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A LENDER-BASED THEORY OF COLLATERAL

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Abstract

We offer a novel explanation for the use of collateral based on the dual function of banks to provide credit and assess the borrower's credit risk. There is no moral hazard or adverse selection on the part of borrowers—the only inefficiency is that banks cannot contractually commit to providing credit as their credit assessment is subjective. Without collateral, a bank may deny credit even if its credit assessment suggests that the project is marginally profitable. Collateral improves the bank's payoffs from financing such marginally profitable projects, thus mitigating the inefficiency arising from discretionary credit decisions. Unlike models of borrower adverse selection, our model suggests that high-quality borrowers post less collateral than low-quality borrowers, which is consistent with the empirical evidence.

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1 Introduction

As a rule of thumb, bank loans are generally secured by specific collateral while bonds are not.[1] As one of the main differences between bank loans and bonds is the type of lender—a single creditor versus small, dispersed bondholders—it would appear natural to rationalize the use of collateral by focusing on the lender. Instead, models of collateral have primarily focused on the borrower by assuming either borrower moral hazard or private information.[2] By implication—and absent any distinction based on lender characteristics—those arguments would then also imply that bonds should be collateralized, contrary to the facts.

This paper provides a *lender-based* theory of collateral based on the dual function of banks to provide credit and assess the borrower's credit risk.[3] We derive our results using the model of discretionary bank lending developed in Inderst and Müller (2003). In that paper, we show that the fact that banks have discretion over credit decisions provides a novel argument for the optimality of debt contracts in borrower-lender relationships. Here, we show that the same intuition also provides a simple, and we believe intuitive, theory of collateral.

At the outset, the bank and the borrower have common information about the project: the borrower lays out his business plan, which provides the bank with information about his business idea, cost and cash flow estimates, and other relevant factors. Building on the notion that banks have specialized expertise in analyzing credit risk, the subsequent credit analysis provides the bank with a more accurate estimate of the project's viability.[4] Almost inevitably,

[1] See, e.g., Brealey and Myers (2003). There are many exceptions to this "rule": Berger and Udell (1990), for instance, find that 30% of commercial and industrial loans in the US are unsecured. On the other hand, utility company bonds and mortgage bonds are often secured by specific assets, as are (by definition) asset-backed bonds.

[2] See Coco (2000) for a survey of the literature.

[3] As for bonds, rating agencies also assess (and publicize) the firm's credit risk. Unlike banks, however, rating agencies do not buy the firm's debt. It is for this reason why our argument does not extend to bonds. While our model is exclusively about bank loans, we provide a brief discussion concerning the distinction between bank loans and bonds in Section 3.

[4] It is unclear to us why, e.g., a young entrepreneur should necessarily have better information about the profitability of his project idea than an experienced lender. Banks' expertise in evaluating projects derives from having granted similar loans in the past (Boot and Thakor (2000), Manove, Padilla, and Pagano (2001)) and the use of credit risk models building on internal (i.e., proprietary) data. Accordingly, Manove et. al argue: "As a result, banks are likely to be more knowledgeable about some aspects of project quality than many of the entrepreneurs they lend to ... This is why banks are, and should be, in the project-evaluation business."

the bank's assessment will be subjective: "[T]he credit decision is left to the local or branch lending officer or relationship manager. Implicitly, this person's expertise, subjective judgement, and his weighting of certain key factors are the most important determinants in the decision to grant credit" (Saunders and Allen (2002)).

We assume that the bank's judgement and beliefs can be represented by a continuous signal. Since the bank's assessement is subjective, this signal is private information, implying that the decision to grant credit is fully discretionary.[5] In our model, the bank's optimal decision rule takes a simple form: approve the loan if and only if the signal is above a certain threshold. The problem is that this threshold is too high relative to a first-best world in which the signal is contractible.[6] In other words, there exists a range of signals where credit is denied even though it should have been granted under the first-best decision rule.

Collateral improves the efficiency of the bank's credit decision: if the project cash flow is low, the bank receives a repayment in excess of the cash flow. In return, the bank can reduce the borrower's repayment at high cash flows. Hence, collateral "flattens" the borrower's repayment schedule. This improves efficiency: since low cash flows are more likely after low signals, shifting more of the borrower's repayment towards low cash flows increases the bank's expected payoff at low signals. Consequently, the bank is more likely to grant credit at low signals, thus lowering its privately optimal acceptance threshold and moving it closer to the first best. As a result, the use of collateral raises the likelihood that credit will be granted.

This argument suggests that we can safely focus on the bank's expected payoff at marginal signals, i.e., signals where the bank's privately optimal decision deviates from the first best. The fact that shifting repayments towards low cash flows (and hence low signals) simultaneously reduces the bank's expected payoff at high signals is inconsequential. This is because the bank's optimal decision rule is a cutoff rule: if the bank finances the project at a certain signal, it is also willing to finance the project at all higher signals.

[5] Precisely, our model is one of lender hidden information, in which a lender takes an observable action (grant or deny credit) after observing a private signal. Due to lack of a sorting variable, there is no point in having the lender choose from a menu of contracts after he observes the signal.

[6] The subjective nature of the credit officer's judgement is frequently viewed as a major problems in credit decisions (Saunders and Allen (2002)). In response to this, banks have developed computerized expert systems such as artificial neural networks and credit scoring. With few exceptions (e.g., credit cards), the practical importance of such computerized systems remains small, however.

Our argument that collateral improves the bank's expected payoff from financing low-signal projects—thus extending the range of signals at which the bank is willing to provide credit—differs markedly from existing (i.e., borrower-based) theories of collateral. It is obviously different from theories based on borrower moral hazard (e.g., Chan and Thakor (1987), Kiyotaki and Moore (1997)). In models of borrower adverse selection, on the other hand, borrowers sort themselves by pledging different amounts of collateral (e.g., Bester (1985), Besanko and Thakor (1987), Stiglitz and Weiss (1986)). In a separating equilibrium, good borrowers pledge more collateral than bad ones—a result that is at odds with the empirical evidence (e.g., Berger and Udell (1990, 1995), Booth (1992)). In our model, by contrast, good borrowers (in terms of ex-ante available information) pledge less collateral than bad ones.[7]

Taking for granted that credit decisions are discretionary, our model argues that collateral improves the incentives of lenders to grant credit after evaluating the project's risk. Rajan and Winton (1995) also examine the effect of collateral on lender incentives, albeit on the incentives of lenders to monitor the borrrower *after* credit has been granted. Precisely, monitoring is valuable because it allows lenders to seek additional collateral if the firm is in financial distress. The question is therefore not whether claims should be collateralized ex ante, but whether lenders will seek to collateralize their claims *after* financing has already been provided. Manove, Padilla, and Pagano (2001), on the other hand, argue that collateral and screening are substitutes. In equilibrium, lenders will either demand collateral *or* screen borrowers. Our model, by contrast, focuses on the incentives of banks to grant credit after having evaluated projects, not on the incentives to evaluate projects as such. To make this point in the simplest possible way, we assume that the project evaluation is costless.[8]

The link between collateral and the amount of credit is an important building block in macro-

[7] Our argument that collateral "flattens" the repayment schedule appears (but is not) related to an argument in the literature on investment financing under adverse selection (Myers and Majluf (1984), Nachman and Noe (1994)). There, the optimal repayment scheme minimizes the underpricing of high-type borrowers, thus ensuring that high types do not break away from the pooling equilibrium. In our model, this is not a concern since loans terms (and thus the "pricing") are determined under symmetric information. (See Section 4 on renegotiation, however.) Rather, "flattening" the repayment schedule allows the lender to commit to financing a larger range of projects after obtaining interim information about the project's risk.

[8] Nothing changes if we introduce a small fixed cost of evaluating the project. For a model in which the structure of financial claims affects the incentives of investors to collect information see, e.g., Fulghieri and Lukin (2001).

economic models studying the propagation and amplification of real and monetary shocks (e.g., Kiyotaki and Moore (1997), Bernanke and Gertler (1989)).[9] Our model offers a microeconomic foundation of this link which, we believe, has desirable properties relative to (moral hazard) models typically used in that literature. For instance, our model suggests that amplification and propagation effects can be large even if investor and creditor protection rights are strong.[10] In fact, they can be perfect as the inefficiency in our model is not with the borrower, but with the lender. All we require is that (i) lenders assess borrowers' credit risk prior to granting credit—an assumption that appears to hold in most cases—and (ii) this assessment is subjective, implying that credit is granted on a discretionary basis. Moreover, microfoundations based on borrower moral hazard are inherently entrepreneurial in the sense that the repayment schedule is designed to provide the owner/entrepreneur with incentives. It is not clear whether these arguments easily extend to firms in which ownership and control are separated, since the ability to design managerial incentive schemes offers an additional degree of freedom.[11] By contrast, it would seem that our argument also applies to public corporations, as the incentive problem is with the lender, not the borrower.

The rest of this paper is organized as follows. Section 2 lays out the model. Section 3 has two parts. The first derives (i) the optimal accept or reject decision given some contract in place, and (ii) the optimal contract taking into account its effect on the subsequent accept or reject decision. The second part examines the comparative statics implications of the optimal contract. In particular, it shows that (i) pledging more collateral increases the likelihood of obtaining credit, and (ii) high-quality borrowers pledge less collateral than low-quality borrowers, which is the opposite result of models based on borrower private information. Section 4 considers renegotiation. Section 5 concludes. All proofs are in the Appendix.

[9] This literature is surveyed in Bernanke, Gertler, and Gilchrist (1999). Introducing a "financial accelerator" to obtain amplification effects is viewed as necessary given that cyclical movements in investments appear too large to be explained by market indicators of expected future profitability.

[10] See La Porta et. al (1998) for a cross-country analysis of investor and creditor protection rights.

[11] For examples of such arguments, see, e.g., Brander and Poitevin (1992) and Dybvig and Zender (1991), who show that the agency problems between firms and investors in Jensen and Meckling (1976) and Myers and Majluf (1984) can be resolved in non-entrepreneurial firms through the optimal design of managerial incentive schemes.

2 The Model

To examine the role of collateral, we extend the model of discretionary bank lending developed in Inderst and Müller (2003). A firm ("the borrower") has a non-divisible project requiring a fixed investment outlay $k > 0$. Financing is provided by a bank ("the lender"). To secure the loan, the borrower can pledge assets, e.g., business property, machines, or receivables due in the future.[12] The total value of pledgeable assets is $w < k$. The project cash flow x is verifiable and random with support $X := [\underline{x}, \overline{x}]$, where it is convenient to assume that $\underline{x} = 0$, albeit this is not crucial. The upper limit $\overline{x}$ can be either finite or infinite.

As laid out in the Introduction, we assume that the borrower and lender initially have common information. Subsequently, the lender obtains a private signal s reflecting his (subjective) assessment of the project's credit risk.[13] The signal is drawn from the unit interval $S := [0, 1]$. The signal distribution $F(s)$ is atomless with density $f(s)$, which is positive everywhere in the interior of S. Each signal gives rise to a (conditional) distribution of project cash flows $G_s(x)$, which is atomless with positive density $g_s(x) > 0$ everywhere. Moreover, $g_s(x)$ is continuous in s for all $x \in X$, while $G_s(x)$ is differentiable in s. The expected project cash flow conditional on s is denoted by $\mu_s := \int_X x g_s(x)dx$.[14] Finally, although only the lender knows s, we assume that the distribution functions $F(s)$ and $G_s(x)$ are common knowledge. The prior (i.e., unconditional) probability of having a cash flow of x is then $\int_S g_s(x)f(s)ds$, while the expected project cash flow based on public information is $\int_X \int_S x g_s(x)f(s)dsdx$.

Observing a high signal is good news as it implies a greater likelihood of high cash flows in the sense of the Monotone Likelihood Ratio Property (MLRP). MLRP is a common assumption in contracting models and satisfied by many distributions (Milgrom (1981)). Moreover, we assume that the conditional project NPV is negative for low signals and positive for high signals, which makes the evaluation of the borrower's project socially desirable.

ASSUMPTION 1. For any pair $(s, s') \in S$ with $s' > s$, the ratio $g_{s'}(x)/g_s(x)$ is strictly increasing in x for all $x \in X$. Moreover, it holds that $\mu_0 < k$ and $\mu_1 > k$.

[12] Since these assets are either needed to undertake the project or become available only at a future date, they cannot be liquidated to finance the investment. Alternatively, we could assume that k is the amount of funds needed after taking into account the firm's liquid funds at the investment stage.

[13] The model can be extended to include a private as well as a public signal.

[14] If $\overline{x} = \infty$, we assume that μ_s is finite for all s.

The timing is as follows. At $\tau = 0$, the lender offers a contract. Given that cash flows are verifiable, the contract specifies a repayment $t(x) \leq x$ out of the project cash flow, an amount $C \leq w$ of collateral to be pledged, and a repayment $c(x) \leq C$ out of the collateralized assets.[15] It is convenient to write $T(x) := t(x) + c(x)$. At $\tau = 1$, the lender evaluates the project, observes the signal s, and then decides whether or not to grant credit. Finally, at $\tau = 2$, the cash flow x is realized, and the lender receives the contractually specified repayment $T(x)$.

Let us briefly comment on two issues. The first concerns a menu of contracts. The standard solution in this sort of setting is to have the lender choose from a prespecified menu after observing s. This solution is of no use here due to lack of a sorting variable.[16] In fact, letting the lender choose from a menu only worsens the efficiency of his credit decision (see Proof of Proposition 4 in the Appendix). Second, there is potentially scope for renegotiation. We consider this in Section 4, where we show that—since renegotiation takes place under asymmetric information—the optimal contract will not be renegotiated in equilibrium.

The following assumption is standard (e.g., Innes (1990), DeMarzo and Duffie (1999)).

Assumption 2. The repayment scheme $T(x)$ is nondecreasing for all $x \in X$.

We also exclude the possibility that the lender "buys" the project *before* assessing the credit risk. Using a standard argument, we assume that upfront payments attract a large pool of fraudulent borrowers, or "fly-by-night operators", i.e., borrowers without a real project (e.g., Rajan (1992), von Thadden (1995)).[17] Assuming that fraudulent borrowers generate a signal $s = 0$ with certainty ensures that they play no role other than ruling out upfront payments.

While the lender makes the contract offer, we assume that the borrower must receive at least $\overline{V} \geq 0$ in expectation at the time of contracting. By gradually increasing $\overline{V}$, we can then trace out the entire frontier of Pareto-optimal contracts. Intuitively, $\overline{V} = 0$ corresponds to a monopolistic credit market where the lender extracts all the surplus. On the other hand, there exists an upper bound $\overline{\overline{V}} > 0$ at which the borrower's utility is maximized, which corresponds to the standard notion of a perfectly competitive credit market.

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[15] To make the problem nontrivial, we assume that varying fractions of C can be liquidated depending on x. This could either be interpreted as selling off liquid assets, e.g., receivables, or as liquidating assets worth C and then handing over a fraction $c(x) \leq C$ of the proceeds to the lender.

[16] We exclude stochastic mechanisms by assuming that it is only verifiable whether credit has been granted or not. By contrast, the probability with which credit has been granted is not verifiable.

[17] This argument also rules out that the lender pays a penalty to the borrower in case credit is denied.

3 Discretionary Credit Decisions and Collateral

3.1 Optimal Contract and Credit Decision

We first characterize the lender's optimal accept or reject decision after observing the signal s. Naturally, this decision depends on the repayment scheme in place, $T(x)$. In a second step, we solve for the optimal repayment scheme, which provides us with solutions for the optimal amount of collateral C and the optimal repayment out of the collateralized assets $c(x)$.

The first-best decision rule—i.e., the optimal decision rule in a world where the signal s is contractible—takes a simple form. Assumption 1, in conjunction with the fact that $g_s(x)$ is continuous in s, implies that the conditional expected cash flow μ_s is continuous and strictly increasing in s, where $\mu_s < k$ for low values of s and $\mu_s > k$ for high values of s. There consequently exists a unique interior cutoff signal $s_{FB} \in (0,1)$ given by $\mu_{s_{FB}} = k$ such that the NPV is positive if and only if $s > s_{FB}$. The first-best decision rule is to grant credit if and only if $s \geq s_{FB}$.

Since s is noncontractible, the lender's credit decision is fully discretionary, however. As a result, the lender provides credit if and only if his conditional expected payoff

$$U_s(T) := \int_X T(x) g_s(x) dx$$

equals or exceeds his investment k.[18,19] As shown in Inderst and Müller (2003), the lender's privately optimal decision rule takes a simple form: provide credit if and only if $s \geq s^*(T)$, where $s^*(T) \in (0,1)$ is the cutoff signal at which the lender breaks even, i.e., $U_{s^*(T)}(T) = k$.[20]

Working backwards, we can now set up the lender's contract design problem at $\tau = 0$, which rationally takes into account the effect of the repayment scheme $T(x)$ on his accept or reject decision at $\tau = 1$. For convenience, we write $s^*(T)$ simply as s^*. The optimal repayment scheme is determined in the usual way by maximizing the utility of one side (here: the lender) subject to providing the other side with a minimum utility of $\overline{V}$. By varying $\overline{V}$, we can trace out the

[18] The lender's expected payoff depends only on the total repayment $T(x)$, not on how this repayment is composed of project cash flows $t(x)$ and pledged assets $c(x)$.

[19] We assume that in case of indifference, the lender approves the loan.

[20] Assumptions 1 and 2 imply that the lender's conditional expected payoff $U_s(T)$ is nondereasing in s. Ignoring cases where the lender approves or rejects the loan for all signals $s \in S$, we have that $T(x) > 0$ on sets of positive measure, which implies that $U_s(T)$ must be strictly increasing in s. The rest is obvious.

entire frontier of Pareto-optimal contracts. The lender maximizes his expected payoff

$$\int_{s^*}^{1} [U_s(T) - k] f(s) ds \tag{1}$$

subject to the constraint that the borrower's expected payoff is at least $\overline{V}$,

$$\int_{s^*}^{1} [\mu_s - U_s(T)] f(s) ds \geq \overline{V}, \tag{2}$$

and the constraint that $U_{s^*(T)}(T) = k$, which characterizes the lender's privately optimal decision rule at $\tau = 1$.

By standard arguments, the borrower's participation constraint (2) binds in equilibrium, implying that the lender receives any surplus in excess of $\overline{V}$. Since the lender is the residual claimant to all surplus, he proposes a contract which incentivizes him to make as efficient as possible a credit decision in $\tau = 1$. If possible, he will thus propose a contract inducing him to employ the first-best decision rule $s^* = s_{FB}$. One situation where this is trivially possible is $\overline{V} = 0$, i.e., if the lender extracts all the surplus. In this case, the lender receives the full cash flow $T(x) = x$ for all $x \in X$, which provides him with first-best incentives. In all other cases, however, the contract design is nontrivial. The following proposition shows that the optimal contract is collateralized debt.

Proposition 1. *The optimal contract stipulates a repayment R and an amount of collateral $C \leq w$ such that the lender receives $T(x) = x + C$ if $x \leq R - C$ and $T(x) = R$ if $x > R - C$.*

Proof. See Appendix.

Proposition 1 also characterizes the optimal repayment out of the collateralized assets $c(x)$. If $x \leq R - C$, the unique optimal repayment out of the collateralized assets is $c(x) = C$. Hence, if the project cash flow is low, the lender seizes the entire collateral. On the other hand, if $x > R - C$, only the total repayment $T(x) = t(x) + c(x)$ is uniquely determined. If there is an arbitrarily small cost of liquidating collateral, however, the unique optimal repayment out of the collateralized assets if $x > R - C$ is $c(x) = \max\{C, R - x\}$. In this case, a transfer is made out of the collateralized assets only if $x < T(x)$, which minimizes the set of cash flows at which collateral is liquidated.[21] The optimal solution is depicted in Figure 1. It shows that payments out of the collateralized assets are made only if the cash flow is low, which accords well with the intuition that "collateral protects lenders from bad outcomes".

[21] In a previous version of this paper, we formally modelled such a preference for minimizing payments out of collateralized assets. This provided no any additional insights, however.

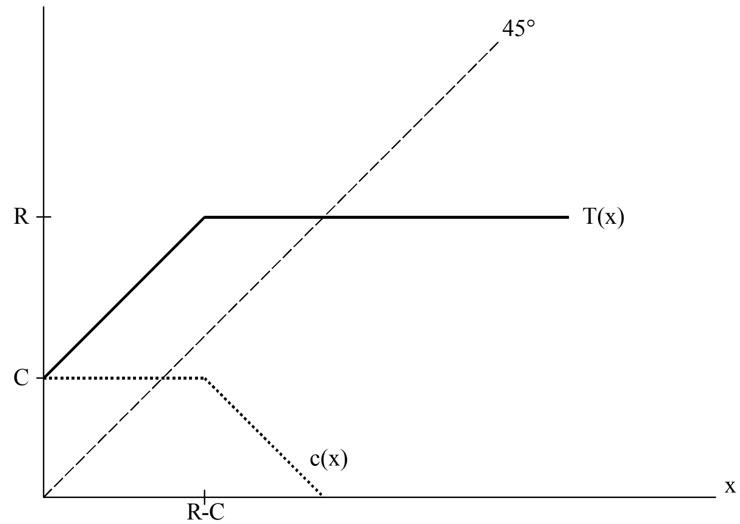

Figure 1: Optimal total repayment $T(x)$, optimal amount of collateralized assets C, and optimal repayment out of the collateralized assets $c(x)$ given by Proposition 1.

Proposition 1 implies that the intuition in Inderst and Müller (2003) for why debt is optimal also suggests a simple—and we believe intuitive—theory of collateral that is different from existing theories based on borrower moral hazard or adverse selection. The easiest way to grasp this intuition is by first considering the case where $w = C = 0$. Without collateral, it must necessarily hold that $T(x) \leq x$ for all x. In the nontrivial case where $\overline{V} > 0$, this immediately implies that $T(x) < x$ for some x on sets of positive measure. Since $g_s(x) > 0$ for all x, this in turn implies that $U_s(T) < \mu_s$ for all s, i.e., the lender's expected payoff is strictly less than the expected project cash flow for all signals. In conjunction with the fact that both μ_s and $U_s(T)$ are strictly increasing in s, this finally implies that $s^* > s_{FB}$.[22] Hence, the lender's cutoff signal is too high relative to the first best. Put differently, at marginal signals $s \in [s_{FB}, s^*)$ the expected project cash flow μ_s exceeds the investment cost k, while the lender's expected payoff $U_s(T)$ falls below it. As a consequence, the lender (inefficiently) rejects the project.

If $w > 0$, pledging collateral mitigates, or even eliminates, this inefficiency. By Proposition

[22] Precisely, at $s = s_{FB}$ it holds that $k = \mu_{s_{FB}} > U_{s_{FB}}(T)$, while at $s = s^*$ it holds that $k = U_{s^*}(T) < \mu_{s^*}$. Given that $U_s(T) < \mu_s$ for all s and $U_s(T)$ and μ_s are both strictly increasing in s, this implies that $s^* > s_{FB}$.

1, the lender receives the entire project cash flow x plus the full collateral C if the project cash flow is low. Since low cash flows are more likely after low signals, this improves the lender's expected payoff $U_s(T)$ at low signals, thereby pushing his cutoff signal s^* down and narrowing the gap between s^* and s_{FB}. In fact, if C is sufficiently large, it is possible to push s^* all the way down to s_{FB}, or equivalently, raise $U_{s_{FB}}(T)$ up to the point where $U_{s_{FB}}(T) = \mu_{s_{FB}} = k$, in which case the first best is attained. Of course, whether C is sufficiently large will depend on the amount of pledgeable assets w. We shall return to this below.

To comments remain in order. First, since $U_s(T)$ is strictly increasing in s, it also holds that $U_s(T) > k$ for all higher signals $s > s^*$. Intuitively, since the lender's optimal decision rule is a cutoff rule, we can safely focus on his cutoff signal. If the lender approves the loan at that signal, he will also approve it at all higher signals $s > s^*$. Second, to maximize $U_s(T)$ at low signals the optimal contract must minimize $U_s(T)$ at high signals, subject to the monotonicity condition in Assumption 2.[23] Otherwise, the borrower cannot obtain $\overline{V}$ in expectation. This is precisely what the optimal contract in Proposition 1 does.

The above discussion suggests that—as long as the lender's credit decision is inefficient (i.e., $s^* > s_{FB}$)—all available assets will be pledged as collateral, i.e., $C = w$. If w is sufficiently large so that $U_{s_{FB}}(T) = \mu_{s_{FB}} = k$ is possible, the first best can be attained. At this point, it is crucial that $U_{s_{FB}}(T)$ is not increased any further, or else we have the opposite inefficiency that $s^* < s_{FB}$. This also implies that—if w is sufficiently large to attain the first best—there is some leeway in structuring the optimal contract. Any optimal contract must have collateral, however.

Let us briefly return to the distinction between bank loans and bonds alluded to in the Introduction. In our model, pledging collateral is useful because the lender's credit decision is inefficient. This inefficiency is due to the fact that (i) the lender's signal is private information, and (ii) the agent observing the signal (i.e., the lender) is also the agent providing the financing. Evidently, if the signal were public information the first best could be attained trivially. Alternatively, if the signal was observed by someone with no financial stake in the project, this person would have no reason to misreport his signal. Again, contracts contingent on the true signal could be written, and the first best could be trivially attained.

Unlike bank loans, bonds are assessed by independent rating agencies such as Moody's and

[23] "Minimizing $U_s(T)$ at high signals" does not mean pushing $U_s(T)$ below k. As we just argued, since $U_s(T)$ is strictly increasing in s, it holds that $U_s(T) > k$ for all signals $s > s^*$.

S&P. Given that rating agencies do not buy the firm's debt, there is prima facie no reason why they should have an incentive to misreport their information. In fact, their entire reputation is based on the notion that their information is truthful. In terms of our model, this implies that the (true) signal s is contractible. But if financial contracts can condition on the true signal, we obtain the first best. Absent any inefficiency, there is then no need for collateral. Hence, while our theory suggests that bank loans should be collateralized, it does not imply that bonds should be collateralized.

If separation of information production and financing can yield the first best, a natural question is why banks provide these functions jointly. Our paper has little to say about this; we merely take as given the notion that banks evaluate borrowers prior to making credit decisions. One commonly found explanation relates to the high fixed cost of a bond issue in combination with the costs and procedural hurdles of obtaining a credit rating. (Indeed, bond issuers commonly need *two* ratings, e.g., one from Moody's and one from S&P.) As a result, issuing bonds tends to be less attractive for smaller firms or firms with smaller financing needs. Another possible explanation (Damodaran (2001)) is that "firms can convey proprietary information to the lending bank that will help in both evaluating and pricing the loan, without worrying about the information getting out to its competitors. This is more difficult to do in a corporate bond issue, where the information provided by the firm will be widely disseminated."

3.2 Pledgeable Assets, Collateral, and Credit Availability

This section examines the comparative statics implications of the optimal contract in Proposition 1. By our previous discussion, the first best cannot be attained unless w is sufficiently large. The resulting inefficiency is minimized by maximizing the lender's expected payoff at low signals, thus maximizing $T(x)$ at low cash flows x. This immediately implies that $C = w$, i.e., all assets are pledged as collateral.

How does a small increase in pledgeable assets, say, from w to w', affect the availability of credit? Assuming we are in the region where the first best cannot be attained, a change from w to w' induces a corresponding change in collateral from $C = w$ to $C' = w'$. To ensure that the borrower's participation constraint (2) remains satisfied, the optimal repayment R stipulated in Proposition 1 must decrease to, say, $R' < R$. The increase in C and corresponding decrease in R further "flattens" the optimal repayment scheme $T(x)$, thereby allowing the lender to further lower his cutoff signal s^*. This holds until the first best is attained. From then on, any further increase in w has no additional effect on either C or R.

Incidentally, it is not optimal to completely "flatten" the repayment schedule (i.e., to set $T(x) = B$ for all x.) If the lender were fully insulated from cash-flow risk, he would either always (if $B \geq k$) or never (if $B < k$) accept the project—irrespective of the signal s. The first-best decision rule, however, prescribes to accept the project if and only if $s \geq s_{FB} \in (0, 1)$, which implies the project should be rejected if the signal is low. Hence, even if the amount of pledgeable assets w is large, the lender must be exposed to some cash-flow variability to induce him to use his private information efficiently.

Proposition 2. *An increase in the amount of pledgeable assets w has the following effect on the required collateral C, the repayment R, and the lender's cutoff signal s^* :*
i) If $\overline{V} = 0$ the optimal contract stipulates $C = 0$, and the credit decision is first-best efficient, i.e., $s^* = s_{FB}$. A change in w has no effect on either C, R, or s^*.
ii) If $\overline{V} > 0$ there exists a critical value w_{FB} such that $s^* > s_{FB}$ if $w < w_{FB}$, which implies the credit decision is inefficient. The optimal contract then requires that all assets be pledged as collateral, i.e., $C = w$. An increase in w increases C by the same amount, while R and s^* both decrease, thereby making it more likely that credit is provided. On the other hand, if $w > w_{FB}$ the credit decision is first-best efficient, the optimal contract stipulates $C = w_{FB} < w$, and any further increase in w beyond w_{FB} has no effect.

Proof. See Appendix.

Proposition 2 is our key result. Unless $\overline{V} = 0$—which corresponds to a credit market in which the lender extracts all the surplus—pledging assets as collateral improves the lender's credit desicion, and thus the availability of credit. If the amount of pledgeable assets w is small, all assets will be collateralized. The greater the amount of pledgeable assets, the higher is the likelihood that credit is provided.[24] Consequently, pledging collateral and screening borrowers are all but substitutes. Quite the contrary: collateral is valuable *because* the lender screens borrowers before granting credit, because it improves the lender's post-screening decision. By contrast, if there was no screening—and the lender had to base his decision solely on public

[24]This is consistent with the empirical evidence. Numerous studies suggest that the availability of credit—and thus the ability to make investments—is positively related to the value of firms' assets (e.g., Fazzari, Hubbard, and Petersen (1988), Kashyap, Lamont, and Stein (1994)).

information—pledging assets as collateral would have no value. Finally, if the amount of pledgeable assets w is sufficiently large to attain the first best ($w = w_{FB}$), any further increase in w has no value, since there is no point in raising the amount of collateralized assets above the efficient level $C = w_{FB}$.

Let us finally address the relation between project quality and collateral. Suppose there is no constraint on the amount of collateral that can be pledged. Under this assumption, theories based on borrower private information (e.g., Bester (1985), Besanko and Thakor (1987)) produce the counterfactual result that in equilibrium high-quality borrowers pledge *more* collateral than low-quality borrowers to induce separation (see Introduction). In our model, the opposite holds: high-quality borrowers pledge *less* collateral than low-quality borrowers, where "quality" is defined in terms of ex-ante available information. Precisely, a project has a higher quality if it has a more favorable ex-ante distribution $F(s)$ in the sense of first-order stochastic dominance (FOSD). This implies, among other things, that high-quality projects have a higher unconditional expected cash flow $\int_X \int_S x g_s(x) f(s) ds dx$ than low-quality projects.

The intuition is straightforward: collateral improves the lender's expected payoff from financing marginally profitable projects, i.e., projects with signals close to s_{FB}. High-quality projects are more likely to generate high signals, which implies they are less likely to be marginally profitable. Consequently, less collateral is required to ensure an efficient credit decision.

Proposition 3. *High-quality borrowers need to pledge less collateral than low-quality borrowers.*

Proof. See Appendix.

4 Renegotiation

If $\overline{V} > 0$ and $w < w_{FB}$, Proposition 2 suggests that the lender's credit decision is inefficient. Precisely, at marginal signals $s \in [s_{FB}, s^*)$ the lender rejects the project even though its (conditional) expected cash flow exceeds the investment cost k. This potentially creates scope for mutually beneficial renegotiations: rather than being denied credit altogether, the borrower might want to propose a new contract under which he obtains a smaller surplus but which allows the lender to break even. This is precisely what would happen if s was commonly observable, in which case renegotiations would eliminate all inefficiencies.

The problem is that the true signal s is not commonly observable. In particular, anticipating that the borrower might propose a more favorable contract (from the lender's point of view), the lender has a strong incentive to claim that the signal is marginal (i.e., $s \in [s_{FB}, s^*))$ even if the true signal is high and he was planning to accept the project anyway. But at signals $s \geq s^*$ where the lender would have accepted the project anyway, replacing the old contract is a pure wealth transfer from the borrower to the lender. As we show below, this implies that the expected value to the borrower from replacing the optimal contract is negative, implying that the optimal contract in Propositions 1-2 is renegotiation-proof.

We first prove an auxiliary result. It shows that, if the lender prefers some new contract $\widetilde{T}(x)$ to the optimal contract $T(x)$ at signal s', he also prefers $\widetilde{T}(x)$ at all higher signals $s > s'$. This confirms our above intuition that the prospects of obtaining a more favorable contract after rejecting the project induces the lender to (strategically) reject the project at high signals.

Lemma 1. *Suppose the credit decision is inefficient (i.e.,* $\overline{V} > 0$ *and* $w < w_{FB}$*) and denote by* $T(x)$ *and* $\widetilde{T}(x)$ *the optimal contract in Propositions 1-2 and some other, arbitrary contract satisfying Assumption 2, respectively. Either the lender prefers* $T(x)$ *to* $\widetilde{T}(x)$ *(or vice versa) for all signals* $s \in S$*, or there exists a threshold signal* $\widetilde{s} \in (0, 1)$ *such that the lender prefers* $\widetilde{T}(x)$ *to* $T(x)$ *if* $s > \widetilde{s}$ *and* $T(x)$ *to* $\widetilde{T}(x)$ *if* $s < \widetilde{s}$*. At* $s = \widetilde{s}$*, he is indifferent between* $T(x)$ *and* $\widetilde{T}(x)$*.*

Proof. See Appendix.

Consider now the renegotiation game. After the lender observes the signal but before a decision is made, a new contract can be offered. (In fact, as the lender can always reverse his decision, it is irrelevant whether the decision has been made or not as it contains no signalling value.) If the lender makes the contract offer, the borrower must agree to it, whereas if the borrower makes the contract offer, the lender must agree.[25] Regardless of who makes the offer, the following proposition shows that the optimal contract will not be renegotiated:[26]

Proposition 4. *Suppose the credit decision is inefficient (i.e.,* $\overline{V} > 0$ *and* $w < w_{FB}$*). Regardless of who makes the contract offer, the optimal contract in Propositions 1-2 will not be renegotiated.*

Proof. See Appendix.

[25] Introducing menus of contracts at the renegotiation stage would not change our results. Absent a sorting variable, the lender would simply choose his most preferred contract from the menu. See the Proof of Proposition 4 in the Appendix for a formal proof of this assertion.

[26] It is straightforward to extend Proposition 4 to any sequence of moves in the renegotiation game provided this does not create a sorting device (e.g., costly delay).

The intuition for Proposition 4 is straightforward. Suppose the lender makes the contract offer and denote the new contract by $\widetilde{T}(x)$. (The argument if the borrower makes the offer is analogous.) By optimality, it must hold that $U_s(\widetilde{T}) > U_s(T)$ for at least some s (or else the lender would not offer $\widetilde{T}$). By Lemma 1, this implies that either (i) $U_s(\widetilde{T}) \geq U_s(T)$ for all s (with strict inequality for some s), or (ii) there exists a threshold signal $\widetilde{s}$ such that $U_s(\widetilde{T}) > U_s(T)$ for all $s > \widetilde{s}$ and $U_s(T) > U_s(\widetilde{T})$ for all $s < \widetilde{s}$.

There are now two cases. In the first case, it holds that $s^*(\widetilde{T}) \geq s^*(T)$, i.e., the new contract does not lead to a lower cutoff signal. This immediately implies that we are in situation (ii). But if $s^*(\widetilde{T}) \geq s^*(T)$ it follows directly from the definitions of $s^*(\widetilde{T}) \geq s^*(T)$ and strict monotonicity of $U_s(T)$ and $U_s(\widetilde{T})$ that $U_{s^*(T)}(\widetilde{T}) \leq U_{s^*(T)}(T)$, and therefore that $\widetilde{s} \geq s^*(T)$. Evidently, if the lender offers to replace T with $\widetilde{T}$, it must be the case that $s \geq \widetilde{s}$, which implies the borrower is strictly worse off by accepting the new contract. He consequently rejects.

In the second case, we have that $s^*(\widetilde{T}) < s^*(T)$, i.e., the new contract *does* lead to a lower cutoff signal. We are then either in situation (i) or (ii). In situation (i), it holds that $U_s(\widetilde{T}) \geq U_s(T)$ for all s (with strict inequality for some s). In situation (ii), by the definitions of $s^*(\widetilde{T})$ and $s^*(T)$, it holds that $U_s(\widetilde{T}) > U_s(T)$ for all $s \in [s^*(\widetilde{T}), s^*(T)]$. But by Lemma 1, this implies that $U_s(\widetilde{T}) > U_s(T)$ for all $s > s^*(T)$. Hence—regardless of whether situation (i) or (ii) applies—the fact that the lender offers $\widetilde{T}$ provides the borrower with absolutely no information as to whether the true signal is $s \in [s^*(\widetilde{T}), s^*(T)]$ (in which case the borrower would like to replace T with $\widetilde{T}$) or $s > s^*(T)$ (in which case the borrower would like to leave T in place, as the project will be accepted anyway and replacing T with $\widetilde{T}$ only transfers wealth to the lender.) Accordingly, the borrower will agree to replace T with $\widetilde{T}$ if and only if

$$\int_{s^*(\widetilde{T})}^{1} [\mu_s - U_s(\widetilde{T})] f(s) ds \geq \int_{s^*(T)}^{1} [\mu_s - U_s(T)] f(s) ds = \overline{V}. \tag{3}$$

But there cannot exist a contract $\widetilde{T}$ leaving the borrower with (weakly) more than $\overline{V}$ while making the lender strictly better off for all $s \geq s^*(\widetilde{T})$ and equally well off for all $s < s^*(\widetilde{T})$ (in which case the project is rejected both under $\widetilde{T}$ and T). If this was the case, T would not be the solution to the lender's original maximization problem (1)-(2), contradicting Propositions 1-2.

5 Conclusion

This paper provides a novel, and we believe intuitive, argument for the use of collateral based on the notion that collateral mitigates excessive conservativism in banks' credit decisions. Our theory is entirely lender-based and does not assume any moral hazard or private information on the part of the borrower. It is valid irrespective of whether investor and creditor protection rights are strong or weak, and whether ownership and control (on the part of the borrower) are separated or not. Moreover, unlike theories based on borrower private information, our theory does not imply that high-quality borrowers pledge more collateral than low-quality borrowers. Rather, it implies the opposite, which is in accord with the empirical evidence.

At the heart of our theory is a multitasking problem on the part of the lender. The lender provides both financing and information about the project's risk and profitability. The fact that the lender provides financing prevents him from truthfully revealing his information, while the fact that the information cannot be contracted upon distorts the lender's financing decision. If information production and financing could be separated, the commitment problem examined in this paper could be overcome—provided the information is revealed truthfully, of course. In practice, banks *do* provide both financing and information, however. While there are many possible reasons, one possible reason is that firms feel more comfortable revealing their proprietary information to their housebank than to a third party such as, e.g., a rating agency, where the information (or at least a coarse aggregate thereof, like a credit rating) would become available to competitors (Damodaran (2001)).

Even if information production and financing cannot be separated, our argument would suggest that efficiency can be improved if at least the lender does not bear the full investment cost. For instance, if the borrower has liquid funds, he could use them to co-finance the investment. (In the paper, we have assumed that the borrower has no liquid funds.) Likewise, additional funds might be provided by a third party who free rides on the (informed) lender's decision. Our theory would then suggest that the lender generating the information (e.g., the housebank) should hold the most senior claim as this maximizes his expected payoff at low signals. Extending our model to multiple lenders appears to be an interesting avenue for future research.

6 Appendix: Proofs

It is convenient to introduce some additional notation. Denote the lender's *ex ante* payoff by $U(T) := \int_{s^*}^1 [U_s(T) - k] f(s) ds$, the borrower's expected payoff for a given signal by $V_s(T) := \mu_s - U_s(T)$, and the borrower's *ex ante* payoff by $V(T) := \int_{s^*}^1 V_s(T) f(s) ds$.

Proof of Proposition 1. The case with $\overline{V} = 0$ is trivial. Here, the lender can achieve $s^* = s_{FB}$ by setting $T(x) = x$ for all $x \in X$, which corresponds to choosing $C = 0$ and $R = \overline{x}$. Suppose thus that $\overline{V} > 0$. To prove Proposition 1 it would be sufficient to show that collateralized debt is *an* optimal contract. However, as this is needed for other proofs below, it is convenient to prove already now that $T(x)$ is uniquely determined if the first-best credit policy can not be achieved. We prove the following claim.

Observation.

i) It always holds (under an optimal contract) that $s^* \geq s_{FB}$.

ii) It is always optimal to offer a contract where $T(x) = \min\{x + C, R\}$.

iii) $T(x)$ *is uniquely pinned down whenever it holds that* $s^* > s_{FB}$.

We focus first on Observations ii) and iii). Observation i) will then follow immediately.

We argue to a contradiction. Suppose the lender offers (t, c) where $T(x)$ does not satisfy the above characteristics. We prove now jointly that the lender is not worse off by offering a contract where the repayment schedule satisfies ii) and that she is even strictly better off to do so in case $s^* \neq s_{FB}$. For the new contract, which satisfies ii), denote the repayment schedule by $\widetilde{T}$, where we choose the level of collateral $\widetilde{C} = w$ and where $\widetilde{R}$ denotes the repayment requirement. Denote the difference in repayments $z(x) := \widetilde{T}(x) - T(x)$. We choose $\widetilde{R}$ such that

$$\int_{s^*(T)}^1 \left[\int_X z(x) g_s(x) dx \right] f(s) ds = 0. \tag{4}$$

Hence, if we hold the cutoff signal fixed at $s^*(T)$, the lender's and borrower's *ex ante* payoffs are unchanged if we replace contracts. Note that existence of a unique value $\widetilde{R}$ solving (4) is immediate by the following observations. First, holding the cutoff signal constant, the lender's payoff is continuous and strictly increasing in $\widetilde{R}$. (Recall that $g_s(x)$ is continuous in s for all x.) Second, the left-hand side of (4) is surely strictly positive at $\widetilde{R} = \overline{x}$ and strictly negative at $\widetilde{R} = 0$.

Note also that, by construction of $\widetilde{T}$ and by Assumption 2, there exists a value $0 < \widetilde{x} < \overline{x}$

such that $z(x) \geq 0$ holds for all $x < \widetilde{x}$ and $z(x) \leq 0$ holds for all $x > \widetilde{x}$, where the inequalities hold strictly for sets of positive measures.

Claim 1. *It holds that* $s^*(\widetilde{T}) < s^*(T)$.

Proof. By (4) and continuity of $g_s(x)$ in s there exists some $s^*(T) < \widetilde{s} < 1$ where $\int_X z(x) g_{\widetilde{s}}(x) dx = 0$. By Assumption 1 and as $\widetilde{s} > s^*(T)$ it holds that $g_{s^*(T)}(x)/g_{\widetilde{s}}(x)$ is strictly decreasing in x. We can thus rewrite $\int_X z(x) g_{s^*(T)}(x) dx$ as

$$\begin{aligned}\int_X z(x) g_{s^*(T)}(x) dx &= \int_{x \leq \widetilde{x}} z(x) g_{\widetilde{s}}(x) \frac{g_{s^*(T)}(x)}{g_{\widetilde{s}}(x)} dx + \int_{x > \widetilde{x}} z(x) g_{\widetilde{s}}(x) \frac{g_{s^*(T)}(x)}{g_{\widetilde{s}}(x)} dx \\ &> \frac{g_{s^*(T)}(\widetilde{x})}{g_{\widetilde{s}}(\widetilde{x})} \int_X z(x) g_{\widetilde{s}}(x) dx = 0.\end{aligned}$$

Since $\int_X z(x) g_{s^*(T)}(x) dx > 0$ and $\int_X T(x) g_{s^*(T)}(x) dx = k$ by definition of $s^*(T)$, we have that $\int_X \widetilde{T}(x) g_{s^*(T)}(x) dx > k$. Then, $s^*(\widetilde{T}) < s^*(T)$ follows immediately from the definition of the cutoff signal and as $U_s(\widetilde{T})$ is strictly increasing in s. **Q.E.D.**

The new cutoff $s^*(\widetilde{T})$ may now lie below the efficient cutoff. In this case, i.e., if $s^*(\widetilde{T}) < s_{FB}$, we carry out another adjustment to the contract. Holding again the original cutoff signal $s^*(T)$ fixed in (4), we decrease $\widetilde{C}$ and increase $\widetilde{R}$ until the true cutoff signal satisfies $s^*(\widetilde{T}) = s_{FB}$.

Claim 2. *If* $s^*(\widetilde{T}) < s_{FB}$ *we can adjust the new contract by decreasing* $\widetilde{C}$ *and increasing* $\widetilde{R}$ *such that* (4) *still holds, while the new cutoff is equal to* s_{FB}.

Proof. We can use an argument analogous to that in the proof of Claim 1. Take first some collateralized debt contract characterized by $(\widehat{R}, \widehat{C})$, where $\widehat{R} > \widetilde{R}$ and $\widehat{C} < \widetilde{C}$, such that (4) holds with $z(x) := \widehat{T}(x) - \widetilde{T}(x)$. From (4)—together with $\widehat{R} > \widetilde{R}$ and $\widehat{C} < \widetilde{C}$—it follows that there exists a value $0 < \widetilde{x} < \overline{x}$ such that $z(x) \geq 0$ holds for all $x > \widetilde{x}$ and $z(x) \leq 0$ holds for all $x < \widetilde{x}$, where the inequalities hold strictly for sets of positive measures. By the argument in Claim 1 this implies $s^*(\widehat{T}) > s^*(\widetilde{T})$.

As we decrease $\widehat{C}$ and adjust $\widehat{R}$ accordingly to satisfy (4), the definition of s^* and continuity of $g_s(x)$ imply that $s^*(\widehat{T})$ increases continuously. As $s^*(\widehat{T}) > s_{FB}$ holds at $\widehat{C} = 0$ this completes the proof. **Q.E.D.**

Summing up, we have constructed a contract with the following characteristics: i) $\widetilde{T}$ satisfies the characterization in the above observation; ii) (4) is satisfied; iii) $s_{FB} \leq s^*(\widetilde{T}) \leq s^*(T)$ holds in case $s^*(T) \geq s_{FB}$, where $s^*(\widetilde{T}) < s^*(T)$ holds strictly in case $s^*(T) > s_{FB}$; and iv) $s^*(T) < s^*(\widetilde{T}) = s_{FB}$ holds in case $s^*(T) < s_{FB}$. If the new contract was accepted by the borrower, the lender would surely be not worse off and even strictly better off if $s^*(\widetilde{T}) \neq s^*(t,c)$. This follows immediately from (4) and as the lender optimally chooses the cutoff signal. To prove the Observation, it only remains to show that the borrower is not worse off under the new contract.

Claim 3. *$V(\widetilde{T}) \geq V(T)$.*

Proof. We distinguish between three cases.

Case 1: $s^*(T) = s_{FB}$. As this implies $s^*(T) = s^*(\widetilde{T})$ by construction of $\widetilde{T}$, the assertion is immediate.

Case 2: $s^*(T) > s_{FB}$. In this case, construction of $\widetilde{T}$ implies $s_{FB} \leq s^*(\widetilde{T}) < s^*(T)$. It follows also from construction of $\widetilde{T}$ that the borrower's expected payoff would remain unchanged if the loan was approved if and only if $s \geq s^*(T)$. Hence, $V(\widetilde{T}) \geq V(T)$ follows if $V_s(\widetilde{T}) \geq 0$ holds for all $s \in [s^*(\widetilde{T}), s^*(T)]$. To see that this is the case, note first that $V_{s^*(\widetilde{T})}(\widetilde{T}) \geq 0$ holds from $U_{s^*(\widetilde{T})}(\widetilde{T}) = 0$ and $s_{FB} \leq s^*(\widetilde{T})$. It thus remains to show that $V_s(\widetilde{T})$ is non-decreasing. To see that this is the case, note first that the borrower's net payoff is $\max\{-\widetilde{C}, \widetilde{R} - x\}$ such that

$$V_s(\widetilde{T}) = \int_{\widetilde{R}-\widetilde{C}}^{\overline{x}} [x - (\widetilde{R} - \widetilde{C})] g_s(x) dx - \widetilde{C},$$

which after partial integration transforms to

$$V_s(\widetilde{T}) = \int_{\widetilde{R}-\widetilde{C}}^{\overline{x}} [1 - G_s(x)]\, dx - \widetilde{C}. \tag{5}$$

Assumption 1 implies that $G_s(x)$ satisfies strict First-Order Stochastic Dominance, i.e., for all $0 < x < \overline{x}$ it holds that $G_s(x)$ is strictly decreasing in s. This proves that $V_s(\widetilde{T})$ is (strictly) increasing in s.

Case 3: $s^*(T) < s_{FB}$. In this case, construction of $\widetilde{T}$ implies $s^*(\widetilde{T}) = s_{FB}$. It then remains to show that $V_s(\widetilde{T}) \leq 0$ holds for all $s \in [s^*(\widetilde{T}), s_{FB}]$. As $s^*(\widetilde{T}) = s_{FB}$, implying $U_{s_{FB}}(\widetilde{T}) = 0$, it follows that $V_{s_{FB}}(\widetilde{T}) = 0$. It thus remains to show that $V_s(\widetilde{T})$ is nondecreasing in s, which holds by the discussion of Case 2. This completes the proof of Claim 3 and thus also of Proposition 1. **Q.E.D.**

Proof of Proposition 2. We can again focus on the non-trivial case $\overline{V} > 0$. Moreover, by Proposition 1 we can focus on collateralized debt contracts, characterized by two variables, (C, R). From the proof of Proposition 1 we also know that it always holds that $s^* \geq s_{FB}$, while for $s^* > s_{FB}$ the lender's repayment schedule $T(x)$ is uniquely characterized. Finally, from the arguments in the main text we know that $s^* > s_{FB}$ holds for $w = 0$. As we now increase w, the lender's program becomes more relaxed, which implies that s^* must be nonincreasing in w. To prove Proposition 2, it thus only remains to show that $s^* > s_{FB}$ implies $C = w$ and that in this case s^* strictly decreases in w.

Claim 1. *If $s^* > s_{FB}$ then $C = w$ and s^* is strictly decreasing in w.*

Proof. We show first that $s^* > s_{FB}$ implies $C = w$. Suppose this was not the case and that a contract specifying $C < w$ was optimal. We can now apply the steps of the proof of Proposition 1. Holding s^* constant, we can choose a new contract characterized by $(\widetilde{R}, \widetilde{C})$ where $w \geq \widetilde{C} > C$ and payoffs remain unchanged if we fix s^*, i.e., (4) holds. As the new contract shifts more of the lender's repayment into low cash-flow states, we have from the argument in Claim 1 of Proposition 1 that $s^*(\widetilde{T}) < s^*(T)$. If we only marginally adjust $\widetilde{C}$ and $\widetilde{R}$, compared to C and R, we can insure that still $s^*(\widetilde{T}) \geq s_{FB}$. Using next Claim 3 of Proposition 1, we also know that the borrower's participation constraint still holds. Finally, by (4) and $s_{FB} \leq s^*(\widetilde{T}) < s^*(T)$ the lender must be strictly better off, contradicting the optimality of the original contract.

A similar argument also proves the second part of the claim. Take a value w where $s^* > s_{FB}$ still holds as we choose $C = w$ under the optimal contract. If the value of assets increases from w to $\widetilde{w} > w$, we can construct a new contract with repayment $\widetilde{T}$ where $\widetilde{w} \geq \widetilde{C} > C$ while the total repayment requirement is lower, $\widetilde{R} < R$. Again, we can choose $\widetilde{C}$ and $\widetilde{R}$ such that $s_{FB} \leq s^*(\widetilde{T}) < s^*(T)$ and the borrower's constraint is still satisfied. This completes the proof of Claim 1 and thus also of Proposition 2. **Q.E.D.**

Proof of Proposition 3. Take two distributions, $F(s)$ and $\widetilde{F}(s)$, where $\widetilde{F}(s)$ strictly dominates $F(s)$ in the sense of strict FOSD. By Proposition 2 we have for $s^* > s_{FB}$ that all assets are posted as collateral. To prove Proposition 3 we now distinguish between two cases. If $s^* > s_{FB}$ holds with $F(s)$, implying $C = w$, then the result is immediate as $\widetilde{C}$ can certainly not be higher. For the second case, where $s^* = s_{FB}$ holds with $F(s)$, we have the following result.

Claim 1. *If $s^* = s_{FB}$ holds with $F(s)$, then we have that $\widetilde{C} < C$.*

Proof. Take the optimal contract (t, c) for $F(s)$, which is characterized by C and R. Note next that the choice of $s^* = s_{FB}$ is independent of the ex ante distribution over s. Moreover, as $V_s(T)$ is strictly increasing in s, the fact that $\widetilde{F}(s)$ strictly dominates $F(s)$ implies that T is also feasible under $\widetilde{F}(s)$. As $s^* = s_{FB}$ holds already under F, optimality then implies that $s^* = s_{FB}$

must also hold under $\widetilde{F}$. Suppose now that a repayment schedule $\widetilde{T}$ is optimal for $\widetilde{F}$. We argue to a contradiction and assume that $\widetilde{C} \geq C$ holds. To ensure that still $s^* = s_{FB}$ holds we must have $V_{s_{FB}}(\widetilde{T}) = V_{s_{FB}}(T)$, implying that $\widetilde{R} \leq R$. It then follows from the argument of Claim 1 in Proposition 1 that $V_{s_{FB}}(\widetilde{T}) \geq V_{s_{FB}}(T)$ for all $s > s_{FB}$. As $\widetilde{F}$ dominates F in the sense of strict FOSD this finally implies that the borrower's participation constraint is not binding if a contract with repayment $\widetilde{T}$ is offered for $\widetilde{F}$. As this is not optimal we obtain a contradiction. This completes the proof of Claim 1 and thus also of Proposition 3. **Q.E.D.**

Proof of Lemma 1. We prove first the following claim.

Claim 1. *Suppose $U_{\widehat{s}}(\widetilde{T}) \geq U_{\widehat{s}}(T)$ holds for some $\widehat{s} < 1$. Then it must hold for all $s > \widehat{s}$ that $U_s(\widetilde{T}) > U_s(T)$.*

Proof. We argue to a contradiction and suppose that this was not the case for some $s > \widehat{s}$ where $U_s(\widetilde{T}) \leq U_s(T)$. Using continuity of $U_s(T)$ and $U_s(\widetilde{T})$, this—together with $U_{\widehat{s}}(\widetilde{T}) \geq U_{\widehat{s}}(T)$—implies existence of some $\widetilde{s}$ satisfying $\widehat{s} < \widetilde{s} < s$ and $U_{\widetilde{s}}(\widetilde{T}) = U_{\widetilde{s}}(T)$. We now show that $U_{\widetilde{s}}(\widetilde{T}) = U_{\widetilde{s}}(T)$ and $U_{\widehat{s}}(\widetilde{T}) \geq U_{\widehat{s}}(T)$ can not hold simultaneously if $\widehat{s} < \widetilde{s}$. For this we can rely on arguments from Proposition 1.

First, construction of $T(x)$ and $U_{\widetilde{s}}(\widetilde{T}) = U_{\widetilde{s}}(T)$ implies existence of some value $0 < \widetilde{x} < \overline{x}$ such that $T(x) \geq \widetilde{T}(x)$ holds for all $x < \widetilde{x}$ and $T(x) \leq \widetilde{T}(x)$ holds for all $x > \widetilde{x}$, where the inequalities hold strictly for sets of positive measures. Second, using that $g_{\widehat{s}}(x)/g_{\widetilde{s}}(x)$ is by Assumption 1 strictly decreasing in x, we have that

$$U_{\widehat{s}}(\widetilde{T}) - U_{\widehat{s}}(T) < \frac{g_{\widehat{s}}(\widetilde{x})}{g_{\widetilde{s}}(\widetilde{x})} \left[U_{\widetilde{s}}(\widetilde{T}) - U_{\widetilde{s}}(T) \right] = 0,$$

which yields a contradiction. **Q.E.D.**

We can now apply the same argument as in Claim 1 to show that also the converse holds. That is, if $U_{\widehat{s}}(\widetilde{T}) \leq U_{\widehat{s}}(T)$ holds for some $\widehat{s} > 0$, then $U_s(\widetilde{T}) < U_s(T)$ must hold for all $s < \widehat{s}$. Lemma 1 follows immediately from these two assertions. **Q.E.D.**

Proof of Proposition 4. Observe first that $s^* > s_{FB}$ implies $C = w$. That is, all assets are posted as collateral. For the proof we make use of the following auxiliary result.

Claim 1. *Take the (commitment) offer from Proposition 1 for the case where $s^* > s_{FB}$. Then the lender would be strictly worse off by offering in $\tau = 1$ a (non-degenerate) menu, from which she would be allowed to pick a contract after observing s.*

Proof. Suppose the lender offers a menu $\{(t_i, c_i)\}_{i \in I}$, where I is an arbitrary index set. As all contracts in the menu must satisfy Assumption 2, it is immediate that there again exists a unique cutoff signal s^*. For simplicity we restrict consideration to the case where the lender only uses pure strategies when picking from the accepted menu. Denote the contract that is chosen at s^* by (t^*, c^*), which gives rise to the repayment schedule T^*.

We are now rather brief as we can build on previous arguments. Suppose we dropped all contracts from the menu besides (t^*, c^*). Then, by the lender's (previously) revealed preferences, the borrower would not be worse off. If the borrower is strictly better off, which relaxes the constraint (2), we can simply adjust (t^*, c^*) and shift more profits to the lender, which reduces s^*. Note that the borrower is indeed strictly better off if the lender strictly preferred some other contracts than (t^*, c^*) from the menu for a set of signals $s > s^*$ with positive measure. Moreover, as $s^* > s_{FB}$ holds by assumption in case a simple contract is offered, we know from Proposition 1 that we can further reduce s^* in case (t^*, c^*) is not collateralized debt with $C = w$.

Summing up, we can reduce the cutoff and, thereby, construct a better offer if either (i) (t^*, c^*) is not collateralized debt with $C = w$ or (ii) the lender strictly prefers other contracts from the menu for a set of signals $s > s^*$ with positive measure.

Note now that if the menu is non-degenerate, the lender must indeed prefer (at least weakly) some other contract (t, c) for some signal $s^* < \widetilde{s} < 1$. As we have shown that (t^*, c^*) must be collateralized debt with $C = w$, this implies from Lemma 1 that she strictly prefers (t, c) to (t^*, c^*) for all higher signals $s > \widetilde{s}$. **Q.E.D.**

It is now convenient to consider first the case where the borrower can offer a new contract. To be more precise, we then have the following game of renegotiations. After the lender observes the signal, the borrower can offer a new contract. The lender can then either accept or reject the new offer. Subsequently, she decides whether to approve the borrower.

Consider some alternative offer $(\widetilde{t}, \widetilde{c})$ made by the borrower, which gives rise to the repayment schedule $\widetilde{T}(x)$. We know from Lemma 1 that, unless the lender prefers one of the contracts for all signals, there exists a critical signal $0 < \widetilde{s} < 1$ such that she prefers $(\widetilde{t}, \widetilde{c})$ for higher and (t, c) for lower signals. We distinguish between two cases. Suppose first that $s^*(\widetilde{T}) < s^*(T)$. Here, the borrower knows that the lender will accept the new offer and approve the loan for $s > s^*(\widetilde{T})$ and reject the loan for $s < s^*(\widetilde{T})$. Thus, the borrower's expected payoff is equal to the payoff that he would obtain if the lender had originally offered $(\widetilde{t}, \widetilde{c})$. As the lender is strictly better off for all

$s > \widetilde{s}$, the borrower's expected payoff must be strictly smaller than $\overline{V}$. Otherwise, the original contract would not have solved the lender's program of Proposition 1. Hence, we showed that for $s^*(\widetilde{T}) < s^*(T)$ the borrower would be strictly worse off by offering the new contract.

Suppose next that $s^*(\widetilde{T}) \geq s^*(T)$. Here, the borrower knows that the lender will accept the new offer and approve the loan for $s > \widetilde{s}$, reject the new offer and still approve the loan for $[s^*(T), \widetilde{s})$, and reject the loan for $s < s^*(T)$. Thus, his expected payoff is as if the lender had originally offered the menu containing the contracts (t, c) and $(\widetilde{t}, \widetilde{c})$. As the lender is again strictly better off with the menu than with the original contract (t, c) for all $s > \widetilde{s}$, Claim 1 implies that the borrower must realize less than $\overline{V}$. Hence, also for $s^*(\widetilde{T}) \geq s^*(T)$ the borrower is strictly worse of by offering the new contract.

Finally, suppose it is the lender who can offer a new contract after observing s. For brevity we restrict again consideration to pure strategies. Hence, in a given (candidate) equilibrium the lender offers at most one new contract for any given signal, while the borrower either accepts or rejects for sure. Denote by s^{**} the lowest signal for which the lender offers a new acceptable contract, which subsequently leads to the approval of the loan. By Lemma 1 we know that by optimality the lender must then offer for all $s \geq s^{**}$ an acceptable contract as well. These contracts can now differ. We denote the set of these contracts by Ω. We distinguish again between two cases.

It is again immediate that there is no equilibrium where $s^{**} \geq s^*(T)$. For $s^{**} < s^*(T)$ the borrower knows that for all $s \geq s^{**}$ the lender offers a contract in Ω, which the borrower is supposed to accept. Given $s \geq s^{**}$, the borrower's payoff in the candidate equilibrium is then equal to that of accepting the menu Ω. If her payoff with the menu was not lower than that from the original contract, $s^{**} < s^*(T)$ would contradict Claim 1. **Q.E.D.**

7 References

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