83	4.50	1.97
Caa-C	21.99	15.46	13.78	7.86									
Investment-Grade	0.06	0.18	0.25	0.36				0.16	0.40	0.50	0.48		0.58
Speculative-Grade	4.73	4.60	3.83	3.09				3.35	3.33	2.88	2.34		1.52
All Corporates	1.54	1.37	1.09	0.89	1.07			1.33	1.37	1.21	0.97		0.88

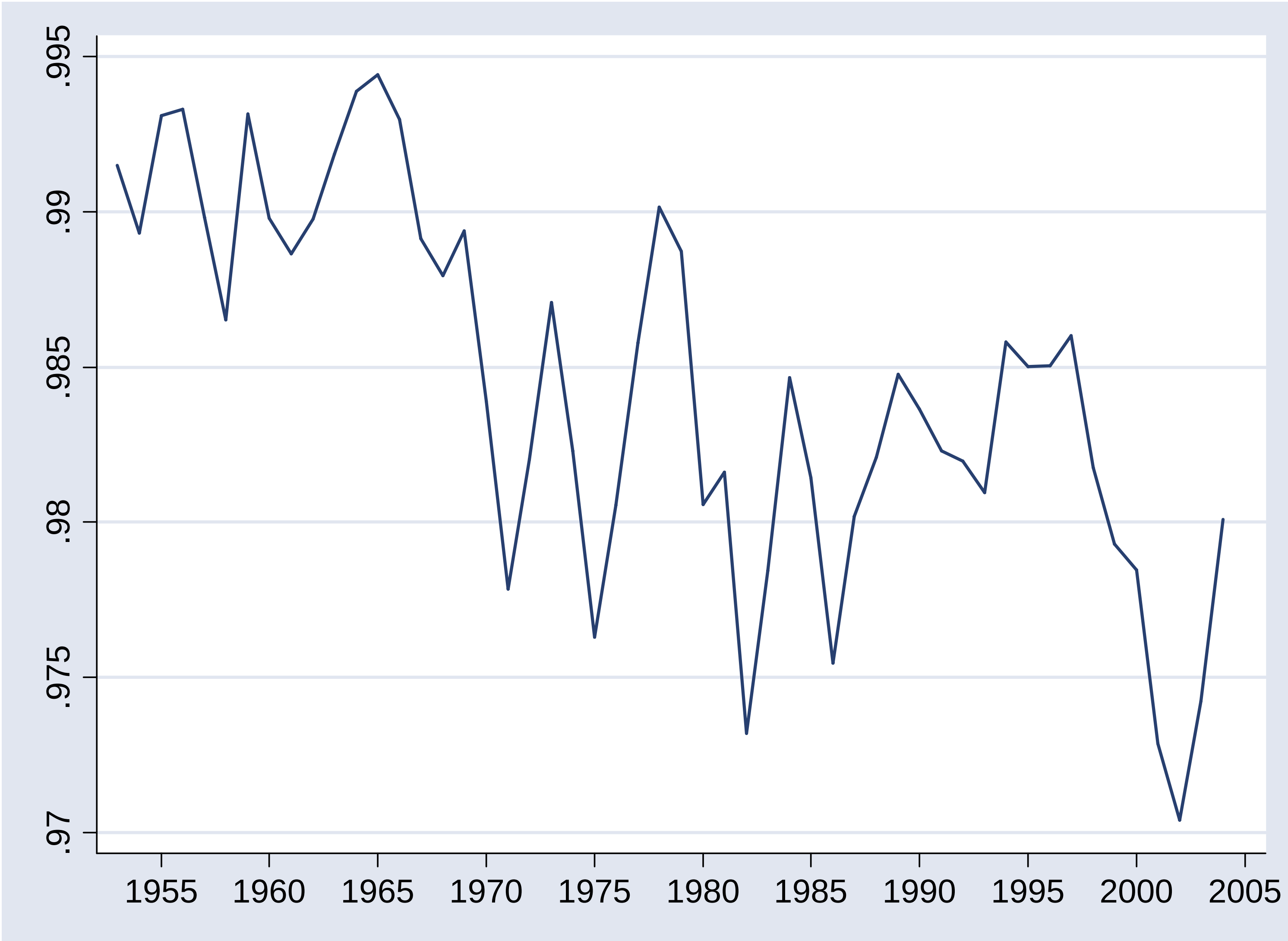
Table 2: Investment Equations

The dependent variable is the change in the investment capital ratio.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vi)
Change in $(1+r^{10}) / (1+y^{Baa})$	1.893	1.772	1.363				1.909
	6.03	*5.91*	*3.62*				*6.82*
Change in $1 / (1+r^{10})$		-0.012					
		-0.08					
Change in Current Liabilities / Assets		0.203	0.209				0.165
		3.15	*3.45*				*2.72*
Change in EBIT over Capital			0.113				
			1.66				
Change in Equity over Capital			0.001				
			0.15				
Change in Mean Asset Volatility							0.192
							2.41
$(1+r^{10}) / (1+y^{Baa})$				2.296	2.274	1.845	
				7.86	*7.59*	*4.8*	
Lagged $(1+r^{10}) / (1+y^{Baa})$				-1.523	-1.55	-1.048	
				-5.26	*-5.6*	*-2.68*	
Current Liabilities / Assets					0.193	0.167	
					3.15	*2.58*	
Lagged Current Liabilities / Assets					-0.131	-0.149	
					-2.04	*-2.32*	
EBIT over Capital						0.107	
						1.73	
Lagged EBIT over Capital						-0.126	
						-1.89	
Equity over Capital						0.001	
						0.16	
Lagged Equity over Capital						0.002	
						0.53	
Constant	0.001	0.001	0.001	-0.761	-0.725	-0.787	0.001
	0.46	*0.66*	*0.77*	*-4.02*	*-3.83*	*-2.65*	*0.67*
N	51	51	51	51	51	51	51
R^2	0.426	0.534	0.563	0.571	0.647	0.687	0.585
Adj. R^2	0.414	0.504	0.525	0.553	0.616	0.627	0.559

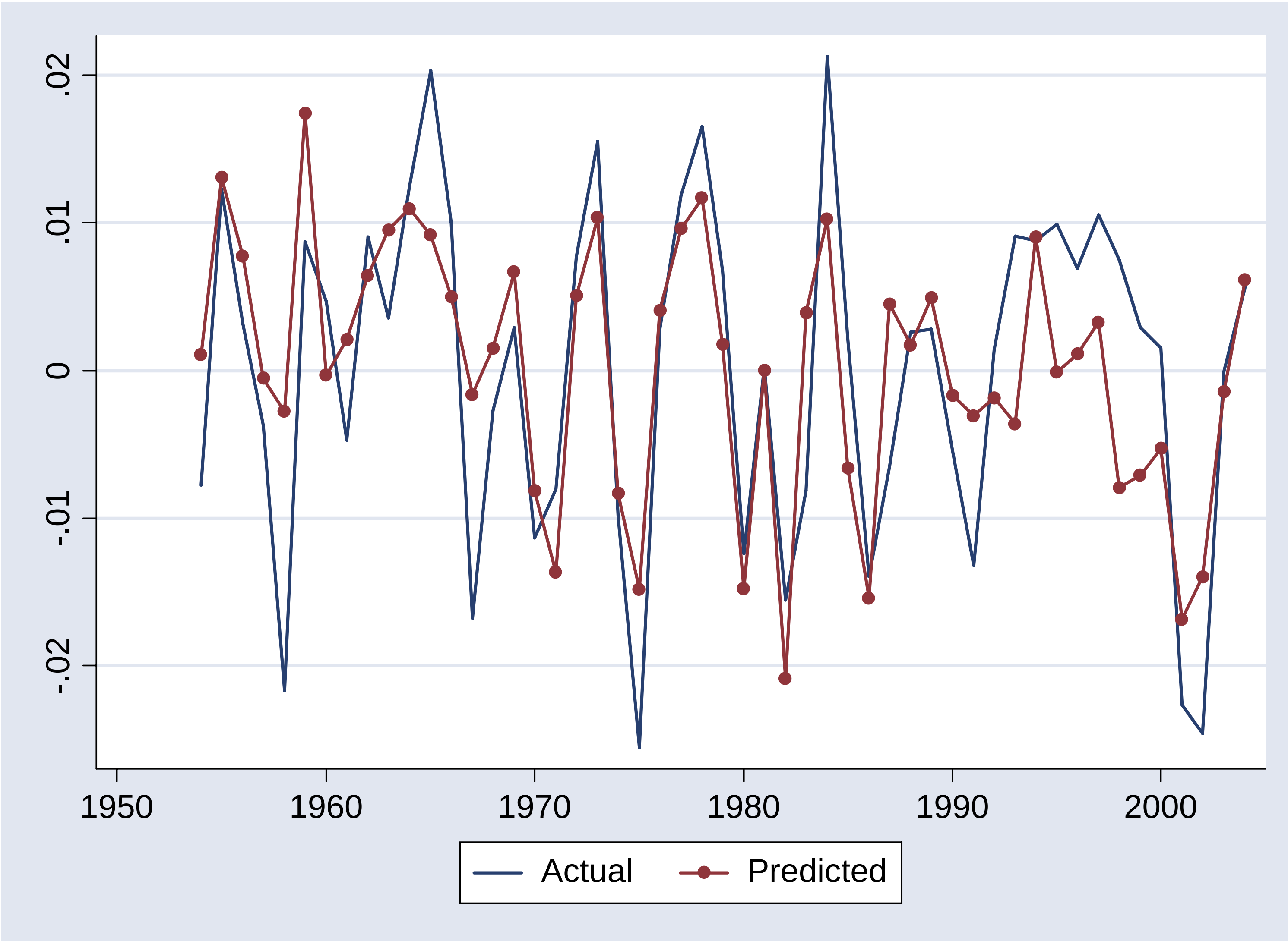
Notes: Annual Data from 1953 to 2004. The capital stock is constructed assuming an annual depreciation rate of 12%. T-statistics are below the coefficients.

Figure 1: Price of Baa Bonds Relative to Treasuries



Note: The figure shows $(1+r^{10}) / (1+y^{Baa})$. The 10-year treasury is from FRED, the yield on Baa bonds is from Moody's.

Figure 2: Annual Change in Investment-Capital Ratio



Note: the predicted value is from the regression reported in column (iv) of Table 2.