Benefits of Broad-Based Option Pay*

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Abstract

Future wage payments drive a wedge between total firm output and the output share received by the firm's owners, thus potentially distorting strategic decisions by the firm's owners such as, e.g., whether to continue the firm, sell it, or shut it down. Using an optimal contracting approach, we show that the unique optimal firm-wide employee compensation scheme from this perspective is a broad-based option plan. Broad-based option pay *minimizes* the firm's expected future wage payments in states of nature where the firm is only marginally profitable, thus making continuation as attractive as possible in precisely those states of nature where, e.g., a high fixed wage would lead the firm's owners to inefficiently exit.

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1 Introduction

Future wage payments drive a wedge between total firm output and the output share received by the firm's owners, thereby distorting strategic decisions by the firm's owners such as, e.g., whether to continue the firm, sell it, or shut it down. For instance, in states of nature where the firm is only marginally profitable, the firm's owners should ideally receive all of the firm's future cash flow to have optimal incentives to continue. But if, e.g., employees are promised a high fixed wage, most of this cash flow will go to the employees. Hence, a high fixed wage may create a severe "wage overhang problem" in states where the firm's expected future cash flow is small, thereby inducing the firm's owners to inefficiently exit.

This suggests a basic, but fundamental principle: When the firm's expected future cash flow is small, expected future wage payments should also be small. According to this principle, any form of variable compensation is better than a fixed wage. But what is the optimal form of variable compensation? We address this question from first principles in an optimal contracting framework with a large class of admissible compensation schemes. We find that the unique optimal aggregate, i.e., firm-wide compensation scheme is a broad-based option plan. (If there is a subsistence requirement, employees additionally receive a base wage.)

The intuition is simple: Broad-based option pay *minimizes* the firm's expected future wage payments in precisely those states of nature where the firm is only marginally profitable, thus shifting more wage payments from low into high states (where they do not matter for the continuation decision) than, e.g., stock or any other form of variable pay.

Our argument might help to understand why firms use option grants to compensate middle- and lower-level employees. Hall and Murphy (2003) find that more than 90 percent of total company stock option grants in the United States are awarded to managers and employees below the top-five executive level.[1] They conclude that "given the increasing prevalence of broad-based plans, a compelling theory of employee stock options must explain not only executive stock options, but also options granted to the rank and file" (p. 54).

We will review some of the main arguments for broad-based option pay in the following section. We do not view our argument as exclusive, but rather as complementary to arguments

[1] See Mehran and Tracy (2001) and Oyer and Schaefer (2003) for related evidence.

based on the decisions made by individual employees, such as retention and sorting arguments. Also, our objective function in this paper is the maximization of firm value, which means our argument has little to say about situations in which employee compensation is set for reasons other than firm value maximization. While our argument appears to be particularly relevant for smaller, owner-controlled firms, we believe it also relevant for larger firms as long as firm value is being maximized.[2]

Let us briefly run through the main arguments of our model. In the center is the owner of a firm who must decide whether or not to continue his business. The owner's optimal decision depends on the "state of nature," which generates a probability distribution over future cash flows. Higher states of nature are associated with "better" cash-flow distributions in the usual sense. The state of nature is indicative of the firm's success, e.g., it may capture how successful the firm has been in the past in finding buyers, striking deals with suppliers and vendors, or establishing a sales and distribution network.

The state of nature is ex ante uncertain, which captures the usual entrepreneurial risk that the owner does not know in advance how successful he will be. At an interim date, the owner privately observes the state of nature and decides whether to continue. The first-best decision is to continue if the expected cash flow from continuation exceeds the opportunity cost from continuation, which holds if and only if the state of nature is sufficiently high. This opportunity cost includes, e.g., forgone revenues from not liquidating or selling the firm, as well as any effort or investment by the owner that is necessary for the firm's continuation.

Since the owner privately observes the state of nature, his decision whether to continue is discretionary. Expected wage payments drive a wedge between the first-best continuation decision and the owner's privately optimal decision. In particular, in marginally profitable states where the firm's expected future cash flow from continuation barely exceeds the cost of continuation, the owner may nevertheless exit as he only receives the firm's expected cash flow

[2] See also Oyer and Schaefer (2003), who take as their "starting point the assertion that broad-based stock option plans are in shareholders' interests" (p. 16). Oyer and Schaefer (2003) and Ittner, Lambert, and Larcker (2003) find that broad-based option pay is particularly prevalent in new-economy firms, which tend to fit the above description. Larger firms in which founders have large ownership stakes, such as, e.g., Microsoft or Oracle, also often have broad option plans.

minus expected wage payments.

The question is therefore what form of employee compensation minimizes the distortions in the owner's decision? We find that the unique optimal aggregate, i.e., firm-wide compensation scheme is a broad-based option plan. If there is a subsistence requirement, employees additionally receive a base wage. Intuitively, broad-based option pay *minimizes* the firm's expected future wage payments in low, and hence marginally profitable, states of nature, thus minimizing the wedge between the first-best continuation decision and the owner's privately optimal decision. By contrast, any other form of employee compensation (e.g., stock grants) implies higher expected wage payments in marginal states of nature, thus making continuation unprofitable in states where it might have been profitable under broad-based option pay.

In an extension of our model, we allow for the possibility that employees additionally receive severance pay. This has no qualitative effects on our results. In another extension, we allow for the possibility that the owner receives private benefits from continuation. As these private benefits also enter into the first best, the inefficiency that the owner exits too often *relative to the first best* remains the same. However, if the owner's private benefits are sufficiently large, it may now be the case that the owner continues even though the firm's continuation value (excluding private benefits) lies below its liquidation value. For an outside observer who does not know the owner's private benefits, it would then appear as if the owner exits *too little*.

The driving force in our model is the owner's inability to commit to a decision rule due to his private information. If the owner could commit to a decision rule based on the true state of nature, the optimal compensation scheme would be trivial. Precisely, there would be an infinite number of compensation schemes (including a fixed wage) that can implement the first best. In this regard, our setting is related to Grossman and Hart (1983) and other models of implicit labor contracting under asymmetric information. In their model, the owner of a firm must decide whether to lay off workers after privately observing the state of nature. Unlike our model, Grossman and Hart *assume* that workers receive a fixed wage. This generates excessive unemployment, which is the primary focus of the implicit labor contracting literature.[3]

The rest of this paper is organized as follows. Section 2 contains a brief review of the literature on broad-based option pay. Section 3 presents the basic model. Section 4 derives

[3] Weitzman (1985) makes a similar point from a general equilibrium perspective.

our main results. Section 5 considers severance pay and private benefits of the owner. It also considers renegotiations, which are potentially important as the owner's decision is inefficient in marginally profitable states of nature. Section 6 concludes.

2 Literature Review

Unlike the firm's CEO, an individual worker's effort or ability has only a small impact on firm output. Accordingly, Hall and Murphy (2003), Lazear (1999), Oyer and Schaefer (2003), and others have argued that it may be difficult to explain the widespread use of broad option grants with traditional arguments for variable pay such as moral hazard or sorting based on ability.[4] On the other hand, broad-based option pay may work as a sorting device if sorting is with respect to employee risk aversion or optimism about the firm's prospects rather than ability (Bergman and Jenter (2003), Oyer and Schaefer (2003, 2004)).

Other explanations for broad-based option pay are the favorable tax and accounting treatment of stock options (Hall and Murphy (2003)) and cash constraints (Core and Guay (2001), Kedia and Mozumdar (2002)). However, Oyer and Schaefer's (2004) calibration analysis casts doubt on the accounting argument, while Ittner, Lambert, and Larcker (2003), Oyer and Schaefer (2004), and Bergman and Jenter (2003) find that broad option grants are unlikely to be driven by cash constraints. In our model, cash constraints play no role.

Oyer (2004) argues that equity-linked pay can be an effective instrument to retain employees. The idea is that if a firm's stock price is positively correlated with the employee's outside opportunities, the value of his compensation package is high precisely when his outside opportunities are good, and vice versa. Empirical support for this argument is provided by Oyer and Schaefer (2003, 2004) and Kedia and Mozumdar (2002). Like this argument, our argument also suggests an efficiency rationale for broad-based option pay, albeit we focus on decisions made by the firm's owners, not on decisions made by individual employees.

Finally, Hall and Murphy (2003) and Bergman and Jenter (2003) offer behavioral expla-

[4] Lazear (1999) cautions that the sorting-by-ability argument "does not explain why some firms give stock options even to very low-level workers".

nations for broad-based option pay.[5] Hall and Murphy argue that boards and firm owners erroneously perceive option plans as "cheap to grant because there is no accounting cost and no cash outlay, and granting decisions are based on this inaccurate "perceived cost" rather than the much-higher economic cost of options" (p. 61). Bergman and Jenter argue that overoptimistic employees are willing to overpay for option grants. Firms rationally take advantage of this overoptimism by paying employees in the form of option grants.

3 The Model

We examine the decision of a firm's owner whether to continue his business. For simplicity, we assume that the firm has a single employee. As there is no (strategic) interaction among employees, we may view this employee as being representative of all employees in the firm. We denote the owner by P (for principal) and the employee by A (for agent).

If the firm is continued, it generates a stochastic cash flow $x \in X = [\underline{x}, \overline{x}]$, where $0 \leq \underline{x} < \overline{x}$, and where $\overline{x}$ can be either finite or infinite. For convenience, we set $\underline{x} = 0$, albeit all our results extend to $\underline{x} > 0$. The cash-flow distribution $G_\theta(x)$ depends on the underlying state of nature $\theta \in \Theta := [\underline{\theta}, \overline{\theta}]$. We assume $G_\theta(x)$ is atomless and the density $g_\theta(x)$ is positive everywhere and continuous in both x and θ. The expected cash flow conditional on the state of nature is denoted by $E[x \mid \theta] := \int_X x g_\theta(x) dx$.[6]

We may also assume that the owner receives private continuation benefits of $B \geq 0$. While this has interesting implications for the owner's continuation decision, it does not affect our qualitative results. For expositional clarity, we initially set $B = 0$. The case where $B > 0$ is considered in Section 5.1.

As argued in the Introduction, the state of nature is indicative of the firm's success. Specifically, we assume that higher states of nature are associated with a more favorable cash-flow distribution in the following sense:

Assumption 1. *The hazard rate* $g_\theta(x)/[1 - G_\theta(x)]$ *is strictly decreasing in* θ *for all* $x \leq \overline{x}$.

[5] See also Weitzman and Kruse (1990, p. 100), who argue that stock-based compensation creates a "corporate culture that emphasizes company spirit."

[6] If $\overline{x}$ is infinite, we assume that $E[x \mid \theta]$ exists.

Assumption 1 is relatively standard in contracting models and known as monotone hazard rate property (MHRP), or log-concavity.[7]

Only the owner observes the true state of nature and therefore the cash-flow distribution G_θ. While employees (and courts) may realistically have some notion of how well the firm is doing, it seems plausible to us that the owner knows more about the firm's future prospects. It is this informational asymmetry that we try to capture here.

The alternative to continuation is exit, i.e., to discontinue the firm's business. We denote the owner's (opportunity) cost of continuation by L. This may include forgone revenues from not liquidating or selling the firm, any effort or investment by the owner that is necessary for the firm's continuation, as well as any profit which the owner forgoes by not pursuing alternative activities. The employee's opportunity cost of continuation consists of forgone unemployment benefits. For simplicity, we assume that these unemployment benefits provide the employee with a minimum subsistence level, which we denote by J. Hence, the total opportunity cost of continuation is $L+J$. While introducing unemployment benefits adds realism to our model, it is unrelated to our economic argument for why options are optimal. Moreover, it is straightforward to extend our model to include unemployment benefits exceeding J or (forgone) wages from alternative employment, which might also exceed J. Any extension along these lines yields no major additional insights, however.

Benchmark: First-Best Decision Rule. Let us briefly state the first-best decision rule. By Assumption 1 and continuity of $g_\theta(x)$, the conditional expected cash flow $E[x \mid \theta]$ is continuous and increasing in θ. To rule out trivial cases where continuation is either always (i.e., in all states of nature) or never optimal, we assume that $E[x \mid \overline{\theta}] > L + J$ and $E[x \mid \underline{\theta}] < L + J$. Consequently, there exists a unique cutoff $\theta_{FB} \in (\underline{\theta}, \overline{\theta})$ given by $E[x \mid \theta_{FB}] = L + J$ such that $E[x \mid \theta] \geq L + J$ if and only if $\theta \geq \theta_{FB}$. The first-best decision rule is therefore to continue if $\theta \geq \theta_{FB}$ and to exit if $\theta < \theta_{FB}$. $\square$

There are three dates: $t = 0$, $t = 0.5$, and $t = 1$. At $t = 0$ the employee receives a wage contract. At this stage, the state of nature is uncertain and represented by the distribution

[7] See Laffont and Tirole (1993) for an economic interpretation. MHRP is implied by, and thus weaker than, the monotone likelihood ratio property, which is satisfied by many standard distributions (Milgrom (1981)).

function $F(\theta)$, which is common knowledge. We assume $F(\theta)$ has no atoms and the density $f(\theta)$ is positive everywhere. At $t = 0.5$ the owner privately observes the state of nature and decides whether to continue. At $t = 1$—if the firm is continued—the cash flow x is realized and the employee receives his wage payment $w(x)$.[8]

Before we proceed, a comment is in order. If the firm is shut down, the employee is formally entitled to any past wage payments that have accrued during his employment period, but not to forgone future wage payments or, e.g., end-of-year bonus pay.[9] We could easily introduce an interim wage payment (e.g., at $t = 0.5$) that the employee receives irrespective of the owner's continuation decision. From the owner's perspective, what matters for his decision is solely the firm's *future* cash flows and wage payments. Past cash flows and wage payments are sunk and thus irrelevant. To capture this intuition in the simplest possible way, we have assumed that there is a single (future) date when cash flows are realized and wages are paid.

We impose the following restriction on the employee's compensation scheme:

Assumption 2. *Both $w(x)$ and $x - w(x)$ are nondecreasing everywhere.*

This restriction is common in contracting models of this sort (e.g., Innes (1990)). It implies, among other things, that neither the employee nor the owner has an incentive to either destroy output or borrow on the market to boost output.

We may also assume that the compensation scheme must guarantee the employee a minimum subsistence level, implying that $w(x) \geq J$.[10] As in the case of the employee's unemployment benefits, this constraint—while adding realism to our model—is unrelated to our economic argument for why options are optimal. To illustrate this, we first derive our main results for the case where $J = 0$. Subsequently, we show that the intuition straightforwardly extends to the

[8] While $w(x)$ cannot *directly* condition on the state of nature, the owner could, in principle, make the employee's wage payment contingent on θ by choosing a wage contract from a prespecified menu after observing the state of nature. Absent any additional sorting variable, such a menu has no benefits. In fact, one can show that a menu of wage contracts is strictly suboptimal in our model.

[9] Employment contracts frequently compensate employees for forgone future wage payments by stipulating severance pay. This is voluntary, however. See Section 5.2. for an analysis of severance pay.

[10] This is possible as we do not assume that the firm is cash constrained. Alternatively, we could have assumed that $\underline{x} \geq J$, in which case the subsistence wage can always be paid out of the firm's cash flow.

case where $J > 0$.

We finally introduce a simple incentive problem implying that the employee's expected compensation under continuation must exceed his unemployment benefits. Between $t = 0$ and $t = 0.5$, the employee can perform a job-related task at private cost Δ. In the broadest sense, we can think of this task as "doing the job well". Whether or not the employee has performed the task is observable only at $t = 1$. Optimality then implies that the employee be compensated if and only if he has performed his task. (The subsistence constraint $w(x) \geq J$ needs to hold only in equilibrium.)

The employee's compensation will be optimally set to make him indifferent between performing and not performing the task. From a $t = 0.5$ perspective, however, such compensation constitutes a *rent*, because at this point the employee has already perfomed his task (in equilibrium). The employee's incentive-compatibility constraint therefore translates into an "efficiency-wage constraint" stating that—to compensate the employee for performing his task—his expected compensation if the firm is continued must exceed his unemployment benefits.[11]

We derive the efficiency-wage constraint from first principles in Section 5.3. At this point, let us merely note that—other than implying that the employee must be compensated for his privately incurred cost Δ—it has no immediate implications for the functional form of $w(x)$ and thus for our results. In fact, we will show in the following section that if the state of nature was observable, there exists an infinite number of optimal compensation schemes (including a flat wage) satisfying the efficiency-wage constraint and implementing the first best.

Second, we do not need to assume that the employee's task has any significant impact on firm output. All we need to assume is that it is efficient to induce the employee to perform the task. For concreteness, suppose if the employee performs the task, the output technology is as described above. By contrast, if he does not perform the task, the firm incurs a small cost C, e.g., it may lose a customer because the employee did not spend enough time to maintain the customer relationship. All we need to assume is that C is sufficiently large relative to Δ.[12]

[11] There are many possible reasons why the employee's expected compensation under continuation ought to exceed his unemployment benefits; the incentive problem here is just one possibility. See Yellen (1994) for an overview of efficiency-wage arguments.

[12] Alternatively, we could have assumed that not performing the task has a (small) negative impact on the cash-

4 Optimality of Broad-Based Option Pay

We proceed in two steps. We first derive the owner's continuation decision at $t = 0.5$ as a function of the state of nature. In a second step, we solve for the optimal compensation scheme $w(x)$ that maximizes the owner's expected payoff at $t = 0$.

Generally, the owner will find it optimal to continue if and only if the expected cash flow minus any promised wage payments exceeds his opportunity cost of continuation, i.e., if $E[x - w(x) \mid \theta] \geq L$. We ignore trivial cases where the owner either always (i.e., for all θ) or never continues; it is straightforward to provide conditions ruling these cases out. If both continuation and exit are optimal for some θ, what is the owner's optimal decision rule? The following lemma shows that the optimal decision rule takes a simple form: continue if and only if the state of nature is sufficiently high.

The intuition for why the owner's decision rule is a cutoff rule is straightforward. By Assumption 2, $x - w(x)$ is nondecreasing everywhere. Moreover, it cannot be the case that $x - w(x) = 0$ for all x, or else the owner would always exit. Consequently, $x - w(x)$ must be strictly increasing for some x on a set of positive measure. In conjunction with Assumption 1 and continuity of $g_\theta(x)$ in θ, this in turn implies that the owner's expected payoff $E[x - w(x) \mid \theta]$ is continuous and strictly increasing in θ. There consequently exists a cutoff state $\theta_P \in (\underline{\theta}, \overline{\theta})$ such that the owner finds it optimal to continue if $\theta \geq \theta_P$ and to exit if $\theta < \theta_P$.[13]

Lemma 1. *There exists a unique state of nature $\theta_P = \theta_P(w(x)) \in (\underline{\theta}, \overline{\theta})$ given by $E[x - w(x) \mid \theta_P] = L$ such that the owner continues if and only if $\theta \geq \theta_P$.*

While the owner's privately optimal cutoff $\theta_P(w(x))$ depends on the compensation scheme $w(x)$, we omit the argument in what follows and simply write θ_P.

Equipped with Lemma 1, we can derive the optimal compensation scheme $w(x)$ offered at $t = 0$. The optimal choice of $w(x)$ maximizes the owner's expected payoff

$$\int_{\theta_P}^{\overline{\theta}} E[x - w(x) \mid \theta] f(\theta) d\theta + LF(\theta_P), \tag{1}$$

flow distribution $G_\theta(x)$. As this concerns out-of-equilibrium behavior, it does not enter into the efficiency-wage constraint. See Section 5.3 for more details.

[13] Without loss of generality, we assume that the owner continues if he is indifferent.

subject to (i) the employee's subsistence constraint $w(x) \geq J$ for all x, and (ii) the efficiency-wage constraint

$$\int_{\theta_P}^{\overline{\theta}} [E[w(x) \mid \theta] - J] f(\theta) d\theta \geq \Delta. \tag{2}$$

(See Section 5.3 for a derivation of this constraint.) We can rewrite (2) as

$$\int_{\theta_P}^{\overline{\theta}} E[w(x) \mid \theta] \frac{f(\theta)}{1 - F(\theta_P)} d\theta \geq J + \frac{\Delta}{1 - F(\theta_P)}, \tag{3}$$

which states that the employee's expected compensation if the firm is continued (left-hand side) must exceed his unemployment benefit J by an amount $\Delta/[1-F(\theta_P)]$.[14] Accordingly, the higher the owner's cutoff θ_P, the lower is the probability $1 - F(\theta_P)$ *that* the firm is continued, and the higher must therefore be the employee's expected compensation *if* the firm is continued.[15]

By standard arguments, (2) must bind at the optimal solution.

Lemma 2. *The efficiency-wage constraint* (2) *must bind at the optimum.*

Proof. See Appendix.

Substituting the binding efficiency-wage constraint (2) into the owner's objective function (1), we have that the owner chooses $w(x)$ to maximize

$$\int_{\theta_P}^{\overline{\theta}} [E[x \mid \theta] - J - L] f(\theta) d\theta + L - \Delta, \tag{4}$$

subject to $w(x) \geq J$, where θ_P depends on $w(x)$ through $E[x - w(x) \mid \theta_P] = L$ by Lemma 1. By inspection, (4) attains its maximum at $\theta_P = \theta_{FB}$. Intuitively, as the owner is the residual claimant to the firm's cash flow, he would ideally like to commit to the (first-best) efficient decision rule. Indeed, if such commitment was possible, the choice of optimal compensation scheme would be trivial, as the following comments illustrate.

Benchmark: Observable States of Nature. If the owner could commit to a particular decision rule, say, $\theta_P = \hat{\theta}$, there exists an infinite number of optimal compensation schemes

[14] While the employee can rationally infer the owner's cutoff θ_P, he does not know the true state θ. All he knows is that $\theta \geq \theta_P$, where $f(\theta)/[1 - F(\theta_P)]$ is the posterior probability of θ given that the firm is continued.

[15] If (2) is satisfied, the employee's ex-ante participation constraint holds trivially, while his interim participation constraint is slack. As both participation constraints are implied by (2), we refrain from writing them down.

satisfying all constraints. For example, under the fixed-wage contract $w(x) = B := J + \Delta/[1 - F(\hat{\theta})]$ the efficiency-wage constraint (2) holds with equality, while the employee's subsistence constraint $w(x) \geq J$ is slack. Hence, if the owner could commit to, say, $\theta_P = \theta_{FB}$, he could trivially implement the first best with a "flat" wage equal to $w(x) = B$.

This confirms our earlier assertion that—unlike a "standard" moral hazard problem—the incentive problem here has no immediate consequences for the form of the employee's compensation scheme $w(x)$. All it does is add a requirement that the employee's expected compensation if the firm is continued must be sufficiently hight to compensate him for his privately incurred cost Δ, implying that it must exceed his unemployment benefit J. However, it says nothing about *how* precisely this compensation ought to be divided across cash flows $x \in [\underline{x}, \overline{x}]$ or continuation states $\theta \geq \theta_P$, which is what our main argument is all about. □

Returning to our original setting where θ is private information, we now derive the solution to the owner's maximization problem. For expositional clarity, we first consider the case where $J = 0$. We show that (i) under *any* admissible compensation scheme $w(x)$ the owner exits in too many states of nature, and (ii) the compensation scheme minimizing this inefficiency is an option on the firm's cash flow. Subsequently, we show that this intuition extends to the case where $J > 0$.

Let us begin by assuming that $J = 0$. In this case, the first-best cutoff is given by $E[x \mid \theta_{FB}] = L$. The efficiency-wage constraint (2), however, requires that $w(x) > 0$ on a set of positive measure, which immediately implies that $E[x - w(x) \mid \theta] < E[x \mid \theta]$ for all θ, which in turn implies that $\theta_P \neq \theta_{FB}$. Hence, the owner's privately optimal continuation decision is inefficient. Indeed, we can say more about this inefficiency: as $E[x \mid \theta]$ and $E[x - w(x) \mid \theta]$ are both continuous and strictly increasing in θ (see above), and $E[x - w(x) \mid \theta] < E[x \mid \theta]$ for all θ, it must be true that $E[x - w(x) \mid \theta_{FB}] < E[x \mid \theta_{FB}] = L$, and hence that $\theta_P > \theta_{FB}$. The inefficiency is thus that the owner's optimal cutoff lies strictly above the first-best cutoff. This holds for *any* admissible compensation scheme $w(x)$. Intuitively, the fact that the employee must be compensated drives a wedge between the expected firm cash flow $E[x \mid \theta]$ and the owner's share of this cash flow, $E[x - w(x) \mid \theta]$. As a result, the owner exits in marginally profitable states $\theta \in [\theta_{FB}, \theta_P)$, which is inefficient.[16]

[16] The fact that the owner exits in too many states of nature holds irrespective of whether Assumption 2 holds.

The owner's problem is consequently one-sided: The optimal compensation scheme $w(x)$ must minimize the owner's incentives to exit, i.e., it must minimize the owner's cutoff θ_P, thereby bringing it closer to the first-best cutoff θ_{FB}. We now argue that the unique solution to this problem is to give the employee an option on the firm's cash flow.

An option minimizes $w(x)$ when x is small. As small x are relatively more likely after low states of nature (Assumption 1), an option thus minimizes the employee's expected compensation $E[w(x) \mid \theta]$ in low states of nature.[17] This implies that it maximizes the owner's expected payoff $E[x - w(x) \mid \theta]$ in low states of nature, thus pushing the critical cutoff state θ_P where the owner is just indifferent between continuing and not continuing as far down as possible towards θ_{FB}.[18] Put simply, an option minimizes the firm's expected future wage costs in low states of nature, thus making it as attractive as possible to continue in these states.

There is a more general principle at work. If the owner continues at $\theta = \theta_P$, he also continues in all higher states $\theta \geq \theta_P$. Hence, we only need to consider the owner's expected payoff in marginally profitable states of nature. His expected payoff in inframarginal states $\theta > \theta_P$ is irrelevant. Accordingly, the optimal compensation scheme must make continuation as attractive as possible for the owner in marginal states. This is precisely what an option does: it shifts more wage costs from low into high states of nature (where wage costs do not matter for efficiency) than any other form of compensation scheme.

Let us now consider the case where $J > 0$. The underlying problem is the same: As the employee must be compensated for performing his task, his expected compensation if the firm is continued must exceed his unemployment benefit.[19] Again, this drives a wedge between the first-best cutoff and the owner's privately optimal cutoff, with the consequence that $\theta_P > \theta_{FB}$. Like above, the employee cares only about his expected compensation if the firm is continued, while

If Assumption 2 holds, however, we can express the inefficiency in a simple way through the difference $\theta_P - \theta_{FB}$.

[17]This is subject to the efficiency-wage constraint (2) and Assumption 2 that $x - w(x)$ be nondecreasing.

[18]There is never a concern that θ_P may be pushed down too far, i.e., below θ_{FB}: As we have shown above, for any admissible compensation scheme $w(x)$ it holds that $\theta_P > \theta_{FB}$.

[19]By (3) we have that

$$\int_{\theta_P}^{\overline{\theta}} [E[w(x) - J \mid \theta] \frac{f(\theta)}{1 - F(\theta_P)} d\theta \geq \frac{\Delta}{1 - F(\theta_P)},$$

where the left-hand side denotes the employee's expected *excess* compensation if the firm is continued.

the owner cares about how precisely this compensation is divided across cash flows $x \in [\underline{x}, \overline{x}]$ and thus across continuation states $\theta \geq \theta_P$. The solution, again, is to shift as much as possible of the employee's compensation into high states of nature as this maximizes the owner's expected payoff in marginally profitable states. This implies that the subsistence constraint $w(x) \geq J$ must bind, which in turn implies that the optimal solution is an option plus a base wage equal to J. Hence, the only effect of introducing $J > 0$ is that it adds a base wage to the optimal compensation scheme.

Proposition 1. *The unique optimal compensation scheme is to pay the employee a base wage plus an option on the firm's cash flow. And yet, under the optimal compensation scheme the owner exits in too many states of nature relative to the first best.*

Proof. See Appendix.

We have assumed that the owner can commit to some compensation scheme $w(x)$ despite the fact that it induces inefficient decisions in some states of nature. As it turns out, such a strong assumption is not necessary. In the following section, we show that the optimal compensation scheme in Proposition 1 is the unique optimal renegotiation-proof compensation scheme if new offers can be made after the state of nature has materialized. The reason, in short, is that renegotiations take place under asymmetric information, which implies the owner will try to use his informational advantage to obtain wage concessions from the employee even in high states $\theta > \theta_P$ where he would have ordinarily continued. As the employee anticipates this, he is better off not renegotiating the original (i.e., optimal) compensation scheme.

5 Extensions and Robustness

5.1 Private Benefits of Continuation

The decision of entrepreneurs to continue their firms may not always be solely guided by profit maximization. Entrepreneurs may also receive private benefits from continuation, i.e., they may gain further valuable experience, build up a reputation vis-à-vis employees and customers, or simply enjoy running their firms.

Suppose continuation entails nonpecuniary private benefits $B > 0$ for the owner. To again rule out trivial cases where continuation is either always or never (first-best) efficient, we assume that $E[x \mid \overline{\theta}] + B > L + J$ and $E[x \mid \underline{\theta}] + B < L + J$. Introducing private benefits changes the optimal continuation decision as follows. The first-best decision is now to continue if and only if $\theta \geq \theta_{FB}$, where θ_{FB} is given by

$$E[x \mid \theta_{FB}] + B = L + J. \tag{5}$$

By the same token, the owner's privately optimal decision rule is to continue if and only if $\theta \geq \theta_P$, where the optimal cutoff θ_P is given by

$$E[x - w(x) \mid \theta_P] + B = L. \tag{6}$$

Let $\widehat{L} := L - B$ denote the owner's opportunity cost of continuation net of his private benefits. Given this transformation, it is immediate that all our results continue to hold, except that we now have $\widehat{L}$ instead of L. In particular, it holds that $\theta_P > \theta_{FB}$, while the unique optimal compensation scheme is again to pay the employee a base wage equal to J plus an option on the firm's cash flow. The crux is that the owner's private benefits enter both into the first-best decision rule and the owner's privately optimal decision rule. From an efficieny standpoint, the owner thus again exits too often (relative to the first best, that is).

Interestingly, if B is sufficiently large, it is possible that $E[x \mid \theta_P] < L$. In this case, there exist states of nature where the owner continues even though the firm's continuation value (i.e., the expected cash flow from continuation *excluding* the owner's private benefits) is less than its liquidation or sales value. From the perspective of an outsider who does not observe the owner's private benefits, it then appears as if the owner exits *too little*.

5.2 Severance Pay

We now introduce the possibility that the employee's compensation scheme includes both a wage payment $w(x)$ if the firm is continued and a (severance) payment S if the firm is discontinued.

Severance pay has both costs and benefits. It is costly for two reasons: first, the owner incurs an additional cost of S if he chooses to exit. Second, the employee's "outside income" at $t = 0.5$ is now $J + S$ instead of merely J. As the employee's expected compensation if the firm is continued must exceed his outside income by an amount $\Delta/[1 - F(\theta_P)]$, this implies that his expected compensation under continuation must also increase by S. Formally, the efficiency-wage constraint (2) becomes

$$\int_{\theta_P}^{\overline{\theta}} [E[w(x) \mid \theta] - J - S] f(\theta) d\theta \geq \Delta, \tag{7}$$

which can be rewritten as

$$\int_{\theta_P}^{\overline{\theta}} E[w(x) \mid \theta] \frac{f(\theta)}{1 - F(\theta_P)} d\theta \geq J + S + \frac{\Delta}{1 - F(\theta_P)}.$$

Hence, the employee's expected compensation if the firm is continued (left-hand side) must increase by an amount S.

Introducing $S > 0$ also entails benefits. While the employee only cares that his expected compensation under continuation increases by S, the owner cares about how precisely this increase is divided across cash flows $x \in [\underline{x}, \overline{x}]$ and thus across continuation states $\theta \geq \theta_P$. Specifically, if $w(x)$ is an option (plus a base wage), the entire increase in the employee's compensation occurs at high cash flows and thus primarily in high states of nature. Consequently, while the employee's expected compensation increases overall by an amount S, it increases by more than S in high states of nature and by less than S in low states of nature. Hence, the owner's expected payoff from continuing decreases by less than S in low states of nature. His payoff from *not* continuing, however, decreases exactly by S, namely, from L to $L - S$. Continuation may now be profitable for the owner in marginally profitable states where it was previously unprofitable. Accordingly, introducing severance pay may *lower* the owner's privately optimal cutoff θ_P.[20]

The next question is whether the owner should set S so high to push θ_P all the way down to θ_{FB}, thus eliminating any inefficiency? The answer is no. At the first-best cutoff $\theta_P = \theta_{FB}$, the efficiency loss due to a marginal increase in θ_P generated by a small decrease in S is zero. The benefit (i.e., the cost saving) of a decrease in S is of first-order magnitude, however. Formally, the owner's objective function if $S > 0$ is

$$\int_{\theta_P}^{\overline{\theta}} E[x - w(x) \mid \theta] f(\theta) d\theta + (L - S) F(\theta_P).$$

[20]While this monotonic relation between S and θ_P holds if $w(x)$ is an option (plus possibly a base wage), it does not necessarily hold for arbitrary (i.e., suboptimal) compensation schemes.

Inserting the (binding) efficiency-wage constraint (7), this transforms to

$$\int_{\theta_P}^{\overline{\theta}} [E[x \mid \theta] - J - L]\, f(\theta) d\theta + L - S - \Delta. \tag{8}$$

By inspection, reducing S affects the owner's expected payoff both directly (positive effect) as well as indirectly via θ_P (negative effect). The total derivative of (8) with respect to S is

$$-\frac{\partial \theta_P}{\partial S} [E[x \mid \theta_P] - J - L] - 1.$$

At the first-best cutoff θ_{FB}, it holds that $E[x \mid \theta_{FB}] = J + L$. Hence, if $\theta_P = \theta_{FB}$ the total derivative is -1, implying that a small reduction in S is strictly profitable.

To summarize, while introducing severance pay *may* mitigate the inefficiency, eliminating it altogether is too costly. In fact, it is not clear whether including severance pay is optimal at all, i.e., whether it is optimal to set $S > 0$. The answer depends on the underlying probability distributions $F(\theta)$ and $G_\theta(x)$. What is clear, however, is that our previously studied inefficiency remains, implying that Proposition 1 continues to hold.

Proposition 2. *Proposition 1 continues to hold if severance pay is possible.*

Proof. See Appendix.

5.3 The Efficiency-Wage Constraint

We now derive the efficiency-wage constraint (2) from first principles. For the sake of brevity, we only present the analysis for our basic model. The analysis with severance pay is, subject to minor modifications, similar.[21]

As we have noted in the main text, optimality prescribes to compensate the employee only if he performed the task. This immediately gives the employee's out-of-equilibrium payoff: If the employee does *not* perform the required task, he will optimally quit at $t = 0.5$ rather than stay in the firm. Hence, the employee's payoff if he does not perform the task is simply his

[21] Precisely, if the employee receives severance pay both if the owner exits and if the employee voluntarily quits, we obtain the efficiency-wage constraint (7) from Section 5.2. By contrast, if the employee receives severance pay only if the owner exits (but not if the employee quits), we obtain our previous constraint (2).

unemployment benefit J.[22] By contrast, his expected payoff if he performs the task is

$$\int_{\theta_P}^{\overline{\theta}} E[w(x) \mid \theta] f(\theta) d\theta + F(\theta_P) J - \Delta,$$

which implies he will perform the task if and only if

$$\int_{\theta_P}^{\overline{\theta}} E[w(x) \mid \theta] f(\theta) d\theta + F(\theta_P) J - \Delta \geq J,$$

which represents the employee's incentive-compatibility constraint. Rearranging yields the efficiency-wage constraint (2) in the main text.

5.4 Renegotiation

The inefficiency that the owner exits in marginally profitable states $\theta \in (\theta_{FB}, \theta_P]$ potentially provides scope for mutually beneficial renegotiations: To make continuation more attractive for the owner, the employee might be willing to take a paycut to secure at least some of his rents. (Recall that the employee's expected compensation under continuation strictly exceeds his unemployment benefit.) As the state of nature is private information, however, such potentially beneficial renegotiations will fail.

We consider the following model of renegotiation. After the state of nature has materialized but before the owner makes his decision, either the owner or the employee can offer a new compensation scheme $w(x)$.[23] If the owner makes the offer, the employee must agree. If the employee rejects the offer, the original compensation scheme remains in effect Conversely, if the employee makes the offer, the owner must agree. As the following proposition shows, irrespective of who makes the offer, the unique optimal compensation scheme derived in Proposition 1 will not be renegotiated in equilibrium.

Proposition 3. *The unique optimal compensation scheme in Proposition 1 is renegotiation-proof. That is, regardless of whether the owner or the employee can make new offers, this*

[22] This clarifies an earlier statement that we are free to specify the possible consequences for the firm if the employee does not perform his task. As the employee then optimally quits at $t = 0.5$, what happens to the firm at $t = 1$ is irrelevant for his (out-of-equilibrium) payoff.

[23] There is no point in renegotiating before the state of nature has materialized. By contrast, we may allow for renegotiations to take place *after* the owner has made his decision provided this decision is reversible. (The decision has no signalling value in this case.) If the exit decision is irreversible, it is too late for renegotiations.

compensation scheme will not be renegotiated with positive probability in any (perfect Bayesian) equilibrium of the renegotiation game.

Proof. See Appendix.

Let us provide the intuition for the case where $S = 0$; the case where $S \geq 0$ is analyzed in the Appendix. Suppose the owner proposes (or accepts) to replace the optimal compensation scheme $w(x)$ with some other compensation scheme $\widetilde{w}(x)$. As the owner knows the true state of nature, it must hold that $E[x - \widetilde{w}(x) \mid \theta] \geq E[x - w(x) \mid \theta]$, or equivalently, $E[w(x) \mid \theta] \leq E[\widetilde{w}(x) \mid \theta]$. Accordingly, in states of nature $\theta \geq \theta_p(w)$ in which the owner would have continued anyway, replacing $w(x)$ with $\widetilde{w}(x)$ merely shifts rents from the employee to the owner, thus making the employee worse off. This implies that, for renegotiations to be mutually beneficial, it *must* hold that (i) the new compensation scheme $\widetilde{w}(x)$ induces continuation in strictly more states of nature than the original (i.e., optimal) compensation scheme, i.e., $\theta_p(\widetilde{w}) < \theta_p(w)$, and (ii) the employee must attach a reasonably high probability that the current state of nature lies between $\theta_p(\widetilde{w})$ and $\theta_p(w)$.

Unfortunately, there is no easy way for the owner to credibly signal that the current state is $\theta \in [\theta_p(\widetilde{w}), \theta_p(w))$, or for the employee to screen the owner's "type" θ. The reason is that, if the owner prefers the new compensation scheme $\widetilde{w}(x)$ to the original, optimal compensation scheme $w(x)$ in state θ, he will also prefer $\widetilde{w}(x)$ to $w(x)$ in all higher states $\widetilde{\theta} > \theta$.[24] Intuitively, the optimal compensation scheme shifts as much as possible of the employee's compensation into high cash-flow states, which implies that any other compensation scheme $\widetilde{w}(x) \neq w(x)$ would make the owner better off. Consequently, the employee will continue to hold his prior beliefs $f(\theta)$, implying that he will prefer $w(x)$ over $\widetilde{w}(x)$ if and only if

$$\int_{\theta_P(\widetilde{w})}^{\overline{\theta}} E[\widetilde{w}(x) \mid \theta] f(\theta) d\theta + F(\theta_P) J \geq \int_{\theta_P}^{\overline{\theta}} E[w(x) \mid \theta] f(\theta) d\theta + F(\theta_P) J,$$

[24] This is because the optimal compensation scheme $w(x)$ is an option (plus a base equal to J). Any other compensation scheme $\widetilde{w}(x)$ satisfying Assumption 2 and $\widetilde{w}(x) \geq J$ must either (i) leave the owner strictly less for all x or (ii) satisfy $\widetilde{w}(x) \geq w(x)$ for all $x \leq \widetilde{x}$ and $\widetilde{w}(x) \leq w(x)$ for all $x > \widetilde{x}$ for some interior $\widetilde{x} \in X$, with strict inequality on a set of positive measure. In case (i) the owner never prefers $\widetilde{w}$ to w. In case (ii), the asserted statement follows directly from Assumption 1.

or

$$\int_{\theta_P(\widetilde{w})}^{\overline{\theta}} [E[\widetilde{w}(x) \mid \theta] - J] f(\theta) d\theta \geq \int_{\theta_P}^{\overline{\theta}} [E[w(x) \mid \theta] - J] f(\theta) d\theta. \tag{9}$$

By Lemma 2, the right-hand side in (9) is equal to Δ, which implies the optimal compensation scheme $w(x)$ satisfies the employee's efficiency-wage constraint (2) with equality. By (9), this in turn implies that the new compensation scheme $\widetilde{w}(x)$ must also satisfy (2). This cannot be true, however: If there exists a compensation scheme $\widetilde{w}(x) \neq w(x)$ satisfying (2) (as well as $w(x) \geq J$) *and* implementing a lower cutoff $\theta_p(\widetilde{w}) < \theta_p(w)$, the compensation scheme $w(x)$ cannot not be optimal. Hence, we have a contradiction, implying that the employee will neither propose nor accept to replace $w(x)$ with $\widetilde{w}(x)$.

6 Concluding Remarks

This paper argues that broad-based option pay maximizes the efficiency of strategic decisions by firm owners, such as, e.g., whether to continue the firm, shut it down, or sell it. In our model, future expected wage payments inefficiently bias the owner of a firm towards exiting. Broad-based option pay shifts most of these future wage payments into states of nature where expected firm profits are high, thereby leaving the owner as much as possible in states where expected firm profits are low. This minimizes the wedge between the first-best continuation decision and the owner's privately optimal decision, thereby maximizing efficiency. While introducing severance pay may (but need not) improve the owner's continuation decision, some inefficiency always remains, implying that broad-based option pay remains uniquely optimal even if severance pay is possible.

While our model has a single, representative employee, it does not insinuate that the employee has any significant impact on firm output. Nor is there any interaction or free-riding among employees. Hence, our argument extends to the firm's employees as a whole, making it indeed a theory of *broad* option pay. Our model does, however, assume that the objective function is to maximize firm value. This suggests that it may be particularly relevant for small, owner-controlled firms (e.g., new economy firms). Empirically, it appears that broad option pay is indeed more pervasive in these kinds of firms (see Introduction). Moreover, our argument requires that there is considerable uncertainty—coupled with asymmetric information—about future firm profits. This may help understand a finding in the empirical literature that broad-based option pay—and high-powered compensation more generally—is more pervasive in volatile industries (Prendergast (2002), Oyer and Schaefer (2003)). Again, new economy firms appear to fit this picture.

An interesting question that we have not explored here is what happens if the continuation decision is made by a manager whose interests diverge from the interests of the firm's owner. Given the owner's bias, it may be optimal to employ such a manager to make more efficient decisions, which would in turn require a hands-off policy by the firm's owner (see Burkart, Gromb, and Panunzi (1997)). On the other hand, delegating decision-making power to a manager with diverging interests may introduce new inefficiencies, such as empire building and entrenchment.

7 Appendix

Proof of Lemma 2. If (2) did not bind, once could adjust $w(x)$ slightly so that (2) still holds while the owner is strictly better off. To prove this, we must first define a feasible adjustment to $w(x)$ such that the employee's payoff does not fall below J for all $x \in X$ and Assumption 2 remains satisfied. By Assumption 2 $w(x)$ is continuous and almost everywhere differentiable, and it satisfies $w'(x) \geq 0$ at points of differentiability. We can distinguish between the following two cases.

In the first case, $w'(x) > 0$ holds strictly for a set of positive measure. In this case, we define a new compensation scheme $\widetilde{w}(x)$ by the requirements that $\widetilde{w}(0) = w(0)$ and that at points of differentiability $\widetilde{w}'(x) = w'(x)(1-\varepsilon)$, where $0 \leq \varepsilon < 1$. By construction, $\widetilde{w}(x)$ satisfies Assumption 2, while $\widetilde{w}(x) \geq K$ holds for all $x \in X$. Note also that $E[\widetilde{w}(x) \mid \theta]$ is continuous and strictly increasing in ε for all θ. By the latter implication, the owner would be strictly better off by offering $\widetilde{w}(x)$ with $\varepsilon > 0$ if (2) was still satisfied. As (2) was originally slack by assumption, it is still satisfied for all sufficiently small values ε in case the employee's payoff is also continuous in ε. By continuity of $E[\widetilde{w} \mid \theta]$ and as $F(\theta)$ has no atoms, this holds surely if the cutoff $\theta_P(\widetilde{w})$ is continuous in ε, which follows immediately from the definition of θ_P and continuity of $E[\widetilde{w}(x) \mid \theta]$ in ε.

In the second case, $w(x)$ is a fixed wage. As $w(x)$ must satisfy (2), this implies $w(0) > J$. We can now simply construct an alternative feasible compensation scheme where $\widetilde{w}(x) = w(x) - \varepsilon$. Again, for sufficiently small $\varepsilon > 0$ (2) is still satisfied while the owner is made strictly better off. Q.E.D.

Proof of Proposition 1. Proposition 1 claims that the optimal compensation scheme satisfies $w(x) = J + \max\{x - s, 0\}$ for some $s \in (\underline{x}, \overline{x})$. We argue to a contradiction. Suppose that some other compensation scheme $w(x)$ that does not satisfy $w(x) = J + \max\{0, x - s\}$ is optimal. Recall now that the constraint (2) binds by Lemma 2, while by the arguments in the main text it follows that $\theta_P(w) > \theta_{FB}$. Moreover, it must hold that $w(x) \geq J$. We now construct a new compensation scheme $\widetilde{w}(x) = J + \max\{0, x - \widetilde{s}\}$ as follows. We keep the original cutoff $\theta_P(w)$ fixed and choose $\widetilde{s}$ such that the constraint (2) is still satisfied with equality, requiring

$$\int_{\theta_P(w)}^{\overline{\theta}} \left(\int_{\widetilde{s}}^{\overline{x}} (x - \widetilde{s}) g_\theta(x) dx \right) f(\theta) d\theta = \Delta. \tag{10}$$

Existence of a unique value $0 < \widetilde{s} < \overline{x}$ solving (10) is immediate.[25] We show now that the true cutoff under the new compensation scheme, $\theta_P(\widetilde{w})$, is strictly lower than the original cutoff $\theta_P(w)$.

Claim 1. *It holds that* $\theta_P(\widetilde{w}) < \theta_P(w)$.

Proof. It is convenient to use the following transformation. Take any function $a(x)$ that is continuous and differentiable almost everywhere on $x \in X$. Partial integration yields

$$\int_X a(x) g_\theta(x) dx = a(0) + \int_X a'(x)[1 - G_\theta(x)] dx. \tag{11}$$

Note next that the derivative of $\widetilde{w}(x)$ satisfies $\widetilde{w}'(x) = 0$ for $x < \widetilde{s}$ and $\widetilde{w}'(x) = 1$ for $x > \widetilde{s}$. Also, by Assumption 2 the original compensation scheme $w(x)$ is continuous and almost everywhere differentiable. Using the transformation (11), we then obtain for all θ

$$E[\widetilde{w}(x) - w(x) \mid \theta] = \int_{\widetilde{s}}^{\overline{x}} [1 - w'(x)] [1 - G_\theta(x)] dx - \int_0^{\widetilde{s}} w'(x) [1 - G_\theta(x)] dx + J - w(0), \tag{12}$$

[25] As (2) holds at the original compensation scheme, the left-hand side of (10) surely exceeds Δ at $\widetilde{s} = 0$. At $\widetilde{s} = \overline{x}$ the left-hand side of (10) is lower than Δ. Moreover, by Assumptions 1 and 2—together with continuity of $g_\theta(x)$—the left-hand side of (10) is also continuous and strictly decreasing in $\widetilde{s}$.

where we used that $\widetilde{w}(0) = J$. As both $w(x)$ and $\widetilde{w}(x)$ satisfy (2), holding $\theta_P(w)$ fixed, there must exist at least one state $\theta_P(w) < \widetilde{\theta} < \overline{\theta}$ such that

$$E[\widetilde{w}(x) - w(x) \mid \widetilde{\theta}] = 0. \tag{13}$$

We next transform (12) into

$$\begin{aligned} E[\widetilde{w}(x) - w(x) \mid \theta] \quad = \quad & \int_{\widetilde{s}}^{\overline{x}} [1 - w'(x)] \left[1 - G_{\widetilde{\theta}}(x)\right] \left[\frac{1 - G_\theta(x)}{1 - G_{\widetilde{\theta}}(x)}\right] dx \\ & - \int_0^{\widetilde{s}} w'(x) \left[1 - G_{\widetilde{\theta}}(x)\right] \left[\frac{1 - G_\theta(x)}{1 - G_{\widetilde{\theta}}(x)}\right] dx + J - w(0). \end{aligned} \tag{14}$$

By Assumption 2 we have $0 \leq w'(x) \leq 1$. Moreover, as $\widetilde{w}(x)$ satisfies (10) and as (2) is binding under $w(x)$, $w'(x) < 1$ holds strictly over a positive measure of values $x > \widetilde{s}$. Likewise, we must either have $w(0) > J$ or $w'(x) > 0$ must hold strictly over a positive measure of values $x < \widetilde{s}$. Note next that Assumption 1 implies for all $\underline{\theta} < \theta < \widetilde{\theta}$ that $\frac{1-G_\theta(x)}{1-G_{\widetilde{\theta}}(x)}$ is strictly decreasing in x.[26] Note also that this implies from $0 < \widetilde{s} < \overline{x}$ that $\frac{1-G_\theta(\widetilde{s})}{1-G_{\widetilde{\theta}}(\widetilde{s})} < 1$. We thus obtain from (14)

$$\begin{aligned} & E[\widetilde{w}(x) - w(x) \mid \theta] \\ < \; & \frac{1 - G_\theta(\widetilde{s})}{1 - G_{\widetilde{\theta}}(\widetilde{s})} \left[\int_{\widetilde{s}}^{\overline{x}} [1 - w'(x)] \left[1 - G_{\widetilde{\theta}}(x)\right] dx - \int_0^{\widetilde{s}} w'(x) \left[1 - G_{\widetilde{\theta}}(x)\right] dx + J - w(0) \right], \end{aligned}$$

which after substitution of (13) transforms into

$$E[\widetilde{w}(x) - w(x) \mid \theta] < \frac{1 - G_\theta(\widetilde{s})}{1 - G_{\widetilde{\theta}}(\widetilde{s})} E[\widetilde{w}(x) - w(x) \mid \widetilde{\theta}] = 0. \tag{15}$$

This implies for all $\theta < \widetilde{\theta}$ that the owner's payoff from continuation is strictly higher under the new compensation scheme, i.e., that $E[x - \widetilde{w} \mid \theta]$ strictly exceeds $E[x - w(x) \mid \theta]$. Recall now that the original cutoff satisfies $\theta_P(w) > \theta_{FB}$ and that, by Lemma 1, it is determined by the indifference condition $E[x - w(x) \mid \theta_P(w)] = L$. This together with the fact that $E[\widetilde{w}(x) \mid \theta]$ is strictly increasing in θ then implies $\theta_P(\widetilde{w}) < \theta_P(w)$. Q.E.D.

Recall now from (10) that $\widetilde{w}(x)$ satisfies (2) with equality if we apply the cutoff from the original compensation scheme, $\theta_P(w)$. Using $\theta_P(\widetilde{w}) < \theta_P(w)$ from Claim 1, inspection of (2)

[26] In fact, Assumption 1 is stronger as it also requires that the distribution function is everywhere absolutely continuous.

reveals that the constraint still holds under the new compensation scheme if we apply the true threshold, $\theta_P(\widetilde{w})$. Hence, to prove that the original compensation scheme was not optimal it only remains to show that the owner is strictly better off under $\widetilde{w}(x)$. This follows immediately from the observation that he would—by construction of $\widetilde{w}(x)$—realize the same expected payoff if he still applied the old cutoff $\theta_P(w)$, while by Claim 1 he strictly prefers a different cutoff under the new compensation scheme.

We have thus shown that an optimal compensation scheme must satisfy $w(x) = J + \max\{0, s - x\}$. Establishing that there is a unique optimal choice of s is straightforward. As the employee's payoff is continuous in s, which follows as θ_P changes continuously in s, there exists a compact set of $s-$values for which (2) binds. As the owner's expected payoff is strictly increasing in s, the largest value in this set uniquely defines the unique optimal compensation scheme. Q.E.D.

Proof of Proposition 2. We now extend the argument of Proposition 1 to the case where $S > 0$. As the cutoff now depends on both $w(x)$ and S we denote it by $\theta_P(S, w)$. (Where this is without ambiguity, we will, however, again abbreviate the cutoff by writing θ_P.) We argue now to a contradiction and assume optimality of a compensation scheme $(S, w(x))$ where $w(x)$ is not given by $w(x) = J + \min\{0, x - s\}$. Note first that the proof of Lemma 2 did not rely on the choice $S = 0$, implying that the constraint (2) must bind irrespective of the choice of S. We now construct a new compensation scheme $(S, \widetilde{w}(x))$ where $\widetilde{w}(x) = J + \min\{0, x - \widetilde{s}\}$ and where

$$\int_{\theta_P(S,w)}^{\overline{\theta}} \left[\left(\int_{\widetilde{s}}^{\overline{x}} (x - \widetilde{s}) g_\theta(x) dx \right) - S \right] f(\theta) d\theta = \Delta. \tag{16}$$

Again, existence of a unique such value $\widetilde{s}$ is immediate. Next, the arguments of Claim 1 in Proposition 1 do not depend on the choice $S = 0$ and, therefore, extend immediately to $S > 0$. We thus have that $\theta_P(S, \widetilde{w}) < \theta_P(S, w)$.

As in the proof of Proposition 1, we are done if $\widetilde{w}(x)$ satisfies (2), once we apply the true cutoff $\theta_P(\widetilde{w})$. We distinguish now between two cases. Assume first that $\theta_P(S, \widetilde{w}) \geq \theta_{FB}$. Note next that, by definition of the optimal cutoff, we have $E[x - \widetilde{w}(x) \mid \theta_P(S, \widetilde{w})] = L - S$, while by $\theta_P(S, \widetilde{w}) \geq \theta_{FB}$ we have $E[x \mid \theta_P(S, \widetilde{w})] \geq L + J$, which together imply $E[\widetilde{w}(x) \mid \theta_P(S, \widetilde{w})] \geq S + J$. By Assumption 1 we then have for all $\theta_P(S, \widetilde{w}) \leq \theta \leq \theta_P(S, w)$, i.e., over the whole new set of states for which the owner changes his decision, that $E[\widetilde{w}(x) \mid \theta] \geq S + J$, which completes the proof.

Take now the other case where $\theta_P(S,\widetilde{w}) < \theta_{FB}$. In this case we have to add an intermediate step. We construct another compensation scheme $(\widehat{S},\widehat{w}(x))$ where $\widehat{S} < S$ and where $\widehat{w}(x) = J + \min\{0, x - \widehat{s}\}$ with $\widehat{s} > \widetilde{s}$. Hence, under the new compensation scheme the employee gets less both when the firm is continued and when the owner decides to exit. We also choose $(\widehat{S},\widehat{w}(x))$ such that (i) the true cutoff satisfies $\theta_P(\widehat{S},\widehat{w}) = \theta_{FB}$ and that (ii) under the original threshold, $\theta_P(S,w)$, the constraint (2) still holds with equality, i.e.,

$$\int_{\theta_P(S,w)}^{\overline{\theta}} \left[\left(\int_{\widehat{s}}^{\overline{x}} (x-\widehat{s}) g_\theta(x) dx \right) - \widehat{S} \right] f(\theta) d\theta = \Delta. \tag{17}$$

We establish first existence of a compensation scheme $(\widehat{S},\widehat{w}(x))$ with these characteristics.

Claim 1. *In case $\theta_P(S,\widetilde{w}) < \theta_{FB}$, we can find a compensation scheme satisfying $(\widehat{S},\widehat{w}(x))$, where $\widehat{S} < S$, $\widehat{w}(x) = J + \min\{0, x - \widehat{s}\}$ with $\widehat{s} > \widetilde{s}$, $\theta_P(\widehat{S},\widehat{w}) = \theta_{FB}$, and the requirement (17).*

Proof. We first argue that a compensation scheme with these characteristics would satisfy $\theta_P(\widehat{S},\widehat{w}) > \theta_P(S,\widetilde{w})$. To see this, note that by (16) and (17) there exists at least one state $\theta_P(S,w) < \widetilde{\theta} < \overline{\theta}$ such that

$$E[\widetilde{w}(x) - S \mid \widetilde{\theta}] = E[\widehat{w}(x) - \widehat{S} \mid \widetilde{\theta}]. \tag{18}$$

Using again the transformation (11), we have next that

$$E[(\widetilde{w}(x) - S) - (\widehat{w}(x) - \widehat{S}) \mid \theta] = \int_{\widetilde{s}}^{\widehat{s}} [1 - G_\theta(x)]\, dx - (\widehat{S} - S),$$

which by Assumption 1 is strictly increasing in θ. (Precisely, this follows from strict First-Order Stochastic Dominance, which is implied by Assumption 1.) Together with (18) this implies $E[\widetilde{w}(x) - S \mid \theta] < E[\widehat{w}(x) - \widehat{S} \mid \theta]$ for all $\theta < \widetilde{\theta}$. Using $\theta_P(S,w) < \widetilde{\theta}$ and $\theta_P(S,\widetilde{w}) < \theta_P(S,w)$, this holds also at $\theta = \theta_P(S,\widetilde{w})$. The assertion that $\theta_P(\widehat{S},\widehat{w}) > \theta_P(S,\widetilde{w})$ follows then immediately from the definition of the cutoff θ_P.

We can now proceed by reducing $\widehat{S}$ and increasing $\widehat{s}$ until $\theta_P(\widehat{S},\widehat{w})$ becomes indeed equal to θ_{FB}. This is feasible as we already know that θ_P is continuous in both $\widehat{S}$ and $\widehat{s}$, while at $\widehat{S} = 0$ it must hold that $\theta_P(\widehat{S},\widehat{w}) > \theta_{FB}$. (Note that $\widehat{s} < \overline{x}$ is needed to satisfy (17).) Q.E.D.

As $\theta_P(\widehat{S}, \widehat{w}) = \theta_{FB}$, we already know from the argument for the case with $\theta_P(S, \widetilde{w}) \leq \theta_{FB}$ that $(\widehat{S}, \widehat{w}(x))$ satisfies (2) also if we apply the true threshold $\theta_P(\widehat{S}, \widehat{w})$. That the owner is strictly better off follows finally again from (17) and as the owner's new optimal cutoff is strictly different from $\theta_P(S, w)$. Q.E.D.

We have thus established that any optimal compensation scheme $w(x)$ must satisfy $w(x) = J + \min\{0, x - s\}$. The fact that $\theta_P > \theta_{FB}$ follows from the argument in the main text. Finally, for a given choice of S we have again a unique corresponding choice of s. Q.E.D.

Proof of Proposition 3. Recall first from Propositions 1-2 that $w(x) = J + \min\{0, x - s\}$. Moreover, for a given choice of $S \geq 0$ the value of s is also uniquely pinned down. It is now convenient to assume that there is also a uniquely optimal level of $S \geq 0$, though the proof can be extended at the cost of adding additional notation. For brevity we refer to the unique optimal (commitment) compensation scheme just as $(S, w(x))$. The following result is now intuitive from the insights of Proposition 1.

Claim 1. *Given some other compensation scheme $\widetilde{w}(x)$ satisfying $E[\widetilde{w}(x) \mid \widehat{\theta}] \leq E[w(x) \mid \widehat{\theta}]$ for some $\widehat{\theta} < \overline{\theta}$, it holds that $E[\widetilde{w}(x) \mid \theta] < E[w(x) \mid \theta]$ for all $\theta > \widehat{\theta}$.*

Proof. We argue to a contradiction and assume that $E[\widetilde{w}(x) \mid \widehat{\theta}] \leq E[w(x) \mid \widehat{\theta}]$ and that $E[\widetilde{w}(x) \mid \theta] \geq E[w(x) \mid \theta]$ for some $\theta > \widehat{\theta}$. Using continuity of $E[\widetilde{w}(x) \mid \theta]$ and $E[w(x) \mid \theta]$ this implies existence of some $\widehat{\theta} \leq \widetilde{\theta} < \widehat{\theta}$ such that $E[\widetilde{w}(x) \mid \widetilde{\theta}] = E[w(x) \mid \widetilde{\theta}]$. We can now fully apply the argument in Claim 1 of Proposition 1 to show that, by construction of $w(x)$ and Assumptions 1-2, this must imply $E[\widetilde{w}(x) \mid \theta] < E[w(x) \mid \theta]$, which yields a contradiction. Precisely, we obtain

$$E[w(x) - \widetilde{w}(x) \mid \theta] > \frac{1 - G_\theta(s)}{1 - G_{\widetilde{\theta}}(s)} E[w(x) - \widetilde{w}(x) \mid \widetilde{\theta}] = 0.$$

Q.E.D.

Consider now first the game where the employee makes some offer $(\widetilde{S}, \widetilde{w}(x))$. By optimality, the owner will then only have to pay $\min\{S, \widetilde{S}\}$ if he decides to exit, while for a given θ he will only have to pay the expected compensation $\min\{E[w(x) \mid \theta], E[\widetilde{w}(x) \mid \theta]\}$ if he decides to continue. It is thus immediate that offering $\widetilde{S} < S$ is not optimal for the employee. Moreover, in case $\widetilde{S} \geq S$ the new severance pay offer is clearly irrelevant as the owner will reject it if he prefers to exit. The new offer is also only profitable for the employee if it increases the set of states for which the firm is continued, i.e., if $\theta_P(\widetilde{S}, \widetilde{w}) < \theta_P(S, w)$. As this implies at $\theta_P(\widetilde{S}, \widetilde{w})$ that $E[\widetilde{w}(x) \mid \theta_P(\widetilde{S}, \widetilde{w})] > E[w(x) \mid \theta_P(\widetilde{S}, \widetilde{w})]$, we have by Claim 1 that $E[\widetilde{w}(x) \mid \theta] > E[w(x) \mid \theta]$ holds for all θ where the owner chooses continuation. As a consequence, offering a new compensation scheme $\widetilde{w}(x)$ that reduces the cutoff is only beneficial for the employee if condition (9) from the main text holds. As we argued in the main text, this is not possible.

Suppose next the owner offers a new compensation scheme. As he knows the true state θ, we have a game of signaling. By specifying optimistic out-of-equilibrium beliefs that put a lot of probability mass on high states θ, it is straightforward to support an equilibrium where no acceptable new offer is made. We show next that there are no successful renegotiations in any (perfect Bayesian) equilibrium. Though the argument holds generally, for brevity's sake we restrict attention to equilibria where the employee accepts a new offer in case he is indifferent, given his beliefs about the proposing types. (Note that if the employee rejects the new offer, the old compensation scheme remains in place.)

We argue again to a contradiction and suppose that, in a given equilibrium, there is a non-empty set of accepted new compensation schemes, which we denote by Ω. By previous arguments it is straightforward that we can restrict consideration to changes in the wage paid in case of continuation: $\Omega = \{w_i(x)\}_{i \in I}$, where I is some index set. Denote $\widetilde{\Omega} = \Omega \cup \{w(x)\}$ and denote the new cutoff, given the compensation schemes in $\widetilde{\Omega}$, by $\theta_P(\widetilde{\Omega})$.[27] We denote one of the compensation schemes that the owner prefers at $\theta = \theta_P(\widetilde{\Omega})$ by $\widetilde{w}(x)$. From our previous arguments we know that we can restrict consideration to the case where $\theta_P(\widetilde{\Omega}) < \theta_P(w)$. Moreover, by Claim 1 we know that for all states $\theta > \theta_P(\widetilde{\Omega})$ the owner strictly prefers to offer a new compensation scheme from Ω. In all states $\theta > \theta_P(\widetilde{\Omega})$ the owner will also offer the most preferred compensation scheme in Ω, which is the least preferred choice for the employee. Consequently, an upper boundary for the employee's payoff in the renegotiation game is given by the case where *only* the compensation scheme $\widetilde{w}(x)$ is offered. But we already know for this case that the employee would be strictly worse off when accepting the offer—a contradiction. Q.E.D.

[27] Formally, the existence of such a cutoff follows as $\max_{i \in I'} E[w_i(x) \mid \theta]$, where we write $\widetilde{\Omega} = \{w_i(x)\}_{i \in I'}$, is by Assumption 2 nondecreasing and continuous in θ.

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