Portfolio Choice with Many Risky Assets, Market Clearing and Cash Flow Predictability

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Abstract

This paper examines portfolio allocations and market clearing prices when the representative agent can allocate across equity portfolios formed on the basis of characteristics like size and book-to-market and portfolio cash flows are predictable. The state space is discrete and price-consumption ratios are obtained portfolio by portfolio simply by inverting an economy-wide matrix and multiplying this matrix by a portfolio-specific vector. The economy-wide matrix has the dimensionality of the state space. The paper calibrates cash flow predictability to the data using the consumption-wealth fraction (*cay*) of Lettau and Ludvigson (2000a) and dividend yield (*div*) as state variables. Annual cash flow processes are calibrated for three stock portfolios and for the aggregate consumption stream. The economy's representative agent possesses a relative risk aversion coefficient of either 5 or 10.

When cash flow predictability is calibrated to the data using *cay* as the predictor and risk aversion is 5, equilibrium excess returns on the four assets are more volatile, more correlated with each other, and have higher means than in the equivalent economy with i.i.d. cash flows. Further, the conditional second moments for returns and the contemporaneous state variable are found to be highly state-dependent. The paper finds much smaller excess return predictability using *cay* in the calibrated economy than in the data, though the relation is positive in both. Conditional Sharpe ratios are virtually invariant to state.

While the representative agent's optimal portfolio is not very state-dependent, her hedging demands are quite large and her optimal portfolio is not minimum-variance. For example, her single-period allocation to the four risky assets is about 75% of the portfolio while her infinite-horizon allocation is 100%. The implication is that the conditional CAPM does not hold in the conditional economy with *cay* as the state variable. However, the spread in CAPM abnormal returns across the three book-to-market portfolios is an order of magnitude smaller in the calibrated economies than in the data. The spread in the data in 5.6% p.a. while the largest spread in the six calibrated economies considered is only 0.6% p.a. Finally, the paper has important implications for partial equilibrium analyses of dynamic portfolio choice.

JEL classification: G11; G12**.**

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1   Introduction

Empirical research indicates that U.S. equity returns are predictable and that investor horizons are longer than a month.[1] Using Merton (1973) and Fama (1970), the implication is that investor's care about more than just mean and standard deviation when choosing their portfolios. Instead, investors are also concerned about the covariance of their portfolios with the state variables that forecast returns. This concern can affect portfolio allocations by generating hedging demands. Lynch (2000) considered this effect for an investor with power utility in a partial equilibrium setting. With the return generating processes calibrated to match U.S. equities, the investor's risky asset portfolio tilts away from the high book-to-market portfolio and toward the low, as she becomes younger and adopts a more multi-period perspective. An obvious concern is that allocations are state-dependent and are unlikely to clear markets, making it difficult to think of the investor as a representative agent. The paper is not alone with this shortcoming. A number of recent papers have performed partial equilibrium analyses of the portfolio allocations by a multi-period investor facing return predictability. While these papers have produced a number of important and intriguing results, they remain partial equilibrium with the limitations that this entails.

This paper remedies this shortcoming by starting with cash flows rather than returns and calculating prices so that markets clear state by state. The discrete state-space economy is carefully specified so that prices can be obtained by inverting just one matrix whose dimension equals that of the state space. The simple solution technique means that a wide range of economies can be easily considered using the setting of this paper. Once market-clearing prices have been determined, the magnitude of any hedging demand can be assessed as in the partial equilibrium papers on portfolio choice. The difference is that my investor can be naturally regarded as a representative agent since markets clear. Consequently, the investor's optimal portfolio can be viewed as the market portfolio and CAPM abnormal returns can be calculated. Lynch (2000) calculated abnormal returns using the young investor's optimal portfolio as the "market" portfolio. However, since his investor's allocations do not clear markets, the investor's portfolio is not really the market portfolio. The

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[1]Campbell (1987) and Fama and French (1989), among others, find that stock return variation can be explained by the one-month Treasury bill rate, the term premium, and the dividend yield.

current paper solves this problem.

The paper is clearly related to a long line of macroeconomics literature dating back to Lucas (1978) that backs out equity returns using a representative agent in economies where the consumption good market clears, and aggregate consumption is calibrated to U.S. data. The current paper extends this literature by describing a setting that allows multiple risky assets and cash flow predictability to be tractably considered simultaneously. As such, this paper is the first to explicitly link the partial-equilibrium portfolio choice literature with the general-equilibrium representative-agent literature. This link would seem crucial in gaining an understanding of how cash flow predictability affects the cross-section of risky asset returns.

Standard representative agent models cannot match asset return data along three important dimensions, at least when the agent has time-separable utility with reasonable risk aversion. In particular, the standard model is unable to explain the high Sharpe ratio for equity, the low riskfree rate and the high equity volatility observed in the data. These shortcomings are known respectively as the "equity premium" puzzle, first documented by Mehra and Prescott (1985), the "risk-free rate" puzzle (see Weil, 1989), and the "equity volatility" puzzle (see Campbell, 2000). This paper provides one of the first opportunities to examine how the standard model fares along another dimension: namely, the cross-section of equity returns.

The paper calibrates cash flow predictability to the data using the consumption-wealth fraction (*cay*) of Lettau and Ludvigson (2000a) and dividend yield (*div*) as state variables. Annual cash flow processes are calibrated for three stock portfolios and for the aggregate consumption stream. The stock portfolios are formed on the basis of book-to-market and together constitute the entire U.S. stock market. The economy's fourth asset is a non-financial wealth portfolio whose cash flow is the difference between the aggregate cash flow and the total stock market cash flow. The economy's representative agent possesses a relative risk aversion coefficient of either 5 or 10. Larger risk aversion values are not considered because of evidence that 10 is an reasonable upper bound for this parameter (see Mehra and Prescott, 1985). There are three possible economies for each risk aversion value: the unconditional economy (UEcon) with i.i.d. cash flows, and two conditional economies (CEcons), one with *cay* as the state variable and the other with *div* as the state variable.

2

The paper has a number of interesting results. Since prices are endogenous, the effect of cash flow predictability on equilibrium asset prices can be examined. When cash flow predictability is calibrated to the data using *cay* as the predictor, equilibrium excess returns on the four assets are more volatile, more correlated with each other, and have higher means than in the equivalent economy with i.i.d. cash flows. Further, the conditional second moments for returns and the contemporaneous state variable are found to be highly state-dependent. The paper finds much smaller excess return predictability using *cay* in the calibrated economy than in the data, though the relation is positive in both. Even so, the excess-return predictability in the calibrated economy is important since it ensures that the conditional Sharpe ratios remain largely invariant to the *cay* state. In particular, the return predictability is such that the cross-state variation in mean excess return exactly offsets the cross-state variation in return volatility to leave the conditional Sharpe ratios constant.

When cash flows are predictable using *cay*, the weights of the assets in the aggregate wealth portfolio are largely invariant to *cay* state, and are similar to the weights in the equivalent economy with i.i.d. cash flows. Since these weights are also the representative agent's optimal portfolio weights, the implication is her optimal portfolio is also not very state-dependent. Even so, her hedging demands are quite large. For example, her single-period allocation to the four risky assets is about 75% of the portfolio while her infinite-horizon allocation is 100%. Thus, when *cay* is the state variable, the representative agent's infinite horizon makes the risky assets more attractive. The reason is that in the economy, asset returns have negative contemporaneous covariances with *cay*, a variable that is positively related to the quality of future opportunity sets. As a consequence, holding the risky assets provides the agent with a hedge against future shifts in the opportunity set, and because her risk aversion is greater than 1, the agent finds this hedge valuable.

Moreover, when cash flows are predictable using *cay*, the representative agent's optimal portfolio is not conditionally minimum-variance. The agent is prepared to accept additional volatility relative to the minimum-variance portfolio to obtain a larger negative contemporaneous covariance with *cay*. Since the representative agent's optimal portfolio coincides with the aggregate market portfolio, the implication is that the conditional CAPM does not hold in the conditional economy with *cay* as the state variable.

The paper also calculates CAPM abnormal returns relative to two benchmarks: the value-weighted stock market portfolio; and, the aggregate wealth portfolio. Abnormal returns are calculated relative to both benchmarks in the calibrated economies but only relative to the stock market portfolio in the data. These calculations are in the same spirit as Campbell and Cochrane's (2000) evaluation of the performance of the consumption CAPM and other pricing models in their habit-persistence economy. However, it is the performance of the conditional CAPM that is the focus of the calculations here, which allows two important comparisons to be performed. The magnitude and pattern of CAPM abnormal returns in the data can be compared to those in the economies, and the effect of using a stock market portfolio to proxy for the total market can be assessed. I find that the spread in abnormal return across the three book-to-market portfolios is an order of magnitude smaller in the calibrated economies than in the data. The spread in the data in 5.6% p.a. while the largest spread in the six calibrated economies is only 0.6% p.a.. The question arises whether this smaller spread is due to the equity volatility puzzle described above. To address this question, the abnormal returns are levered up under the assumption that the lower volatility in the calibrated economies is due to lower firm leverage. The levered-up spreads are still less than 30% of the spread in the data. This finding suggests that standard representative-agent models are unable to generate the spread in CAPM abnormal returns in the data, at least for reasonable risk aversion values for the representative agent.

Finally, the utility cost of ignoring cash flow predictability is found to be non-zero for an individual investor with the same utility function as the representative agent. This cost is increasing in the representative agent's risk aversion and is larger when *div* is the predictive variable rather than *cay*. However, this partial equilibrium cost is not indicative of the cost to the entire economy of the cash flows becoming i.i.d.. Rather, because her utility function is time separable and the unconditional cash flow distribution remains the same, the representative agent receives the same utility irrespective of whether cash flows are predictable or i.i.d..

The paper has important implications for partial equilibrium analyses of dynamic portfolio choice. First, it shows that market clearing typically requires state dependent second moments, even when the cash flows are calibrated to be homoscedastic. In so doing, the paper highlights the importance of allowing second moments to be state-dependent when examining portfolio choice in

4

a partial equilibrium setting. Moreover, recent papers by Lynch and Balduzzi (2000), Liu (2000), and Chacko and Viceira (1999) find that heteroscedasticity calibrated to the data can have large effects on portfolio allocations. Second, utility cost calculations reported in these papers cannot be used to assess the benefits of predictability for an entire economy. The main reason is the shift in equilibrium returns that occurs when the economy's cash flows become predictable is not incorporated into these partial equilibrium cost calculations.

A number of recent papers address the issue of portfolio choice by an investor facing return predictability.[2] Almost all of these papers have a single risky asset available to the investor, and only a couple of these papers allow the investor to allocate across portfolios of stocks formed on the basis of equity characteristics like size and book-to-market. Recent papers by Pastor (2000) and Pastor and Stambaugh (2000) examine portfolio allocations to size and book-to-market equity portfolios by a Baysian investor facing uncertainty about the true pricing model. However, the investor is myopic and returns are not predictable. Brennan and Xia (2000) allow a dynamic investor who is learning about return means to allocate across book-to-market portfolios. Any return predictability in their setting is driven by the investor learning about the mean from recent return realizations. Moreover, all these portfolio choice papers specify the return-generating process exogenously, which means that the analysis is always partial equilibrium in nature. The current paper does not suffer from this limitation since prices are endogenous and clear markets.

The standard representative agent paradigm has been extended in several directions in an

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[2]Kandel and Stambaugh (1996) explore the effects of ignoring predictability in a myopic setting, while Brennan and Schwartz (1996), Brennan, Schwartz, and Lagnado (1997) and Barberis (2000) analyze numerically the impact of myopic versus dynamic decision-making. Campbell and Viceira (1999) use log-linear approximations to solve the investor's multi-period discrete-time problem, while Kim and Omberg (1996) and Liu (1998) obtain exact analytical solutions for a range of continuous-time problems with predictability. Brandt (1999) uses the investor's Euler equations and U.S. stock returns to estimate the investor's portfolio allocation to stocks. Balduzzi and Lynch (1999) and Lynch and Balduzzi (1999) solve numerically the investor's multi-period problem with transaction costs. Brennan, Schwartz and Lagnado (1997) and Campbell and Viceira (2000) allow investors to hold long-term bonds in addition to stocks while Ang and Bekaert (1999) consider the portfolio allocation problem when investors can invest in country funds. Campbell, Chan and Viceira (1999) and Ait-Sahalia and Brandt (2000) examine dynamic portfolio allocation with more than one predictive variable.

effort to explain the three asset pricing puzzles described earlier. A number of papers have advanced friction-based explanations. Grossman and Laroque (1990) and Marshall and Parekh (1999) use consumption adjustment costs (specifically for durable goods), Lynch (1996) allows for unsynchronized and infrequent decision-making by investors, and He and Modest (1995), Luttmer (1992) and Cochrane and Hansen (1992) all use some combination of borrowing and short sale restrictions and asset market transaction costs. Idiosyncratic and uninsurable labor income risk is another mechanism that can generate larger Sharpe ratios for equities (see Mankiw, 1986, Lucas, 1994, Telmer, 1993, and Heaton and Lucas,1996). Finally, time and state nonseparabilities have been introduced into the representative agent's utility function. Examples of this approach include the habit formation papers of Constantinides (1990), Sundaresan (1989), and more recently Campbell and Cochrane (1999). In fact, recent work by Dai (2000) and Wachter (2000) suggests that an appropriately specified habit utility function for the representative agent allows aggregate consumption to simultaneously explain the equity premium and riskfree rate puzzles, as well as the "expectations" puzzle documented for the term structure. The current paper suggests that the cross-section of equity returns provides yet another challenge for the standard model that these more general models must also be asked to explain.

The current paper's setup is perhaps most closely related to two papers by Kandel and Stambaugh (1990 and 1991) that examine how return moments for equity and the riskfree asset are affected by consumption predictability calibrated to the data. While the structure of the economies are similar in all three, their papers only consider a single equity portfolio, and pay little attention to the implications of the equilibrium for the agent's portfolio allocation decisions. In contrast, the current paper has multiple equity portfolios formed on the basis of book-to-market and considers a range of portfolio allocation issues.

Finally, a number of recent empirical papers have made contributions to our understanding of the cross-section of expected equity returns, particularly the well-documented relation between expected return and book-to-market, which remains after controlling for CAPM Beta. Campbell (1996) examines empirically whether cross-sectional variation in expected returns can be explained using the Euler equation from the multi-period investor's problem. Fama and French (1993) and (1995) explore risk-based explanations for the book-to-market effect, while Ferson, Sarkissian and

6

Simin (1999) show that this evidence does not always imply a risk-based explanation. Daniel and Titman (1997) show that the book-to-market effect is being driven by the book-to-market characteristic and not the Fama-French risk loading, while Lakonishok, Shleifer and Vishny (1994) argue that the effect is due to market inefficiency or suboptimal investor behavior. Jagannathan and Wang (1996), Ferson and Harvey (1999) and Lettau and Ludvigson (2000b) test conditional versions of various pricing models, while Lamont (1999) and Liew and Vassalou (2000) examine the ability of stocks to hedge economic risks. None of these papers examine how the cash flow predictability in the data affects asset returns and the representative agent's portfolio allocation in equilibrium as the current paper does.

The paper is organized as follows. Section 2 describes the setup of the artificial economies and how equilibrium prices are calculated, while section 3 discusses the portfolio allocation issues addressed by the paper. Section 4 describes the data and the calibration technique employed, while the paper's main results are presented in section 5. Conclusions and directions for future work are contained in section 6.

2   The Economy

2.1   The Cash Flows

Consider an endowment economy where N risky assets plus a riskless asset are available for investment. Without loss of generality, I assume that there is a single Markovian state variable Z_t for the economy with K possible realizations at each date t. The variable k_t provides an index for the state of Z_t: i.e., Z_t is in the (k_t)th state. The number of state variables is not important. Rather, it is the assumption of a discrete state space that allows market clearing prices to be backed out so easily. Thus, the state of the economy at time t can be summarized by k_t. The transition probability matrix Π is a K×K matrix with $(k, \underline{k})$ element, $\pi_{k,\underline{k}}$, that gives the probability of the $\underline{k}$th state occurring tomorrow given the kth state today.

Define D^i_{t+1} to be the dividend from asset i in period $t+1$, and d^i_{t+1} to be the dividend from asset i in period $t+1$ scaled by aggregate dividend in period t. The joint distribution of $\{d^i_{t+1}|\ i=1,$ ..., $N\}$, is assumed to depend only on the Z_t-state for any t. This is why Z_t is the economy's state variable at time t. Let $d^i(k_t, k_{t+1})$ be the random variable of possible realizations of d^i_{t+1}. Without loss

of generality, I assume a finite number of joint realizations of $\{\boldsymbol{d^i}(k_t, k_{t+1})|\ i=1, ..., N\}$, and that the number, S, is the same for all possible (k_t, k_{t+1}) pairs. Thus, for each (k_t, k_{t+1}) pair:

1)   there are S possible joint realizations with s_t referring to the realization at time t,

2) $\boldsymbol{d^i}(k_t, k_{t+1})$ is an S×1 vector whose sth element, $d^i(k_t, k_{t+1}, s)$ gives the d^i_{t+1} value for the sth realization, and,

3)   there is an associated Sx1 conditional density vector $\boldsymbol{p}(k_t, k_{t+1})$ whose sth element $p(k_t, k_{t+1}, s)$ gives the probability of the sth realization conditional on k_t: the elements of $\boldsymbol{p}(k_t, k_{t+1})$ sum to $\pi_{k,k}$.

Finally, let D^{Ag}_{t+1} denote the aggregate dividend in period $t+1$, which by construction must equal the sum of the D^i_{t+1} over $i = 1,..., N$. Similarly, d^{Ag}_{t+1} denotes the aggregate dividend in period $t+1$ scaled by aggregate dividend in period t, and $\boldsymbol{d^{Ag}}(k_t, k_{t+1})$ denotes the vector of its S possible realizations given (k_t, k_{t+1}). The riskfree asset is assumed to be in zero net supply. The set-up of the economy follows Kandel and Stambaugh (1990) except that scaled cash flow is a continuous random variable in their set-up.

The scaling of asset dividends by last period's aggregate dividend is not innocuous. It is this assumption that ensures that Z_t is the only state variable for the economy. However, the assumption is less restrictive than it may appear at first. In particular, while there is considerable empirical evidence that the recent performance by a stock contains information about the stock's future performance, this dependence can be incorporated by allowing $\{d^i_t, i=1, ..., N\}$ to be state variables at time-t.

2.2   The Representative Agent's Problem

The representative agent is infinitely lived with time separable, power utility and a rate of time preference equal to β. The agent knows the state of the economy, Z, at any time t. Expected lifetime utility is given by

$$E\left[\sum_{t=1}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}|Z_1\right] \tag{1}$$

where c_t is agent's consumption at time t, and γ is the agent's relative-risk-aversion coefficient. Let

R_{t+1} denote the Nx1 vector of risky asset returns from time t to $t+1$, whose ith element, R^i_{t+1}, is the return on the ith asset from time t to $t+1$. Let R^f_t denote the riskfree rate available at t. The law of motion of the agent's wealth, W, is given by

$$W_{t+1} = (W_t - c_t) \left[\boldsymbol{\alpha}'_t (\boldsymbol{R}_{t+1} - R^f_t \, \boldsymbol{i}_N) + R^f_t \right] = W_t (1 - \kappa_t) \, R^W_{t+1} \qquad (2)$$

where $\boldsymbol{\alpha}_t$ is the Nx1 vector of portfolio weights chosen for the risky assets at t, R^W_{t+1} is the portfolio return from t to $t+1$, and κ_t is the fraction of wealth consumed at t. The ith element of $\boldsymbol{\alpha}_t$, α^i_t, is the portfolio weight chosen for the ith risky asset at time t. Short-selling is allowed. Hakansson (1970) considering the case without return predictability is one of the first papers to characterize the solution to this type of dynamic problem.

2.3   Market Clearing

Let P^i_t be the value of asset i at time t for $i = 1, \dots , N$, and P^{Ag}_t be the value of the market at time t. By definition, summing P^i_t over $i = 1, \dots , N$, gives P^{Ag}_t. Also, the value of the market at time t plus the aggregate time-t dividend must equal the agent's wealth at time t:

$$P^{Ag}_t + D^{Ag}_t = W_t \qquad (3)$$

Finally, let f_t be the value of asset i at time t scaled by the aggregate dividend at time t for $i = 1, \dots , N$, and f^{Ag}_t be the value of the market at time t scaled by the aggregate dividend at time t.

Market clearing in the goods market requires that consumption equals aggregate dividend:

$$D^{Ag}_t = c_t \iff \kappa_t = \frac{1}{f^{Ag}_t + 1}. \qquad (4)$$

The reason for expressing the clearing condition in terms of κ_t and f^{Ag}_t will be made clear in the next subsection when equilibrium prices are characterized. Turning to the asset market, market clearing requires that agent's portfolio weights equal the weights of the assets in the aggregate market portfolio:

$$\alpha^i_t = \frac{P^i_t}{P^{Ag}_t} = \frac{f^i_t}{f^{Ag}_t}, \quad i = 1, \dots , N. \qquad (5)$$

9

Notice that the zero net supply condition for the riskfree asset is implicitly satisfied by condition **(5)**.

2.4   Equilibrium Prices

This subsection characterizes market-clearing prices in this economy. I first assume that market-clearing prices are such that f^i_t depends only on the Z-state at time t. In particular, for any time t, if Z_t is in the (k_t)th state, then

$$f^i_t = f^i(k_t) \quad i = 1, \dots , N \tag{6}$$

where $f^i(.)$ is a function that does not depend on t. The same must be true for f^{Ag}_t by construction and I define the function $f^{Ag}(.)$ accordingly. The pricing function **(6)** implies the following expression for the return on asset i from time t to time $t+1$:

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\begin{aligned}
R^i_{t+1} &= \frac{P^i_{t+1} + D^i_{t+1}}{P^i_t} \\
&= \frac{f^i_{t+1} \, d^{Ag}_{t+1} + d^i_{t+1}}{f^i_t} = \frac{f^i(k_{t+1}) \, d^{Ag}(k_t,k_{t+1},s_{t+1}) + d^i(k_t,k_{t+1},s_{t+1})}{f^i(k_t)}
\end{aligned} \tag{7}
$$

Thus, the return on asset i from time t to time $t+1$ depends on (k_t,k_{t+1},s_{t+1}) and so can be represented by the function $R^i(.)$ that satisfies $R^i_{t+1} = R^i(k_t,k_{t+1},s_{t+1})$. The riskless asset available at time t is assumed to be a function of only k_t:

$$R^f_t = R^f(k_t). \tag{8}$$

The market clearing conditions **(4)** and **(5)** imply that the agent's choice of κ_t and α_t must only depend on the Z-state at time t. Thus, for the assumed price and return functions **(6)** and **(8)** to clear markets, they must induce the agent to make κ_t and α_t choices that only depend on the Z-state at time t. So the next step is to take the price functions in **(6)** and **(8)** as given and show the representative agent's first order conditions imply choices of κ_t and α_t that only depend on k_t. This is equivalent to showing that k_t is the only state variable for the agent's problem. To show this, I assume that the value function for the agent at time $(t+1)$ is

$$\frac{a(k_{t+1}) \ W_{t+1}^{1-\gamma}}{1-\gamma} \tag{9}$$

and show that the agent's Bellman equation implies a value function at time t that has the same form. Given **(9)**, the Bellman equation for the agent's value function at time t is:

$$\max_{\kappa_t, a_t} \left\{ \frac{\kappa_t^{1-\gamma} W_t^{1-\gamma}}{1-\gamma} + \beta(1-\kappa_t)^{1-\gamma} W_t^{1-\gamma} \frac{1}{1-\gamma} E\Big[a(k_{t+1})R_{W,t+1}^{1-\gamma}|k_t\Big] \right\}$$

$$= \max_{\kappa_t, a_t} \left\{ \frac{\kappa_t^{1-\gamma} + \beta(1-\kappa_t)^{1-\gamma} E\Big[a(k_{t+1})[\ a_t'\ (\boldsymbol{R}(k_t,k_{t+1},s_{t+1})-R^f(k_t)\ \boldsymbol{i}_N)+R^f(k_t)\ ]^{1-\gamma}|k_t\Big]}{1-\gamma} \right\} W_t^{1-\gamma}. \tag{10}$$

It follows immediately from **(10)** that the agent's value function at t has the same form as **(9)** since the expression being maximized in parentheses only depends on k_t.

Having shown that the equilibrium price functions satisfy **(6)** and **(8)**, the last task is to obtain analytic expressions for these functions. Since Z can take on K possible values, it follows that for any i (including $i = Ag$), the function $f^i(.)$ can be represented by a $K \times 1$ vector, $\boldsymbol{f}^i$, with $f^i(k)$ as its kth element. Similarly, the riskfree rate function $R^f(.)$ can be represented as a $K \times 1$ vector, $\boldsymbol{R}^f$, with $R^f(k)$ as its kth element. The first order conditions for the agent's problem are given by

$$E\left[\beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R_t^f \mid k_t\right] = 1, \tag{11}$$

and, for any risky asset i,

$$E\left[\beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R_{t+1}^i \mid k_t\right] = 1. \tag{12}$$

The market-clearing condition for the goods market **(4)** together with **(6)** and **(8)** can be used to rewrite the conditions:

$$E\left[\beta \left(d^{Ag}(k_t,k_{t+1},s_{t+1})\right)^{-\gamma} R^f(k_t) \mid k_t\right] = 1, \tag{13}$$

and, for any risky asset i,

11

$$E\left[\beta \left(d^{Ag}(k_t,k_{t+1},s_{t+1})\right)^{-\gamma} \frac{f^i(k_{t+1})\ d^{Ag}(k_t,k_{t+1},s_{t+1})\ +\ d^i(k_t,k_{t+1},s_{t+1})}{f^i(k_t)}\ \Big|\ k_t \right] = 1. \qquad (14)$$

The first order condition for asset i can be rearranged to obtain:

$$E\left[\beta\left(d^{Ag}(k_t,k_{t+1},s_{t+1})\right)^{1-\gamma} f^i(k_{t+1})\ \Big|\ k_t \right] - f^i(k_t) = -E\left[\beta\left(d^{Ag}(k_t,k_{t+1},s_{t+1})\right)^{-\gamma} d^i(k_t,k_{t+1},s_{t+1})\ \Big|\ k_t \right] (15)$$

An expression for the riskfree rate in state $k_t=k$ can be obtained for $k = 1, \ldots , K$:

$$R^f(k) = \frac{1}{\displaystyle\sum_{\hat{k}=1}^{K}\ \beta\ p(k,\hat{k})^T\left(d^{Ag}(k,\hat{k})^{-\gamma}\right)}. \qquad (16)$$

where a vector raised to a power means each element of the vector is raised to the power. Similarly, for a given risky asset i, the first order condition for state $k_t=k$ can be obtained for $k = 1, \ldots ,K$:

$$\sum_{\hat{k}=1}^{K}\ \beta\ p(k,\hat{k})^T\left(d^{Ag}(k,\hat{k})^{1-\gamma}\right)f^i(\hat{k})\ -\ f^i(k)\ =\ -\sum_{\hat{k}=1}^{K}\ \beta\ p(k,\hat{k})^T\left(d^{Ag}(k,\hat{k})^{-\gamma}.*\ d^i(k,\hat{k})\right) \qquad (17)$$

where $.*$ is element by element multiplication. By defining $Q_{k,\hat{k}}$ and q_k^i appropriately, equation **(17)** can be written more compactly:

$$\sum_{\hat{k}=1}^{K}\ Q_{k,\hat{k}}\ f^i(\hat{k})\ -\ f^i(k)\ =\ -q_k^i \qquad (18)$$

for $k = 1, \ldots , K$. Stacking these K equations gives

$$(\boldsymbol{Q}\ -\ \boldsymbol{I_K})\ \boldsymbol{f}^i\ =\ -\boldsymbol{q}^i \qquad (19)$$

where $\boldsymbol{I_K}$ is the identity matrix, the $(k,\ \hat{k})$th element of $\boldsymbol{Q}$ is $Q_{k,\hat{k}}$, and the kth element of $\boldsymbol{q}^i$ is q_k^i. Finally, an expression for the equilibrium price function for asset i, $\boldsymbol{f}^i$, can be obtained

$$\boldsymbol{f}^i\ =\ -(\boldsymbol{Q}\ -\ \boldsymbol{I_K})^{-1}\ \boldsymbol{q}^i \qquad (20)$$

where the matrix to be inverted is the same for all risky assets.

2.5   Economies Considered

An economy is fully described by the distribution of Z_t and the specification of the joint distribution of the scaled cash flows for the N assets $\{\vec{d}^i(k_t,\ k_{t+1})|\ i=1, \ldots, N\}$. Moreover, such a

specification implies an unconditional distribution for the scaled cash flows $\{d^i|\ i=1, ..., N\}$. For a given distribution of the state variable and the scaled cash flows, equilibrium prices for the conditional economy (CEcon) are calculated, as well as for the economy for which the scaled cash flows are i.i.d. over time with their unconditional distribution (UEcon).

3   Portfolio Allocation Issues.

3.1   Return Generating Processes Considered.

Portfolio choice papers often consider the impact of return predictability on portfolio choice. The investor's allocation when using the predictive variable is contrasted with her allocation when facing i.i.d. returns with the same unconditional distribution. This paper performs the same comparison using the equilibrium return generating process obtained for the conditional economy (CEcon). The one complication is that the riskfree rate is a function of the state. Keeping the generating process for the riskfree rate in the CEcon, the excess risky-asset returns from t to $t+1$ conditional on the state at t are assumed to follow the joint unconditional distribution for excess returns in the CEcon, and be uncorrelated with the riskfree rate at $t+1$. I refer to this return generating process as the conditional economy not using Z. Because prices are endogenous in this paper, I can also compare the allocation made by the investor in the unconditional economy (UEcon) with the these two allocations for the associated conditional economy.

3.2   Hedging Demands.

The Bellman equation of the representative agent is given by **(10)**. An infinitely-lived investor's hedging demand is the difference between her infinite-horizon allocation and her allocation when faced with a single-period horizon. Given this definition, the representative agent's hedging demand can be calculated by comparing the optimal allocation for **(10)** to the optimal allocation for the representative agent's single period problem:

$$\max_{\kappa_{1,t},\alpha_{1,t}}\left\{\frac{\kappa_{1,t}^{1-\gamma}W_t^{1-\gamma}}{1-\gamma} + \beta(1-\kappa_{1,t})^{1-\gamma}W_t^{1-\gamma}\frac{1}{1-\gamma}E\left[R_{W,t+1}^{1-\gamma}|Z_t\right]\right\}. \tag{21}$$

The solution to this problem is an $\alpha_{1,t}$ vector that only depends on k_t. Consequently, the single-period

portfolio allocation rule can be characterized by a function $\alpha_1(.)$ which satisfies $\alpha_{1,t} = \alpha_1(k_t)$ for all k_t.

3.3   Abnormal Return Calculation.

With return predictability, the resulting hedging demands can cause an investor's optimal portfolio to be mean-variance inefficient. As a consequence, abnormal returns calculated relative to the investor's optimal portfolio can be non-zero, where conditional abnormal return is the intercept from a condition regression of excess asset return on excess portfolio return. For return generating processes calibrated to be i.i.d, and in the data, unconditional abnormal returns are calculated analogously. Performing a partial equilibrium analysis that calibrates returns to U.S. data, Lynch (2000) reports abnormal returns calculated in this manner. However, it is difficult to say too much about the numbers he reports since the investor's optimal portfolio is not the market portfolio, especially since the optimal allocations are often negative.   Here, the investor is the representative agent whose allocations clear markets so her optimal portfolio is truly the market portfolio for the economy.

Abnormal returns relative to two benchmarks are presented. The first is the representative agent's optimal portfolio and the second is her optimal portfolio of financial wealth. Many papers testing the CAPM use a market portfolio of U.S. stocks (see, for a recent example, Fama and French, 1993).   The financial wealth portfolio is the appropriate benchmark to use when attempting to compare abnormal returns for my calibrated economies to those obtained by these studies. A few papers have attempted to use a broader market portfolio (see, for a recent example, Jagannathan and Wang, 1996) which in one reason why abnormal returns relative to the representative agent's optimal portfolio are also reported. Another reason is to assess the sensitivity of abnormal return to the use of a stock market proxy for the market portfolio rather than the aggregate market itself. While it is well understood that the using a proxy for the aggregate market can distort abnormal returns (see Roll, 1977), I am able to quantify the extent of the distortion from using a value-weighted stock portfolio as the proxy in the economies that I calibrate.

3.4   Utility Cost Calculation.

Portfolio choice papers also report utility costs associated with ignoring predictability (see for example, Campbell and Viceira, 1999 and 2000, Balduzzi and Lynch, 1999, and Lynch, 2000). The cost number calculated here gives the fraction of her wealth that the investor would give up to be given access to the state variable. This paper reports the cost of ignoring predictability in the conditional economy by using the optimal allocation for the associated i.i.d return generating process. The number is calculated assuming the investor currently using the i.i.d.-return allocation does not know the current state of the economy and that the investor's wealth is the same irrespective of the state.

However, this number does not represent the cost to the representative agent of being in the unconditional economy rather than the conditional economy for two reasons. First, the return generating processes differ for the unconditional and conditional economies. Second, the representative agent's wealth is different in the two economies. Incorporating both effects in the cost calculation must result in a zero cost since the unconditional distribution of the scaled cash flows is the same in the two economies and the representative agent has a time-separable utility function. A partial equilibrium comparison would calculate the cost of being in the unconditional rather than the conditional economy, holding wealth fixed. Results for this partial-equilibrium comparison are reported.

4   Calibration.

This section describes the data, the VARs used to measure predictability, and the quadrature approximation used to calibrate the economies' scaled cash flows to the data.

4.1   Data.

The calibrated economies have four risky assets: three stock portfolios and a non-financial wealth port folio. The three stock portfolios are formed from the six value-weighted portfolios SL, SM, SH, BL, BM, and BH from Fama and French (1993) and Davis, Fama and French (2000). The notation S (B) indicates that the firms in the portfolio are smaller (larger) than 50% of NYSE stocks. The notation L indicates that the firms in the portfolio have book-to-market ratios that place them in the bottom three deciles for all stocks; analogously, M indicates the middle four deciles and H

15

indicates the top three deciles. The high book-to-market portfolio, B3, is a value-weighted portfolio of SH and BH; B2 and B1 are formed similarly.

The choice of firm characteristic to form the stock portfolios is predicated by the aim of achieving a wide dispersion in expected return across the portfolios. Empirical work by Stattman (1980), and Fama and French (1992), (1993) among others finds that average return depends on this variable even after controlling for market Beta. However, a natural concern is data snooping which can result in portfolios with an in-sample dispersion in average return that overstates the dispersion in expected return (Lo and Mackinlay, 1990). Work by Berk (1995) provides a theoretical rationale for using variables that depend on price to obtain dispersion in expected return, and book-to-market satisfies this criterion.

The cash flow for a stock portfolio for a given calendar year is the within-portfolio sum of earnings for the fiscal year ending in that calendar year, scaled by the average dividend payout ratio for the portfolio. The earnings are before extraordinary items but after interest, depreciation, taxes and preferred dividends, while the dividend payout ratio scales the sum of dividends paid in the calendar year by the earnings number. The earnings number is the EI(t) variable used by Fama and French (1994), multiplied by the number of firms in the portfolio. I use earnings scaled by average dividend payout as the cash flow variable rather than dividends, due to concern about dividend smoothing by firms.[3] The cash flow for the aggregate market is taken to be the seasonally adjusted total Personal Consumption Expenditures reported by the U.S. Dept of Commerce, Bureau of Economic Analysis. The nominal quarterly consumption numbers are summed to obtain annual numbers.

All cash flow variables are scaled by last year's aggregate consumption and deflated using annual CPI inflation from CITIBASE to give the scaled cash flows used to calibrate the economies. The sum of the scaled cash flows for the four assets must equal the scaled cash flow for the aggregate market. Thus, the scaled cash flow for the non-financial asset is obtained by taking the scaled cash flow for the aggregate market and subtracting the scaled cash flows for the three stock portfolios.

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[3]I would like to Gene Fama and Ken French for kindly providing me with the cash flow and return data for the three book-to-market portfolios .

Cash flow data is available annually from 1963 to 1998 for the stock portfolios and quarterly from 1947 to 1998 for consumption. The *cay* variable, as recently updated by Lettau and Ludvigson, is available at a quarterly frequency from 1952:2 to 1998:4:, while the dividend yield variable, *div*, is available from 1927 to 1998. Return data for the three stock portfolios is also available back to 1927 (see Davis, Fama and French, 1999). The annual riskfree rate is taken to be the return from a rolling investment in 30 day T-bills aver the year.

4.2   Measuring Predictability.

Following Balduzzi and Lynch (1999) and Lynch and Balduzzi (1999), a VAR is estimated to assess the empirical ability of each state variable (Z) to predict the scaled cash flows (d). The (N-1)x1 vector of scaled cash flows for the stock portfolios, d^{N-1}, is converted to a continuously compounded basis for the VAR. This specification for the stock cash flows allows them to go negative. However, with power utility for the representative agent, aggregate consumption must not go negative. Hence, the change in consumption as a fraction of current consumption is continuously compounded for the VAR rather than consumption growth, d^{Ag}, itself. This ensures that consumption growth is always positive. Without loss of generality, the predictive variable, *cay* or *div*, is normalized to be mean zero with unit variance. The VAR is estimated using OLS. With $\bar{d}_{t+1} \equiv [\ \ln(1 + (d_{t+1}^{Ag}-1)) \ \ \ln(1 + d_{t+1}^{N-1})' \ ]'$, the VAR can be written as follows:

$$\bar{d}_{t+1} = a_d + b_d Z_t + e_{t+1} \tag{22}$$

$$Z_{t+1} = a_Z + b_Z Z_t + v_{t+1} \tag{23}$$

where a_d, an Nx1 vector, and a_Z, a scalar, are intercepts; b_d, an N x1 vector, and b_Z, a scalar, are coefficients; and, $[e_{t+1}' \ \ v_{t+1}']'$ is an i.i.d., mean zero disturbance vector with covariance matrix Σ_{ev}; This specification assumes that any cash flow predictability is fully captured by Z_t. The VAR implies the following expression for scaled cash flows:

$$\bar{d}_{t+1} = a_d + b_d Z_t + \eta \ v_{t+1} + u_{t+1} \tag{24}$$

where η is a NxK vector of coefficients from a regression of e_{t+1} on v_{t+1}, and u_{t+1} is a i.i.d., mean-zero disturbance vector with covariance matrix Σ_u that is uncorrelated with v_{t+1}. The disturbance vector

$[u_{t+1}' \quad v_{t+1}']'$ is assumed to be multivariate normally distributed but with truncation for extreme realizations.[4]

Once equilibrium prices are calculated for a calibrated economy, the resulting return generating process can be compared to that in the data. To this end, the following VAR is calculated for the economy and estimated in the data:

$$\breve{r}_{t+1} = a_r + b_r Z_t + \omega_{t+1} \tag{25}$$

where $\breve{r}_{t+1}$ is a vector raw and excess returns and can include the riskless rate.

4.3   Quadrature Approximation.

The data VAR for scaled cash flows is approximated using a variation of the Gaussian quadrature method described by Tauchen and Hussey (1991). First, Tauchen and Hussey's method is used to discretize the predictive variable, Z_t, treating it as a first-order autoregressive process as in (23). The quadrature method is then used to calibrate a discrete distribution for the innovation u. I can then calculate a discrete distribution for $\bar{d}_{t+1}$ for each $\{Z_{t+1}, Z_t\}$ pair from the discretization of Z, since $v_{t+1} = Z_{t+1} - a_Z - b_Z Z_t$. This approach ensures that Z is the only state vector. I chose a specification with 15 quadrature points for the state variable, and 3 points for the innovations in scaled cash flows. Following Lynch (2000), this study implements the discretization in such a way as to ensure that moments important for portfolio choice are matched exactly. In particular, the procedure matches both the conditional mean vector and the covariance matrix for log cash flows at all grid points of the predictive variables, as well as the unconditional volatilities of the predictive variables and the correlations of log cash flows with the predictive variables.[5]

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[4]Truncation is assumed so that short-selling is not ruled out by extreme realizations of e_{t+1} that have positive probability under the normal distribution but are in fact implausible. This issue is less important in the general equilibrium setting developed here, since market clearing means that assets with positive prices have positive weights in the representative agent's portfolio.

[5]The data values for Σ_{ev} are taken to be the covariance matrix for the associated untruncated Normal distributions when performing the quadrature approximation. But since the truncation typically uses extreme cutoffs, the resulting misstatement of Σ_{ev} by the approximation is likely to be small.

I find that increasing the number of grid points for the state variable from 7 to 15 has almost no effect on the results. Moreover, with 7 grid points for the state variable, increasing the number of grid points for scaled cash flows from 3 to 7 has virtually no effect either. There are two implications of this insensitivity to the number of grid points. Since 7 grid points for scaled cash flows implies that the largest realization for u_{t+1} is more than 3.75 standard deviations from zero, implausibly large deviations from the mean by scaled cash flow are needed for the possibility of negative wealth to affect the representative agent's portfolio choice. Second, the agent's optimal portfolio is largely unaffected by the severity of a symmetric truncation of scaled cash flow that is sufficient to ensure that the possibility of negative wealth does not drive the investor's portfolio choice.

4.4   Scaled Cash Flows and *cay* as the Predictor: Data and Quadrature VAR.

This subsection examines the ability of the quadrature approximation to incorporate important features of the data VAR estimated for the scaled cash flows with *cay* as the predictor. The data VAR is estimated using 36 non-overlapping observations and Newey-West standard errors with 1 lag are used for testing. In the data, there is weak support for *cay* as a predictive variable for cash flows. The point estimates of the slope coefficients are all negative for the 3 stock portfolios but positive for the aggregate wealth portfolio. A test of the hypothesis that all four slope coefficients are 0 can be rejected at the 10% level. The contemporaneous conditional correlation between *cay* and scaled cash flow is -0.191 for the aggregate wealth portfolio , but is never more than 0.05 in magnitude for any of the three stock portfolios. Contemporaneous correlation with *cay* must be present in asset returns for the representative agent to have non-trivial hedging demands in the CEcon with *cay* as the state variable.

Turning to the VAR estimated for the discretization, it is apparent that discretization is able to match the important features of the data. In particular, the approximation is able to match the VAR parameters, intercepts and slope coefficents, as well as the covariance matrix for both the variables and residuals from the VAR.

5   Results.

Moments and parameters for prices and returns in both the UEcon and the CEcon with *cay* as the state variable are presented, and compared to empirical returns. In both the calibrated economies, the representative agent has a risk aversion parameter (γ) of 5, a rate of time preference (β) of 0.99, and is infinitely lived. The section also contains optimal portfolio allocations and characteristics of the optimal portfolio for a power utility investor with identical risk aversion and time preference as the representative agent are also reported. The investor's horizon is either 1 year or infinite. While the allocations are plotted for the two economies described earlier (UEcon for γ=5 and CEcon for γ=5 and *cay*), the portfolio characteristics are presented for a wider range of economies in which the representative agent's risk aversion can be either 5 or 10 and the state variable in CEcon can be either *cay* or *div*. Abnormal returns relative to a couple of benchmarks are reported for this larger set of economies, togther with some utility comparisons.

5.1   Prices in Equilibrium.

Figure 1 plots prices scaled by the current aggregate cash flow, f^i , for the three book-to market portfolios and the aggregate wealth portfolio in both the unconditional economy and in the cconditional economy with *cay* as the state variable. In both economies, the representative agent has a risk aversion parameter (γ) of 5, a rate of time preference (β) of 0.99, and is infinitely lived. For all 4 assets, scaled price in the CEcon is decreasing in *cay* and the relation appears linear.

5.2   Unconditional Return Moments in Equilibrium.

Table 2 reports unconditional moments and VAR parameters for returns in the two economies from the previous subsection and for empirical returns. The data VAR is estimated using OLS, has 180 overlapping observations from 1954:1 to 1998:4 with a rolling quarterly window, and uses Newey-West standard errors with 4 lags. While *cay* is the predictive variable in the VAR, up to 7 dependent variables are used: raw or excess return on B1, B2, B3, the portfolio of financial wealth (Fi) and the aggregate wealth portfolio (Ag) as well as the riskless asset and contemporaneous *cay*.

Since *cay* is normalized to be mean zero, the VAR intercept gives the unconditional mean of the dependent variable.   Panel A of Table 2 reports VAR parameters and shows that the mean

excess returns in both the UEcon and CEcon are at least an order of magnitude lower than in the data. The mean excess return on the value-weighted stock market (Fi) is 8.199% in the data, but only 0.201% in UEcon and 0.252% in CEcon. The unconditional Sharpe ratios reported in Panel B show that the higher mean returns in the data than in the calibrated economies persist after controlling for return volatility differences.

The lower mean equity returns in the calibrated economies than in the data is a manifestation of the well-known "equity premium" puzzle, first documented by Mehra and Prescott (1985). Put simply, aggregate consumption is not sufficiently volatile or correlated with stock dividends to generate the observed equity premium in an economy whose representative agent has power utility with a reasonable risk aversion coefficient. The artificial economies here calibrate cash flows to the data, with the aggregate cash flow calibrated to aggregate consumption. Moreover, the representative agent has standard time-separable power utility with a risk aversion of 5. Thus, it is not surprising that an "equity premium" puzzle is observed. Decomposing the financial wealth portfolio into three book-to-market portfolios does not alter the mean return on the financial wealth portfolio in an endowment economy.

The mean riskfree rate is much higher in the calibrated economies than in the data. The average 1-year riskfree rate is 1.342% in the data but is more than16% in both the calibrated economies. This discrepancy illustrates the "riskfree rate" puzzle which has been documented by Weil (1989) and others.

Comparing the two calibrated economies indicates that the mean excess returns for the conditional economy are higher than for unconditional. However, an examination of Panel B indicates that the higher mean excess return is largely being driven by more volatile returns, at least unconditionally: unconditional Sharpe ratios, except for the high book-to-market portfolio, are about the same or slightly higher in the conditional economy. At least two effects determine the variation in mean returns across the two economies. In a partial equilibrium setting, predictability makes stocks less volatile, which leads to higher Sharpe ratios. This effects makes stocks more attractive. With a multi-period investor, there is also a hedging demand, which can make stocks more or less attractive depending on the covariance of stock returns this period with the opportunity set at the start of the next period. I will return to this issue when I discuss the results for conditional Sharpe ratios

in the CEcon.

The pattern of mean excess returns across the three book-to-market portfolios is very different in the data and in the two economies. In the data, expected excess return is increasing in book-to-market going from B1 to B2 to B3. And while B3 also has the highest expected excess return in the economies, B2 and not B1 has the lowest in both. Controlling for volatility does not help since the unconditional Sharpe ratio is increasing in book-to-market in the data, but decreasing in both economies.

Turning to the covariance matrices reported in Panel C of Table 2, we see that equity return volatilities are much higher in the data than in either economy. This result is a manifestation of the "equity volatility" puzzle (see Campbell, 2000). Looking at the covariance matrix for raw returns and the state variable on the left-hand side of Panel C, we see that the four equity portfolios have volatilities ranging from 16.65% for the B2 portfolio to 19.05% for the B3. The volatility of the financial wealth portfolio is 16.75%. However, the largest volatility is 9.68% in the conditional economy and only 8.54% in the unconditional economy, for the B3 portfolio in each. At the same time, the rank ordering of the stock portfolio volatilities in both economies matches that for the data. This finding indicates that the earnings and dividend payout data for the three stock portfolios used to calibrate the cash flows for the two economies have some relation to the cash flow information used to determine prices in the data. The volatility of the riskfree rate is also reported and the number in the conditional economy (1.61%) comes close to matching the number in the data (1.94%).

Both economies generate lower correlations between stock returns than the data, but in all three, the sign is positive for all pairwise combinations of the stock portfolios. The riskfree rate has negative contemporaneous correlations with raw risky-asset returns and the state variable in the data, but these correlations are positive in the conditional economy.

Turning to covariance matrices for the VAR residuals on the right-hand side of Panel C, and focusing on the excess return results, we again see that the excess return volatilities are lower in the conditional economy than in the data, but exhibit the same pattern across the stock portfolios. Similarly, the correlations between the excess stock return residuals are slightly lower in the conditional economy than in the data, though all pairwise combinations are again positive in both.

Probably the most interesting comparison is of the contemporaneous covariance between the residuals for excess stock return and the state variable. These are all negative both in the data and in the conditional economies, though the magnitudes of the correlations are larger for the conditional economy. This covariance is important for determining the direction and magnitude of any hedging demand by a multiperiod investor.

Finally, the unconditional return covariance matrices for the two economies can be compared to assess how cash flow predictability affects the equilibrium return generating process for an economy. We see that the return volatilities are higher in the conditional economy. Further, while the return correlations are always positive in both economies, the correlation for any pair is always higher in the conditional economy. Examining excess rather than raw returns does not alter any of these conclusions regarding the unconditional distributions for the two economies. The implication is that partial-equilibrium utility comparisons designed to assess the benefits of predictability may not be appropriate when attempting to assess the benefits of predictability for an economy as a whole. This issue is explored in more detail below.

5.3   Conditional Return Moments in Equilibrium.

5.3.1   Predictability.

Panel B in Table 2 reports slope coefficients from regressions of excess return on the lagged state variable *cay* both in the data and in the conditional economies. The magnitudes of these coefficients for excess stock returns are much larger in the data than in the conditional economy. Consistent with Lettau and Ludvigson (2000a), the slopes in the data are all positive and highly significant and the regressions explain between 14% and 20% of the variation in excess return, depending on the portfolio. In contrast, while the excess return slopes are all positive in the conditional economy, the R^2s are all virtually zero. This result is particularly striking when it is noted that stock portfolios have much lower volatility in the economies than in the data.

One concern is that the regression R^2s are low because the relation between excess return and the state variable *cay* is nonlinear. To explore this possibility, Figure 2 plots conditional excess return as a function of cay for the three book-to-market portfolios, the financial wealth portfolio and the aggregate wealth portfolio. For all 5 assets, the relation is monotonic and increasing, Moreover,

in unreported results, the unconditional volatilities of the regression residuals are virtually identical to the unconditional volatilities of the excess return deviations from the condition mean. While the state variable *cay* may explain only a small fraction of the variation in stock excess returns, the explained variation may still be important. This issue is discussed further when we examine conditional Sharpe ratios below.

Turning to the regression of the riskfree rate on *cay*, we see that the wide disparity between the data and the conditional economy continues. In the data, the slope coefficient is negative while the R^2 is less than 1%. In contrast, the conditional economy produces a positive slope and the regression explains virtually 100% of the variation in the riskfree rate. By construction, the riskfree rate can be represented as a function of *cay* in the conditional economy, but this result says that the function is essentially linear.

One final point is worth making. Since the slope coefficients on the excess return to aggregate wealth, the three stock portfolios and on the riskless asset are all positive, it follows that the opportunity set available to the representative agent at time *t* is improving as cay_t increases. Consequently, the representative agent, whose risk aversion is greater than 1, likes a portfolio with negative contemporaneous covariance with the state variable.

5.3.2   Heteroscedasticity.

Market clearing may induce variation in conditional return volatility across states. Figure 3 plots conditional return volatility as a function of *cay* for the three book-to-market portfolios, the financial wealth portfolio and the aggregate wealth portfolio. Volatility is an increasing, monotonic function of *cay* for all 5 assets. The variation in volatility as a function of cay can be quite large, particularly for the B3 portfolio. For example, conditional volatility for B3 ranges from 8.7% to 10.3% as *cay* ranges from a -2.4 to a +2.4 standard-deviation realization. This result indicates the likely importance of allowing second as well as first moments of asset returns to be state dependent when examining portfolio allocation in a partial equilibrium setting.

5.3.3   Conditional Sharpe Ratios.

Both the first and second moments of excess returns are state dependent in the conditional

economy. Figure 4 examines whether conditional Sharpe ratios are also state dependent for the three book-to-market portfolios, the aggregate wealth portfolio, and the financial wealth portfolio. Interestingly, the plots of the conditional Sharpe ratio as a function of cay are essentially flat for all 5 portfolios. So while the across-state variation in expected excess return explains only a minuscule fraction of the total variation in excess return, this variation in expected excess return does ensure that the conditional Sharpe ratio is flat as a function of the state.

Figure 4 also plots the Sharpe ratio in the unconditional economy for the same 5 assets. By construction, this ratio is independent of *cay*. The Sharpe ratio for aggregate wealth is lower in the conditional economy than in the unconditional economy. The implication is that stocks are more attractive to the representative agent , after controlling for differences in volatility, in the conditional economy than in the unconditional economy.   The reason is the negative contemporaneous covariance of the risky asset returns with the state variable which means that holding the risky assets help the infinitely-lived agent hedge against future shifts in the opportunity set.

5.3.4 Conditional Correlations Across Asset Returns and With the State Variable.

Figure 5 plots average conditional correlations and covariances between three book-to-market portfolios and the nonfinancial wealth portfolio. The right-hand graph shows that the average pairwise covariance is increasing in the state variable, current *cay*.   Since the return volatilities of these portfolios are also increasing in current *cay*, the question arises whether the increasing volatilities are driving the increasing average covariance. In addressing this question, the left-hand graph shows that the average pairwise correlation is a decreasing function of lagged *cay*.

The conditional covariances between these portfolios' excess returns and contemporaneous *cay* is also worth examining, since these covariances affects the representative agent's hedging demands. Figure 6 plots the conditional contemporaneous correlations and covariances of the state variable *cay* with the excess returns on these four portfolios. The right-hand graph shows that excess return's conditional covariance with contemporaneous *cay* is negative and a decreasing function of current *cay* for all four portfolios, but especially for B3, the high book-to-market portfolio.   On the other hand, the left-hand graph shows that the correlations are essentially flat for all 4 portfolios which suggests that the heteroscedasticity in excess return documented above can explain the

relation between covariance and lagged *cay*. In particular, since cay is homoscedastic and excess return volatility is increasing in lagged *cay*, approximately constant negative correlation translates into negative covariance that becomes even more negative as lagged *cay* increases.

The immediate implication of these results is the same as the implication of the return heteroscedasticity in the conditional economy. Specifically, papers examining portfolio allocation in a partial equilibrium setting may find it important to allow both the first and second moments of assets returns to be state dependent when calibrating return generating processes to the data.

5.4   Portfolio Decision-making by the Representative Agent.

This subsection examines portfolio allocations by a power utility investor with the same risk aversion coefficient and rate of time preference as the representative agent. This investor has either a single-period horizon or an infinite-period horizon. The infinite-horizon investor is the representative agent and the difference between her allocations and those of the single-period investor gives the hedging demands of the representative agent. In the unconditional economy, these two investors make the same allocation decisions. This subsection also examines portfolio allocations by the representative agent when confronted with the conditional economy not using *Z* Recall from subsection 3.1 that the conditional economy not using *Z* has a riskfree rate that follows the same process as in the conditional economy but has excess returns that are i.i.d. with the same unconditional distribution as in the conditional economy. Moreover, the excess returns are uncorrelated with next period's riskfree rate, and so hedging demands are zero in the conditional economy not using *Z*. Consequently, for a given risk aversion and rate of time preference for the representative agent and a given state variable, there are four allocations of interest: the representative agent's allocation in UEcon, in CEcon and in CEcon not using *Z*, and the 1-year investor's allocation in CEcon. The investor always has access to the three book-to-market portfolios, the non-financial wealth portfolio and the riskfree rate.

5.4.1   Portfolio Allocations and Hedging Demands.

Figure 7 presents these four allocations when the representative agent's risk aversion parameter is 5, her rate of time preference is 0.99, and the conditional economy's state variable is

cay. Portfolio weights under the four allocations are plotted as a function of *cay* for the three book-to-market portfolios, the non-financial wealth portfolio and the riskfree asset. The total weight of the 4 risky assets in the portfolio is also plotted.

A number of observations are worth making. First, the 1-year investor and the representative agent make allocations in the CEcon that are remarkably invariant to the state of the economy. This result is in stark contrast to partial equilibrium portfolio choice papers that typically report allocations that vary widely with the applicable state variable. Second, the representative agent's allocations in the UEcon and CEcon are virtually identical. Notice that the representative agent's allocations in the UEcon and CEcon are equal to value weighting in the applicable economy. Thus, the similarity of these two allocations implies that the value weights in the CEcon are invariant to state and similar to the value weights in the UEcon. Interestingly, the representative agent's UEcon allocation is closer to her CEcon allocation than her allocation for CEcon not using *Z*.

Finally, there are large differences between the representative agent's CEcon allocation and the 1-year investor's allocation. The implication is that the representative agent's hedging demands are large despite the fact that her optimal allocation in the CEcon is largely invariant to the state. In particular, the total allocation to the 4 risky assets goes from about 70% for the 1-year investor to 100% for the infinitely-lived investor. Thus, hedging demand accounts for about 30% of the representative's risky-asset demand. This result is consistent with the earlier argument that the negative covariance of the risky-asset returns with the state variable makes the risky-assets attractive to a multi-period power utility investor with risk aversion coefficient greater than 1.

5.4.2 Comparison of Optimal Portfolios to Minimum Variance Portfolios.

Table 3 compares conditional moments for the optimal portfolios of the representative agent and the 1-year investor to those for minimum variance portfolios in the same economy. In particular, the optimal portfolio is compared to the conditional minimum variance portfolio with the same conditional expected return. The first three columns describe the economy. The state variable is either *cay* or *div* and the representative agent's risk aversion coefficient is either 5 or 10. The fourth column of the table gives the horizon of the investor which is either infinite (the representative agent) or 1-year (the 1-year investor). The next three columns report the unconditional mean, the

27

average conditional volatility and average conditional covariance with the predictive variable of the investor's optimal portfolio. Averaging is performed using the unconditional distribution for the predictive variable. The final two columns report the average conditional volatility and average conditional covariance with the predictive variable for the conditional minimum variance portfolio with the same conditional mean as the optimal portfolio.

The optimal portfolios for the representative agent in the UEcon and for the 1-year investor in the CEcon have average conditional volatilities that are virtually identical to those for the same-mean minimum-variance portfolio. The implication is that a single-period power utility investor confronted with either the UEcon or CEcon return distributions, only cares about mean and variance when choosing her portfolio. In the case of CEcon, it is *conditional* mean and variance that matters to the 1-year investor. Recall that the returns are endogenous to the relevant economy which raises the possibility of higher moments being important to the investor. However, the results in Table 3 indictate that is not the case for the cash flow distributions employed here.

Turning to the optimal portfolio for the infinitely-lived representative agent in the CEcon, we see that the average volatility is always higher than for the minimum variable portfolio, irrespective of state variable or risk aversion. These results provide another measure of the potential importance of hedging demands in a general equilibrium setting. Focusing on the conditional economies with *cay* as the state variable, we already know that the infinitely-lived representative agent likes negative covariance with *cay* because her risk aversion coefficient is greater than 1 and *cay* is positively related to the quality of future opportunity sets. Because of this liking, the investor is willing to accept additional volatility to obtain a portfolio with larger negative contemporaneous covariance with *cay*.

Another interesting comparative static is with respect to risk aversion. In both the UEcon and CEcon, irrespective of the state variable, increasing risk aversion from 5 to 10 increases the mean of the representative agent's optimal portfolio. When the return distribution is held fixed and risk aversion is increased, the mean of the optimal portfolio typically decreases, because the more risk-averse investor holds less of the risky asset. However, in the general equilibrium setting here, the more risk-averse representative agent must still be induced to hold the aggregate wealth portfolio. The only way to do this is to increase the mean return on the risky assets per unit of volatility. This

explains why equilibrium Sharpe ratios and the mean of the representative agent's optimal portfolio are both increasing functions of risk aversion in the general equilibrium setting of this paper.

5.5   Abnormal Returns in Equilibrium.

Having documented the optimal portfolio allocation by the representative agent, I now turn to the CAPM abnormal returns implied by this allocation. Of particular interest is the magnitudes of the abnormal returns for the three book-to-market portfolios obtained using a stock market proxy for the aggregate wealth portfolio. These magnitudes can be compared to the abnormal return magnitudes for the three portfolios in the data obtained using the value-weighted stock market portfolio as the benchmark. This comparison allows us to assess whether standard representative agent models using aggregate consumption and cash flow predictability can explain the magnitude of the CAPM abnormal returns documented in the data. The other question that can be addressed is whether the pattern of the abnormal returns in the data can be captured by such a model. This is a much tougher task for the model to accomplish.

Table 4 reports abnormal returns relative to two benchmark portfolios for the 3 book-to-market portfolios, B1 (low), B2, and B3 (high), both in the data and in the calibrated economies. In the data, the benchmark is the value weighted market portfolio, and in the calibrated economies, it is either the aggregate wealth portfolio or the financial wealth portfolio, the latter being a value-weighted portfolio of the three stock portfolios. By construction, the aggregate wealth portfolio is the representative agent's optimal portfolio. Abnormal return is calculated as the average intercept from a conditional regression of each asset's excess return on the excess return of the investors portfolio is reported. In the data, and in the unconditional economies (UEcon), this regression is unconditional. The data regression is estimated using OLS, has 180 overlapping observations from 1954:1 to 1998:4 with a rolling quarterly window, and uses Newey-West standard errors with 4 lags.

Starting with the data, we see clear evidence of the well-documented book-to-market effect. The high book-to-market portfolio has a per-annum abnormal return relative to the value-weighted stock market portfolio of 4.700% while the low portfolio's abnormal return is -0.906% per annum. Turning to the calibrated economies, the abnormal returns calculated relative to the representative agent's optimal stock portfolio are an order of magnitude smaller than in the data. This result holds

29

irrespective of the type of economy (UEcon or CEcon), the risk aversion of the representative agent (5 or 10) or the state variable calibrated in the CEcon (*cay* or *div*). While the abnormal return spread in the data is 5.607%, the largest such spread in any of economies is only 0.621%. The implication is that standard representative-agent economies calibrated to aggregate consumption data appear unable to replicate the magnitude of the CAPM abnormal returns found in the data, at least using reasonable risk aversion values for the representative agent.

One concern is that this result is just a manifestation of the equity volatility puzzle. In other words, the larger abnormal returns in the data may be due to the higher excess return volatilities in the data relative to those in the calibrated economies. To explore this possibility, the abnormal return spreads in the calibrated economies are recalculated using levered abnormal returns rather than raw abnormal returns. In a given economy, a stock portfolio's levered abnormal return is obtained by multiplying its raw abnormal return by the ratio of its excess-return volatility in the data over its excess-return volatility in the calibrated economy. This adjustment to the portfolio's abnormal return is appropriate if the portfolio's lower volatility in the economy than the data is due to a difference in firm leverage in the data and the economy. Note that the stock portfolio cash flows are calibrated to earnings after interest, so leverage effects have already been incorporated in the cash flow calibration. Even so, the levered spread results can indicate whether the larger stock abnormal return magnitudes in the data can be explained by the higher stock return volatilities in the data. The last column of Table 4 shows that the levered-up abnormal return spreads are still all less than 30% of the spread in the data. Consequently, it is unlikely that the CAPM abnormal return puzzle documented here is being driven by the equity volatility puzzle.

Table 4 shows that the abnormal returns in the unconditional economies are virtually zero when measured relative to the true aggregate market portfolio. This result is consistent with evidence in Table 3 that the representative agent's optimal portfolios in the unconditional economies are indistinguishable from minimum-variance portfolios with the same mean. In contrast, in the conditional economies, abnormal returns relative to the true aggregate market portfolio are non-zero Moreover, for a given state variable, the magnitude of these abnormal returns is increasing in the representative agent's risk aversion. For example, with *cay* as the state variable, the spread in abnormal return increases from 0.081% to 0.302% as risk aversion increases from 5 to 10.

The use of a value-weighted stock market portfolio as a proxy for the aggregate market portfolio can distort CAPM abnormal returns calculations. This abnormal return distortion is the well-known Roll (1977) critique of CAPM testing. Table 4 can be used to assess the impact of Roll's (1977) critique on the measured abnormal returns of the 3 book-to-market portfolios in the calibrated economies. Interestingly, when abnormal returns are measured relative to the financial wealth portfolio, the spread is similar in the unconditional and the two conditional (*cay* and *div*) economies for a given risk aversion coefficient. So using the value-weighted stock portfolio as a market proxy appears to have a larger effect on abnormal return calculations in the UEcon than the two CEcons, at least for the calibrated economies considered here.

Finally, Table 4 can be used to examine the pattern of abnormal returns across the three book-to-market portfolios both in the data and in the calibrated economies. However, none of calibrated economies, conditional or unconditional are able to generate the same pattern as the data. This result is yet another strike against the standard representative-agent model as a descriptor of asset prices, since the two state variables used are among the better stock return predictors available.

5.6   Utility Comparisons.

Table 5 performs the three utility comparisons described in subsection 3.4. The cost of treating excess return as i.i.d. and uncorrelated with next period's riskfree rate in the CEcon is reported in the last column. The reported cost is for an investor with the same preferences as the representative agent. The table shows that this cost is higher when that variable is *div* rather than *cay* and that the cost is increasing in the representative agent's risk aversion coefficient. The investor with risk aversion of 10 who is not using *div* is prepared to give up 5.69% of her wealth to find out about the predictive ability of *div*.

The above comparison is partial equilibrium in nature, since it ignores the change in the equilibrium return generating process that occurs when cash flows become predictable. This approach is appropriate for assessing the cost to an individual investor of not using Z in the CEcon. On the other hand, it is not appropriate for assessing the cost to the economy of eliminating the cash flow predictability. As explained in subsection 3.4, the unconditional cash flow distribution is held fixed going from UEcon to CEcon, and so the representative agent's utility is unchanged going

31

UEcon to CEcon since her utility is time-separable. Thus, the partial equilibrium cost discussed above clearly overstates the cost to the economy's representative agent of the cash flows becoming unpredictable.

Even if one is careful to use the UEcon return distribution to calculate the representative agent's utility in the absence of cash flow predictability, utility comparisons can still get distorted. Such a distortion occurs if the agent's wealth is held fixed going from UEcon to CEcon. Utility costs calculated in this partial equilibrium fashion are reported in the next-to last column. These costs are non-zero, especially when *div* is the state variable. While fixing wealth is appropriate for an individual investor, it is not for the representative agent, since the price of the aggregate cash flow becomes state-dependent going from UEcon to CEcon. In fact, when the calculation incorporates this state-dependent wealth effect, the utility cost of going from CEon to UEcon becomes zero, as intuition dictates (see the first column of costs in Table 5). The message of this section is that partial equilibrium utility cost calculations are likely to be misleading when used to assess the value of cash flow predictability for an entire economy.

6   Conclusion.

This paper examines portfolio allocations and market clearing prices when the representative agent can allocate across equity portfolios formed on the basis of characteristics like size and book-to-market, and portfolio cash flows are predictable. The state space is discrete and price-consumption ratios are obtained portfolio by portfolio simply by inverting an economy-wide matrix and multiplying this matrix by a portfolio-specific vector. The economy-wide matrix has the dimensionality of the state space. The paper calibrates cash flow predictability to the data using the consumption-wealth fraction (*cay*) of Lettau and Ludvigson (2000a) and dividend yield (*div*) as state variables. Annual cash flow processes are calibrated for three stock portfolios and for the aggregate consumption stream. The economy's representative agent possesses a relative risk aversion coefficient of either 5 or 10.

When cash flow predictability is calibrated to the data using *cay* as the predictor and risk aversion is 5, equilibrium excess returns on the four assets are more volatile, more correlated with each other, and have higher means than in the equivalent economy with i.i.d. cash flows. Further,

the conditional second moments for returns and the contemporaneous state variable are found to be highly state-dependent. The paper finds much smaller excess return predictability using *cay* in the calibrated economy than in the data, though the relation is positive in both. Conditional Sharpe ratios are virtually invariant to state.

While the representative agent's optimal portfolio is not very state-dependent, her hedging demands are quite large and her optimal portfolio is not minimum-variance. For example, her single-period allocation to the four risky assets is about 75% of the portfolio while her infinite-horizon allocation, by construction, is 100%. The implication is that the conditional CAPM does not hold in the conditional economy with *cay* as the state variable. However, the spread in CAPM abnormal returns across the three book-to-market portfolios is an order of magnitude smaller in the calibrated economies than in the data. The spread in the data in 5.6% p.a. while the largest spread in the six calibrated economies considered is only 0.6% p.a. Finally, the paper has important implications for partial equilibrium analyses of dynamic portfolio choice.

A number of extensions are of interest. While the current paper deliberately treated the asset cash flows as homoscedastic, it would be useful to examine the effects of allowing the cash flows to exhibit heteroscedasticity. Another interesting extension incorporates parameter uncertainty. Allowing for more general preferences, especially habit persistence, is another interesting direction.

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Table 1. Moments and VAR parameters for scaled cash flows and the predictive variable, *cay*: Data and Discretization.

Next year's scaled cash flow is earnings after interest, multiplied by average dividend payout (see Fama and French, 1994, for details), and scaled by this year's aggregate consumption. The variable *cay* is aggregate consumption as a fraction of financial and non-financial wealth (see Lettau and Ludvigson, 2000a, for details). Scaled cash flow is available for 3 book-to-market portfolios and an aggregate wealth portfolio (Ag). B1, B2 and B3 are the low, middle and high book-to-market portfolios, respectively. Panel A reports VAR parameters and R^2 both for the data and the discretization (labelled Econ). Panel B reports covariance matrices for the variables and the VAR residuals, again for the data and the discretization (again labelled Econ). Scaled cash flow is continuously compounded for all portfolios but aggregate wealth. For this portfolio, the scaled change in cash flow is continuously compounded. Gaussian quadrature is used to calibrate the discretization (Econ) to the data VAR. The data VAR is estimated using OLS, has 36 non-overlapping observations from 1963 to 1998, and uses Newey-West standard errors with 1 lag.

Panel A: VAR Coefficients

VAR Coefficients	Data				Econ		
	Mean (*a*)	*b*	t-test (*b*)	R^2	Mean (*a*)	*b*	R^2
Ag	3.035	0.275	0.56	1.62	3.035	0.275	1.62
B1	0.226	-0.000	-0.03	0.00	0.226	-0.000	0.00
B2	0.280	-0.001	-0.13	0.05	0.280	-0.001	0.05
B3	0.131	-0.019	-1.68	6.26	0.131	-0.019	6.26
cay	-0.084	0.248	1.33	6.15	-0.000	0.248	6.15
Hypothesis Test		χ^2	d.f.	p-value			
$b^{Ag}_d = b^{B1}_d = b^{B2}_d = b^{B3}_d = 0$		8.20	4	0.084			

Table 1 cont

Panel B: Covariance Matrix. Standard Deviation (on diagonal), Correlation (below), Covariance (above).

	Unconditional					VAR Residual				
	Ag	B1	B2	B3	cay	Ag	B1	B2	B3	cay
Data										
Ag	2.159	-0.002	-0.039	0.006	-0.411	2.141	-0.002	-0.038	0.011	-0.480
B1	-0.018	0.059	0.002	-0.001	0.001	-0.018	0.059	0.002	-0.001	0.001
B2	-0.271	0.430	0.067	0.002	-0.000	-0.270	0.430	0.066	0.002	0.000
B3	0.036	-0.167	0.444	0.076	-0.008	0.071	-0.174	0.453	0.074	-0.003
cay	-0.191	0.022	-0.002	-0.107	1.000	-0.231	0.025	0.004	-0.048	0.969
CEcon										
Ag	2.159	-0.002	-0.002	-0.002	-0.411	2.141	-0.002	-0.038	0.011	-0.480
B1	-0.018	0.059	-0.002	-0.002	0.001	-0.018	0.059	0.002	-0.001	0.001
B2	-0.271	0.430	0.067	-0.002	-0.000	-0.270	0.430	0.066	0.002	0.000
B3	0.036	-0.167	0.444	0.076	-0.008	0.071	-0.174	0.453	0.074	-0.003
cay	-0.191	0.022	-0.002	-0.107	1.000	-0.231	0.025	0.004	-0.048	0.969

Table 2. Moments and VAR parameters for returns and the predictive variable, *cay*: Data and Calibrated Economies

Returns for the calibrated economies are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of 5 and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data. The unconditional economy (UEcon) assumes that the scaled cash flows are i.i.d. over time with a joint distribution equal to their unconditional distribution in the CEcon. The variable *cay* is aggregate consumption as a fraction of financial and non-financial wealth (see Lettau and Ludvigson, 2000a, for details). Return moments are reported for 3 book-to-market portfolios, B1 (low), B2, and B3 (high), the market portfolio of financial wealth, Fi, the aggregate wealth portfolio, Ag, and the riskless asset, fr. Panel A reports VAR parameters and R^2 and Panel B reports unconditional Sharpe ratios, for the data and the two calibrated economies. Panel C reports covariance matrices for the variables and the VAR residuals, again for the data and the two calibrated economies. Results for both raw returns and returns in excess of the riskfree rate are reported. The data VAR is estimated using OLS, has 180 overlapping observations from 1954:1 to 1998:4 with a rolling quarterly window, and uses Newey-West standard errors with 4 lags.

Panel A: VAR Coefficients

Return	Asset	Data				CEcon			UEcon
		Mean (*a*)	*b*	t-test (*b*)	R^2	Mean (*a*)	*b*	R^2	Mean
Excess	Ag					0.303	0.004	0.00	0.273
	Fi	8.199	8.130	4.92	20.01	0.252	0.004	0.00	0.201
	B1	7.817	8.240	4.33	17.25	0.266	0.004	0.00	0.233
	B2	8.954	8.304	5.91	20.92	0.178	0.002	0.00	0.143
	B3	13.067	7.903	4.20	14.45	0.388	0.013	0.00	0.272
Raw	fr	1.342	-0.163	-0.75	0.62	16.903	1.608	99.99	16.881

Panel B: Unconditional Sharpe Ratios

Asset	Data	CEcon	UEcon
Ag		0.086	0.108
Fi	0.490	0.054	0.056
B1	0.426	0.054	0.055
B2	0.538	0.041	0.041
B3	0.686	0.040	0.032

Table 2 cont

Panel C: Covariance Matrix. Standard Deviation (on diagonal), Correlation (below), Covariance (above).

	Unconditional							VAR Residual						
	Ag	Fi	B1	B2	B3	fr	cay	Ag	Fi	B1	B2	B3	fr	cay
	Data, Excess Return							Data, Excess Return						
Fi		16.98	306.65	278.59	294.11	-5.77	-0.46		15.18	248.20	219.69	238.05	-4.61	-3.01
B1		0.97	18.53	283.45	292.39	-5.34	-1.26		0.97	16.86	223.74	235.56	-4.16	-3.85
B2		0.97	0.90	16.96	296.62	-7.05	0.51		0.96	0.88	15.08	239.36	-5.86	-2.10
B3		0.89	0.81	0.90	19.42	-9.06	-0.92		0.87	0.78	0.88	17.96	-7.93	-3.40
fr		-0.17	-0.15	-0.21	-0.24	1.94	-0.37		-0.16	-0.13	-0.20	-0.23	1.94	-0.32
cay		-0.03	-0.07	0.03	-0.05	-0.19	1.00		-0.21	-0.24	-0.15	-0.20	-0.17	0.94
	Data, Raw Return							Data, Raw Return						
Fi		16.75	299.31	269.55	283.06	-1.99	-0.83		15.00	243.17	212.96	229.26	-0.86	-3.33
B1		0.97	18.34	274.83	281.76	-1.57	-1.63		0.97	16.72	217.46	227.22	-0.42	-4.16
B2		0.97	0.90	16.65	284.29	-3.28	0.14		0.96	0.88	14.81	229.31	-2.12	-2.41
B3		0.89	0.81	0.90	19.05	-5.29	-1.29		0.87	0.77	0.88	17.62	-4.18	-3.72
R^f		-0.06	-0.04	-0.10	-0.14	1.94	-0.37		-0.03	-0.01	-0.07	-0.12	1.94	-0.32
cay		-0.05	-0.09	0.01	-0.07	-0.19	1.00		-0.24	-0.26	-0.17	-0.22	-0.17	0.94
	CEcon, Raw Return							CEcon, Excess Return						
Ag	3.54	12.04	11.50	9.63	18.18	2.59	-1.52	3.15	9.44	8.91	7.04	15.57	0.00	-1.92
Fi	0.73	4.65	17.20	18.79	35.49	2.59	-2.03	0.69	4.36	14.60	16.20	32.88	0.00	-2.43
B1	0.66	0.75	4.90	14.21	11.87	2.59	-1.44	0.61	0.72	4.62	11.62	9.26	0.00	-1.84
B2	0.63	0.94	0.68	4.29	27.59	2.59	-1.57	0.56	0.93	0.63	3.98	24.98	0.00	-1.97
B3	0.53	0.79	0.25	0.66	9.68	2.61	-4.05	0.52	0.79	0.21	0.66	9.54	0.00	-4.45
fr	0.46	0.35	0.33	0.38	0.17	1.61	0.40	0.00	0.00	0.00	0.00	0.00	0.02	-0.00
cay	-0.43	-0.44	-0.29	-0.37	-0.42	0.25	1.00	-0.63	-0.57	-0.41	-0.51	-0.48	-0.00	0.97
	UEcon, Raw Return													
Ag	2.53	4.72	5.46	3.36	6.38									
Fi	0.52	3.61	10.00	11.30	22.00									
B1	0.51	0.65	4.24	8.01	0.49									
B2	0.38	0.91	0.55	3.46	15.62									
B3	0.30	0.71	0.01	0.53	8.54									

Table 3. Comparison of Optimal and Minimum-variance Portfolios in the Calibrated Economies

Parameters for a power utility investor's optimal portfolio in the conditional (CEcon) and unconditional (CEcon) economies are reported. For the conditional economies, the state variable is calibrated to either consumption as a fraction of total wealth, *cay,* or to dividend yield, *div.* Returns for the calibrated economies are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of either 5 or 10, and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data. The unconditional economy (UEcon) assumes that the scaled cash flows are i.i.d. over time with a joint distribution equal to their unconditional distribution in the CEcon. The power utility investor exhibits the same risk aversion as the representative agent and has either a 1-year or an infinite horizon. With an infinite horizon, the investor is the representative agent, and so her optimal portfolio is the aggregate wealth portfolio. The expected excess return on the portfolio is reported $E[r_{W,t+1}]$ together with the average conditional volatility ($\sigma_t[R_{W,t+1}]$) and the average conditional covariance with the predictive variable $\sigma_t[R_{W,t+1},Z_{t+1}]$. These last two statistics are also reported for the minimum-variance portfolio with the same conditional mean as the investor's portfolio in each state. In the unconditional economy (UEcon), only portfolio volatility is reported.

Z	γ	Econ	Hor-izon	Optimal Portfolio			Minimum-variance Portfolio	
				$E[r_{W,t+1}]$	av. $\sigma_t[R_{W,t+1}]$	av. $\sigma_t[R_{W,t+1},Z_{t+1}]$	av. $\sigma_t[R_{W,t+1}]$	av. $\sigma_t[R_{W,t+1},Z_{t+1}]$
cay	5	U	∞	0.273	2.53	na	2.53	na
		C	1-yr	0.221	2.28	-1.32	2.28	-1.32
			∞	0.303	3.15	-1.92	3.12	-1.81
	10	U	∞	0.624	2.90	na	2.90	na
		C	1-yr	0.388	2.29	-1.61	2.29	-1.61
			∞	0.766	4.66	-3.61	4.51	-3.18
div	5	U	∞	0.273	2.53	na	2.53	na
		C	1-yr	0.105	1.57	0.02	1.57	0.02
			∞	0.113	4.22	1.79	1.68	0.02
	10	U	∞	0.624	2.90	na	2.90	na
		C	1-yr	0.144	1.39	-0.03	1.39	-0.03
			∞	0.117	6.67	3.10	1.13	-0.02

Table 4. Abnormal Returns: Data and Economies.

Abnormal returns relative to two benchmark portfolios are reported for 3 book-to-market portfolios, B1 (low), B2, and B3 (high). In the data, the benchmark is the value weighted market portfolio, and in the calibrated economies, it is either the aggregate wealth portfolio or the financial wealth portfolio, the latter being a value-weighted portfolio of the three stock portfolios. By construction, the aggregate wealth portfolio is the representative agent's optimal portfolio. For the conditional economies, the state variable is calibrated to either consumption as a fraction of total wealth, *cay,* or to dividend yield, *div*. Returns for the calibrated economies are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of either 5 or 10, and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data. The unconditional economy (UEcon) assumes that the scaled cash flows are i.i.d. over time with a joint distribution equal to their unconditional distribution in the CEcon. Abnormal return is calculated as the average intercept from a conditional regression of each asset's excess return on the excess return of the benchmark. In the data, and in the unconditional economies (UEcon), this regression is unconditional. The data regression is estimated using OLS, has 180 overlapping observations from 1954:1 to 1998:4 with a rolling quarterly window, and uses Newey-West standard errors with 4 lags. The Spread measures the maximum difference in abnormal return over the three possible pairwise combinations of the 3 book-to-market portfolios. The Levered Spread is defined analogously, except the abnormal return of each stock portfolio is first multiplied by the ratio of the portfolio's excess return volatility in the data over that in the calibrated economy.

Z	γ	Econ	Relative to Aggregate Wealth Portfolio				Relative to Financial Wealth Portfolio				
			B1	B2	B3	Spread	B1	B2	B3	Spread	Levered Spread
Data							-0.906	1.029	4.700	5.607	5.607
							(-1.62)	(1.86)	(4.38)		
cay	5	U	-0.000	-0.000	0.000	0.000	0.078	-0.031	-0.068	0.146	0.793
		C	-0.007	-0.037	-0.088	0.081	0.072	-0.036	-0.047	0.120	0.535
	10	U	-0.000	-0.001	0.000	0.001	0.244	-0.221	0.057	0.465	1.538
		C	-0.064	-0.297	-0.365	0.302	0.231	-0.251	0.145	0.482	1.294
div	5	U	-0.000	-0.000	0.000	0.000	0.078	-0.031	-0.068	0.146	0.793
		C	-0.047	-0.215	-0.272	0.225	0.068	-0.056	0.002	0.124	0.371
	10	U	-0.000	-0.001	0.000	0.001	0.244	-0.221	0.057	0.465	1.538
		C	-0.189	-0.695	-0.395	0.506	0.115	-0.257	0.364	0.621	0.867

Table 5. Utility Cost Calculations for the Calibrated Economies

The cost number reported gives the fraction of her wealth that the representative agent would be prepared to give up to be given access to a different environment. For each conditional economy (CEcon), the last column reports the cost of ignoring predictability in the conditional economy by using the optimal allocation for the i.i.d excess return generating process with the same unconditional distribution as the excess return in the CEcon. The number is calculated assuming the investor currently using the i.i.d.-return allocation does not know the current state of the economy and that the investor's wealth is the same irrespective of the state. The cost of being in the unconditional economy (UEcon) rather than the equivalent conditional economy is also reported. Two costs numbers are reported. The Partial Equilibrium comparison calculates the cost holding wealth fixed, while the General Equilibrium comparison recognizes that the representative agent's wealth becomes state-dependent going from UEcon to CEcon. For the conditional economies, the state variable is calibrated to either consumption as a fraction of total wealth, *cay,* or to dividend yield, *div.* Returns for the calibrated economies are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of either 5 or 10, and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data. The unconditional economy (UEcon) assumes that the scaled cash flows are i.i.d. over time with a joint distribution equal to their unconditional distribution in the CEcon.

Z	γ	UEcon vs CEcon		CEcon
		General Equilibrium	Partial Equilibrium	Not Using Z
cay	5	0.00	0.15	0.12
	10	0.00	0.04	0.62
div	5	0.00	-3.46	2.10
	10	0.00	-8.75	5.69

B1
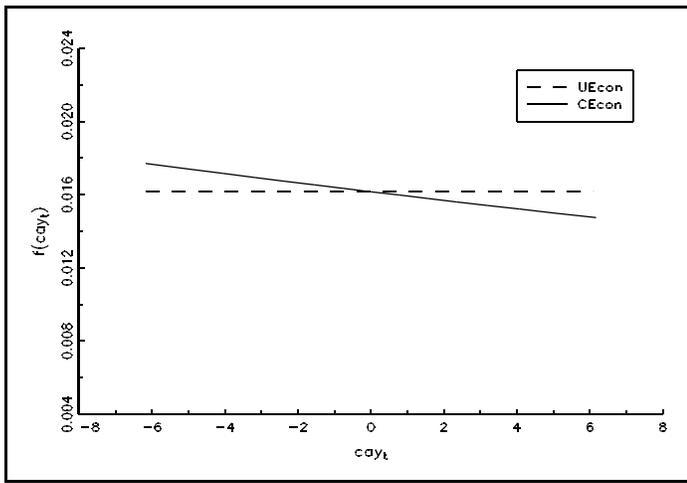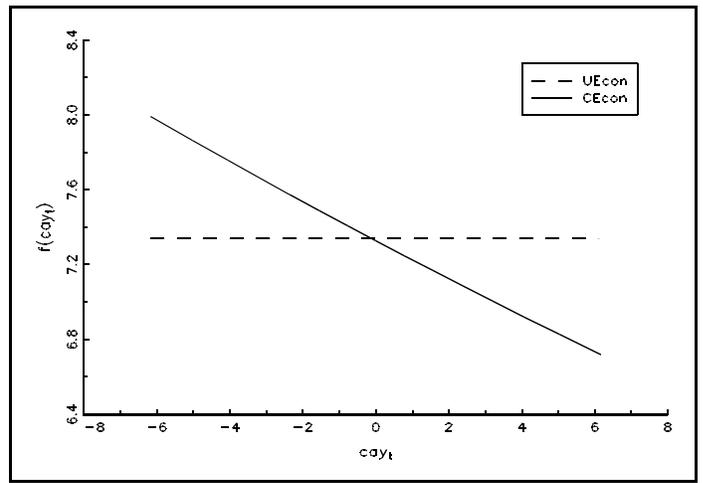
Ag - Aggregate Wealth
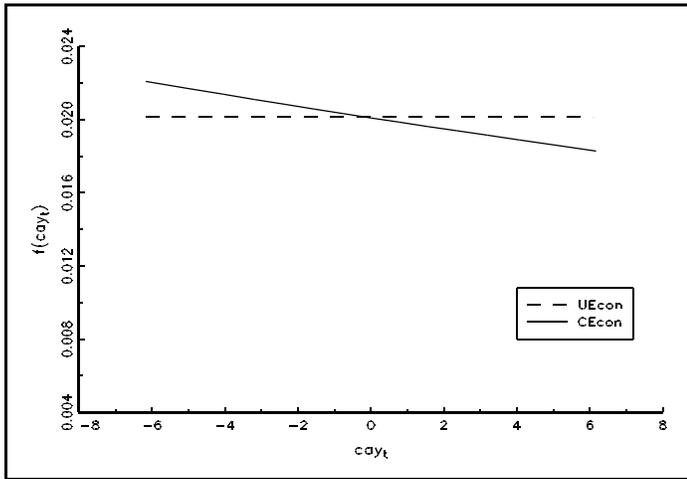
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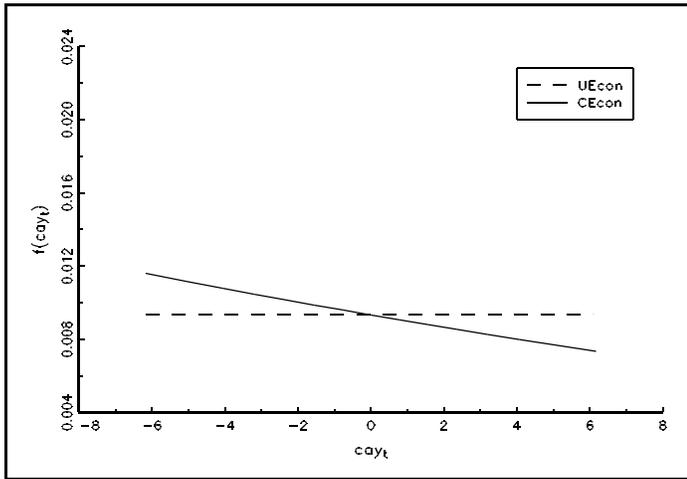
B3

Figure 1.

Figure 1 presents price scaled by current aggregate consumption for 3 book-to-market portfolios, B1 (low), B2, and B3 (high), and the aggregate wealth portfolio in two calibrated economies. Returns in the economies are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of 5 and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data with *cay* as the predictive variable. The variable *cay* is a measure of consumption as a fraction of total wealth. The unconditional economy (CEcon) assumes that the scaled cash flows are i.i.d. over time with a joint distribution equal to their unconditional distribution in the CEcon.

B1

Ag - Aggregate Wealth

B2

Fi - Financial Wealth

B3

Figure 2.

Figure 2 plots conditional expected excess returns for 3 book-to-market portfolios, B1 (low), B2, and B3 (high), the financial wealth portfolio (Fi), and the aggregate wealth portfolio (Ag) in two calibrated economies. Returns in the economies are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of 5 and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data with *cay* as the predictive variable. The variable *cay* is a measure of consumption as a fraction of total wealth. The unconditional economy (CEcon) assumes that the scaled cash flows are i.i.d. over time with a joint distribution equal to their unconditional distribution in the CEcon.

B1

Ag - Aggregate Wealth

B2

Fi - Financial Wealth

B3

Figure 3.

Figure 3 plots conditional return volatilities for 3 book-to-market portfolios, B1 (low), B2, and B3 (high), the financial wealth portfolio (Fi), and the aggregate wealth portfolio (Ag) in two calibrated economies. Returns in the economies are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of 5 and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data with *cay* as the predictive variable. The variable *cay* is a measure of consumption as a fraction of total wealth. The unconditional economy (CEcon) assumes that the scaled cash flows are i.i.d. over time with a joint distribution equal to their unconditional distribution in the CEcon.

B1

Ag - Aggregate Wealth

B2

Fi - Financial Wealth

B3

Figure 4.

Figure 4 plots conditional Sharpe ratios for 3 book-to-market portfolios, B1 (low), B2, and B3 (high), the financial wealth portfolio (Fi), and the aggregate wealth portfolio (Ag) in two calibrated economies. Returns in the economies are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of 5 and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data with *cay* as the predictive variable. The variable *cay* is a measure of consumption as a fraction of total wealth. The unconditional economy (UEcon) assumes that the scaled cash flows are i.i.d. over time with a joint distribution equal to their unconditional distribution in the CEcon.

Average Correlation *Average Covariance*

Figure 5.

Figure 5 plots the average conditional return correlation and covariance between 3 book-to-market portfolios, B1 (low), B2, and B3 (high), and between the 3 book-to-market portfolios and a nonfinancial wealth portfolio (nF) in a calibrated economy. The averaging is performed over all pairwise combinations. Returns in the economy are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of 5 and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data with *cay* as the predictive variable. The variable *cay* is a measure of consumption as a fraction of total wealth.
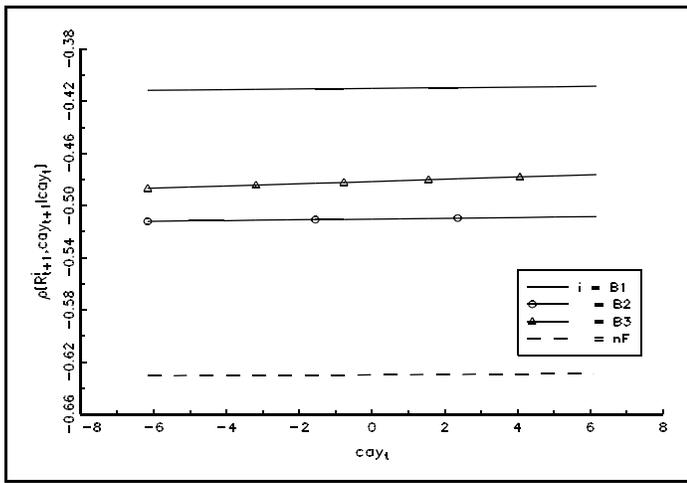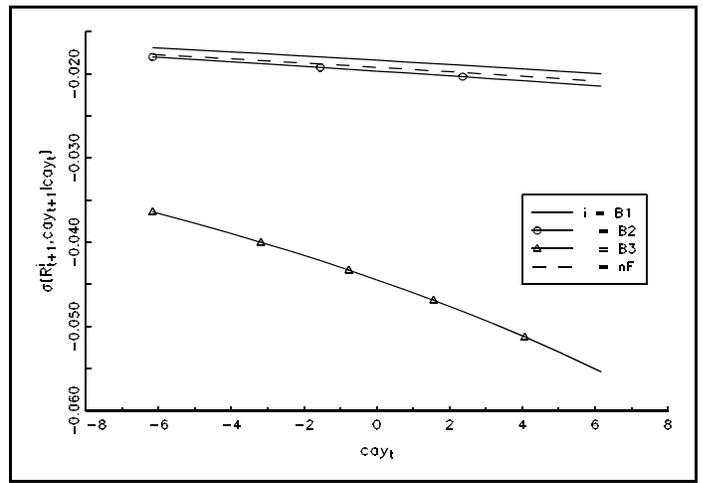
Conditional Correlation   *Conditional Covariance*

Figure 6.

Figure 6 plots the conditional contemporaneous correlations and covariances between the *cay* variable and the returns on 3 book-to-market portfolios, B1 (low), B2, and B3 (high), and a nonfinancial wealth portfolio (nF) in a calibrated economy. Returns in the economy are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of 5 and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data with *cay* as the predictive variable. The variable *cay* is a measure of consumption as a fraction of total wealth.
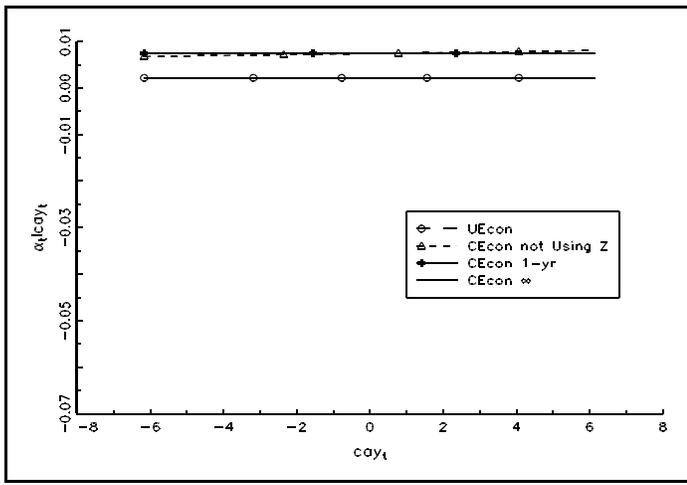
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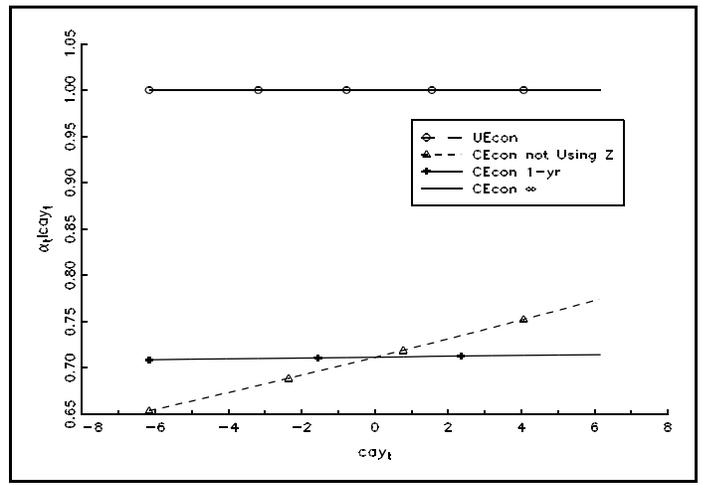
Total Allocation to B1, B2, B3 and nF
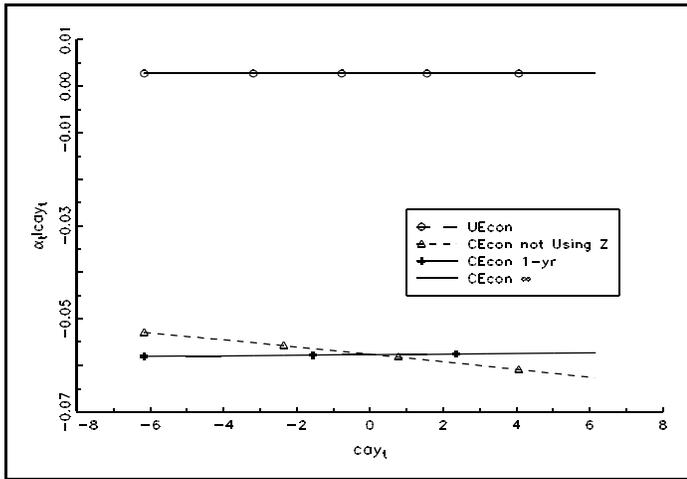
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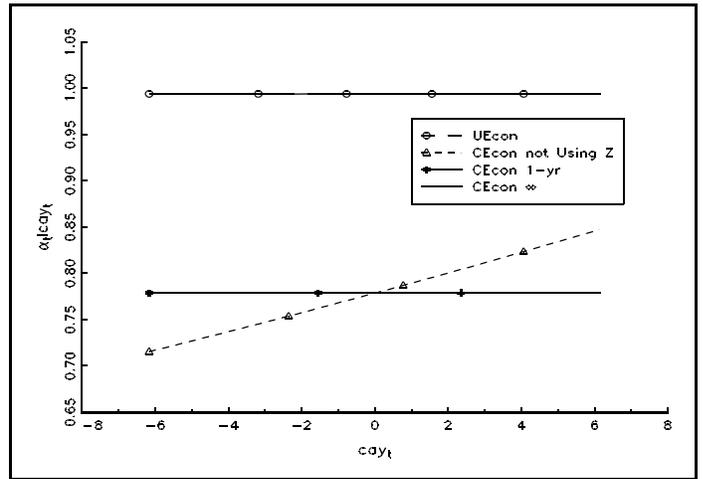
nF - Non-financial Wealth
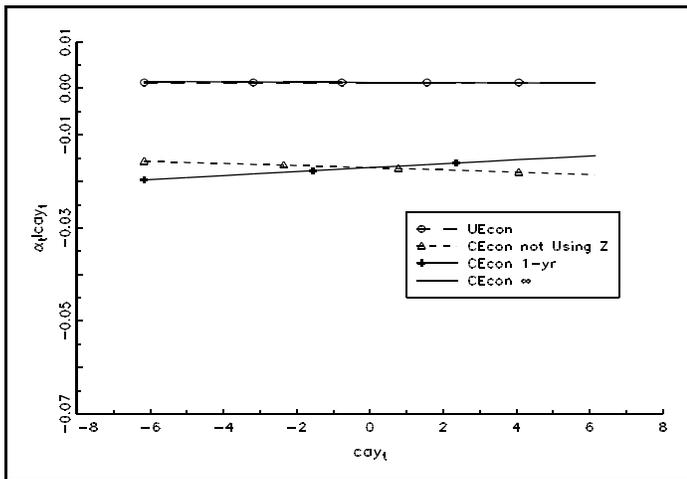
B3
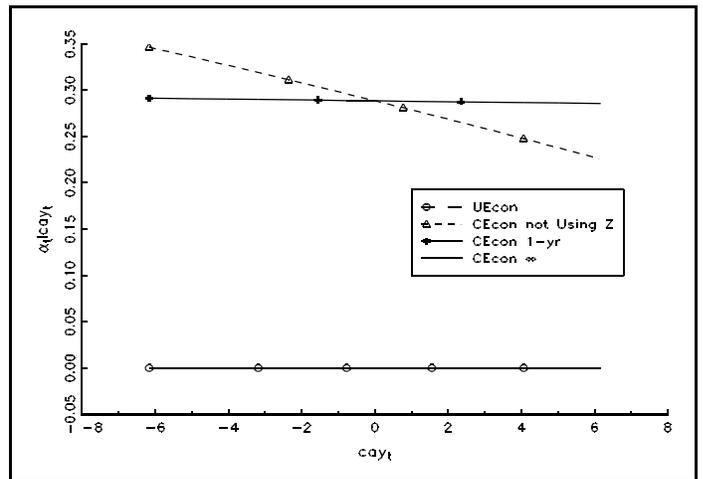
fr - Riskfree Asset

Figure 7.

Figure 7 plots portfolio allocations by a variety of power utility investors to 3 book-to-market portfolios, B1 (low), B2, and B3 (high), a nonfinancial wealth portfolio (Fi), and the riskless asset. The total allocation to the 4 risky assets is also plotted. Returns in two calibrated economies are obtained taking the generating process for scaled cash flows as exogenous and solving for asset prices that clear markets. The representative agent is infinitely lived and exhibits power utility with a risk aversion coefficient, γ, of 5 and a rate of time preference of 0.99. The conditional economy (CEcon) uses the generating process for scaled cash flows obtained when the quadrature approximation is applied to the data with *cay* as the predictive variable. The variable *cay* is a measure of consumption as a fraction of total wealth. The unconditional economy (CEcon) assumes that the scaled cash flows are i.i.d. over time with a joint distribution equal to

their unconditional distribution in the CEcon. The power utility investor has a risk aversion coefficient, γ, of 5 and a rate is of time preference of 0.99, just like the representative agent. The investor is faced with either the UEcon return process, the CEcon return process, or an i.i.d excess return process with the same unconditional distribution as the excess return in the CEcon (labelled CEcon Not Using Z). The investor facing the CEcon return process has either a 1 year horizon (labelled CEcon 1-yr) or an infinite horizon (labelled CEcon ∞). Finally, by construction, the investor facing UEcon, and the infinite-horizon investor facing CEcon, are the representative agent's for the UEcon, and the CEcon, respectively. Thus, their allocations coincide with value weighting in the relevant economy.