Investor Uncertainty and Order Flow Information

Gideon Saar[1]

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[1]Stern School of Business, New York University, 44 West Fourth Street Suite 9-93, New York, NY 10012-1126. Tel: 212-998-0318. Fax: 212-995-4233. E-mail: gsaar@stern.nyu.edu. I wish to thank Anat Admati, Yakov Amihud, David Brown, Henry Cao, David Easley, Michael Fishman, Joel Hasbrouck, Soeren Hvidkjaer, Robert Jarrow, Charles Lee, Richard Lyons, Roni Michaely, Maureen O'Hara, Mark Ready, Bhaskaran Swaminathan, Ingrid Werner, Lei Yu and seminar participants at Cornell University, Dartmouth College, New York University, University of California at Berkeley, University of Texas at Austin, and the University of Wisconsin at Madison for helpful comments.

Abstract

This paper proposes an alternative explanation for the price impact of trades created by information that is carried in the order flow. Unlike models that consider information asymmetry about the future cash flows (or liquidation value) of the asset, the approach here postulates uncertainty about the distribution of preferences and endowments of investors. This "investor uncertainty" results in prices moving on trades and therefore creates a spread between the bid and the ask. Greater investor uncertainty increases the spread, decreases expected trading volume, and lowers the welfare of all investors in the market. Hence, all investors are better off if market makers are expert in assessing the distribution of preferences and endowments of the investor population. The information content of the order flow is further investigated by applying an econometric spread decomposition procedure to data generated by simulating the model. The results indicate that a significant adverse selection component of the spread can arise solely due to the informational effects of investor uncertainty.

Investor Uncertainty and Order Flow Information

Why do trades move prices? A leading explanation in the market microstructure literature involves information asymmetry among investors about the future cash flows of assets. When some investors have private information about an asset and can potentially trade on it to make a profit, others attempt to infer the private information from the order flow and prices adjust to reflect the information. This inference problem has been analyzed in numerous papers, mostly following general modeling frameworks developed by Glosten and Milgrom (1985) and Kyle (1985). In these models, risk neutral and competitive market makers receive orders from informed investors (who are endowed with information about the liquidation payoffs of an asset) and uninformed investors. These models characterize the manner in which information about future cash flows is incorporated into prices, and in particular establish information asymmetry as a cause for the price impact of trades that creates the spread between the bid and ask prices.

This paper advances an alternative explanation for the price impact of trades: uncertainty about the preferences and endowments of investors in the market (henceforth "investor uncertainty"). Conceptually, an asset's price is determined jointly by the future cash flows associated with the asset and the preferences and endowments of the investors who demand the asset. Differential information about either future cash flows or the preferences and endowments of the investors can and should affect that price. The assumption of uncertainty in the market about the distribution of investors' preferences and endowments seems rather intuitive since these attributes of investors are inherently unobservable. In addition, different investors arrive to financial markets at different time, further complicating the task of learning about the overall distribution of preferences and endowments of the investor population. This uncertainty about the investor population creates a problem with respect to pricing the asset. Order flow communicates the trading desires of investors and can be used to extract information about the preferences and endowments of investors and hence about the value of the asset.

Investor uncertainty can be viewed in terms of differential information by examining the information sets of investors. Even if at any point in time past order flow and the fundamentals of the asset are known to all, each investor has a piece of information only he knows—his own preferences and endowments, and hence his optimal demand for the asset. This creates a situation in which there are different information sets for different investors and prices must adjust as investors arrive in the market and reveal their demand.[1] The objective of this paper is to examine how uncertainty about the preferences and endowments of investors introduces information content into the order flow and how it affects prices, volume, and the welfare of investors.

In principle, uncertainty about investors in the market can reflect uncertainty about many different attributes such as endowments, preferences, information sets, and private investment opportunities. The term "investor uncertainty" is defined more narrowly in this paper to describe uncertainty about the distribution of preferences and endowments of investors to differentiate it from the extant literature. While other papers examine how different sources of uncertainty affect prices in the market, the driving force behind all these models is an information imperfection about the future cash flows of the asset. For example, Easley and O'Hara (1992) add to the basic structure of information asymmetry also "event uncertainty," whereby investors do not know if an information event about the firm has occured. Similarly, Avery and Zemsky (1998) add uncertainty about the proportions of traders who receive signals of different precisions about the future cash flows of the firm. What sets this paper apart is that all investors (and market makers) have the same information about the firm. The uncertainty that generates the information imperfection has nothing to do with the firm but rather only with attributes of investors.

I develop a simple sequential trade model with two types of investors who differ with respect to risk aversion and endowments. Their demand for the risky stock depends on

[1] Yet another way to view investor uncertainty is to think about it as information about future order flow. The arrival of an investor can be used to make an inference about the entire population of investors like the inference from a sample about a population. Hence, the arrival of an order provides information about the nature of future order flow and therefore causes a change in the price of the asset.

their preferences, endowments, and the prevailing price when they arrive to trade. While traditional information-asymmetry-driven sequential trade models featured market makers with preferences over the liquidation payoff of the stock at the end of the economy, prices here are set by market makers who care about supply and demand during the trading period.[2] As Mayer (1988) notes, "In general, NYSE specialists do not take a view of where a stock is going over time. They are in business not to maximize the value of their inventory but to maximize the turnover of their capital" (p. 211). Market makers are therefore assumed to search for the price that approximately equates the flow of shares bought and sold by investors, and behave differently from investors who hold the stock to benefit from its future prospects.

I formalize this assumption by having the market makers maximize expected profit per unit time subject to a constraint that their inventory has no drift. This is similar to the specification in Garman (1976) and Brock and Kleidon (1992). With uncertainty about the preferences and endowments of the investor population, the arrival of orders changes the market makers' information set used for pricing the stock. For example, an investor who submits a buy order reveals that he is less risk averse or has a smaller endowment (or both) than an investor who submits a sell order. Market makers then update their beliefs about the investor population and raise prices to reflect the information that there may be more investors who are less risk averse or have small endowments. I adopt the rational expectations requirement of the traditional information-asymmetry-driven sequential trade models that each order is executed at a price that reflects its information content. Market makers determine at the beginning of each trading period an equilibrium strategy that specifies the prices for different incoming orders. This equilibrium strategy constitutes the market makers' quote, where the bid price for executing an arriving sell order is lower than the ask price for executing an arriving buy order.

[2] I am using the term "traditional sequential trade models" to denote the models of Glosten and Milgrom (1985), Easley and O'Hara (1987, 1991, 1992), Diamond and Verrecchia (1987), and others who follow a similar framework. An exception is Leach and Madhavan (1993) where market makers solve a dynamic program maximizing total trading profits.

Greater investor uncertainty is shown to cause a higher ask and a lower bid (i.e., a larger spread). Since investors face worse prices when investor uncertainty is greater, they choose to trade fewer shares thus expected trading volume decreases. The model provides a clear welfare implication: greater investor uncertainty lowers the welfare of all investors in the market. This result highlights the importance of expertise of market makers. The better they are in assessing the nature of the investor population, the tighter is the distribution over investors' preferences and endowments they use for pricing. This allows them to set prices that make all investors better off and increase volume in the market.

The model is simulated to examine whether investor uncertainty alone can generate informational effects in prices that empirical work has traditionally attributed to information asymmetry about future cash flows. I show that the "adverse selection" component of the spread estimated using the methodology of Madhavan, Richardson, and Roomans (1997) picks up information about investors. Hence, current methodologies are unable to distinguish between information about the firm's future cash flows and information about the preferences and endowments of investors in the market.

The uncertainty about investors modeled in this paper adversely affects liquidity.[3] These results are therefore related to the literature that investigates the effect of market participation on the prices of assets.[4] In particular, Kraus and Smith (1989) stress that uncertainty about future prices can reflect the beliefs, preferences and endowments of the participants in the economy. They refer to this uncertainty as "market created risk" to emphasize that its source is the investors themselves rather than the future cash flows of a firm.[5] The approach taken here differs from theirs along several dimensions, among which are the sequential arrival of investors and, most importantly, the recognition that information about the investor

[3]The implication that larger demand variability increases trading costs is also discussed in Spiegel and Subrahmanyam (1995). They use simulations to investigate the effects of exogenous supply shocks on the intraday risk premium.

[4]See Merton (1987), Pagano (1989), Allen and Gale (1994), Orosel (1997), and Shapiro (2001).

[5]There is also a literature that considers the effects of random preferences and endowments in Walrasian exchange economies. See Hildenbrand (1971), Bhattacharya and Majumdar (1973), and Mendelson (1985).

population is carried by the order flow and is affecting prices in the market.[6]

A few recent papers emphasize the existence of information in financial markets other than information about future cash flows. Madrigal (1996) studies a market in which insiders coexist with traders who do not have information on fundamental values, but who possess superior knowledge of the market trading process or environment. Such knowledge allows them to estimate fundamental information from public data more accurately than the market at large. His model differs from the approach taken here in that the source of information in the model is still the future cash flows of the firm.[7]

Lyons (1997) and Cao and Lyons (1999) generate private information in the foreign exchange multiple-dealer setting. Each dealer has sole knowledge of his customers' orders, and this inventory information gives rise to speculative trading and thus affects prices in the market. Their models differ from the approach taken here in that they describe a simultaneous trading game of dealers in contrast to the sequential trading of investors used in this paper. The price effects of inventory information in their settings are temporary, while here the price effects of investor uncertainty are or can be permanent. In addition, since customers are not optimizing in their models, prices do not reflect the preferences of investors, just the risk bearing capacity of dealers. While the exposition here is done in terms of an equity market, the idea of investor uncertainty applies to other financial assets as well. In particular, the implications of the model can be used to explain informational effects in the prices of assets like closed-end funds, bonds, futures on indexes, and foreign exchange where postulating private information about future cash flows is less plausible than in the equity market.

[6] The investor uncertainty model also relates to papers that attempt to explain, within the framework of rationality, how prices seem to change substantially without significant external news (e.g., Romer, 1993; Coval and Hirshleifer, 1998). While this literature focuses on transaction costs and uncertainty about the precision of signals, the model here can generate similar implications using uncertainty about preferences and endowments.

[7] In Brown and Zhang (1997), speculators trade on future cash flows information but dealers are able to aggregate information from different speculators and hence can be viewed as trading on order flow information. This informational advantage is profitable and can be used to construct an equilibrium level of dealer services.

The rest of the paper is organized as follows. Section 1 describes the economy and establishes the existence of an equilibrium. Section 2 investigates the implications of uncertainty about the preferences and endowments of investors vis-a-vis prices, spreads, expected volume, and the welfare of investors. Section 3 shows how econometric estimates of the adverse selection component of the spread can arise solely from the informational effects of investor uncertainty. Section 4 concludes with a discussion of the approach pursued in the paper, limitations of the model, and possible extensions.

1 Economy

1.1 Assets

There are two assets in the economy: a risky asset (a stock) that pays $\tilde{u}$ dollars at time T', where $\tilde{u}$ is normally distributed with mean θ and variance σ^2, and a riskless bond that pays R dollars at time T'. Trading in the stock takes place in discrete intervals of time denoted $t = 1, 2, \cdots, T$, where trading ends before the liquidating dividend of the stock is realized ($T < T'$). As in traditional sequential trade models, each interval is long enough to accommodate at most one trade (see Easley and O'Hara, 1992).

1.2 Investors

There is a continuum of investors in the economy with unit mass. All investors maximize Constant Absolute Risk Aversion (CARA) expected utility of their wealth at time T' (when the liquidating dividends of the assets are realized). There are two types of investors in the population indexed by $i \in \{1, 2\}$ who differ with respect to their endowments ($\bar{X}_i$ of the risky asset and $\bar{M}_i$ of the riskless bond) and their coefficient of absolute risk aversion, α_i. The relative population weight of type 1 is q and the relative population weight of type 2 is $1-q$.

As in the traditional sequential trade models, each period an investor is randomly selected to trade from among the pool of investors. The probability that an investor who arrives to trade belongs to a certain type is equal to that type's relative population weight. In other words, the probability that an investor who arrives to trade belongs to type 1 is q and the probability that he belongs to type 2 is $1-q$.[8] Since investors are being selected from a continuum, the probability that any individual will be selected twice is zero, and therefore an investor who arrives to the market trades to optimally rebalance his portfolio believing that he will not be able to return to the market to trade again. Investors behave competitively in that they take market prices as given and decide on the fraction of their wealth to be invested in the stock and the fraction to be invested in the riskless bond.

Under these assumptions, a type i investor who arrives to the market in period t solves the following problem:

$$\max_{D_{i,t}} \quad E\left[-e^{\alpha_i W_{i,T'}}\right] \tag{1}$$

$$s.t. \quad RM_{t,i} + \tilde{u} D_{i,t} = W_{i,T'} \tag{2}$$

$$M_{t,i} + P_t D_{i,t} = \bar{M}_i + P_t \bar{X}_i \tag{3}$$

where $D_{i,t}$ is the investor's demand for the stock, P_t is the price at which the investor can transact in the stock, and the price of the riskless bond is set to unity. The solution to this problem is well known and the optimal demand is:[9]

$$D^*_{i,t} = \frac{\theta - RP_t}{\alpha_i \sigma^2} \tag{4}$$

When an investor arrives to the market, he submits an order,

$$X_{i,t} = D^*_{i,t} - \bar{X}_i \tag{5}$$

where $X_{i,t} > 0$ ($X_{i,t} < 0$) is interpreted as a buy (sell) order.

[8] Nothing changes in the results if the model is extended to allow periods without trading. I can assume that each period an investor arrives to the market with probability δ. If an investor arrives, his type is chosen according to the relative weights of type 1 and type 2. As it will be shown later, prices are only adjusting when an investor arrives and his order reveals his type (in the absent of public information arrival). Unlike in Easley and O'Hara (1992), prices here do not change in periods without trading.

[9] See, for example, Grossman and Stiglitz (1980).

The focus of this paper is on the effects of uncertainty about the distribution of investors' preferences and endowments. This investor uncertainty is modeled by assuming that no one knows the value of the relative population weight q.

1.3 Market Makers

Like in traditional sequential trade models, trading in the market is facilitated by competitive and risk neutral market makers. However, the traditional models defined market makers' preferences over their wealth at the end of the economy, when the liquidating dividends of the assets are realized. This forced the equilibrium price of the stock to be the conditional expected value of the liquidating dividend. In other words, the preferences and endowments of investors did not matter for pricing. In contrast, investor uncertainty affects prices in this paper since market makers care about supply and demand of shares by investors.

The notion of prices that are determined by equating the flow of shares demanded and supplied rather than by forecasting the future cash flows of a firm seems to correspond rather well to the activity of market makers. Bagehot (1971) writes that "it is well known that market makers of all kinds make surprisingly little use of fundamental information. Instead, they observe the relative pressure of buy and sell orders and attempt to find a price that equilibrate these pressures" (p. 14). Mayer (1988) also notes that market makers are not interested in taking a position in the stock based on long-term forecasts. Rather, they are constantly searching for the prices at which the flows of shares bought and sold are approximately equal. Their constant search for a market clearing price keeps them in business since their inventories do not drift without bound. This behavior sets them apart from investors who trade infrequently to re-balance a portfolio of investments they hold for prolonged periods of time.

I adopt a specification of a market maker's objective similar to the one used by Garman (1976) and Brock and Kleidon (1992). Each market maker in the economy maximizes expected profit per unit time subject to the constraint that the expected number of shares bought and sold per unit time is equal to zero.[10] In other words, market makers set prices every trading period by maximizing the expected revenue from selling shares to investors minus the expected cost of buying shares from investors subject to the constraint that the number of shares bought and sold is the same on average.

Here is where investor uncertainty enters since it creates a problem with respect to assessing the demand and supply of shares by investors. If no one in the economy actually knows the distribution of the investor population, the prices set by market makers cannot reflect that information. While the market makers' information set does not include q, I assume that they have a prior on q at time zero denoted by $f^0(q)$. The prior distribution can be rather general but its support should be in $[0,1]$. The prior can be interpreted as the experience market makers develop by regularly observing the investor clientele who trades the stock. Each time an investor arrives and submits an order, market makers use Bayes rule to update their beliefs about the distribution of investors' preferences and endowments.

The rules of the trading game are similar to those of the traditional sequential trade models. At the beginning of each period, market makers are required to post binding prices and depths at which investors can trade. Then, an investor arrives, optimizes taking the market makers' prices as given, and submits his order. The order is executed by the market makers at the quoted price. Trading is anonymous in the sense that the only information market makers have about an arriving investor is the order the investor submits. Since orders convey information but the market makers have to commit to prices before they observe the orders, the equilibrium pricing strategy of the market makers should reflect their rational expectations about the incoming orders. Market makers post "regret-free" prices in that the information inferred from different order sizes is used to calculate the prices at which the orders will be executed (so that the depth that accompanies a price is equal to the order size) before the orders arrive at the market. Since this situation is similar to the one described by traditional sequential trade models, it will be instructive to compare the manner in which

[10] Amihud and Mendelson (1980) use a similar specification but impose bounds on the allowable inventory position of the market maker instead of requiring a zero drift.

equilibrium prices are determined in the two approaches.

1.4 Equilibrium Prices and Strategies

Traditional sequential trade models define the transaction price as the equilibrium price that reflects all information available to the market makers, including the information contained in the transaction itself (i.e., semi-strong price efficiency when the public's information set is the same as the one used by market makers). In these models, the price that market makers set to execute a transaction for a single quantity of shares, Q, is just the conditional expectation of the liquidation payoff when the information set includes the arrival of the order, $P_Q = \frac{E[\tilde{u}|Q]}{R}$. Hence, at any point in time there is only one equilibrium price—the price of the most recent transaction. Looking forward, market makers can determine in advance what prices they will set for different incoming orders. Since a buy order (B) contains different information than a sell order (S), the equilibrium prices conditional on these orders will be different, $P_B = \frac{E[\tilde{u}|B]}{R} \neq \frac{E[\tilde{u}|S]}{R} = P_S$. Note, however, that these two prices never exist at the same time—each is conditional on a different information set. Nonetheless, if market makers are asked at the beginning of a trading interval what will be the prices at which they will execute incoming buy or sell orders, they quote these two prices. The market makers' quote therefore describes potential equilibrium prices.

A similar situation exists in this paper. I begin by defining the Equilibrium Price conditional on a single information set, which is the equivalent of the conditional expectation of the liquidating dividend in the traditional sequential trade models. Then I define the market makers' Equilibrium Strategy, which is the equivalent of the pair of potential equilibrium prices that constitute the market makers' quote in the traditional sequential trade models.

Definition (Equilibrium Price): If there is trading in the economy, an *Equilibrium Price* conditional on the market makers' information set Φ, $P(\Phi)$, is the unique price determined by competitive market makers who maximize conditional expected profit per period subject to the constraint that the conditional expected number of shares bought and sold in the period is equal to zero.

The basic problem solved by each market maker is similar to that in Garman (1976) and Brock and Kleidon (1992):

$$\max_{\{P_1,P_2\}} \quad E\left[qP_1X_1 + (1-q)P_2X_2 \mid \Phi\right] \tag{6}$$

$$s.t. \quad E\left[qX_1 + (1-q)X_2 \mid \Phi\right] = 0 \tag{7}$$

where P_i is the price the market maker is quoting to an investor belonging to type $i \in \{1,2\}$ who arrives to the market and submits an order X_i. So, each market maker maximizes expected profit for the period subject to the constraint that the expected number of shares bought and sold is the same.

The market is populated by multiple, competitive market makers.[11] The convention used here is that each arriving order is divided equally among all market makers who quote the best price, and for simplicity of exposition I assume that there is a continuum of market makers with unit mass. This leads to a Bertrand competition where each market maker has an incentive to improve prices in order to capture order flow. Note that if a market maker quotes different prices (P_1 and P_2) for the different investor types, improving only one of the prices by a small amount will capture the order flow of one type of investors and lead to a violation of the constraint in (7). By improving both P_1 and P_2, a market maker will capture the entire order flow and make positive expected profit for a small enough price improvement. The steady state of the Bertrand competition occurs when expected profit is zero. The result that price competition among market makers leads to zero expected profit is shared by many other sequential trade models (see the discussions in Glosten and Milgrom (1985), Easley and O'Hara (1987), Glosten (1989), and Madhavan (1992)).[12]

[11]Similar implications about the effects of investor uncertainty on the information content of prices can also be derived in the model with a monopolistic market maker. The details are available from the author upon request.

[12]The assumptions of risk neutrality and no capital constraints in these papers lead immediately to the result that expected profits are zero. Dennert (1993) shows under a different set of assumptions (notably that informed investors trade with all market makers while uninformed investors trade with only one market maker) that multiple market makers can quote a larger, rather than a smaller spread.

If I do not impose the constraint in (7), there is an infinite number of solutions to the market makers' problem when expected profit is equal to zero. The assumption I make in the spirit of the aforementioned quotes from Bagehot (1971) and Mayer (1988) is that market makers' inventories cannot have either a positive or a negative drift for the entire duration of the economy. All solutions of the unconstrained problem except for two result in a permanent drift. In principle, however, prices in a secondary market must adjust over sufficiently long horizons such that the number of shares bought and sold by investors is the same (without the firm issuing additional shares). Market makers are needed since investors arrive one at a time, but what market makers do is basically to balance the expected flow of shares bought and sold. Hence, the solutions that result in a drift in one direction for the entire duration of the economy seem unrealistic.

Two solutions to the unconstrained problem result in expected excess demand of zero: one in which market makers set two different prices but all investors submit orders for zero shares, and another where the constraint in (7) is satisfied and in which market makers set a single price and investors trade for risk sharing. The first solution is not interesting since there is no trading in the economy, and so this paper focuses on the second solution. Note that in any given period only one order arrives, and therefore the realization of shares bought or sold will not be zero. The constraint in (7) is best viewed as implementing market clearing in expectations, and is a parsimonious way for formalizing the assumption I make that the market makers' inventory should not have a drift.[13] While this form of inventory management may seem a bit restrictive, it is very simple and very useful for the task at hand.[14]

[13]The issue of inventory drift is further discussed in Section 2.

[14]My goal here is not to examine how market makers manage their inventories. In Stoll (1978) and Ho and Stoll (1981), inventory control is driven by risk aversion of a market maker who seeks to maintain an optimal portfolio position. In Amihud and Mendelson (1980), the market maker must not let the level of inventory get above or below certain bounds. Here, I abstract from specific characteristics of market makers such as their degree of risk aversion or wealth constraints since my interest is not in evaluating how these characteristics affect pricing. Rather, I want to investigate the influence of uncertainty about the distribution of preferences and endowments of the investor population on prices in a sequential trade model. Therefore, the market maker's problem satisfies the simple requirement that on average, inventory will not have a drift and so supply and demand of shares in the market will be approximately the same. While this setting

To solve for the Equilibrium Price, we can use the constraint in (7) to write X_1 in terms of X_2, and then plug it into the objective function. Then, imposing zero expected profit results in the following equation:

$$(1 - E[q \mid \Phi])X_2(P_2 - P_1) = 0 \tag{8}$$

Hence, in every economy with trading (i.e., $X_2 \neq 0$ and $X_1 \neq 0$), there is a single equilibrium price $P(\Phi) = P_1 = P_2$. We can plug the optimal demands of the two types of investors from (4) and (5) into (7),

$$E[q \mid \Phi]\left(\frac{\theta - RP(\Phi)}{\alpha_1\sigma^2} - \bar{X}_1\right) + (1 - E[q \mid \Phi])\left(\frac{\theta - RP(\Phi)}{\alpha_2\sigma^2} - \bar{X}_2\right) = 0 \tag{9}$$

and solve for the equilibrium price:

$$P(\Phi) = \frac{\theta}{R} - \frac{X(\Phi)\alpha_1\alpha_2\sigma^2}{R\alpha(\Phi)} \tag{10}$$

where $X(\Phi) = q_\Phi\bar{X}_1 + (1 - q_\Phi)\bar{X}_2$, $\alpha(\Phi) = q_\Phi\alpha_2 + (1 - q_\Phi)\alpha_1$, and $q_\Phi = E[q \mid \Phi]$.

The equilibrium price set by the market makers has the usual structure from asset pricing models: a risk neutral component, the mean of the asset's payoffs divided by the risk free rate, and a risk premium that depends on the relative population weight q. Note, however, that since market makers do not know q, the aggregate demand and the harmonic mean of the risk aversion coefficients are calculated using the conditional expectation of q rather than the parameter itself. This price also preserves an important notion of optimality in that if market makers have full information (they know q), it is equal to the competitive equilibrium price in the economy. Hence, market makers are just a conduit through which shares are transferred from sellers to buyers and the market price depends solely on the characteristics of the investor population. Such a specification is therefore closer in spirit to the usual formulation in the asset pricing literature whereby prices are determined using a market clearing argument.

abstracts from many constraints that market makers have in the real world, it provides a powerful tool for investigating investor uncertainty.

With the Equilibrium Price defined, I turn to the Equilibrium Strategy of market makers that represents the quote to which they commit at the beginning of each trading period (in a manner similar to the definition of quotes in traditional sequential trade models). Let $\mathcal{M}_{t,1}$ be the information set that includes the information in the next arrival of an order for $X_{1,t}$ shares in addition to all public information and past order flow at the beginning of period t. Since market makers post their binding prices and depths schedule at the beginning of a trading period before an investor arrives, they would like to post an Equilibrium Price to execute an arriving order for $X_{1,t}$ shares that reflects the information that can be inferred from such an order. In other words, calculating an execution price for this order conditioning on $\mathcal{M}_{t,1}$ would result in a "regret-free" price. Similarly, let $\mathcal{M}_{t,2}$ be the information set that includes all past information at the beginning of period t and the information in the next arrival of an order for $X_{2,t}$ shares.

Definition (Equilibrium Strategy): The market makers' *Equilibrium Strategy* is a pair of prices and depths $P_t \equiv \{P(\mathcal{M}_{t,k}), X_{k,t}\}_{k\in\{1,2\}}$ such that P($\mathcal{M}_{t,k}$) is an Equilibrium Price conditional on the information set $\mathcal{M}_{t,k}$ that includes all information up to time t and the next arrival of an order for $X_{k,t}$ shares.

The Equilibrium Strategy is therefore a schedule of prices and quantities or a "quote" that the market makers post publicly before an investor arrives in period t. It specifies at which prices the market makers will execute trades for different quantities of shares. Since there are two types of investors in the economy, it is natural to restrict attention to the order sizes that the two types of investors will find optimal.

1.5 Equilibrium

Definition (Fully-Revealing Equilibrium): A *Fully-Revealing Equilibrium* in the market is when:

1. At the beginning of each period, market makers commit to an Equilibrium Strategy

assuming that they can identify an arriving investor's type from his order.[15]

2. An optimizing investor belonging to type i who arrives in the market at period t chooses to trade using the pair $(P(\mathcal{M}_{t,i}), X_{i,t})$, where $X_{i,t}$ is his optimal order size, and $X_{1,t} \neq X_{2,t}$.

To simplify the exposition, assume that the parameters of the economy are such that $\Delta\alpha\bar{X} = \alpha_1\bar{X}_1 - \alpha_2\bar{X}_2 > 0$. This will result in an equilibrium where type 1 investors are sellers and type 2 investors are buyers (since type 1 investors are more risk averse and/or have a larger endowment of the stock then type 2 investors). The case of $\Delta\alpha\bar{X} < 0$ is completely symmetrical, and in equilibrium type 1 are buyers and type 2 are sellers.[16] This assumption is without loss of generality since we could always rename the two types of investors.[17] The following proposition establishes the existence of a fully-revealing equilibrium:

Proposition 1 *There exists a fully-revealing equilibrium where (i) the market makers' Equilibrium Strategy is:*

$$P_t = \begin{cases} \frac{\theta}{R} - \frac{\alpha_1\alpha_2\sigma^2\bar{X}(M_{t,2})}{R} & \text{for all orders } X = X_{2,t} \\ \infty & \text{for all orders } X \neq X_{2,t}, X > 0 \\ \frac{\theta}{R} - \frac{\alpha_1\alpha_2\sigma^2\bar{X}(M_{t,1})}{R} & \text{for all orders } X = X_{1,t} \\ 0 & \text{for all orders } X \neq X_{1,t}, X < 0 \end{cases} \tag{11}$$

(ii) An arriving type 1 investor submits the order:

$$X_{1,t} = -\frac{(1 - q_{1,t})\Delta\alpha\bar{X}}{\alpha(M_{t,1})} \tag{12}$$

and an arriving type 2 investor submits the order:

$$X_{2,t} = \frac{q_{2,t}\Delta\alpha\bar{X}}{\alpha(M_{t,2})} \tag{13}$$

[15] This requirement, that market makers can extract information only from the order size, makes the market makers' problem more realistic. Modifying the problem by giving market makers additional information about the identity of arriving investors does not materially affect the analysis.

[16] When $\Delta\alpha\bar{X} = 0$, investors choose not to trade for risk sharing. This is also the case in which there will be no risk sharing among investors in an equivalent market cleared by a Walrasian auctioneer. This case is less interesting and will not be pursued further in the paper.

[17] The proofs of all propositions in Section 2 (that establish the implications of the investor uncertainty model) are done for the general case of $\Delta\alpha\bar{X} \neq 0$.

where $q_{1,t} = E[q \mid \mathcal{M}_{t,1}]$, $q_{2,t} = E[q \mid \mathcal{M}_{t,2}]$, $\bar{X}(M_{t,1}) = q_{1,t}\bar{X}_1 + (1-q_{1,t})\bar{X}_2$, $\bar{X}(M_{t,2}) = q_{2,t}\bar{X}_1 + (1-q_{2,t})\bar{X}_2$, $\alpha(M_{t,1}) = q_{1,t}\alpha_2 + (1-q_{1,t})\alpha_1$, *and* $\alpha(M_{t,2}) = q_{2,t}\alpha_2 + (1-q_{2,t})\alpha_1$.

All proofs are provided in the appendix. The proof of this proposition follows the standard structure. I start by assuming that market makers can identify the type (preferences and endowments) of an investor who arrives in the market from his optimal order. They use that information to calculate the Equilibrium Strategy that specifies the prices at which investors can transact. Then, the investors' participation and incentive compatibility conditions are analyzed to show that an investor will in fact self-select to trade using the price and depth that the market makers had set for investors of his type.

Proposition 1 shows that, like in traditional sequential trade models, market makers post two prices: one at which investors can buy the stock (an "ask") and another at which investors can sell the stock (a "bid"). These prices depend on the information set of the market makers and are "regret-free". This is achieved by letting the market makers condition on the order when they commit to the quote before the order arrives to the market, just like in the traditional framework.

This equilibrium, however, differs from the one in traditional sequential trade models with respect to two important attributes. First, the quantities investors trade depend on prices and therefore change as the market makers update their prices. This means that the welfare of investors is affected by the existence of investor uncertainty in the market and by the expertise of market makers that is reflected in their prior beliefs about q. The next section will develop specific results that demonstrate the effects of investor uncertainty on both market statistics such as prices and volume and on the welfare of investors. Second, the equilibrium in the traditional framework is a partially revealing equilibrium. Market makers do not learn all information that is known to the informed investors from the arrival of an order. The existence of noise traders in these models enables the slow adjustment of prices to private information about the liquidating dividends. In contrast, the equilibrium here is fully-revealing: the order size chosen by an investor reveals to the market makers all the private information that the investor possesses—his own preferences and endowments. However, this information is not sufficient to resolve all uncertainty in the economy since no one knows the true value of the population parameter q. The orders of investors provide market makers with pieces of information that they can use to update their beliefs about the distribution of preferences and endowments in the population.

2 Implications of the Investor Uncertainty Model

In this section, I show how investor uncertainty affects prices, volume, and the welfare of investors. Before turning to the results, it will be useful to establish how market makers update their beliefs about the distribution of preferences and endowments in the population as investors arrive to the market and trade. More specifically, the next proposition shows that the arrival of an investor belonging to type i causes market makers to believe that there are more investors of type i in the market.

Proposition 2

$$q_{1,t} \equiv E\left[q \mid \mathcal{M}_{t,1}\right] = q_t + \frac{V_t[q]}{q_t} > q_t \tag{14}$$

$$q_{2,t} \equiv E\left[q \mid \mathcal{M}_{t,2}\right] = q_t - \frac{V_t[q]}{1-q_t} < q_t \tag{15}$$

where $q_t = E\left[q \mid \mathcal{M}_t\right]$, $V_t[q] = E\left[q^2 \mid \mathcal{M}_t\right] - q_t^2$, *and* $\mathcal{M}_t$ *is the information set of market makers at the beginning of period* t*, before the arrival of any investor.*

This proposition holds for general priors (continuous or discrete) that the market makers may have about the relative population weight q. Proposition 2 states that the arrival of a type 1 (type 2) investor increases the market makers' expectation of the relative population weight of type 1 (type 2). The intuition behind this result is that of learning from a sample about a population. Each time an investor of type i arrives, his type is considered a random draw from the true distribution. Bayes Rule then dictates that the market makers update their beliefs about the population of investors giving more weight to type i. The learning is not about the type of the arriving investor, which is a zero-one event that is fully known to the market makers in equilibrium when the investor submits his order. Rather, the learning is about the true distribution of investors in the population.

Proposition 3 *There exists a positive spread between the bid and ask prices:*

$$S_t = \frac{\alpha_1 \alpha_2 \sigma^2 \left| \Delta \alpha \bar{X} \right| V_t[q]}{R \alpha(M_{t,1}) \alpha(M_{t,2}) q_t (1 - q_t)} \tag{16}$$

In this model, like in the traditional sequential trade models, the spread is the difference between the price at which market makers are willing to buy shares and the price at which they are willing to sell shares. What is the intuition behind the spread? Uncertainty about the preferences and endowments of investors harms liquidity in the market. The reason for the illiquidity is that without knowledge of the overall demand for the asset, it is difficult to set a price that clears the market. Market makers try to learn about the population of investors from the arriving orders, and the updating of their beliefs is moving prices with each order.

When market makers observe a sell order for $X_{1,t}$ shares, they learn that a type 1 investor arrived to the market. We can think about it as if the private information of an investor is his own preferences and endowments, and the order he submits results in full revelation of his private information. Since $\alpha_1 \bar{X}_1 > \alpha_2 \bar{X}_2$, a type 1 investor is more risk averse and/or has a larger endowment of the stock than a type 2 investor. Market makers then update their beliefs about the distribution of investors so that their expectation of q increases. Hence, prices must decrease to reflect the information that there are more investors in the market with large endowments and/or higher risk aversion. The arriving type 1 order suffers from a price impact as the equilibrium price adjusts downward. Similarly, an arriving buy order will identify the investor as belonging to type 2 and will increase the market makers' expectation of the relative population weight of type 2 investors. This will cause an increase in the equilibrium price to reflect the beliefs that there are more investors who are less risk averse or who have smaller endowments. The spread is the sum of these two price impacts, much like in traditional sequential trade models where the spread is the sum of the price impacts of a buy order and a sell order (both for a single quantity of shares).[18]

Extending the model to accommodate $n > 2$ types of investors who differ with respect to their preferences and endowments is conceptually straightforward. While the proof of the equilibrium will be much more complex due to the greater number of incentive compatability conditions, the result will be a schedule of prices and associated depths at which investors can trade. A larger buy order, for example, will imply that more investors are characterized with much lower risk aversion or much smaller endowments and therefore will create a larger price impact than a smaller buy order.

Proposition 4 *Prices are decreasing functions of σ^2 (the variance of the asset's liquidation payoff). The spread and the relative spread increase with σ^2*

As is generally true in asset pricing models with risk averse investors, higher expected return is demanded from riskier assets and hence the prices of these assets are lower. The prediction that riskier stocks would have larger spreads can also be found in inventory models with risk averse market makers (Ho and Stoll (1981)) and in information-asymmetry-driven models (Copeland and Galai (1983); Easley and O'Hara (1992)).

The next five results examine how differences in the extent of investor uncertainty affect the market. While market makers may have a rather general prior on the distribution of the population parameter q, (14) and (15) show that only the first two moments of that distribution matter for pricing the stock. The extent of uncertainty about investors in the market can therefore be represented by the variance of the market makers' beliefs about the population parameter. The smaller $V_t[q]$, the tighter the distribution of types around the mean, which implies less uncertainty about the distribution of preferences and endowments

[18] While the result that prices move on trades may cause us to question the price taking assumption, there is a sense in which investors do take into account their impact on the price. The price charged of an investor is adjusted for the investor's private information—his preferences and endowments. When the investor arrives, he calculates his optimal demand using the price that already reflects his order. Hence, this rational expectations feature of the prices set by the market makers creates a situation in which price taking is consistent with a quasi-strategic behavior in which an investor's demand reflects his impact on the market's price.

in the market. The following propositions consider the effect of changing the variance of q, holding its expected value constant.

Proposition 5 *The ask price increases and the bid price decreases (i.e., the spread increases) with the variance of the market makers' beliefs about q.*

Higher uncertainty about demand in the market is costly to investors since it causes the ask to be higher and the bid to be lower. This result is driven by the learning process of the market makers. Proposition 2 shows how the larger is $V_t[q]$, the greater is the "distance" between the market makers' prior and posterior expectations of q. Greater revisions in the market makers' estimate of q cause larger price impacts and therefore the ask is higher and the bid is lower. Another implication of this result is that the volatility of transaction prices increases with investor uncertainty. Standing at the beginning of the period, the conditional variance of transaction prices in period t can be written as:

$$
\begin{aligned}
V\left[P_t^{tr} \mid \mathcal{M}_t\right] &= q_t\left[P(M_{t,1}) - E\left[P_t^{tr} \mid \mathcal{M}_t\right]\right]^2 + (1-q_t)\left[P(M_{t,2}) - E\left[P_t^{tr} \mid \mathcal{M}_t\right]\right]^2 \\
&= S_t^2 q_t(1-q_t)
\end{aligned}
$$

where P_t^{tr} is the transaction price and $E\left[P_t^{tr} \mid \mathcal{M}_t\right] = q_t P(M_{t,1}) + (1-q_t)P(M_{t,2})$. There is a sense in which part of this volatility can be viewed as a manifestation of a bid-ask bounce. However, the larger price impact of trades when investor uncertainty is higher moves the subsequent quote further up or down, and therefore volatility calculated from midquotes will also be greater.

It is also interesting to note that transaction prices need not be a martingale. While martingale prices are an implication of traditional sequential trade models (due to the fact that prices are just conditional expected values of an exogenous random variable), other market microstructure models demonstrate how market frictions can result in non-martingale prices. For example, Amihud and Mendelson (1980) show how inventory control considerations cause non-martingale prices. Leach and Madhavan (1993) present a sequential trade model where market makers implement price experimentation to try to profit from discovering the private information of traders. Prices in their model do not follow a martingale. Investor uncertainty also creates a market imperfection that results in a similar implication.[19] Hasbrouck and Ho (1987) perform empirical tests controlling for bid-ask bounce using both transaction prices and quote midpoints and find that prices do not follow a martingale.

Proposition 6 *$V_t[q]$ decreases on average when orders arrive. $V_t[q]$ approaches zero and the bid and ask prices converge to a single price as T goes to infinity.*

The decrease in investor uncertainty as orders arrive is a consequence of the Bayesian updating. A similar learning process also takes place in traditional sequential trade models that utilize information asymmetry about future cash flows to generate the spread. In these models, it is learning about the signal that informed investors observe that drives down the spread as more orders arrive. Here, the learning is about preferences and endowments of the investor population. In both cases informational effects in prices should disappear without renewed uncertainty or private information. If we continue to observe the effects of investor uncertainty in the market, it has to be the case that the trading environment is constantly changing.

For example, one way to think about the manner in which markets operate is that every day there is a different subset of investors who trade: some suddenly need money, others have found the time to go over their finances, and so on. This subset of investors determines the price path on that day as market makers try to learn the distribution of types. Of course, prices that go up or down by a large amount will attract the attention of other subsets of investors who did not plan to arrive in the market on that day. But within some bounds on the price movement, prices are determined at every point in time by this search for the preferences and endowments of a subset of the investor population. Another way in which investor uncertainty arises is when the market learns about changes in tastes and income of

[19] In the model presented here, transaction prices will still follow a martingale if all investors have the same coefficient of absolute risk aversion.

the population of investors as a whole. This hierarchy of inferences, both on a subset and on the population, creates a situation in which market makers can never stop learning from the order flow and the information effects of investor uncertainty are always present.[20]

When the bid and ask prices converge to a single price, the market makers' pricing problem ensures that their inventories, which are absorbing the excess demand of investors, would not drift. However, as long as market makers learn about the investor population, their inventory can drift in one direction or the other depending on the relation between their prior expected value of q and the true population parameter. This situation is similar to that of market makers in traditional sequential trade models. As Glosten and Milgrom (1985) note, the inventory of the specialist in their model would not drift only in the limit. As long as some investors have private information and they trade on it, inventory may drift. A positive or a negative inventory drift in the traditional models and here is due to the assumption of risk neutrality on the part of market makers and the assumption of no inventory carrying costs. Inventory models such as Ho and Stoll (1981) and O'Hara and Oldfield (1986) show how risk aversion results in the market makers adjusting prices to avoid inventory accumulation.[21]

Proposition 7 *Expected trading volume per period decreases with* $V_t[q]$*.*

[20]Note that uncertainty about preferences and endowments can exist in markets even when all investors are present. However, it is intuitively clear that such uncertainty would be greater in sequential markets where not all investors are in the market at any given time. It is therefore reasonable to believe that the cost of trading (manifested by this spread) is intensified in sequential markets where there is greater uncertainty about the preferences and endowments of the investor population. While a limit order book is not modeled explicitely in this paper, it also seems reasonable to conjecture that arrival of limit orders can provide information about preferences and endowments of investors. However, as the discussion in the text emphasizes, investor uncertainty is constantly created in the market and therefore a limit order book could not eliminate it.

[21]In a previous version of the paper, I examined how investor uncertainty produces similar implications to those described here in the presence of inventory control. In particular, if the market makers' objective function is replaced with one that requires that they end trading at time T with expected inventory of zero, they will adjust prices to affect the order sizes of arriving investors so that their inventory will be brought back to zero. Even with this alternative specification of the market makers' objective function, uncertainty about investor preferences and endowments produces a spread that increases in the variance of q and decreases as market makers learn about the population of investors.

Investor uncertainty creates an inverse relation between the volume of shares traded and illiquidity (as manifested by the price impact of trades). Volume in the market is determined by the optimal quantities of shares, or order sizes, chosen by investors. When the variance of the market makers' beliefs is high, a buyer is forced to transact at a high price and a seller at a low price. Faced with worse prices, investors want to buy or sell smaller quantities of shares and hence volume decreases.[22]

A major difference between the traditional sequential trade models that investigate information asymmetry about future cash flows and the model presented here, which generates an information imperfection using uncertainty about investor preferences and endowments, is the ability of the latter to examine the welfare implications of the information imperfection. The traditional models are unable to provide a welfare analysis since informed investors profit at the expense of uninformed investors (and the uninformed investors in most models do not have an explicit utility functions that can be evaluated). The investor uncertainty model spells out a clear welfare result:

Proposition 8 *The welfare of all investors in the economy is decreasing in $V_t[q]$.*

Since prices and hence optimal demands are functions of $V_t[q]$, investor uncertainty enters the indirect utility functions of both types of investors. Investor uncertainty hurts all investors in the market. It therefore follows that design and regulation of trading venues aimed at reducing investor uncertainty will make all investors in the market better off. In particular, the aforementioned results suggest that market makers who are experts in assessing the nature of the investor population (i.e., have tighter priors on q) can offer better prices to investors and increase trading volume. Hence, the expertise of market makers constitutes

[22]This implication of the model may be difficult to test since the average transaction size when investor uncertainty increases must be compared with the average transaction size of the same investors when investor uncertainty is lower. However, the event that causes the increase in investor uncertainty may include the entrance into the market of investors belonging to different types who previously where on the sideline. The preferences and endowments of these investors may lead them trade larger quantities than the investors who were previously in the market. Hence, just comparing the average transaction size before and after an event that is conjectured to change the level of investor uncertainty in the market may fail to detect the effect described in this proposition.

a positive externality that benefits all investors in the market.

3 Investor Uncertainty and Spread Decomposition

The previous section showed how uncertainty about the preferences and endowments of the investor population can introduce informational effects into prices. Market makers learn from the order flow about the investor population and change prices to accommodate the information revealed by the orders. Traditional sequential trade models postulated the existence of informed investors with private information about the future cash flows of the firm. Market makers in these models extracted information about the liquidating payoff of the stock and adjusted prices accordingly. Hence, both types of information imperfections, investor uncertainty and information asymmetry about future cash flows, introduce informational effects into prices.

Much empirical work in the market microstructure literature has been devoted to identifying and investigating informational effects in prices. Econometric spread decomposition procedures were developed and used extensively to measure the "adverse selection" component of the spread that is attributed to information asymmetry about the firm (e.g., Glosten and Harris, 1988; Stoll, 1989; George, Kaul, and Nimalendran, 1991; Affleck-Graves, Hedge, and Miller, 1994; Lin, Sanger and Booth, 1995; Krinsky and Lee, 1996; Huang and Stoll, 1997; Madhavan, Richardson and Roomans, 1997; Neal and Wheatley, 1998).[23] In general, these methodologies identify the "permanent" component of the price changes and attribute it to information (as opposed to the temporary component that is attributed to order processing costs and inventory costs). Since investor uncertainty generates informational effects similar to those described by the traditional sequential trade models but for a completely different reason, a question arises as to what these methodologies exactly capture.

This issue is illustrated in Neal and Wheatley (1998) who show that the spread of closed-

[23] Hasbrouck (1991) develops a variance decomposition procedure to quantify trade informativeness, and Easley, Kiefer and O'Hara (1997a, 1997b) estimate a structural trading model that includes a measure of information-based trading.

end funds contains a large adverse selection (or information) component. This component of the spread is identified by established empirical methods despite the fact that there is very little information asymmetry about the value of closed-end funds (the market value of their holdings is published every week). Moreover, Neal and Wheatley are unable to relate cross-sectional variation in the adverse selection component to takeover activity and closed-end fund discounts. They end up concluding that either the methodologies that identify the adverse selection component are misspecified or that they pick up something other than information asymmetry about the liquidation value. The investor uncertainty model can be used to interpret the results documented by Neal and Wheatley: the spread component they find arises from information about the preferences and endowments of the investor population carried in the order flow.

To examine the question whether investor uncertainty effects are picked up by the "adverse selection" component of the spread, I simulate the investor uncertainty model and apply an econometric spread decomposition procedure to the simulated data. Since there is no private information about future cash flows in the model, any information effects that will be picked up by the spread decomposition procedure must be due to investor uncertainty. The parameters of the model that I use for the simulation are: θ=1.3, σ^2=0.09, R=1.05, α_1=2, α_2=1, $\bar{X}_1$=3, and $\bar{X}_2$=2. I assume that market makers start the day with a $Beta(3,3)$ prior on the population parameter. The type of an arriving investor is drawn from a Bernoulli distribution with parameter $q = 0.6$. The economy is in the fully-revealing equilibrium described by Proposition 1, where market makers are setting prices according to (11) and investors submit orders according to (12) and (13). The length of a trading day is taken to be 100 periods, and 250 independent days are simulated (so that market makers begin each day with the same prior on q). Panel A of Table 1 contains summary statistics of the simulated data. The average transaction price is 0.926421 and the average spread is 0.009294 (about 1% of the price).

As a representative of the spread decomposition techniques I use Madhavan, Richardson, and Roomans (1997). Let x_t denote an indicator variable taking the value of 1 if the transaction in period t is buyer initiated and -1 if it is seller initiated, and let μ_t denote the post-trade expected value of a stock. Madhavan *et al.* specify the revision of beliefs following a trade as the sum of the change in beliefs due to public information and the change in beliefs due to the order flow innovation:

$$\mu_t = \mu_{t-1} + \nu\left(x_t - E[x_t|x_{t-1}]\right) + \epsilon_t \tag{17}$$

where ν is the permanent impact of the order flow innovation and is a measure of the degree of information asymmetry about the firm (the "adverse selection" component of the half spread), and ϵ_t is the innovation in beliefs between times t–1 and t due to public information. Let p_t denote the transaction price at time t, and ϕ denote the market makers' cost per share of supplying liquidity (compensating them for order processing costs, inventory costs, and so on). The transaction price can then be expressed as:

$$p_t = \mu_t + \phi x_t + \xi_t \tag{18}$$

Equations (17) and (18) can be used to obtain:

$$u_t = p_t - p_{t-1} - (\phi + \nu)x_t + (\phi + \rho\nu)x_{t-1} \tag{19}$$

where ρ is the first-order autocorrelation of the trade initiation variable. Then, the measure of information asymmetry ν, alongside ϕ, ρ, and a constant δ can be estimated using GMM applied to the following moment conditions:[24]

$$E\begin{pmatrix} x_t x_{t-1} - x_{t-1}^2\rho \\ u_t - \delta \\ (u_t - \delta)x_t \\ (u_t - \delta)x_{t-1} \end{pmatrix} = 0 \tag{20}$$

Panel B of Table 1 presents the GMM estimates from the Madhavan, Richardson, and Roomans (1997) procedure applied to the simulated data. The estimate of ν, the information

[24] Madhavan *et al.* note that trades with prices between the bid and the ask can be viewed as both buyer and seller initiated, and set for those $x_t = 0$. They then estimate an additional parameter, λ, which is the unconditional probability that a transaction occurs within the quoted spread. Since all transactions in the model occur at the quotes, $\lambda = 0$ by construction and need not be estimated.

component of the half spread, is large (and very significant) in comparison with the estimate of ϕ, the order processing component. These estimates show that over 90% of the spread is attributed to the permanent component of the spread.[25] If we were presented with these results without knowing which model generated the data, applying the traditional interpretation we would say that there is a high degree of information asymmetry about the firm in the market or that there is a high likelihood of encountering investors who are informed about the future cash flows of the firm. In this case, however, the model that generated the data involved no information asymmetry about the firm. The informational effects picked up by the procedure are therefore solely due to investor uncertainty.[26]

The results of this simulation suggest that spread decomposition methodologies identify an information or adverse selection component that can be either due to information asymmetry about future cash flows or due to uncertainty about investor preferences and endowments. In fact, it is most likely that both information imperfections exist in the market and hence the estimated information component of the spread is the sum of the spread components they create. It follows that caution should be exercised when using the information component of the spread as a measure of information asymmetry about future cash flows or the activity of informed traders, since investor uncertainty may complicate the relationship between the measure and the information asymmetry environment.

[25]I also applied the two-way spread decomposition from Huang and Stoll (1997) to the simulated data, which is essentially equivalent to the Madhavan *et al.* procedure if the first-order autocorrelation (ρ) is assumed to be zero. The following relationship is taken from equation (5) in Huang and Stoll (1997):

$$P_t - P_{t-1} = \frac{S}{2}(x_t - x_{t-1}) + \gamma \frac{S}{2} x_{t-1} + e_t \tag{21}$$

where S is the estimated spread and γ is the fraction of the spread attributed to adverse selection (in the absence of inventory control). When equation (21) is estimated using GMM, S is equal to 0.008059 (t-statistic 61.92) and the estimate of γ is 0.952715 (t-statistic 174.25). The estimate of γ shows that more than 95% of the spread is due to adverse selection.

[26]It is interesting to note that the estimation procedure does attribute a small portion of the spread to order processing cost. This is a bit unsettling since the model does not include any order processing costs and therefore we should have seen the entire spread attributed to information about investors. This result is probably due to the discrepancy between the specification of the empirical equations and the model. For example, the effects of buys and sells on prices in the simulated model are not symmetric while the econometric specification assumes that they are, and therefore some informational effects are perhaps being picked up by the order processing component.

Attributing the estimated informational effects in prices to either investor uncertainty or information asymmetry about future cash flows therefore requires looking at the economic context. In particular, the investor uncertainty model provides an alternative way of interpreting empirical evidence of informational effects in prices of assets that do not fit easily into the asymmetric information paradigm. One such example is the foreign exchange market. Since "inside information" in the usual sense is less relevant in the foreign exchange market, much of the day-to-day pricing reflects the demand of different users. A foreign exchange trader receives orders, infers the demand, and sets quotes much like the market makers in this paper (see Lyons, 1995). Hence, we can expect that investor uncertainty would play an important role in this market.[27] Similarly, evidence of informational effects in prices of mutual funds (Neal and Wheatley, 1998) or Treasury securities (Green, 1999) may reflect investor uncertainty in these markets.

4 Concluding Remarks

This paper promotes the idea that uncertainty about the preferences and endowments of investors in the market—termed investor uncertainty—introduces information into the order flow. The underlying reason for the informational effects in prices here is therefore very different from the one promoted by traditional market microstructure models, where information in the order flow is due to the trading of a subset of investors with private information about the future cash flows of the asset. Investor uncertainty has two attractive properties as a potential explanation for informational effects in prices. First, the preferences and endowments of investors are usually unobservable, providing an intuitive appeal to the explicit modeling of the learning process about them. Second, investor uncertainty provides a bridge to asset pricing models with risk averse investors where prices are determined jointly by the distribution of future cash flows of the asset and the preferences and endowments of investors who demand the asset. This, in contrast to the traditional sequential trade models where

[27] For an argument in support of the traditional form of adverse selection in the foreign exchange market see Naranjo and Nimalendran (2000).

the possibility that risk aversion plays a role in informational effects has not been explored.

The key insight that comes out of the model is that the process of learning about the preferences and endowments of investors from the orders they submit causes prices in the market to change in response to order flow. For example, observing a buy order for a certain quantity can indicate that the pool of investors who are interested in buying at the prevailing price, perhaps those who are less risk averse, may be larger than previously believed. This, in turn, causes market makers to raise prices to reflect their new beliefs about the risk premium. While the source of the information imperfection differs substantially from that postulated by traditional sequential trade models, the learning process of the market makers that translates the uncertainty into price impacts is similar. In particular, it is the rational expectations property of the market makers' equilibrium pricing strategy that creates the price impact of trades both here and in the traditional sequential trade models.

The model shows how uncertainty about preferences and endowments in the market creates a situation whereby prices move with the order flow, and how this price impact increases with the extent of the uncertainty. Furthermore, I show that the welfare of all investors in the economy decreases with the extent of investor uncertainty. While there is no welfare analysis in the paper that includes market makers in addition to the investor population, it seems very likely that investor uncertainty in actual markets would lower market makers' utility as well. This is because investor uncertainty is shown to decrease trading volume, a variable that is tightly associated with market makers' trading revenues. The model, therefore, provides a mechanism that relates liquidity to welfare, and shows how investor uncertainty can cause both worsened liquidity and lower welfare.

An additional point that the paper is making has to do with the spread decomposition procedures that are being used to estimate the extent of informed trading or adverse selection problem in the market. Using data generated by simulating the model (which does not feature information asymmetry about future cash flows), I show that the adverse selection component of the spread produced by these procedures picks up the effects of investor uncertainty. This raises questions as to the interpretation of the output of these econometric techniques, as they seem to bundle information about investor preferences and endowments with private information about the firm.

The approach pursued in this paper, however, is subject to several limitations. The first limitation, which is also shared by the traditional sequential trade models, is the probabilistic selection of investors that precludes dynamic trading strategies. The second limitation has to do with the static problem of the market makers. The model therefore does not reflect possible intertemporal strategies market makers can adopt. This drawback is also shared by most traditional sequential trade models, where the requirement (formalized in Glosten and Milgrom, 1985) that the price of each trade reflects a zero expected profit condition results in a convenient pricing rule but does not lend itself easily to dynamic considerations. Leach and Madhavan (1993), for example, show how more complex patterns of price experimentation can arise when market makers solve dynamic problems. While the simplified assumptions of the model here are helpful in the introduction of investor uncertainty and its effects on the information content of the order flow, I believe that the intuition behind investor uncertainty is more general than the model itself and would hold in more complex settings.

Extending the model to consider additional features of actual markets can offer interesting venues for future work. One could look at a market in which multiple risky assets are priced and traded. If an investor does not rebalance his entire portfolio on a single arrival to the market, a transaction in one stock could reveal information about the preferences and endowments of the investor population and hence affect the values of other stocks. Another interesting extension may involve looking at derivative securities. If investors implement complex trading strategies in which they take offsetting positions in the underlying asset and in the derivative security, order flow in one market can be used to infer information about the population of investors in the other market.

While this paper demonstrates how investor uncertainty can serve as an alternative to the paradigm of information asymmetry about future cash flows, these two need not be mutually exclusive. For example, Saar (2000) shows that when information asymmetry about future cash flows is introduced into the investor uncertainty model, there exists a pooling equilibrium in which market makers learn from the arrival of orders both on the preferences and endowments of investors and on the future cash flows. A challenge for future work is the construction of a more general model of information imperfections. Such a model could be used to examine how information about investors' preferences and endowments, information about the future cash flows of the firm, and information about market conditions interact in a market populated by different groups of investors and market professionals.

Appendix

Proof of Proposition 1:
Assume that market makers can identify the type of an investor from his order size, and that type 1 (type 2) investors submit orders for $X_{1,t}$ ($X_{2,t}$) shares. First I will establish the Equilibrium Price conditional on $\mathcal{M}_{t,1}$. Since the same information set is used in the objective function and in the expected market clearing constraint, I can perform the same manipulation as in (8) and get a single equilibrium price conditional on the information that can be inferred from the arrival of an order of a type 1 investor. $P(M_{t,1})$ can be found by plugging the optimal orders into the expected market clearing condition in (7) as follows:

$$q_{1,t}\left(\frac{\theta - RP(M_{t,1})}{\alpha_1\sigma^2} - \bar{X}_1\right) + (1 - q_{1,t})\left(\frac{\theta - RP(M_{t,1})}{\alpha_2\sigma^2} - \bar{X}_2\right) = 0 \tag{22}$$

where $q_{1,t} = E\left[q \mid \mathcal{M}_{t,1}\right]$. Solving for $P(M_{t,1})$ we get,

$$P(M_{t,1}) = \frac{\theta}{R} - \frac{\bar{X}(M_{t,1})\alpha_1\alpha_2\sigma^2}{R\alpha(M_{t,1})} \tag{23}$$

where $\bar{X}(M_{t,1}) = q_{1,t}\bar{X}_1 + (1-q_{1,t})\bar{X}_2$ and $\alpha(M_{t,1}) = q_{1,t}\alpha_2 + (1-q_{1,t})\alpha_1$. The Equilibrium Price conditional on $\mathcal{M}_{t,2}$ can be found in a similar fashion. Let $q_{2,t} = E\left[q \mid \mathcal{M}_{t,2}\right]$. Then,

$$P(M_{t,2}) = \frac{\theta}{R} - \frac{\bar{X}(M_{t,2})\alpha_1\alpha_2\sigma^2}{R\alpha(M_{t,2})} \tag{24}$$

where $\bar{X}(M_{t,2}) = q_{2,t}\bar{X}_1 + (1-q_{2,t})\bar{X}_2$ and $\alpha(M_{t,1}) = q_{2,t}\alpha_2 + (1-q_{2,t})\alpha_1$. Since a type 1 investor will face the price $P(M_{t,1})$ when he arrives to trade, his optimal order is:

$$X_{1,t} = \frac{\theta - RP(M_{t,1})}{\alpha_1\sigma^2} - \bar{X}_1 = -\frac{(1 - q_{1,t})\Delta\alpha\bar{X}}{\alpha(M_{t,1})} < 0 \tag{25}$$

Similarly, a type 2 investor will be charged the price $P(M_{t,2})$ when he will trade, and so his optimal order is:

$$X_{2,t} = \frac{\theta - RP(M_{t,2})}{\alpha_1\sigma^2} - \bar{X}_2 = \frac{q_{2,t}\Delta\alpha\bar{X}}{\alpha(M_{t,2})} > 0 \tag{26}$$

Market makers restrict trading in the economy to the optimal orders of the two types of investors, and so their Equilibrium Strategy specifies a depth that is equal to type 1's optimal order as the appropriate depth for a sell order that will be executed at the price $P(M_{t,1})$. A sell order for any other size can execute only at a price of zero. Similarly, market makers set the depth associated with $P(M_{t,2})$ equal to the size of the type 2 buy order in (26). Any other buy order can execute only at an infinite price. This Equilibrium Strategy therefore causes investors to transact only using these two order sizes. Note that $X_{1,t} \neq X_{2,t}$ (one is a buy order and the other a sell order), and hence market makers can differentiate between them. This completes the first requirement of the fully-revealing equilibrium.

Since investors face two prices with associated depths and are free to choose between them, I still need to check that type 1 investors will choose $(P(M_{t,1}), X_{1,t})$ while type 2 investors will choose $(P(M_{t,2}), X_{2,t})$. For that consider the following participation and incentive compatability conditions:

$$\begin{array}{lrcl} \text{P.1.} & U_1\left(P(M_{t,1}), X_{1,t}\right) & \geq & U_1\left(P, 0\right) \\ \text{IC.1.} & U_1\left(P(M_{t,1}), X_{1,t}\right) & \geq & U_1\left(P(M_{t,2}), X_{2,t}\right) \\ \text{P.2.} & U_2\left(P(M_{t,2}), X_{2,t}\right) & \geq & U_2\left(P, 0\right) \\ \text{IC.2.} & U_2\left(P(M_{t,2}), X_{2,t}\right) & \geq & U_2\left(P(M_{t,1}), X_{1,t}\right) \end{array} \quad (27)$$

where P in the participation conditions denotes any arbitrary price. The participation condition P.1. can be written as follows:

$$R\bar{M}_1 - RP(M_{t,1})X_{1,t} + (\bar{X}_1 + X_{1,t})\theta - \frac{\alpha_1}{2}(\bar{X}_1 + X_{1,t})^2\sigma^2 \geq \quad (28)$$

$$R\bar{M}_1 - RP0 + (\bar{X}_1 + 0)\theta - \frac{\alpha_1}{2}(\bar{X}_1 + 0)^2\sigma^2 \quad (29)$$

which simplifies to:

$$-RP(M_{t,1})X_{1,t} + X_{1,t}(\theta - \alpha_1\sigma^2\bar{X}_1) - \frac{\alpha_1\sigma^2 X_{1,t}^2}{2} \geq 0 \quad (30)$$

Denote the left-hand-side of the above expression by **A**. Using (23) and (25):

$$\begin{aligned} \mathbf{A} &= X_{1,t}\left[-R\left(\frac{\theta}{R} - \frac{\alpha_1\alpha_2\bar{X}(M_{t,1})\sigma^2}{R\alpha(M_{t,1})}\right) + \theta - \alpha_1\sigma^2\bar{X}_1 - \frac{\alpha_1\sigma^2 X_{1,t}}{2}\right] && (31) \\ &= \frac{X_{1,t}\alpha_1\sigma^2}{2}\left[\frac{2\alpha_2\bar{X}(M_{t,1})}{\alpha(M_{t,1})} - 2\bar{X}_1 - X_{1,t}\right] && (32) \\ &= \frac{X_{1,t}\alpha_1\sigma^2}{2\alpha(M_{t,1})}\left[(1-q_{1,t})(-\Delta\alpha\bar{X})\right] = \frac{(1-q_{1,t})^2\alpha_1\sigma^2(\Delta\alpha\bar{X})^2}{2\alpha(M_{t,1})^2} > 0 && (33) \end{aligned}$$

Hence, P.1. always holds. The incentive compatibility condition I.C.1. can be written as follows:

$$R\bar{M}_1 - RP(M_{t,1})X_{1,t} + (\bar{X}_1 + X_{1,t})\theta - \frac{\alpha_1}{2}(\bar{X}_1 + X_{1,t})^2\sigma^2 \geq \quad (34)$$

$$R\bar{M}_1 - RP(M_{t,2})X_{2,t} + (\bar{X}_1 + X_{2,t})\theta - \frac{\alpha_1}{2}(\bar{X}_1 + X_{2,t})^2\sigma^2 \quad (35)$$

which simplifies to:

$$\left[-RP(M_{t,1})X_{1,t} + X_{1,t}(\theta - \alpha_1\sigma^2\bar{X}_1) - \frac{\alpha_1\sigma^2 X_{1,t}^2}{2}\right] \quad (36)$$

$$+\left[RP(M_{t,2})X_{2,t} - X_{2,t}(\theta - \alpha_1\sigma^2\bar{X}_1) + \frac{\alpha_1\sigma^2 X_{2,t}^2}{2}\right] \geq 0 \quad (37)$$

The first term in the left-hand-side of the above expression is **A** that was shown to be positive. Denote the second term by **B**. It can be rewritten using (24) and (26) as follows:

$$\begin{aligned} \mathbf{B} &= X_{2,t}\left[R\left(\frac{\theta}{R} - \frac{\alpha_1\alpha_2\bar{X}(M_{t,2})\sigma^2}{R\alpha(M_{t,2})}\right) - \theta + \alpha_1\sigma^2\bar{X}_1 + \frac{\alpha_1\sigma^2 X_{2,t}}{2}\right] && (38) \\ &= \frac{X_{2,t}\alpha_1\sigma^2}{2\alpha(M_{t,2})}\left[(2-q_{2,t})\Delta\alpha\bar{X}\right] = \frac{q_{2,t}(2-q_{2,t})\alpha_1\sigma^2(\Delta\alpha\bar{X})^2}{2\alpha(M_{t,2})^2} > 0 && (39) \end{aligned}$$

Hence, I.C.1. always holds. The participation condition P.2. can be written as follows:

$$R\bar{M}_2 - RP(M_{t,2})X_{2,t} + (\bar{X}_2 + X_{2,t})\theta - \frac{\alpha_2}{2}(\bar{X}_2 + X_{2,t})^2\sigma^2 \geq \tag{40}$$

$$R\bar{M}_2 - RP0 + (\bar{X}_2 + 0)\theta - \frac{\alpha_2}{2}(\bar{X}_2 + 0)^2\sigma^2 \tag{41}$$

which simplifies to:

$$-RP(M_{t,2})X_{2,t} + X_{2,t}(\theta - \alpha_2\sigma^2\bar{X}_2) - \frac{\alpha_2\sigma^2 X_{2,t}^2}{2} \geq 0 \tag{42}$$

Denote the left-hand-side of the above expression by **C**. Using (24) and (26):

$$\mathbf{C} = \frac{X_{2,t}\alpha_2\sigma^2}{2}\left[\frac{2\alpha_1\bar{X}(M_{t,2})}{\alpha(M_{t,2})} - 2\bar{X}_2 - X_{2,t}\right] \tag{43}$$

$$= \frac{X_{2,t}\alpha_2\sigma^2}{2\alpha(M_{t,2})}\left[q_{2,t}\Delta\alpha\bar{X}\right] = \frac{q_{2,t}^2\alpha_2\sigma^2(\Delta\alpha\bar{X})^2}{2\alpha(M_{t,2})^2} > 0 \tag{44}$$

Hence, P.2. always holds. The incentive compatibility condition I.C.2. can be written as follows:

$$R\bar{M}_2 - RP(M_{t,2})X_{2,t} + (\bar{X}_2 + X_{2,t})\theta - \frac{\alpha_2}{2}(\bar{X}_2 + X_{2,t})^2\sigma^2 \geq \tag{45}$$

$$R\bar{M}_2 - RP(M_{t,1})X_{1,t} + (\bar{X}_2 + X_{1,t})\theta - \frac{\alpha_2}{2}(\bar{X}_2 + X_{1,t})^2\sigma^2 \tag{46}$$

which simplifies to:

$$\left[-RP(M_{t,2})X_{2,t} + X_{2,t}(\theta - \alpha_2\sigma^2\bar{X}_2) - \frac{\alpha_2\sigma^2 X_{2,t}^2}{2}\right] \tag{47}$$

$$+\left[RP(M_{t,1})X_{1,t} + X_{1,t}(\theta - \alpha_2\sigma^2\bar{X}_2) - \frac{\alpha_2\sigma^2 X_{1,t}^2}{2}\right] \geq 0 \tag{48}$$

The first term in the left-hand-side of the above expression is **C** that was shown to be positive. Denote the second term by **D**. It can be rewritten using (23) and (25) as follows:

$$\mathbf{D} = X_{1,t}\left[R\left(\frac{\theta}{R} - \frac{\alpha_1\alpha_2\bar{X}(M_{t,1})\sigma^2}{R\alpha(M_{t,1})}\right) - \theta + \alpha_2\sigma^2\bar{X}_2 + \frac{\alpha_2\sigma^2 X_{1,t}}{2}\right] \tag{49}$$

$$= \frac{X_{1,t}\alpha_2\sigma^2}{2\alpha(M_{t,1})}\left[-(1+q_{1,t})\Delta\alpha\bar{X}\right] = \frac{(1-q_{1,t})(1+q_{1,t})\alpha_2\sigma^2(\Delta\alpha\bar{X})^2}{2\alpha(M_{t,1})^2} > 0 \tag{50}$$

Hence, I.C.2. always holds. These four conditions show that investors self-select to the pairs of prices and order sizes that reveal their types to the market makers, and hence the second requirement of the fully-revealing equilibrium is satisfied.
Q.E.D.

Proof of Proposition 2:
Let $f^{\mathcal{M}_t}(q)$ be the prior distribution of q given all public information including the order flow up to time t, and $M_{t,i} = \{\mathcal{M}_t, X_i\}$ be the information set that also includes an incoming order of type i. By Bayes Law,

$$f^{\mathcal{M}_{t,1}}(q) = \frac{q f^{\mathcal{M}_t}(q)}{\int_0^1 q f^{\mathcal{M}_t}(q) dq} \tag{51}$$

$$\begin{aligned} E\left[q \mid \mathcal{M}_{t,1}\right] &= \int_0^1 q f^{\mathcal{M}_{t,1}}(q) dq \\ &= \frac{\int_0^1 q^2 f^{\mathcal{M}_t}(q) dq}{\int_0^1 q f^{\mathcal{M}_t}(q) dq} = \frac{E_t\left[q^2\right]}{q_t} \\ &= \frac{V_t[q] + q_t^2}{q_t} = q_t + \frac{V_t[q]}{q_t} \end{aligned} \tag{52}$$

where $E_t[\cdot] = E[\cdot \mid \mathcal{M}_t]$, $q_t = E_t[q]$, and $V_t[q] = E_t[q^2] - q_t^2$. Similarly,

$$f^{\mathcal{M}_{t,2}}(q) = \frac{(1-q) f^{\mathcal{M}_t}(q)}{\int_0^1 (1-q) f^{\mathcal{M}_t}(q) dq} \tag{53}$$

$$\begin{aligned} E\left[q \mid \mathcal{M}_{t,2}\right] &= \int_0^1 q f^{\mathcal{M}_{t,2}}(q) dq \\ &= \frac{\int_0^1 q(1-q) f^{\mathcal{M}_t}(q) dq}{\int_0^1 (1-q) f^{\mathcal{M}_t}(q) dq} = \frac{q_t - E_t\left[q^2\right]}{1 - q_t} \\ &= \frac{q_t - V_t[q] - q_t^2}{1 - q_t} = q_t - \frac{V_t[q]}{1 - q_t} \end{aligned} \tag{54}$$

Q.E.D

Proof of Proposition 3:
From (25) and (26), type 2 investors are buyers (sellers) and type 1 investors are sellers (buyers) if and only if $\Delta\alpha\bar{X} = \alpha_1 \bar{X}_1 - \alpha_2 \bar{X}_2 > 0 (< 0)$. Hence, the spread S_t can be defined as:

$$S_t = (P(M_{t,2}) - P(M_{t,1})) \mathrm{sign}\left(\Delta\alpha\bar{X}\right) \tag{55}$$

Using (23), (24) and Proposition 2,

$$\begin{aligned} S_t &= \frac{\alpha_1 \alpha_2 \sigma^2}{R} \left[\frac{\bar{X}(M_{t,1})}{\alpha(M_{t,1})} - \frac{\bar{X}(M_{t,2})}{\alpha(M_{t,2})}\right] \mathrm{sign}\left(\Delta\alpha\bar{X}\right) \\ &= \frac{\alpha_1 \alpha_2 \sigma^2}{R\alpha(M_{t,1})\alpha(M_{t,2})} \left|\Delta\alpha\bar{X}\right| \left(\mathrm{sign}\left(\Delta\alpha\bar{X}\right)\right)^2 (q_{1,t} - q_{2,t}) \\ &= \frac{\alpha_1 \alpha_2 \sigma^2 \left|\Delta\alpha\bar{X}\right| V_t[q]}{R\alpha(M_{t,1})\alpha(M_{t,2}) q_t (1 - q_t)} > 0 \end{aligned} \tag{56}$$

Q.E.D

Proof of Proposition 4:
Using (23), (24) and (16),

$$\frac{\partial P(M_{t,1})}{\partial \sigma^2} = -\frac{\alpha_1\alpha_2\bar{X}(M_{t,1})}{R\alpha(M_{t,1})} < 0 \tag{57}$$

$$\frac{\partial P(M_{t,2})}{\partial \sigma^2} = -\frac{\alpha_1\alpha_2\bar{X}(M_{t,2})}{R\alpha(M_{t,2})} < 0 \tag{58}$$

$$\frac{\partial S_t}{\partial \sigma^2} = \frac{\alpha_1\alpha_2\left|\Delta\alpha\bar{X}\right|V_t[q]}{R\alpha(M_{t,1})\alpha(M_{t,2})q_t\left(1-q_t\right)} > 0 \tag{59}$$

Define the relative spread as the spread divided by the mid-quote:

$$\frac{2S_t}{P(M_{t,1})+P(M_{t,2})} = \frac{2\alpha_1\alpha_2\sigma^2\left|\Delta\alpha\bar{X}\right|V_t[q]}{q_t\left(1-q_t\right)} \cdot \frac{1}{\left[2\theta\alpha(M_{t,1})\alpha(M_{t,2}) - \alpha_1\alpha_2\sigma^2\left(\bar{X}(M_{t,1})\alpha(M_{t,2})+\bar{X}(M_{t,2})\alpha(M_{t,1})\right)\right]} \tag{60}$$

Then,

$$\frac{\partial\frac{2S_t}{P(M_{t,1})+P(M_{t,2})}}{\partial\sigma^2} = \frac{4\alpha_1\alpha_2\theta\alpha(M_{t,1})\alpha(M_{t,2})\left|\Delta\alpha\bar{X}\right|V_t[q]}{q_t\left(1-q_t\right)} \cdot \frac{1}{\left[2\theta\alpha(M_{t,1})\alpha(M_{t,2}) - \alpha_1\alpha_2\sigma^2\left(\bar{X}(M_{t,1})\alpha(M_{t,2})+\bar{X}(M_{t,2})\alpha(M_{t,1})\right)\right]^2} > 0 \tag{61}$$

Q.E.D

Proof of Proposition 5:
Using (23) and Proposition 2,

$$\frac{\partial P(M_{t,1})}{\partial V_t[q]} = -\frac{\alpha_1\alpha_2\sigma^2\Delta\alpha\bar{X}}{R\alpha(M_{t,1})^2 q_t} \tag{62}$$

Hence, if $\Delta\alpha\bar{X} < 0$ and type 1 investors are buyers, the ask increases with $V_t[q]$. If $\Delta\alpha\bar{X} > 0$ and type 1 investors are sellers, the bid decreases with $V_t[q]$. Using (24) and Proposition 2,

$$\frac{\partial P(M_{t,2})}{\partial V_t[q]} = \frac{\alpha_1\alpha_2\sigma^2\Delta\alpha\bar{X}}{R\alpha(M_{t,2})^2\left(1-q_t\right)} \tag{63}$$

Hence, if $\Delta\alpha\bar{X} < 0$ and type 2 investors are sellers, the bid decreases with $V_t[q]$. If $\Delta\alpha\bar{X} > 0$ and type 2 investors are buyers, the ask increases with $V_t[q]$. Using the definition of the spread from (55),

$$\frac{\partial S_t}{\partial V_t[q]} = \left[\frac{\alpha_1\alpha_2\sigma^2}{R\alpha(M_{t,2})^2(1-q_t)} + \frac{\alpha_1\alpha_2\sigma^2}{R\alpha(M_{t,1})^2 q_t}\right]\left|\Delta\alpha\bar{X}\right| > 0 \tag{64}$$

Q.E.D

Proof of Proposition 6:
These are implications of standard Bayesian results. Using the Law of Iterated Expectations, $E_{X_i}[V(q|X_i)] = V(q) - V_{X_i}[E(q|X_i)] \leq V(q)$. Also, since q_t converges almost surely to q, $V_t[q]$ goes in the limit to zero. Using Proposition 2 and the definitions of $\bar{X}(M_{t,1})$, $\alpha(M_{t,1})$, $\bar{X}(M_{t,2})$, and $\alpha(M_{t,2})$:

$$\begin{aligned}\lim_{t\to\infty} P(M_{t,1}) &= \frac{\theta}{R} - \frac{\lim_{t\to\infty}\bar{X}(M_{t,1})\alpha_1\alpha_2\sigma^2}{\lim_{t\to\infty}R\alpha(M_{t,1})} = \frac{\theta}{R} - \frac{\bar{X}^*\alpha_1\alpha_2\sigma^2}{R\alpha^*} \\ \lim_{t\to\infty} P(M_{t,2}) &= \frac{\theta}{R} - \frac{\lim_{t\to\infty}\bar{X}(M_{t,2})\alpha_1\alpha_2\sigma^2}{\lim_{t\to\infty}R\alpha(M_{t,2})} = \frac{\theta}{R} - \frac{\bar{X}^*\alpha_1\alpha_2\sigma^2}{R\alpha^*}\end{aligned}$$

where $\bar{X}^* = q\bar{X}_1 + (1-q)\bar{X}_2$ and $\alpha^* = q\alpha_2 + (1-q)\alpha_1$.
Q.E.D

Proof of Proposition 7:
One way to represent expected volume per period in the market is:

$$\begin{aligned}\overline{VOL}_t &= \text{sign}\Big(\Delta\alpha\bar{X}\Big)\left[q_t(-X_{1,t}) + (1-q_t)\,X_{2,t}\right] \\ &= \Big|\Delta\alpha\bar{X}\Big|\left[\frac{q_t(1-q_{1,t})}{\alpha(M_{t,1})} + \frac{(1-q_t)\,q_{2,t}}{\alpha(M_{t,2})}\right]\end{aligned} \tag{65}$$

where the last equality follows from (25) and (26). Using Proposition 2,

$$\frac{\partial\overline{VOL}_t}{\partial V_t[q]} = \Big|\Delta\alpha\bar{X}\Big|\left[-\frac{\alpha_2}{(\alpha(M_{t,1}))^2} - \frac{\alpha_1}{(\alpha(M_{t,2}))^2}\right] < 0 \tag{66}$$

Q.E.D

Proof of Proposition 8:
The indirect utility function of a type 1 investor is:

$$U_1 = -\exp\left\{-\alpha_1\left[R\bar{M}_1 + R\bar{X}_1P(M_{t,1}) + \frac{(\theta - RP(M_{t,1}))^2}{2\alpha_1\sigma^2}\right]\right\} \tag{67}$$

Let $X(M_t) = q_t\bar{X}_1 + (1-q_t)\,\bar{X}_2$ and $\alpha(M_t) = q_t\alpha_2 + (1-q_t)\,\alpha_1$. Using (23) and Proposition 2, $P(M_{t,1})$ can be written as:

$$P(M_{t,1}) = \frac{\theta}{R} - \frac{\left(X(M_t) - \frac{V_t[q](\bar{X}_2 - \bar{X}_1)}{q_t}\right)\alpha_1\alpha_2\sigma^2}{R\left(\alpha(M_t) + \frac{V_t[q](\alpha_2-\alpha_1)}{q_t}\right)} \tag{68}$$

Plugging the price into the indirect utility function and differentiating with respect to $V_t[q]$,

$$\begin{aligned}\frac{\partial U_1}{\partial V_t[q]} &= -\exp\left\{-\alpha_1\left[R\bar{M}_1 + R\bar{X}_1P(M_{t,1}) + \frac{(\theta - RP(M_{t,1}))^2}{2\alpha_1\sigma^2}\right]\right\} \\ &\quad \frac{\alpha_1^2\alpha_2\sigma^2q_t(\Delta\alpha\bar{X})^2(1-q_{1,t})}{\alpha(M_{t,1})\left[q_t\alpha(M_t) + V_t[q](\alpha_2-\alpha_1)\right]^2} < 0\end{aligned} \tag{69}$$

Similarly for a type 2 investor,

$$P(M_{t,2}) = \frac{\theta}{R} - \frac{\left(X(M_t) + \frac{V_t[q](\bar{X}_2 - \bar{X}_1)}{1-q_t}\right)\alpha_1\alpha_2\sigma^2}{R\left(\alpha(M_t) - \frac{V_t[q](\alpha_2-\alpha_1)}{1-q_t}\right)} \tag{70}$$

$$\begin{aligned} \frac{\partial U_2}{\partial V_t[q]} &= -\exp\left\{-\alpha_2\left[R\bar{M}_2 + R\bar{X}_2 P(M_{t,2}) + \frac{(\theta - RP(M_{t,2}))^2}{2\alpha_2\sigma^2}\right]\right\} \\ & \frac{\alpha_1\alpha_2^2\sigma^2\,(1-q_t)\,(\Delta\alpha\bar{X})^2 q_{2,t}}{\alpha(M_{t,2})\,[(1-q_t)\,\alpha(M_t) - V_t[q](\alpha_2-\alpha_1)]^2} < 0 \end{aligned} \tag{71}$$

Q.E.D

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Table 1
Spread Decomposition Applied to Simulated Data

Panel A presents summary statistics from simulating 250 trading days of the investor uncertainty model. The parameters used for the simulation are: $R = 1.05, \theta = 1.3, \sigma^2 = 0.09, \alpha_1 = 2, \alpha_2 = 1, \bar{X}_1 = 3, \bar{X}_2 = 2, q = 0.6, T = 100, f^0(q) = Beta(3,3)$. Panel B presents the GMM estimates of the Madhavan, Richardson, and Roomans (1997) spread decomposition procedure applied to the simulated data. ν is the permanent impact of the order flow innovation or the "adverse selection" component of the half spread. ϕ is the temporary component of the half spread that incorporates order processing costs and inventory costs. ρ is the first-order autocorrelation of the trade initiation variable, and δ is a constant.

Panel A: Summary Statistics of Simulated Data

	Trade Price	**Trade Size**		**Bid Price**	**Ask Price**	**Bid-Ask Spread**
		Sells	**Buys**			
Mean	0.926421	-1.158447	1.568042	0.922618	0.931912	0.009294
Median	0.926984	-1.166667	1.573770	0.923810	0.931602	0.006091
Std.	0.024628	0.139294	0.286190	0.023737	0.024692	0.008132
Obs.	25000	14809	10191	25000	25000	25000

Panel B: Results of Spread Decomposition

	Estimate	t-Statistic
δ	0.005331	0.81
ρ	0.000418	17.93
ϕ	0.000176	5.10
ν	0.004211	58.47