Empirical tests of interest rate model pricing kernels

May 1999

Joshua V. Rosenberg
Department of Finance
NYU - Stern School of Business
44 West 4th Street, Suite 9-190
New York, New York 10012-1126
(212) 998-0311

Abstract

This paper estimates and tests consumption-based pricing kernels used in common equilibrium interest rate term structure models. In contrast to previous papers that use return orthogonality conditions, estimation in this paper is accomplished using moment conditions from a consumption-based option pricing equation and market prices of interest rate options. This methodology is more sensitive to preference misspecification over states associated with large changes in consumption than previous techniques. In addition, this methodology provides a large set of natural moment conditions to use in estimation and testing compared to an arbitrary choice of return orthogonality conditions (e.g. instruments selected) used in GMM estimation.

Eurodollar futures option prices and an estimated joint model of quarterly aggregate consumption and three month Eurodollar rates suggest are used to estimate and test pricing kernels based on logarithmic, power, and exponential utility functions. Using the market prices of interest rate options, evidence is found which is consistent with the equity premium puzzle; very high levels of risk aversion are needed to justify the observed premium associated with an investment position positively correlated with aggregate consumption. In addition, evidence is found which is consistent with the riskfree rate puzzle: at high levels of risk-aversion for power or exponential utility, negative rates of time preference are needed to fit the observed low riskless interest rates.

These results suggest that typical term structure models are misspecified in terms of assumed preferences. This may have deleterious effects on model estimates of the interest rate term structure estimates and interest rate option prices.

This paper has benefited from the suggestions of Joel Hasbrouck, Jose Lopez, Anthony Lynch, seminar participants at New York University - Stern School of Business, and the 1999 Derivatives Securities Conference. The author wishes to thank David Hait for providing data used in an earlier version of the paper.

I. Introduction

Interest rate term structure models depend on an explicit or implicit specification of investor preferences. Examples of explicit preference specifications include: specification of the pricing kernel, the market price of risk, or the utility function of a representative agent. Implicit preference specifications are reflected in the choice of a risk-neutral interest rate process, which, together with the original interest rate process, implies a set of supporting preferences.

The specification of investor preferences plays an important role in predictions of the interest rate term structure. Campbell (1986) shows how the term structure is a function of the coefficient of relative risk aversion under constant relative risk aversion utility. Stanton (1997) non-parametrically estimates the market price of risk as a function of the interest rate level, and finds that the market price of risk has a significant effect on the prices of long term bonds.

The power of consumption-based pricing kernels in explaining characteristics of the interest rate term structure has been investigated in the existing literature. For example, Boudoukh (1993) as well as Canova and Marrinan (1996) consider pricing kernel based on a representative investor with a power utility function. Using restrictions on bond returns, Ferson (1983) finds stronger evidence against exponential utility than power utility. Dunn and Singleton (1986) find that non-separable utility explains some bond return characteristics, but they also find evidence against this model.

Outside the specific context of interest rate term structure models, consumption-based pricing kernels are estimated and tested using return orthogonality conditions by Hansen and Singleton (1982, 1993), Cochrane and Hansen (1992), Gallant, Hansen, and Tauchen (1990), and Chapman (1997). Pricing kernel estimation approaches that utilize asset returns instead of consumption data to identify the pricing kernel include Bansal and Viswanathan (1993) and Backus, Gregory, and Zin (1997).

Several papers use option price data to identify pricing kernels defined over equity return states. These papers estimate the pricing kernel as the ratio of state prices measured with option data and state probabilities measured with historical returns data. Jackwerth (1997) and Ait-Sahalia and Lo (1997) estimate average pricing kernels, while Rosenberg and Engle (1999) estimate a time-varying pricing kernel. While these papers demonstrate the usefulness of option price data in identifying the pricing kernel, they do not link estimated pricing kernels to aggregate consumption states, so they cannot be used to directly test consumption-based utility specifications.

In contrast to previous papers that use return orthogonality conditions, estimation in this paper is accomplished using moment conditions from a consumption-based option pricing equation and market prices of interest rate options. This methodology offers several advantages over previous approaches.

First, this methodology is more sensitive to preference misspecification over states associated with large changes in consumption than previous techniques. Previous papers utilize moment conditions based on equity or bond returns. These moment conditions are based on expectations taken over all future return and consumption states. Hence, they reflect average risk aversion over all consumption states. Moment conditions based on out-of-the-money option prices depend on preferences (and probabilities) for large changes in the underlying asset price, which are associated with large changes in consumption. Prices of assets that provide insurance against low consumption states are very sensitive to risk aversion; thus, these assets are especially useful in identifying preference misspecification.

Second, this methodology provides a large set of natural moment conditions to use in estimation and testing compared to an arbitrary choice of return orthogonality conditions (e.g. instruments selected) used in GMM estimation. Each observed option price along with a consumption-based derivative pricing formula provides a moment condition. The magnitude of derivative pricing errors also provides a useful metric to evaluate the pricing kernel specification; this metric has a more intuitive interpretation than errors from return orthogonality conditions.

This paper solves the inverse of the equilibrium asset pricing problem to identify preference parameters; preference parameters are estimated such that they are consistent with observed prices and estimated payoff probabilities. In particular, preference parameters are selected to minimize interest rate option pricing error, given the estimated joint density of future consumption and interest rates. This technique is similar to Heaton's (1995) two-stage simulated method of moments approach. Evidence is provided (in section IV.d) that the results in the paper are robust with respect to the specification of the joint density function.

Pricing kernel specifications are then tested by analyzing the magnitude of derivative pricing error and predictability of the pricing errors. Under the null hypothesis of a correctly specified pricing kernel and probability model, the pricing errors — due to bid-ask spread and price discretization — should be white noise. This methodology extends the research on consumption-based asset pricing beyond equities and bonds to derivative assets revealing the impact of interest rate model pricing kernel misspecification on interest rate derivative prices.

The important empirical findings of this paper are as follows. First, evidence is found which is consistent with the equity premium puzzle: very high levels of risk aversion are needed to justify the observed premium associated with owning an asset positively correlated with aggregate consumption. While the equity premium puzzle is typically framed using aggregate consumption data and equity returns, this paper provides another perspective using aggregate consumption data and market prices of interest rate options. This finding also points to a possible source of error in interest rate term structure forecasts and interest rate option prices from log-utility-based models such as Cox, Ingersoll, and Ross (1986) or Longstaff and Schwartz (1992).

Second, evidence is found which is consistent with the riskfree rate puzzle: at high levels of absolute or relative risk-aversion, negative rates of time preference are needed to fit the observed low riskless interest rates. This finding is especially troubling in the context of interest rate term structure models which are very sensitive to the rate of time preference.

Third, none of the estimated pricing kernels passes a specification test based on the predictability of pricing errors; all of the models underprice interest rate calls. Even with a high level of risk aversion (which increases the fitted value of interest rate calls which insure against low consumption states), the power and exponential specifications are inadequate.

This paper is organized as follows. Section II examines the use of consumption-based pricing kernels in interest rate derivative models and interest rate term structure models. Section III describes the pricing kernel estimation and testing technique. Section IV presents the pricing kernel estimation results and specification tests. Section V concludes the paper.

II.a. Pricing kernels and interest rate derivative valuation

Equilibrium term structure models typically posit a representative investor maximizing his or her expected utility subject to a budget constraint. In a representative agent framework, the following first order condition — which defines a general asset pricing equation — is obtained. See, for example, Constantinides (1989).

(1) $$X_t = E_t[K_{t,T}(C_T)g(X_T)]$$

Equation (1) states that the current price of an asset (X_t) is the expectation of it's pricing-kernel-weighted payoffs. The pricing kernel — $K_{t,T}(C_T)$ — is defined over future consumption-states (C_T) and is decreasing in consumption, while the payoff function — $g(X_T)$ — depends on the characteristics of the asset (e.g. the identity function for a linear asset) and the future asset value (X_T).

When the representative investor has time-separable utility, the pricing kernel may be written as the marginal rate of substitution between current and future consumption multiplied by a subjective discount factor. The marginal rate of substitution embodies the risk-discounting of payoffs, while the subjective discount factor embodies the time-discounting of payoffs.

Equation (1) may be used for derivative pricing by replacing general payoff function with the call or put payoff function. The (real) price of a call or put option (D_t), with an underlying asset with terminal price r_T, and which has a date T payoff given by $g(r_T)$ is:

$$(2) \qquad D_t = E_t[K_{t,T}(C_T)g(r_T)] = \iint K_{t,T}(C_T)g(r_T)f_{C,r,t}(C_T, r_T)dC_T dr_T$$

As written, evaluation of the equation (1) or equation (2) consumption-based pricing kernel valuation formula requires specification of the pricing kernel, the joint density of consumption and the payoff random variable, and evaluation of the double integral. This pricing formula is considered in Rubinstein (1976), Brennan (1979), Stapleton and Subramanyam (1984), Amin and Ng (1993), Duan (1995), and Satchell and Timmerman (1997). In all of these papers, preference parameters are substituted out of the final pricing formula for observable market variables.

A convenient feature of these "preference-free" pricing formulas is that preference parameters and the moments of the consumption process do not need to be estimated to obtain option prices. However, the preference and probability assumptions are restrictive. Usually, constant relative risk aversion and joint-lognormality of consumption and the underlying asset price is required. In the last three papers, certain types of stochastic volatility are permitted. However, these papers require that the innovations to the underlying asset price process are normal. Thus, over a single time period, price jumps and other "fat-tailed" behavior are ruled out.

The interest rate option pricing problem has been solved in "preference-free" form in the context of specific continuous-time interest rate dynamics and (logarithmic) preferences of a representative agent. See, for example, Courtadon (1982), Cox, Ingersoll, and Ross (1985), and Longstaff and Schwartz (1992). In some sense, the “preference-free”-ness of these models is by assumption, since log-utility is a special case of constant relative risk aversion with the coefficient of relative risk aversion set to unity.

In a general stochastic setting, preference parameters will appear in the option pricing formula. Thus, equation (2), along with an observed option price, provides a moment condition that may be used to identify a preference parameter. With an arbitrary specification of the stochastics as well as pricing kernel state variables and functional form, the pricing formula may be numerically inverted with the double integral evaluated by simulation to obtain estimates of pricing kernel parameters.

Equations (1) and (2) also show how the choice of asset affects pricing kernel estimates. Assets such as bonds or equities are priced by integrating over the entire range of future prices (payoffs). Options — which have non-zero payoffs only over a segment of the range of future underlying prices — are priced by integrating over the range of in-the-money states. Hence, option prices depends only on preferences (and probabilities) over a particular payoff state range.

Particular state ranges of interest include those associated with large changes in the underlying price (e.g. an interest rate or equity index level). Since these states may be associated with low consumption, these are the states for which investors may demand insurance. Hence, the prices of the insuring assets will be highly sensitive to investor state preferences. Pricing kernel estimates over equity index return states by Ait-Sahalia and Lo (1997) and Rosenberg and Engle (1998) indicate that some of characteristics of the cross-section of equity index option prices are due to exceptionally high demand for payoffs in low consumption (large negative equity index return) states.

Interest rates are negatively correlated with aggregate consumption, so high levels of risk aversion should have the greatest effect on out-of-the-money interest rate calls. These options have positive payoffs when there is a large increase in interest rates, which is associated with a large decrease in consumption.

While the equation (1) and (2) pricing formulas are appropriate for pricing assets in real terms, market prices are quoted in nominal terms. Using the cash-in-advance model of Lucas (1980) — see also Boudoukh (1993) and Canova and Marrinan (1996) — equation (1) may be generalized to give the nominal price of an asset as:

(3) $$X_t = E_t[K_{t,T}(C_T)g(X_T)I_{t,T}^{-1}]$$

Equation (3) incorporates the effects of inflation by including the reciprocal of the realized gross inflation rate ($I_{t,T}^{-1}$) in the pricing formula. A higher expected inflation rate (when inflation is uncorrelated with consumption and the payoff variable) will result in a lower current asset price. In practice, adding an additional stochastic variable considerably complicates pricing, since a triple integral, rather than a double integral, must be evaluated.

However, if inflation is sufficiently predictable, it may suffice to replace inflation as a random variable with it's realization. This is equivalent to assuming inflation is perfectly forecastable. This motivates the equation (4) approximation to equation (3), which states that the nominal asset price is the realized-inflation adjusted real price. Equation (4) will be referred to as the CPK (consumption-based pricing kernel) formula.

(4) $$X_t \cong I_{t,T}^{-1}E_t[K_{t,T}(C_T)g(X_T)]$$

In the empirical work in this paper, the condition of strong predictability of inflation is satisfied. The estimation period (1991-1998) is a period of low inflation (average rate of quarterly CPI growth of .42%) and low inflation volatility (standard deviation of quarterly CPI growth of .15%). This data is summarized in Table 1. Over one-quarter, which is the time-horizon for estimation, inflation is highly predictable. A regression of the quarterly inflation rate on eight lags using monthly data for 1947 - 1999 generates a standard deviation of prediction error (RMSE) of .28% and adjusted r-squared of 80%.

II.b. Pricing kernel specifications and interest rate models

Pricing kernels used in interest rate models are typically chosen for analytical convenience rather than empirical validity. This section describes the pricing kernels used in several of these models.

Consider the Cox, Ingersoll, and Ross (1985) or the Longstaff and Schwartz (1992) model of the interest rate term structure. In both models, it is assumed that there is a representative investor with a logarithmic consumption-based utility function. Let C_t be the representative investor's consumption on date t. Then, the pricing kernel (with rate of time preference ρ) is:

(5) $$K_{C,t}(C_T; r) = e^{-r(T-t)}(C_T / C_t)^{-1}$$

Campbell (1986) derives an interest rate term structure model based on a representative investor with power (constant relative risk aversion, CRRA) utility over consumption states. In this model, the pricing kernel (with rate of time preference ρ and coefficient of relative risk aversion γ) is:

(6) $$K_{C,t}(C_T; r,\ g) = e^{-r(T-t)}(C_T / C_t)^{-g}$$

Constantinides (1992) derives an interest rate term structure model based on a pricing kernel specification (and utility function) which allows an arbitrary number of unspecified state variables. A special case of the Constantinides pricing kernel is defined by an investor with exponential (constant absolute risk aversion, CARA) utility over consumption states. The pricing kernel (with rate of time preference ρ and coefficient of absolute risk aversion γ) is:

(7) $$K_{C,t}(C_T; r,\ g) = e^{-r(T-t)}e^{-g(C_T-C_t)}$$

III. Pricing kernel estimation and testing

Given the correct probability model and payoff function, a pricing kernel specification should be rejected if it misprices traded securities. This paper uses the CPK option pricing errors in pricing kernel specification tests.

Empirical implementation of the CPK valuation formula requires estimates of pricing kernel preference parameters (ρ and γ) and probabilities (the joint density of consumption and interest rates). Since aggregate consumption and interest rates are observable, there are a variety of estimation techniques that may be used to estimate their joint density.

In this paper, a bivariate time-series model is used to estimate the joint consumption and interest rate process. The forecast joint density is then obtained by monte-carlo simulation. Details of the model specification are given in the next section. The joint density of future consumption (C_T) and interest rates (r_T) — conditioned on the information set available today (t) and indexed by the parameter vector θ_1 — is denoted $f_{C,r,t}(C_T, r_T; q_1)$.

The pricing kernel parameter vector (θ_2) is obtained by numerical inversion of the CPK formula, in which the pricing kernel is denoted $K_{C,t}(C_T; q_2)$. Consider the CPK formula with a white noise error term reflecting observational error in derivative prices due to price discretization, bid-ask spread, and non-synchronous prices of the derivative and underlying asset.

(8) $$D_t = \int\int K_{C,t}(C_T; q_2) g(r_T) f_{C,r,t}(C_T, r_T; q_1) dC_T dr_T + e_t$$

Non-linear least squares regression of interest rate derivative prices on fitted values using the CPK formula will identify the preference parameter vector (θ_2). Let $D_{t,i}$ be the observed price of an interest rate derivative on date t and $\hat{D}_{t,i}(q_2)$ be the fitted derivative price using the CPK formula. The double integral in the CPK pricing equation may be evaluated numerically by averaging the pricing kernel weighted payoff at each (j=1..J) simulated terminal consumption and interest rate pair. This type of simulation-based option pricing is an extension of the approach of Boyle (1977).

(9) $$\hat{D}_{i,t}(q_2) \cong \frac{1}{J} \sum_{j=1...J} K_{C,t}(C_{T,j}; q_2) g_{i,r}(r_{T,j})$$

Then, the NLS regression may be written as:

(10) $$D_{t,i} = \hat{D}_{t,i}(q_2) + e_{t,i}$$

When the error term ($\varepsilon_{t,i}$) is normally distributed, the non-linear least squares estimator will obtain the maximum likelihood estimates of the pricing kernel parameter vector (θ_2). Otherwise, the NLS estimate will be a consistent estimate of the parameter vector.

Pricing kernel specification tests are based on the N (i=1...N) estimated pricing errors. The pricing error variance is used to test the goodness of fit of each proposed pricing kernel. The unbiasedness of pricing model estimates is tested using the t-statistic for the average pricing error for calls and puts. In addition, the economic plausibility of the estimated preference parameters is analyzed.

IV. Pricing kernel estimation and testing: empirical results

In order to implement the CPK formula, a model of the joint density of future consumption and interest rates is required, along with a dataset of interest rate option prices, and realized inflation rates. The CPK formula requires that the forecast dates for the consumption and interest rate density match the expiration dates of the options to be priced. For this reason, a discrete-time model of quarterly consumption and interest rates is estimated, and the one-quarter-ahead joint density forecast is used in the CPK equation to price options with one quarter until expiration.

Since Eurodollar futures options traded on the Chicago Mercantile Exchange are some of the most liquid interest rate option contracts, prices of these options are used in the estimation procedure. Eurodollar futures options have payoffs at expiration that are determined by the three-month Eurodollar spot interest rate, motivating the choice of the interest rate series used in the empirical section. Adjustments are made to synchronize the option expiration dates as needed and to exclude the value of early exercise due to the American exercise style of these options.

IV.a. The consumption and interest rate data and model

The dynamic relationship between consumption and interest rates is modeled in a bivariate framework, which allows for correlated errors. Table 1 summarizes the properties of quarterly consumption and interest rates over the period from 1985:2 until 1998:2. The starting date of the sample coincides with the listing date for Eurodollar options on futures.

The consumption series is the quarterly, per-capita, real consumption of non-durable goods and services over the quarter as reported in the CITIBASE database. The Eurodollar interest rate series is the British Banker's Association three-month London Interbank Offer Rate (LIBOR) as reported the first business day of the quarter.

There is evidence for a unit root in the consumption and interest rate series as well as their logs, based on results from an augmented Dickey-Fuller test. This suggests that the series should be modeled in differences or that an error correction model is required. A Johansen (1991) test for cointegration is performed and no cointegrating vectors are found. There is evidence of stochastic volatility in the interest rate series based on the p-value of the ARCH(1) coefficient. Figure 1 plots the consumption and interest rate series over the sample period.

The consumption and interest rate models used are based on the existing literature and statistical evidence from this dataset. Log-differenced quarterly consumption is typically found to obey a first-order vector autoregressive process with constant volatility and normal residuals, which suggests the following model:

(11) $$\Delta C_t^* = a_1 + b_1 \Delta C_{t-1}^* + e_{1,t} \qquad e_{1,t} \sim N(0, s_1^2)$$

Estimation of this AR(1) model results in an autoregressive parameter estimate of .3077, a standard error of .1334, and an adjusted r-squared of 9.45%. These and other model estimation results are reported in Table 2. Prior to model estimation, the consumption data is filtered using a single moving average term at the third lag to remove spurious autocorrelation due to sampling error, as noted by Wilcox (1992).

This specification provides an adequate fit to the consumption data; model residuals and their squares are not autocorrelated as reported in Table 3. As postulated, residuals are generated by a normal density as indicated by the results of the Jarque-Bera (1980) test. Since the normal density is obtained in the limit from a student's-t density as the number of degrees of freedom approaches infinity, the innovation density for the consumption process is rewritten in this manner. Residuals are also found to be uncorrelated with lagged log-differenced interest rates.

The interest rate model incorporates the finding that forward rates are often significant predictors of future spot rates. See, for example, Fama (1984). This suggests the following model for the Eurodollar spot rate, in which f^*_{t-1} is the log of lagged three-month Eurodollar forward rate.

(12) $$\Delta r_t^* = a_2 + b_2 (f_{t-1}^* - r_{t-1}^*) + e_{2,t}$$

This model is estimated, and the term spread is found to be statistically significant with a coefficient of .6434 and a standard error of .2910. The adjusted-r-squared from the regression is 7.08%. For this regression, the forward rate is estimated using the price of the Eurodollar futures contract with one quarter until expiration.

The squared residuals from the interest rate conditional mean equation exhibit autocorrelation, which is evidence for stochastic volatility. Amin and Ng (1997) find that implied volatilities are superior to historical squared returns in predicting Eurodollar interest rate volatility. This motivates the use of a conditional volatility model in which implied volatility at the start of the quarter for a Eurodollar futures option with approximately three months until expiration ($\sigma_{i,t-1}^2$) is used to predict the interest rate innovation volatility over the quarter. Estimation is performed using maximum likelihood and a student's-t likelihood function with ν_2 degrees of freedom.

(13) $$\sigma^2_{2,t|t-1} = \gamma\sigma^2_{i,t-1} \qquad \varepsilon_{2,t} \sim student's-t(0, \sigma^2_{2,t|t-1}, \nu_2)$$

The estimated coefficient on lagged implied volatility is 1.10 and the number of degrees of freedom is 12.41. Figure 2 plots the estimated annualized interest rate innovation volatility, which ranges from approximately 5% to 35%, and the estimated annualized consumption innovation volatility, which is .70%. The magnitude of ν_2 shows a significant deviation from normality.

Specification tests in Table 3 indicate that this model fits the data well. Again, (standardized) residuals and their squares are uncorrelated. The excess skewness and kurtosis relative to a normal density are accommodated using a student's-t innovation density.

The interest rate and consumption processes are related by their joint error density, which is assumed to be bivariate student's-t (Fang, Kotz, Ng, 1990). This density allows for excess kurtosis relative to the bivariate normal, and it incorporates the normal as a limiting case. The marginal densities are student's-t and are related by their correlation (ρ), which is estimated using the sample correlation of the standardized residuals (-.3760). Negative correlation is consistent with the notion that increases in interest rates are associated with lower levels of consumption.

(14) $$\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \sim student-t\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_{2,t|t-1} \\ \rho\sigma_1\sigma_{2,t|t-1} & \sigma_{2,t}^2 \end{pmatrix}, \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}\right]$$

Table 4 presents summary statistics for simulations of the estimated joint consumption and interest rate process. The simulations are based on initial values of consumption and interest rates at the start of April from 1991 until 1998. The simulated values of consumption and interest rates at the start of July are obtained using the model described above with 50,000 simulations per year.

The simulated data reflects the characteristics of the original data and the estimated model. The simulated correlation of the log-differenced consumption each year is nearly identical to the estimated value. The impact of the term spread in the interest rate equation is relatively small, except in 1994, when the term spread is unusually large. The simulated interest rate changes exhibit time-varying volatility and excess kurtosis as desired.

IV.b. The interest rate option data

The interest rate option data used in the estimation of the pricing kernels is based on closing prices of Eurodollar futures options traded on the Chicago Mercantile Exchange. One advantage of using the option pricing equation for pricing kernel estimation is that each observed option price provides a moment condition. To reduce the number of moment conditions to a manageable size, the initial database of closing prices is subsetted, so that attention is restricted to out-of-the-money options with reported closing prices on the first trading day of April from 1991 until 1998, at least 100 contracts traded, and expirations in June of the same year.

The CME prices, which are for contracts with American exercise-style and mid-month expiration, are used to estimate the price of options with European exercise-style and beginning of the month expiration. It is necessary that the options have beginning of the quarter expiration to synchronize timing of the payoff variable in the option pricing equation with the estimated consumption and interest rate density. European exercise-style is necessary to eliminate path dependence in the option payoff.

The adjusted prices are estimated as follows. First, the implied standard deviation corresponding to each option price is calculated by numerically inverting the Black (1976) model with the Barone-Adesi and Whaley (1987) adjustment for the early exercise premium. The riskless rate used is the three-month constant maturity treasury rate from the Federal Reserve Board's FRED database, the time-to-expiration is measured in calendar time, and the option strike price and futures settlement price are converted to rates by taking the difference between one-hundred and the original price. The option type is reversed, since call (put) bond option contracts correspond to put (call) interest rate option contracts.

Then, the prices of out-of-the-money European Eurodollar interest rate options with beginning of July expiration are estimated using the Black (1976) model. The implied standard deviation for each strike rate is calculated as above. The same spot rate and riskless rate are used as above, and time until expiration is measured in calendar time until the first trading day of July.

Table 5 presents the estimated option prices using this procedure. There are a total of forty-five options that meet the screening criteria, with a minimum of three in 1997 and a maximum of nine in 1992. The options are roughly half puts and half calls in each year. The estimated prices of the European-style beginning of the month expiration options are consistently higher than the original option prices due to the value of an additional two weeks of existence. The loss of the early exercise feature results in a small decrease in option value, holding time-until-expiration constant.

Figures 3 and 4 plot the contract-specific option-implied volatility in each year against the centered strike rate. The centered strike rate is the difference between the original strike rate and the implied forward rate (from the June Eurodollar contract) rounded to the nearest 25 basis points. A "volatility smile," i.e. an upward parabolic shape centered on the at-the-money contract is observed in most years. The level and curvature of the smile exhibit time variation.

The existence of the smile indicates that the Black (1976) model is not the correct model for interest rate option pricing. However, it is still a useful device for interpolation, when the contract specific implied volatility is used. Stochastic volatility and non-normal innovations, both characteristics found in the quarterly interest rate process, would generate higher prices for away-from-the-money options resulting in a smile. In addition, higher demand for insurance in high interest rate states (which occur in low consumption states) may generate higher prices for out-of-the-money interest rate calls. This would be the counterpart to the equity volatility skew induced by higher demand for insurance against low S&P500 states (which occur in low consumption states).

IV.c. Pricing kernel estimation and specification tests

Consumption-based pricing kernels are estimated using a non-linear least squares regression of Eurodollar interest rate option prices on fitted prices using the CPK formula, as described in section III.

An additional moment condition is added to the estimation, so that the estimated models are consistent with market interest rates as well as interest rate option prices. In particular, the rate of time preference is chosen so that the fitted yield of a riskless three-month bond is equal to the contemporaneous three-month constant maturity treasury rate.

The important empirical findings of this paper, reported in Table 6, are as follows. First, evidence is found which is consistent with the equity premium puzzle: very high levels of risk aversion are needed to justify the observed premium associated with an investment position with returns positively correlated with aggregate consumption. For interest rate options, ceteris paribus, higher levels of risk aversion increase the price of interest rate calls which provide a hedge against low consumption (high interest rate) states. The effect on interest rate puts is in the opposite direction.

The CPK estimation results show that very high levels of risk aversion are needed to fit market prices of interest rate options: the CPK estimate of the coefficient of relative risk aversion is 193.2. This is substantially higher than the level of risk aversion found in previous studies, and significantly different from 1, which corresponds to log utility. It is likely that the high estimated level of risk aversion is due to the increased sensitivity of the CPK estimation technique to preferences over states associated with a large decrease in consumption, compared to previous tests which average across all consumption states.

The rejection of log utility is significant, because the Cox, Ingersoll, and Ross (1985) and Longstaff and Schwartz (1992) models are based on this assumption. Violation this assumption indicates a possible source of error in term structure predictions or option price estimates using these models. The magnitude of the estimated coefficient of relative risk aversion suggests that preference effects on term structure forecasts are likely to be substantial. However, it is also the case that the very high level of estimated risk aversion for the CRRA pricing kernel is economically implausible also indicating misspecification.

These results in Table 6 also show that the choice of preference specifications has a significant impact on estimated option prices. The CRRA and CARA pricing kernels increase model pricing accuracy by 50% versus the log and linear specifications. The CRRA and CARA pricing error standard deviations are about 1.4 basis points, compared to about 3 basis points for the log and linear pricing kernels. This confirms the assertion that out-of-the-money option prices are highly sensitive to preference specifications.

The second panel of Table 6 reports the estimated rates of time preference for each model. While the estimates for the linear and log utility specifications are reasonable (on the order of the annualized riskless interest rate), the estimates for the power (CRRA) and exponential specifications (CARA) are large and negative. This finding is consistent with the riskfree rate puzzle: at high levels of absolute or relative risk-aversion, negative rates of time preference are needed to fit the observed low riskless interest rates. Since large negative rates of time preference are economically implausible, this is evidence for misspecification of the CRRA and CARA pricing kernels.

The first panel of Table 6 also reports a test of pricing model unbiasedness using the t-statistic for the average proportional pricing error for calls and puts from each model. Positive t-statistics indicate the model is underpricing on average and negative t-statistics indicate that the model is overpricing on average. All of the pricing kernels significantly underprice interest rate calls, which is a further indication of misspecification.

IV.d. Robustness tests

The estimation procedure in this paper requires the complete specification of the joint density of consumption and interest rates. In this section, a distribution-free alternative estimation technique, which utilizes option returns, is proposed and implemented. The results from this technique are qualitatively similar to the fully-specified estimation confirming the reliability of the previously stated results.

Consider the moment condition obtained by dividing both sides of equation (3) by the current asset price and subtracting one from each side.

(15) $$0 = E_t[I_{t,T}^{-1} K_{t,T}(C_T) r_{t,T} - 1]$$

Equation (15) states that, for all assets, real conditional risk-adjusted expected returns are zero. This equation also holds unconditionally:

(16) $$0 = E[I_{t,T}^{-1} K_{t,T}(C_T) r_{t,T} - 1]$$

And, the sample version of equation (16) may be used for estimation by minimizing the squared error for the sample moment conditions:

$$(17) \qquad 0 \cong \frac{1}{J}\sum_{j=1}^{J}\left[I_{t,T}^{-1} K_{t_j,T_j}(C_{T_j}) r_{t_j,T_j} - 1 \right]$$

The assets used in estimation are interest rate calls and puts that are 1% or 5% out-of-the-money with one-quarter until expiration over the period 1991 - 1998. Option returns ($r_{t,T} = D_T/D_t$) are calculated using the quarterly interpolated European interest rate option price based on the cross-section of Eurodollar options with closest to 90 days until expiration on the first day of each quarter (D_t). The option payoff (D_T) is determined using the option payoff function and the quoted BBA LIBOR rate on the option expiration date.

The inflation rate ($I_{t,T}$) is the quarterly CPI growth rate (urban: all items) from the Federal Reserve's FRED database. The consumption data and pricing kernel specifications are the same as those used in the CPK estimation. An additional condition used in estimation, analogous to that of the CPK technique, is that rate of time preference is chosen so that the fitted yield of a riskless three-month bond is equal to the average three-month constant maturity treasury rate over this period.

The estimation results are reported in the third column of table 6. The coefficients of relative and absolute risk aversion using the distribution-free technique are 114 and .03. While these are lower than using the CPK technique, they are still very high. The corresponding estimated rates of time preference are -1.64 and -1.70. Thus, the qualitative results using the CPK and distribution-free methods are similar; namely, very high levels of risk-aversion are needed to characterize interest rate option prices and interest rate option returns. To fit the observed low riskless interest rates, high levels of risk-aversion require negative rates of time preference.

V. Conclusions

This paper estimates and tests consumption-based pricing kernels used in common equilibrium interest rate term structure models. In contrast to previous papers that use return orthogonality conditions, estimation in this paper is accomplished using moment conditions from a consumption-based option pricing equation and market prices of interest rate options.

Evidence is found which is consistent with the equity premium and risk-free rate puzzles. Very high levels of risk aversion are needed to characterize market prices of interest rate options; at these high levels of risk aversion, negative rates of time preference are needed to fit observed riskless interest rates. In terms of the tested pricing kernels, the linear and log specifications do not have high enough risk aversion to fit the data. The power and exponential specifications require rates of time preference which are economically implausible.

These results suggest that typical term structure models are misspecified in terms of assumed preferences. This may have deleterious effects on interest rate term structure and option price estimates.

References

Ait-Sahalia, Y. and A. Lo, 1997, "Nonparametric Risk Management and Implied Risk Aversion," Manuscript.

Amin, K. and V. Ng, 1997, "Inferring Future Volatility from the Information in Implied Volatility in Eurodollar Options: A New Approach," *Review of Financial Studies*, 10, 333-367.

Amin, K. I. and V. K. Ng, 1993, "Option Valuation with Systematic Stochastic Volatility," *Journal of Finance*, 48, 881-910.

Backus, D., A. Gregory and S. Zin, 1997, "Reverse Engineering the Yield Curve," National Bureau of Economic Research, Working Paper, 4676 (Revised).

Bansal, R. and S. Viswanathan, 1993, "No Arbitrage and Arbitrage Pricing - A New Approach," *Journal of Finance*, 48, 1231-1262.

Barone-Adesi, G. and R. E. Whaley, 1987, "Efficient Analytic Approximation of American Option Values," *Journal of Finance*, 42, 301-320.

Black, F., 1976, "The Pricing of Commodity Contracts," *Journal of Financial Economics*, 3, 167-179.

Boudoukh, J., 1993, "An Equilibrium Model of Nominal Bond Prices With Inflation-Output Correlation and Stochastic Volatility," *Journal of Money Credit and Banking*, 25, 636-665.

Boyle, P. P., 1977, "Options: A Monte-Carlo Approach," *Journal of Financial Economics*, 4, 323-338.

Brennan, M., 1979, "The Pricing of Contingent Claims in Discrete-time Models," *Journal of Finance*, 34, 53-68.

Campbell, J., 1986, "Bond and Stock Returns in a Simple Exchange Economy," *Quarterly Journal of Economics*, 101, 785-804.

Canova, A. and J. Marrinan, 1996, "Reconciling the Term Structure of Interest Rates with the Consumption-Based ICAP Model," *Journal of Economic Dynamics and Control*, 20, 709-750.

Chapman, D. A., 1997, "Approximating the Asset Pricing Kernel," *Journal of Finance*, 52, 1383-1410.

Cochrane, J. H. and L. P. Hansen, 1992, "Asset Pricing Explorations for Macroeconomics," *NBER Macroeconomics Bulletin*, 115-165.

Constantinides, G. M. "Theory of Valuation: Overview." In *Theory of Valuation: Frontiers of Modern Financial Theory*, edited by S. Bhattacharya and G. M. Constantinides, 1-24. Totowa: Rowman & Littlefield, 1989.

Constantinides, G. M., 1992, "A Theory of the Nominal Term Structure of Interest Rates," *Review of Financial Studies*, 5, 531-552.

Courtadon, G., 1982, "The Pricing of Options of Default Free Bonds," *Journal of Financial and Quantitative Analysis*, 17, 75-100.

Cox, J. C., J. E. Ingersoll, Jr. and S. Ross, 1985, "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53, 385-407.

Cox, J. C., J. E. Ingersoll, Jr. and S. A. Ross, 1985, "An Intertemporal General Equilibrium Model of Asset Prices," *Econometrica*, 53, 363-385.

Duan, J. C., 1995, "The GARCH Option Pricing Model," *Mathematical Finance*, 5, 13-32.

Fama, E., 1984, "The Information in the Term Structure," *Journal of Financial Economics*, 13, 509.

Fang, K.-T., S. Kotz and K. W. Ng. "Symmetric Multivariate and Related Distributions." , x, 220 p. London: Chapman and Hall, 1990.

Ferson, W., 1983, "Expectations of Real Interest Rates and Aggregate Consumption: Empirical Tests," *Journal of Financial and Quantitative Analysis*, 18, 477-497.

Gallant, A. R., L. P. Hansen and G. Tauchen, 1990, "Using Conditional Moments of Asset Payoffs to Infer the Volatility of Intertemporal Marginal Rates of Substitution," *Journal of Econometrics*, 45, 141-179.

Hansen, L. P. and K. J. Singleton, 1982, "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica*, 50, 1269-1286.

Hansen, L. P. and K. J. Singleton, 1983, "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," *Journal of Political Economy*, 91, 249-265.

Heaton, J., 1995, "An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications," *Econometrica*, 63, 681-717.

Jackwerth, J. C., 1997, "Recovering Risk Aversion from Option Prices and Realized Returns," London Business School, Manuscript.

Longstaff, F. A. and E. S. Schwartz, 1992, "Interest Rate Volatility and the Term Structure - a Two-Factor General Equilibrium Model," *Journal of Finance*, 47, 1259-1282.

Lucas, R., 1980, "Equilibrium in a Pure Currency Economy," *Economic Inquiry*, 18, 203-220.

Rosenberg, J. and R. F. Engle, 1999, "Empirical Pricing Kernels," NYU - Stern School of Business, Manuscript.

Rubinstein, M., 1976, "The Valuation of Uncertain Income Streams and the Pricing of Options," *Bell Journal of Economics and Management Science*, 7, 407-425.

Satchell, S. and A. Timmerman, 1995, "Option Pricing with GARCH and Systematic Consumption Risk," *Derivatives Use,Trading and Regulation*, 1, 279-291.

Stanton, R., 1997, "A Nonparametric Model of Term Structure Dynamics and the Market Price of Interest Rate Risk," *Journal of Finance*, 52, 1973-2002.

Stapleton, R. C. and M. G. Subrahmanyam, 1984, "The Valuation of Multivariate Contingent Claims in Discrete Time Models," *Journal of Finance*, 39, 207-228.

Wilcox, D., 1992, "The Construction of U.S. Consumption Data: Some Facts and Their Implications for Empirical Work," *American Economic Review*, 82, 922-941.

Table 1
Data summary

	Consumption	Log consumption	Log differenced consumption	Eurodollar rate	Log Eurodollar rate	Log differenced Eurodollar rate	Inflation rate (1991-1998)
Number obs.	53	53	53	53	53	53	32
Mean	$3,664	8.21	0.0042	6.38%	-2.80	-0.01	0.42%
Maximum	$4,082	8.31	0.0112	10.25%	-2.28	0.23	0.71%
Minimum	$3,294	8.10	-0.0077	3.25%	-3.43	-0.30	0.19%
Std. Dev.	$193	0.05	0.0039	1.81%	0.31	0.09	0.15%
Skewness	0.12	0.01	-0.76	-0.02	-0.58	-0.22	0.07
Kurtosis	2.46	2.44	4.20	2.26	2.55	3.81	1.95
Augmented Dickey-Fuller test statistic	0.56	0.03	-3.90	-1.47	-2.16	-4.68	-5.17
First-order autocorrelation	0.91	0.91	0.25	0.93	0.92	-0.09	0.08
ARCH(1) p-value			0.3971			0.0292	0.7801

This table presents summary statistics for the consumption and interest rate data used in the estimation of the joint model of their time-series processes. Consumption is the quarterly per capita non-durable goods and services consumption from the National Income and Product (NIPA) accounts as recorded in the CITIBASE database. Consumption is seasonally adjusted and deflated to constant 1992 dollars. Per capita consumption is obtained by dividing by the total monthly population reported by the U.S. Census Bureau in the CITIBASE database. Consumption statistics are reported by the Bureau of Economic Analysis in the Personal Income and Outlays news release.

The Eurodollar rate is the 3 month Eurodollar offer rate (LIBOR) reported by the British Banker’s Association (BBA) at 11 AM London time on the first business day of each quarter. BBA data is available beginning in January of 1986. Prior to this date, the Eurodollar bid rate (EDBID) —as reported on the Federal Reserve H.15 form is available beginning in January 1971. EDBID is transformed into a fitted offer rate using parameters from a linear regression of daily LIBOR on EDBID over the period from 1986 through 1998. The estimated model (with LIBOR in percent terms) is LIBOR = 0.128743 + 0.998897*EDBID. The regression adjusted r-squared is .9997, and the root mean squared error is 3.1 basis points.

Log consumption and log Eurodollar rate are the natural logarithms of the respective variables. Kurtosis is reported in total rather than excess units. The augmented Dickey-Fuller test statistic is from a regression of the level on a constant, the lagged level, and one lagged difference. In this case, the 5% critical value to reject a unit root is -2.92. The ARCH(1) p-value is the probability value for the coefficient on a single lagged squared residual from an ARCH(1) model estimated with a constant term in the mean equation (and a single lagged difference for differenced consumption). Low p-values are evidence for ARCH.

The inflation rate — reported in the rightmost column —is not used in the simulation, but it is used in CPK option price estimation as defined in equation (4). Only data for the same period as the observed option prices (1991-1998) is used in estimation, so this is the data for which summary statistics are reported. The inflation rate is defined as the net growth rate of the seasonally adjusted quarterly consumer price index for all urban consumers: all items as measured by the Bureau of Labor Statistics and reported in the Federal Reserve’s FRED database.

Table 2

Model of the joint consumption and interest rate process

$$\Delta C_t^* = \alpha_1 + \beta_1 \Delta C_{t-1}^* + \varepsilon_{1,t}$$
$$\Delta r_t^* = \alpha_2 + \beta_2 (f_{t-1}^* - r_{t-1}^*) + \varepsilon_{2,t}$$
$$\sigma_{2,t|t-1}^2 = \gamma \sigma_{i,t-1}^2$$

$$\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \sim student-t \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_{2,t|t-1} \\ \rho \sigma_1 \sigma_{2,t|t-1} & \sigma_{2,t}^2 \end{pmatrix}, \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \right]$$

Consumption mean equation

	Parameter estimate	Standard error
α1	0.0030	0.0007
β1	0.3077	0.1334

Interest rate mean equation

	Parameter estimate	Standard error
α1	-0.0149	0.0130
β1	0.6434	0.2910

Consumption variance equation

	Parameter estimate	Standard error
σ1	0.0035	
ν1	∞	

Interest rate variance equation

	Parameter estimate	Standard error
γ	1.10	0.2524
ν2	12.41	15.1475

Joint error density

	Parameter estimate	Standard error
ρ	-0.3760	

Estimation summary statistics

Adj. R-sqr.	0.0945
F-probability	0.0251

Estimation summary statistics

Adj. R-sqr.	0.0708
F-probability	0.0317

This table presents the estimated joint model for the consumption and interest rate process. The estimation is performed using log-differences of the original series (C*=log(C), r*=log(r)). The consumption process is estimated as an AR(1) in log-differences with constant volatility and normal residuals. The consumption data is prefiltered to remove spurious seasonal correlation at the third lag as noted by Wilcox (1992). The interest rate process is estimated with the spread between the log spot rate and the log three month forward rate (implied by the Eurodollar futures price) as an independent variable in the mean equation as presented in Fama (1984). The interest rate conditional variance dynamics are estimated by maximum likelihood using a t-density with ν2 degrees of freedom and the residuals from the mean equation. The implied volatility of Eurodollar futures options with approximately three months until expiration (σ_i) is used as a predictor of future expected volatility. See, for example, Amin and Ng (1997). The correlation parameter (ρ) is the sample correlation of the standardized residuals from the two models. Specification tests for the models are presented in Table 3.

Table 3

Specification tests for the consumption and interest rate model

Tests of standardized residuals

	Consumption equation (p-value)	Interest rate equation (p-value)
Error autocorrelation		
Autocorrelation Q-test	0.5221	0.1646
Autocorrelation in squares Q-test	0.6325	0.6333
Error distribution		
Jarque-Bera normality test p-value	0.3824	
Skewness	-0.0968	-0.1394
Kurtosis	3.9951	4.7067

This table presents results of several specification tests on the standardized residuals (e_t/σ_t) from the estimated consumption and interest rate model. All test statistics (except for skewness and kurtosis) are reported as p-values such that a p-value less than .05 represents a rejection of the null hypothesis of correct specification. The Q-test statistics are p-values from a Ljung-Box (1978) test based regression of the standardized residuals (or their squares) on 4 lags. The Jarque-Bera (1980) normality test is based on the skewness and kurtosis of the residuals. Kurtosis is reported in total, rather than excess units.

Table 4

Summary statistics from simulation of consumption and interest rate processes

1991	Mean	Std Dev	Skewness	Kurtosis	Minimum	Maximum
Simulated consumption	3608.45	12.55	0.00	3.00	3557.14	3660.72
Simulated cons. growth (gross)	1.0049	0.0035	0.00	3.00	0.99	1.02
Simulated interest rate	6.39%	0.53%	0.34	3.96	3.81%	9.68%
Simulated interest rate (change)	0.01%	0.53%	0.34	3.96	-2.56%	3.30%
Initial consumption	3590.88					
Initial interest rate	6.38%					
Initial forward rate	6.51%					
Simulated correlation (of log diff.)	-0.38					
1992	Mean	Std Dev	Skewness	Kurtosis	Minimum	Maximum
Simulated consumption	3664.74	12.74	0.01	3.02	3611.63	3720.11
Simulated cons. growth (gross)	1.0040	0.0035	0.01	3.02	0.99	1.02
Simulated interest rate	4.38%	0.46%	0.42	4.18	2.31%	8.14%
Simulated interest rate (change)	0.07%	0.46%	0.42	4.18	-2.00%	3.83%
Initial consumption	3590.88					
Initial interest rate	4.31%					
Initial forward rate	4.49%					
Simulated correlation (of log diff.)	-0.37					
1993	Mean	Std Dev	Skewness	Kurtosis	Minimum	Maximum
Simulated consumption	3713.42	12.93	0.00	2.99	3659.83	3765.38
Simulated cons. growth (gross)	1.0042	0.0035	0.00	2.99	0.99	1.02
Simulated interest rate	3.26%	0.33%	0.38	3.91	1.83%	5.49%
Simulated interest rate (change)	0.01%	0.33%	0.38	3.91	-1.42%	2.24%
Initial consumption	3698.03					
Initial interest rate	3.25%					
Initial forward rate	3.32%					
Simulated correlation (of log diff.)	-0.37					
1994	Mean	Std Dev	Skewness	Kurtosis	Minimum	Maximum
Simulated consumption	3793.82	13.19	0.00	3.03	3738.42	3847.09
Simulated cons. growth (gross)	1.0046	0.0035	0.00	3.03	0.99	1.02
Simulated interest rate	4.26%	0.41%	0.37	3.99	1.87%	7.55%
Simulated interest rate (change)	0.32%	0.41%	0.37	3.99	-2.06%	3.61%
Initial consumption	3776.36					
Initial interest rate	3.94%					
Initial forward rate	4.53%					
Simulated correlation (of log diff.)	-0.37					
1995	Mean	Std Dev	Skewness	Kurtosis	Minimum	Maximum
Simulated consumption	3850.00	13.40	0.00	2.98	3794.19	3910.22
Simulated cons. growth (gross)	1.0052	0.0035	0.00	2.98	0.99	1.02
Simulated interest rate	6.34%	0.41%	0.25	3.74	4.47%	8.96%
Simulated interest rate (change)	0.02%	0.41%	0.25	3.74	-1.84%	2.65%
Initial consumption	3830.16					
Initial interest rate	6.31%					
Initial forward rate	6.48%					
Simulated correlation (of log diff.)	-0.37					

Table 4 (continued)

Summary statistics from simulation of consumption and interest rate processes

1996	Mean	Std Dev	Skewness	Kurtosis	Minimum	Maximum
Simulated consumption	3926.26	13.69	0.03	2.99	3869.46	3979.59
Simulated cons. growth (gross)	1.0058	0.0035	0.03	2.99	0.99	1.02
Simulated interest rate	5.37%	0.34%	0.27	3.84	3.79%	7.82%
Simulated interest rate (change)	-0.09%	0.34%	0.27	3.84	-1.67%	2.35%
Initial consumption	3903.66					
Initial interest rate	5.46%					
Initial forward rate	5.43%					
Simulated correlation (of log diff.)	-0.37					
1997	Mean	Std Dev	Skewness	Kurtosis	Minimum	Maximum
Simulated consumption	3979.50	13.88	0.01	3.04	3924.44	4038.64
Simulated cons. growth (gross)	1.0041	0.0035	0.01	3.04	0.99	1.02
Simulated interest rate	5.84%	0.25%	0.19	3.78	4.77%	7.23%
Simulated interest rate (change)	0.02%	0.25%	0.19	3.78	-1.05%	1.42%
Initial consumption	3963.44					
Initial interest rate	5.81%					
Initial forward rate	5.98%					
Simulated correlation (of log diff.)	-0.38					
1998	Mean	Std Dev	Skewness	Kurtosis	Minimum	Maximum
Simulated consumption	4122.50	14.34	0.03	2.99	4067.83	4180.14
Simulated cons. growth (gross)	1.0062	0.0035	0.03	2.99	0.99	1.02
Simulated interest rate	5.63%	0.24%	0.19	3.79	4.26%	7.33%
Simulated interest rate (change)	-0.08%	0.24%	0.19	3.79	-1.44%	1.62%
Initial consumption	4097.28					
Initial interest rate	5.71%					
Initial forward rate	5.71%					
Simulated correlation (of log diff.)	-0.38					

This table presents summary statistics for the simulated consumption and interest rate processes using the joint model presented in Table 2. 50,000 simulation replications are used per year.

Simulated consumption is the simulated quarterly consumption choice for the third quarter of each year at the beginning of July, and consumption growth is the ratio of simulated consumption and the second quarter actual consumption choice (initial consumption).

The simulated interest rate is the simulated interest rate on the first business day of July. The simulated interest rate change is the difference between the simulated interest rate and LIBOR on the first business day of April (initial interest rate). The initial forward rate is the implied three month forward rate from the June Eurodollar futures contract on the first business day of April. The simulated correlation is the Pearson correlation coefficient of the simulated consumption and interest rate log differences.

Table 5
Eurodollar interest rate option data

Date	Implied June forward rate	Strike rate	Option type (as an interest rate option)	Estimated option price	Original option price	Implied standard deviation
April-91	6.51%	5.75%	PUT	0.0281	0.0200	19.37%
April-91	6.51%	6.00%	PUT	0.0512	0.0400	17.46%
April-91	6.51%	6.25%	PUT	0.0940	0.0800	15.57%
April-91	6.51%	6.50%	PUT	0.1962	0.1800	15.67%
April-91	6.51%	6.75%	CALL	0.1047	0.0900	15.38%
April-91	6.51%	7.00%	CALL	0.0510	0.0400	15.74%
April-91	6.51%	7.25%	CALL	0.0280	0.0200	16.91%
April-92	4.49%	3.75%	PUT	0.0159	0.0100	24.50%
April-92	4.49%	4.00%	PUT	0.0282	0.0200	20.49%
April-92	4.49%	4.25%	PUT	0.0732	0.0600	19.12%
April-92	4.49%	4.50%	CALL	0.1551	0.1400	17.96%
April-92	4.49%	4.75%	CALL	0.0849	0.0700	20.31%
April-92	4.49%	5.00%	CALL	0.0531	0.0400	23.30%
April-92	4.49%	5.25%	CALL	0.0294	0.0200	24.70%
April-92	4.49%	5.50%	CALL	0.0303	0.0200	29.98%
April-92	4.49%	5.75%	CALL	0.0168	0.0100	30.35%
April-93	3.32%	3.00%	PUT	0.0277	0.0200	20.51%
April-93	3.32%	3.25%	PUT	0.0698	0.0600	15.43%
April-93	3.32%	3.50%	CALL	0.0615	0.0500	19.48%
April-93	3.32%	3.75%	CALL	0.0284	0.0200	22.53%
April-93	3.32%	4.00%	CALL	0.0161	0.0100	25.95%
April-94	4.37%	4.00%	PUT	0.0347	0.0200	18.43%
April-94	4.37%	4.25%	PUT	0.0978	0.0800	17.51%
April-94	4.37%	4.50%	CALL	0.1026	0.0800	18.04%
April-94	4.37%	4.75%	CALL	0.0446	0.0250	18.88%
April-94	4.37%	5.00%	CALL	0.0203	0.0150	20.24%
April-95	6.48%	6.00%	PUT	0.0143	0.0150	11.36%
April-95	6.48%	6.25%	PUT	0.0485	0.0400	10.51%
April-95	6.48%	6.50%	CALL	0.1532	0.1400	12.72%
April-95	6.48%	6.75%	CALL	0.0604	0.0500	12.21%
April-95	6.48%	7.00%	CALL	0.0393	0.0300	14.95%
April-95	6.48%	7.50%	CALL	0.0154	0.0100	18.42%
April-96	5.41%	4.75%	PUT	0.0148	0.0100	17.89%
April-96	5.41%	5.00%	PUT	0.0266	0.0200	14.51%
April-96	5.41%	5.25%	PUT	0.0587	0.0500	11.56%
April-96	5.41%	5.50%	CALL	0.0792	0.0700	10.98%
April-96	5.41%	5.75%	CALL	0.0262	0.0200	11.88%
April-97	5.98%	5.75%	PUT	0.0277	0.0200	8.99%
April-97	5.98%	6.00%	CALL	0.0911	0.0800	8.50%
April-97	5.98%	6.25%	CALL	0.0152	0.0150	7.78%
April-98	5.71%	5.50%	PUT	0.0131	0.0150	6.71%
April-98	5.71%	5.63%	PUT	0.0450	0.0450	6.82%
April-98	5.71%	5.75%	CALL	0.0592	0.0550	7.02%
April-98	5.71%	5.88%	CALL	0.0268	0.0250	7.63%
April-98	5.71%	6.00%	CALL	0.0150	0.0150	8.60%

This table contains estimated European Eurodollar interest rate option prices (with beginning of July expiration) at the end of the first business day of April from 1991 through 1998. These prices are extrapolated from the market closing prices of June Eurodollar futures options with the same strike rate. Closing prices are from a database created by the Chicago Mercantile Exchange and distributed by the Futures Industry Institute.

The specifics of the price estimation are as follows. The closing prices of June Eurodollar futures options are collected on the first trading day of April each year. Options with volume less than 100 contracts are discarded. Implied standard deviations are estimated by numerically inverting the Black (1976) futures options pricing formula with Barone-Adesi and Whaley (1987) adjustment for the early exercise premium. The 3 month constant maturity treasury rate (as reported by the Federal Reserve in the FRED database) is used as the riskless rate (r), the time until expiration is measured as a fraction of a year using calendar days (T-t), and the spot price is the June Eurodollar futures settlement price which is converted to a forward rate (St) by taking the difference of 100 and the futures price. The strike rate (E) is the difference between 100 and the Eurodollar futures option strike price. The type of a Eurodollar futures option call (put) as an interest rate options is put (call).

The prices of out-of-the-money European Eurodollar interest rate options with beginning of July expiration are estimated using the Black model and the following parameters: σ_i = implied standard deviation for the strike calculated as above, the spot rate (S_t) and riskless rate (r) as above, and time until expiration (T-t) is the fraction of a year until the first trading day of July. The strike rate (E) is 100 minus the strike price of the original Eurodollar futures contract from which the implied standard deviation was calculated, and the option type is selected so that all contracts are out-of-the-money.

Table 6
Pricing kernel estimation results

Utillity function	Risk aversion parameter: CPK estimation	Risk aversion parameter: Distribution-free estimation	Standard deviation of pricing error	Standard error of standard deviation	Average pricing error T-statistic (puts)	Average pricing error T-statistic (calls)
Linear			0.0298	0.0031	-0.77	3.45
Log			0.0296	0.0031	-0.75	3.44
CRRA	193.2281	114.0054	0.0141	0.0015	1.22	2.12
CARA	0.0519	0.0307	0.0139	0.0015	1.22	2.08

Estimated annual rate of time preference

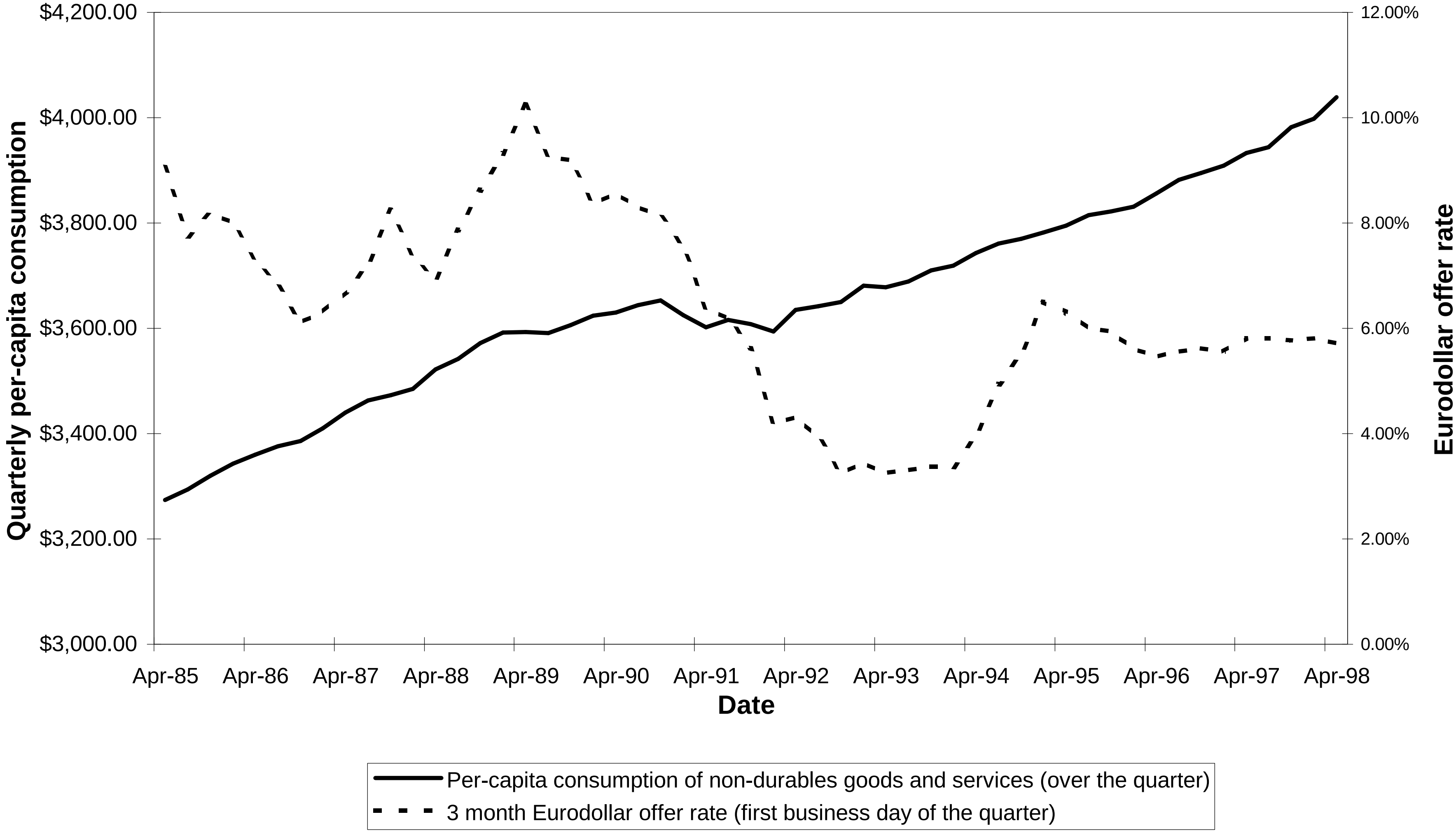
Year	Linear	Log	CRRA	CARA
1991	0.0638	0.0443	-2.7976	-2.7325
1992	0.0431	0.0273	-2.1133	-2.0931
1993	0.0325	0.0159	-2.2658	-2.2612
1994	0.0394	0.0210	-2.6127	-2.6427
1995	0.0631	0.0425	-3.0179	-3.0857
1996	0.0546	0.0316	-3.4945	-3.6280
1997	0.0581	0.0420	-2.1505	-2.2354
1998	0.0571	0.0326	-3.7806	-4.0758

This table contains the estimated pricing kernel parameters and two pricing kernel specification tests. CPK pricing kernel parameters are estimated using a non-linear least squares regression of observed Eurodollar option prices on estimated Eurodollar option prices. The option dataset is described in Table 5. An additional condition in the estimation sets the the rate of time preference such that the riskless interest rate (contemporaneous constant maturity three month treasury bill yield as reported in the Federal Reserve Board's FRED database) is replicated by the model. The second panel reports the annualized rate of time preference for each model.

For comparison, pricing kernel parameters are also estimated using a distribution-free technique described in section IV.d. of the paper. The moment condition used in this case is that expected quarterly risk-adjusted real net returns for 1% and 5% out-of-the-money calls and puts with 90 days until expiration are equal to zero. Estimation is accomplished by minimizing the squared error for the sample version of this equation.

The standard errors of the standard deviations are asymptotic standard errors for the normal case. The pricing error T-statistic is the student's-t statistic for the sample average of the proportional pricing errors [(market price - model price) / model price] of out-of-the-money calls or puts using each model. T-statistics greater than 2 in absolute value are indication that the pricing model provides biased estimates and that the pricing kernel is misspecified. Positive t-statistics indicate the model is underpricing on average and negative t-statistics indicate that the model is overpricing on average.

Figure 1
Quarterly consumption and interest rates



This figure plots the time-series of quarterly per capita consumption (real seasonally adjusted) by U.S. residents along with the 3 month Eurodollar rate (LIBOR) over the period from 1971:2 to 1998:2. The log-differences of these series are used to estimate the joint consumption and interest rate process. The scales for each series are along opposite sides of the chart.

Figure 2
Consumption and interest rate innovation volatility

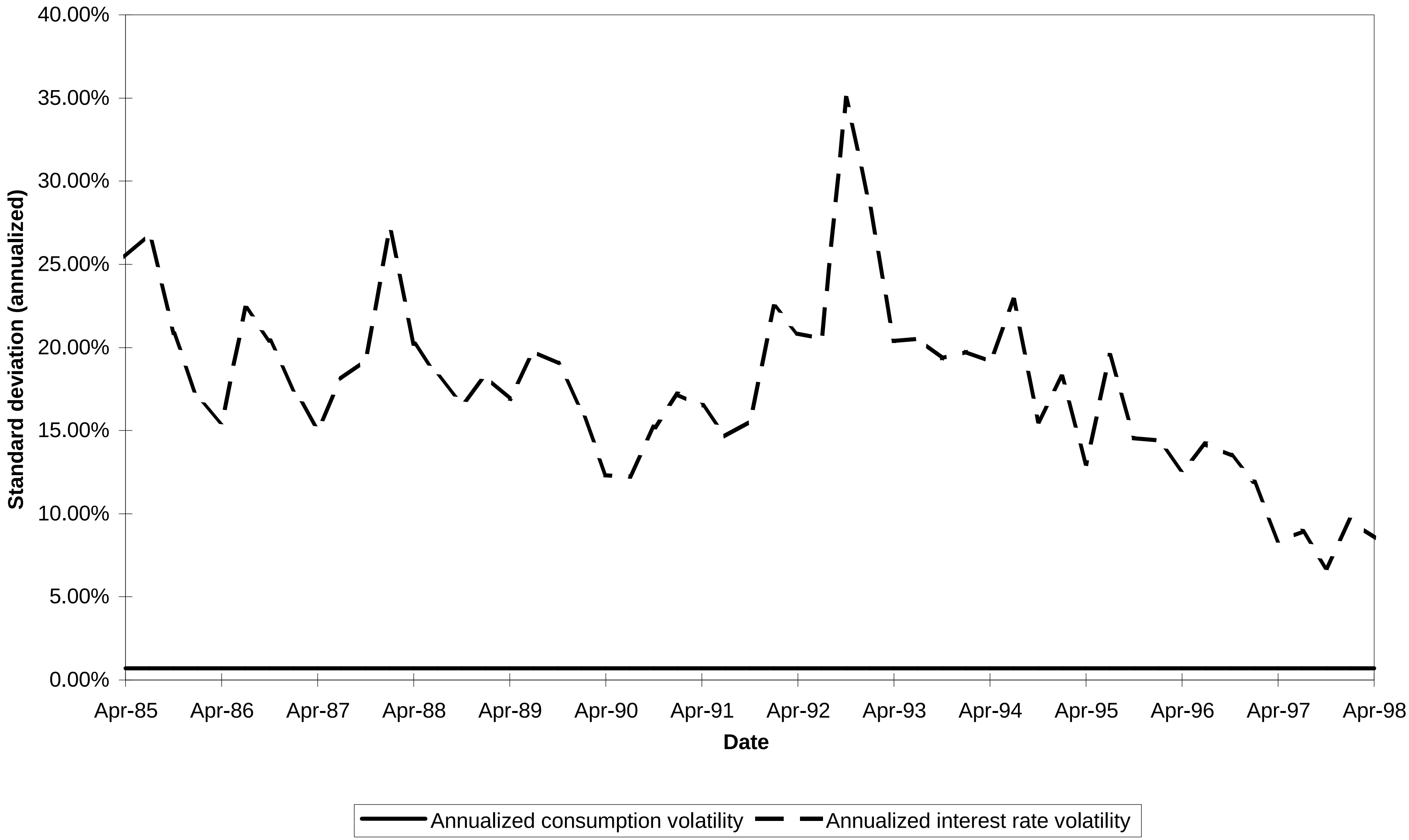
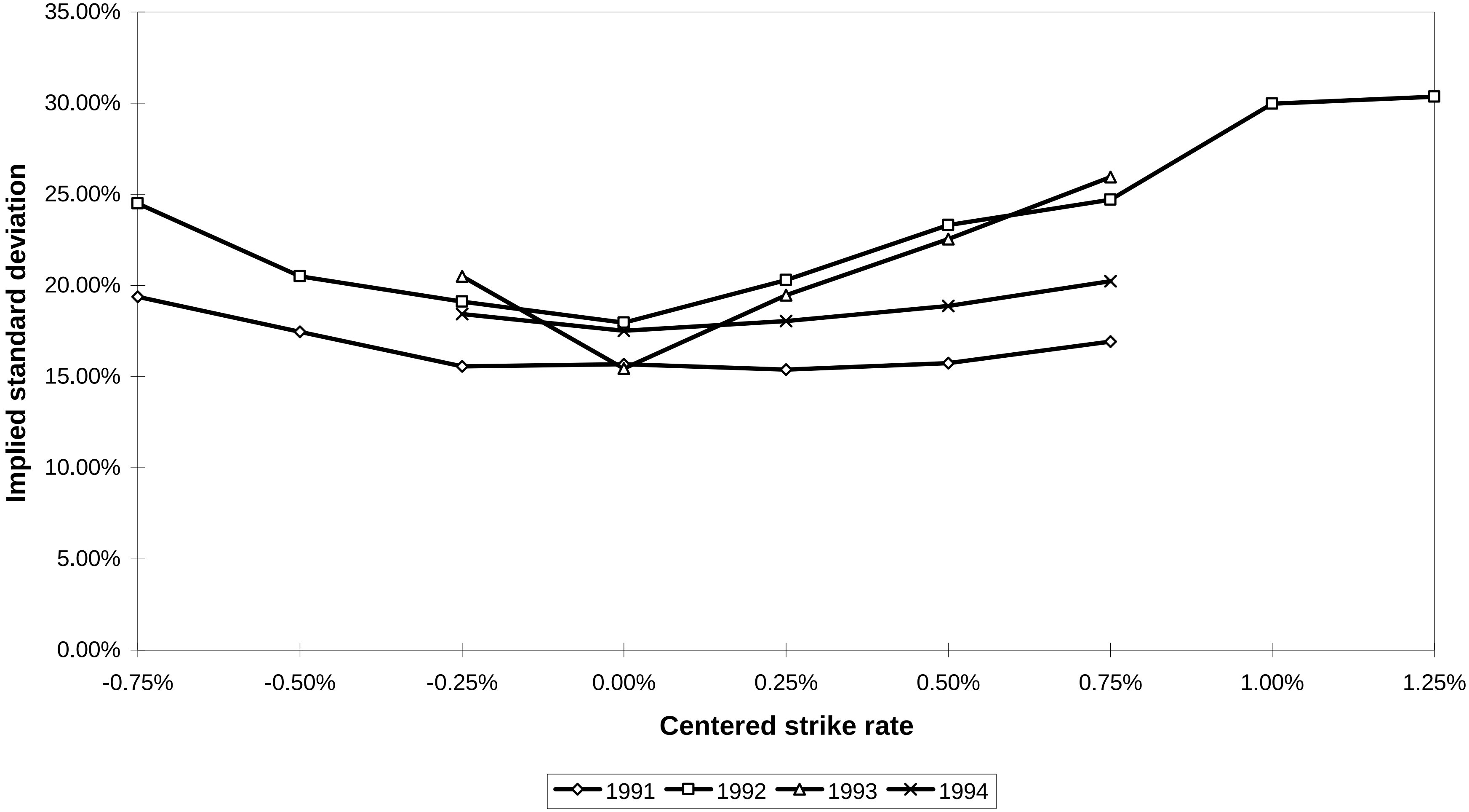


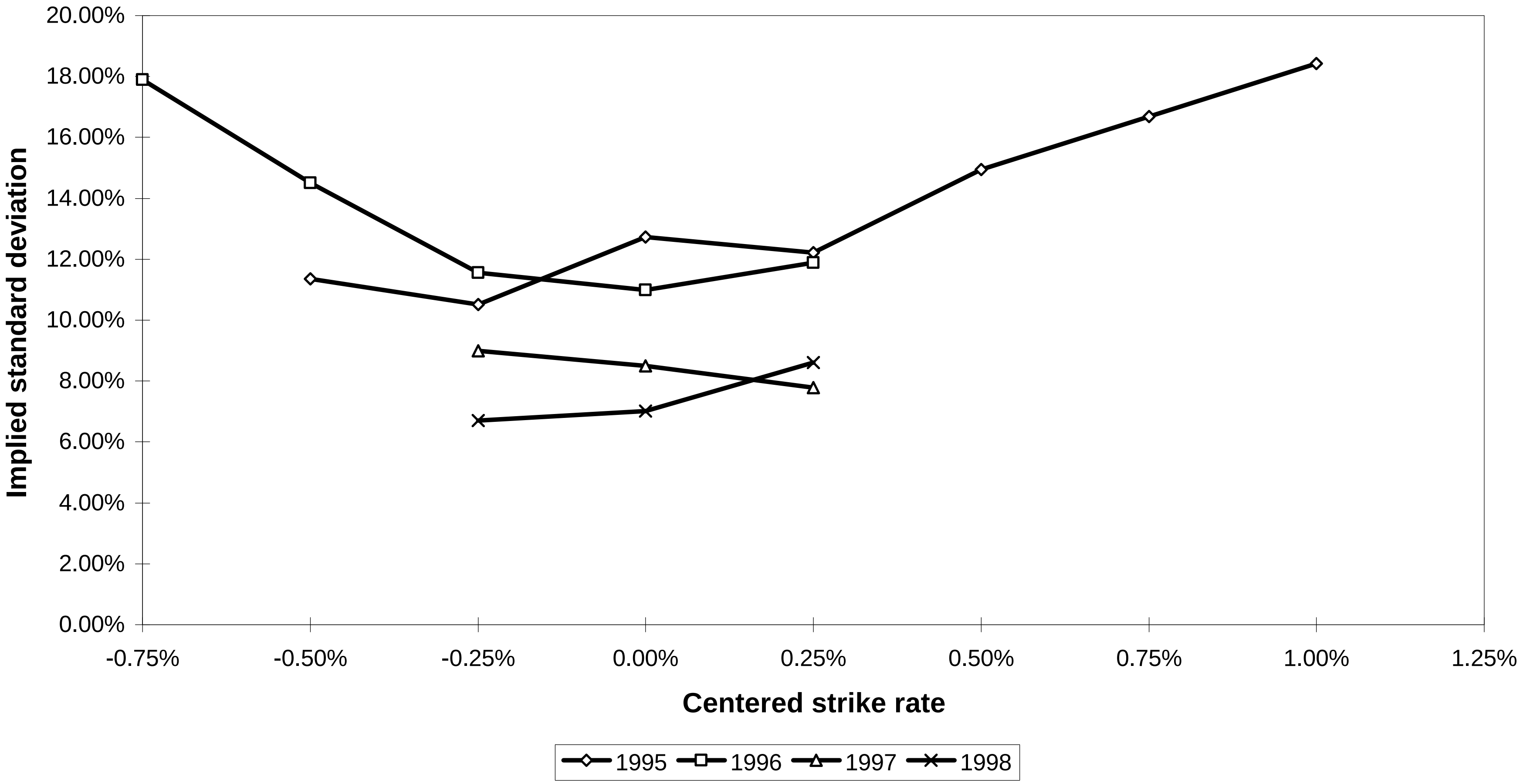
Figure 2 plots the time-series of volatilities (annualized standard deviations) of the consumption and interest rate innovations. The volatility models are described in Table 2.

Figure 3
June Eurodollar futures options
Implied volatility, April (1991 - 1994)



This figure plots implied volatilities against centered strike rates for June Eurodollar futures options at the beginning of April from 1991 to 1994. Implied volatilities are calculated using the Barone-Adesi and Whaley (1987) model using closing June Eurodollar futures options prices on the first business day of April. Implied volatilities are reported as standard deviations in annualized percentage terms. The x-axis is defined as a strike rate, i.e. 100 - Eurodollar futures option strike price, which is centered by subtracting the three month implied forward rate on the first trading day of April (rounded to the nearest 25 basis points).

Figure 4
June Eurodollar futures options
Implied volatility, April (1995 - 1998)



This figure plots implied volatilities against centered strike rates for June Eurodollar futures options at the beginning of April from 1995 to 1998. Implied volatilities are calculated using the Barone-Adesi and Whaley (1987) model using closing June Eurodollar futures options prices on the first business day of April. Implied volatilities are reported as standard deviations in annualized percentage terms. The x-axis is defined as a strike rate, i.e. 100 - Eurodollar futures option strike price, which is centered by subtracting the three month implied forward rate on the first trading day of April (rounded to the nearest 25 basis points).