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Modern Portfolio Theory, 1950 to Date

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MODERN PORTFOLIO THEORY, 1950 TO DATE

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One of the key issues facing an individual is how to allocate wealth among alternative assets. Almost all financial institutions have the same problem with the added complication that they need to explicitly include the characteristics of their liabilities in the analysis. While the structure of these problems varies somewhat, they are similar enough that we classify both as portfolio theory.

Portfolio theory is a well-developed paradigm. There are excellent textbooks on the subject. Of course, we are especially partial to our own *Modern Portfolio Theory and Investment Analysis.*[1] There are also good reviews in more advanced doctoral-level texts such as Ingersoll (1987) or Huang and Litzenberger (1988). There are also some careful mathematical treatments (Szego (1980)). Finally, good review articles such as Constantinides and Malliaris (1995) exist. Therefore, instead of writing one more general review article, we will be more selective and our discussion will be rather eclectic. This paper will present four topics we find of particular interest. Rather than attempting to survey all articles or all issues on each topic, we will discuss what we find of special interest and importance. We will attempt to convey where the field is today and where it is headed in the future.

This paper is divided into four sections, each covering one topic. The first section presents a historical review of the basic theory and its current state of development. The second discusses issues in estimating the key inputs for portfolio theory. The third

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1 Elton, Edwin J. and Gruber, Martin J. (1995).

discusses the special issues that arise when portfolio theory is applied to financial institutions.[2] The final section will review portfolio evaluation techniques.

I. Historical Development and Current State of Theory

Markowitz (1952 and 1959) is the father of modern portfolio theory. His original book and article on the subject clearly delineated, for the first time, modern portfolio theory. The book was filled with insights and suggestions that anticipated many of the subsequent developments in the field. Markowitz formulated the portfolio problem as a choice of the mean and variance of a portfolio of assets. He proved the fundamental theorem of mean variance portfolio theory, namely holding constant variance, maximize expected return, and holding constant expected return minimize variance. These two principles led to the formulation of an efficient frontier from which the investor could choose his or her preferred portfolio, depending on individual risk return preferences. The important message of the theory was that assets *could not* be selected only on characteristics that were unique to the security. Rather, an investor had to consider how each security co-moved with all other securities. Furthermore, taking these co-movements into account resulted in an ability to construct a portfolio that had the same expected return and less risk than a portfolio constructed by ignoring the interactions between securities.

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2 One of the early discussions of this problem is presented by Szego (1980), chapters 15 and 16.

Considering just the mean return and variance of return of a portfolio is, of course, a simplification relative to including additional moments that might more completely describe the distribution of returns of the portfolio. Early work developed necessary conditions on either the utility function of investors or the return distribution of assets that would result in mean variance theory being optimal (see, for example, Tobin (1958)). In addition, researchers (see Lee (1977) and Litzenberger and Kraus (1976)) offered alternative portfolio theories that included more moments such as skewness or were accurate for more realistic descriptions of the distribution of return (Fama (1969) and Elton and Gruber (1974)). Nevertheless, mean variance theory has remained the cornerstone of modern portfolio theory despite these alternatives. This persistence is not due to the realism of the utility or return distribution assumptions that are necessary for it to be correct. Rather, we believe there are two reasons for its persistence. First, mean variance theory itself places large data requirements on the investor, and there is no evidence that adding additional moments improves the desirability of the portfolio selected. Second, the implications of mean variance portfolio theory are well developed, widely known, and have great intuitive appeal. Professionals who have never run an optimizer have learned that correlations as well as means and variances are necessary to understand the impact of adding a security to a portfolio. Borrowing a concept from the next section, risk measures such as beta, which have been developed based on mean variance analyses, add information and are recognized and used by investors who have no idea of the theory behind them. The precepts of mean variance theory work. Thus, we will concentrate on mean variance portfolio theory.

Mean variance portfolio theory was developed to find the optimum portfolio when an investor is concerned with return distributions over a single period. An investor is assumed to estimate the mean return and variance of return for each asset being considered for the portfolio over the single period.[3] In addition, the correlations or covariances between all pairs of assets being considered need to be estimated. Once again, these estimates are for the single decision period. One of the major theoretical problems that has been analyzed is how the single-period problem should be modified if the investor's true problem is multi-period in nature. Papers by Fama (1970), Hakansson (1970 and 1974), Merton (1969), and Mossin (1969) have all analyzed this problem under various assumptions. The papers found that under several sets of reasonable assumptions, the multi-period problem can be solved as a sequence of single-period problems. However, the optimum portfolio would be different from that selected if only one period was examined. The difference arises because the appropriate utility function in the multi-period case is a derived utility function that takes into account multiple periods, and this differs from the utility function that is appropriate for decision-making over a single period.

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[3] The appropriate length of the period in a single-period solution is a topic of significance on which very little research has been done.

One assumption underlying most multi-period portfolio analysis is independence of returns between periods. There has been a substantial amount of research in the last decade showing that mean returns and variances are related over time and are functions of easily observable variables (Fama and French (1989); Campbell and Shiller (1988)). A major research topic for the future will be how this empirical literature should effect optimum multi-period portfolio decisions.

Another strand of theoretical research has been the study of separation theorems. It is easy to show that if an investor has access to a riskless asset, the choice of the optimum portfolio of risky assets is unequivocal and independent of the investor's taste for expected return or variance. This is the separation theorem. It has three implications. First, it facilitates calculation in that the portfolio problem can be stated as finding the tangency portfolio to a ray passing through the riskless asset in expected return standard deviation space. The tangency portfolio is the portfolio that maximizes the ratio of expected return minus the return on the riskless asset to the standard deviation. Second, it leads to a mutual fund theorem, namely that all investors can obtain their desired portfolio by mixing two mutual funds, one made up of the riskless asset and one representing the tangency portfolio. One of the areas of theoretical research deals with how many mutual funds are needed and what is the nature of the portfolios that comprise them under alterative assumptions about the nature of asset returns or utility functions (see for example Ross (1978)). This is important because it provides guidance to investors and the mutual fund industry on what kinds of portfolios should be attractive. Furthermore, for financial institutions such as banks or insurance companies, mutual fund theorems provide guidance with respect to which types of commingled funds to offer. As the assumption of a constant riskless lending and borrowing rate is relaxed, are other assumptions, more funds and new types of funds enter the decision set. For example, if there are different riskless lending and borrowing rates but short sales of risky assets are allowed, four funds are needed. Two of the portfolios are any two portfolios of risky assets that lie on the efficient frontier and the remaining two portfolios are the instruments which yield the riskless lending and borrowing rates. While a substantial amount of work has been undertaken on mutual fund separation, and we will discuss it again in later sections, more still needs to be done. In particular, a reconciliation between theory and the actual array of mutual funds which exist would be useful.

There are two other types of theoretical research that have received substantial attention in the literature but have not had a major impact on the implementation of portfolio management. First, a number of articles have been written that analyze the portfolio problem in continuous time (see Merton (1969)). In the continuous-time formulation, the portfolio problem and consumption investment problem are solved simultaneously. The primary result of the continuous-time framework has been to confirm the discrete-time results. The confirmation in some cases assumes a somewhat more realistic set of assumptions such as lognormal rather than normal returns and more general utility functions, but the results constitute in large part a confirmation of standard discrete-time results. The major exception is Merton's results concerning hedging portfolios. Merton finds that in the intertemporal continuous-time framework an investor needs to hold hedging portfolios to protect against changes in the state variables.

The other type of theoretical research that has received some attention is attempts to understand how current holdings and transaction costs should affect portfolio rebalancing. Again, this analysis, while interesting, has had little impact on practice.

An applied area of research that has been of concern is computational efficiency. With the enormous improvement in computer speed and memory, early computational concerns about solving large-scale problems have disappeared. What still has value is the insights into the composition of the optimum portfolio that can be obtained by simple solutions to the portfolio problem. In a series of papers, Elton, Gruber and Padberg ((1976), (1977), (1978a), (1978b)) showed how optimum portfolios could be selected by a simple ranking device. This ranking device clarified the characteristics of securities that would lead to their inclusion in an optimum portfolio. The basic insight that made the EG&P algorithms work is the following: most analysts who solve portfolio problems use a model to estimate the covariances between securities. Portfolio problems can be solved by inverting a matrix. Often the structure of the model used to estimate the covariances is such that the covariance matrix can be explicitly inverted and a closed form solution written down. This solution clarifies why a security enters into the optimum portfolio and provides a simple ranking device that allows easy solution. In the single-index case, the ranking is by the excess return (expected return minus riskless rate) divided by the security's beta. If the covariance is estimated by assuming that the correlation among securities is constant, the ranking is by excess return to standard deviation of return. Similar ranking devices exist for more complicated assumptions about the how the covariance is estimated. A number of papers have elaborated on the original EG&P analysis. These have suggested ways of speeding up the computations and incorporating more realistic short sales assumptions, and have led to improved understanding of the effect of estimation error on optimum portfolios (see Chen and Brown (1983)). A useful extension would be to pursue the solution where multi-index models describe the return-generating process. EG&P have done some of the analysis (see EG&P (1977) and (1979)). However, they made some simplifying assumptions that, while consistent with models used at the time they derived their results, are less typical of models used today. The use of multi-index return-generating models is standard in today's investment community. A solution that is consistent with the types of models used today would be useful.

The final area that has received a great deal of attention is estimation of the inputs to the portfolio selection problem. We will not attempt to discuss all of this literature. Rather, we will concentrate on the literature that is unique to the portfolio selection area, namely the estimation of the covariance or correlation structure. It is to this discussion that we now turn.

II. Index Models and Covariance Estimates

After mean variance portfolio theory was first developed, there was an enormous amount of work on estimating inputs. For the first time in the literature of financial economics, estimates of correlation coefficients (or alternatively covariances) were required. The principal tool developed for estimating covariances was index models.[4] These models have found wide application beyond estimating covariance structures, and are worth reviewing on their own.

A. Index Models

The earliest index model that received wide attention was the single index model, and in particular one variant of the single-index model, the market model. This was first discussed in Markowitz, but was developed and popularized by Sharpe (1967). The market model is

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}$$

where

1. R_{it} is the return of stock I in period t.

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[4] Index models fitted to historic data produce the same estimates of historic average returns and historic variance as do using the raw data itself. The only difference is in the estimates of correlations.

2. α_i is the unique expected return of security I.

3. β_i is the sensitivity of stock I to market movements.

4. R_{mt} is the return on the market in period t.

5. e_{it} is the unique risky return of security I in period t and has a mean of zero and variance $\sigma^2_{e_i}$.

From the point of view of portfolio inputs, the important characteristics of the use of the market model were that the number of estimates required was reduced and the accuracy was increased. In particular, if there are *N* securities, direct estimates of all pairwise covariances require $\frac{N\,(N-1)}{2}$ estimates, while using the market model to estimate covariances required N betas and one market variance, or N+1 estimates. For 100 securities, this reduced the estimation requirement from 4,950 estimates to 101. In addition, even using historical data to estimate the market model, the accuracy of the market model in estimating covariances was higher than direct estimation (see, for example, Elton and Gruber (1973)). Furthermore, if subjective estimates are used or if subjective modification of historical data is used, the reduced data requirements of the single-index model should lead to improved forecasting. Finally, a steel analyst can better understand the relationship between steel and the market than between steel and General Foods, and therefore is likely better able to meaningfully estimate betas than covariances.

The single-index model quickly took on a life well beyond its use in estimating inputs. A large number of firms were started whose principal business was estimating betas. These firms developed sophisticated techniques that improved estimation over simply fitting a regression to historical data.[5]

In addition, many financial institutions estimate or use others' estimates of beta. Although many of the estimates found their way into portfolio algorithms, more commonly they were used to understand and manage the market exposure of the portfolio. In particular, since betas are additive, security betas help a manager ascertain the impact of adding a security to the overall market exposure of the portfolio.

Shortly after the market model was developed, a number of researchers started to explore whether multi-index models better explained reality. The prototype multi-index model is

$$R_{it} = \alpha_i + \sum_{j=1}^{J} \beta_{ij} I_{jt} + e_{it} \qquad i = 1,\ldots,N$$

where

1. β_{ij} is sensitivity of security I to index j.
2. I_j is the jth index.
3. J is the total number of indexes employed.

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[5] See Elton and Gruber (1995), Ch. 5.

4. Other terms as before.

The basic issue was what were the indexes and how many were there. Early work was primarily statistical in nature. Indexes were extracted from the variance-covariance matrix of returns using either factor analysis or principal components analysis (see Roll and Ross (1980), Dhrymes, Friend and Gultekin (1984), Brown and Weinstein (1993), and Cho, Elton and Gruber (1984)). The alternative way to estimate a multi-index model was to pre-specify a structure. Three types of pre-specifications are used:

1. Market plus industry indexes (see Cohen and Pogue (1967)).
2. Surprises in basic economic indexes (e.g., production, inflation) (see Chen, Roll and Ross (1986)).
3. Portfolios of traded securities (e.g., an index of small minus large securities) (see Fama and French (1992)).

The use of models in which indexes are pre-specified is gaining acceptance and is likely to be the dominant form of multi-index models in the future. Statistical estimates of factors are likely to be used primarily to help confirm that the pre-specified indexes capture all of the major influences and that they are important in explaining returns.

Multi-index models can be used to provide inputs for a portfolio optimization technique. The analyst needs to have estimates of N times J betas and the variances of the J indexes. However, as with single-index models, multi-index models are widely used in other ways. Multi-index models are the building blocks for arbitrage pricing theory.

Multi-index models are also used by portfolio managers to understand the sensitivity of the portfolio to various economic influences and to allow the manager to make active bets on how the indexes will change in the next period. As shown in a later section, multi-index models are the basic tool for evaluating fund managers. Finally, multi-index models can be used to reformulate mean-variance portfolio theory in a way that may be more meaningful to managers. This use is discussed in the next section.

Currently, different types of multi-index models have been used for different applications, and this will likely continue in the future. A manager interested in understanding the sensitivity of the portfolio to basic economic influences needs a multi-index model that captures these influences. The same manager is likely to use a model expressed in terms of tradeable portfolios of securities when evaluating performance, and may use a market-plus-industry model in deciding on sector rotation.

Multi-index models also are useful in understanding and visualizing the portfolio choice. This is the topic to which we now turn.

B. Portfolio Analysis in a Multi-Index World

In the previous section we saw that single and multi-index models not only simplify the inputs to the portfolio selection problem, they also can lead to better forecasts and make the selection process easier to comprehend. If we accept index models as a sufficient description of reality, they also allow us to simplify the construction of the optimal portfolio.

In the previous section we reviewed portfolio selection when the single-index model is employed. If we make the additional assumption that the Capital Asset Pricing Model holds and we ignore Fama's (1968) insight that the residuals from the market model cannot be uncorrelated and we allow short sales, then the Elton, Gruber and Padberg (1976) solution reduces to the Treynor and Black (1975) solution that the investment in any stock should be proportional to the ratio of alpha to the variance of residual risk.

The reformulation of the portfolio problem for multi-index models was first analyzed by Ross (1978) and Ingersoll (1987). When returns are generated by a multi-index model, the portfolio problem can be viewed as choosing among portfolios in multi-dimensional space with one dimension for expected return and an additional dimension for each beta. When some securities are mispriced, an additional dimension must be added for residual risk. Although this is equivalent to the mean variance problem, managers may relate better to this choice. In particular, a manager may find it easier to think in terms of choosing exposures to risks and how much residual risk they will accept rather than total risk. The general case where indexes cannot be exactly replicated (e.g., they may be macroeconomic influences), where securities exist which are priced out of equilibrium and indexes are not necessarily normally distributed, is analyzed by Elton and Gruber (1989).[6] They show that in this case each investor can form an optimum portfolio by choosing from among an investment in the riskless asset plus a number of portfolios equal to the number of indexes generating returns plus one. The solution involves forming one replicating portfolio for each index (simple rules are presented for forming those portfolios) plus one portfolio which is made up of mispriced securities. Elton and Gruber (1992b) prove that the optimum proportions in the portfolio of mispriced securities are proportional to the product of the inverse of the variance-covariance matrix of residuals and the vector of alphas. If we assume that the coviarance between residuals is equal to zero, this reduces to the ratio of each alpha to the variance of each residual risk.[7]

Fama (1994) develops models of equilibrium and portfolio optimization in the world of Merton's Intertemporal Capital Asset Pricing Model. While Fama's results are analogous to those presented above, the multi-period nature of the problem requires the addition of one more portfolio, the market portfolio, to be included in the set of portfolio choices given to the investor.

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[6] When indexes cannot be replicated exactly, there is no mathematical inconsistency in assuming residuals from the model are not correlated.

[7] This solution reduces to that presented by Ross (1978) and Ingersoll (1987) when indexes can be replicated perfectly (e.g., when they are tradeable portfolios).

III. Portfolio Optimization with Investor Liabilities

We believe that one of the important future directions for portfolio theory involves the explicit inclusion of liabilities into the asset allocation decision. While conceptually easy to solve, the implementation of a system to include both liabilities and assets in a manner that produces real insight is much more difficult.

At one extreme, liabilities have been treated in the immunization literature. The idea of simply holding a set of bonds that has the same duration as, or that will have the same cash flows as, a set of liabilities has been treated in great detail in the literature of financial economics. At the other extreme, the idea of simply treating liabilities as assets with negative cash flows and constraining them to be present in the QPS solution to a portfolio optimization problem has been proposed. The first idea assumes that the user is interested in a minimum variance solution. The second provides little insight into what is of necessity a problem with a multi-period time dimension. Neither takes into consideration the fact that cash flows from liabilities are uncertain and subject to systematic risks which are similar to those for asset returns.

Elton and Gruber (1992) formulate the asset liability problem, where both assets and liabilities are related to a one-index model, and develop an equilibrium model and a portfolio theory where equilibrium exists but some assets are out of equilibrium. The special role of duration and cash flow matching is developed and the analysis shows robust conditions under which cash flow matching some, but not all, of the liabilities is desirable.

We believe that the extension of this analysis employing a multi-index framework has great potential. Liabilities are usually not certain, but often depend on influences such as inflation and changes in interest rates. Similar influences affect the return in assets. By formulating both liabilities and asset returns as a function of indexes which affect these returns (and in the case of liabilities affect the amounts), solutions to the portfolio problem can be reached that allow investors to make tradeoffs in terms of not only expected return and total risk but in terms of the type of risk to which they will be exposed.

IV. Portfolio Evaluation

Up to this point we have been concerned with how to construct an optimal portfolio. We have also discussed the fact that the payoff from portfolio management models is in large part a function of the type and quality of the data used. It would be inappropriate to end this review article without a discussion of portfolio evaluation. Not only do evaluation models follow naturally from the theories of portfolio management, it is only by employing evaluation measures that we can find whether the combination of portfolio management techniques and the type of data used add value. If managers cannot add value to passive portfolios, why bother with portfolio management models at all?

Concern with portfolio performance predates the advent of modern portfolio theory. One of the early interesting studies on performance evaluation was published by Alfred Cowles (1933). In this study he compared the average performance of a set of managed portfolios to a passive portfolio and concluded that the managed portfolios underperformed the passive benchmark. Cowles examined return but ignored any consideration of risk.

What modern theory has taught us is that we need to be concerned with risk as well as return in examining performance. It wasn't long after Markowitz's path-breaking contribution that techniques were developed and applied (primarily to mutual fund data) for evaluating performance based on risk and return.

Early studies employed a variety of evaluation techniques. These included the Sharpe ratio (Sharpe (1966)), the Treynor ratio (Treynor (1965)), Jensen's alpha ((1968) and (1969)), and Friend, Blume and Crockett's (1970) use of randomly generated passive portfolios of the same risk. Each of these studies evaluated performance adjusting for a measure of risk. Some used total risk (Sharpe, and Friend, Blume, and Crockett) as the correct measure. Others (Treynor, Jensen, and Friend, Blume and Crockett) used beta as the correct measure of risk. While Friend, Blume and Crockett evaluated a managed portfolio against randomly generated unmanaged portfolios of the same risk, each of the other authors used the fact that combinations of any portfolio and the riskless asset lie along a straight line in either expected return beta space or expected return standard deviation space to evaluate performance.

Because these measures are discussed in most books of portfolio management or investment analysis, we will not review them in detail.[8] Instead, we will use Jensen's measure as a benchmark for further discussion. Jensen's alpha is the intercept from the following time series regression:

$$R_{Pt} - R_{Ft} = \alpha_p + \beta_p (R_{mt} - R_{Ft}) + e_{Pt}$$

where

1. R_{pt} is the return on the portfolio being evaluated at time t.
2. R_{Ft} is the riskless rate in period t.
3. R_{mt} is the return on the reference portfolio.
4. β_p is the sensitivity to the reference portfolio.
5. e_{Pt} is the mean zero random error.

The alpha from this regression can be defined as the abnormal return above what would be earned if the Capital Asset Pricing Model (CAPM) held and thus the non-equilibrium return the manager earned. There is a simpler and more intuitive explanation. Alpha represents the return the manager earned over a combination of an index fund (the market portfolio) and Treasury bills, where the combination is selected to have the same

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[8] See, for example, Elton and Gruber (1995), Chapter 24.

risk as the managed portfolio. Since the investor is free to place some money in an index fund and some in riskless assets, this is an appealing way to justify this model.

In the remainder of this article we will discuss some of the problems and progress that have occurred in employing the early models of portfolio performance.

A. Jensen's Measure and Benchmark Identification

The problem with identifying the correct index to use in the Jensen model has been discussed in detail in Roll (1978). The efficiency of the index is of key importance. If the index is not efficient, then performance attributed to any fund becomes a function of the particular index selected. Relying on the defense that alpha simply represents the abnormal return over holding the portfolio of an index fund (e.g., the S&P 500 Index) and Treasury bills with the same beta also fails because it ignores the choices available to the manager. For example, small stocks (low market value stocks) have outperformed the S&P 500 Index for long periods of time. A small stock mutual fund manager with no selection ability or even negative selection ability would show superior performance during such periods when compared to a combination of riskless assets and an S&P 500 index fund.

The problems inherent in the use of a single-index model have led to the development of multi-index performance measures. In answer to Roll's critique, we know that if returns are generated by no more than N factors, then N diversified portfolios are sufficient to describe relative returns and a linear combination of the N diversified portfolios will be efficient (see Ross (1978) and Grinblatt and Titman (1987)). A second defense for a multi-index model comes directly from arbitrage pricing theory, which postulates that expected returns can be expressed as a linear function of sensitivities to more than one index. Thus, deviations from this linear function are a measure of a manager's selection ability.

Finally, returning to viewing the manager relative to a passive fund or funds, it is clear that comparing active funds in total or a particular active fund to a passive strategy is inappropriate if these funds hold very different types of securities from the passive portfolios with which they are being compared.

This is made very clear in a series of recent articles. Ippolito (1989), using data for 1965-1984, found that a sample of mutual funds had positive alphas from the Jensen model. In addition, mutual funds with higher expenses had at least as good performance as those with lower expenses, even after fees were deducted in computing performance and, finally, load funds outperformed no-load funds. An examination of Ippolito's sample showed that funds in his sample had much more money in small stocks than that implied by the composition of the S&P 500 Index and the weighting of stocks in the S&P 500 Index. In fact, a number of the funds had as their objective the construction of a portfolio of small stocks. It is easy to correct for small stocks by introducing another index into a Jensen-type model. In the simplest case the model becomes

$$R_{Pt} - R_{Ft} = \alpha_p + \beta_p (R_{mt} - R_{ft}) + B_{ssp} (R_{sst} - R_{Ft}) + e_{pt}$$

where

1. R_{sst} is the return on an index of small stocks in period t.

2. Other terms as before.

During the period studied by Ippolito, small stocks had a much bigger return than large stocks, in excess of 10% annually. Furthermore, the average mutual fund had a large positive beta with the small stock index. When this multi-index model is used to determine abnormal returns (α), the results indicate that funds on average have negative alphas and funds which have higher expenses tend to do worse than funds with smaller expenses, and load funds underperform no-load funds.[9] Thus, the use of an appropriate model reversed the conclusions.

On a theoretical and practical basis, multi-index versions of the Jensen measure are more appropriate than single-index versions. This still leaves us with the problem of which multi-index models to use. In a sense this is a return to our earlier section on multi-index models where we discussed alternative ways of specifying them. Several authors (e.g., Lehmann and Modest (1987) and Connor and Korajzcyk (1991)) have examined the use of multi-index models derived statistically to evaluate mutual fund performance.

A second approach to developing multiple indexes is to find portfolios of securities which span the types of investments held by mutual funds. This approach has

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[9] See Elton, Gruber, Das and Hlavka (1993).

been used by Sharpe (1992), Elton, Gruber and Blake (1996a), and Blake, Elton and Gruber (1993). For example, Elton, Gruber and Blake (1996a) employ a four-index model involving the S&P 500 Index, a size-related index, a bond index, and a growth-value index to explain the return on domestic non-specialized mutual funds. Sharpe uses a 12-index model to explain the return on a broader set of mutual funds including domestic as well as international bond funds and stock funds.[10]

The appropriate set of indexes to use to evaluate portfolio performance has not been completely resolved. However, recent papers by Sharpe (1992) and Elton, Gruber and Blake (1996c) indicate that good models already exist. In particular, Elton, Gruber and Blake (1996c) show that no more than five indexes are sufficient to explain the returns on domestic stock funds and suggest that performance can be meaningfully evaluated in terms of these five easily identifiable indexes.

B. Return Ratio

Up to now we have been describing multi-index forms of Jensen's alpha as the appropriate measure to use for portfolio evaluation. In fact, Sharpe (1994) has formalized a generalized version of the Sharpe ratio as a useful alternative for performance

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[10] Another advantage of using these models is that the measured betas indicate the type of securities each fund is holding. Elton, Gruber and Blake fit their model via regression analysis. Sharpe uses quadratic programming. Sharpe's approach, like regression analysis, fits the betas to minimize squared deviation from the regression plane, but by using QPS, Sharpe introduced constraints so that the sum of the betas equals 1 and no beta is negative. He interprets the betas from these constrained regressions as being investment proportions.

measurements in a multi-index world. Rather than measure the ratio of difference between average return on a portfolio and the riskless rate to the standard deviation of the portfolio as in the original Sharpe ratio, the generalized ratio suggested by Sharpe uses the ratio of the difference between the average return on the portfolio and the benchmark portfolio (which can be a combination of several portfolios, e.g., a multi-index model) to the standard deviation of the difference.[11] While there are several ways to identify the benchmark portfolio and estimate sensitivities to it, if this is done via a time series analysis of a multi-index model, the Sharpe measure becomes $\frac{\alpha_p}{\sigma_{ep}}$.

Notice that both the multi-index Sharpe measure and a multi-index version of the Jensen measure will identify the same funds as having superior performance. However, they can rank funds in different order.[12]

C. Forecasting

We have examined some alternative measures of performance, but we have not yet faced an important issue. The major purpose in analyzing past performance is to gain insight into the future. If past performance is unrelated to future performance, then

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[11] Sharpe uses QPS rather than regression analysis to identify differential return and risk, and requires the betas to sum to one. The results from this estimation will be somewhat different, since the weights are different and this affects both α_p and σ_{ep}.

[12] It is worth noting that Elton and Gruber (1992b) show that under reasonable sets of assumptions the optimal amount to invest in an actively managed fund in an actively managed portfolio of funds is proportional to $\frac{\alpha_p}{\sigma_{ep}^2}$.

performance evaluation is of no help when selecting a fund or a manager. Four recent studies have found persistence in mutual fund performance: Hendricks, Patel and Zeckhauser (1993), Grinblatt and Titman (1992), Sharpe (1994), and Elton, Gruber and Blake (1996a). The last two studies use multi-index models based on portfolios of securities. Elton, Gruber and Blake (1996a) document that the top 10% of funds ranked on past alpha over a one- and three-year period not only provide superior risk-adjusted return in future one- and three-year periods, they produce positive alphas in a period when alphas are negative for the fund industry as a whole. Finally, when modern portfolio theory is followed and funds are weighted proportionally in the top decile by $\frac{\alpha_p}{\sigma^2_{ep}}$ rather than equally weighted, the subsequent alpha is significantly increased.

Past performance when risk adjustment is done carefully through the use of an appropriately designed multi-index model allows investors to select funds that have superior future performance.

D. Timing

The use of a single or multiple index model to judge performance and to forecast performance assumes that betas (sensitivities) are reasonably constant and that management does not change them to gain added return through market timing. There are a number of papers (e.g., Treynor and Mazuy (1966), Lehmann and Modest (1987), and Hendricksson (1984)) that examine the reasonableness of this assumption. The general conclusion is that timing does not increase risk adjusted returns and may even lower them.

The more immediate question for this article is whether changing betas cause the betas on multi-index models to be so unreliable as to be of no value. The answer appears to be "no." Large changes in the portfolio betas in the multi-index model would cause betas to be poorly estimated and should cause past alphas to be unrelated to future alphas. This does not seem to be the case for most funds. Both Sharpe (1994) and Elton, Gruber and Blake (1996a) find that past alphas have some predictive power. However, Elton, Gruber and Blake (1996a) show that alphas are more predictable if funds for which the model has low correlation (R^2<.80) during the fit period are eliminated. This does suggest that these funds for which the model does not fit well, which may include funds attempting to time the market, should be examined separately and are worthy of further study.

E. Why Bother With Performance?

In closing this section, we want to discuss why we bother with performance. In a perfectly efficient market we would expect performance to be random over time. While some funds might outperform an appropriate passive strategy and others underperform it, the difference would be strictly random over time. On the other hand, if superior management exists, then unless this performance is reflected in higher fees, we would expect to find persistence in performance. The facts that mutual funds (and, we suspect, managers in general) do not raise fees to reflect performance and that fees as a percent of assets tend to be lower for good performing funds mean that, if superior management exists, it should be reflected in persistence in performance. The fact that performance shows persistence suggests that at least some managers have superior information and gives us hope that modern portfolio theory can be used to benefit the investor.

Final Comments

In this article we have attempted to review modern portfolio theory and to highlight some topics and areas of future research. Because of contraints on time and space, we have not included references to many areas of interesting research. We apologize to the authors of the hundreds of interesting papers in portfolio theory that we have not cited in this paper.

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