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Macroeconomic Foundations of Higher Moments in Bond Yields*

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Abstract

Kurtosis in asset prices and returns has been so widely documented it hardly bears comment. Equally interesting, in our view, is the relatively modest kurtosis in consumption growth and inflation. The question is how to reconcile the two: Is kurtosis in asset prices inherited from macroeconomic fundamentals, or does some feature of the economy generate leptokurtotic returns internally? We describe a model that reconciles the two by generating leptokurtotic interest rates from a near-normal pricing kernel.

JEL Classification Codes: E43, G12, G13.

Keywords: term structure; skewness and kurtosis; jumps; multi-factor affine models; pricing kernels; consumption growth; inflation.

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1 Introduction

One of the features of bond yields we find interesting is their higher moments: both changes in bond yields and differences between long and short rates exhibit substantial kurtosis. Although some of this interest probably stems from psychological sources best left untouched, we think higher moments may shed some light on the relation between macroeconomic fundamentals and asset prices. We find it puzzling that the kurtosis commonly found in asset prices and returns has little counterpart in other aggregate variables. Changes in bond yields, for example, are highly leptokurtotic, but consumption growth and inflation are not. If there is a connection between interest rates and macroeconomic variables, what is it? We explore this issue in a parametric model of bond pricing in which the pricing kernel can be identified from properties of bond yields alone.

2 Notation and Evidence

We start by documenting some of the properties of bond yields and macroeconomic variables. In what follows, the continuously-compounded yield on an n-period bond is

$$y_t^n = -n^{-1} \log b_t^n, \tag{1}$$

where b_t^n is the dollar price at date t of a claim to one dollar at $t+n$. Forward rates are

$$f_t^n = \log(b_t^n / b_t^{n+1}),$$

so that yields are averages of forward rates:

$$y_t^n = n^{-1} \sum_{j=1}^{n} f_t^{j-1}.$$

Yields are the variable of choice in most studies, but the theory is often most transparent for forward rates.

In Tables 1 and 2, we summarize some of the properties of bond yields using a monthly time interval. The data were provided by McCulloch and Kwon (1993). In Table 1 we see that bond yields with maturities between one month and ten years are highly persistent, with autocorrelations well over 0.9. Skewness and kurtosis, however, are modest: the classical γ_1 (skewness) and

γ_2 (kurtosis) are greater than zero (the value for normal random variables) but less than one (an arbitrary number we take to be small). These small departures from normality gain greater prominence when we look at yield spreads (the difference between a yield and the short rate y^1) or monthly changes, both of which are described in Table 2. The spread between the three- and one-month yields has a sample kurtosis of 7, and the monthly change in the one-month rate has a sample kurtosis over 10.

Other aggregate variables exhibit much less in the way of higher moments. As we see in Table 3, neither consumption, inflation, money growth, nor industrial production exhibits much skewness: values of γ_1 range from –0.687 for industrial production to +0.573 for the growth rate of the consumption deflator (inflation, in other words). Kurtosis is largely similar, although there is some evidence of leptokurtosis (positive γ_2) in the growth rates of durables consumption (3.847), industrial production (3.126), and the nondurables deflator (2.074). None of these estimates appear to be the result of a small number of extreme observations. Even for these three, the evidence of kurtosis is much weaker than for yield spreads and changes (Table 2). The question is how to reconcile Tables 2 and 3. If there is a link between bond yields and these macroeconomic variables as there is most theories, it appears not to be a simple one. Apparently we need a theory in which moderate kurtosis in fundamentals produces substantial kurtosis in yield spreads.

3 A Theoretical Benchmark

In the absence of a widely accepted general equilibrium model of bond prices, we turn to finance and consider a multifactor gaussian interest rate model in the spirit of Brennan and Schwartz (1979) and Vasicek (1977). This model, along with its cousin the Cox-Ingersoll-Ross (1985) model, serves as the standard benchmark against which other models of bond pricing are measured.

The Model

The gaussian interest rate model, like many others in finance, bases bond prices on a single random variable m that we call the pricing kernel. Given a pricing kernel, bond prices of all maturities $n \geq 0$ satisfy the pricing relation

$$b_t^{n+1} = E_t\left(m_{t+1} b_{t+1}^n\right). \tag{2}$$

In representative agent settings, equation (2) is one of the agent's first-order conditions. More generally, there exists a positive pricing kernel m that satisfies (2) in any environment that is free of pure arbitrage opportunities (Duffie 1992, for example). Given a pricing kernel, we compute bond prices recursively starting with $b_t^0 = 1$ (a dollar to day is worth one dollar).

We construct a pricing kernel for our multifactor gaussian interest rate model in two steps. In the first step, independent state variables or "factors" z_i are specified as normal first-order autoregressions:

$$z_{it+1} = \varphi_i z_{it} + \varepsilon_{it+1}, \tag{3}$$

with innovations ε_{it} normally distributed with mean zero and variance σ_i^2 and independent across i and t. Equation (3) is the discrete-time analog of the diffusions commonly used in continuous-time models of asset pricing. The second step is the pricing kernel:

$$-\log m_{t+1} = \delta + \sum_i \left[(\lambda_i \sigma_i)^2/2 + z_{it} + \lambda_i \varepsilon_{it+1} \right]. \tag{4}$$

The logarithm guarantees that m is positive, as required by the theory. The minus sign and choice of intercept define the short rate as

$$y_t^1 = \delta + \sum_i z_{it}. \tag{5}$$

With this strcuture, each parameter has a clear interpretation. Equation (5) implies that δ is the mean short rate. The variance and autocorrelation of the short rate (and other rates, as well) are governed by $\{\sigma_i\}$ and $\{\varphi_i\}$. Finally, the λ_i's govern the correlation between innovations in the state variables and the pricing kernel: risk, in other words.

One of the convenient features of this model is that bond prices are log-linear functions of the state variables. For each maturity n, the price can be expressed

$$-\log b_t^n = A_n + \sum_i B_{in} z_{it}, \tag{6}$$

for some choice of parameters $\{A_n, B_{in}\}$. From the pricing relation, we derive the parameters recursively from

$$\begin{aligned} A_{n+1} &= A_n + \delta + \sum_i \left[\lambda_i^2 - (\lambda_i + B_{in})^2 \right] \sigma_i^2/2 \\ B_{in+1} &= 1 + B_{in}\varphi_i, \end{aligned}$$

starting with $A_0 = B_{i0} = 0$. The solution in this case can be computed analytically, and implies forward rates of

$$f_t^n = \delta + \sum_i \left[\lambda_i^2 - \left(\lambda_i + \frac{1 - \varphi_i^n}{1 - \varphi_i} \right)^2 \right] \sigma_i^2 / 2 + \sum_i \varphi_i^n z_{it} \qquad (7)$$

Thus more persistent factors (those with larger φ's) have relatively greater influence on long forward rates and yields. If a particular φ_i is close to one, then its effect is similar across maturities and state variable i has little effect on a yield spread. In the two-factor case, we can estimate (roughly speaking) the parameters of the more persistent factor from long rates and those of the less persistent factor from yield spreads.

Parameter Values

We estimate the parameters by the method of moments, or just-identified GMM. This provides us with a quantitative version of the model and — perhaps more important — indicates the features of the data that are needed to identify specific parameters.

The one-factor model illustrates this procedure in a simple setting and establishes the need for a model with two or more factors. With one factor we have four parameters: δ, φ, σ, and λ. The short rate is

$$y_t^1 = \delta + z_t.$$

Since z has a mean of zero, we find δ from the mean of the short rate. The evidence in Table 1 implies $\delta = 5.314/1200 = 0.004428$. The autoregressive parameter φ is also implied by the behavior of the short rate in Table 1, namely its autocorrelation $\varphi = 0.976$. The variance is the short rate is

$$Var\left(y^1\right) = \frac{\sigma^2}{1 - \varphi^2},$$

implying $\sigma = 0.000556$. The risk parameter, λ, governs the risk premiums on long bonds and must be identified from long yields. Mean yields in this model are

$$E\left(y^n\right) = n^{-1} A_n.$$

To match the mean ten-year bond yield in Table 1, we need $\lambda = -147.5$.

The most striking discrepancy between theory and data in the one-factor case was noted by Gibbons and Ramaswamy (1993) for the closely related one-factor Cox-Ingersoll-Ross (1985) model. As we see in Figure 1, our model reproduces — by construction — mean yields for maturities of one month and ten years, but it understates mean yields for intermediate maturities. The source of the problem is φ. With a smaller value the model generates greater curvature of the mean yield curve, but it then understates the autocorrelation of the short rate. This tension between the cross-section and time-series evidence on φ is an indication that the one-factor gaussian model is inadequate.

Another issue is the autocorrelation of yield spreads. In the one-factor model, all yields and yields spreads are linear functions of z, and therefore have the same autocorrelation. In the data they do not. Yield spreads, in particular, exhibit substantially less persistence than yields (Table 2).

A two-factor model provides a more accurate approximation to the data along both dimensions. The model has seven parameters, which we estimate with seven moment conditions:

- We estimate δ from the mean short rate and use the same value, $\delta = 0.004428$.

- Volatility (σ_i) and persistence (φ_i) parameters are now intertwined. The theory implies that an arbitrary yield or yield spread s is a linear function of the state variables:

  $$s_t = c_0 + c_1 z_{1t} + c_2 z_{2t},$$

  with parameters $\{c_0, c_1, c_2\}$ related to $\{A_n, B_{in}\}$ as indicated by (1, 6). Note from the recursions for B_{ni} that each c_i is a function of the autoregressive parameter φ_i alone. The spread has variance

  $$Var(s) = c_1^2\, Var(z_1) + c_2^2\, Var(z_2) \tag{8}$$

  and autocorrelation

  $$Auto(s) = \frac{c_1^2\, Var(z_1)}{c_1^2\, Var(z_1) + c_2^2\, Var(z_2)}\, \varphi_1 + \frac{c_2^2\, Var(z_2)}{c_1^2\, Var(z_1) + c_2^2\, Var(z_2)}\, \varphi_2. \tag{9}$$

  Observations of variances and autocorrelations for two spreads allow us, in principle, to compute two σ's and two φ's. The difficulty is that the parameters must be computed simultaneously. Given the φ's, we can compute c_1 and c_2 and, from (8), the variances of the state variables and the volatility parameters σ_i. And given the variances of the state variables, we can compute the autoregressive parameters φ_i from (9). We compute them together using the variances and autocorrelations of two "spreads": the short rate y^1 and the five-year spread $y^{60} - y^1$. The result is $\varphi_1 = 0.997$, $\sigma_1 = 0.000177$, $\varphi_2 = 0.858$, and $\sigma_2 = 0.000511$.

- We estimate the λ's from the mean yields for maturities of 60 and 120 months. The intermediate 60-month rate is the "Gibbons-Ramaswamy" moment that the one-factor model failed to reproduce. The implied values are $\lambda_1 = -135.7$ and $\lambda_2 = -564.1$.

These estimates imply that the second factor accounts for most of the conditional variance in the short rate ($\sigma_2^2 > \sigma_1^2$), but the first factor accounts for most of its unconditional variance $[\sigma_1^2/(1-\varphi_1^2) > \sigma_2^2/(1-\varphi_2^2)]$. In this sense, the first factor generates the highly persistent movements in interest rates that are sometimes attributed to similar behavior in inflation.

This model also makes progress toward correcting two of the anomalous properties of the one-factor model. We mitigate the Gibbons-Ramaswamy anomaly by using a small λ on the more persistent factor (the first one) and a larger one (in absolute value) on the second factor. This comes considerably closer to mean yields than the one-factor model (see Figure 1), but does not reproduce the steep slope at the short end. With respect to the difference in autocorrelations, the short rate in this model is dominated by the persistent factor (its variance is 5.6 times that of the second) and therefore inherits its persistence. The spread, on the other hand, emphasizes the less persistent factor and is therefore less highly autocorrelated. Both are common in the literature: Chen and Scott (1993) and Duffie and Singleton (1996) report estimates with similar properties for different versions of the two-factor Cox-Ingersoll-Ross model.

4 Das's Jump Model

Despite success along other dimensions, the two-factor benchmark model generates normal bond yields and is therefore incapable of addressing issues related to higher moments. We turn to an extension developed by Das (1994) that adds non-normal innovations ("jumps") to an otherwise similar structure. Jumps strike us as a sensible way to incorporate non-normality: they generate (by construction) kurtosis in yield changes, but substantially less in yields themselves, which is exactly what we see in the data (compare Tables 1 and 2B). Stochastic volatility models, on the other hand, have the opposite property: they tend to generate greater kurtosis in yields than yield changes. Another useful feature of Das's model is that it retains the linearity of the benchmark: yields are linear functions of the state variables.

The Model

In our version of Das's jump model the innovation to one of the two factors, which we arbitrarily designate as the second, is a mixture of zero-mean normals. An alternative that is equally tractable is a Gram-Charlier expansion (Backus, Foresi, and Telmer, 1996). The state variable z_2 follows

$$z_{2t+1} = \varphi_2 z_{2t} + \varepsilon_{2t+1},$$

where ε_2 is a mixture of two normals,

$$\varepsilon_{2t} = \begin{cases} \eta_{1t} & \text{with probability } 1-\pi \\ \eta_{2t} & \text{with probability } \pi \end{cases}$$

and each η_{it} is an independent normal with mean zero and variance σ_{2i}^2. The aggregate variance is $Var(\varepsilon_{2t}) = \sigma_2^2 = (1-\pi)\sigma_{21}^2 + \pi\sigma_{22}^2$. This structure is similar to continuous-time jump models in which there is a small probability of a discontinuous "jump" in the path of the state variable z. In our discrete-time setting, there is a probability π that the innovation is drawn from a normal distribution with a different variance. The result is innovations with greater kurtosis. Within this structure, a natural pricing kernel is

$$\begin{aligned} -\log m_{t+1} &= \delta + \lambda_1^2/2 + z_{1t} + \lambda_1 \varepsilon_{1t+1} \\ &\quad + \log\left[(1-\pi)e^{(\sigma_{21}\lambda_{21})^2/2} + \pi e^{(\sigma_{22}\lambda_{22})^2/2}\right] + z_{2t} + \varepsilon'_{2t+1}, \end{aligned}$$

where

$$\varepsilon'_{2t} = \begin{cases} \lambda_{21}\eta_{1t} & \text{with probability } 1-\pi \\ \lambda_{22}\eta_{2t} & \text{with probability } \pi \end{cases} .$$

As before, the minus sign and intercept define the short rate by (5).

This model allows non-normal innovations in interest rates. It also allows (since λ_{21} need not equal λ_{22}) the two components of the mixture to have different risk parameters. Thus we might see, for example, that large shocks (draws from the high variance component) are more highly correlated with the pricing kernel than small shocks, and that the risk premium associated with large shocks is larger as a result. Or we might see the reverse. The point is that the model allows us to infer the answer from the data.

Despite the non-normal innovations, bond yields in this model are linear functions of the state variables. Bond prices again satisfy (6) for some choice of $\{A_n, B_{in}\}$. The coefficients satisfy

$$\begin{aligned} A_{n+1} &= A_n + \delta + \left[\lambda_1^2 - (\lambda_1 + B_{1n})^2\right] \sigma_1^2/2 \\ &\quad + \log\left[(1-\pi)e^{(\lambda_{21}+B_{2n})^2\sigma_{21}^2/2} + \pi e^{(\lambda_{22}+B_{2n})^2\sigma_{22}^2/2}\right] \\ B_{in+1} &= 1 + B_{in}\varphi_i, \end{aligned}$$

starting with $A_0 = B_{i0} = 0$.

Parameter Values

This model has three additional parameters that we can use to reproduce additional features of the data. We estimate them as follows:

- We estimate δ from the mean short rate, so again $\delta = 0.004428$.
- Since the variances and autocorrelations of this model are the same as our previous model, we estimate them the same way. The estimates are $\varphi_1 = 0.997$, $\sigma_1 = 0.000177$, $\varphi_2 = 0.858$, and $\sigma_2 = 0.000511$.
- The new ingredients are the mixtures. We follow Das (1994) in fixing the probability π at the outset. Our value of $\pi = 0.01$ corresponds to roughly one ocurrence every three months. Given π, we choose volatilities σ_{2i} to reproduce the variance of the second factor and the kurtosis of innovations in the short rate,

  $$y_{t+1}^1 - E_t y_{t+1}^1 = \varepsilon_{1t+1} + \varepsilon_{2t+1}.$$

  The former implies

  $$\sigma_2^2 = (1-\pi)\sigma_{21}^2 + \pi\sigma_{22}^2,$$

with σ_2 given above. The latter implies kurtosis for the short rate innovation of

$$\gamma_2 = 3 \frac{(1-\pi)\sigma_{21}^4 + \pi\sigma_{22}^4 + \sigma_1^4}{[(1-\pi)\sigma_{21}^2 + \pi\sigma_{22}^2 + \sigma_1^2]^2} - 3.$$

We set this equal to the kurtosis in the change in the short rate in Table 2 (10.224), which turns out to be a good approximation. These two conditions imply $\sigma_{21} = 0.000455$ and $\sigma_{22} = 0.00238$, so the standard deviation of the second component is more than five times as large as that of the first.

- We estimate the λ's from mean yields for maturities of 12, 60, and 120 months, which imply $\lambda_1 = -162.4$, $\lambda_{21} = 9684$, and $\lambda_{22} = -2118$.

In many respects, this model is like the two-factor benchmark. It differs, first, in the substantial kurtosis of the short rate. This kurtosis applies to short rate innovations: The short rate itself has little kurtosis in either the data and the model.

In short, a small probability of a large innovation in the short rate allows the model to reproduce the kurtosis in short rate changes as well as the steep slope of the mean yield curve at the short end. Both are improvements on the two-factor gaussian interest rate model.

5 Interpretation

This method of estimating a pricing kernel leaves open the question of where it comes from, but its properties are fertile ground for speculation.

Although its shortcomings are well known, much of our intuition about the relation between asset prices and macroeconomic variables stems from the representative agent model with power utility. In this model, the pricing kernel satisfies

$$-\log m_{t+1} = \rho + \alpha \log(c_{t+1}/c_t) + \log(p_{t+1}/p_t), \tag{10}$$

where c is consumption, p is the price level, ρ is the discount rate, and α is the risk aversion parameter.

Consider, by way of comparison, our estimated pricing kernels. The analog of the Hansen-Jagannathan (1992) bound in our setting is the variance of $\log m$. For the benchmark model, the conditional variance is

$$Var_t(\log m_{t+1}) = (\lambda_1 \sigma_1)^2 + (\lambda_2 \sigma_2)^2 = 0.28945^2.$$

The unconditional variance is

$$Var(\log m_{t+1}) = Var(\log m_{t+1}) + Var(z_1) + Var(z_2) = 0.28946^2.$$

We refer to both numbers as lower bounds, since we have excluded variation in the kernel that is uncorrelated with interest rates. More interesting, we find, as did Cochrane and Hansen (1992) by other methods, that most of the variance in the pricing kernel is conditional. It therefore has very little autocorrelation.

For the jump model the variance of the kernel is much larger. The unconditional variance is

$$\begin{aligned} Var(\log m_{t+1}) &= Var(z_1) + Var(z_2) + (\lambda_1 \sigma_1)^2 + (1-\pi)(\lambda_{21}\sigma_{21})^2 + \pi(\lambda_{22}\sigma_{22})^2 \\ &= 4.414^2. \end{aligned}$$

This number strikes us as extreme, but it illustrates how evidence of higher moments can affect estimates of the variability of the kernel. Snow (1991) makes a similar point using very different methods.

Now consider the representative agent model summarized by equation (10). One problem is that consumption growth is much less variable than our estimated pricing kernel. If α is less than 100, the pricing kernel is less variable than even our benchmark estimates indicate. Inflation is also much less variable than the estimated kernel, and has no parameter to magnify its effects. Moreover, its dynamics are wrong: the pricing kernel is close to white noise, with a small negative autocorrelation. Inflation has strong positive autocorrelation (Table 3).

A popular solution to at least some of these problems is to posit exotic preferences, like habit persistence or non-expected utility. These models generate greater variability in the kernel from given variation in consumption growth. Many of them also introduce skewness and kurtosis in the pricing kernel, even if consumption growth is normal.

Our model suggests a different solution: that a normal pricing kernel might give rise to non-normal asset prices. We derived parameters that reproduced the kurtosis in short rate innovations and conjectured that "jumps" might have greater risk than small changes: that, in the notation of the model, $|\lambda_{21}| < |\lambda_{22}|$. In fact the opposite is true. As a result, the kernel contains a mixture of normals with standard deviations of $|\lambda_{21}\sigma_{21}| = 4.41$ and $|\lambda_{22}\sigma_{22}| = 5.03$. Since these are similar, their mixture has little kurtosis and innovations in the pricing kernel have substantially less kurtosis than those in the short rate.

This exercise is at best an illustration, but it demonstrates that there need be no difficulty in principle reconciling leptokurtosis in bond yields with its relative absence from macroeconomic variables. A near-normal mixture for inflation or consumption growth, for example, can reproduce the leptokurtotic interest rate changes we see in the data.

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Table 1
Properties of US Government Bond Yields

Maturity	Mean	St Dev	Skewness	Kurtosis	Auto
1 month	5.314	3.064	0.886	0.789	0.976
3 months	5.640	3.143	0.858	0.691	0.981
6 months	5.884	3.178	0.809	0.574	0.982
9 months	6.003	3.182	0.776	0.480	0.982
12 months	6.079	3.168	0.730	0.315	0.983
24 months	6.272	3.124	0.660	0.086	0.986
36 months	6.386	3.087	0.621	-0.066	0.988
48 months	6.467	3.069	0.612	-0.125	0.989
60 months	6.531	3.056	0.599	-0.200	0.990
84 months	6.624	3.043	0.570	-0.349	0.991
120 months	6.683	3.013	0.532	-0.477	0.992

The data are monthly estimates of annualized continuously-compounded zero-coupon US government bond yields computed by McCulloch and Kwon (1993), January 1952 to February 1991 (470 observations). Mean is the sample mean, St Dev the sample standard deviation, Skewness an estimate of the skewness measure γ_1, Kurtosis an estimate of the kurtosis measure γ_2, and Autocorr the first autocorrelation. The skewness and kurtosis measures are defined, specifically, in terms of central moments μ_j: $\gamma_1 = \mu_3/\mu_2^{3/2}$ and $\gamma_2 = \mu_4/\mu_2^2 - 3$. Both are zero for normal random variables. Our estimates replace population moments with sample moments.

Table 2
Properties of Yield Spreads and Changes

Maturity	Mean	St Dev	Skewness	Kurtosis	Auto
		A. Spreads Over Short Rate			
3 months	0.326	0.303	2.036	7.079	0.353
6 months	0.570	0.437	1.457	5.350	0.556
9 months	0.689	0.521	1.362	5.032	0.630
12 months	0.765	0.593	1.271	4.964	0.686
24 months	0.959	0.796	0.531	2.606	0.793
36 months	1.073	0.927	0.275	1.988	0.831
48 months	1.154	1.011	0.098	1.554	0.851
60 months	1.217	1.078	0.032	1.333	0.864
84 months	1.305	1.178	-0.001	1.092	0.879
120 months	1.369	1.237	-0.087	0.815	0.885
		B. Monthly Changes in Yields			
1 month	0.008	0.644	-1.172	10.224	0.023
3 months	0.009	0.575	-1.751	14.008	0.110
6 months	0.009	0.570	-1.619	15.618	0.150
9 months	0.009	0.571	-1.240	14.680	0.148
12 months	0.010	0.547	-0.783	12.824	0.152
24 months	0.011	0.487	-0.398	11.474	0.132
36 months	0.011	0.441	-0.032	8.128	0.100
48 months	0.011	0.409	0.052	6.359	0.087
60 months	0.011	0.382	0.077	5.142	0.077
84 months	0.012	0.340	0.040	3.548	0.069
120 months	0.012	0.309	-0.205	3.288	0.068

See Table 1 for notes.

Table 3
Properties of Other Aggregate Variables

Variable	Mean	St Dev	Skewness	Kurtosis	Auto
Consumption Growth					
Consumption	0.002	0.006	-0.316	1.562	-0.162
Durables	0.003	0.030	-0.517	3.847	-0.132
Nondurables	0.001	0.007	0.122	0.974	-0.364
Services	0.002	0.004	-0.218	1.906	-0.203
Inflation					
Consumption	0.004	0.003	0.573	0.249	0.560
Durables	0.002	0.003	0.271	1.549	0.367
Nondurables	0.003	0.004	0.646	2.074	0.575
Services	0.004	0.003	0.030	-0.095	0.367
Other Growth Rates					
Money (M1)	0.001	0.006	-0.179	1.278	0.491
Money (M2)	0.002	0.005	0.124	1.460	0.648
Ind Production	0.003	0.008	-0.687	3.126	0.370

See Table 1 for labels. Data are monthly from January 1960 to September 1996. Growth rates, including inflation, are differences in logarithms. All series from Datastream.

Figure 1
Mean Yields in Data and Vasicek Models

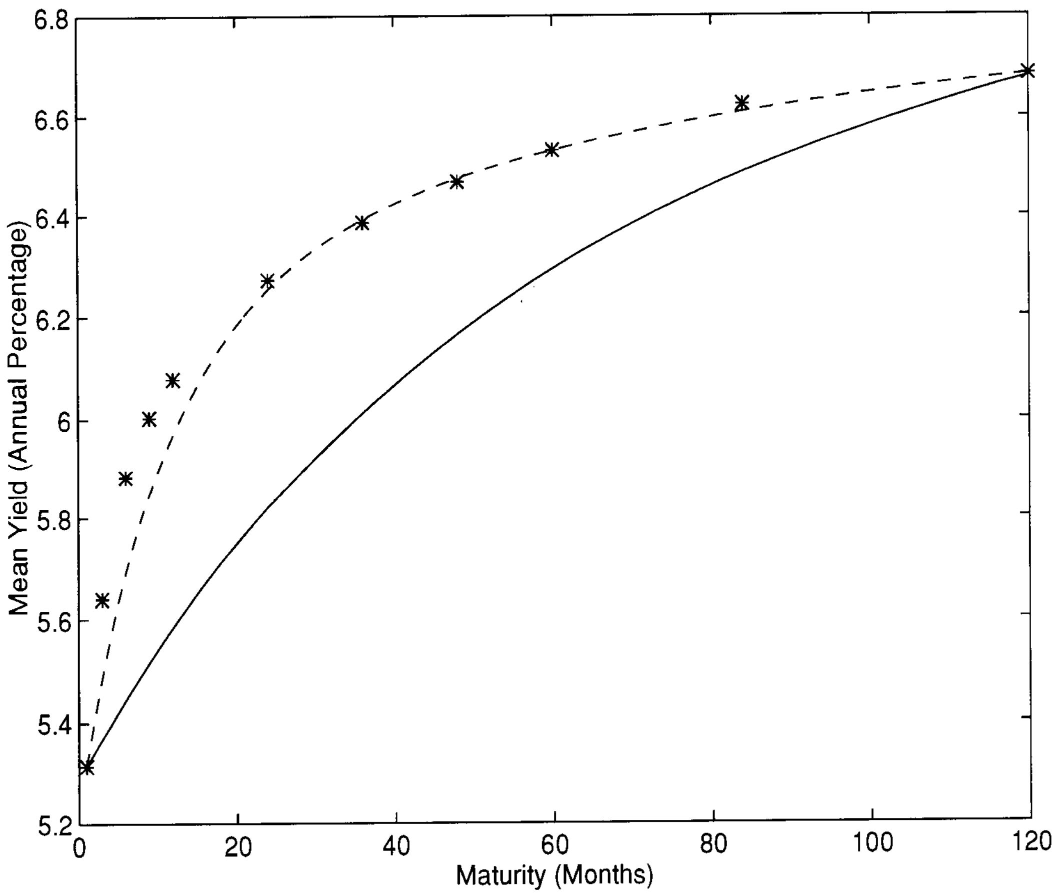

Figure 2
Mean Yields in Data and Jump Model