Interest Rate Option Markets: The Role of Liquidity in Volatility Smiles*

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ABSTRACT

We investigate the interaction of volatility smiles and liquidity in the euro (€) interest rate option markets, using daily bid and ask prices of interest rate caps/floors. We find that liquidity variables have significant explanatory power for both curvature and asymmetry of the implied volatility smiles. This effect is generally stronger on the ask side, indicating that ask-prices are more relevant for these markets. In addition, the shape of the implied volatility smile has some information about future levels and volatility of the term structure. Our results have important implications for the modeling and risk management of fixed income derivatives.

JEL Classification: G10, G12, G13, G15

Keywords: Volatility smiles; liquidity; interest rate options; euro interest rate markets; Euribor market.

Over-the-counter interest rate options such as caps/floors and swaptions are among the most liquid options that trade in the global financial markets, with about $20 trillion of notional principal outstanding, as of December 2003.[1] Given the large size of these markets, significant effort has been devoted, both in academia and in industry, to the development and testing of valuation models to accurately price and hedge these claims.[2] However, most of the models developed and, consequently, the empirical tests of these models, have assumed a competitive, frictionless framework, where there are no liquidity costs. The problems in assuming away liquidity costs are exacerbated for options that are away-from-the-money, especially if there are volatility smiles/skews across strike rates in interest rate option markets.[3] It is likely that differential liquidity effects across strike rates confound the commonly observed volatility smile/skew, at least partly. Specifically, it is well known that at-the-money options are typically much more liquid than those that are away-from-the-money. Therefore, it is possible that at least part of the volatility smile pattern can be ascribed to differential liquidity effects across strike rates. The question is how much. A further question is whether changes in liquidity affect the smile. By the same token, a related issue is whether the smile has a feedback effect on the liquidity of these options. To our knowledge, there is no study that has examined these, or other related issues in the interest rate option markets.

Although there has been some work in the existing literature that examines some of these issues, virtually all the analysis and evidence so far has been in the context of equity options.[4] The conclusions from equity option markets cannot be extended to interest rate option markets since these markets differ significantly from each other for several reasons. In contrast to equity options, interest rate option markets are almost entirely institutional, with hardly any retail presence. Most interest rate options, particularly the long-dated ones such as caps, floors and swaptions, are sold over-the-counter (OTC) by large market makers, typically international banks. The customers are usually on one side of the market (the ask-side), and the size of individual trades is relatively large. Many popular interest rate option products, such as caps, floors and collars are portfolios of options, from relatively short-dated to extremely long-dated ones. These features lead to significant issues relating to supply/demand and asymmetric information about the order flow, potentially resulting in larger bid-ask spreads than those for exchange traded equity options. Since interest rate options are traded in an OTC market, there are also important credit risk issues that may influence the pricing of these options, especially during periods of crisis. Therefore, inferences drawn from studies in the equity option markets

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[1] BIS Quarterly Review, September 2004, Bank for International Settlements, Basel, Switzerland.

[2] These include the studies by Driessen, Klaasen and Melenberg (2003), Fan, Gupta and Ritchken (2003), Gupta and Subrahmanyam (2004), Longstaff, Santa-Clara and Schwartz (2001), Peterson, Stapleton and Subrahmanyam (2003), Jarrow, Li and Zhao (2003), and others.

[3] Using the Black model, the volatilities implied from option prices are different across different strike rates. In particular, the pattern is typically U-shaped, and skewed to one side; it is, therefore, referred to as the volatility smile or skew.

[4] See Ederington and Guan (2002), Mayhew (2002), and Pena, Rubio and Serna (1999, 2001), and Bollen and Whaley (2004), for example.

are not directly relevant for interest rate option markets, although there may be some broad similarities. Unlike exchange traded option markets, in the OTC interest rate option markets, the only metric of liquidity available is the bid-ask spread – there are no volume, depth, or open interest data available. Therefore, in spite of its potential shortcomings, we are constrained to rely on the bid-ask spread alone for all our liquidity analyses, which still lead to interesting findings.

In the interest rate option markets, an important question that is unanswered, as yet, relates to the economic determinants of volatility smiles, and their relationship to liquidity factors. In this paper, we investigate the determinants of these volatility smiles in the cap/floor markets, specifically including transaction costs, for which the bid-ask spread may serve as a proxy, as one of the explanatory variables. The bid-ask spreads in these markets may be indicators of the liquidity risk premium built into option prices. This liquidity risk premium may itself be dependent on various factors such as the depth-in-the-money or "moneyness," market volatility, lagged trends in interest rates, the slope of the term structure, supply-demand factors and asymmetric information about the order flow. Thus, we examine the interaction of liquidity effects and other economic variables with the implied volatility smiles in the euro interest rate option market.

We contribute to the literature in three distinct ways. First, we present an extensive documentation of the volatility smile patterns in the interest rate option markets for different maturities, in the presence of bid-ask spreads. Second, we explore the determinants of volatility smiles in these markets, in terms of macro-economic and liquidity variables. We control for the level of volatility and the level and the slope of the yield curve in an effort to separate the effect of liquidity from the effects arising out of an alternate model for the interest rate process. Typically, in models such as those based on stochastic volatility, which are able to generate some volatility smile/skew patterns across strike prices or rates, the current level of volatility and interest rates are determinants of the future distribution of the interest rates. Third, we examine the bidirectional Granger-causality relationships between volatility smiles and the various liquidity and economic variables, to understand whether any of these variables have power in predicting smile patterns, or whether the volatility smiles have any information about the future values of these variables.

We find that there are clearly perceptible volatility smiles in caps and floors, across all maturities. The pattern of these volatility smiles, however, varies depending on the maturity of the option. Short-term caps and floors exhibit smiles that are significantly steeper than those for longer-term caps and floors. Long-term floors display more of a “smirk” than a smile. A principal components analysis of changes in the cap and floor volatility surface indicates that more than one dominant factor explains the variations in this surface. The number of primary principal components is different on the ask side (four) from the bid side (two), suggesting that there may be significant differences in the information content of the volatility smile curve on the askversus the bid-side. We estimate parametric functional forms for the volatility smiles for caps and floors separately, as well as for caps and floors pooled together, and find that they display significant curvature, as well as an asymmetry in the slope of the smile. When we analyze these smile patterns separately for the ask-side and the bid-side for these options, we find that the smiles are much steeper on the ask-side than on the bid-side.

We also find that liquidity variables, as proxied by bid-ask spreads, affect the implied volatility smile patterns in this market. Interestingly, these results are somewhat stronger on the ask side. indicating that the ask-side is adjusted more frequently than the bid side in these markets. This is consistent with the reality, since most of the corporate/institutional investor customers for these options are buyers of caps, floors and collars. Our results also show that, in general, the economic variables have more explanatory power on the ask-side than on the bid-side. Thus, there is reason to believe that the ask side has more relevant information in these markets.

The bivariate Granger-causality tests are used to see if lagged values of any of the explanatory variables can predict the curvature and asymmetry of the volatility smile and vice-versa. We find that the curvature of the volatility smile Granger-causes the at-the-money volatility in the interest rate options market and the slope of the term structure at the long end, while there is no feedback effect in the other direction. There is evidence that the asymmetry of the smile for long maturity options predicts the slope of the yield curve at the short end and the long end. Thus, it appears that the shape of the smile has some information about the future levels of uncertainty in the market and the medium and long-term interest rates.

The results of our paper have three major implications for the modeling and risk management of interest rate derivatives, especially options. First, it is important to incorporate liquidity effects in the valuation model in order to explain the volatility smile across strike rates more completely. Second, interest rate option models should be calibrated using option prices across a range of strike rates, not just using ATM options, since, as we show in this paper, the shape of the volatility smile curve has important information about the future term structures and their volatility. Third, using the mid-price (the average of the bid- and the ask-price) of options, especially OTC options, may significantly distort the estimation of the models, since the shape of the volatility smile, and the information contained therein, are significantly different on the ask-side from those on the bid-side. In particular, the prices from the ask-side of the price curve should be given more weight since they seem to contain more information than the bid-side. While it is common practice to calibrate interest rate option models using data across strike rates, it is not often that the ask- and bid-sides are dealt with *separately*. Our research suggests that taking the mid-prices to calibrate models may lead to losing important information about the dynamic evolution of market prices in response to liquidity effects.

The structure of our paper is as follows. Section 1 presents an overview of prior research that has tried to explain the patterns of volatility smiles, mainly in equity option markets. Section 2 describes the data set and presents summary statistics. Section 3 presents the empirical patterns of the volatility smile that we observe in the data. Section 4 relates the patterns of volatility smiles to economic and liquidity variables. Section 5 presents the results of the bivariate Granger-causality tests. Section 6 concludes with a summary of the main results and directions for future research.

1. Related Literature

Early empirical work on the pricing of equity options concluded that the standard Black-Scholes model fitted to the data gives rise to volatility smiles/skews.[5] Several approaches have been proposed for explaining these volatility smile patterns, many of them being *ad hoc* in nature. The formal approaches to modeling this empirical phenomenon rely on specifying alternative dynamics for the underlying asset price process, such that the process exhibits skewness and excess kurtosis in the underlying asset return distributions. These approaches implicitly assume that skewness and excess kurtosis in the underlying asset returns distributions are the only source of volatility smiles/skews in option prices. Excess kurtosis makes extreme observations more likely than in the Black-Scholes case, thereby increasing the value of away-from-the-money options relative to the at-the-money options, creating the smile. The presence of skewness has the effect of accentuating just one side of the smile.

Within the broad array of modeling approaches, three classes of models have been proposed in the area of equity options. The first consists of models where the volatility of the underlying security returns is assumed to evolve deterministically through time.[6] However, Dumas, Fleming and Whaley (1998) show that these models have highly unstable parameters, and cannot explain the shape of the implied volatility functions, even compared to the naïve Black-Scholes model. The second class of models consists of stochastic volatility models that allow the volatility of the return process to evolve randomly over time. In these models, the correlation between the Brownian motions associated with the underlying asset and the volatility affects the skewness of returns, while the "volatility of volatility" determines the kurtosis. However, this modification only provides, at best, a partial explanation of the shape of the implied volatility function.[7] The third class of models includes jump-diffusion models, where the underlying asset return process

[5] See, for example, Macbeth and Merville (1979), Heynen (1993), Duque and Paxson (1993), and Heynen, Kemna and Vorst (1994).
[6] These include the constant elasticity of variance (CEV) models (see, for example, Emanuel and MacBeth (1982)), and the implied tree models of Dupire (1994), Derman and Kani (1994), and Rubinstein (1994), where volatility is a deterministic function of asset price and time.
[7] This is documented by Hull and White (1987), Wiggins (1987), Amin and Ng (1993) and Heston (1993).

is augmented by a Poisson-driven jump process.[8] However, these models, by themselves, are also unable to explain volatility smiles. For instance, Heynen (1993) finds that the observed smile pattern is inconsistent with various stochastic volatility models. Jorion (1988) concludes that jump processes cannot explain the smile, while Bates (1996) concludes the same for stochastic volatility models. Das and Sundaram (1999) find that the implied volatility smiles implied by stochastic volatility models are too shallow and that jump-diffusion models imply a smile only at short maturities. For these reasons, Bakshi, Cao, and Chen (1997) advocate the use of a stochastic volatility model with jumps for valuing S&P 500 index options, with somewhat mixed results. Similarly, Bates (2000) finds that the inclusion of a jump process in stochastic volatility models improves the model's ability to generate implied volatility functions consistent with market prices, but in order to do so, the parameters of the process must be set to unreasonable values. In a similar spirit, Jackwerth (2000) concludes that the risk aversion functions implied by S&P 500 index option prices (across strike rates) are irreconcilable with reasonable preferences for the representative investor.

All these studies show that even these broad classes of models have been unsuccessful in accurately describing the behavior of the observed volatility smiles in the equity options markets – the empirically observed volatility smiles are typically much larger than those predicted by the theory. Clearly, some other economic phenomenon is missing from the models so far, particularly since, with frictions in the market, option-pricing models may not satisfy the martingale restriction (i.e., the price of the underlying asset implied by the option pricing model must equal its actual market value).[9] In such a situation, the no-arbitrage framework can only place bounds on option prices and hence, cannot explain the observed patterns in option prices either across strike rates or across maturities. This discussion points towards liquidity being one of the factors that might cause, or at least influence, the volatility smiles across strike rates. In this context, Constantinides (1997) concurs that, with transaction costs, the concept of the no-arbitrage price of a derivative is replaced by a range of prices, which may differ across strike rates for options. However, he distinguishes between plain-vanilla, exchange-traded derivatives (such as equity options) and customized, over-the-counter derivatives - many interest rate options fall in the latter category. From a theoretical standpoint, he argues that transaction costs are more likely to play an important role in the pricing of the customized, over-the-counter derivatives, as opposed to plain-vanilla exchange-traded contracts, since the seller has to incur higher hedging costs to cover short positions, if they are customized contracts. This issue clearly needs empirical verification and amplification.

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[8] This type of model was proposed by Merton (1976), Ball and Torous (1985), Jarrow and Rosenfeld (1984), Amin (1993) and Bates (1996), where the basic Black and Scholes (1973) model is augmented by a Poisson-driven jump process.

[9] See, for example, Longstaff (1995), Brenner and Eom (1997) and others.

Another way to think about liquidity effects is to focus on the feasibility of arbitrage trades if option prices deviate from their theoretical values. In theory, the demand for an option should not affect its price (or implied volatility), since derivatives are redundant assets. However, in reality, there may be limits to arbitrage. Liu and Longstaff (2004) show that investors may underinvest in a profitable arbitrage opportunity than what may be allowed by collateral constraints, partly due to the possibility of interim losses. In such a situation, potential mispricing could persist or even become wider, and theoretical valuation arguments based on the absence of arbitrage may not always hold.[10] In effect, the no-arbitrage band within which prices can fluctuate can be quite wide, allowing prices to be affected by supply and demand considerations. If the market demand is more on one side – the ask-side – than the other, this can lead to different shapes for the implied volatility functions, and different shapes of this function on the ask-side compared to its shape on the bid-side. This is another way in which the slope and curvature of the implied volatility functions can be affected by the liquidity in the option markets.

Overall, the focus of the literature in equity options has been to document systematic strike price biases in the standard Black-Scholes framework, extend the Black-Scholes framework to incorporate stochastic volatility, jumps, etc., and test whether these extended models can remove the strike price biases. Most studies have confirmed that significant biases remain, even after modeling stochastic volatility and jumps into the underlying asset price process. Based on this motivation, Pena, Rubio and Serna (1999), examine the impact of liquidity factors on the volatility smiles in the Spanish index options market. They find that liquidity variables have a significant impact on the shape of the smile. In another related paper, Bollen and Whaley (2004) examine the effect of a liquidity variable, defined by net buying pressure, on the shape of the implied volatility function, also known as the volatility smile.[11] They define the net buying pressure as the difference between the number of buyer-motivated contracts and seller-motivated contracts, scaled by trading volume. Their results show that net buying pressure influences the shape of the volatility smile, particularly for index options, but these effects are transitory. They also show that these effects yield profitable trading opportunities by dynamically hedging out-of-the-money index options.

In the area of interest rate options, Jarrow, Li and Zhao (2003) find that smiles exist in the US interest rate cap markets, but the models of the term structure cannot capture them. In particular, they report that even the most sophisticated LIBOR market models, with stochastic volatility and jumps, cannot capture the volatility smile in interest rate options. Similarly, Gupta and Subrahmanyam (2004) find that several alternative one-factor and two-factor term structure

[10] These limits to arbitrage may also explain the empirical results in Ofek, Richardson and Whitelaw (2004).

[11] Bollen and Whaley analyze the (very different) market for equity options, both on the S&P 500 index and certain individual stocks. In addition, since their data are for exchange-traded options, they are able to use data on trading volume.

models (spot rate, forward rate, as well as market models) are unable to capture the volatility smile in caps and floors.

It is important to note that so far, there has been no work on exploring the impact of liquidity factors on interest rate option prices, and on volatility smiles. The evidence in the equity option markets suggests that liquidity effects are likely to be stronger for interest rate options due to the over-the-counter nature of the markets and the asymmetry in the demand for these options. The evidence in the interest rate option pricing literature also supports this motivation since just modeling the term structure of interest rates does not seem to capture the smiles. Ultimately, it would be important to include liquidity effects in the modeling of the term structure of interest rates and the pricing and hedging of interest rate derivatives. However, this effort must await the establishment of stylized facts based on detailed empirical research. Our work can, therefore, be seen in this context. In particular, we begin by documenting volatility smile effects in the Euro interest rate option markets, separately for the ask-side and the bid-side for different maturities. Then, we concentrate on analyzing the influence of liquidity and other economic factors on the shape of the smile. We use the Black model to obtain the implied volatilities from the option prices. We then control for the level of volatility and interest rates in our subsequent analysis. Thus, we can control for the effects arising out of a model of interest rates with skewness and excess kurtosis without tying ourselves to any particular model. Our evidence suggests that in the interest rate option markets, there are liquidity effects in the volatility smiles that are *separate* from the effects that could be broadly attributed to an alternate stochastic process for interest rates.

2. Data

The data for this study consist of an extensive collection of euro (€) cap and floor prices over the 29-month period, January 1999 to May 2001, obtained from WestLB (Westdeutsche Landesbank Girozentrale) Global Derivatives and Fixed Income Group. These are daily bid and offer quotes for nine maturities (2 years to 10 years, in annual increments) across twelve different strike rates ranging from 2% to 8% (prices are not available for all of the maturity-strike combinations each day.)[12] These caps and floors are portfolios of European interest rate options on the 6-month Euribor with a 6 monthly reset frequency.[13] Therefore, this dataset allows us to examine strike price biases as well as liquidity effects in caps and floors. Along with the options data, we also collected data on € swap rates and the daily term structure of euro interest rates curve from the

[12] It is important to note that these quotes may only be indicative in the sense that the market-maker is not *obliged* to transact at these prices at any significant volume. However, there is no bias in the quotes from an econometric perspective. Also, this being an OTC market, it is very hard to obtain transaction prices and volume. In other words, the quotes are the best indicator of liquidity in the market, but may be observed with an error. In what follows, we adjust for this potential error-in-variables problem.

[13] In Appendix A.1, we provide details of the contract structure for these options as well as the Black (1976) model for converting cap and floor prices into implied volatilities.

same source. These are key inputs necessary for checking cap-floor parity, as well as for conducting the subsequent empirical tests.

Table 1 provides descriptive statistics on the midpoint of the bid and ask prices for caps and floors over our sample period. The prices of these options can be almost three orders of magnitude apart, depending on the strike rate and maturity of the option. For example, a deep out-of-the-money two-year cap may have a market price of just a few basis points, while a deep in-the-money ten-year cap may be priced above 1500 basis points. Since interest rates have varied substantially during our sample period, the data have to be reclassified in terms of moneyness ("depth in-the-money") to be meaningfully compared over time. In Table 1, the prices of options are grouped together into "moneyness buckets," by estimating the Log Moneyness Ratio (LMR) for each cap/floor. The LMR is defined as the logarithm of the ratio of the par swap rate (for a non-standard swap as discussed in Appendix A.2) to the strike rate of the option. Since the relevant swap rate changes every day, the moneyness of the same strike rate, same maturity, option, measured by the LMR, also changes each day. The average price, as well as the standard deviation of these prices, in basis points, is reported in the table. It is clear from the table that cap/floor prices display a fair amount of variability over time. Since these prices are grouped together by moneyness, a large part of this variability in prices over time can be attributed to changes in volatilities over time, since term structure effects are largely taken into account by our adjustment.

We also document the magnitude and behavior of the liquidity costs in these markets over time, for caps and floors across strike rates. We use the bid-ask spreads for the caps and floors as a proxy for the illiquidity of the market. It is important to note that these are measures of the liquidity costs in the interest rate options market and not in the underlying market for swaps. Although the liquidity costs in the two markets may be related, the bid-ask spreads for caps and floors directly capture the effect of various frictions in the interest rate options market, in addition to the transaction costs in the underlying market, as well as the imperfections in hedging between the option market and the underlying swap market. Thus, the liquidity costs in the option market are more relevant for this study since they are a comprehensive measure of all the costs that matter.[14] In Table 2, we present the bid-ask spreads scaled by the average of the bid and ask price of the option, grouped together into moneyness buckets by the LMR. Close-to-the-money caps and floors have proportional bid-ask spreads of about 8% - 9%, except for some of the shorter-term caps and floors that have higher bid-ask spreads. Since deep in-the-money

[14] The objective of this paper is to analyze liquidity effects directly in the interest rate options market, not in the market for underlying swaps or bonds. The liquidity in the underlying swap markets may have its own effect on the options market. However, the underlying "assets" for all the interest rate options analyzed in this paper are the same (since they are all options on the same yield curve), unlike equity options, where different options have different underlying assets. Therefore, the effects of liquidity in the underlying swap markets are likely to be more homogeneous across all interest rate options analyzed in this paper; hence they are less likely to affect the cross-sectional comparisons that we make in this paper.

options (low strike rate caps and high strike rate floors) have higher prices, they have lower proportional bid-ask spreads (3% - 4%). Some of the deep out-of-the-money options have large proportional bid-ask spreads - for example, the two year deep out-of-the-money caps, with an average price of just a couple of basis points, have bid-ask spreads almost as large as the price itself, on average about 80.9% of the price. Part of the reason for this behavior of bid-ask spreads is that some of the costs of the market makers (transactions costs on hedges, administrative costs of trading, etc.) are absolute costs that must be incurred whatever may be the value of the option sold. However, some of the other costs of the market maker (inventory holding costs, hedging costs, etc.) would be dependent on the value of the option bought or sold. It is also important to note that, in general, these bid-ask spreads, are much larger than those for most exchange traded options.

Next, we examine the no-arbitrage relation between caps and floors. Details of the put-call parity relationship between caps and floors are provided in Appendix A.2. If put-call parity holds due to arbitrage restrictions, the implied volatility of a cap and a floor at the same strike rate and maturity must be identical. However, the implied volatilities of caps and floors at different strike rates or maturities need not be the same. Indeed, the pattern of these volatilities across strike rates is the subject of investigation in this paper.

We examine the efficiency of the euro interest rate cap/floor market in exploiting profit opportunities, using the bid and ask prices for caps and floors and the values of the relevant swaps calculated using the method explained in Appendix A.2. We calculate the difference on the left hand side of inequalities (A.5) and (A.6), average them, and check if the average difference is significantly less than 0. The tests are conducted in a manner such that the actual feasibility of implementing the arbitrage, in case of any violations, can be examined. If the net price of these portfolios is negative, it is potentially a violation of put-call parity in these options, since it implies that the cost of setting up this completely hedged portfolio is negative. The results are presented in Table 3. In 3 out of 18 cases, the mean difference is negative and significant. Also, if we look at the last column, it can be seen that, in several cases, the difference is less than 0.

However, in these calculations, we have not accounted for the bid-ask spreads for the off-market swaps used in the tests since this information is not available. Typically, the bid-ask spread for "plain vanilla" market (par) Euro swaps is about 4-5 basis points. As pointed out above, the swaps that would be needed for these arbitrage transactions would, in general, be off-market, non-standard swaps (not par swaps), which are likely to have bid-ask spreads significantly greater than 4-5 basis points. From Table 3, it can be seen that, the maximum average violation is about 5 basis points. Therefore, from a practical perspective, many of the potential violations (defined as cases where the net cost of these portfolios is negative) are not arbitrageable, and are unlikely to be true violations.[15] Without precise information on the bid-ask spreads for off-market

[15] Furthermore, since these are the quotes posted by a particular dealer, it is unlikely that the dealer would

swaps, it is not possible to determine how many of these potential violations are actually arbitrageable.

Since the average difference is well within potential bid-ask spreads for off-market non-standard swaps, we argue that the cap floor parity holds *on average*; hence, pooling the data for caps and floors is appropriate for further analysis. Pooling the data from caps and floors allows us to obtain data for a wider range of strike rates, covering rates that are both in-the-money and out-of-the-money for both caps and floors. The parity computations are also a consistency check, which assures us about the integrity of the dataset.[16]

Even if the no-arbitrage relation holds on average, the fact that it does not hold for several individual observations indicates that these prices might be measured with error. In our subsequent analysis, the implied volatilities calculated from the prices of these options are the dependent variables. We are not concerned about the errors-in-variables problem in the dependent variables, since they do not bias the coefficients in the regressions. However, we also use the bid-ask spread scaled by mid-price as one of the explanatory variables. Thus, the scaled bid-ask spreads are contaminated by the errors-in-variable problem. We observe in the data that the bid and ask prices of the options move together. So there is strong reason to believe that even though individual bid and ask prices are measured with error, the unscaled bid-ask spread is not affected. However, dividing by the mid-price, which may be measured with error, introduces a potential error in the scaled bid-ask spreads. To account for this problem, we use the instrumental variable approach, where unscaled bid-ask spreads are used as instruments for the scaled bid-ask spreads.

3. Volatility Smiles in Interest Rate Option Markets

We use implied volatilities from the Black-BGM model throughout the analysis from here on. Although there may be a complex model that explains at least part of the smile/skew or the term structure of volatility, it is necessary to first obtain a sense of the empirical regularities using the standard model. In other words, we need to first document the characteristics of the smile before attempting to model it formally.[17] Furthermore, the evidence in the equity option markets suggests that even such complex models may not explain the volatility smile adequately, without considering other effects such as liquidity.

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allow a potential customer to create an arbitrage position based on his quotes. In our discussions with dealers in this market, we were told that these quotes are likely to reflect the best information available with the dealer at the time of posting these quotes, and any error in prices is likely to be unbiased.

[16] Since caps and floors are traded in the market and not caplets and floorlets, it makes sense to conduct the arbitrage test for caps and floors. However, we also conducted the tests for individual caplets and floorlets, whose prices were calculated by bootstrapping. The results were similar.

[17] This is in line with the approach of Bollen and Whaley (2004), who also use the implied volatilities from the Black-Scholes model.

We document volatility smiles/skews in euro interest rate caps and floors across a range of maturities over the sample period. Figure 1 presents scatter plots of the implied flat volatilities of caps and floors over our sample period. The vertical axis in the plots corresponds to the implied volatility of the mid-price (average of bid and ask price) of the option, scaled by the at-the-money volatility for the option of the same maturity (Scaled IV). We divide by the implied volatility of at-the-money option to account for the effect of changes in the level of implied volatilities over time. The horizontal axis in the plots corresponds to the *log moneyness ratio* (LMR), our measure of the moneyness of the option.

We first examine the overall shape of the implied volatility smile. The plots are presented for three representative maturities - 2-year, 5-year, and 10-year, for the pooled cap and floor data.[18] These plots clearly show that there is a significant smile curve in interest rate options in this market, across strike rates. The smile curve is steeper for shorter-term options, while for longer-term options, it is flatter and not symmetric around the at-the-money strike rate.

In addition, we analyze the principal components of the changes in the Black volatility surface (across strike rates and maturities) for caps and floors. If there is information in away-from-the-money option prices that is different from that in ATM option prices, then we should not observe a very high proportion of the variation in these implied volatilities being explained by just one principal component. Indeed, we find that for caps, on the ask-side, there are four significant principal components that together explain 91.7% of the daily variation in the volatility surface (the first four components explaining 31.7%, 29.2%, 17.1%, and 13.9% respectively). However, on the bid-side, we find only two significant principal components that explain 80.5% of the daily variation – the first four principal components together explain 89.9% of the daily variation in the volatility surface (42.8%, 37.7%, 5.1%, and 4.3% respectively). The structure of the principal components is similar for floors. In addition, when we analyze the principal components using weekly changes in implied volatilities, we find an even stronger effect in terms of four primary components on the ask-side but only two primary components on the bid-side. *Prima facie*, this indicates that there are four types of primary shocks that affect the volatility surface on the ask-side, while only two types of shocks appear to impact the bid-side. This empirical finding may be related to the institutional nature of the market, where most of the customers of caps/floors are on the ask-side of the market.[19] Therefore, potentially, there may be more information in the shape of the volatility smile curve on the ask-side than on the bid-side.

Next, we estimate various functional forms for volatility smiles using pooled time-series cross-sectional regressions, in order to understand the overall form of the volatility smile over our

[18] The separate plots for caps and floors, and the plots for other maturities, show similar characteristics; hence, they have not been presented in the paper.

[19] Our conversations with dealers in this market confirmed that the demand is usually on one side – while corporate and institutional investors are buyers of these options, banks are on the other side as sellers of these options. Hedge funds step in when they perceive the bid-ask spreads to be too wide.

entire sample period.[20] The most common functional forms for the volatility smile are quadratic functions of either moneyness or the logarithm of moneyness. The scatter plots in Figure 1 also support a quadratic form. In order to account for the asymmetry, if any, in the smile curve, we allow the slope to differ for in-the-money and out-of-the-money options. We also estimate the linear and quadratic functional forms without the asymmetry term. In addition, we present the volatility smiles on the bid-side and the ask-side separately. Using the mid-point of the bid-ask prices may not accurately display the true smile in the implied volatility functions, given that bid-ask spreads differ across strike rates. We avoid this pitfall and separately report the smiles for the bid-side and the ask-side.

The specific models that we estimate, using ordinary least squares estimation in the pooled time series and cross sectional regression, are as follows:

$$Scaled\ IV = c1 + c2 * LMR \tag{1}$$

$$Scaled\ IV = c1 + c2 * LMR + c3 * LMR^2 \tag{2}$$

$$Scaled\ IV = c1 + c2 * LMR + c3 * LMR^2 + c4 * 1_{LMR<0} * LMR \tag{3}$$

In Table 4, we report the results for caps and floors separately only for (3), the quadratic functional form with the asymmetric slope term, since it fits the observed volatility smiles the best.[21] The regression coefficients in all the specifications are highly significant. In addition, the quadratic functional form with asymmetric slope term explains a fairly high proportion of the variability in the scaled implied volatilities. For example, for 5 year caps, this specification explains about 96% of the variability in the scaled IV on the ask-side and 67% on the bid-side. In most specifications, the asymmetry term for the slope of the smile is significant, indicating that the shape of the volatility function is different for in-the-money options, as compared to that for out-of-the-money options.[22] In addition, for most maturities, the explanatory power of the model is greater for caps than for floors, and within caps, it is greater on the ask side than on the bid side. This relates to the nature of the institutional market for these options – since most dealers are net sellers of these options, the ask side of the price curve is likely to be more sensitive to the moneyness of the option than the bid side. Also, there is typically more demand from customers

[20] Unfortunately, we do not have enough observations each day to estimate these relationships cross-sectionally, since caps and floors tend to be traded at strike rates that are either ATM or out-of-the-money. Hence, the observations for each contract – caps and floors - tend to be for strike rates that are on one side of the ATM strike rate.

[21] We also tested a specification with an asymmetric term for the curvature of the smile, but it did not add any significant explanatory power over the specification with the asymmetric term for just the slope of the smile. We got similar results when we tested a polynomial specification with higher order terms, which turned out to be statistically unimportant.

[22] We also conducted the same exercise with spot volatilities i.e. using inferred prices of caplets and floorlets, obtained after "bootstrapping" the data as described in Appendix A.1. Model (3) fits well there also. Those results are not presented here to save space.

for caps rather than floors, since most of them tend to hedge floating rate borrowing, rather than lending.

Figure 2 presents the plots of these fitted functions for caps and floors for different maturities. These plots clearly show the shape of the smile curve for these specific options. These implied volatility smiles display some interesting patterns. Caps always display a smile, for all maturities, although the smile flattens as the maturity of the cap increases. In-the-money caps (caps with LMR>0) have a significantly steeper smile than out-of-the-money caps, which is indicative of the asymmetric slope of the smile on either side of the at-the-money strike. More interestingly, the ask-side of the smile is steeper than the bid-side, the difference being significantly larger for in-the-money caps. Floors display somewhat similar patterns. The smile gets flatter as the maturity of the floor increases. In-the-money floors (floors with LMR<0) exhibit a significantly steeper smile, especially for short-term floors. Long-term floors display almost a "smirk", instead of a smile. As with caps, the smile curve for floors is steeper on the ask-side, as compared to that on the bid-side.

Next, we pool the data for caps and floors to fit the functional form in equation (3). As discussed earlier, the advantage of using pooled data is that we have observations across a wider range of strike rates (moneyness). This allows us to estimate the true functional form for the smile more accurately. Table 5 presents the results for this functional form for pooled data on caps and floors for all maturities. The results are broadly similar to those in the separate regressions for caps and floors, although the explanatory power (adjusted R^2) is somewhat lower, perhaps due to the additional measurement errors resulting from pooling of data. In addition, for shorter term caps and floors (2, 3, and 4 year maturity), the asymmetric slope functional form explains the observed smile patterns better on the ask side than on the bid side. The fitted smiles for these specifications using the pooled data are presented in Figure 3. These results show that there is a significant volatility smile for short-term caps and floors, which is quite asymmetric across both sides of the strike rates (i.e., for out-of-the-money and in-the-money options). In addition, the smile curve is significantly steeper on the ask-side than on the bid-side. However, as the maturity of these options increases, the smile flattens, and eventually converts into a "smirk" when we reach the 10-year maturity.

4. The Determinants of Volatility Smiles

In the previous section, we document the smile patterns for euro interest rate caps and floors. However, the implied volatility smile patterns observed in this market may be significantly related to the liquidity of these interest rate options, as well as to other economic variables. A clear understanding of the determinants of these smile patterns can help in developing models that eventually explain the entire smile. We explore these issues in this section.

4.1 The impact of liquidity on volatility smiles

We use the bid-ask spread as a proxy for liquidity in the market for those options. It can be questioned whether the bid-ask spread is an appropriate proxy for liquidity. However, given the nature of the OTC market for caps and floors, it is extremely difficult to obtain other measures of liquidity, common in exchange-traded markets, such as volume, depth, market impact etc. The data on even the bid-ask spreads is not widely available for the market as a whole. In our sample, we do observe the bid-ask spread for a particular dealer for each option every day. Therefore, we settle for using this metric as a meaningful, although potentially imperfect, proxy of liquidity. As a first cut, before more elaborate analysis of the dependence of the shape of the smile on the liquidity factors over time, we consider the original model (3). We then compare the same specification with another specification where the bid-ask spread of each option (scaled by its mid-price) is added as an explanatory variable:

$$\begin{aligned} &Scaled\ IV = c1 + c2 * LMR + c3 * LMR^2 + c4 * 1_{LMR<0} * LMR \\ &Scaled\ IV = d1 + d2 * LMR + d3 * LMR^2 + d4 * 1_{LMR<0} * LMR + d5 * ScaledBA \end{aligned} \tag{4}$$

This approach allows us to examine the *incremental* effect of the bid-ask spreads in explaining the observed volatility smile patterns, *after* controlling for the dependence of the implied volatilities on functional forms of moneyness.

We did consider the possibility of including other option "Greeks" in the above specifications.[23] We did not do so for two reasons. First, the squared term for the LMR included above is an approximate proxy for the convexity term. Second, introducing other option Greeks explicitly may introduce potential collinearity, since, to a first order approximation, these risk parameters can be modeled as linear functions of volatility and the square root of the time to expiration.[24]

We use unscaled bid-ask spreads as instruments to address the errors-in-variable problem in scaled bid-ask spreads as discussed in section 2. We estimate this system of equations using the technique of two-stage least squares. In the first stage, we regress each of the explanatory variables on the unscaled bid-ask spreads and on the independent variables, other than the scaled bid-ask spreads. Then, in the second stage, we use the fitted values from the first stage as regressors. Scaling by the mid-price presents another problem. For in-the-money options, the large prices result in very small bid-ask spreads in percentage terms. For out-of-the money options, the small prices result in large scaled bid-ask spreads. This denominator effect seems to

[23] In equity markets, Jameson and Wilhelm (1992) show that the bid-ask spreads for options are explained by option Greeks.
[24] See, for example, Brenner and Subrahmanyam (1994), who provide, in the context of the Black-Scholes model, approximate values for the risk parameters of options that are close to being at-the-money on a forward basis.

contaminate the regression results. We get around this problem by running the regressions *separately* for in-the-money and out-of-the money options. The results are presented in Table 6.

We find that the coefficient for the scaled bid-ask spread (d5) is significant 17 out of 18 times on the ask-side. On the bid-side, the coefficient is significant 15 out of 18 times. Thus, bid-ask spreads are useful in explaining the level of implied volatilities across strike rates. We test the joint hypothesis that the smile coefficients are the same across the two models (c2=d2, c3=d3, c4=d4). The p-values for this joint hypothesis are presented in the last but one column of Table 6. Equality of these smile coefficients is rejected at the 1% level for almost all the maturities on the ask-side but only 50% of the times on the bid-side. Therefore, there is strong evidence that the smile, after correcting for bid-ask spreads, is different from the uncorrected smile on the ask–side, but not as much on the bid-side. In addition, the coefficient for the scaled bid-ask spread is positive and significant for almost all maturities on the ask-side, implying that higher implied volatilities of these options are associated with higher scaled bid-ask spreads. This is intuitive as the market-maker tries to pass on the higher liquidity costs of away-from-the-money options by increasing the ask-price. The more interesting part of the table is the comparison of the results on the ask-side with the results on the bid-side. The results for rejecting the hypothesis that the smile coefficients with and without the scaled bid-ask spreads are equal are much weaker on the bid-side. Also, while the sign of the coefficient for the liquidity proxy is consistently positive on the ask-side, this is not the case on the bid-side. Thus, there is a strong reason to believe that the market-maker adjusts the ask-side much more in response to the changes in the liquidity of options in this market. The bid-side, in contrast, is left pretty much unaffected by changes in option market liquidity. This would be a natural course of action for by the market makers if the demand in this market were mainly on the ask-side.

4.2 Time-series analysis of volatility smiles

In the previous sub-section, we demonstrate that in a pooled time-series and cross-sectional setting, liquidity affects the volatility smiles in interest rate option markets. In this sub-section, we explore the time-series behavior of the volatility smile, since there is considerable time-variation in term structures, implied volatilities, and the factors that affect them.

In figure 4A, we present the surface plots for the implied volatilities over time, by moneyness represented by LMR.[25] The shapes of these surface plots show similar trends – the 2-year maturity options display a large curvature in the volatility smile, while the smile flattens out and turns into more of a skew as we move towards the longer maturity options, especially at the 10-

[25] These plots are presented for representative maturities of 2-, 5-, and 10-years, since the plots for the other maturities are similar. In addition, since 3-D plots require the data to be complete over the entire grid, we present the volatility smiles over the LMR range from -0.3 to +0.3, which is the subset of strikes over which complete data are available over a substantial number of days in our dataset.

year maturity. More importantly, both the curvature and the slope of the volatility smile show significant time-variation, sometimes even on a daily basis. The changes in the curvature and slope over time are more pronounced for the 2-year maturity options, although they are also perceptible for the longer maturity options.

Figure 4B presents the surface plot of the Euro spot rates from one to ten years maturity over our sample period. Similar to the volatility surfaces, the Euro term structure surface also shows significant time variation. It is clear that there is an increase in spot rates in the early part of our sample, followed by a flattening of the term structure due to an increase, primarily in the rates at the shorter end of the term structure, during the latter part of our sample period. Therefore, both the level of interest rates and the slope of the term structure exhibit significant time variation over our sample period.

The natural question is whether on a time-series basis, certain economic variables exhibit a significant relationship with the implied volatility smile patterns. We examine this issue in this sub-section. In order to do that, we first need appropriate measures of the asymmetry and curvature of the smile curves each day. There are two ways of estimating these measures. The first method is to posit a functional form for the volatility smile, and estimate the parameters of that functional form every day (cross-sectionally), using implied volatilities across strike rates. Unfortunately, we do not have enough observations across strike rates each day to reliably estimate four parameters of the functional form in (3). So we adopt an alternative method, which is to explicitly define empirical proxies for these attributes and estimate them using the volatility smile curves. For example, a measure of the asymmetry of the implied volatility curve, widely used by practitioners is the "risk reversal," which is the difference in the implied volatility of the in-the-money and out-of-the-money options (roughly equally above and below the at-the-money volatility). A measure of the curvature is the "butterfly spread," which is the difference between the average of the implied volatilities of two away-from-the-money volatilities and the at-the-money volatility.[26] The advantage of using these empirical measures is that they do not require the estimation of any specific model; hence, they can be used when there are fewer observations across strike rates.

We construct the two variables explained above, the butterfly spread and the risk reversal, to proxy for the curvature and asymmetry of the daily smile in the interest rate options. Figure 5 explains the calculation of the butterfly spread and the risk reversal. Each day, for each maturity, we consider the scaled implied volatilities of caps and floors against the LMR. We obtain scaled implied volatilities at +0.25 and -0.25 LMR by linearly interpolating between the closest options on the either side of these points. When cap as well as floor data are available at the same level of

[26] These structures involve option-spread positions and are traded in the OTC interest rate and currency markets as explicit contracts. These prices are often used in the industry for calibrating interest rate option models. See, for example, Wystup (2003).

moneyness, the average of the two scaled implied volatilities is used. The butterfly spread and risk reversal are calculated as follows

$$\begin{aligned} &Risk\ Reversal = Scaled\ IV_{+0.25LMR} - Scaled\ IV_{-0.25LMR} \\ &Butterfly\ Spread = \left(Scaled\ IV_{+0.25LMR} + Scaled\ IV_{-0.25LMR}\right)/2 - Scaled\ IV_{ATM} \end{aligned} \tag{5}$$

The butterfly spread captures the average curvature of the implied volatility smile at 0.25 LMR away-from-the-money on either side of 0. It is, therefore, a proxy for the curvature of the smile. The risk reversal represents the difference between the implied volatility of in-the-money options and out-of-the-money options. Thus, it is a proxy for the asymmetry of the smile. Naturally, since the estimation of these two measures of the shape of the volatility smile requires data across the entire spectrum of available strikes, all of the time-series analysis from this point onwards is combined analysis including in-the-money, at-the-money, and out-of-the-money options on each day.

We explore the relationship between the slope and curvature of the daily smiles and liquidity and economic variables. We consider the following economic variables: the level of volatility of at-the-money interest rate options, the slope of the term structure at the short end (6 months to 5 years) and at the long end (5 years to 10 years), the spot 6-month Euribor, the 6-month Treasury-Euribor spread, the scaled ATM bid-ask spread, the average, scaled away-from-the-money bid-ask spread and the difference between in-the-money and out-of-the-money bid-ask spreads. These are time-series regressions of curvature and asymmetry measures calculated using data across all the strikes each day. The regression specifications are as follows:[27]

$$\begin{aligned} Butterfly\ Spread = {} & c1 + c2 * ATMVol + c3 * 6Mrate + c4 * 5yr6Mslope \\ & + c5 * 10yr5yrslope + c6 * DefSpread + c7 * atmBAspread \\ & + c8 * Average\ \ awayBAspread \\ Risk\ \ Reversal = {} & d1 + d2 * ATMVol + d3 * 6Mrate + d4 * 5yr6Mslope \\ & + d5 * 10yr5yrslope + d6 * DefSpread + d7 * atmBAspread \\ & + d8 * Difference\ \ in\ \ awayBAspread \end{aligned} \tag{6}$$

The intuition for examining these variables is as follows. First, the volatility variable is added to examine whether the patterns of the smile vary significantly with the level of uncertainty in the market. During uncertain times, information asymmetry issues are likely to be more important than during periods of lower volatility. If there is significantly greater information asymmetry, market makers may charge higher than normal prices for away-from-the-money options, since

[27] This time series regression is estimated by including AR(2) error terms to correct for serial correlation. We check the residual plots to see if the correction for serial correlation is appropriate, and find no patterns or trends in those plots. In addition, for all maturities, the Durbin-Watson statistic is insignificantly different from 2. Therefore, the inclusion of the AR(2) error terms, indeed, takes care of any serial correlation in the regression model.

they may be more averse to taking short position at these strike rates. This will lead to a steeper smile, especially on the ask side of the smile curve. Also, during times of greater uncertainty, a risk-averse market maker may demand higher compensation for providing liquidity to the market, which would affect the shape of the smile. Since we have divided the volatility of each option by the volatility of the corresponding ATM cap to obtain the scaled IV, we use the ATM swaption volatility as an explanatory variable here, in order to avoid having the same variable on both sides of the regression equation. The ATM swaption volatility can be interpreted as a general measure of the future interest rate volatility.[28] Second, we include the spot 6-month Euribor as another explanatory variable. The absolute level of interest rates is also indicative of general economic conditions, as well as the direction of interest rate changes in the future - for example, if interest rates are mean-reverting, very low interest rates are likely to be followed by rate increases. This would manifest itself in a higher demand for out-of-the-money caps in the market, thus affecting the prices of these options, and possibly the shape of the implied volatility smile itself. Third, the slope of the yield curve is added as an explanatory variable, as it is widely believed to proxy for general economic conditions, in particular the stage of the business cycle. The slope of the yield curve is also an indicator of future interest rates, which affects the demand for away-from-the-money options: if interest rates are expected to increase steeply, there will be a high demand for out-of-the-money caps, resulting in a steepening of the smile curve. We use the difference between the 10 year spot rate and the 5 year spot rate as a proxy for the slope of the yield curve at the long end, and the difference between the 5 year spot rate and the 6 month spot rate as the proxy for the slope of the yield curve at the short end.

The ATM volatility and the term structure variables act as approximate controls for a model of interest rates displaying skewness and excess kurtosis. Typically, in such models the future distribution of interest rates depends on today's volatility and the level of interest rates. Thus, by including the contemporaneous volatility and interest rate variables in the regression, we try to ensure that the relationship of the shape of the smile to liquidity is separate from the effect arising out of a more sophisticated model for the interest rates.

Fourth, we examine the relationship of the volatility smile to the 6-month Treasury-Euribor spread. This variable is often used as a measure of aggregate liquidity as well as the default risk of the constituent banks in the Euribor fixing. A wider spread should indicate a higher liquidity risk premium, which could affect the prices of away-from-the-money options more than the prices of ATM options, thus affecting the shape of the smile. Fifth, we include two measures of the relative bid-ask spreads of these options – ATM and away-from-the-money. The objective of including these two variables is to directly control for the explicit liquidity of these options, while

[28] Although swaption implied volatilities are not exactly the same as the cap/floor implied volatility, they both tend to move together. Hence, swaption implied volatilities are a valid proxy for the perceived uncertainty in the future interest rates. The data on the ATM swaption volatility in the Euro market was obtained from DataStream.

examining the relationship of the other economic variables to the volatility smile. The relative bid-ask spreads of ATM options capture the general level of liquidity in the market. The relative bid-ask spreads for away-from-the-money options, which are averages of the relative bid-ask spreads of options with +0.25 and –0.25 LMR, capture the average liquidity of the options away-from-the-money. The difference between ITM and OTM scaled bid-ask spreads capture the asymmetry in the liquidity.

Since scaled bid-ask spreads are measured with error, we use a two-stage least square approach for estimating these regressions. The ATM and away-from-the-money unscaled bid-ask spreads are used as instruments for the corresponding measures of the scaled bid-ask spreads. As the other economic variables are measured correctly, they are used as instruments for themselves. In the first stage, we regress all the explanatory variables on unscaled bid-ask spread measures and variables other than scaled bid-ask spread measures. Then, we use the fitted values from the first stage as regressors in the second stage. The standard errors correctly account for the fact that the explanatory variables in the second stage regression are fitted values from a previous regression.

The results for this regression analysis are presented in Table 7. We find that, except for very short maturities, the away-from-the-money spread has a positive impact on the curvature of the smile, i.e., when the away-from-the-money spread is large, the curvature of the volatility smile is also greater. This implies that at least some of the increase in the curvature of the smile is attributable to an increase in the bid-ask spreads for away-from-the-money options in this market. Therefore, in modeling option prices across strike rates, it is important to account for liquidity effects, in addition to the other factors that may contribute to the smile. In addition, the only other variable that is significant is the slope of the term structure at the short end. However, it appears that the curvature of the volatility smile increases when the slope of the term structure decreases, i.e., when the yield curve is flatter. This is opposite to the expected effect. We expected that a higher slope of the term structure would result in a higher curvature of the smile. One possible explanation is that a higher slope of the term structure captures the expectation that interest rates will rise in future. This increases the demand for out-of-the-money caps, but reduces the demand for in-the-money caps. Since curvature is the average implied volatility of in-the-money and out-of-the-money options, it is the *net* effect that matters. Possibly, the effect of a reduction in the implied volatility of in-the-money caps in greater than the effect on out-of-the-money caps, resulting in the reduction of the curvature. However, this also implies that the asymmetry of the smile (the difference between implied volatility positive LMR options and negative LMR options) should decrease with a rise in the slope of the yield curve. This is exactly what we observe in Table 7. The risk reversal has a negative relationship with the slope of the yield curve at the short end.[29] It is important to note that these effects are more consistent on the ask-side than on the bid-side, in line with our earlier results.

[29] In addition to aggregate tests over our entire sample, we split our sample into two parts (Jan 01, 1999 - May 14, 2000, and May 15, 2000 – May 31, 2001), in order to examine whether our findings hold within sub-

Our results are similar in spirit to what Bollen and Whaley (2004) find in the equity index options market. They find that the net buying pressure in this market affects the implied volatility smile. We find that the liquidity variables that would depend on the capacity of the market maker to absorb the demand pressure in the market have significant explanatory power for the shape of the smile. We also find that other economic variables that can potentially influence the demand for interest rate options also affect the curvature and asymmetry of the smile. Since it is extremely difficult to get volume data for OTC interest rate option markets, we have to rely on the indirect ways to explore the potential effect of demand pressure on the shape of the volatility smile in these markets.

5. Granger Causality Tests and the Volatility Smile

In the previous section, we show that liquidity variables are significantly related to the shape of the *contemporaneous* smile. In this section, we examine the relationship between the lagged values of liquidity variables and the shape of the smile, and vice-versa. We use Granger causality tests, which can provide useful information on whether knowledge of the past values of a variable improves the short-run forecasts of the current and future values of another variable. Although this analysis may not explain causality *per se*, it may throw light on the linkages between liquidity and the volatility smile in a dynamic predictive sense.

The general formulation of the Granger (1969) causality tests for the case of two scalar-valued, stationary and ergodic time series X_t and Y_t is defined as follows. Let $F(X_t \mid I_{t-1})$ be the conditional probability distribution of X_t, given the bivariate information set I_{t-1}, consisting of a Lx-length lagged vector of X_t and a Ly-length vector of Y_t. Given lags Lx and Ly, the time series Y_t does *not* strictly Granger-cause X_t if:

$$F\left(X_t \middle| I_{t-1}\right) = F\left(X_t \middle| \left(I_{t-1} - Y_{t-Ly}^{Ly}\right)\right), \qquad t = 1,2,... \tag{7}$$

If this equality does not hold, then knowledge of past values of Y helps to predict current and future values of X, and Y is said to strictly Granger-cause X.

The linear Granger causality test is estimated as a reduced-form, bivariate, vector auto regression (VAR) model as follows,

$$\begin{aligned} X_t &= A(L)X_t + B(L)Y_t + \varepsilon_{X,t} \\ Y_t &= C(L)X_t + D(L)Y_t + \varepsilon_{Y,t} \qquad t = 1,2,... \end{aligned} \tag{8}$$

samples. We find that, in some cases, the results are weaker in the sub-samples. We attribute this to the shorter time series in the sub-samples.

where *A(L), B(L), C(L),* and *D(L)* are lagged polynomials of the same order in the lag operator L. The error terms are assumed to be mutually independent and individually i.i.d. processes with zero mean and constant variance. The test of whether *Y* strictly Granger-causes *X* is a standard F-test of the joint restriction that all the coefficients contained in the lagged polynomial *B(L)* are zero. Similarly, the null hypothesis that *X* does not Granger-cause *Y* is rejected, if the coefficients of *C(L)* are jointly significantly different from zero. Bidirectional feedback, or causality running in both directions, exists if the elements in both the lag polynomials *B(L)* and *C(L)* are jointly significantly different from zero.

We estimate bivariate VARs for the butterfly spread and the two liquidity variables – ATM scaled bid-ask spreads and average away-from-the-money scaled bid-ask spread. We repeat this for the risk reversal and ATM spreads and difference in the ITM and OTM scaled bid-ask spreads. Since the butterfly spread, risk reversal and the scaled bid-ask spreads are measured with error, we use the two stage least squares approach to estimate each equation in the bivariate VAR.[30] We use all the other economic variables and unscaled bid-ask spreads with appropriate lags as the instruments. The appropriate number of lags in each case is determined using the Akaike information criterion.

Table 8 presents the results of the linear, bivariate, Granger-causality tests. There is no evidence, consistent across maturities, of the liquidity variables predicting the shape of the smile or vice-versa. However, as we saw in section 4, the ask-side and the bid-side do not exhibit similar relationships to the economic variables examined in this paper. Therefore, it is possible that some of the effects of liquidity variables on the shape of the volatility smile are manifested through these economic variables. To analyze this, we estimate bivariate VARs separately for the bid and the ask sides, for the butterfly spread and each of the economic variables. The economic variables are the same as those in the previous section – ATM volatility, term structure variables and default spread. We repeat the same analysis for risk reversal as well. The results of these analyses are presented in Table 9.

We find that the shape of the volatility smile plays a role in predicting some of the economic variables. In particular, the butterfly spread Granger-causes the ATM volatility and the slope of the yield curve (at the long end), but the causality does not flow in the other direction, i.e., these variables do not Granger-cause the butterfly spread. This indicates that these two variables (ATM volatility and slope of the term structure at the long end) are influenced by the curvature of volatility smiles in interest rate option markets, but there is no feedback effect in the other direction. The one-way Granger causality from butterfly spreads to the slope of the yield curve and the volatility of interest rates indicates the information content of the volatility smiles.

[30] We thank Rob Engle for insightful discussions on the econometric procedure used in this section.

The risk reversal for options at longer maturities appears to Granger-cause the slope of the yield curve at the short- as well as the long-end. Thus, the asymmetry in the volatility smile curves for long-maturity options is useful for predicting medium term as well as long term interest rates. There is no evidence of causality in the other direction. This is intuitive since the option prices are forward looking. The shape of the smile incorporates information about the market's perception about the future distribution of the interest rates.

Based on this analysis, the curvature and asymmetry of the implied volatility smiles appear to have information that is useful in predicting future slopes of the term structure. This implies that information from the interest rate option markets can be useful in estimating future yield curves. This also points to the need for calibrating interest rate option pricing models to option prices across strike rates, not just to ATM options. In addition, one-way Granger causality from the butterfly spread to the ATM volatility indicates that there may be an underlying stochastic volatility process, about which there is some information in the curvature of the volatility smile. Again, it appears that future volatility can be predicted, to some extent, by analyzing the curvature of the volatility smile in interest rate option markets. At a minimum, predictions of future volatility may be improved by analyzing the entire volatility smile curve in interest rate options, rather than just analyzing ATM options. Both these effects point to the need for calibrating interest rate option models, especially stochastic volatility models, to option price data across strike rates, not just to ATM option prices.

One problem with linear causality testing is that such tests may have low power in detecting certain kinds of nonlinear predictive relationships. In addition, even after removal of the linear predictive power using a linear VAR model, there may be significant incremental predictive power of one time series to another. We test for nonlinear Granger causality as well, using logarithmic and quadratic terms in the lagged time series of each variable. These results are similar to the results reported for linear causality and are not reported here to conserve space.

6. Conclusions

We examine the patterns of implied volatility in the euro interest rate option markets, using data on bid and ask prices of interest rate caps and floors across strike rates. We document the pattern of implied volatility across strike rates for these options, separately on the bid-side and the ask-side, and find that the volatility smile curve is clearly evident in the euro interest rate cap and floor market. Furthermore, the smile curve on the ask-side appears to be steeper than the bid-side smile.

We further examine the impact of bid-ask spreads along with other economic variables on the volatility smile curves. We include the level of volatility and interest rates to control for the effects arising out of a more elaborate model of interest rates. We find a significant relationship between liquidity and the shape of the smile. Away-from-the-money bid-ask spreads have a significant influence on the curvature and asymmetry of the smile, especially on the ask-side. The effect on the bid-side is less significant. The slope of the yield curve, especially at the short end, also has explanatory power for the curvature and asymmetry of the volatility smile, with a stronger impact on the ask side. Thus, the ask side is generally more responsive to the changes in the economic variables. The evidence on the difference in the responsiveness of the ask-side vis-à-vis the bid-side points towards the institutional structure of the market, where most of the customers are on the ask-side; hence, the dealers who set prices are more likely to adjust the ask-side of the price curve than the bid-side in response to new information. Therefore, the ask-prices appear to be more relevant for calibrating interest rate models to caps and floors.[31]

In Granger causality tests, we find that the curvature of the volatility smile in these markets has power in predicting the ATM volatility as well as the slope of the term structure at the long-end. The asymmetry in the smiles for long maturity options has some predictive power for the slope of the yield curve at the short- and the long-end. This has important implications for developing and calibrating interest rate models – it is critical that these models be calibrated to option prices across strike rates, not just to ATM options.

To summarize, liquidity effects are important in explaining the shape of the implied volatility smiles in the interest rate option markets. These effects are present even after controlling for the effects arising out of an alternate model of interest rates. Ask-prices are more relevant in these markets, given the evidence that they move more in response to various economic variables. The shape of the volatility smile has predictive power for the future slope of the yield curve and the future level of uncertainty in the interest rate option markets. All of these issues are critical to the appropriate modeling and risk management of an array of interest rate derivatives.

In future research, we intend to explore seasonal patterns in the smile curves, to see if there are day-of-the week effects in the market that affect the volatility smile curve.[32] Also, the term structure of implied volatility of interest rate options will have implications for interest rate modeling, since this dimension is also of considerable interest to researchers and practitioners. In addition, the shape of implied volatility smile in the swaption market and its relation to the smile in the caps and floors market would throw further light on the volatility dynamics in the interest rate option markets. This line of research could eventually lead to the development of interest rate option pricing and hedging models that incorporate such liquidity effects.

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[31] We would also conclude that the bid-curve can then be computed relative to this curve, after taking into account market liquidity and the existing positions of the dealer.

[32] Return seasonality has been explored in the equities markets. It has been observed that in equity markets, bid-ask spreads are the highest on Mondays, implying higher trading costs. This, coupled with the reduced demand from liquidity traders who fear increased adverse selection on Mondays, results in lower trading volumes on Mondays.

What are the implications of these results for the modeling of interest rate derivatives? First, it is necessary to use data from options at different strike rates for calibration of the models, since there is a lot of information in the volatility smile itself. Second, it may be somewhat misleading to take the mid-point of the bid and ask quotes as inputs. Rather, it may be better to fit the bid- and the ask-side of the volatility smiles separately, since the bid-ask spread is informative and varies over time. Third, although factors such as jumps and fat-tails may be important in determining the supply and demand for interest rate options, it may be fruitful to model the liquidity aspect by fitting the bid- and ask- sides with relatively simple models before introducing complexities into the model formulation.

Appendix A

A.1. Implied Volatility in the Black Model and the Pricing of Caps and Floors

The standard model used for dealer quotations for interest rate caps and floors is the Black (1976) model of pricing of options on futures/forward contracts. The model is a variant of the basic Black and Scholes (1973) option-pricing model. Applied to the interest rate option context, the model assumes that interest rates are lognormally distributed and relates the price of a European call option (C) and a put option (P), at time *0*, on an interest rate forward rate agreement (FRA) to the underlying variables as follows:[33]

$$
\begin{aligned}
C &= \left[fN(d_1) - kN(d_2) \right] \times m \times B_{0,t+m} \\
P &= \left[-kN(-d_2) - fN(-d_1) \right] \times m \times B_{0,t+m} \\
d_1 &= \frac{\ln f/k + t\sigma^2/2}{\sqrt{t}\sigma} \\
d_2 &= d_1 - \sqrt{t}\sigma
\end{aligned}
\qquad \text{(A.1)}
$$

where

f = forward interest rate for the period t to $t+m$,
σ = annualized volatility of the forward interest rate t on the maturity date,
m= the maturity period of the underlying loan,
t = maturity date of the option,
k = strike rate of the option,
$B_{0,t+m}$= the zero bond price at time *0*, for the bond maturing at date $t+m$.

Of course, the key variable in the above equations, which is not observable, but about which market participants may have differing views, is the volatility. The quotations of interest rate options are usually for the implied volatility that reflects the market price, rather than the price directly.

An interest rate cap (floor) is a collection of caplets (floorlets). A caplet (floorlet), in turn, is a single European call (put) option on a reference interest rate, expiring on a specific date. Hence, a cap (floor) can be regarded as a portfolio of European call (put) options on interest rates, or equivalently, put (call) options on discount bonds. Typically, an interest rate cap is an agreement

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[33] This formula is also consistent with the model proposed by Brace, Gatarek and Musiela (1997) [BGM] and Miltersen, Sandmann and Sondermann (1997), which is popular among practitioners. BGM derive the processes followed by market quoted rates within the HJM framework, and deduce the restrictions necessary to ensure that the distribution of market quoted rates of a *given* tenor under the risk-neutral forward measure is lognormal. With these restrictions, caplets of that tenor satisfy the Black (1976) formula for options on forward/futures contracts.

between a cap writer and a buyer (for example, a borrower) to limit the latter's floating interest payments to a specific level for a given period of time. The cap is structured on a specific reference rate (usually the 3- or the 6- month Libor (London Interbank Offer Rate) or Euribor (Euro Interbank Offer Rate)) at a predetermined strike level. The reference rate is reset at periodic intervals (usually 3- or 6- months). In a similar manner, an interest rate floor contract sets a *minimum* interest rate level for a floating rate *lender*. The cap and floor contracts are defined on a pre-specified principal amount.[34]

A caplet with maturity t_i and strike rate k, pays at date t_i, an amount based on the difference between the rate (r_i) at time t_i and the strike rate, if this difference is positive, and zero otherwise. The amount paid is based on the notional amount and the reset period of the caplet and is paid on a discounted basis at time t_i. The payoff of this caplet at date t_i, on a notional principal of €A, is given by

$$c_{t_i} = A(t_{i+1} - t_i)\max\left[\frac{r_i - k}{1 + r_i(t_{i+1} - t_i)}, 0\right] \quad \text{(A.2)}$$

The payoff from a floorlet can be described in a similar manner.

Since the interest rate over the first period is known, there is no caplet corresponding to the first period of the cap. For example, a 2-year cap on the 6-month Euribor rate, with 4 semiannual periods over its life, would consist of 3 caplets, the first one expiring in 6 months, and the last one in 1 year and 6 months. Thus, the underlying interest rate for the first period is the 6-month Euribor rate on the date 6 months from initiating the cap contract.

Each caplet or floorlet has to be valued separately, using a valuation model such as the Black or BGM model in equation (1), (the same model that is generally used by the market for quotation purposes), with the price of the cap or floor being the sum of these prices. The volatilities used for each caplet or floorlet, which are generally different, across strike rates and maturities, are sometimes called *spot volatilities*. The market quotation for interest rate caps and floors, however, is based on the *same* volatility for all the caplets in a particular cap (or the floorlets in a particular floor). In other words, the market price of a cap (or floor) can be derived by plugging in this constant volatility for all the component caplets (or floorlets) in the contract. This constant volatility is referred to as the *flat volatility* for the particular cap (or floor) and varies with the maturity of the contract. Since caps are portfolios of caplets, the implied flat volatilities of caps reflect some average of the implied spot volatilities of individual caplets. In this paper, one of our

[34] Interest rate caps and floors for various maturities and reference rates in all the major currencies are traded in the over-the-counter (OTC) markets. The most common reference rate in the case of U.S. dollar caps/floors is the 3-month Libor. In the euro markets, the most common reference rate is the 6-month Euribor.

primary objectives is to examine liquidity effects in interest rate options. For doing that, we need to focus on traded assets, which are caps and floors. Therefore, we use the flat volatilities of caps and floors, since spot volatilities would correspond to caplets and floorlets, which are, untraded assets. We also checked the prices of the individual caplets/floorlets, which are obtained by "bootstrapping" and found that the smile patterns are broadly similar.

A.2. Put-Call Parity in Caps and Floors

The put call parity condition for interest rate caps and floors is:

Price of a Cap = Price of a Floor + Value of the Swap (Pay fixed receive floating) (A.3)

If the cap and floor have a strike rate of *k,* the swap, in this case, is an agreement to make fixed payments at a rate *k* and receive floating payments in exchange, with no exchange of payments on the first reset date. This is a non-standard swap that is defined to ensure conformity with caps and floors, which are structured so that there is no payment resulting from the first period of the contract. The other, more important, reason the swap is non-standard is that it is an "off-market" swap, since the fixed payment at a rate *k* is related to the strike rates of the cap and floor rather than to the market swap rate. Thus, the swap in equation (A.3) has a non-zero value unlike the standard market swap.

By definition, the relevant ATM swap rate is strike rate *k* such that the value of the non-standard swap is 0. Hence, the ATM non-standard swap rate (sw) is given by following formula:

$$sw = \frac{\sum f_{t,t+m} \times m \times B_{0,t+m}}{\sum m \times B_{0,t+m}} \qquad \text{(A.4)}$$

where

$f_{t,t+m}$ = the forward rate, at time *0,* for the period *t* to *t+m*,

m = underlying period of the loan,

$B_{0,t+m}$ = the zero bond price, at time *0,* for the bond maturing at time *t+m*.

The summation is for the all the maturity dates of individual caplets/floorlets. This formula takes care of the fact that there is no exchange of payments at the first reset date of the swap, to be consistent with the definition of the caps/floors. The number of cash flows exactly matches the number of individual options in the cap/floor contracts.

However, in the presence of the bid-ask spread, the parity relationship in (A.3) has to be modified as a pair of inequalities. For the same strike price and maturity,[35]

Ask Price of a Cap– Bid Price of a Floor – Bid Price of non-standard Swap ≥ 0 (A.5)

Ask Price of a Floor– Bid Price of a Cap + Ask Price of non-standard Swap ≥ 0 (A.6)

where the swap contract is defined as a "pay-fixed, receive-floating" contract at the same strike rate as the cap and the floor. We have bid and ask prices for caps and floors. But we do not have prices for the relevant swaps because they are non-standard, off-market contracts. So instead, we compute the value of the off-market swap (pay-fixed, receive -floating) as follows:

$$Value\ of\ swap = \sum f_{t,t+m} \times m \times B_{0,t+m} - \sum k \times m \times B_{0,t+m} \quad (A.7)$$

where

$f_{t,t+m}$ = the forward rate, at time *0*, for the period *t* to *t+m*,

m = underlying period of the loan,

$B_{0,t+m}$ = the zero bond price, at time *0*, for the bond maturing at time *t+m*.

k = fixed rate on the swap.

[35] These are the conditions for an arbitrageur who has no existing position. The bounds may be somewhat tighter for agents who already have an existing position in one or more of the contracts in the arbitrage trade implied by the inequalities.

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Table 1

Descriptive statistics for cap and floor prices

This table presents descriptive statistics on euro interest rate cap and floor prices across maturities and strike rates, over the sample period Jan 99 - May 01, obtained from WestLB Global Derivatives and Fixed Income Group. The caps and floors are grouped together by moneyness into five categories. The moneyness for these options is expressed in terms of the Log Moneyness Ratio (LMR), defined as the log of the ratio of the par swap rate to the strike rate of the cap/floor. All prices are averages, reported in basis points, with the standard deviations of these prices in parenthesis.

Maturity	Caps					Floors				
	Deep OTM	OTM	ATM	ITM	Deep ITM	Deep ITM	ITM	ATM	OTM	Deep OTM
	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3
2-year	2.1	11.1	43.2	107.7	250.5	250.5	153.7	55.5	13.6	3.6
	(0.5)	(5.8)	(19.8)	(30.9)	(58.8)	(48.1)	(50.7)	(25.4)	(7.9)	(2.0)
3-year	10.7	37.7	91.9	209.6	481.3	529.1	285.3	111.3	32.7	6.9
	(10.0)	(20.0)	(33.8)	(52.3)	(133.4)	(114.2)	(74.7)	(44.6)	(18.0)	(4.6)
4-year	22.3	72.6	152.7	311.3	674.4	728.3	406.4	176.1	62.1	12.0
	(12.5)	(32.2)	(49.7)	(78.3)	(193.1)	(138.7)	(98.9)	(64.8)	(27.8)	(7.9)
5-year	42.7	119.4	221.7	409.1	872.3	910.8	519.5	244.7	94.3	19.2
	(16.3)	(48.6)	(67.2)	(95.4)	(252.2)	(161.2)	(122.5)	(84.5)	(35.2)	(13.9)
6-year	66.9	163.7	286.6	507.9	1,006.6	1,093.1	663.8	323.7	128.6	27.2
	(20.2)	(64.4)	(84.6)	(109.5)	(257.4)	(173.2)	(133.1)	(101.9)	(43.5)	(18.7)
7-year	93.7	210.9	355.8	610.8	1206.4	1,239.0	809.3	393.3	164.1	36.9
	(25.4)	(82.2)	(99.3)	(125.3)	(275.5)	(147.0)	(127.5)	(115.2)	(51.9)	(33.0)
8-year	123.9	264.2	433.2	706.8	1,248.2	1,284.7	924.7	425.2	199.2	46.8
	(31.4)	(98.1)	(115.9)	(162.8)	(253.4)	(120.8)	(139.3)	(108.3)	(59.6)	(32.8)
9-year	152.1	309.6	509.9	811.8	1,310.3	NA	997.1	482.3	235.0	58.9
	(35.6)	(103.2)	(128.7)	(172.2)	(205.3)		(150.2)	(120.9)	(69.6)	(41.5)
10-year	179.6	347.8	598.0	881.3	1,493.4	NA	815.5	541.7	242.9	71.3
	(39.8)	(106.7)	(140.0)	(153.4)	(275.3)		(31.1)	(139.6)	(61.9)	(50.1)

Table 2

Scaled bid-ask spreads for caps and floors

This table presents summary statistics on the bid-ask spreads for euro interest rate caps and floors, scaled by the average of the bid and ask prices for the options, across strike rates, for different maturities. The statistics are presented for the entire sample period, Jan 99 - May 01, based on data obtained from WestLB Global Derivatives and Fixed Income Group. The caps and floors are grouped together by moneyness into five categories. The moneyness for these options is expressed in terms of the Log Moneyness Ratio (LMR), defined as the log of the ratio of the par swap rate to the strike rate of the cap/floor. All the spreads are averages, reported as percentages, with the standard deviations of the scaled spreads in parenthesis.

Maturity	Caps					Floors				
	Deep OTM LMR < -0.3	OTM -0.3 < LMR < -0.1	ATM -0.1 < LMR < 0.1	ITM 0.1 < LMR < 0.3	Deep ITM LMR > 0.3	Deep ITM LMR < -0.3	ITM -0.3 < LMR < -0.1	ATM -0.1 < LMR < 0.1	OTM 0.1 < LMR < 0.3	Deep OTM LMR > 0.3
2-year	80.9%	32.4%	14.7%	7.1%	3.8%	2.5%	4.5%	13.3%	30.8%	77.2%
	(21.2%)	(14.3%)	(4.8%)	(2.4%)	(0.5%)	(1.3%)	(1.3%)	(7.9%)	(11.7%)	(24.1%)
3-year	44.2%	19.0%	11.4%	7.0%	3.8%	2.9%	4.7%	11.2%	31.6%	72.0%
	(22.9%)	(5.7%)	(3.2%)	(2.5%)	(0.6%)	(1.1%)	(1.1%)	(6.1%)	(18.1%)	(25.2%)
4-year	26.1%	14.4%	9.1%	6.2%	4.1%	2.9%	4.5%	8.4%	22.2%	59.9%
	(9.4%)	(4.7%)	(2.5%)	(2.2%)	(1.0%)	(1.0%)	(1.0%)	(2.5%)	(14.5%)	(28.7%)
5-year	20.0%	12.6%	8.6%	6.1%	4.1%	3.1%	4.7%	8.2%	19.8%	59.5%
	(5.5%)	(3.9%)	(2.3%)	(2.1%)	(0.9%)	(1.0%)	(1.1%)	(2.3%)	(13.2%)	(27.4%)
6-year	18.3%	12.1%	8.5%	5.7%	4.1%	3.3%	4.7%	7.9%	15.8%	50.2%
	(4.8%)	(3.6%)	(2.2%)	(1.4%)	(0.9%)	(0.9%)	(1.2%)	(2.0%)	(7.5%)	(24.6%)
7-year	17.6%	11.5%	8.4%	5.5%	4.1%	3.4%	4.6%	7.8%	14.0%	45.3%
	(4.4%)	(3.4%)	(2.1%)	(1.3%)	(3.9%)	(0.9%)	(1.1%)	(1.9%)	(5.0%)	(24.6%)
8-year	17.1%	11.1%	8.3%	5.6%	4.0%	3.2%	4.5%	8.1%	14.0%	42.3%
	(3.8%)	(3.3%)	(2.0%)	(1.1%)	(0.3%)	(1.0%)	(1.1%)	(2.0%)	(5.1%)	(21.9%)
9-year	17.1%	11.0%	8.3%	6.0%	4.2%	NA	4.8%	8.3%	14.0%	40.0%
	(3.4%)	(3.1%)	(1.9%)	(0.7%)	(0.3%)		(1.0%)	(2.0%)	(5.2%)	(20.8%)
10-year	17.1%	11.2%	7.9%	6.2%	4.1%	NA	4.7%	8.1%	14.9%	38.6%
	(2.9%)	(3.0%)	(1.8%)	(0.6%)	(0.3%)		(1.2%)	(2.2%)	(5.5%)	(20.6%)

Table 3

Put-call parity tests for caps and floors

This table presents the results for put-call parity tests for euro interest rate caps and floors, for various strike rates and maturities. The statistics are presented for the entire sample period, Jan 99 - May 01, based on data obtained from WestLB Global Derivatives and Fixed Income Group. The test results are presented as follows:

Panel A: Cap (ask) - Floor (bid) - swap
Panel B: Floor (ask) - Cap (bid) + swap

For each panel, we report the mean price, in basis points, for constructing the portfolio, for each maturity. A negative value indicates a potential arbitrage opportunity, since it implies that the hedged portfolio can be constructed at a negative cost, but only by ignoring the bid-offer spread of an off-market swap.

Maturity	Mean Difference (basis points)	Total Number of observations	Number of observations where the difference is		
			< 0	< -5 bp	< -10 bp
Panel A					
2-year	-4.5	1360	1023	659	257
3-year	-4.9	2968	1977	1,548	1,078
4-year	-1.3	3095	1725	1,477	1,087
5-year	10.7	3509	1486	1,180	854
6-year	14.3	3248	1079	737	520
7-year	25.8	2681	443	310	198
8-year	35.6	1837	169	112	77
9-year	46.6	1490	101	70	23
10-year	62.7	641	0	0	0
Panel B					
2-year	13.9	1468	16	0	0
3-year	24.2	3098	41	2	0
4-year	29.4	3099	97	21	1
5-year	34.5	3509	283	135	38
6-year	34.3	3248	378	222	110
7-year	33.3	2681	469	368	233
8-year	30.3	1837	339	278	232
9-year	30.8	1490	322	253	195
10-year	17.9	641	222	197	179

Table 4

Functional forms for implied volatility smiles (Caps and floors separately)

This table presents regression results when the scaled implied flat volatility for euro interest rate caps and floors, for various maturities, is regressed on a quadratic function of the Log Moneyness Ratio (LMR) with an asymmetric slope term, as follows:

$$Scaled\ IV = c1 + c2 * LMR + c3 * LMR^2 + c4 * 1_{LMR<0} * LMR$$

The statistics are presented for the entire sample period, Jan 99 - May 01, based on data obtained from WestLB Global Derivatives and Fixed Income Group. The coefficient and regression statistics are presented for caps and floors, separately for bid and ask quotes, for all maturities. Asterisk implies significance at the 5% level.

Panel A: Caps

Maturity	c1	c2	c3	c4	Adj R^2
Ask					
2-year	0.98*	-0.36*	3.70*	1.07*	0.91
3-year	1.03*	0.00	1.97*	0.63*	0.93
4-year	1.04*	0.14*	1.55*	0.40*	0.95
5-year	1.05*	0.21*	1.31*	0.24*	0.96
6-year	1.04*	0.33*	0.96*	-0.01	0.93
7-year	1.06*	0.43*	0.98*	-0.13	0.60
8-year	1.04*	0.31*	0.74*	-0.02	0.92
9-year	1.04*	0.36*	0.59*	-0.11*	0.88
10-year	1.11*	0.51*	0.79*	-0.16	0.81
Bid					
2-year	0.84*	-0.80*	2.73*	1.25*	0.35
3-year	0.90*	-0.35*	1.09*	0.57*	0.50
4-year	0.95*	-0.55*	1.29*	1.00*	0.68
5-year	0.97*	-0.48*	1.14*	0.85*	0.67
6-year	0.97*	-0.29*	1.01*	0.65*	0.81
7-year	0.98*	0.13*	0.68*	0.07	0.85
8-year	0.95*	0.19*	0.47*	-0.03	0.88
9-year	0.95*	0.19*	0.50*	-0.02	0.81
10-year	1.01*	0.33*	0.66*	-0.07	0.71

Panel B: Floors

Maturity	c1	c2	c3	c4	Adj R^2
<u>Ask</u>					
2-year	1.27*	-1.33*	3.38*	0.69*	0.81
3-year	1.19*	-0.34*	0.92*	-0.73*	0.68
4-year	1.14*	-0.05	0.45*	-0.79*	0.57
5-year	1.12*	0.19*	0.17*	-0.94*	0.51
6-year	1.10*	0.18*	0.12*	-0.76*	0.38
7-year	1.09*	0.33*	0.00	-0.81*	0.17
8-year	1.06*	0.20*	0.07*	-0.31*	0.41
9-year	1.04*	0.26*	0.05*	-0.65*	0.49
10-year	1.09*	0.30*	0.05*	-0.46*	0.60
<u>Bid</u>					
2-year	1.13*	-1.17*	2.61*	0.59*	0.82
3-year	1.07*	-0.36*	0.67*	-0.43*	0.68
4-year	1.05*	-0.06*	0.27*	-0.51*	0.54
5-year	1.04*	0.06*	0.15*	-0.50*	0.47
6-year	1.01*	0.11*	0.07*	-0.43*	0.32
7-year	1.01*	0.21*	0.02	-0.40*	0.55
8-year	0.98*	0.17*	0.03	-0.13*	0.47
9-year	0.95*	0.22*	0.01	-0.41*	0.56
10-year	1.00*	0.25*	0.01	-0.31*	0.71

Table 5

Functional forms for implied volatility smiles (Caps and floors pooled)

This table presents regression results when the scaled implied flat volatility for euro interest rate caps and floors, for various maturities, is regressed on a quadratic function of the Log Moneyness Ratio (LMR) with an asymmetric slope term, as follows:

$$Scaled\ IV = c1 + c2 * LMR + c3 * LMR^2 + c4 * 1_{LMR<0} * LMR$$

The statistics are presented for the entire sample period, Jan 99 - May 01, based on data obtained from WestLB Global Derivatives and Fixed Income Group. The coefficient and regression statistics are presented for caps and floors pooled together, separately for bid and ask prices, for all maturities. Asterisk implies significance at the 5% level.

Maturity	c1	c2	c3	c4	Adj R^2
Ask					
2-year	1.15*	-1.43*	4.92*	1.55*	0.65
3-year	1.15*	-0.67*	2.45*	0.98*	0.59
4-year	1.13*	-0.41*	1.78*	0.67*	0.63
5-year	1.08*	0.25*	0.68*	-0.64*	0.33
6-year	1.04*	0.62*	-0.06*	-1.05*	0.46
7-year	1.05*	0.73*	-0.19*	-1.10*	0.27
8-year	1.04*	0.44*	-0.14*	-0.53*	0.49
9-year	1.04*	0.36*	-0.07*	-0.40*	0.53
10-year	1.11*	0.37*	-0.04	-0.26*	0.59
Bid					
2-year	1.00*	-1.32*	3.55*	1.18*	0.53
3-year	0.99*	-0.35*	0.92*	0.08	0.30
4-year	1.01*	-0.42*	0.98*	0.47*	0.34
5-year	1.00*	-0.18*	0.61*	0.13*	0.40
6-year	0.97*	0.12*	0.21*	-0.27*	0.40
7-year	0.98*	0.38*	-0.04*	-0.55*	0.55
8-year	0.96*	0.31*	-0.09*	-0.35*	0.56
9-year	0.95*	0.28*	-0.06*	-0.33*	0.61
10-year	1.01*	0.31*	-0.05*	-0.27*	0.66

Table 6

Effects of bid-ask spread on volatility smiles

This table presents regression results for two models - first when the scaled implied flat volatility for euro interest rate caps and floors is regressed on a quadratic function of the Log Moneyness Ratio (LMR) with an asymmetric slope term, and second when the scaled bid-ask spread is added to the first model, as follows:

$$Scaled\ IV = c1 + c2 * LMR + c3 * LMR^2 + c4 * 1_{LMR<0} * LMR$$

$$Scaled\ IV = d1 + d2 * LMR + d3 * LMR^2 + d4 * 1_{LMR<0} * LMR + d5 * ScaledBA$$

The statistics are presented for the entire sample period, Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group. The coefficient and regression statistics are presented for the pooled sample of caps and floors, separately for bid and ask prices, for all maturities. Asterisk implies significance at the 5% level. The system of equations is estimated using two-stage least squares. The p-values presented are for the joint hypothesis that c2=d2, c3=d3, c4=d4 .

Panel A: In the money caps and floors

Maturity	c1	c2	c3	c4	d1	d2	d3	d4	d5	p-value
Ask										
2-year	1.14*	-1.13*	4.42*	0.46*	0.77*	-0.02	3.24*	-1.94*	3.2*	0.000
3-year	1.12*	-0.41*	2.36*	-0.26*	0.58*	0.64*	1.59*	-2.61*	5.71*	0.000
4-year	1.1*	-0.13*	1.79*	-0.35*	0.54*	0.57*	1.39*	-2.1*	7.23*	0.000
5-year	1.09*	0.03	1.46*	-0.44*	0.67*	0.54*	1.18*	-1.74*	5.59*	0.000
6-year	1.07*	0.2*	1.09*	-0.58*	0.59*	0.88*	0.65*	-2.18*	6.33*	0.000
7-year	1.08*	0.3*	1.04*	-0.51*	0.84*	0.71*	0.73*	-1.43*	3.19*	0.000
8-year	1.06*	0.25*	0.77*	-0.2*	0.81*	0.71*	0.36*	-1.23*	3.26*	0.000
9-year	1.04*	0.32*	0.68*	-0.55*	0.7*	0.89*	0.24*	-1.88*	4.51*	0.000
10-year	1.11*	0.52*	0.81*	-0.46*	1.15*	0.46*	0.87*	-0.28	-0.52	0.610
Bid										
2-year	0.99*	-1.47*	3.08*	0.6*	1.16*	-2.06*	3.86*	1.87*	-1.39*	0.000
3-year	1*	-0.94*	1.69*	0.37*	0.92*	-0.77*	1.55*	0	0.8*	0.170
4-year	1.02*	-0.86*	1.57*	0.69*	0.87*	-0.64*	1.42*	0.17	1.95*	0.000
5-year	1.01*	-0.67*	1.3*	0.55*	1.07*	-0.74*	1.34*	0.73*	-0.76*	0.120
6-year	1*	-0.43*	1.14*	0.41*	0.96*	-0.37*	1.1*	0.27*	0.58*	0.140
7-year	0.99*	0.05*	0.75*	-0.11*	0.99*	0.06	0.75*	-0.12	0.02	1.000
8-year	0.97*	0.13*	0.51*	0	0.89*	0.27*	0.38*	-0.31*	0.99*	0.000
9-year	0.96*	0.12*	0.64*	-0.17*	0.75*	0.47*	0.37*	-0.99*	2.74*	0.000
10-year	1.01*	0.34*	0.69*	-0.28*	1.11*	0.18*	0.82*	0.15	-1.24*	0.000

Panel B: Out of the money caps and floors

Maturity	c1	c2	c3	c4	d1	d2	d3	d4	d5	p-value
Ask										
2-year	1.1*	0.19*	0.7*	0.21	0.98*	-0.03	-2.25*	0.92*	0.88*	0.000
3-year	1.08*	0.31*	0.1*	-0.31*	0.98*	-0.4*	-0.9*	0.7*	0.95*	0.000
4-year	1.07*	0.34*	0.05	-0.25*	1.02*	0.14*	-0.37*	-0.04	0.49*	0.000
5-year	1.08*	0.37*	0.02	-0.26*	1.04*	0.02	-0.11*	0.18*	0.49*	0.000
6-year	1.07*	0.28*	0.03	-0.14*	1.03*	0.02	-0.13*	0.2*	0.54*	0.000
7-year	1.09*	0.3*	0.03	-0.19*	1.05*	0.14*	-0.09*	0.03	0.41*	0.000
8-year	1.05*	0.27*	0.02	-0.18*	1.01*	0.07*	-0.09*	0.1*	0.49*	0.000
9-year	1.03*	0.27*	0.04*	-0.2*	1*	0.06*	-0.04*	0.1*	0.51*	0.000
10-year	1.1*	0.28*	0.06*	-0.19*	1.05*	0.02	-0.01	0.21*	0.64*	0.000
Bid										
2-year	0.96*	0.42*	-0.33*	-0.29*	0.94*	0.38*	-0.86*	-0.17	0.16*	0.180
3-year	0.97*	0.29*	-0.15*	-0.4*	0.95*	0.16*	-0.33*	-0.22*	0.17*	0.000
4-year	0.98*	0.3*	-0.1*	-0.31*	0.98*	0.3*	-0.11*	-0.3*	0.01	0.950
5-year	0.99*	0.24*	0	-0.19*	0.99*	0.22*	-0.01	-0.16*	0.03*	0.220
6-year	0.99*	0.23*	-0.02	-0.15*	0.98*	0.17*	-0.05*	-0.07*	0.12*	0.000
7-year	1*	0.27*	-0.04*	-0.23*	1*	0.27*	-0.04*	-0.22*	0.01	0.920
8-year	0.96*	0.24*	-0.03*	-0.2*	0.95*	0.2*	-0.05*	-0.14*	0.1*	0.000
9-year	0.95*	0.24*	-0.01	-0.22*	0.94*	0.19*	-0.02*	-0.16*	0.11*	0.000
10-year	1.01*	0.24*	0.02	-0.19*	1.01*	0.26*	0.02*	-0.21*	-0.03*	0.340

Table 7

Effects of economic variables on volatility smiles

This table presents regression results for the impact of economic and liquidity variables on the curvature of the volatility smile (as proxied by the butterfly spread - BS) and asymmetry in the volatility smile (as proxies by risk reversal - RR):

$$Butterfly\ Spread = c1 + c2 * ATMVol + c3 * 6Mrate + c4 * 5yr6Mslope + c5 * 10yr5yrslope$$
$$+ c6 * DefSpread + c7 * atmBAspread + c8 * Average\ awayBAspread$$

$$Risk\ \mathrm{Re}versal = d1 + d2 * ATMVol + d3 * 6Mrate + d4 * 5yr6Mslope + d5 * 10yr5yrslope$$
$$+ d6 * DefSpread + d7 * atmBAspread + d8 * Difference\ in\ awayBAspread$$

The statistics are presented for the entire sample period, Jan 99 - May 01, for various maturities, based on data obtained from WestLB Global Derivatives and Fixed Income Group. The coefficients and regression statistics are presented for the pooled sample of caps and floors, separately for bid and ask prices, for all maturities. Lagged error terms are included in the regression equation to correct for serial correlation. For bid-ask spread variables, we use the fitted values using 2-stage least square to correct for the errors-in-variables problem in these two variables. Asterisk indicates statistical significance at the 5% level.

Panel A: Ask Side

BS	c1	c2	c3	c4	c5	c6	c7	c8	Adj R^2
2-year	1.52*	0.01	-13.14*	-28.47*	-30.23*	0.00	2.48*	-1.40*	0.79
3-year	1.05*	0.00	0.77	-11.29*	-8.14	0.00	-0.30	-0.59	0.88
4-year	0.66*	0.01	-4.13*	-5.90*	-2.57	0.00	-0.30	1.38*	0.81
5-year	0.94*	0.00	-3.37	-8.72*	-14.05	0.00	-0.48	0.22*	0.79
6-year	0.56*	0.00	1.41	-3.39*	4.11*	0.00	-0.54	0.96*	0.98
7-year	1.21*	0.00	-0.30	-8.31*	-5.62	0.00	-5.99*	0.93*	0.73
8-year	0.53*	0.00	0.74	0.30	0.25	0.00	-0.12	0.68*	0.94
9-year	0.68*	-0.01*	2.07	-0.20	-3.09	0.00	-1.21	0.43*	0.91
10-year	0.66*	0.01	-2.79*	-4.66*	15.79*	0.00	-0.92	0.62*	0.87
RR	**d1**	**d2**	**d3**	**d4**	**d5**	**d6**	**d7**	**d8**	**Adj R^2**
2-year	0.60*	0.01	-17.66*	-33.10*	-27.08*	0.00	-0.23	0.33	0.54
3-year	-0.09	0.02*	-11.12*	-1.21	-3.98	0.00*	-0.52	1.29*	0.82
4-year	0.12	0.01	-5.14	-17.41*	-14.96*	0.00	0.26	0.58	0.84
5-year	-0.10	0.01	-5.63*	0.59	5.76	0.00	0.22	1.23*	0.70
6-year	0.28	0.01	-13.45*	-7.51	-17.44*	0.00	3.36	0.60*	0.84
7-year	0.25	0.00	-5.52*	-10.49*	9.74*	0.00	0.77	-0.69*	0.96
8-year	0.14	0.00	-1.50	-2.00	-0.17	0.00*	-0.25	-0.41*	0.93
9-year	0.34*	0.00	-3.94*	-9.88*	-0.11	0.00*	0.71	-1.01*	0.97
10-year	1.01*	-0.01	-13.80*	-13.12*	-5.76	0.00	0.00	-1.70*	0.96

Panel B: Bid Side

BS	c1	c2	c3	c4	c5	c6	c7	c8	Adj R^2
2-year	1.94*	-0.02	-7.87	-14.61	-14.52	0.00	-0.03	-1.42*	0.87
3-year	0.87*	-0.01	1.08	-9.01*	-7.31	0.00	-0.87	-0.22	0.86
4-year	0.56	-0.01	6.17	0.16	6.31	0.00	2.84	-3.17*	0.63
5-year	0.55*	0.00	-1.74	0.66	-5.73	0.00	0.06	-0.10	0.55
6-year	0.52*	0.00	1.16	-2.24*	4.88	0.00	-0.18	-0.02	0.88
7-year	0.99*	0.00	1.24	-5.70*	-3.32	0.00	-5.74*	0.53*	0.54
8-year	0.41*	0.00	2.44*	1.79	0.87	0.00	-0.63	0.55*	0.93
9-year	0.50*	0.00	2.49*	1.31	-2.39	0.00	-1.14*	0.41*	0.90
10-year	0.40*	0.01	-2.03*	-1.50	16.79*	0.00	-0.49	0.69*	0.83
RR	**d1**	**d2**	**d3**	**d4**	**d5**	**d6**	**d7**	**d8**	**Adj R^2**
2-year	2.49	0.00	-67.74	-37.06*	-27.90	0.00	-2.00*	1.16*	0.52
3-year	-0.36*	0.01	4.03	8.32*	10.40*	0.00*	-1.34*	-0.09	0.89
4-year	-0.32*	0.00	2.43	7.38*	11.78*	0.00	0.61	-0.56	0.66
5-year	0.17	-0.01	-6.52*	3.31	5.08	0.00	0.34	0.00	0.88
6-year	0.03	0.01	-1.38	1.10	10.17*	0.00*	-2.97*	0.72*	0.94
7-year	0.19	0.01	-5.55*	-6.32*	3.87	0.00	0.13	0.08	0.95
8-year	0.10	0.01*	-1.86	-2.18	-1.60	0.00	-0.59	-0.10	0.92
9-year	0.58*	0.00	-8.72*	-11.83*	-3.68	0.00*	0.52	-0.90*	0.97
10-year	0.77*	0.00	-9.67*	-9.27*	-6.27	0.00	-0.23	-1.31*	0.95

Table 8

Linear Bivariate Granger Causality Tests – Smile and Liquidity

This table presents results for linear, bivariate, Granger-causality tests, for various maturities. The p-values for rejecting the null hypothesis of "No Granger Causality" are given below. Asterisk represents p-value less than or equal to 5%. Three bid-ask spread variables are used as proxy for liquidity – ATM, Average of ITM & OTM and Difference between ITM & OTM. We use the fitted values using 2-stage least square to correct for the errors-in-variables problem in these variables.

Null Hypothesis 1 – the liquidity variables do not individually Granger cause the butterfly spread (BS) / risk reversal (RR) on the ask / bid side
Null Hypothesis 2 – Butterfly spread (BS) / risk reversal (RR) on the ask / bid side does not Granger cause each of the liquidity variables

	Liquidity to Smile (Null Hypothesis 1)				Smile to Liquidity (Null Hypothesis 2)			
	Ask		Bid		Ask		Bid	
BS	ATM	Average of ITM & OTM	ATM	Average of ITM & OTM	ATM	Average of ITM & OTM	ATM	Average of ITM & OTM
2-year	0.37	0.87	0.99	0.83	0.16	0.45	0.29	0.92
3-year	0.00*	0.01*	0.01*	0.01*	0.41	0.36	0.24	0.37
4-year	0.09	0.06	0.20	0.63	0.01*	0.34	0.00*	0.13
5-year	0.16	0.00*	0.22	0.00*	0.14	0.00*	0.66	0.02*
6-year	0.15	0.14	0.12	0.06	0.00*	0.18	0.01*	0.09
7-year	0.01*	0.07	0.04*	0.20	0.43	0.56	0.65	0.54
8-year	0.56	0.76	0.67	0.92	0.35	0.45	0.57	0.48
9-year	0.51	0.24	0.62	0.38	0.02*	0.46	0.46	0.57
10-year	0.00*	0.10	0.01*	0.14	0.23	0.28	0.23	0.20
RR	ATM	Difference between ITM & OTM	ATM	Difference between ITM & OTM	ATM	Difference between ITM & OTM	ATM	Difference between ITM & OTM
2-year	0.45	0.13	0.96	0.49	0.30	0.43	0.81	0.37
3-year	0.45	0.84	0.36	0.54	0.89	0.72	1.00	0.47
4-year	0.13	0.15	0.38	0.09	0.00*	0.40	0.74	0.10
5-year	0.17	0.04*	0.91	1.00	0.05*	0.03*	0.10	0.01*
6-year	0.00*	0.78	0.00*	0.78	0.90	0.32	0.87	0.35
7-year	0.66	0.44	0.98	0.69	0.12	0.00*	0.00*	0.14
8-year	0.22	0.55	0.55	0.64	0.03*	0.00*	0.06	0.41
9-year	0.00*	0.06	0.00*	0.01*	0.01*	0.39	0.00*	0.01*
10-year	0.00*	0.07	0.10	0.03*	0.00*	0.89	0.01*	0.48

Table 9

Linear Bivariate Granger Causality Tests – Smile and other variables

This table presents results for linear, bivariate, Granger-causality tests, for various maturities. The p-values for rejecting the null hypothesis of "No Granger Causality" are given below. Asterisk represents p-value less than or equal to 5%.

Panel A: Null Hypothesis – presented variables do not individually Granger cause the butterfly spread (BS) / risk reversal (RR) on the ask / bid side

	Ask					Bid				
BS	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	10yr rate - 5yr rate	Default Spread (6m)	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	10yr rate - 5yr rate	Default Spread (6m)
2-year	0.10	0.38	0.16	0.86	0.61	0.10	0.21	0.18	0.78	0.79
3-year	0.02*	0.39	0.31	0.18	0.66	0.19	0.56	0.28	0.38	0.54
4-year	0.00*	0.50	0.01*	0.07	1.00	0.11	0.87	0.95	0.35	0.53
5-year	0.00*	0.87	0.09	0.02*	0.88	0.22	0.69	0.57	0.22	0.74
6-year	0.83	0.04*	0.00*	0.00*	0.73	0.93	0.15	0.32	0.00*	0.33
7-year	0.03*	0.77	0.78	0.01*	0.49	0.05*	0.86	0.89	0.00*	0.07
8-year	0.21	0.65	0.58	0.96	0.88	0.14	0.08	0.38	0.94	0.97
9-year	0.31	0.83	0.92	0.57	0.76	0.57	0.52	0.94	0.92	0.55
10-year	0.05*	0.90	0.17	0.14	0.14	0.10	0.90	0.24	0.13	0.14
RR	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	10yr rate - 5yr rate	Default Spread (6m)	ATM Vol.	6 m Rate	5 yr rate– 6 m Rate	10yr rate - 5yr rate	Default Spread (6m)
2-year	0.94	0.56	0.76	0.42	0.39	0.75	0.09	0.13	0.03*	0.21
3-year	0.63	0.31	0.16	0.72	0.29	0.70	0.08	0.04*	0.23	0.31
4-year	0.02*	0.16	0.08	0.02*	0.11	0.76	0.39	0.63	0.37	0.86
5-year	0.35	0.18	0.34	0.04*	0.69	0.87	0.66	0.98	0.97	0.26
6-year	0.15	0.19	0.51	0.00*	0.34	0.18	0.16	0.28	0.00*	0.29
7-year	0.75	0.75	0.95	0.55	0.12	0.97	0.01*	0.39	0.31	0.05*
8-year	0.02*	0.01*	0.70	0.24	0.21	0.03*	0.06	0.93	0.96	0.18
9-year	0.41	0.00*	0.05	0.01*	0.25	0.12	0.31	0.13	0.18	0.20
10-year	0.52	0.00*	0.87	0.86	0.01*	0.24	0.00*	0.39	0.38	0.02*

Panel B: Null Hypothesis – Butterfly spread (BS) / risk reversal (RR) on the ask / bid side does not Granger cause each of the presented variables

	Ask					Bid				
BS	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	10yr rate - 5yr rate	Default Spread (6m)	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	10yr rate - 5yr rate	Default Spread (6m)
2-year	0.90	0.27	0.01*	0.33	0.39	0.42	0.07	0.00*	0.03*	0.45
3-year	0.00*	0.14	0.93	0.48	0.54	0.03*	0.09	0.29	0.32	0.87
4-year	0.00*	0.32	0.52	0.00*	0.04*	0.16	0.03*	0.01*	0.04*	0.42
5-year	0.01*	0.54	0.13	0.00*	02	0.03*	0.30	0.11	0.00*	0.64
6-year	0.00*	0.48	0.08	0.00*	0.55	0.00*	0.74	0.70	0.00*	0.97
7-year	0.00*	0.36	0.49	0.02*	0.09	0.01*	0.64	0.56	0.01*	0.06
8-year	0.02*	0.11	0.01*	0.00*	1.00	0.00*	0.18	0.06	0.00*	0.45
9-year	0.50	0.26	0.06	0.00*	0.78	0.57	0.60	0.07	0.00*	0.59
10-year	0.00*	0.64	0.80	0.01*	0.11	0.00*	0.59	0.68	0.00*	0.25
RR	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	10yr rate - 5yr rate	Default Spread (6m)	ATM Vol.	6 m Rate	5 yr rate – 6 m Rate	10yr rate - 5yr rate	Default Spread (6m)
2-year	0.17	0.39	0.22	0.82	0.13	0.76	0.56	0.49	0.79	0.34
3-year	0.31	0.20	0.17	0.18	0.04*	0.59	0.46	0.14	0.09	0.44
4-year	0.09	0.70	0.53	0.29	0.23	0.00*	0.39	0.41	0.01*	0.16
5-year	0.82	0.88	0.65	0.86	0.02*	0.12	0.24	0.28	0.01*	0.17
6-year	0.52	0.44	0.08	0.62	0.58	0.52	0.96	0.22	0.77	0.33
7-year	0.02*	0.02*	0.01*	0.00*	0.07	0.39	0.13	0.03*	0.00*	0.18
8-year	0.09	0.10	0.00*	0.01*	0.32	0.03*	0.00*	0.00*	0.00*	0.83
9-year	0.19	0.26	0.01*	0.00*	0.76	0.12	0.01*	0.01*	0.00*	0.61
10-year	0.33	0.29	0.05	0.00*	0.42	0.18	0.10	0.01*	0.00*	0.81

Figure 1

Implied volatility smiles in interest rate caps and floors

This figure presents scatter plots of the implied flat volatilities of euro interest rate caps and floors over our sample period. The vertical axis in the plots corresponds to the implied volatility of the mid-price (average of bid and ask price) of the option, scaled by the at-the-money volatility for the option of similar maturity. The horizontal axis in the plots corresponds to the logarithm of the moneyness ratio, defined as the ratio of the par swap rate to the strike rate of the option. The plots are for three representative maturities - 2-year, 5-year, and 10-year for the entire sample period, Jan 99 - May 01, based on data obtained from WestLB Global Derivatives and Fixed Income Group.

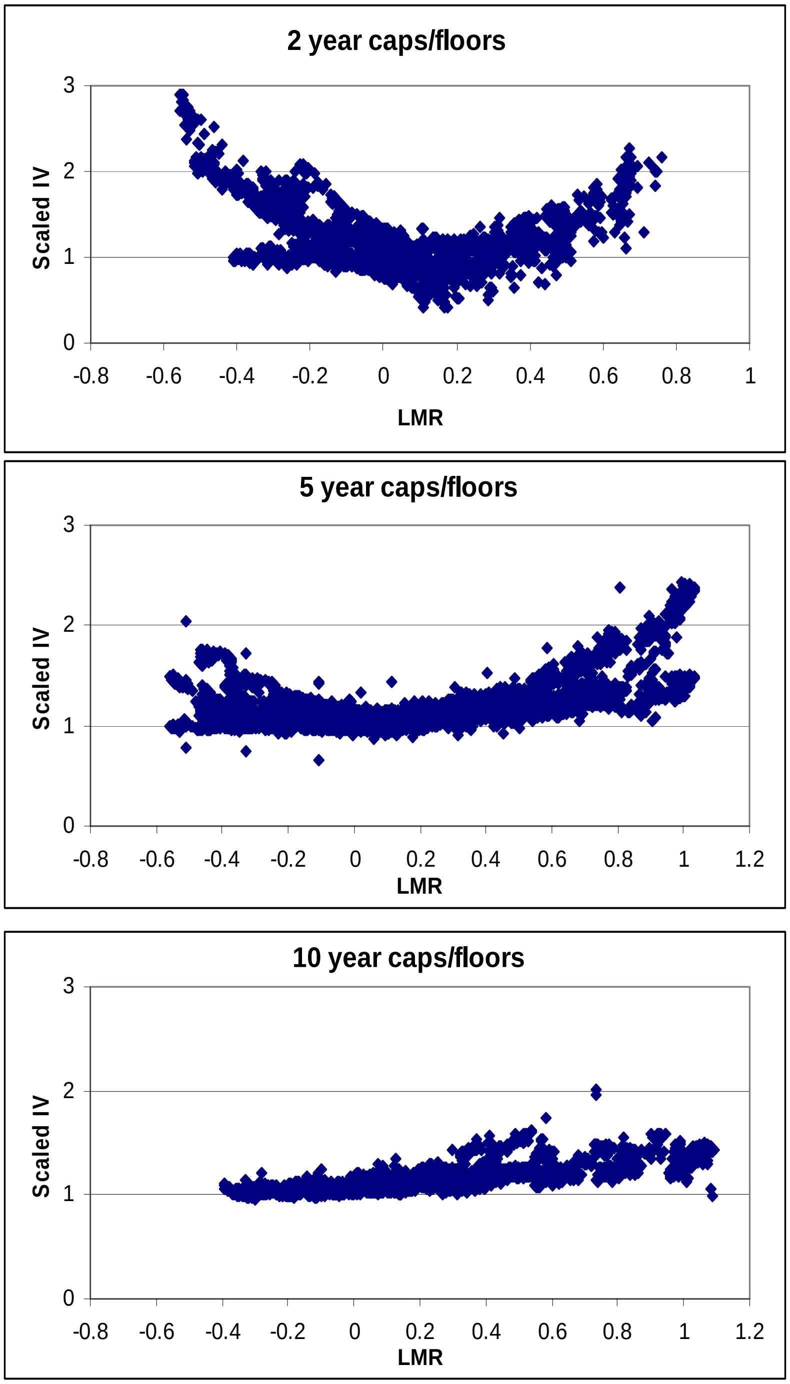


Figure 2

Functional forms of implied volatility smiles in interest rate caps and floors

This figure presents the fitted smile functions for the bid and ask implied flat volatilities of euro interest rate caps and floors separately, across different maturities. The vertical axis in the plots corresponds to the implied flat volatility of the bid and ask prices of the option, scaled by the at-the-money volatility for the option of similar maturity (Scaled IV) calculated using the regression model in Table VI. The horizontal axis in the plots corresponds to the logarithm of the moneyness ratio (LMR), defined as the ratio of the par swap rate to the strike rate of the option. The plots are three representative maturities - 2-year, 5-year, and 10-year for the entire sample period, Jan 99 - May 01, based on data obtained from WestLB Global Derivatives and Fixed Income Group.

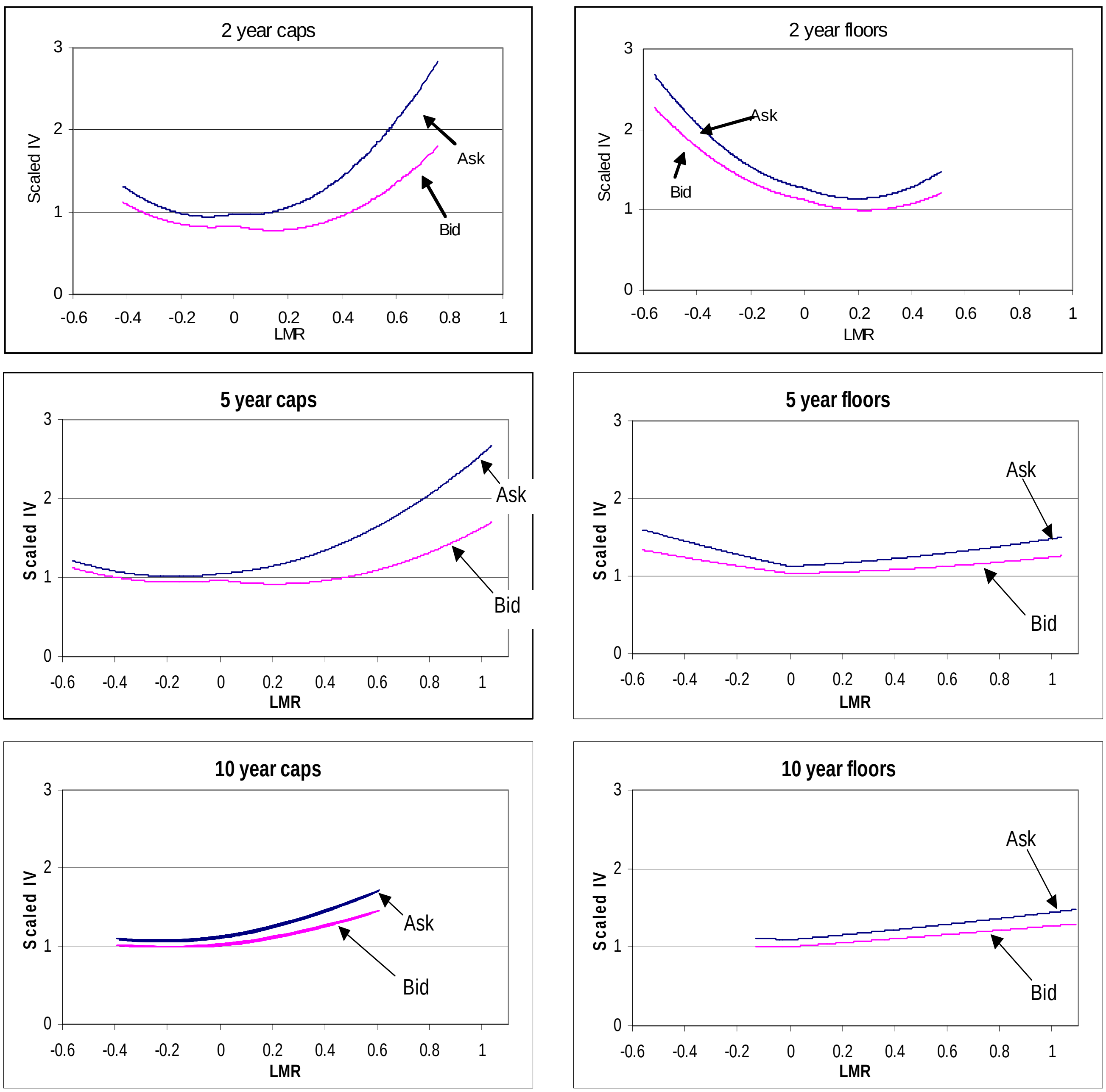


Figure 3

Functional forms of implied volatility smiles using pooled data from interest rate caps and floors

This figure presents the fitted smile functions for the bid and ask implied flat volatilities of euro interest rate caps and floors pooled, for different maturities. The vertical axis corresponds to the implied flat volatility of the bid and ask prices of the option, scaled by the at-the-money volatility for the option of similar maturity (Scaled IV) calculated using the regression model in Table VII. The horizontal axis in the plots corresponds to the logarithm of the moneyness ratio (LMR), defined as the ratio of the par swap rate to the strike rate of the option. The plots are for three representative maturities - 2-year, 5-year, and 10-year for the entire sample period, Jan 99 - May 01, based on data obtained from WestLB Global Derivatives and Fixed Income Group.

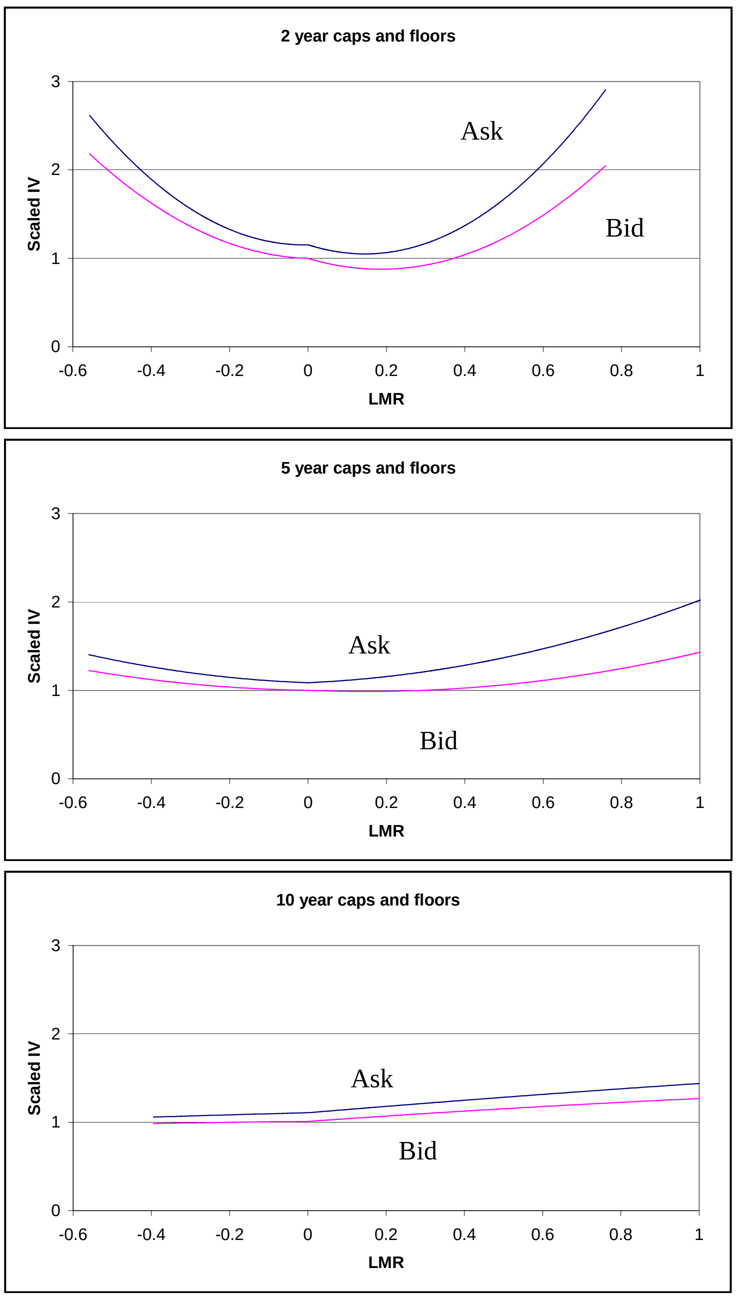
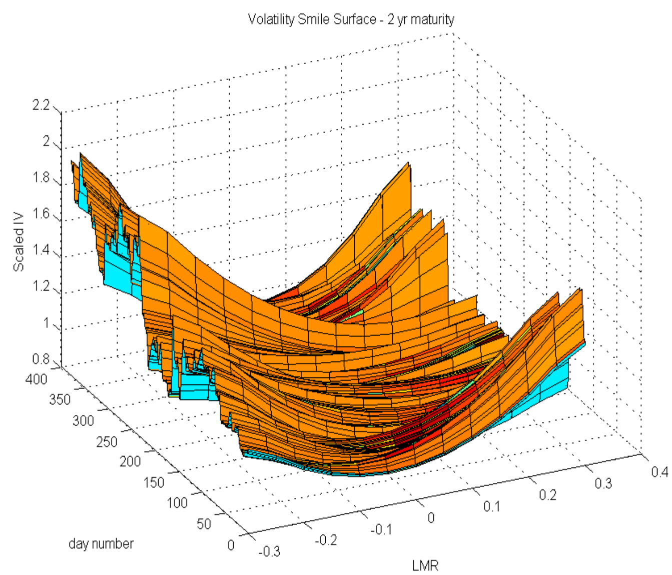


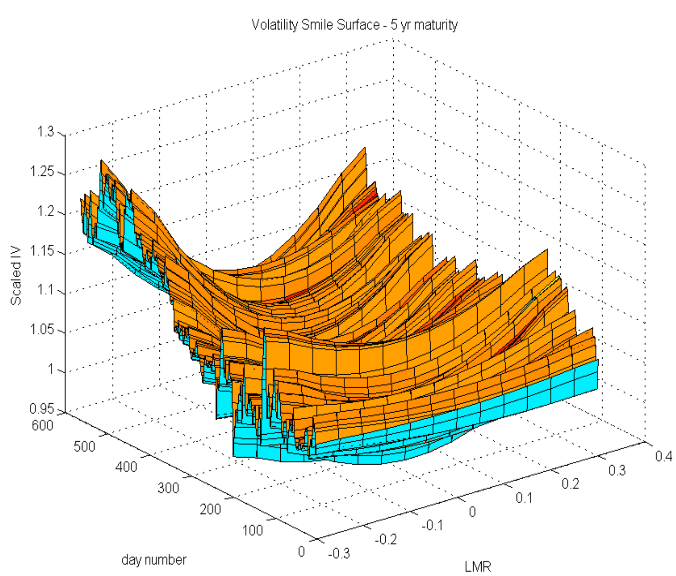
Figure 4

Time variation in volatility smiles and the Euro term structure

This figure presents surface plots showing the time variation in the implied flat volatilities of euro interest rate caps and floors as well as the Euro term structure over our sample period (Jan 99 - May 01), using data obtained from WestLB Global Derivatives and Fixed Income Group. In figure 4A, the first three plots (for three representative maturities - 2-year, 5-year, and 10-year), the vertical axis corresponds to the implied volatility of the mid-price (average of bid and ask price) of the option, scaled by the at-the-money volatility for the option of similar maturity. The horizontal axes in these plots correspond to the logarithm of the moneyness ratio (defined as the ratio of the par swap rate to the strike rate of the option), and time. In figure 4B, the fourth plot depicts the Euro spot rate surface by maturity (in years) over time (daily).

Figure 4A





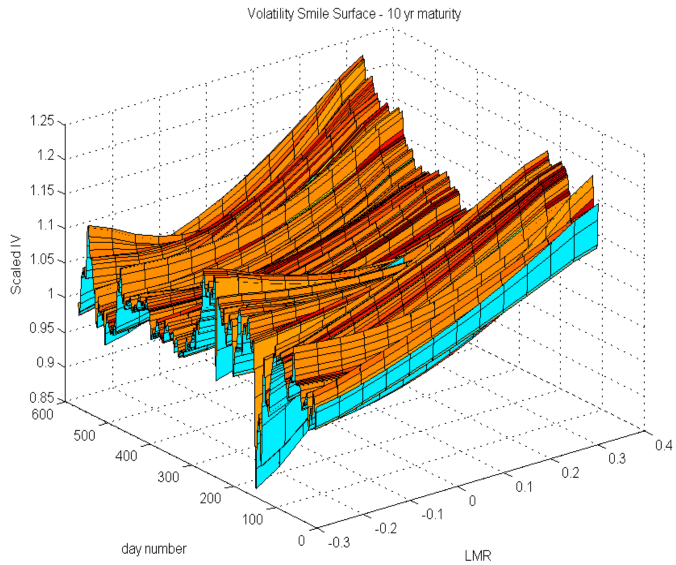

Volatility Smile Surface - 10 yr maturity
Scaled IV
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
600
500
400
300
200
100
0
day number
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
LMR

Figure 4B

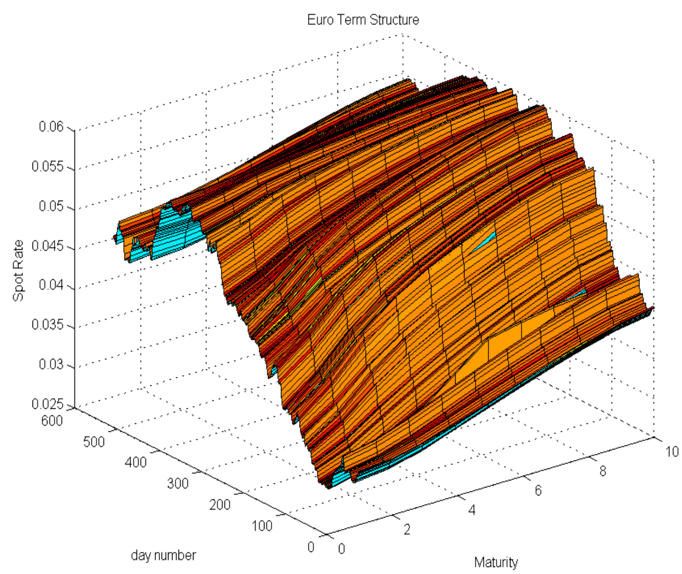

Euro Term Structure
Spot Rate
0.06
0.055
0.05
0.045
0.04
0.035
0.03
0.025
600
500
400
300
200
100
0
day number
0
2
4
6
8
10
Maturity

Figure 5

Calculation of the butterfly spread and risk reversal

This figure explains the calculation of the "butterfly spread" and the "risk reversal" for each day for each maturity. The points marked "Cap" and "Floor" are the observed scaled implied volatilities from cap and floor prices for that day. They are plotted against their LMR. The scaled IV at -0.25 LMR is calculated by linearly interpolating scaled IVs at two adjacent points. Similarly, the scaled IV at +0.25 LMR is calculated. The risk reversal is the difference between scaled IV at +0.25 LMR and -0.25 LMR. The butterfly spread is the difference between the average of these two away-from-the-money volatilities and the at-the-money volatility