# FACTORS AFFECTING THE VALUATION OF CORPORATE BONDS

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The valuation of corporate debt is an important issue in asset pricing. While there has been an enormous amount of theoretical modeling of corporate bond prices, there has been relatively little empirical testing of these models[1]. Recently there has been extensive development of rating based reduced form models. These models take as a premise that bonds when grouped by ratings are homogeneous with respect to risk. For each risk group the models require estimates of several characteristics such as the spot yield curve, the default probabilities and the recovery rate. These estimates are then used to compute the theoretical price for each bond in the group. The purpose of this article is to examine the pricing of corporate bonds when bonds are grouped by ratings, and to investigate the ability of characteristics, in addition to bond ratings, to improve the performance of rating based models. Most of our testing will be conducted in models which are in the spirit of the theory developed by Duffie and Singleton (1997) and Duffie (1999).

The paper is divided into three sections. In the first section, we discuss various reduced form models that have been suggested in the literature. In the second section we examine how well standard classifications serve as a metric for forming homogeneous groups. In this section we show that using standard classifications results in errors being systematically related to specific bond characteristics. Finally, in the last section we take account of these specific bond characteristics in our estimation

[1] Most testing of theoretical models has been performed using other types of debt. Cumby and Evans (1997 ) examine Brady bonds, Merrick (1999 ) examines Russian bonds and Madan and Unal (1996 ) examine Certificates of Deposit.

procedure for determining spot prices and show how this lead to improved estimates of corporate bond prices.

## I. ALTERNATIVE MODELS

There are two basic approaches to the pricing of risky debt: reduced form models and models based on option pricing. Reduced form models are found in Elton, Gruber, Agrawal, and Mann (2001), Duffie and Singleton (1997), Duffee (1999 ), Jarrow, Lando and Turnbull (1997), Lando (1997), Das and Tufano (1996). Option-based models are found in Merton (1974) and Jones, Mason, and Rosenfeld (1984). In this paper we will deal with a subset of reduced form models, those that are ratings based. Discussion of the efficacy of the second approach can be found in Jones, Mason, and Rosenfeld (1984).

The basic structure of reduced form models assumes that the value of a bond is the certainty equivalent cash flows (at risk neutral probabilities) brought back at risk free rates. For a two-period bond that has a face value of $1, value can be expressed as follows:

$$Value_o = \frac{C\left(1-{}_t l_1\right)+a\,{}_t l_1}{\left(1+r_1\right)} + \frac{\left(C+1\right)\left(1-{}_t l_1\right)\left(1-{}_t l_2\right)+a\,{}_t l_2\left(1-{}_t l_1\right)}{\left(1+r_2\right)^2} \qquad (1)$$

Where:

(1) $C$ is the coupon

(2) $a$ is the recovery rate

(3) $r_t$ is the riskless rate from 0 to t

(4) ${}_t l_j$ are the term structure of risk neutral probabilities of default at time t which capture the probability of default, the risk premium, and taxes for all periods j.

The issue is how to estimate the risk neutral probabilities. Risk neutral probabilities are either estimated for an individual firm using the bonds the firm has outstanding or for a group of firms that are believed to be homogeneous. This latter method uses all bonds in the homogeneous risk class. When individual firms are employed to estimate ${}_t l_j$ 's one is constrained in the type of estimation that can be done because of the limited number of observations (bonds of the same firm) which exist. Because of this, authors who estimate a new term structure of risk neutral probabilities at each point in time either assume that risk neutral probabilities at a point in time do not change for different horizons (${}_t l_j = {}_t l$) or that the shape of the term structure of risk premiums at any moment follows a particular simple shape (can be estimated with a very small number of parameters).

Examples of research that assume that all elements in the term structure of risk neutral probabilities are the same at any moment in time (${}_t l_j = {}_t l$ ) include Yawitz (1977 ), Bierman and Hass (1975), and Cumby and Evans (1997 ). Examples of papers using a simple model to describe the term structure of risk neutral probabilities are Merrick (1999), who assumes that any moment in time risk neutral probabilities are a linear function of the time until a payment, Claessens and Pennachhi (1996), Madan and Unal (1996) or Cumby and Evans (1997), who assume risk neutral probabilities follow a standard stochastic process or Cumby and Evans (1997), who, in addition to their other model, assume a random walk with mean reversion. Another variation in modeling applied to individual firms assumes that the spread between corporates and treasuries follows a particular stochastic process both at each point in time and over time (see Duffee (1998)). This intertemporal model provides an ability to price option features on bonds which is not possible with the prior model.

The alternative to assuming that each firm has a unique set of risk neutral probabilities is to assume that each firm in a homogeneous class of firms has the same set of risk neutral probabilities. This allows the estimation of much less constrained term structures of risk neutral probabilities. This approach has been modeled in Duffie and Singleton (1997) and Jarrow, Lando, Turnbull (1994). Both of these studies show how the term structure of risk neutral probabilities can be obtained by using the difference in corporate and government spot rates. The difference in their estimates comes about because of a difference in assumption about recovery rates. Duffie and Singleton (1997) assume that the recovery rate as a percentage of a like maturity non-defaulted bond of the same risk class is a constant across the homogeneous group. Jarrow, Lando and Turnbull (1994) assume that the recovery rate is constant across the homogeneous group when expressed as a fraction of a like maturity government bond.

Models that estimate the term structure of risk neutral probabilities from the bonds of a single firm will have errors because small sample sizes mean that the model used to estimate the term structure of risk neutral probabilities is likely to be estimated with substantial error and because the model is likely to be oversimplified. The major source of errors for models that use a homogeneous group of bonds comes from the possibility that investors view the bonds within a group as having different risk.

In this paper we will explore how to determine a homogeneous group to minimize risk differences. Like Jarrow, Lando and Turnbull (1997), we will initially assume that Moody's or S&P rating classes are a sufficient metric for defining a homogeneous group. We will then show that this group has variations in risk and what these variations depend on. How these variations can be dealt with and the improvement that comes from accounting for these differences will then be explored.

In a separate paper we explore the pricing errors from applying the Jarrow, Lando and Turnbull (1997) and Duffie and Singleton (1997) models to price corporate bonds. In that paper we have shown that the Duffie Singleton model produces smaller errors.[2] Thus, in this paper we will explore this model

[2]See Elton, Gruber, Agrawal and Mann (2000).

further. One of the nice features of the model is that with this model using risk neutral probabilities and riskless rates is equivalent to discounting promised cash flows at corporate spot rates. In this article we will use this equivalent form.

## II ANALYSIS BASED ON RATING CLASS

In this section we accept Moody's rating as a sufficient metric for homogeneity and investigate the pricing of bonds under this assumption. We start by describing our sample and the method used to extract spot rates for corporate bonds. We then examine the pricing errors for bonds when this technique is applied.

### A. Data

Our bond data is extracted from the Lehman Brothers Fixed Income database distributed by Warga (1998). This database contains monthly price, accrued interest, and return data on all investment grade corporate bonds and government bonds. In addition, the database contains descriptive data on bonds including coupon, ratings, and callability.

A subset of the data in the Warga database is used in this study. First, any bond that is matrix-priced rather than trader-priced in a particular month is eliminated from the sample for that month. Employing matrix prices might mean that all our analysis uncovers is the formula used to matrix-price bonds rather than the economic influences at work in the market. Eliminating matrix-priced bonds leaves us with a set of prices based on dealer quotes. This is the same type of data contained in the standard academic source of government bond data: the CRSP government bond file.

Next, we eliminate all bonds with special features that would result in their being priced differently. This means we eliminate all bonds with options (e.g., callable or sinking fund), all corporate floating rate debt, bonds with an odd frequency of coupon payments, government flower bonds and index-linked bonds.[3] Next, we eliminate all bonds not included in the Lehman Brothers bond indexes because researchers in charge of the database at Shearson-Lehman indicated that the care in preparing the data was much less for bonds not included in their indexes. Finally we eliminate bonds where the data is problematic.[4] For classifying bonds we use Moody's ratings. In the few cases where Moody's ratings do not exist, we classify using the parallel S&P rating.

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3 The alternative was to construct a model which explicitly prices the option like features. While this is an interesting project, it is helpful to understand the determination of risk and homogeneity before dealing with option pricing.

4 Slightly less than 3% of the sample was eliminated because of problematic data. The eliminated bonds had either a price that was clearly out of line with surrounding prices (pricing error) or involved a company or bond undergoing a major change.

Our final sample covered the ten year period: 1987-1996. Details on sample size are presented in the accompanying tables. The basic sample varied from an average of 42 bonds for the industrial Aa category to 278 bonds for the financial A category.

## B. Extracting Spot Rates

In this section we discuss the methods of extracting spots from bond prices and apply it to our sample when Moody's ratings are used to define a homogeneous risk class.

Calculating model prices using the discounting of promised cash flows is relatively straightforward. First, spot rates must be estimated. In order to find spot rates, we used the Nelson Siegel (1987) procedure for estimating spots from a set of coupon paying bonds. For each rating category, including governments, spots can be estimated as follows:[5]

[5] See Nelson and Siegel (1987). For comparisons with other procedures, see Green and Odegaard (1997) and Dahlquist and Svensson (1996). We also investigated the McCulloch cubic spline procedures and found substantially similar results throughout our analysis. The Nelson and Siegel model was fit using standard Gauss-Newton non-linear least squared methods. The Nelson and Siegel (1987) and McCulloch (1971) procedures have the advantage of using all bonds outstanding within any rating class in the estimation procedure, therefore lessening the effect of sparse data over some maturities and lessening the effect of pricing errors on one or more bonds. The cost of these procedures is that they place constraints on the shape of the yield curve. We used Moody's categories where they existed to classify bonds. Otherwise we used the equivalent S&P categories.

$$D_t = e^{-r_{0t}t}$$

$$r_{0t} = a_0 + (a_1 + a_2)\left[\frac{1-e^{-a_3 t}}{a_3 t}\right] - a_2 e^{-a_3 t} \qquad (2)$$

Where

$D_t$ is the present value as of time zero for a payment that is received t periods in the future

$r_{0t}$ is the spot rate at time zero for a payment to be received at time t

$a_0, a_1, a_2$ and $a_3$ are parameters of the model.

Discounting the promised cash flows on each bond in a particular rating class at the estimated spot rates for that rating class produces the model price for that bond. Table 1 (Panel A) presents the pricing errors when this technique is used. For all rating classes the average error is close to zero. The average error is less than 1 cent per $100 of the face value of the bond over the sample period. This is not surprising because the Nelson Siegel procedure should give unbiased estimates of the appropriate spot rates for each rating class. Of more interest is the absolute error. This is a measure of the dispersion of errors across bonds within one rating class and thus, of how homogeneous the rating class is. To the extent that rating classes are not homogeneous or there is a lot of noise in the dealer prices on which we measure errors, the dispersion of pricing errors (the average absolute pricing error) would be quite large. The results in Table 1 (Panel B) show that while Moody's rating classes do an excellent job of pricing the "average bond' there are large errors in pricing individual bonds. The errors vary from 34 cents per $100 for Financials Aa's to over $1.17 for Baa industrials. This suggests that there are other variables that systematically effect bond prices and by studying pricing errors we can uncover the additional influences. In the next section we will explore this issue.

## II OTHER FACTORS THAT AFFECT RISK

When estimating spot rates, one has to make a decision as to how to construct a group of bonds that is homogeneous with respect to risk. In the prior section, like others, we accepted the major classifications of rating agencies. In this section we explore the use of additional data to form more meaningful groups and to understand what effects corporate bond prices.

In general, when dividing bonds into subsets, one faces a difficult tradeoff. The more subsets one has, the less bonds are present in any subset. Bond prices are subject to idiosyncratic noise as well as systematic influences. The more bonds in a subset, the more the idiosyncratic noise is averaged out. This suggests larger groupings. However, if the subset is not homogeneous, one may be averaging out important differences in underlying risk and mis-estimating spot rates because they are estimated for a group of bonds where subsets of the group have different yield curves.

What are the characteristics of bonds that vary within a rating class that could lead to price differences? We will examine the following possibilities:

(A) Default Risk

(B) Liquidity

(C) Tax Liability

(D) Recovery Rates

(E) Age

## A. Differential Default Risks

All bonds within a rating class may not be viewed as equally risky. There are several characteristics of bonds which might be useful in dividing bonds within a rating class into new groups. We will examine several of these in this section. We start by examining the subcategories within a rating class which Moody's and Standard & Poors have both introduced. We then examine whether either past changes in rating or a difference in rating by Moody's and Standard & Poors convey information.

We start by examining the finer breakdown of ratings produced by the rating agencies themselves. Standard & Poors and Moody's have introduced plus and minus categories within each letter rating class. One obvious possibility is that bonds that are rated as a plus or a minus are viewed as having different risk than bonds that receive a flat letter rating. If this is true, then estimating one set of spot rates for all bonds in a class should result in consistent pricing errors for bonds rated "plus" (too low a model price and hence negative errors) or bonds rating "minus" (too high a model price and hence positive errors).

Tables II A and II B explore this possibility. For each rating class the table is split into two sections. The top section shows the number of bond months in each rating class for varying maturity and across all maturities.[6] The bottom section shows the average of the model price minus the invoice price (market price plus accrued interest) for each rating category. For all rating categories, plus-rated bonds have, on average, too low a model price, and minus-rated bonds too high a model price. The difference between the pricing error of plus rated, flat and negative rated bonds is statistically significant at the 5% level. Furthermore, the differences are of economic significance (e.g., for minus versus flat Baa industrial bonds the overall difference is over 1% of the invoice price). The same pattern is present for most of the maturities with some tendency for the magnitude of errors to increase with the maturity. In addition, the size of the average pricing error increases as rating decreases. Thus, it is most important for Baa bonds. This would suggest that one should estimate a separate spot curve for these subclasses of ratings. However, for much of the sample, the paucity of bonds in many of the subclasses makes it difficult to

[6] For all bonds rated by Moody's we use Moodys' classification. For the few bonds not rated by Moody's, we use S&P's classification.

estimate meaningful spot rates for a subclass. In a latter section we will explore how these differences can be built into an estimation procedure for spot rates.

There is a second reason why investors might consider bonds within the same rating class to have different risk. Investors might believe that a particular bond is likely to be downgraded or upgraded. One predictor of this might be past rating changes. Past rating changes might predict future rating changes, either because rating agencies tended to make changes in steps or because a company whose risk has increased or decreased in the past is more likely to experience similar changes in the future. In Table III we explore whether past rating changes contain information about future rating changes. As shown in the table, bonds that have been upgraded in the past are more than twice as likely to be upgraded in the future than they are to be downgraded, and bonds that have been downgraded in the past are about twice as likely to be downgraded than upgraded in the future.

Although there is evidence that past rating changes predict future rating changes, it is unclear if the tendency is strong enough to show up in price data. We examined differences between model price and invoice price for all bonds which had a past change in ratings. Pricing errors were examined in the month of the change, the next three months after the change, and the period 4 to 15 months after the change. These results are shown in Table IV. Despite the fact that past rating changes contain information about future rating changes, we find no evidence that bonds with past rating changes have prices that are systematically different from model prices. Our sample of bonds with rating changes was quite small, for there were few bonds which had rating changes. Thus the failure to find a relationship between past rating changes and errors could arise either because investors do not take the predictability of past rating changes into account when they price bonds, or simply because the number of rating changes is so small that the effect is swamped by random pricing errors.  In any case, examining past rating changes provides no evidence that the Markov assumption used in calculating the transition probability matrix found in many studies is violated.

In Table V we explore whether bonds that are given a higher (lower) rating by S&P than by Moody's are considered less (more) risky by investors.  Recall that our yield curves are derived using Moody's ratings.  The question is whether when Standard & Poor's gives the bond a higher rating that Moody's does this indicate that the bond is less risky: does an investor believe that the second rating conveys information not contained in  the first rating. In considering differences we use pluses and minuses.  Thus, if Moody's rates a bond as Baa and S&P rates the bond BBB+, we count this as a difference in ratings. Once again the upper half of the table shows the number of bonds in each category, and the lower half the difference between model price and invoice price. In presenting the data we do not sub-classify by maturity since we found no pattern in pricing errors across maturity.

Investors clearly take the difference in rating into account. If the S&P rating is lower than Moody's, then investors act as if the bond is higher risk than is implied by the Moody's rating and they will set a lower market price, and this results in a model price above invoice price and a positive error. Likewise, if S&P rates the bond higher than Moody's the bond is considered by investors as lower risk compared to bonds where they agree and the pricing error is negative. The errors when the rating agencies disagree is statistically different from the errors when they agree.

## B. Different Liquidity

The second reason why bonds within a rating class might be valued differently is because they have different liquidity. Data is not available on bid/ask spread, the most direct measure of liquidity, nor is there data on trading volume which is a natural proxy for liquidity. We used three indirect measures of liquidity: dollars value outstanding, the percentage of months a bond was matrix priced, and whether a bond was recently issued. Our logic behind the second measure was that dealers priced the more active issues more often. Thus bonds that were always dealer-priced were likely to be more liquid than bonds that were dealer-priced only part of the time. Neither of the first two measures showed any significant patterns, and so we have not presented a table of results. The third measure rests on the belief that newly issued bonds are more liquid than bonds which have been in the market for a longer period of time. We defined newly issued bonds as bonds that were brought to the market within the previous year. Table VI shows the difference between newly issued (first-year bonds) and older bonds. Once again the top half is the number of bond months in each cell, and the bottom half is the average difference between model price and invoice price. As shown in Table VI, newly issued bonds sell at a premium compared to model prices. These results are consistent with newly issued bonds being more liquid.

## C. Different Tax Treatment

The third possible reason why bonds within a risk class might be viewed by investors differently is because they have different after tax value because of the way coupons and capital gains are taxed. Throughout most of the period used in our study the tax rates on capital gains and interest income were the same. However, since capital gains are paid at the time of sale, bonds with lower coupons may be more valuable because some taxes are postponed until the time of sale and because the holder of the bond has control over when these taxes are paid (tax timing option). In order to examine the effect of taxes, we group bonds by coupon and examined the model errors. Table VII shows the results for Baa rated industrial bonds. The results for other ratings are similar. The entries in Panel B represent model prices minus invoice price across six coupon categories and different maturities. Panel A shows the number of bond months in each category.

If taxes matter, we would expect to see a particular pattern in this table. Recall that for any risk class, spot rates are fitted across all bonds. This means that for the average bond the tax effect on pricing errors should be zero (because it is averaged out), and if taxes don't matter errors should not vary with maturity. If taxes matter, high coupon bonds should be disadvantaged relative to the average bond, and these bonds would have to offer the investor a higher pre-tax return. But since we are discounting all bonds in a risk class at the same rate, this implies that if taxes matter we are discounting high coupon bonds at too low a rate, and thus are computing a model price which is too high. This translates into a positive value for the pricing error, and this is what we see in Table VII. In addition, as shown in Table VII, the longer the maturity of the bond, the more significant the pricing error becomes. For bonds with coupons below the average coupon in a risk class we should get the opposite sign (a negative sign) on the pricing error and the size of the error should become more negative with the maturity of the bond. This is the pattern shown in Table VII.

### D. Different Recovery Rates

The fourth reason investors might rate bonds differently within a risk class is because of different expectations about recovery. Firms go bankrupt, not individual bonds. Bonds of the same firm with different ratings imply that the rating agency believes they will have different recovery rates. Thus investors should believe that an A bond of an Aa firm has different expected recovery rate than an Aa bond of the same firm.

Moodys (or S&P) ratings for any bond are a combination of their estimate of default risk for the company issuing the bond and their estimate of the recovery rate on the bond if the firm goes bankrupt. If their implicit weighting is the same as investors, then sorting a bond rating class by different company ratings should not result in pricing errors being related to the company rating. Examining Table VIII shows that bonds where the bond rating is higher than the company rating have model prices above invoice prices. When the model price is above the invoice price, investors are requiring a higher rate of return in pricing the bond. Bonds whose ratings are above companies ratings (e.g., Aa and A respectively), have more default risk and higher recovery rates than bonds which have company and bond rating both equal to that of the bond (e.g. both AA). Since, from Table VIII, investors price these bonds lower, investors are placing more weight on bankruptcy probability and less on estimated recovery rates than Moody's does. The same logic holds for bonds ranked below the company rating.

This raises another question. Could pricing be improved by discounting bonds at spot rates estimated from groups formed by using company rating rather than formed by bond rating? When we use company ratings to form groups and estimate spots the pricing errors are much larger. Bonds should be priced from discount rates estimated from groups using bond rating. However, taking into account the difference between bond rating and company rating adds information.

### E. Bond Age

We explore one other reason why bonds in a particular rating class might be viewed differently by investors: age of the bond. While the finance literature presents no economic reason why this might be true except for liquidity effects with new issues, it is a common way to present data in the corporate bond area, and it is an important consideration if one wants to model rating drift as a Markov process.[7] The issue is whether a bond with 15 years to maturity rated A, and ten years old, is different from a bond with the same characteristics but two years old. When we examined this issue, except for new issues, there was no age effect. Thus there is no definitive evidence that the Markov assumption is being violated, and no definitive evidence that age of the bond is an important characteristic for classification.

## IV Adjusting for Differences

[7] For example, Moody's typically presents data on the default rates as a function of the age of the bonds.

We have now shown that a number of factors combined one at a time cause bonds within the same Moody's classification to have systematic price differences. The next step is to examine what proportion of the variation in errors across bonds that can be explained by these factors and whether they are important when considered jointly. In addition, we do more formal statistical testing in this section.

Our prior analysis has shown the following influences are important:

1. A plus or minus rating within each risk letter classification. Furthermore, the importance is a function of maturity.

2. Differences in S&P and Moody's rating.

3. The coupon on a bond.

4. Differences in bond and company ratings.

5. An age of less than one year.

To estimate the adjustment function we regressed model errors on a series of variables to capture simultaneously the impact of the influences discussed above. The variables are discreet except for coupon which is continuous. The regression we estimated is

$$E_j = a + \sum_{i=1}^{8} B_i V_{ij} + e_j \qquad (3)$$

Where

$E_j =$ the error measured as model price minus invoice price for bond j

$V_{1j} =$ the maturity of a bond if it is rated plus otherwise zero

$V_{2j} =$ the maturity of a bond if it is rated minus, otherwise zero

$V_{3j} =$ dummy variable which is 1 if S&P rates a bond higher than Moody's, otherwise zero

$V_{4j} =$ dummy variable which is 1 if Moody's rates a bond higher than S&P, otherwise zero

$V_{5j} =$ the coupon on the bond minus the average coupon across all bonds[8]

$V_{6j} =$ dummy variable which is 1 if the company has a higher rating than the bond, otherwise zero

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[8] This variable was demeaned as not to transfer the average tax effect to the intercept.

$V_{7j}$ = a dummy variable which is 1 if the bond has a higher rating than the company, otherwise zero

$V_{8j}$ = a dummy variable which is 1 if the bond is less than 1 year of age, otherwise zero

$B_i$ = the sensitivity of errors to variable i.

The regression is estimated for bonds within each rating class for industrials and financials separately. Results are shown in Table IX. Almost all regression coefficients are statistically significant at the 1% level in every sample and have the sign that we would expect to see. The adjusted $R^2$ vary between .05 and .3 and average .18.

If we examine the regression coefficients one at a time we see very strong results. For plus rating the regression has the right sign for all rating categories and five of the six coefficients are significant at the 1% level. For minus ratings the coefficient has the right sign and is significant for five of the six categories. In the one group where the sign is inconsistent with what we would expect the coefficient is both small and not statistically significantly different from zero at the 5% level. When interpreting the signs, recall that plus rated bonds are expected to have a negative error since the model price overestimates their risk.

Turning to bonds which have a S&P rating different from their Moody's rating, we find that the S&P rating contains added information about prices. For differences in ratings in either direction, the coefficient has the appropriate sign in all cases and is significantly different from zero at the 1% level in all but one case.

We have hypothesized that high coupon bonds were less desirable due to taxes. The coupon variable has the correct sign in all cases and a coefficient which is significantly different from zero (at the 1% level) in five of the six case. While we reasoned that the impact of company and bond ratings were ambiguous because it depends on the weight the investor places on recovery rate versus probability of bankruptcy, the results tell a very consistent story. Of the 11 groups examined, 10 had consistent signs and of these 10, 7 had coefficients which were statistically significantly different from zero at the 1% level. The one coefficient with the inconsistent sign was not significantly different from zero at the 5% level. These results indicate that investors place more emphasis on bankruptcy risk than the relative weight it is given in bond ratings. Finally, new bonds sell at a premium. All the estimates have the right sign and are statistically different from zero at the 1% level.

The next logical step would be to take the influences discussed above into account in defining new classifications (homogeneous groups) of bonds that exist within each Moody's risk classification and to estimate new spot curves within each classification. Unfortunately, this would result in such fine classifications that we would have too few bonds within each classification to estimate spot curves with any accuracy.

An alternative is to introduce these variables directly into the procedure for estimating spot curves so that the spot rates determined for any bond are not only a function of the Moody's risk class to which the bond belongs, but the rates are conditional on all of the variables we have found important in the previous section. The spot rates developed from this procedure can then be used to price bonds and the resulting model prices compared with model prices arrived at only using Moody's ratings.

We modify the Nelson-Siegel estimation approach to take added influences into account. Because of the number of influences we found important and the number of parameters, as well as ratios and cross products of parameters in the Nelson-Siegel procedure we needed to make some simplifying assumptions about the nature of changes in the term structure caused by adding these influences. We assumed that each of the variables discussed in the previous section of this paper could effect the level but not the shape of the corporate term structure. For example, are estimation procedure assumes that the Baa+ and Baa- spot term structure curves are parallel to each other and the Baa spot term structure curve. To the extent that this simplification of the effect of variables is inappropriate it will bias our results against attributing importance to the influences we examine.

The new equation used to estimate the term structure for any bond with a particular Moody's rating is found by using the following modification of equation (3)

$$r_{ot} = a_o + (a_1 + a_2)\left[\frac{1-e^{-a_3 \cdot t}}{a_3 t}\right] - a_2 e^{-a_3 t} + \sum_{j=1} b_i V_{ij} \qquad (4)$$

This equation was estimated within each Moody's risk class for industrial and financial bonds separately. This allowed us to estimate a spot curve for any bond and to arrive at a model price based on these spots.

The results of this analysis are shown in Table X. In this table we show the average absolute errors from using equation (4) to value Baa, A and Aa rated bonds for industrial and financial companies for two five-year periods and the overall ten-year period. The average absolute error varies from 36 cents per 100 bond for the financial Aa category up to 92.4 cents for the industrial Baa rated category.

How can we judge the improvement from incorporating these additional factors? One way is to compare these errors with the errors when rating is accepted as a metric for homogeneous risk. In each of the six categories for the ten-year period and for 11 of the 12 five-year categories the error has been reduced. In each of these eleven cases, the reduction in model error is statistically significant at the one percent level[9].

We wish to get a better measure of the improvement the estimates of the spot yield curve with our added set of variables. When we only employ risk class as a measure of homogeneity pricing errors will tend to persist over time for three reasons: (1) because the additional qualities of a bond not captured by risk class would be expected to impact the price and since these qualities change slowly over time, if at all, we should observe persistence, (2) firm effects may be present and (3) dealer prices may be sticky since dealers may not correct their misestimation quickly over time.

One way to correct for all three of these reasons is to adjust the price predicted for a bond by past errors in pricing the bond[10]. To measure this we used the average of the last six months' errors.

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[9] The t's associated with the differences in errors average 5.1 to 17.67 with the typical one about 10.

[10] The model price is reduced (increased) by the amount that the model price overestimated (underestimated) the bonds actual price.

Table X  shows that introducing past errors in the analysis reduces the error based on Moodys ratings by a significant amount.  For example, for Baa industrial bonds the size of the average absolute error is reduced from $1.17 per $100 bond to $0.61.  Recall that this reduction occurred because of omitting bond characteristics which should have been included in estimating bond spot rates, firm affects, and stickiness in dealer prices.  We know estimate what percentage of this reduction is just due to omitting the set of bond characteristics we have been examining (equation 3).  This is shown in the last column of table 10.  For industrial bonds incorporating our set of fundamental characteristics into the estimates of spot rates accounts for a decrease of between 38% and 49% of the aggregate impact of the three influences discussed above.  We have not been quite as successful for financial bonds but we have reduced the error by 2% to 45%.  This analysis shows that the set of variables we have examined are important influences in determining the risk structure of corporate bonds and capture a significant portion of the influences that affect bond prices beyond that captured by rating class.

**Conclusion**

In this paper we explore the characteristics of corporate bonds that effect their price.  All rating-based techniques involve working with a homogeneous population of bonds.  We explore what characteristics of bonds are priced differently by the market. We find that several characteristics of bonds and bond rating beyond the simple rating categories of Moody's and Standard and Poor convey information about the pricing of corporate bonds. In particular the following five influences are important:

1. The finer rating categories introduced by both rating agencies when combined with maturity measures.
2. Differences between S&P and Moody's ratings.
3. Differences in the rating of a bond and the rating of the company which issued that bond.
4. The coupon on the bond.
5. Whether a bond is new and has traded for more than one year.

We adjust for these characteristics and show the improvement in pricing error. Bond pricing models which are based on ratings whether the models involve discounting cash flows or determining risk neutral probabilities need to be adjusted for these influences.

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**Table I**
**Pricing Errors based on Rating Classes**

This table shows the average pricing errors when promised payments are discounted at the corporate rates. Discounted rates on promised payments were fitted each month separately for each rating category of bonds. Errors are the fitted prices minus the invoice prices of coupon bonds. Errors are expressed in dollars on bonds with face value of 100 dollars.

| | Financial Sector | | | Industrial Sector | | |
|---|---|---|---|---|---|---|
| | Aa | A | Baa | Aa | A | Baa |
| Average pricing errors | -0.0094 | -0.0104 | -0.0149 | -0.0162 | -0.0082 | 0.0094 |
| Average absolute pricing errors | 0.335 | 0.593 | 0.884 | 0.475 | 0.625 | 1.172 |

**Table II (a)**

## Model Errors due to Maturity and Gradations within Ratings
## Industrial Sector

Moody's rates bonds using broad categories as well as finer gradations (+, 0, and -.) Plus securities are designated as less risky than minus securities. This table separates bonds into groups according to these finer gradations (along the left-hand side.) It further separates the bonds according to maturity (in years from left to right.) The first column represents bonds with maturity between 1.0 and 2.0 years, inclusive. Model price is calculated by discounting promised cash flows at estimated corporate spot rates. Average error is defined as model price minus invoice price.

AA

Number of Bonds

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | 34 | 130 | 129 | 108 | 172 | 18 | 591 |
| 0 | 360 | 634 | 509 | 365 | 398 | 62 | 2328 |
| - | 228 | 452 | 448 | 502 | 559 | 75 | 2264 |

Average Error

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | 0.112 | -0.152 | 0.360 | 0.255 | 0.517 | -0.113 | 0.245 |
| 0 | 0.045 | -0.015 | 0.004 | 0.065 | 0.009 | -0.216 | 0.010 |
| - | 0.084 | 0.030 | 0.061 | -0.095 | -0.227 | 0.378 | -0.038 |

A

Number of Bonds

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | 707 | 1364 | 1425 | 1176 | 1173 | 178 | 6023 |
| 0 | 752 | 1549 | 1692 | 1423 | 1641 | 200 | 7257 |
| - | 511 | 1092 | 1423 | 1481 | 1613 | 275 | 6395 |

Average Error

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | 0.171 | 0.288 | 0.504 | 0.524 | 0.622 | 0.531 | 0.443 |
| 0 | -0.005 | -0.111 | -0.078 | -0.145 | -0.133 | 0.160 | -0.096 |
| - | -0.095 | -0.237 | -0.225 | -0.279 | -0.391 | -0.355 | -0.277 |

BBB

Number of Bonds

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | 361 | 866 | 889 | 864 | 1257 | 66 | 4303 |
| 0 | 324 | 938 | 1068 | 965 | 1255 | 149 | 4699 |
| - | 393 | 1037 | 1039 | 1094 | 1236 | 93 | 4892 |

Average Error

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | 0.374 | 0.684 | 0.932 | 0.839 | 1.009 | 1.415 | 0.846 |
| 0 | 0.242 | 0.039 | 0.116 | 0.266 | 0.278 | 0.500 | 0.196 |
| - | -0.391 | -0.567 | -0.662 | -1.013 | -1.287 | -1.509 | -0.873 |

**Table II (b)**

## Model Errors due to Maturity and Gradations within Ratings
## Financial Sector

AA

Number of Bonds

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | 218 | 207 | 36 | 47 | 44 | 0 | 552 |
| 0 | 306 | 616 | 642 | 420 | 294 | 12 | 2290 |
| - | 1284 | 2081 | 1283 | 705 | 551 | 44 | 5948 |

Average Error

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | -0.044 | -0.055 | -0.131 | -0.283 | -0.369 | - | -0.100 |
| 0 | -0.049 | 0.014 | -0.066 | -0.055 | 0.046 | -0.707 | -0.029 |
| - | -0.025 | 0.056 | -0.062 | -0.024 | 0.166 | 0.064 | 0.014 |

A

Number of Bonds

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | 1838 | 3131 | 2146 | 1486 | 1475 | 110 | 10186 |
| 0 | 2100 | 4014 | 2604 | 2134 | 2378 | 222 | 13452 |
| - | 903 | 2112 | 2352 | 2352 | 2168 | 262 | 10149 |

Average Error

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | -0.112 | -0.179 | -0.491 | -0.575 | -0.646 | -0.288 | -0.359 |
| 0 | -0.065 | -0.025 | -0.143 | -0.127 | -0.038 | -0.163 | -0.075 |
| - | 0.163 | 0.460 | 0.368 | 0.417 | 0.608 | 0.173 | 0.426 |

BBB

Number of Bonds

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | 843 | 1562 | 1092 | 1157 | 1499 | 123 | 6276 |
| 0 | 333 | 568 | 831 | 758 | 836 | 64 | 3390 |
| - | 131 | 228 | 254 | 350 | 365 | 4 | 1332 |

Average Error

| | 1.0 - 2.0 | 2.01-4.0 | 4.01-6.0 | 6.01-8.0 | 8.01-10.0 | 10.01-10.99 | Overall |
|---|---|---|---|---|---|---|---|
| + | -0.168 | 0.020 | -0.255 | -0.227 | -0.224 | -0.128 | -0.160 |
| 0 | 0.062 | 0.118 | -0.231 | -0.135 | 0.110 | 0.142 | -0.031 |
| - | 0.225 | 0.349 | 0.982 | 0.799 | 1.036 | 0.766 | 0.765 |

## Table III
# Predictability of Rating Changes by Past Rating Changes

This table examines whether the direction of rating change (i.e. upgrade or downgrade) in year *t-1* can predict the direction of rating change in year *t*. Each year, each issuer was put into one of the nine cells depending on the direction of rating change in year *t-1* and year *t*. This procedure was repeated for all the active issuers in a given year to arrive at a 3 by 3 table showing the number of issuers in each cell. The table shown below is the average of these tables over the 10-year period 1987 to 1996. It shows the average number of issuers per annum undergoing the particular type of rating transitions.

| | year t upgrade | year t no change | year t downgrade |
|---|---|---|---|
| year t-1 upgrade | 24.7 | 123.4 | 9.4 |
| year t-1 no change | 135.2 | 1192.9 | 197.0 |
| year t-1 downgrade | 25.9 | 157.2 | 56.7 |

**Table IV**

## Model Errors due to Recent Company Rating Changes

For each risk class (e.g., Financial Sector, AA bonds), specific bonds were chosen for which the issuing company experienced a rating change. The pricing errors, model price minus invoice price (using equation 4), were then placed into six bins. The bins are separated along two diffent dimensions: direction of rating change (Up or Down) and number of months after rating change (month of rating change, the following three months, and the subsequent fifteen months.) Panel A gives the number of observations in each bin. Because this table covers the entire sample of ten years, some bonds are observed many times. This results in the number of bonds in a given risk class being roughly proportional to the number of months in the bin. Panel B gives the average error for each bin. Units are dollars per $100 bond.

| | | Financial Sector | | | Industrial Sector | | |
|---|---|---|---|---|---|---|---|
| Direction | Months after Change | Aa | A | Baa | Aa | A | Baa |
| | Panel A: Number of Pricing Error Observations in Bin | | | | | | |
| Up | 4 to 15 | 24 | 1904 | 476 | 258 | 645 | 422 |
| Up | 1 to 3 | 6 | 506 | 138 | 58 | 139 | 131 |
| Up | 0 | 2 | 161 | 53 | 12 | 42 | 50 |
| Down | 0 | 1 | 104 | 19 | 12 | 38 | 62 |
| Down | 1 to 3 | 3 | 307 | 63 | 36 | 112 | 213 |
| Down | 4 to 15 | 12 | 1296 | 267 | 162 | 475 | 737 |
| | Panel B: Average Error in Bin | | | | | | |
| Up | 4 to 15 | 0.937 | -0.081 | -0.469 | -0.241 | -0.120 | 0.065 |
| Up | 1 to 3 | 0.838 | -0.078 | -0.486 | -0.137 | -0.043 | -0.344 |
| Up | 0 | 1.306 | -0.019 | -0.502 | -0.446 | -0.015 | -0.166 |
| Down | 0 | 0.465 | 0.096 | -0.569 | -0.185 | 0.567 | 0.431 |
| Down | 1 to 3 | 0.413 | -0.057 | -0.090 | -0.231 | 0.725 | 0.263 |
| Down | 4 to 15 | 0.201 | 0.113 | 0.065 | -0.157 | 0.847 | 0.174 |

**Table V**

## Model Errors due to Differences between Moody's and Standard and Poors

This table examines whether bonds whose S&P rating is different from Moody's rating are viewed by the market as having different risks. Model errors are model price minus invoice price. Units are dollars per $100 bond.

| | Financial Sector | | | Industrial Sector | | |
|---|---|---|---|---|---|---|
| | Aa | A | Baa | Aa | A | Baa |
| Panel A: Number of Pricing Error Observations | | | | | | |
| S&P Lower | 2075 | 4557 | 1720 | 841 | 4281 | 3111 |
| S&P Same | 5198 | 18537 | 3481 | 2906 | 9459 | 6639 |
| S&P Higher | 1456 | 10465 | 5702 | 1432 | 5875 | 4062 |
| Panel B: Average Error | | | | | | |
| S&P Lower | 0.015 | 0.253 | 0.117 | 0.080 | 0.010 | 0.212 |
| S&P Same | -0.020 | -0.085 | 0.009 | 0.063 | 0.052 | 0.000 |
| S&P Higher | -0.086 | -0.000 | -0.066 | -0.232 | -0.138 | -0.237 |

**Table VI**

## Model Errors due to Bond Age

This table examines the effect of age on pricing errors. The data was separated into two groups: one whose bonds were issued within a year of observation and another whose bonds were older than a year. Panel A gives the number of bonds in each group. Panel B gives the average error for each group. Error is defined as model price minus invoice price. Units are dollars per $100 bond.

| | Financial Sector | | | Industrial Sector | | |
|---|---|---|---|---|---|---|
| | Aa | A | Baa | Aa | A | Baa |
| Panel A: Number of Pricing Error Observations | | | | | | |
| First Year | 2663 | 8501 | 2786 | 1116 | 4294 | 3298 |
| Older | 6071 | 24692 | 8021 | 3912 | 14728 | 10288 |
| Panel B: Average Error | | | | | | |
| First Year | -0.121 | -0.189 | -0.059 | -0.210 | -0.201 | -0.268 |
| Older | 0.047 | 0.051 | 0.009 | 0.039 | 0.045 | 0.058 |

**Table VII**

## Errors for Industrial Baa Bonds sorted by coupon and maturity

Panel (B) of this table shows the errors from discounting the promised payments for Baa rated bonds of industrial category. The errors are model prices minus the invoice prices. The columns are different maturity ranges and the rows are different coupon ranges. Panel (A) shows the number of bonds over which the averaging was done in each cell. Units are dollars per $100.

Panel (A): Number of bonds

| | [1,2) years | [2,4) years | [4,6) years | [6,8) years | [8,10) years | [10,11) years |
|---|---|---|---|---|---|---|
| [0,5)% | 57 | 58 | 0 | 0 | 0 | 0 |
| [5,6.5)% | 112 | 279 | 156 | 84 | 190 | 1 |
| [6.5,8)% | 144 | 501 | 584 | 774 | 1562 | 115 |
| [8,9.5)% | 470 | 1200 | 1185 | 1149 | 1273 | 125 |
| [9.5,11)% | 258 | 624 | 954 | 853 | 722 | 103 |
| [11,15)% | 69 | 179 | 116 | 70 | 12 | 2 |

Panel (B): Average errors

| | [1,2) years | [2,4) years | [4,6) years | [6,8) years | [8,10) years | [10,11) years |
|---|---|---|---|---|---|---|
| [0,5)% | -0.4363 | -0.6707 | . | . | . | . |
| [5,6.5)% | -0.0381 | -0.5762 | -1.1603 | -0.9723 | -1.3549 | -1.4769 |
| [6.5,8)% | -0.0575 | 0.2403 | -0.1202 | -0.1021 | -0.3126 | -0.2746 |
| [8,9.5)% | 0.0497 | 0.0646 | -0.0820 | -0.0968 | 0.0789 | -0.6200 |
| [9.5,11)% | -0.0937 | -0.0415 | 0.0991 | 0.4165 | 1.0066 | 0.6395 |
| [11,15)% | 0.2479 | 0.4590 | 0.7475 | 1.5713 | 2.5329 | 2.4079 |
| Weighted Average | -0.0190 | 0.0192 | -0.0558 | 0.0660 | 0.0298 | -0.1153 |

**Table VIII**

## Model Errors due to Differences in Bond and Company Rating

Each risk class is separated into three groups, one in which the bond is rated higher than the issuing company, one in which the bond is rated lower than the issuing company, and one in which the bond and the issuing company are equally rated. Panel A gives the number of bond price observations for each group of bonds. Panel B gives the average error, defined as model price minus invoice price. Units are dollars per $100 bond.

| | Financial Sector | | | Industrial Sector | | |
|---|---|---|---|---|---|---|
| Bond Rating is … | Aa | A | Baa | Aa | A | Baa |
| Panel A: Number of Pricing Error Observations | | | | | | |
| Higher | 3385 | 1737 | 145 | 1211 | 4355 | 1108 |
| Same | 5086 | 19261 | 1839 | 3420 | 14201 | 9537 |
| Lower | 2 | 11396 | 8344 | 0 | 888 | 2604 |
| Panel B: Average Error | | | | | | |
| Higher | 0.006 | 0.588 | 0.887 | 0.306 | 0.147 | 0.854 |
| Same | -0.040 | -0.025 | 0.427 | -0.168 | -0.027 | 0.093 |
| Lower | -0.097 | -0.105 | -0.135 | - | -0.615 | -0.866 |

**Table IX**

This table presents regression results … Age < 1.0 is one if the bond age is less than 1.0 years. Company > Bond is one if the company rating is better than the bond rating. Bond > Company is one if the bond rating is better than the company rating. Plus is one if the bond has a plus rating (eg., Aa+). Minus is one if the bond has a negative rating. S&P > Moody's is one if Standard and Poor rated the bond as less risky than Moody's did. Moody's > S&P is one if Moody's rated the bond as less risky than Standard and Poor did. Coupon is the bond's coupon rate.

| | Financial Sector | | | Industrial Sector | | |
|---|---|---|---|---|---|---|
| Variable | Aa | A | Baa | Aa | A | Baa |
| Panel A: Number of Pricing Error Observations | | | | | | |
| Intercept | -0.022* | -0.018* | 0.423* | -0.093* | 0.082* | -0.195* |
| Plus * maturity | -0.008 | -0.055* | -0.005* | -0.010* | -0.069* | -0.071* |
| Minus * maturity | 0.014* | 0.061* | 0.123* | -0.003 | 0.030* | 0.159* |
| S&P > Moody's | -0.274* | -0.283* | -0.124* | -0.109* | -0.257* | -0.086* |
| Moody's > S&P | 0.035** | 0.147* | 0.456* | 0.333* | 0.167* | 0.982* |
| Coupon | 0.051* | 0.059* | 0.071* | 0.110* | 0.101* | 0.155* |
| Age < 1.0 | -0.135* | -0.119* | -0.083* | -0.224* | -0.155* | -0.210* |
| Company > Bond | 0.059 | -0.010 | -0.570* | - | -0.222* | -0.407* |
| Bond > Company | 0.018 | 0.487* | 0.183 | 0.379* | 0.075* | 0.686* |
| Adjusted $R^2$ | 0.053 | 0.219 | 0.109 | 0.182 | 0.184 | 0.325 |

* indicates the coefficient is different from zero at the 1% level of significance ** 5% level of significance

Table X

Reduction in pricing errors by incorporating the bond characteristics information

This table shows the extent of improvement that can be achieved in the model prices of bonds by using the information on bond characteristics. The pricing error is defined as $(P_{mi} - P_{ai})$, where $P_{mi}$ is the model price of bond $i$ obtained by discounting its cash flows by spot rates derived from spline fitting and $P_{ai}$ is the actual invoice price of this bond. Column (a) shows the mean absolute pricing error obtained when the model did not use the information on the bond characteristics. Mean absolute pricing error for a given risk category (e.g. Industrial BBB) was obtained by averaging the absolute pricing errors of all the bonds in that risk category across all the months in the time period mentioned. Column (b) shows the mean absolute pricing errors obtained when the model incorporated the information about the bond characteristics (Characteristics adjustment). Column (c) shows the mean absolute pricing error when the model price of each bond was adjusted by subtracting back the average of last six months' pricing errors on that bond (Time Series correction). Column (d) shows the reduction in mean absolute errors obtained by Time Series adjustment as a fraction of the reduction obtained through Characteristics adjustment.

Panel (A) : Mean absolute pricing errors over the full time period (1/1987 to 12/1996)

| Risk Category | Unadjusted Errors (a) | Errors adjusted by Bond characteristics (Characteristics Adjustment) (b) | Errors adjusted by previous months' errors (Time Series Adjustment) (c) | Fraction of error reduction obtained by Time Series adjustment d = (a-c)/(a-b) |
|---|---|---|---|---|
| Financial AA | 0.378 | 0.282 | 0.367 | 11.72% |
| Financial A | 0.618 | 0.412 | 0.527 | 44.49% |
| Financial BBB | 0.899 | 0.598 | 0.812 | 28.77% |
| Industrial AA | 0.482 | 0.320 | 0.415 | 41.57% |
| Industrial A | 0.648 | 0.410 | 0.553 | 39.99% |
| Industrial BBB | 1.182 | 0.646 | 0.924 | 48.05% |

Panel (B) : Mean absolute pricing errors over the first half time period (1/1987 to 12/1991)

| Risk Category | Unadjusted Errors (a) | Errors adjusted by Bond characteristics (Characteristics Adjustment) (b) | Errors adjusted by previous months' errors (Time Series Adjustment) (c) | Fraction of error reduction obtained by Time Series adjustment d = (a-c)/(a-b) |
|---|---|---|---|---|
| Financial AA | 0.379 | 0.294 | 0.399 | -23.25%* |
| Financial A | 0.718 | 0.503 | 0.611 | 49.81% |
| Financial BBB | 0.954 | 0.731 | 0.897 | 25.34% |
| Industrial AA | 0.511 | 0.359 | 0.417 | 61.73% |
| Industrial A | 0.694 | 0.458 | 0.596 | 41.80% |
| Industrial BBB | 1.155 | 0.742 | 0.895 | 62.88% |

Panel (C) : Mean absolute pricing errors across the second half time period (1/1992 to 12/1996)

| Risk Category | Unadjusted Errors (a) | Errors adjusted by Bond characteristics (Characteristics Adjustment) (b) | Errors adjusted by previous months' errors (Time Series Adjustment) (c) | Fraction of error reduction obtained by Time Series adjustment d = (a-c)/(a-b) |
|---|---|---|---|---|
| Financial AA | 0.377 | 0.270 | 0.335 | 39.27% |
| Financial A | 0.518 | 0.321 | 0.442 | 38.69% |
| Financial BBB | 0.844 | 0.464 | 0.727 | 30.78% |
| Industrial AA | 0.453 | 0.282 | 0.412 | 23.62% |
| Industrial A | 0.602 | 0.362 | 0.510 | 38.21% |
| Industrial BBB | 1.209 | 0.550 | 0.953 | 38.74% |

*In this case, the Time Series adjustment leads to a bigger error rather than a reduction in error.