Asset pricing puzzles:

Evidence from options markets

September 2000

Joshua V. Rosenberg
Department of Finance
NYU - Stern School of Business
44 West 4th Street, Suite 9-190
New York, New York 10012-1126
(212) 998-0311
jrosenb0@stern.nyu.edu

Abstract

This paper proposes and implements a consumption-based pricing kernel (stochastic discount factor) testing methodology that focuses on the covariance between the pricing kernel and asset squared excess returns. This covariance has an intuitive economic interpretation as a risk-neutral variance risk-premium, i.e. the difference between the risk-neutral return variance and the objective return variance. In the same way that an asset risk-premium puzzle is due to a failure of the pricing kernel to adequately covary with asset excess returns, a risk-neutral variance puzzle is due to a failure of the pricing kernel to adequately covary with asset squared excess returns.

This paper tests a consumption-based pricing kernel specification that is compatible with habit formation, consumption durability, and constant relative risk-aversion over a range of plausible preference parameter values. The difference between consumption-based and semi-parametric option-based estimates of unconditional risk-neutral S&P500 return variance is used as a pricing kernel specification test statistic.

Evidence is found of a risk-neutral S&P500 return variance puzzle if constant relative risk-aversion is assumed. The puzzle is resolved when the pricing kernel is allowed to exhibit habit formation. The acceptable habit pricing kernels exhibit higher habit levels, higher utility function concavity, and lower rates of time-preference than estimates in related papers. When the full history of consumption data is used, the preference parameter estimates are more similar to those of related papers.

This paper has benefited from the comments of Jerome Detemple, John Heaton, Martin Lettau, René Stulz, an anonymous referee at the Journal of Finance, seminar participants at the 10th Annual Derivative Securities Conference, and seminar participants the Federal Reserve Bank of New York. The most recent version of this paper is available at http://www.stern.nyu.edu/~jrosenb0/J_wpaper.htm.

I. Introduction

Is there an aggregate consumption-based pricing kernel that can rationalize the characteristics of financial market data? This question is the focus of a substantial body of asset pricing literature; see, for example, Hansen and Singleton (1982, 1983), Ferson (1983), Cochrane and Hansen (1992), Gallant, Hansen, and Tauchen (1990), Ferson and Constantinides (1991), Hansen and Jagannathan (1991), Cecchetti, Lam, and Mark (1994), Dunn and Singleton (1995), Heaton (1995), Chapman (1997), Campbell and Cochrane (1999), and many others.

The standard setting to address this question begins with a time-series of aggregate consumption data, a postulated utility function specification (or a nonparametric estimation procedure), a time-series of asset returns (equities, bonds, currencies), and a set of orthogonality conditions implied by the Euler equation. Various specification tests are used to measure the adequacy of the model fit to the data.

The finding that consumption-based pricing kernel specifications at economically plausible preference parameter values provide a poor fit to financial market data is referred to as an "asset pricing puzzle." The equity premium puzzle of Mehra and Prescott (1985) refers to result that a consumption-based constant relative risk-aversion (CRRA) pricing kernel must have a high coefficient of relative risk-aversion (γ) to characterize average equity returns. The risk-free rate puzzle of Weil (1989) refers to the result that a consumption-based CRRA pricing kernel must have a rate of time-preference (ρ) greater than one to accurately predict the average riskless interest rate level, when the coefficient of relative risk-aversion is high enough to replicate average equity returns.

This paper proposes and implements a consumption-based pricing kernel (stochastic discount factor) testing methodology that focuses on the covariance between the pricing kernel and asset squared excess returns. This covariance has an intuitive economic interpretation as a risk-neutral variance risk-premium, i.e. the difference between the risk-neutral return variance and the objective return variance.[1] In the same way that an asset risk-premium puzzle is due to a failure of the pricing kernel to adequately covary with asset excess returns, a risk-neutral variance puzzle is due to a failure of the pricing kernel to adequately covary with asset squared excess returns.

[1] The risk-neutral variance is the variance of the risk-neutral return density, i.e. $E_t^*(R_{t+1} - RF_t)^2$, where R_{t+1} is the asset return and RF_t is the riskless interest rate. The risk-neutral return density is also referred to as the equivalent martingale measure. The objective density is sometimes also referred to as the subjective density. Note that the risk-neutral variance is only equal to the Black-Scholes (1973) implied variance when the risk-neutral density is lognormal.

The expected value of the product of the pricing kernel and asset squared excess return is proportional to the risk-neutral asset return variance; so different consumption-based pricing kernel specifications estimate different levels of unconditional risk-neutral asset return variance. The difference between the estimated unconditional risk-neutral asset return variance using a particular consumption-based pricing kernel and the "true" level provides a measure of the pricing kernel adequacy.

In practice, the "true" risk-neutral asset return variance cannot be directly observed, but it may be estimated using option data. Option data is helpful in this context, because it is possible to develop a "model-free" option-based estimator that does not depend on a particular specification of consumer preferences or asset price stochastics. Since risk-neutral return variance is often measured using option data, this paper links the literature focusing on estimation of risk-neutral densities using option data — e.g. Shimko (1993), Rubinstein (1994), Longstaff (1995), and Ait-Sahalia and Lo (1998) — with the consumption-based asset pricing literature.

Option-based risk-neutral return variance is also a summary statistic that captures an important characteristic of historical option prices. Thus, this paper provides a technique to incorporate data from options markets in tests of consumption-based pricing kernels. Previous papers have analyzed consumption-based pricing kernels using equity, bond, and currency data, but not option data.

This paper also extends the existing literature that characterizes necessary properties of a consumption-based pricing kernel. For example, Hansen and Singleton (1982) utilize expectations of the product of the pricing kernel, asset returns, and instrumental variables to estimate pricing kernel preference parameters. Hansen and Jagannathan (1991) characterize the minimum acceptable ratio of the pricing kernel standard deviation and mean, while Cochrane and Hansen (1992) show that low correlation between the pricing kernel and asset returns is often the source of asset pricing puzzles.

Several existing papers — Ait-Sahalia and Lo (2000), Jackwerth (2000), and Rosenberg and Engle (2000) — estimate a pricing kernel defined over equity return states using option data. However, these papers do not propose a methodology to link the pricing kernel defined over equity return states to a consumption-based pricing kernel. So, these papers cannot directly analyze consumption-based specifications.

This paper tests a consumption-based pricing kernel specification that is compatible with habit formation, consumption durability, and constant relative risk-aversion over a range of plausible preference parameter values. The pricing kernels are evaluated using aggregate monthly consumption data from the National Income and Product Accounts, and the consumption-based unconditional risk-neutral S&P500 return variance is estimated using the time-series of one-month pricing kernel realizations and squared S&P500 one-month excess returns over the period from 1988:01 to 1997:12.

The option-based unconditional risk-neutral S&P500 return variance is estimated using S&P500 futures option data over the same period. On the first trading day of each month, the conditional risk-neutral S&P500 one-month return variance is estimated semi-parametrically. The average of these conditional risk-neutral S&P500 return variances provides an estimate of the option-based unconditional risk-neutral S&P500 return variance. The statistical significance of the difference in the consumption-based and option-based unconditional risk-neutral variance estimates is measured using the bootstrap percentile technique of Efron and Tibshirani (1993).

This paper presents several important empirical results. Evidence is found of a risk-neutral S&P500 return variance puzzle if constant relative risk-aversion is assumed. The puzzle is resolved when the pricing kernel is allowed to exhibit habit formation. The acceptable habit pricing kernels exhibit higher habit levels, higher utility function concavity, and lower rates of time-preference than estimates in related papers. When the full history of consumption data is used, the preference parameter estimates are more similar to those of related papers.

The remainder of the paper is organized as follows. Section II develops the theory of risk-neutral variance in consumption-based asset pricing models and its relationship to option valuation. Section III presents estimation techniques for consumption-based and option-based unconditional risk-neutral variance as well as a technique to evaluate the statistical significance of the difference in the variance estimates. Section IV describes the data used in estimation of consumption-based and option-based unconditional risk-neutral S&P500 variance and the estimation results. Section V evaluates the consumption-based pricing kernel specifications using the unconditional risk-neutral variance criterion, and Section VI concludes the paper.

II. Risk-neutral variance: Theory

II.a. Risk-neutral variance and consumption-based asset pricing models

Consumption-based asset pricing models typically assume the existence of a representative consumer with time-separable preferences who chooses to maximize lifetime expected utility subject to a budget constraint (e.g.

Lucas, 1978). In this setting, the equilibrium real price of a traded asset is its expected pricing-kernel-weighted payoff. The pricing kernel acts as a weighting function that increases (decreases) the subjective value of a payoff in states where the marginal utility of consumption is high (low).

Consider a general asset, which might be a derivative security, with a payoff that depends on a future "underlying asset" price. Let d_t be the price of the asset in units of consumption, let $g(p_{t+1})$ be the asset's payoff as a function of a future realized "underlying asset" price (p_{t+1}), and let $m_{t+1} = e^{-\rho}U'(C_{t+1})/U'(C_t)$ be the pricing kernel or stochastic discount factor which depends on current consumption (C_t) and next period consumption (C_{t+1}). Then, the consumption-based asset pricing equation is:[2]

$$(1) \qquad d_t = E_t\left[g(p_{t+1})m_{t+1}\right]$$

The pricing kernel may be generalized to incorporate habit formation or consumption durability by adding a second state variable to the utility function to measure the impact of habit on utility, i.e. $U_t = U(C_t, X_t)$ where X_t represents the habit. Then, $m_{t+1} = [\partial U(C_{t+1}, X_{t+1})/\partial C_{t+1}]/[\partial U(C_t, X_t)/\partial C_t]$. See, for example, the survey of habit models in Campbell, Lo, and MacKinlay (1997, section 8.4).

Now, consider the nominal pricing equation for an asset with a payoff function that depends on a future nominal asset value. Following the analysis of Boudoukh (1993), inflation enters the pricing equation exogenously as a numeraire. Letting D_t represent the nominal asset price, I_{t+1} represent the gross inflation rate, and $M_{t+1}= I_{t+1}^{-1}m_{t+1}$ represent the nominal pricing kernel: [3]

$$(2) \qquad D_t = E_t\left[g(P_{t+1})m_{t+1}I_{t+1}^{-1}\right] = E_t\left[g(P_{t+1})M_{t+1}\right]$$

The nominal riskless one-period bond price (B_t), nominal riskless one-period interest rate (RF_t), and unconditional nominal riskless one-period interest rate ($E[RF_t]$) are obtained by evaluating equation (2) with a payoff function of unity.

---

[2] The familiar Euler equation of Hansen and Singleton (1982, equation 2.6) — that the expected pricing-kernel-weighted net return is zero — is obtained by dividing both sides of equation (1) by the current asset price and subtracting one, so that $E_t[M_{t+1}r_{t+1}] = 0$, where $r_{t+1}= g(p_{t+1})/d_t - 1$ is the asset net return. If the asset to be priced is the underlying asset, then $r_{t+1} = p_{t+1}/p_t - 1$, since $d_t = p_t$ and $g(p_{t+1})=p_{t+1}$.

(3) $$B_t = E_t\left[M_{t+1}\right] \qquad RF_t = E_t\left[M_{t+1}\right]^{-1} - 1 \qquad E[RF_t] = E\left[M_{t+1}\right]^{-1} - 1$$

If $R_{t+1}=P_{t+1}/P_t - 1$ represents the nominal one-period net return for the underlying asset with a current price of P_t, then the underlying asset's nominal expected return and unconditional nominal expected return are:[4]

(4) $$E_t[R_{t+1}] = -E_t\left[M_{t+1}\right]^{-1} Cov_t[R_{t+1}, M_{t+1}] \qquad E[R_{t+1}] = -E\left[M_{t+1}\right]^{-1} Cov[R_{t+1}, M_{t+1}]$$

Equation (4) is a version of the consumption capital asset pricing model (e.g. Merton, 1973 or Breeden, 1979), since an asset's expected return depends linearly on its covariance with the pricing kernel. Assets with large negative covariance with the pricing kernel are more risky (less desirable) and require higher expected returns, because they have the highest payoffs when the marginal utility of a unit payoff is lowest.

Equation (2) may also be rewritten in "risk-neutral density" form, in which the risk-weighting of the pricing kernel is incorporated into the density function. In particular, the conditional risk-neutral density is the bivariate conditional density function of the pricing kernel and the asset price scaled by the pricing kernel and then renormalized.[5]

(5) $$D_t = E_t[M_{t+1}] \iint g(P_{t+1})\, f_t^*(M_{t+1}, P_{t+1}) dM_{t+1} dP_{t+1}$$
$$f_t^*(M_{t+1}, P_{t+1}) = E_t[M_{t+1}]^{-1} M_{t+1} f_t(M_{t+1}, P_{t+1})$$

---

[3] Suppose the price index is given by i_t. Then, the real price of the asset is the expected pricing-kernel-weighted real payoff. $D_t/i_t = E_t[(g(P_{t+1})/i_{t+1})m_1]$. So, the nominal price of the asset is given by multiplying through by the current price index: $D_t = E_t[g(P_{t+1})m_{t+1}(i_t/i_{t+1})] = E_t[g(P_{t+1})m_{t+1}I_{t+1}^{-1}] = E_t[g(P_{t+1})M_{t+1}]$.

[4] Using the identity $Cov(X,Y) = E[XY] - E[X]E[Y]$, $Cov_t[R_{t+1},M_{t+1}] = E[R_{t+1}M_{t+1}] - E[R_{t+1}]E[M_{t+1}]$. Equation (2) implies that $E_t[M_{t+1}R_{t+1}] = 0$, so $Cov_t[R_{t+1},M_{t+1}] = - E[R_{t+1}]E[M_{t+1}]$ or $E[R_{t+1}] = -E[M_{t+1}]^{-1}Cov_t[R_{t+1},M_{t+1}]$.

[5] Since $\iint M_{t+1} f_t(M_{t+1}, P_{t+1}) dM_{t+1} dP_{t+1} = E_t[M_{t+1}]$, $f_t^*(M_{t+1}, P_{t+1}) = E_t[M_{t+1}]^{-1} M_{t+1} f_t(M_{t+1}, P_{t+1})$ is a valid density function, since it is continuous and integrates to unity. Using equation (2),
$D_t = E_t\left[g(P_{t+1}) M_{t+1}\right] = \iint g(P_{t+1}) M_{t+1} f_t(M_{t+1}, P_{t+1}) dM_{t+1} dP_{t+1} =$
$\iint g(P_{t+1}) E_t[M_{t+1}] E_t[M_{t+1}]^{-1} M_{t+1} f_t(M_{t+1}, P_{t+1}) dM_{t+1} dP_{t+1}$, so
$D_t = E_t[M_{t+1}] \iint g(P_{t+1})\, f_t^*(M_{t+1}, P_{t+1}) dM_{t+1} dP_{t+1}$.

The bivariate conditional risk-neutral density defined in equation (5) may be simplified to a univariate conditional density that contains sufficient information to price any asset whose payoff depends only on P_{t+1}. First, define the conditional pricing kernel as $M'_{t+1} = E_t[M_{t+1}|P_{t+1}]$. Then, rewriting equation (2) by factoring the joint density into the product of the conditional and marginal densities:[6]

$$(6) \qquad D_t = \int g(P_{t+1}) M'_{t+1} f_t(P_{t+1}) dP_{t+1} = E_t[g(P_{t+1}) M'_{t+1}]$$

And, the univariate risk-neutral pricing equation and univariate conditional risk-neutral density are:[7]

$$(7) \qquad D_t = E_t[M_{t+1}] \int g(P_{t+1}) f_t^*(P_{t+1}) dP_{t+1}$$

$$f_t^*(P_{t+1}) = E_t[M_{t+1}]^{-1} M'_{t+1} f_t(P_{t+1})$$

The univariate conditional risk-neutral density defined in equation (7) may be used for valuation of all assets whose payoff function depends on the underlying price P_{t+1}. Assets with payoffs that depend on another underlying price (e.g. Q_{t+1}) will require the conditional risk-neutral density of this price (e.g. $f_t^*(Q_{t+1})$) for valuation. Notice that equation (7) does not require the underlying asset price (P_{t+1}) to be a proxy for aggregate consumption or aggregate wealth. If the asset to be priced has a payoff function that depends on aggregate consumption, then the appropriate conditional risk-neutral density to use for pricing would be the conditional risk-neutral density of aggregate consumption.

The consumption-based conditional risk-neutral variance ($\sigma^2_{c,t}$) is the variance of the conditional risk-neutral density of the one-period-ahead underlying asset return. Evaluation of this variance requires the conditional risk-

---

[6] Notice that $f_t(M_{t+1},P_{t+1}) = f_t(M_{t+1}|P_{t+1})f(P_{t+1})$, so $D_t = E_t\left[g(P_{t+1})M_{t+1}\right] = \iint g(P_{t+1}) M_{t+1} f_t(M_{t+1}, P_{t+1}) dM_{t+1} dP_{t+1} =$
$\iint g(P_{t+1}) [M_{t+1} f_t(M_{t+1} \mid P_{t+1}) dM_{t+1}] f_t(P_{t+1}) dP_{t+1} = \int g(P_{t+1}) E_t[M_{t+1} \mid P_{t+1}] f_t(P_{t+1}) dP_{t+1} =$
$\int g(P_{t+1}) M'_{t+1} f_t(P_{t+1}) dP_{t+1}$

[7] Since $\int M'_{t+1} f_t(P_{t+1}) dP_{t+1} = E_t[M'_{t+1}] = E_t[E_t[M_{t+1} \mid P_{t+1}]] = E_t[M_{t+1}]$,
$f_t^*(P_{t+1}) = E_t[M_{t+1}]^{-1} M_{t+1} f_t(P_{t+1})$ is a valid density function, because it is continuous and integrates to unity.
Then, $D_t = E_t\left[g(P_{t+1})M'_{t+1}\right] = \int g(P_{t+1}) M'_{t+1} f_t(P_{t+1}) dP_{t+1} = \int g(P_{t+1}) E_t[M_{t+1}] E_t[M_{t+1}]^{-1} M'_{t+1} f_t(P_{t+1}) dP_{t+1}$,
so $D_t = E_t[M_{t+1}] \int g(P_{t+1}) f_t^*(P_{t+1}) dP_{t+1}$.

neutral density of the underlying asset return $f_t^*(R_{t+1})$, instead of the conditional risk-neutral density of the underlying price, $f_t^*(P_{t+1})$. However, there is a one-to-one mapping between these two densities. The probability of the outcome P_{t+1} is identical to the probability of the outcome $R_{t+1} = P_{t+1}/P_t - 1$ since P_t is known at date t.

Two additional facts are required for evaluation of the conditional risk-neutral variance. First, given a particular return (R_{t+1}), the corresponding underlying price that would generate that return is $P_{t+1} = P_t(1+R_{t+1})$. Second, the mean of the conditional risk-neutral return density is RF_t.[8] Hence, the consumption-based conditional risk-neutral variance is given by:

$$(8) \qquad s_{c,t}^{*2} = \int (R_{t+1} - RF_t)^2 f_t^*(R_{t+1}) dR_{t+1} = E_t^*[(R_{t+1} - RF_t)^2]$$

$$f_t^*(R_{t+1}) = f_t^*(P_{t+1})\Big|_{P_t(1+R_{t+1})}$$

And, the consumption-based unconditional risk-neutral variance is defined as the unconditional expectation of the conditional risk-neutral variance.

$$(9) \qquad s_c^{*2} = E[s_{c,t}^{*2}]$$

The conditional risk-neutral variance ($\sigma^{*2}_{c,t}$) is related to the conditional objective return variance ($\sigma_t^2 = E_t[(R_{t+1} - RF_t)^2]$) and the conditional risk-neutral variance risk-premium ($\lambda_{c,t}$).[9] Rewriting equation (8):[10]

$$(10) \qquad s_{c,t}^{*2} = s_t^2 + l_{c,t} \qquad\qquad l_{c,t} = E_t[M_{t+1}]^{-1} Cov_t[(R_{t+1} - RF_t)^2, M_{t+1}]$$

[8] Using equation (7), the underlying asset price is given by $P_t = E_t[M_{t+1}]^{-1}E_t^*[P_{t+1}]$, i.e. the payoff function is $g(P_{t+1}) = P_{t+1}$. So, it is the case that $E_t^*[P_{t+1}] = E_t[M_{t+1}]P_t$, i.e. the mean of the conditional risk-neutral price density is $E_t[M_{t+1}]^{-1}P_t$. Then, the mean of the conditional risk-neutral return density is $E_t[M_{t+1}]^{-1}P_t/P_t - 1 = RF_t$.

[9] The definition of the conditional objective variance as $\sigma_t^2 = E_t[(R_{t+1} - RF_t)^2]$ is nonstandard in that the returns are centered around the riskless rate rather than the expected return. However, for convenience, this quantity is referred to as the "conditional objective variance," i.e. return variance under the conditional objective probability measure. This may also be interpreted as the conditional expected squared excess return.

[10] Substituting $f_t^*(P_{t+1}) = E_t[M_{t+1}]^{-1}M'_{t+1}f_t(P_{t+1})$ into equation (8), $\sigma^{*2}_{c,t} = E_t^*[R_{t+1} - RF_t]^2 = E_t[M_{t+1}]^{-1}E_t[(R_{t+1} - RF_t)^2M'_{t+1}]$. Using the identity $E[XY] = Cov(X,Y) + E[X]E[Y]$, $E_t[M_{t+1}]^{-1}E_t[(R_{t+1} - RF_t)^2M'_{t+1}] = E_t[M_{t+1}]^{-1}[Cov_t[(R_{t+1} - RF_t)^2, M'_{t+1}] + E_t[M_{t+1}]E_t[R_{t+1} - RF_t]^2$, so $\sigma^{*2}_{c,t} = \sigma_t^2 + E_t[M_{t+1}]^{-1}[Cov_t[(R_{t+1} - RF_t)^2, M'_{t+1}]$, with $\sigma_t^2 = E_t[(R_{t+1} - RF_t)^2]$. So, $\lambda_{c,t} = E_t[M_{t+1}]^{-1}[Cov_t[(R_{t+1} - RF_t)^2, M'_{t+1}]$.

Or, in unconditional terms:

$$(11) \qquad \sigma_c^{*2} = \sigma^2 + \lambda_c \qquad \sigma^2 = E[\sigma_t^2] \qquad \lambda_c = E[\lambda_{c,t}] = \sigma_c^{*2} - \sigma^2$$

Equation (10) states that the consumption-based conditional risk-neutral variance is equal to the conditional objective return variance plus a conditional risk-neutral variance risk-premium. The conditional risk-neutral variance risk-premium is equal to the scaled covariance of the squared excess return and the pricing kernel. So, an asset with a positive (negative) conditional covariance of squared excess returns with the pricing kernel has a conditional risk-neutral variance that is larger (smaller) than its conditional objective variance. In equation (11), the unconditional risk-neutral variance is equal to the sum of the unconditional objective variance and the unconditional risk-neutral variance risk-premium.

The consumption-based risk-neutral variance provides insight into consumption-based pricing kernel specifications by measuring the predicted covariance between squared excess returns and the pricing kernel. This provides an extension of standard analyses of consumption-based pricing kernels that focus on the predicted asset return risk-premium, which is related to the covariance of the asset excess return and the pricing kernel or on analyses based on the mean and standard deviation of the pricing kernel.

The risk-neutral variance risk-premium often arises in the context of option pricing under stochastic volatility. Since options are priced as their expected payoff under the risk-neutral density of the underlying price, the risk-neutral variance is relevant (rather than the objective variance) for valuation. The objective and risk-neutral variance are not identical when the squared excess returns are correlated with the pricing kernel, i.e. the risk-neutral volatility risk-premium is non-zero. Hull and White (1987) require that volatility changes have a beta of zero (i.e. squared-returns are uncorrelated with the pricing kernel) in order for the risk-neutral variance risk-premium to be zero and their pricing results to hold. Amin and Ng (1993) show how the option pricing formula is affected when the underlying asset return volatility is correlated with consumption growth volatility, so that the risk-neutral variance risk-premium is non-zero.

II.b. Risk-neutral variance and option valuation

Notice that the risk-neutral variance may be equivalently written in terms of the original pricing kernel (M_{t+1}) rather than the conditional expectation of the pricing kernel (M'_{t+1}), so that $\sigma^{*2}_{c,t} = E_t^*[R_{t+1} - RF_t]^2 = E_t[M_{t+1}]^{-1}E_t[(R_{t+1} - RF_t)^2 M_{t+1}]$. Then, $\sigma^{*2}_{c,t} = \sigma_t^2 + E_t[M_{t+1}]^{-1}[Cov_t[(R_{t+1} - RF_t)^2, M_{t+1}]$, with $\sigma_t^2 = E_t[(R_{t+1} - RF_t)^2]$. So, $\lambda_{c,t} = E_t[M_{t+1}]^{-1}[Cov_t[(R_{t+1} - RF_t)^2, M_{t+1}]$.

Equation (7) shows that the conditional risk-neutral density of the underlying asset price may be used to value any other asset whose payoff depends on the underlying asset price. Consider the valuation of a one-period call option with an exercise price of K. The payoff function is $g(P_{t+1}) = Max[0, P_{t+1} - K]$, so the current call price (C_t) is given by:

$$\text{(12)} \qquad C_t = E_t[M_{t+1}] \int Max[0, P_{t+1} - K] f_t^*(P_{t+1}) dP_{t+1}$$

$$f_t^*(P_{t+1}) = E_t[M_{t+1}]^{-1} M'_{t+1} f_t(P_{t+1})$$

The slope and curvature of the call option pricing formula as a function of the exercise price are closely related to the conditional risk-neutral density of the underlying asset price. Let $F_{o,t}^*(P_{t+1})$ be the option-based conditional cumulative risk-neutral density, and let $f_{o,t}^*(P_{t+1})$ be the option-based conditional risk-neutral density, where both densities are expressed in terms of derivatives of the call price function. Breeden and Litzenberger (1978) show that:[11]

$$\text{(13)} \qquad F_{o.t}^*(P_{t+1}) = (1 + RF_t) \left. \frac{\partial C_t}{\partial K} \right|_{K=P_{t+1}} + 1$$

And,[12]

[11] This relationship is derived by differentiating the consumption-based call option pricing formula in equation (12) and solving for the conditional cumulative risk-neutral density function.

$$C_t = E_t[M_{t+1}] \int Max[0, P_{t+1} - K] f_t^*(P_{t+1}) dP_{t+1} = E_t[M_{t+1}] \int_K^{\infty} [P_{t+1} - K] f_t^*(P_{t+1}) dP_{t+1} \text{, so}$$

$$\frac{\partial C_t}{\partial K} = -E_t[M_{t+1}] \int_K^{\infty} f_t^*(P_{t+1}) dP_{t+1} = -E_t[M_{t+1}] F_t^*(P_{t+1} \geq K) = -E_t[M_{t+1}](1 - F_t^*(K)) .$$

Thus, $F_t^*(P_{t+1}) = E_t[M_{t+1}]^{-1} \left. \frac{\partial C_t}{\partial K} \right|_{K=P_{t+1}} + 1 = (1 + RF_t) \left. \frac{\partial C_t}{\partial K} \right|_{K=P_{t+1}} + 1$, since $(1 + RF_t) = E_t[M_{t+1}]^{-1}$.

[12] The second derivative of the call pricing formula in equation (12) is:

$$\frac{\partial^2 C_t}{\partial K^2} = \frac{\partial}{\partial K} \left[\frac{\partial C_t}{\partial K} \right] = \frac{\partial}{\partial K} \left[- E_t[M_{t+1}] \int_K^{\infty} f_t^*(P_{t+1}) dP_{t+1} \right] = E_t[M_{t+1}] f_t^*(K) .$$

(14) $$f^*_{o,t}(P_{t+1}) = (1+RF_t)\left.\frac{\partial^2 C_t}{\partial K^2}\right|_{K=P_{t+1}}$$

Also, notice that conditional risk-neutral underlying asset return density $f^*_t(R_{t+1})$ is identical to the conditional risk-neutral underlying price density $f^*_t(P_{t+1})$ with $R_{t+1} = P_{t+1}/P_t - 1$, since the probability of P_{t+1} is equal to the probability of $P_{t+1}/P_t - 1$. So:[13]

(15) $$f^*_{o,t}(R_{t+1}) = (1+RF_t)\left.\frac{\partial^2 C_t}{\partial K^2}\right|_{K=P_t(1+R_{t+1})}$$

Breeden and Litzenberger (1978) develop an example where the underlying asset price corresponds to the level of aggregate consumption, and they derive the prices of consumption-state securities. This assumption and several others are necessary in order to value *all* securities using a single conditional risk-neutral density. However, as is shown in the derivations of equations (13)-(15), no particular correspondence between consumption and the underlying asset price is required to derive $f^*_t(P_{t+1})$ from the function that defines the prices of call options on the underlying asset. In this paper, it is not assumed that the underlying asset corresponds to aggregate wealth, aggregate consumption, or the value of a market portfolio, because there is no need to derive risk-neutral density of future consumption. Instead, this paper is concerned with the underlying risk-neutral density of future underlying asset prices.

There are now two equations that define the underlying asset conditional risk-neutral density. First, equation (7) expresses the conditional risk-neutral density in terms of the consumption-based pricing kernel and the objective density function. This is referred to as the consumption-based conditional risk-neutral density and is used to obtain the consumption-based conditional risk-neutral variance. Second, equation (14) expresses the conditional risk-neutral density in terms of the call price function and the riskless discount rate. This is referred to

---

Thus, $f^*_t(P_{t+1}) = E_t[M_{t+1}]^{-1}\left.\frac{\partial^2 C_t}{\partial K^2}\right|_{K=P_{t+1}}$.

[13] To evaluate the probability at a given return (R_{t+1}), notice that the underlying price (P_{t+1}) that would generate this return is $P_{t+1} = P_t(1 + R_{t+1})$. Then, substituting into equation (14), $K = P_t(1 + R_{t+1})$.

as the option-based conditional risk-neutral density, and is used to obtain the option-based conditional risk-neutral variance.

Using the equation (15) definition of the option-based conditional risk-neutral return density and the fact that the mean of the conditional risk-neutral return density is $E_t[M_{t+1}]^{-1} - 1 = RF_t$, the option-based conditional risk-neutral variance is given by:

(16) $$s_{o,t}^{*2} = \int (R_{t+1} - RF_t)^2 f_{o,t}^{*}(R_{t+1}) dR_{t+1} = E_{o,t}^{*}[(R_{t+1} - RF_t)^2]$$

And, the option-based unconditional risk-neutral variance is defined as the unconditional expectation of the conditional risk-neutral variance.

(17) $$s_o^{*2} = E[s_{o,t}^{*2}]$$

From equation (11), it is clear that the unconditional risk-neutral variance risk-premium is equal to the difference between the unconditional risk-neutral variance and unconditional asset return variance. Hence, an option-based expression for the unconditional risk-neutral variance risk-premium is given by:

(18) $$l_o = s_o^{*2} - s^2$$

III. Unconditional risk-neutral variance: estimation and testing

III.a. Estimation of consumption-based unconditional risk-neutral variance

The population estimator of consumption-based unconditional risk-neutral variance is given in equation (9) as the unconditional expectation of the consumption-based conditional risk-neutral variance. A sample estimator is obtained from the population estimator by replacing each variable with its realization and by replacing integration with the sample average. Applying a suitable law of large numbers, the average will converge to the unconditional expected value, providing a consistent estimate.[14]

(19) $$\hat{s}_c^{*2} = \frac{1}{T}\sum_{t=1}^{T}\hat{s}_{c,t}^{*2} = \frac{1}{T}\sum_{t=1}^{T}(1+RF_t)(R_{t+1}-RF_t)^2 M_{t+1}$$

Using equation (11), a sample estimator for the consumption-based unconditional risk-neutral variance risk-premium is given by:[15]

(20) $$\hat{l}_c = \hat{s}_c^{*2} - \hat{s}^2 = \frac{1}{T}\sum_{t=1}^{T}(1+RF_t)(R_{t+1}-RF_t)^2 M_{t+1} - \frac{1}{T}\sum_{t=1}^{T}(R_{t+1}-RF_t)^2$$

The population estimator for the consumption-based unconditional consumption-based riskless rate is given in equation (3). A sample estimator of the consumption-based unconditional consumption-based riskless rate is:[16]

(21) $$\hat{RF}_c = \left[\frac{1}{T}\sum_{t=1}^{T}M_{t+1}\right]^{-1} - 1$$

Implementation of these estimators requires a specification for the pricing kernel. Suppose that the utility function is a power function and that the representative consumer derives utility from the amount that

[14] White (1984, Ch. III.4) and Hamilton (1994, Ch. 7) present laws of large numbers that apply to non-IID stochastic processes. Using the equation 5 definition of the risk-neutral density and noticing that $E_t[R_{t+1}]^{-1} =$ $(1+RF_t)$, $s_{c,t}^{*2} = \int(1+RF_t)(R_{t+1}-RF_t)^2 M_{t+1} f_t(M_{t+1},P_{t+1})dM_{t+1}dP_{t+1}$. So,

$\hat{s}_{c,t}^{*2} = (1+RF_t)(R_{t+1}-RF_t)^2 M_{t+1}$ and $\hat{s}_c^{*2} = \frac{1}{T}\sum_{t=1}^{T}\hat{s}_{c,t}^{*2} = \frac{1}{T}\sum_{t=1}^{T}(1+RF_t)(R_{t+1}-RF_t)^2 M_{t+1}$

[15] From equation (11), $l_c = s_c^{*2} - s^2$. A consistent estimator of the unconditional risk-neutral return variance is given in equation (19), and a consistent estimator of the unconditional squared excess return is the average square excess return. So, $\hat{l}_c = \hat{s}_c^{*2} - \hat{s}^2 = \frac{1}{T}\sum_{t=1}^{T}(1+RF_t)(R_{t+1}-RF_t)^2 M_{t+1} - \frac{1}{T}\sum_{t=1}^{T}(R_{t+1}-RF_t)^2$.

[16] $E[RF_t] = E[M_{t+1}]^{-1} - 1$, so $\hat{RF}_c = \left[(1/T)\sum_{t=1}^{T}M_{t+1}\right]^{-1} - 1$.

consumption (C_t) exceeds the level of habit (X_t).[17] Examples of such "difference models" include Sundaresan (1989), Constantinides (1990), and Campbell and Cochrane (1999). Then, the representative consumer maximizes the following expected utility function:[18]

$$(22) \qquad U_t = E_t \sum_{i=0}^{I} r^i \frac{(C_{t+i} - X_{t+i})^{1-g} - 1}{(1-g)}$$

Habit is typically defined as a function of previous consumption levels. For example, in Constantinides (1990), habit is an exponentially-weighted sum of past consumption. In Ferson and Constantinides (1991), habit is a weighted average of past consumption services, where consumption services are a weighted average of past consumption levels.

Let δ be a parameter that scales the impact of the habit on current utility, and let the habit level be a weighted sum of past consumption levels. Then:

$$(23) \qquad X_t = d \sum_{j=1}^{J} a_j C_{t-j}$$

When δ is positive, the utility function exhibits habit formation, since utility is obtained from the difference between current and past consumption. When δ is negative, the utility function exhibits consumption durability, since past consumption increases current utility. When δ is zero, the utility function exhibits constant relative risk-aversion. With J = 1 and $a_1 = 1$, this habit specification is the same as the one-lag habit model of Hansen and Jagannathan (1991, equations 30-31).

The pricing kernel also depends on whether the habit is internal or external. An internal habit model allows the current consumption decision to affect future levels of the habit. An external habit model treats the habit as

---

[17] For utility to be well defined in a difference model, consumption must always remain above the level of habit. For this reason, habit may be interpreted as the subsistence level of consumption.

[18] In this model, γ is no longer equal to the coefficient of relative risk-aversion. Campbell and Cochrane (1999) utilize a measure of risk-aversion based on the local curvature of the utility function. In this case, the local curvature is defined as $\gamma/[(C_t - X_t)/C_t]$, where the denominator is the ratio of current surplus consumption to current consumption. An estimate of average local curvature is given by $g \,/ \left[\overline{(C_t - X_t)/C_t}\right]$, where the denominator is the average surplus consumption ratio.

exogenous, i.e. habit is determined by aggregate consumption, rather than individual consumption. In the case of an external habit (with I_{t+1} equal to the gross inflation rate), the nominal pricing kernel is:

(24) $$M_{t+1} = r\left(\frac{C_{t+1} - X_{t+1}}{C_t - X_t}\right)^{-g} I_{t+1}^{-1}$$

Given fixed values of ρ and γ, it is possible to estimate the habit level (δ) that is consistent with a pre-specified level of unconditional risk-neutral variance (s^{*2}) or a pre-specified unconditional risk-neutral variance risk-premium (λ). This level of habit will be referred to as the "implied δ." Estimation requires the solution of the univariate nonlinear equation $\hat{s}_c^{*2}(d) - s^{*2} = 0$ or $\hat{l}_c(d) - l = 0$ To find the delta that solves this equation, the IMSL optimization routine DUVMIF is used to minimize the squared deviation of the fitted and pre-specified risk-neutral variance or of the fitted and pre-specified risk-neutral variance risk-premium.

III.b. Estimation of option-based unconditional risk-neutral variance

Suppose that the pricing function for call options on particular underlying asset with price P_t depends on N state variables and the option exercise price (K). Consider an approximation to the true call pricing function that is estimated by minimizing a distance criterion between observed one-period option prices and fitted option prices.

(25) $$\hat{C}_t(X_{1,t},..., X_{N,t}, K) \cong C_t(X_{1,t},..., X_{N,t}, K)$$

Using numerical differentiation of the estimated call price function:

(26) $$\frac{\partial^2 \hat{C}_t}{\partial K^2} = \frac{\hat{C}_t(K+e) - 2\hat{C}_t(K) + \hat{C}_t(K-e)}{e^2} \cong \frac{\partial^2 C_t}{\partial K^2}$$

Then, using equation (15):

(27) $$\hat{f}^*_{o,t}(R_{t+1}) = (1+RF_t)\frac{\partial^2 \hat{C}_t}{\partial K^2}\bigg|_{K=P_t(1+R_{t+1})}$$

And, the estimated option-based conditional risk-neutral variance may be obtained by numerically integrating the squared excess return with respect to the estimated option-based conditional risk-neutral return density. This paper uses the IMSL routine DQDAG for numerical integration.

(28) $$\hat{s}^{*2}_{o,t} = \int (R_{t+1} - RF_t)^2 \hat{f}^*_{o,t}(R_{t+1}) dR_{t+1}$$

If an option-based conditional risk-neutral density is estimated on a sequence of dates (t = 1…T) using a contemporaneous set of one-period call prices on each date, then a sequence of option-based conditional risk-neutral variances may be obtained using equation (28). The sample estimator that corresponds to the population estimator in equation (17) uses the sample average of the time-series of conditional risk-neutral variances to estimate the option-based unconditional risk-neutral variance.

(29) $$\hat{s}^{*2}_{o} = \frac{1}{T}\sum_{t=1}^{T} \hat{s}^{*2}_{o,t}$$

Equation (18) expresses the option-based unconditional risk-neutral variance risk-premium as the difference between the option-based unconditional risk-neutral variance (σ_o^{*2}) and the unconditional asset return variance (σ^2). Recall that the unconditional asset return variance is estimated as the average of squared excess returns. Hence, a sample estimator is given by:

(30) $$\hat{l}_o = \hat{s}^{*2}_{o} - \hat{s}^{2}$$

Implementation of the option-based estimators requires a technique to obtain the call price function, given a sample of observed option prices. Ait-Sahalia and Lo (1998) suggest a nonparametric technique for estimation of the call price function and its derivatives using kernel regression of observed call prices on observable state variables. This technique has the advantage of imposing minimal assumptions on the characteristics of the risk-neutral density, so that the estimated risk-neutral density reflects the expectations and preferences of market participants as revealed through observed prices. Using this technique, no specific restrictions are placed on the underlying asset price process or the preferences of a representative consumer.

In their empirical work, Ait-Sahalia and Lo (1998) adopt a semi-parametric technique in which "… the call-pricing function is given by the parametric Black-Scholes formula … except that the implied volatility parameter for that option is a nonparametric function …" (p. 510). This technique has the advantage of dimensionality reduction (i.e. there are fewer regressors than in a fully nonparametric technique), and it imposes additional smoothness on the call pricing function, which improves the behavior of the derivatives of the call pricing function.[19]

Following Ait-Sahalia and Lo (1998, equations 9, 13). Let BS(•) be the Black-Scholes (1973) formula using the Merton (1973) dividend adjustment. In addition, let $\sigma_{i,t}$ be the Black-Scholes implied volatility for a particular option, let K be the option exercise price, let T-t be the time until expiration, and let F_t be the price of a futures contract with identical expiration and underlying asset as the option contract.[20] Also, let $k(\cdot)$ be the Nadaraya-Watson kernel estimator with a bandwidth of h. Then:

(31) $$\hat{C}_t = BS(F_t, K, T-t, \hat{s}_t(F_t, K, T-t))$$

(32) $$\hat{s}_t(F_t, K, T-t) = \frac{\sum_{i=1}^{n} k_F\left((F_t - F_{t_i})/h_F\right) k_K\left((K-K_i)/h_K\right) k_{T-t}(((T-t)-(T-t)_i)/h_{T-t}) s_{i,t}}{\sum_{i=1}^{n} k_F\left((F_t - F_{t_i})/h_F\right) k_K\left((K-K_i)/h_K\right) k_{T-t}(((T-t)-(T-t)_i)/h_{T-t})}$$

[19] In empirical option research, it is common to convert option prices to implied volatilities (usually using the Black-Scholes formula) prior to analysis. Performing estimation using implied volatilities rather than the original option prices amounts to a non-linear transformation of the original data, but does not impose the assumptions of the particular pricing formula on the data.

When the Black-Scholes assumptions are valid for options on a particular asset, identical Black-Scholes implied volatilities are obtained for options of all exercise prices and maturities. When the Black-Scholes assumptions are not valid, Black-Scholes implied volatilities for options on a particular asset exhibit different implied volatilities across exercise prices and maturities. The pattern of implied volatilities as a function of exercise price and maturity defines the deviation of the empirical risk-neutral density from the lognormal risk-neutral density of the Black-Scholes model.

[20] When the cost-of-carry model holds, $F_t = P_t e^{(r-\delta)(T-t)}$ so that the futures price (F_t) combines the effects of the spot price (P_t), riskless rate (r), and payout rate (δ) on the call option price. Using the futures price in the kernel regression instead separately using the underlying price, dividend yield, and riskless rate reduces the dimensionality of the problem with little loss of generality.

The fitted one-period option price function is given by evaluating equations (31) and (32) with T-t set equal to the time interval defined by one period. In this paper, a Gaussian kernel is used for estimation with bandwidth selection as given by Silverman (1996, equation 3.31). The "Silverman" bandwidth is equal to $.9N^{-1/5}$Min(standard deviation, interquartile range/1.34). N represents the number of observations, and the standard deviation and interquartile range are calculated as sample statistics of the regressor. The sensitivity to bandwidth selection is measured by performing the same estimation procedure with bandwidth equal to 75% or 125% of the Silverman bandwidth.

A difficulty in implementation of the Ait-Sahalia and Lo (1998) technique is that it is often not effective for estimation of the tails of the conditional risk-neutral density. This is because the kernel estimator is not informative outside of the range of prices (P_{t+1}) bounded below by K_{min} and bounded above by K_{max}, where K_{min} and K_{max} are the lowest and highest exercise prices of traded options on date t. Shimko (1993) proposes a technique for estimation of the tails of the conditional risk-neutral density beyond the range of traded exercise prices, which involves selecting a lognormal density function that matches the estimated cumulative conditional risk-neutral density and the estimated conditional risk-neutral density at the exercise price boundaries. This paper adopts the Shimko (1993) technique.

In particular, one lognormal density is selected to match the kernel-estimated conditional cumulative density and density at the lowest exercise price and another lognormal density is selected to match the kernel-estimated conditional cumulative density and density at the highest exercise price. The final estimated conditional density function uses the kernel-estimated conditional risk-neutral density for $K_{min} \leq P_{t+1} \leq K_{max}$, the lower bound lognormal density for $P_{t+1} < K_{min}$, and the upper bound lognormal density for $P_{t+1} > K_{max}$. The IMSL routine DUMINF to identify the lognormal density parameters that minimize the sum of squared deviations of the actual and fitted values of the density and cumulative density functions.

III.c. Testing differences in option-based and consumption-based unconditional risk-neutral variance

The unconditional option-based risk-neutral variance is a summary statistic that measures a stationary characteristic of the historical option prices, in the same way that the unconditional equity index return or unconditional equity index return standard deviation measures a stationary characteristic of equity returns. While estimation of option-based unconditional risk-neutral variance is more involved than estimation of the unconditional equity index return, minimal assumptions are used in the option-based risk-neutral variance calculation. Option-based unconditional risk-neutral variance is obtained without imposing significant restrictions on the utility function of the representative consumer or the time-series process of the underlying asset price.

Since option-based unconditional risk-neutral variance is an important summary statistic for option prices, it is natural to evaluate the performance of a consumption-based pricing kernels based on success in fitting this statistic. Both consumption-based and option-based unconditional risk-neutral variances are estimated with error, so it is necessary to test whether the difference between the two estimates is statistically significant. In this paper, the bootstrap percentile technique (Efron and Tibshirani, 1993, Chapter 13) is used to calculate confidence intervals for the difference between the unconditional risk-neutral variance estimates. If the confidence interval (at a given significance level) includes zero, then the null hypothesis that the consumption-based and option-based unconditional risk-neutral variance are equal cannot be rejected at that significance level.

The bootstrap percentile technique involves the simulation of the distribution of unconditional risk-neutral variance differences. A single simulation replication is obtained using the following procedure. A vector of N dates is constructed by randomly sampling with replacement from the vector of N dates in the estimation period. Then, the option-based and consumption-based unconditional risk-neutral variances are estimated using data on the dates given by the simulated date vector. The simulated unconditional risk-neutral variance difference is the difference between the two simulated risk-neutral unconditional variance estimates.

This procedure is repeated B times to construct a simulated distribution of unconditional risk-neutral variance differences. In this paper, B is set equal to 10,000. The empirical .5%, 2.5%, 5%, 95%, 97.5%, and 99.5% percentiles are calculated using this distribution. Then, the 1%, 5%, and 10% confidence intervals are constructed using the appropriate percentiles. The significance level of the unconditional variance difference is obtained by selecting the smallest confidence interval (greatest significance level) than includes zero. For example, if the 1% and 5% confidence levels include zero, then the null hypothesis is rejected at the 5% level. In addition, the significance level for the difference between the estimated consumption-based unconditional consumption-based riskless interest rate and the estimated unconditional riskless interest rate is obtained using an analogous procedure.

IV. Tests of consumption-based and option-based unconditional risk-neutral S&P500 variance

For any "underlying asset," a correctly specified consumption-based model should estimate an unconditional risk-neutral variance that is close to the measured option-based unconditional risk-neutral variance. So, there are many possible risk-neutral variances that could be compared, e.g. the risk-neutral variance of foreign currency returns, bond returns, or equity returns. Since traded options are required to estimate the option-based risk-neutral variance for a particular underlying asset, the universe of possible candidates is somewhat narrowed. In addition, it may be most interesting to focus on an asset that exhibits a large risk-premium (high negative covariance of returns with the pricing kernel), since this asset may also exhibit a large risk-neutral variance risk-premium (high covariance of squared excess returns with the pricing kernel).

In this paper, the S&P500 portfolio is selected as the underlying asset, and risk-neutral S&P500 one-month return variance is analyzed. S&P500 returns and other diversified equity portfolio returns have been subjects of research related to the equity premium puzzle.[21] This paper determines whether there is a risk-neutral S&P500 variance puzzle by comparing the estimated unconditional risk-neutral S&P500 variance from a consumption-based model with the unconditional risk-neutral S&P500 variance estimated semi-parametrically using S&P500 futures option data.

The S&P500 is also a particularly appropriate underlying asset to analyze because of the liquidity and active trading of S&P500 options across a range of exercise prices and maturities. This facilitates accurate estimation of option-based unconditional risk-neutral S&P500 variance. Empirical results in this paper provide additional insight into characteristics of risk-neutral S&P500 variance, building on the literature that analyzes the option-based S&P500 risk-neutral density, e.g. Bates (1991), Jackwerth and Rubinstein (1996), and Ait-Sahalia and Lo (1998).

IV.a. Data used in estimation of consumption-based risk-neutral S&P500 return variance

Following the existing literature, this paper uses monthly real, seasonally adjusted nondurable goods and services consumption data from the National Income and Product Accounts to estimate aggregate consumption. Then, dividing aggregate consumption by the U.S. population provides an estimate of the consumption (C_t) of the representative consumer. Consumption data is obtained from the Federal Reserve Bank's FRED database.[22] The sample period chosen is 1988:01 – 1997:12.[23]

The first, second, and fifth rows of Panel A of Table 1 describe the consumption data. Over this period, monthly consumption in annual terms averages $14,879. Consumption generally increases over the period with an average monthly consumption change in annual terms of $17, and the volatility of monthly consumption changes in annual terms is $47. Figure 1 graphs the time-series of monthly consumption over the full consumption history and shows that the only sustained decline in consumption during the sample period is during the 1990 – 1991 recession.

In this paper, the habit (X_t) is defined as the five-year moving average of historical consumption scaled by δ (equation 23, J = 60 months, a_j = 1/60, j = 1…60). This definition of the habit provides similar results to the slow-moving external habit estimated in Campbell and Cochrane (1999, Figure 8), and it guarantees that consumption is greater than habit with $\delta \leq 1$ over the period from 1964:01 – 1997:12. As illustrated in Figure 1, consumption approaches habit during the 1990–1991 recession, but always remains above habit. The results of Table 1 also show that changes in habit with δ=1 are less volatile than changes in consumption.

The inflation rate (I_t) is estimated using the monthly percentage change in the Consumer Price Index.[24] During the sample period, monthly inflation averages .28% with a standard deviation of .20%. These statistics correspond to an annualized mean of 3.40% and an annualized standard deviation of .69%. Since the inflation rate is relatively low and stable, the difference between the real pricing kernel (m_t) and the nominal pricing kernel (M_t) is relatively small.

---

[21] For example, Hansen and Jagannathan (1991) investigate the equity premium puzzle using S&P500 index returns and New York Stock Exchange index returns.

[22] The variables as given in the FRED database are real, seasonally-adjusted nondurable goods consumption (pcendc92), real, seasonally-adjusted services consumption (pcesc92), and total U.S. population (pop). Measured consumption over the month (yymm) is taken to represent beginning of month consumption. See, e.g., Campbell, Lo, and MacKinlay, (1997), p. 308.

[23] S&P500 futures options begin trading in February of 1983, so this is the first month that the option-based risk-neutral variance may be calculated. Thus, to match the consumption sample to the option sample, the earliest starting date would be 1983:02. However, because of the potentially large effect of the 1987 crash on the option and return data, the beginning of the sample is chosen to be 1988:01.

This paper also provides comparisons of consumption-based pricing kernels estimated using the full consumption history (1964:01 – 1997:12) with estimates over the sample period. See, for example, Section IV.c and Table 4 as well as Section V.c and Table 7. The "full consumption history" is defined as starting in 1964:01, because, while the NIPA consumption estimates begin in 1959:01, five years of data is required to estimate the habit.

[24] The price index variable as given in the FRED database is the consumer price index for all urban consumers (cpiaucns). The price index for a given month (i_{yymm}) is taken to represent the beginning of the month price level, so that the net inflation rate over the month of yymm is equal to i_{yymm+1}/i_{yymm} - 1.

The riskless interest rate over the month (RF_t) is derived from the one-month Treasury Bill rate reported in the CRSP Risk-Free Rates file.[25] During the sample period, the monthly riskless rate averages .43% with a standard deviation of .14%. These correspond to an annualized mean of 5.31% and an annualized standard deviation of .47%. The highest annualized riskless rate over the period is 9.36% in April 1989, and the lowest annualized riskless rate is 2.54% in March 1993.

The underlying asset is chosen to be the S&P500 portfolio (without dividends), so the asset return (R_{t+1}) is set equal to the monthly capital-appreciation return on the S&P500 portfolio as reported in the CRSP US Indices database. The average monthly return for the S&P500 portfolio is 1.21% (or 15.47% on an annual basis). However, during the 1990–1991 recession, there is a sustained decline in equity value.

The monthly S&P500 return standard deviation is 3.47%, which is equal to 12.01% on an annual basis. Over the sample period, the largest monthly return decrease is –9.49% in August 1990 and the largest monthly return increase is 11.15% in December 1991. S&P500 returns exhibit negative skewness (-.11) and positive kurtosis (3.37). The means of the total and excess S&P500 returns are different (1.21% versus .77%), but the other moments (standard deviation, skewness, and kurtosis) are similar.

Panel B of Table 1 presents sample statistics for the same variables measured over the full consumption history (1964:01 – 1997:12). Over the full history, the monthly S&P500 excess return has a lower mean (.20% versus .77%) and a higher standard deviation (4.22% versus 3.46%) than in the sample period. The monthly consumption growth rate over the full history has a higher mean (.17% versus .11%) and a higher standard deviation (.38% versus .32%) than in the sample period.

IV.b. Data used in estimation of option-based risk-neutral S&P500 variance

This paper uses S&P500 futures option prices to estimate the option-based risk-neutral density and risk-neutral variance of S&P500 returns.[26] The S&P500 futures option and S&P500 futures data is end-of-day data

[25] The CRSP Risk-Free Rate file contains continuously-compounded annualized one-month Treasury yields measured on the last trading day of each month. To obtain the one-month riskless rate defined as $1/B_t - 1$, where B_t is the one-month riskless bond price measured at the beginning of month t, the following procedure is used. First, the average of the bid and asked one-month Treasury Bill yields from the CRSP file is extracted for each month of the sample. Then, this is converted to a riskless one-month bond price using the expression $B_t = \exp[(-\text{yield}*\text{days})/365]$, where days represents the actual number of days in the month. The riskless rate over the month yymm is taken to be the one-month riskless rate is measured at the end of the month yymm – 1. This is annualized using the formula $RF_{annual} = (1 + RF_{monthly})^{12} - 1$.

[26] S&P500 futures option data from the Chicago Mercantile Exchange (CME) is potentially preferable to S&P500 option data from the Chicago Board Options Exchange (CBOE), because the CME reports official futures option closing (settlement)

obtained from the Futures Industry Institute (FII) for all contracts traded from 1988:01 through 1997:12. A cross-section of futures options contracts is extracted on the first trading day of each month. Futures options with trading volume of at least five contracts, with two weeks but no longer than three months until maturity, and with moneyness (exercise price / futures price – 1) between –20% and 20% are included in the monthly estimation procedure.

Estimation of the conditional risk-neutral S&P500 density using the kernel regression technique requires Black-Scholes (1973) implied volatilities obtained from European S&P500 index options. In this paper, the BBSR American option pricing formula of Broadie and DeTemple (1996, Appendix B.5) is used to estimate a BBSR implied volatility for each S&P500 futures option.[27] The BBSR implied volatility for an S&P500 futures option approximates the Black-Scholes implied volatility for an S&P500 option with identical contract terms.[28] To obtain each implied volatility, the BBSR formula is numerically inverted with the number of time-steps set to 100, which is the maximum number tested by Broadie and Detemple (1996).

---

prices, while there is no official closing price reported by the CBOE. Other papers have used transactions data to artificially construct a cross-section of time-synchronous S&P500 option closing prices, e.g. Dumas, Fleming, Whaley (1998). The Chicago Mercantile Exchange utilizes a methodology to maintain the accuracy of futures option (and underlying futures) settlement prices due to their role in determining daily mark-to-market payments. For additional details, see rule 813 of the CME rulebook.

[27] The BBSR formula is constructed as a computationally-efficient implementation of the Cox, Ross, and Rubinstein (1979) binomial pricing model for American options on the spot asset. Several modifications are made to apply this model to futures options. In place of the up and down returns u and d, $u_F = u{*}e^{-c\Delta}$ and $d_F = d{*}e^{-c\Delta}$ are used, where Δ is the amount of calendar time represented by one time-step. In place of the risk-neutral probability p, the risk-neutral probability $p_F = (1-d_F)/(u_F - d_F)$ is used. These parameters are derived as follows. The cost-of-carry model states that $F_t = e^{c(T-t)}P_t$, where F_t is the futures price, P_t is the spot price, T-t is the number of years until the futures contract expires, and c is the continuously-compounded annual cost of carry. So, $u_F = F_u / F_t = e^{c(T-t-\Delta)}P_u / e^{c(T-t)}P_t = u{*}e^{-c\Delta}$. And, $d_F = F_d / F_t = e^{c(T-t-\Delta)}P_d / e^{c(T-t)}P_t = d{*}e^{-c\Delta}$ To price futures options, the risk-neutral probabilities (p, 1-p) are written in terms of the futures returns rather than the underlying returns to obtain p_F and $1-p_F$. The appropriate formula is $p_F = (1-d_F)/(u_F - d_F)$. This parameter is derived as follows. The BBSR algorithm defines p as $p = (e^{c\Delta} - d) / (u - d)$. Using the definitions above, $u = u_F e^{c\Delta}$ and $d = d_F e^{c\Delta}$. Substituting into p to define p_F, $p_F = (1-d_F)/(u_F - d_F)$.

[28] When early exercise is not optimal, the BBSR S&P500 futures option implied volatility is equal to the BBSR S&P500 option volatility for contracts with identical terms. This is because the S&P500 futures option and S&P500 option prices are identical, and the BBSR formula for an American futures option is equivalent to the BBSR formula for a European spot option. When the binomial model assumptions are satisfied and early exercise is potentially optimal, then the BBSR S&P500 futures option implied volatility is equal to the BBSR S&P500 option volatility for contracts with identical terms (same exercise price and expiration date). If this is not the case, then there is an arbitrage opportunity.

In general, the BBSR S&P500 futures option implied volatility is approximately equal to the BBSR S&P500 option volatility for contracts with identical terms, since the BBSR formula incorporates the effect of the early-exercise premium on the futures option price. The BBSR S&P500 option implied volatility converges to the Black-Scholes (1973) S&P500 option implied volatility, since the BBSR formula is an implementation of the Cox, Ross, and Rubinstein (1979) binomial formula, which converges to the Black-Scholes formula. Hence, the BBSR S&P500 futures option implied volatility approximates the Black-Scholes (1973) S&P500 option implied volatility.

Brenner, Courtadon, and Subrahmanyam (1985) and Ramaswamy and Sundaresan (1985) discuss the relationship between the prices of futures option and option contracts. And, Brenner, Courtadon, and Subrahmanyam (1989) show that price

For the BBSR implied volatility calculation, the riskless rate is extracted from the term-structure defined by the one and three month Treasury Bill rates from the CRSP Risk-Free Rates file.[29] The cost-of-carry is set equal to the implied cost-of-carry that is consistent with the observed S&P500 futures price and S&P500 index level.[30] And, the option time until expiration is measured as a fraction of a calendar year.

Panel A of Table 2 describes this data. On average over the sample period, the short-term continuously-compounded riskless interest rate term structure is upward sloping with one-month riskless rates averaging 5.17% and three-month riskless rates averaging 5.50%. The average cost of carry is 5.55% with a standard deviation of 5.57%.[31]

To eliminate potential data errors from the sample, contracts with annualized implied volatilities outside of the range from 5% to 90% are deleted. A single implied volatility is obtained for each contract on each date by averaging the implied variances of puts and calls with identical contract terms on the same trading date. Then, for each contract, a fitted European S&P500 call option price is obtained using the estimated implied volatility and the Black-Scholes (1973) formula with the Merton (1973) dividend adjustment.

In this calculation, underlying price is set equal to the closing value of the S&P500 index on the estimation date, and the dividend yield is set equal to the monthly difference (annualized) between the S&P500 return with dividends and without dividends as reported in the CRSP US Indices database. Panel A of Table 2 reports that the average annualized S&P500 one-month dividend yield is 3.06% and the lowest and highest dividend yields over the sample period are 1.32% to 8.46%. The riskless rate and time until expiration are calculated as before.

---

difference between futures option and option contracts is typically a small fraction of the option premium, except for long-maturity or deep in-the-money options.

[29] Interest rates for maturities between one and three months are obtained by linear interpolation of the observed one and three month rates. Interest rates for maturities between two weeks and one month are set equal to the observed one-month rate.

[30] The "implied" cost of carry is directly estimated from the spot and futures price by solving the cost-of-carry model for the cost-of-carry in terms of the spot and futures price. From $F_t = e^{c(T-t)}P_t$, $c = [\ln(F_t) - \log(P_t)]/(T-t)$ where F_t is the futures price for a contract with time until expiration of T-t and P_t is the spot price. For stock index futures, $c = r - \delta$, where r is the riskless interest rate and δ is the dividend yield until the futures expiration.

[31] The cost-of-carry may be either positive or negative depending on the relative levels of the dividend yield and the riskless rate. Over the sample period, the range for the cost-of-carry is from -5.51% to 28.83%. This is a wider range than would be expected given the fluctuations in riskless rates and dividend yields over this period, which may reflect some deviations from the cost-of-carry pricing model for S&P500 futures. See, for example, Brennan and Schwartz (1990).

The resulting cross-section of fitted European S&P500 call prices on the first trading day of each month is screened for violations of no-arbitrage monotonicity or convexity conditions of Merton (1973).[32] Contracts that violate these conditions are deleted.

The S&P500 futures option data is summarized in the second panel of Table 2. There are a total of 4501 option contracts that meet the screening criteria. On the first trading day of each month, there are contracts from at least two expiration dates, where one expiration is less than one month and one expiration is greater than one month. Average time-until-expiration for these contracts is .11 years or about 40 days. The moneyness of the contracts ranges from -20.00% to 18.98%, and the annualized implied volatility of the contracts ranges from 7.54% to 61.09%.

IV.c Estimation of consumption-based risk-neutral S&P500 variance

Applying equation (19), the unconditional consumption-based risk-neutral S&P500 one-month return variance may be estimated as the average scaled product of the squared excess S&P500 return and the pricing kernel. In this case, the excess return ($R_{t+1} - RF_t$) is defined by the difference between the S&P500 one-month return and the one-month riskless rate. And, using equation (24), the nominal one-month pricing kernel is defined as $M_{t+1} = \rho[(C_{t+1} - X_{t+1})/ (C_t - X_t)]^{-\gamma} I_{t+1}^{-1}$ where C_t is the level of consumption at date t, X_t is the level of habit at date t, and I_{t+1} is the gross inflation rate from date t to date t+1. The habit is defined as the five-year moving average of historical consumption scaled by δ.

Table 3 reports the characteristics of the nominal monthly pricing kernel at a range of levels of utility function concavity (γ = 1, 5, 10, 15, 20, 25, 30), levels of habit (δ = -.9, -.7, -.5, -.3, -.1, 0, .1, .3, .5, .7, .9), and a fixed rate of time preference (ρ = .999). Panels A, B, and C provide pricing kernel summary statistics using consumption data from 1988:01 to 1997:12, which is the sample period used for estimation of unconditional risk-neutral variance. Panels D, E, and F report the same statistics for the sample using the full history of

[32] The monotonicity condition is that the call price function is decreasing in exercise price, i.e. $C_t(K_1, T\text{-}t) \geq C(K_2, T\text{-}t)$ for $K_1 < K_2$. This corresponds to the condition that the cumulative risk-neutral return density function is increasing in returns. The monotonicity condition is tested by comparing the prices of pairs of adjacent call options. If the condition is violated, the higher exercise price option is deleted from the sample. This procedure is repeated twice.

The convexity condition is that the call option pricing function is a convex function of exercise price, i.e. $C_t(K_2, T\text{-}t) \leq \lambda C_t(K_1, T\text{-}t) + (1\text{-}\lambda)C_t(K_3, T\text{-}t)$ for $K_1 < K_2 < K_3$. This corresponds to the condition that the risk-neutral return density function is non-negative. Setting $\lambda = .5$, multiplying both sides of the equation by two, and rearranging, $C_t(K_1, T\text{-}t) - 2C_t(K_2, T\text{-}t) - C_t(K_3, T\text{-}t) \geq 0$ for $K_1 < K_2 < K_3$. The monotonicity condition is tested by comparing the prices of triplets of adjacent call options. If the convexity condition is violated, the highest exercise price option is deleted from the sample. This procedure is repeated twice.

consumption data from 1964:01 to 1997:12. The actual consumption history begins in 1959:01, but the first habit calculation, which requires five years of data, is 1964:01.

Consider the middle column of panel A (δ = 0), which corresponds to a constant relative risk-aversion pricing kernel. The monthly pricing kernel mean for γ = 1 and δ = 0 is .9951, and the consumption-based unconditional annualized riskless rate is 6.07%.[33] Moving down the column, the monthly pricing kernel mean for γ = 30 and δ = 0 is .9680, and the consumption-based unconditional annualized riskless rate is 47.74%.

For the full consumption history reported in panel D, the pricing kernel mean for γ = 1 and δ = 0 is .9933 and for γ = 30 and δ = 0 is .9510. The corresponding riskless rates are 8.40% and 82.74%. These results are consistent with the risk-free rate puzzle, since estimated riskless rates are unrealistically high using a reasonable rate of time-preference and a high level of risk-aversion.

For the habit model reported in the right-most column of Panel A (δ = .9), the pricing kernel mean increases and the riskless rate decreases as gamma increases. Four of the five pricing kernel mean estimates are greater than one for these habit specifications, and the corresponding unconditional riskless rates range from –3.07% at γ = 5 to –96.19% at γ = 30. The results are similar in Panel D using the full consumption history. Thus, the habit model has the ability to maintain lower interest rates at higher levels of risk-aversion and a reasonable rate of time-preference than the CRRA model. However, a related problem with the habit model is the prediction of negative interest rates at high levels of risk-aversion and habit.

For the consumption durability pricing kernel reported in the leftmost column of Panel A (δ = -.9), the pricing kernel means are very close to the pricing kernel means for the constant relative risk-aversion model. The results are the same for the full consumption history reported in Panel D. The consumption durability specification does not solve the risk-free rate puzzle.

Panels B and E report monthly pricing kernel standard deviations. The results in Panels B and E demonstrate that the pricing kernel standard deviation is increasing in the level of habit. For example, in Panel B, as δ increases from -.9 to .9 at γ = 1, and the pricing kernel standard deviation increases from .0024 to .0250. The pricing kernel standard deviation is also increasing in the level of γ, as illustrated by the column corresponding to δ = 0 in Panels B and E. Finally, pricing kernel standard deviations are higher in the full history than in the sample period, e.g. .0034 in the sample period and .0043 in the full history with γ = 1 and δ = 0.

[33] The consumption-based unconditional riskless interest rate is estimated as $RF_{c,monthly}$ = 1/average(M_t) – 1, where M_t is the monthly consumption-based pricing kernel realization. This is annualized using the formula $RF_{c,annual} = (1 + RF_{c,monthly})^{12} - 1$.

Panels C and F report the ratio of the pricing kernel standard deviation to the pricing kernel mean. The characteristics of the ratios are similar to the characteristics of the pricing kernel standard deviations. In other words, the ratios are increasing in δ and increasing in γ. Also, the ratios are higher in the full history than in the sample period.

Hansen and Jagannathan (1991) show that an acceptable pricing kernel must have a pricing kernel standard deviation and mean ratio that is at least as large as the unconditional excess asset return mean and standard deviation ratio. This relationship may be informally examined using sample data for S&P500 excess returns.[34] Panel G indicates that over the sample period, the mean-standard deviation ratio for excess S&P500 returns is .2958, and over the full history the ratio is .1224.

Using the sample period data (Panel C), pricing kernel parameter values that generate a ratio of at least .2958 are $\gamma = 30$ and $\delta = .7$ or $\gamma \geq 15$ and $\delta = .9$. These results suggest that habit formation and utility function concavity must be very large to satisfy the Hansen and Jagannathan (1991) bound. Using the full sample data (Panel F), pricing kernel parameter values that generate a ratio of at least .1224 are $\gamma = 30$ and $\delta = .1$, $\gamma \geq 25$ and $\delta = .3$, $\gamma \geq 20$ and $\delta = .5$, $\gamma \geq 15$ and $\delta = .7$, or $\gamma \geq 5$ and $\delta = .9$. So, using the full history of consumption and returns data, a larger set of acceptable habit models satisfy the Hansen and Jagannathan (1991) bound. This informal evidence indicates that a habit specification is preferred to a consumption durability or constant relative risk-aversion specification, and that the sample period provides a more stringent test of the pricing kernel specification than the full consumption history.

The consumption-based unconditional risk-neutral S&P500 variances for a range of constant relative risk-aversion and habit pricing kernels are reported in Table 5 and Table 6. Analogous results using the full consumption history are reported in Table 7. Section V provides a detailed discussion of these results.

IV.d Estimation of option-based risk-neutral S&P500 variance

On the first trading day of each month in the sample period, the option-based risk-neutral S&P500 one-month return variance is obtained by numerically integrating squared excess returns with respect to the estimated

[34] In this analysis, the S&P500 return is the S&P500 one-month return including dividends from the CRSP US Indices database, the riskless rate is the one-month Treasury Bill rate from the CRSP Risk-Free Rates file, and the excess return is the difference between the S&P500 return and the one-month riskless rate. The following comparisons do not account for the standard errors of the measured ratios. For a rigorous approach that utilizes the appropriate test statistics, see Cecchetti, Lam, and Mark (1994).

conditional risk-neutral S&P500 one-month return density (equation 28 with R_{t+1} equal to the S&P500 one-month return). The option-based unconditional risk-neutral S&P500 variance is estimated as the average of conditional variance estimates over the sample period (equation 29 with the risk-neutral variance equal to the risk-neutral variance of S&P500 one-month returns).

The conditional risk-neutral S&P500 one-month return density is calculated by numerically differentiating the estimated S&P500 one-month call price function with respect to the exercise price (equation 27 with P_t equal to the S&P500 level, R_{t+1} equal to the S&P500 one-month return, and K equal to the exercise price for an S&P500 index option). In this paper, the estimated S&P500 one-month call price function is given by the semi-parametric representation in equations 31-32 with T-t set equal to 1/12 year, where the S&P500 implied volatility function is estimated by a kernel regression of S&P500 option implied volatilities on S&P500 futures price, exercise price, and time until expiration.

Figure 2 graphs the nonparametrically-estimated S&P500 one-month implied volatility functions on five representative dates of the sample (the first trading day of June 1988, June 1990, June 1992, June 1994, and June 1996). This figure shows that the S&P500 implied volatility functions exhibit a volatility skew, i.e. implied volatilities are decreasing as a function of exercise price (or as a function of the S&P500 return where S&P500 return = exercise price / S&P500 level - 1). Time-variation in the level and slope of the volatility skew are also apparent.

The conditional risk-neutral S&P500 return densities are graphed in Figure 3 for the same five estimation dates. Changes in probability expectations and preferences over S&P500 return states are reflected in changes in the estimated conditional risk-neutral S&P500 return density. For example, the estimated conditional risk-neutral annualized standard deviation on the first trading day of June 1988 is 19.57%, and the conditional risk-neutral skewness is –.10. On the first trading day of June 1990, the estimated conditional risk-neutral annualized standard deviation is 15.55%, and the conditional risk-neutral skewness is –.23.

Table 4 provides summary statistics for the estimated conditional risk-neutral S&P500 return variances and risk-neutral standard deviations. The key statistic reported in Panel A of Table 4 is the estimate of the option-based unconditional risk-neutral S&P500 annualized variance (.0278), which is obtained by averaging the monthly conditional risk-neutral annualized variance estimates. This corresponds to an average risk-neutral annualized standard deviation of 16.67%.

The second row of Panel A reports the characteristics of the option-based conditional risk-neutral standard deviation, which is a more familiar statistic.[35] Over the sample period, the conditional risk-neutral annualized standard deviation ranges from 10.93% to 29.02%. The highest conditional risk-neutral standard deviation is estimated in 1988:01 and the lowest is estimated in 1994:02. The time-series of conditional risk-neutral S&P500 return standard deviations is graphed in Figure 4.

Panel B of Table 4 presents bootstrap confidence intervals for the average risk-neutral variance and average risk-neutral standard deviation.[36] The 95% confidence interval for the average risk-neutral variance is [.0256, .0301], and the 95% confidence interval for the average risk-neutral standard deviation is [15.71%, 16.93%].

Panel C of Table 4 reports sample statistics for estimated option-based risk-neutral standard deviations using different choices of bandwidth for the kernel regression. The first, second, and third rows show estimation results using 75%, 100%, and 125% of the Silverman bandwidth. The average risk-neutral standard deviation is very similar for all three bandwidth choices (16.41%, 16.31%, and 16.52%). These results suggest that the option-based unconditional risk-neutral variance estimates are robust to the selection of bandwidth.

V. Tests of consumption-based pricing kernels using unconditional risk-neutral S&P500 return variance differences

V.a. Constant relative risk-aversion

In Table 5, consumption-based unconditional risk-neutral S&P500 return variances are estimated using a constant relative risk-aversion pricing kernel, and they are compared with option-based unconditional risk-neutral S&P500 return variances. The tested pricing kernels are estimated at a range of levels of risk-aversion (γ = 1, 5, 10, 15, 20, 25, 30) and a fixed rate of time-preference (ρ=.999).[37]

---

[35] The risk-neutral standard deviation time-series is obtained by taking the square-root of each monthly estimate of the risk-neutral variance. By Jensen's inequality, the square-root of the average variance ($\sqrt{.0278}$ = 16.67% from the first row of Panel A) is not equal to the average of the square-root of each variance (16.31% from the second row of Panel A).

[36] These confidence intervals are obtained using the bootstrap percentile technique. The distribution of sample averages is constructed by sampling 120 data points (with replacement) from the monthly dataset, calculating the average of the variable of interest, and repeating 10,000 times. The confidence intervals are the empirical percentiles of the distribution of sample averages.

[37] The unconditional consumption-based risk-neutral variance is not sensitive to the choice of ρ. The selection of ρ = .999 is similar to the estimation results of Hansen and Singleton (1982), and this relatively large value for ρ tends to mitigate the risk-free rate puzzle for the CRRA specification. The ρ that is required to match the estimated unconditional riskless interest rate over the sample period is also calculated.

Column two of this table reports that the option-based estimate of unconditional risk-neutral S&P500 return variance is .0278. The subsequent columns report that consumption-based unconditional risk-neutral variances for the CRRA specification range from .0145 to .0151 (12.06% to 12.30% in standard deviation terms). As shown in the column 4 of the table, all consumption-based and option-based variance differences are significantly different from zero at the 1% confidence level. The CRRA unconditional risk-neutral variance is decreasing in risk-aversion, so higher levels of risk-aversion do not improve estimates of CRRA unconditional risk-neutral variance.

Since the CRRA pricing kernel is unable to replicate the option-based unconditional risk-neutral variance at economically plausible preference parameter values, there is a risk-neutral variance puzzle for this pricing kernel specification. The high levels of risk-aversion that are required to solve the equity premium puzzle worsen the risk-neutral variance puzzle.

Comparisons of the consumption-based and option-based unconditional risk-neutral variance risk-premia are presented in the fifth column. The estimated option-based unconditional risk-neutral variance risk-premium (annualized value of .0127) is equal to the difference between the estimated option-based unconditional risk-neutral variance (annualized value of .0278) and the estimated unconditional squared S&P500 excess return (annualized value of .0151). The estimated consumption-based unconditional variance risk-premia are substantially lower with a range of -.0000 to -.0006.

Columns 6 through 9 document the risk-free rate puzzle for the CRRA specification. The estimated unconditional riskless rate over the sample period is 5.30%.[38] In contrast, the estimated unconditional consumption-based riskless rate using the CRRA model ranges from a low of 5.88% at $\gamma = 1$ to a high of 39.65% for $\gamma = 30$. Other than for $\gamma = 1$, the estimated unconditional consumption-based riskless rate is significantly different from the estimated unconditional riskless rate at the 1% level.

Column 9 presents the rate of time preference (ρ) that is necessary for a particular CRRA specification to match the estimated unconditional riskless rate.[39] The required ρ is greater than one for all CRRA models except for $\gamma = 1$. A rate of time preference greater than one corresponds to a preference for future consumption

[38] The unconditional riskless rate is estimated as 1/average(B_t) – 1 where B_t is the one-month Treasury Bill price derived from the one-month Treasury Bill rate reported in the CRSP Risk-Free Rates file. This is slightly different than the average riskless rate reported in Table 1 (5.31%), which is estimated as average(RF_t).

[39] The required ρ to match the estimated unconditional riskless rate is equal to (model ρ) / (pricing kernel mean * (1 + estimated unconditional riskless rate)).

over current consumption. This typically considered to be economically implausible, although Cecchetti, Lam, and Mark (1994, pp. 135-136) provide arguments to support negative time preference.

V.b. Habit formation and consumption durability

Table 6 compares option-based and consumption-based unconditional risk-neutral S&P500 return variances using a pricing kernel that is consistent with habit formation ($\delta > 0$), consumption durability ($\delta < 0$), and constant relative risk-aversion ($\delta = 0$). Consumption-based pricing kernels are estimated using a range of levels of utility function curvature (γ = 1, 5, 10, 15, 20, 25, 30) and a fixed rate of time-preference (ρ = .999). Instead of testing a range of possible levels of habit (δ), an "implied δ" is calculated that minimizes the difference between consumption-based and option-based unconditional risk-neutral S&P500 variance at fixed levels of γ and ρ. The implied delta is restricted to the range of [-.99, .99], since values outside this range imply extreme time-inseparability.

As reported in column 1 of this table, the implied δ ranges from a high of .99 at $\gamma = 1$ to a low of .95 at γ = 30. The fact that all implied δ's are positive suggests that habit formation is a preferred specification to consumption durability or constant relative risk-aversion. Column 4 shows that the equality of the consumption-based and option-based unconditional risk-neutral variance cannot be rejected at concavity parameters (γ) between 10 and 30. So, at suitably high levels of concavity and habit, the risk-neutral variance puzzle is resolved.

As shown in column 8, at a rate of time preference of .999, equality of estimated unconditional consumption-based riskless rates and estimated unconditional riskless rates is not rejected for $\gamma = 1$. However, equality is rejected at the 5% level for $\gamma = 5$ and at the 1% level for $\gamma \geq 10$. Column 9 shows that the consumption-based interest rate predictions may be salvaged by using low rates of time-preference for the otherwise acceptable habit models, e.g. ρ between .44 and .58 for $\gamma \geq 10$.

The acceptable habit models have somewhat higher concavity and habit parameters and lower rates of time-preference than habit models estimated in the existing literature.[40] For example, Ferson and Constantinides (1991) use GMM to estimate a one-lag habit model with financial instrumental variables, monthly equity and

[40] Not all papers find that habit formation is preferred to consumption durability. Dunn and Singleton (1986) and Eichenbaum and Hansen (1990) find evidence for consumption durability using monthly data.

bond returns, and monthly consumption data. They obtain a habit parameter of .717, a concavity parameter of 8.437, and a rate of time-preference of .838.[41]

Also, Cecchetti, Lam, and Mark (1994) use Hansen-Jagannathan (1991) bounds to test a one-lag habit model with monthly consumption data and monthly equity and bond returns. They find that a habit model with a habit parameter of .5, rate of time-preference of .9992, and a concavity parameter between 4 and 10 cannot be rejected at the 5% level. Also, with a rate of time-preference of 1.0017, a one-lag habit model with a habit parameter of .5 and a concavity parameter between 2 and 9 cannot be rejected at the 5% level.[42]

The fact that there are preference parameters that allow a consumption-based pricing model to fit the characteristics of traded assets does not necessarily "solve" an asset pricing puzzle. It is also required that the postulated preference parameter values be economically plausible. Plausibility is a subjective criterion, and there is some debate about what values of γ are too high (e.g. Kandel and Stambaugh, 1991), what rates of time preference are reasonable (e.g. Cecchetti, Lam, and Mark, 1994), and what level of habit is reasonable. This paper simply notes that the acceptable habit models do not require preference parameters that are typically considered to be unreasonable, i.e. negative rates of time-preference ($\rho > 1$), negative utility function curvature ($\gamma < 0$), or extreme levels of habit ($\delta > 1$).

V.c. Habit formation and consumption durability using the full history of consumption data

This section presents an analysis of unconditional consumption-based risk-neutral variance risk-premia estimated using the full history of consumption data (1964:01 – 1997:12). Since there are some differences in

---

[41] The reported estimates in the text are from Ferson and Constantinides (1991, Table 4, p. 216, Panel 1, last row), which seems to be the most comparable model to this paper. The sample period is May 1959 – October 1986. Ferson and Constantinides (1991) also estimate a number of other models using different instruments (and lagged instruments), different frequencies of consumption data (quarterly and annual), and different assumptions about the error process. Other estimates of the habit parameter using monthly data and financial instruments (Table 4, p. 217, Panels 2 and 3) correspond to δ of .642 and .361 with rates of time-preference (ρ) of .837 and .999.

Ferson and Constantinides (1991) write the consumption habit difference as $C_t + X_t$ ($X_t = b_1 C_{t-1}$), and their estimated habit parameter is $b_1 = -.717$. Hence, an estimated b_1 of -.717 in Ferson and Constantinides (1991) corresponds to an estimated δ of .717 in this paper with the consumption habit difference as $C_t - X_t$ with $X_t = \delta C_{t-1}$.

[42] The reported estimates in the text are from Cecchetti, Lam, and Mark (1994, Table IV, p. 140, Panel C), which seems to be the most comparable model to this paper. The sample period is 1964 to 1988. Cecchetti, Lam, and Mark (1994) also estimate models using annual equity and bonds returns, monthly Treasury Bill term structures, and monthly foreign currency returns. In some cases, the CRRA model is not rejected, but in all cases, the consumption durability model is rejected.

Cecchetti, Lam, and Mark (1994) write the consumption-habit difference as $C_t + X_t$ ($X_t = dC_{t-1}$), so an estimated d of -.5 in Cecchetti, Lam, and Mark (1994) corresponds to a δ of .5 in this paper. They also report the rate of time-preference in annualized terms so their time-preference parameters of .99 and 1.02 correspond to ρ of .9992 and 1.0017 in this paper.

the characteristics of consumption-based pricing kernels estimated over the sample period and the full history (see Section IV.c.), the estimated unconditional risk-neutral variance risk-premia could be different over these periods.

Panel A of Table 7 reports consumption-based unconditional risk-neutral S&P500 variance risk-premia estimated using a constant relative risk-aversion pricing kernel and the full consumption history. The consumption-based unconditional risk-neutral variance risk-premia displayed in column 2 are all negative, while the option-based unconditional risk-neutral variance risk-premium (.0064) is positive.[43] Since the CRRA specification is unable to replicate the option-based risk-neutral variance risk-premium, there is an unconditional risk-neutral variance risk-premium puzzle that is analogous to the risk-neutral variance puzzle found in the shorter sample period.

The CRRA predictions of unconditional riskless interest rates are not consistent with estimated unconditional riskless rates. Column 5 indicates that equality of the estimates is rejected at the 1% level for all tested levels of risk-aversion. And, column 6 shows that a rate of time-preference greater than one is required to match the estimated unconditional riskless rate. These results are consistent with the risk-free rate puzzle found in the shorter sample period.

Panel B of Table 7 reports consumption-based unconditional risk-neutral S&P500 variance risk-premia estimated using a general pricing kernel and the full consumption history. For these pricing kernels, the level of habit (implied δ) is estimated from the data by minimizing the distance between the consumption-based unconditional variance risk-premium estimated from 1964:01 – 1997:12, and the option-based unconditional risk-neutral variance risk-premium estimated from 1988:01 – 1997:12.[44]

The implied δ's in column 1 range from .86 to .99, and all specifications with $\gamma \geq 10$ are able to exactly match the option-based unconditional risk-neutral variance risk-premium. These estimated habit levels are somewhat lower in the full consumption sample than in the sample period.

Equality of the estimated unconditional consumption-based riskless rate and estimated unconditional riskless rate is rejected at the 10% level for $\gamma \geq 10$, as reported in column 5. To obtain accurate riskless rate predictions

[43] The option-based risk-neutral variance risk-premium is lower in the full consumption history (.0064) than in the sample period (.0127), since the average squared excess return is higher in the full consumption history (.0214) than in the sample period (.0151).

[44] Notice that it is not possible to construct option-based risk-neutral variance over the longer time period, since option data is not available. For estimation of the implied δ, it seems more reasonable to assume that the average option-based risk-neutral variance risk-premium is approximately the same over the full history and the sample period, than that the average option-based risk-neutral variance is approximately the same over the full history and the sample period.

using otherwise acceptable habit models ($\gamma \geq 10$), the required rates of time-preference range from .85 to .87. These rates of time-preference are closer to those estimated in previous papers than the required rates of time-preference in the shorter sample period.

VI. Conclusions

This paper explores the adequacy of consumption-based pricing kernels using a new testing methodology that focuses on the covariance between the pricing kernel and squared excess returns. This covariance determines the risk-neutral variance risk-premium and is closely related to the risk-neutral variance of asset returns.

Sample estimators for the unconditional consumption-based and option-based risk-neutral variance and risk-neutral variance risk-premium are developed. While the consumption-based estimators depend on the pricing kernel specification and preference parameter values, the option-based estimators are "model-free." The difference between the consumption-based and option-based unconditional risk-neutral variance is used to measure the adequacy of the consumption-based pricing kernel.

This paper tests a consumption-based pricing kernel specification that is compatible with habit formation, consumption durability, and constant relative risk-aversion over a range of plausible preference parameter values. The difference between consumption-based and option-based estimates of unconditional risk-neutral S&P500 return variance is used as a pricing kernel specification test statistic.

Using monthly consumption and S&P500 returns data over the period 1988:01 – 1997:12, a consumption-based CRRA pricing kernel is unable to replicate the unconditional option-based risk-neutral S&P500 return variance. In contrast, a pricing kernel that incorporates habit formation reproduces the option-based risk-neutral S&P500 return variance and variance risk-premium over this time period. The acceptable habit pricing kernels exhibit higher habit levels, higher utility function concavity, and lower rates of time-preference than in related papers. When the full history of consumption data is used, the parameter estimates are more similar to those of related papers.

Bibliography

Ait-Sahalia, Y. and A. W. Lo. "Nonparametric Estimation of State-Price Densities Implicit in Financial Asset Prices," *Journal of Finance,* 53 (1998), pp. 499-547.

Ait-Sahalia, Y. and A. W. Lo. "Nonparametric Risk Management and Implied Risk-aversion," *Journal of Econometrics,* 94 (2000), pp. 9-51.

Amin, K. I. and V. K. Ng. "Option Valuation with Systematic Stochastic Volatility," *Journal of Finance,* 48 (1993), pp. 881-910.

Bates, D. S. "The Crash of 87 - Was It Expected? - the Evidence from Options Markets," *Journal of Finance,* 46 (1991), pp. 1009-1044.

Black, F. and M. Scholes. "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy,* 81 (1973), pp. 637-654.

Boudoukh, J. "An Equilibrium Model of Nominal Bond Prices With Inflation-Output Correlation and Stochastic Volatility," *Journal of Money Credit and Banking,* 25 (1993), pp. 636-665.

Breeden, D. T. "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics,* 7 (1979), pp. 265-296.

Breeden, D. T. and R. H. Litzenberger. "Prices of State-Contingent Claims Implicit in Option Prices," *Journal of Business,* 51 (1978), pp. 621-651.

Brennan, M. J. and E. S. Schwartz. "Arbitrage in Stock Index Futures," *Journal of Business,* 63 (1990), pp. S7-S31.

Brenner, M., G. Courtadon and M. Subrahmanyam. "Options on the Spot and Options on Futures," *Journal of Finance,* 40 (1985), pp. 1303-1317.

Brenner, M., G. Courtadon and M. Subrahmanyam. "Options on Stock Indices and Options on Futures," *Journal of Banking and Finance,* 13 (1989), pp. 773-782.

Broadie, M. and J. Detemple. "American Option Valuation: New Bounds, Approximations, and a Comparison of Existing Methods," *Review of Financial Studies,* 9 (1996), pp. 1211 - 1250.

Campbell, J. and J. Cochrane. "By Force of Habit: A Consumption Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy,* 107 (1999), pp. 205-251.

Campbell, J., A. Lo and C. MacKinlay. "The Econometrics of Financial Markets." Princeton, Princeton University Press, 1997.

Cecchetti, S. C., P. S. Lam and M. C. Nelson. "Testing Volatility Restrictions on Intertemporal Marginal Rates of Substitution Implied by Euler Equations and Asset Returns," *Journal of Finance,* 49 (1994), pp. 123-152.

Chapman, D. A. "Approximating the Asset Pricing Kernel," *Journal of Finance,* 52 (1997), pp. 1383-1410.

Cochrane, J. H. and L. P. Hansen. "Asset Pricing Explorations for Macroeconomics," *NBER Macroeconomics Bulletin,* (1992), pp. 115-165.

Constantinides, G. M. "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy,* 98 (1990), pp. 519-543.

Cox, J. C., S. A. Ross and M. Rubinstein. "Option Pricing: A Simplified Approach," *Journal of Financial Economics,* 7 (1979), pp. 229-263.

Dumas, B., J. Fleming and R. E. Whaley. "Implied Volatility Functions: Empirical Tests," *Journal of Finance,* (1998), pp. 2059-2106.

Dunn, K. B. and K. J. Singleton. "Modeling the Term Structure of Interest Rates Under Non-Separable Utility and Durability of Goods," *Journal of Financial Economics,* 17 (1986), pp. 27-55.

Efron, B. and R. J. Tibshirani. "An Introduction to the Bootstrap." New York, Chapman and Hall, 1993.

Eichenbaum, M. S. and L. P. Hansen. "Estimating Models with Intertemporal Substitution using Aggregate Time Series Data," *Journal of Business and Economic Statistics,* 8 (1990), pp. 53-69.

Ferson, W. "Expectations of Real Interest Rates and Aggregate Consumption: Empirical Tests," *Journal of Financial and Quantitative Analysis,* 18 (1983), pp. 477-497.

Ferson, W. and G. Constantinides. "Habit Formation and Durability in Aggregate Consumption: Empirical Tests," *Journal of Financial Economics,* 29 (1991), pp. 199-240.

Gallant, A. R., L. P. Hansen and G. Tauchen. "Using Conditional Moments of Asset Payoffs to Infer the Volatility of Intertemporal Marginal Rates of Substitution," *Journal of Econometrics,* 45 (1990), pp. 141-179.

Hamilton, J. "Time Series Analysis." Princeton, Princeton University Press, 1994.

Hansen, L. P. "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica,* 50 (1982), pp. 1029-1054.

Hansen, L. P. and R. Jagannathan. "Implications of Security Market Data for Models of Dynamic Economies," *Journal of Political Economy,* 99 (1991), pp. 225-262.

Hansen, L. P. and K. J. Singleton. “Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models,” *Econometrica,* 50 (1982), pp. 1269-1286.

Hansen, L. P. and K. J. Singleton. “Stochastic Consumption, Risk-aversion, and the Temporal Behavior of Asset Returns,” *Journal of Political Economy,* 91 (1983), pp. 249-265.

Heaton, J. “An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications,” *Econometrica,* 63 (1995), pp. 681-717.

Hull, J. and A. White. “The Pricing of Options on Assets with Stochastic Volatilities,” *Journal of Finance,* 42 (1987), pp. 281-301.

Jackwerth, J. “Recovering Risk-aversion from Option Prices and Realized Returns,” *Review of Financial Studies,* 13 (2000), pp. 433-451.

Jackwerth, J. and M. Rubinstein. “Recovering Probability Distributions from Option Prices,” *Journal of Finance,* 51 (1996), pp. 1611-1631.

Kandel, S. and R. Stambaugh. “Asset Returns and Intertemporal Preferences,” *Journal of Monetary Economics,* 27 (1991), pp. 39-71.

Longstaff, F. A. “Option Pricing and the Martingale Restriction,” *Review of Financial Studies,* 8 (1995), pp. 1091-1124.

Lucas, R. “Asset Prices in an Exchange Economy,” *Econometrica,* 46 (1978), pp. 1429-1445.

Mehra, R. and E. C. Prescott. “The Equity Premium: A Puzzle,” *Journal of Monetary Economics,* 15 (1985), pp. 145-161.

Merton, R. C. “An Intertemporal Capital Asset Pricing Model,” *Econometrica,* 41 (1973), pp. 867-887.

Merton, R. C. “Theory of Rational Option Pricing,” *Bell Journal of Economics and Management Science,* 4 (1973), pp. 141-183.

Ramaswamy, K. and S. Sundaresan. “The Valuation of Options on Futures Contracts,” *Journal of Finance,* 40 (1985), pp. 1319-1341.

Rosenberg, J. V. and R. F. Engle. “Empirical Pricing Kernels,” NYU - Stern School of Business, Manuscript, (2000).

Rubinstein, M. “Implied Binomial Trees,” *Journal of Finance,* 49 (1994), pp. 771-818.

Shimko, D. C. “Bounds of Probability,” *Risk,* 6 (1993), pp. 33-37.

Silverman, B. W. "Density Estimation for Statistics and Data Analysis." London, England: Chapman and Hall, 1996.

Sundaresan, S. "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth," *Review of Financial Studies,* 2 (1989), pp. 73-88.

Weil, P. "The Equity Premium Puzzle and the Risk-Free Rate Puzzle," *Journal of Monetary Economics,* 24 (1989), pp. 401-421.

White, H. "Asymptotic Theory for Econometricians." San Diego, Academic Press, 1984.

Table 1

Description of data used to estimate consumption-based unconditional risk-neutral S&P500 return variance

Panel A. Sample period, 1988:01 - 1997:12

	Number of observations	Mean	Std. Dev.	Skewness	Kurtosis	Minimum	Maximum
Consumption (annualized)	120	$14,879	$515	0.57	2.16	$14,057	$16,011
Change in consumption (annualized)	120	$17	$47	0.24	4.02	-$113	$168
Habit (annualized)	120	$14,360	$539	-0.38	2.31	$13,191	$15,288
Change in habit (annualized)	120	$18	$7	0.40	1.89	$8	$30
Consumption growth rate (monthly)	120	0.11%	0.32%	0.25	4.08	-0.77%	1.12%
Surplus consumption ratio (monthly)	120	3.48%	1.46%	0.15	2.26	0.65%	6.49%
Inflation rate (monthly)	120	0.28%	0.20%	0.80	4.55	-0.12%	1.03%
Riskless interest rate (monthly)	120	0.43%	0.14%	0.28	2.33	0.21%	0.75%
S&P500 return (monthly)	120	1.21%	3.47%	-0.11	3.37	-9.49%	11.15%
S&P500 excess return (monthly)	120	0.77%	3.46%	-0.15	3.42	-10.13%	10.81%

Panel B. Full consumption history, 1964:01 - 1997:12

	Number of observations	Mean	Std. Dev.	Skewness	Kurtosis	Minimum	Maximum
Consumption (annualized)	408	$12,152	$2,236	-0.07	1.82	$7,951	$16,011
Change in consumption (annualized)	408	$20	$45	-0.03	3.71	-$152	$168
Habit (annualized)	408	$11,563	$2,279	-0.08	1.83	$7,530	$15,288
Change in habit (annualized)	408	$19	$6	-0.09	2.20	$7	$31
Consumption growth rate (monthly)	408	0.17%	0.38%	0.11	3.74	-1.20%	1.58%
Surplus consumption ratio (monthly)	408	5.09%	2.08%	-0.20	2.19	0.65%	9.33%
Inflation rate (monthly)	408	0.41%	0.33%	0.87	4.17	-0.46%	1.81%
Riskless interest rate (monthly)	408	0.51%	0.21%	1.34	5.04	0.21%	1.38%
S&P500 return (monthly)	408	0.72%	4.20%	-0.28	5.36	-21.72%	16.43%
S&P500 excess return (monthly)	408	0.20%	4.22%	-0.31	5.32	-22.26%	15.92%

This table reports characteristics of the data used to estimate consumption-based unconditional risk-neutral S&P500 return variance. Panel A reports data over the sample period, and Panel B reports data over the full consumption history. All statistics are given by their standard definitions. Kurtosis is reported in total (rather than excess) terms, so the value of 3 would reflect the kurtosis from a Gaussian density.

Consumption of the representative consumer is estimated using monthly per-capita consumption (non-durable goods and services, seasonally adjusted, in real terms) from the Federal Reserve Bank's FRED database. The habit is defined as the five-year moving average of historical consumption scaled by δ with $\delta = 1$. The consumption growth rate is the log differenced monthly consumption level. The surplus consumption ratio is equal to the ratio of the consumption-habit difference and consumption: $(C_t - X_t)/C_t$.

The inflation rate is the monthly proportional change in the CPI (Consumer Price Index For All Urban Consumers) as reported in the FRED database. The riskless interest rate is the calculated as $1/B_t - 1$, where B_t is the one-month Treasury Bill price derived from the one-month Treasury Bill rate reported in the CRSP Risk-Free Rates file. The S&P500 return is the monthly nominal capital-appreciation return for the S&P500 portfolio as reported in the CRSP US Indices database, and the S&P500 excess return is the difference between the S&P500 return and the riskless rate.

Table 2

Data used to estimate option-based unconditional risk-neutral S&P500 return variance

Panel A. Data used in implied volatility and fitted option price calculations

First trading day of each month, 1988:01-1997:12

	N	Mean	Std. dev.	Minimum	Maximum
Riskless interest rate (1 month, annualized)	120	5.17%	1.62%	2.67%	8.78%
Riskless interest rate (3 month,annualized)	120	5.50%	1.69%	2.73%	9.12%
Implied cost of carry (annualized)	314	5.55%	5.57%	-5.51%	28.83%
S&P500 dividend yield (annualized)	120	3.06%	1.38%	1.32%	8.46%

Panel B. S&P500 futures option data

First trading day of each month, 1988:01-1997:12

	N	Mean	Std. dev.	Minimum	Maximum
Time until expiration (years)	4501	0.11	0.06	0.03	0.22
Trading volume (contracts)	4501	172.38	305.79	5	6682
Moneyness (K/F_t - 1)	4501	-2.71%	6.54%	-20.00%	18.98%
Implied volatility (annualized)	4501	18.74%	6.75%	7.54%	61.09%

This table reports characteristics of the data used to estimate option-based unconditional risk-neutral S&P500 one-month return variance. The first panel of the table reports data that is used in the calculation of the BBSR implied volatilities and the fitted call option prices.

The riskless rate used is extracted from the term-structure defined by the one and three-month continuously compounded Treasury Bill rates from the CRSP Risk-Free Rates file. The implied cost of carry is equal to the difference between the dividend yield and riskless rate over the remaining life of the futures contract. The implied cost of carry is equal to $[\ln(F_t) - \log(P_t)]/(T-t)$ where F_t is the futures price for a contract with time until expiration of T-t and P_t is the contemporaneous S&P500 level. The S&P500 dividend yield is the monthly difference (annualized) between the S&P500 return with dividends and without dividends as reported in the CRSP US Indices database.

There are 120 months in the sample, which accounts for the number of data points for the riskless interest rate and the S&P500 dividend yield. The implied cost of carry is calculated for each expiration date every month, so there are at least two data points per month and sometimes three, resulting in a total of 314 data points.

The second panel of the table reports the characteristics of the S&P500 futures option data. The S&P500 futures option and S&P500 futures data is end-of-day data obtained from the Futures Industry Institute (FII) for all contracts traded from 1988:01 through 1997:12. A cross-section of futures options is extracted on the first trading day of each month. Futures options with trading volume of at least five contracts, with two weeks but no longer than three months until maturity, with moneyness (K/F_t – 1) between –20% and 20%, and implied volatilities (annualized) between 5% and 90% are used. Contracts that violate the no-arbitrage monotonicity or convexity conditions are deleted. Time until expiration is reported as a fraction of a calendar year. An implied volatility is calculated for each contract by numerically inverting the BBSR American option pricing formula (Broadie and DeTemple, 1996) adjusted to apply to futures options contracts.

Table 3
Characteristics of the consumption-based pricing kernel

Sample period 1988:01-1997:12
Panel A. Pricing kernel mean (monthly)

	δ = -.9	δ = -.7	δ = -.5	δ = -.3	δ = -.1	δ = 0	δ = .1	δ = .3	δ = .5	δ = .7	δ = .9
γ = 1	0.9951	0.9951	0.9951	0.9951	0.9951	0.9951	0.9951	0.9952	0.9953	0.9954	0.9962
γ = 5	0.9905	0.9905	0.9906	0.9907	0.9908	0.9909	0.9910	0.9912	0.9918	0.9933	1.0026
γ = 10	0.9848	0.9849	0.9851	0.9853	0.9856	0.9858	0.9860	0.9868	0.9883	0.9927	1.0252
γ = 15	0.9792	0.9794	0.9797	0.9801	0.9806	0.9809	0.9814	0.9828	0.9858	0.9946	1.0648
γ = 20	0.9738	0.9741	0.9745	0.9750	0.9758	0.9764	0.9771	0.9793	0.9842	0.9989	1.1235
γ = 25	0.9684	0.9688	0.9694	0.9701	0.9713	0.9721	0.9731	0.9764	0.9835	1.0055	1.2045
γ = 30	0.9632	0.9637	0.9644	0.9654	0.9669	0.9680	0.9694	0.9739	0.9837	1.0147	1.3130

Panel B. Pricing kernel standard deviation (monthly)

	δ = -.9	δ = -.7	δ = -.5	δ = -.3	δ = -.1	δ = 0	δ = .1	δ = .3	δ = .5	δ = .7	δ = .9
γ = 1	0.0024	0.0025	0.0026	0.0028	0.0031	0.0034	0.0036	0.0045	0.0060	0.0096	0.0250
γ = 5	0.0083	0.0092	0.0104	0.0119	0.0140	0.0153	0.0170	0.0217	0.0301	0.0487	0.1262
γ = 10	0.0164	0.0183	0.0206	0.0237	0.0280	0.0307	0.0340	0.0435	0.0602	0.0974	0.2604
γ = 15	0.0246	0.0273	0.0309	0.0355	0.0418	0.0459	0.0509	0.0650	0.0901	0.1465	0.4166
γ = 20	0.0326	0.0363	0.0410	0.0471	0.0555	0.0610	0.0676	0.0865	0.1199	0.1966	0.6108
γ = 25	0.0406	0.0451	0.0510	0.0586	0.0691	0.0759	0.0842	0.1078	0.1500	0.2487	0.8632
γ = 30	0.0484	0.0539	0.0609	0.0700	0.0826	0.0907	0.1006	0.1291	0.1803	0.3033	1.2010

Panel C. Ratio (PK standard deviation / PK mean)

	δ = -.9	δ = -.7	δ = -.5	δ = -.3	δ = -.1	δ = 0	δ = .1	δ = .3	δ = .5	δ = .7	δ = .9
γ = 1	0.0024	0.0025	0.0026	0.0028	0.0032	0.0034	0.0037	0.0045	0.0060	0.0097	0.0251
γ = 5	0.0084	0.0093	0.0105	0.0120	0.0141	0.0155	0.0172	0.0219	0.0304	0.0490	0.1259
γ = 10	0.0167	0.0186	0.0210	0.0241	0.0284	0.0311	0.0345	0.0441	0.0609	0.0981	0.2540
γ = 15	0.0251	0.0279	0.0315	0.0362	0.0426	0.0468	0.0519	0.0662	0.0914	0.1473	0.3913
γ = 20	0.0335	0.0372	0.0420	0.0483	0.0569	0.0624	0.0692	0.0883	0.1219	0.1969	0.5436
γ = 25	0.0419	0.0466	0.0526	0.0604	0.0711	0.0781	0.0865	0.1104	0.1525	0.2473	0.7166
γ = 30	0.0503	0.0559	0.0631	0.0725	0.0854	0.0937	0.1038	0.1325	0.1833	0.2989	0.9147

Full consumption history 1964:01 - 1997:12
Panel D. Pricing kernel mean (monthly)

	δ = -.9	δ = -.7	δ = -.5	δ = -.3	δ = -.1	δ = 0	δ = .1	δ = .3	δ = .5	δ = .7	δ = .9
γ = 1	0.9932	0.9932	0.9932	0.9932	0.9933	0.9933	0.9933	0.9933	0.9933	0.9933	0.9937
γ = 5	0.9864	0.9865	0.9865	0.9865	0.9866	0.9866	0.9866	0.9868	0.9871	0.9882	0.9958
γ = 10	0.9781	0.9782	0.9782	0.9784	0.9785	0.9787	0.9788	0.9794	0.9807	0.9846	1.0147
γ = 15	0.9699	0.9701	0.9702	0.9705	0.9709	0.9711	0.9715	0.9727	0.9755	0.9843	1.0530
γ = 20	0.9619	0.9622	0.9624	0.9629	0.9635	0.9640	0.9647	0.9668	0.9717	0.9871	1.1132
γ = 25	0.9541	0.9544	0.9549	0.9556	0.9566	0.9573	0.9583	0.9616	0.9691	0.9932	1.1996
γ = 30	0.9465	0.9469	0.9476	0.9485	0.9499	0.9510	0.9524	0.9571	0.9678	1.0026	1.3186

Panel E. Pricing kernel standard deviation (monthly)

	δ = -.9	δ = -.7	δ = -.5	δ = -.3	δ = -.1	δ = 0	δ = .1	δ = .3	δ = .5	δ = .7	δ = .9
γ = 1	0.0034	0.0035	0.0036	0.0038	0.0041	0.0043	0.0046	0.0054	0.0071	0.0109	0.0260
γ = 5	0.0101	0.0111	0.0125	0.0143	0.0167	0.0183	0.0202	0.0256	0.0351	0.0555	0.1333
γ = 10	0.0200	0.0221	0.0249	0.0285	0.0335	0.0366	0.0405	0.0514	0.0704	0.1116	0.2780
γ = 15	0.0299	0.0331	0.0373	0.0427	0.0501	0.0548	0.0606	0.0770	0.1056	0.1686	0.4493
γ = 20	0.0396	0.0440	0.0495	0.0567	0.0665	0.0728	0.0805	0.1024	0.1408	0.2275	0.6683
γ = 25	0.0492	0.0546	0.0615	0.0704	0.0827	0.0906	0.1002	0.1277	0.1764	0.2894	0.9655
γ = 30	0.0587	0.0651	0.0733	0.0840	0.0987	0.1082	0.1198	0.1530	0.2126	0.3556	1.3870

Panel F. Ratio (PK standard deviation / PK mean)

	δ = -.9	δ = -.7	δ = -.5	δ = -.3	δ = -.1	δ = 0	δ = .1	δ = .3	δ = .5	δ = .7	δ = .9
γ = 1	0.0034	0.0035	0.0037	0.0038	0.0041	0.0043	0.0046	0.0055	0.0071	0.0110	0.0262
γ = 5	0.0102	0.0113	0.0126	0.0145	0.0169	0.0185	0.0205	0.0260	0.0356	0.0562	0.1339
γ = 10	0.0204	0.0226	0.0255	0.0292	0.0342	0.0374	0.0414	0.0525	0.0718	0.1134	0.2740
γ = 15	0.0308	0.0341	0.0384	0.0440	0.0516	0.0565	0.0624	0.0791	0.1082	0.1713	0.4267
γ = 20	0.0412	0.0457	0.0514	0.0588	0.0690	0.0755	0.0835	0.1059	0.1449	0.2305	0.6004
γ = 25	0.0516	0.0572	0.0644	0.0737	0.0864	0.0946	0.1046	0.1328	0.1820	0.2914	0.8048
γ = 30	0.0620	0.0688	0.0774	0.0886	0.1039	0.1138	0.1258	0.1598	0.2197	0.3546	1.0519

Table 3 (continued)

Characteristics of the consumption-based pricing kernel

Panel G. Characteristics of S&P500 excess return

	N	Mean	Std. dev.	Minimum	Maximum	Mean / Std. dev.
Sample	120	1.03%	3.47%	-9.72%	11.07%	0.2958
Full history	408	0.52%	4.21%	-22.04%	16.25%	0.1224

This table reports characteristics of the consumption-based pricing kernel for a range of levels of utility function concavity (γ = 1, 5, 10, 15, 20, 25, 30), levels of habit (δ = -.9, -.7, -.5, -.3, -.1, 0, .1, .3, .5, .7, .9), and a fixed rate of time preference (ρ = .999). The nominal pricing kernel is defined as $M_{t+1} = \rho[(C_{t+1} - X_{t+1})/ (C_t - X_t)]^{-\gamma} I_{t+1}^{-1}$ where C_t is the level of consumption, X_t is the level of habit, and I_{t+1} is the gross inflation rate. The habit is defined as the five-year moving average of historical consumption scaled by δ. The pricing kernel is compatible with habit formation ($\delta > 0$), consumption durability ($\delta < 0$), or constant relative risk aversion ($\delta = 0$).

Panels A, B, and C report the pricing kernel mean, pricing kernel standard deviation and the ratio of the pricing kernel standard deviation to the pricing kernel mean for the sample period (1988:01 – 1997:12). The pricing kernel mean and standard deviation are calculated using monthly data and reported in monthly terms. The ratio reported in Panel C is the ratio that is bounded below by the ratio of the benchmark portfolio unconditional expected return and unconditional standard deviation (Hansen and Jagannathan, 1991).

Panels D, E, and F report the same statistics for the sample using the full history of consumption data (1964:01 – 1997:12). The actual consumption history begins in 1959:01 but the first habit calculation, which requires five years of data, is 1964:01.

Panel G reports sample characteristics of the monthly S&P500 excess return used to calculate the Hansen and Jagannathan (1991) bound. The S&P500 return is the S&P500 one-month return including dividends from the CRSP US Indices database, the riskless rate is the one-month Treasury Bill rate from the CRSP Risk-Free Rates file, and the excess return is the difference between the S&P500 return and the riskless rate. The statistics for the “sample” are obtained using the period 1988:01 – 1997:12 and the statistics for the “full history” are obtained using the period 1964:01 – 1997:12. The excess return mean and standard deviation ratio provides the Hansen and Jagannathan (1991) lower bound on the pricing kernel mean and standard deviation ratio.

Table 4

Option-based risk-neutral S&P500 return variance estimation

Panel A. Sample statistics (1988:01-1997:12)

	N	Mean	Std. Dev.	Min	Max
Conditional risk-neutral variance (annualized)	120	0.0278	0.0126	0.0119	0.0842
Conditional risk-neutral standard deviation (annualized)	120	16.31%	3.45%	10.93%	29.02%
Number of options per estimation date	120	37.51	16.03	16	96

Panel B. Confidence intervals for sample averages

	Percentile of simulated distribution of sample averages					
	0.5%	2.5%	5.0%	95.0%	97.5%	99.5%
Average risk-neutral variance (annualized)	0.0250	0.0256	0.0260	0.0297	0.0301	0.0309
Average risk-neutral standard deviation (annualized)	15.52%	15.71%	15.81%	16.83%	16.93%	17.15%

Panel C. Comparison using different bandwidths

	N	Mean	Std. Dev.	Min	Max
Conditional risk-neutral std. dev. 0.75*(Silverman bandwidth)	120	16.52%	3.33%	10.78%	28.38%
Conditional risk-neutral std. dev. 1.00*(Silverman bandwidth)	120	16.31%	3.45%	10.93%	29.02%
Conditional risk-neutral std. dev. 1.25*(Silverman bandwidth)	120	16.41%	3.58%	11.10%	30.03%

The first panel of the table reports sample statistics for the option-based conditional risk-neutral one-month variance estimates. The conditional risk-neutral variances are obtained by numerical integration of the option-based conditional risk-neutral S&P500 one-month return density, which is estimated on the first trading day of each month. The conditional risk-neutral standard deviation is the square-root of the conditional risk-neutral variance. Conditional variances are annualized by multiplying by 12, and conditional standard deviations are annualized by multiplying by $\sqrt{12}$.

The second panel of the table reports confidence intervals for the average conditional risk-neutral variance and average conditional risk-neutral standard deviation. These confidence intervals are obtained using the bootstrap percentile method. The distribution of sample averages is constructed by sampling 120 data points (with replacement) from the monthly dataset, calculating the average of the variable of interest, and repeating 10,000 times. The confidence intervals are the empirical percentiles of the distribution of sample averages.

The third panel of the table reports sample statistics for conditional option-based risk-neutral standard deviations using different choices of bandwidth. The Silverman bandwidth (Silverman, 1996) is equal to $.9{*}N^{(-1/5)}$Min(standard deviation, interquartile range/1.34). N represents the number of observations, and the standard deviation and interquartile range are calculated as sample statistics of the regressors. The sensitivity to bandwidth selection is measured by performing the same estimation procedure with bandwidth equal to 75% or 125% of the Silverman bandwidth.

Table 5

Option-based and consumption-based unconditional risk-neutral S&P500 return variance comparison

Constant relative risk aversion pricing kernel specification (δ=0), 1988:01 - 1997:12

	(1) Delta	(2) Estimated unconditional risk-neutral variance	(3) Estimated unconditional risk-neutral standard deviation	(4) Significance of unconditional risk-neutral variance difference	(5) Estimated unconditional risk-neutral variance risk premium	(6) Estimated unconditional consumption-based riskless rate	(7) Estimated unconditional riskless rate	(8) Signficance of unconditional riskless rate difference	(9) Required ρ to replicate estimated unconditional riskless rate
Option-based		0.0278	16.67%		0.0127				
Consumption-based (γ = 1, ρ = .999)	0.00	0.0151	12.30%	1%	-0.0000	5.88%	5.30%	10%	0.9996
Consumption-based (γ = 5, ρ = .999)	0.00	0.0150	12.26%	1%	-0.0001	11.08%	5.30%	1%	1.0039
Consumption-based (γ = 10, ρ = .999)	0.00	0.0149	12.19%	1%	-0.0002	17.34%	5.30%	1%	1.0091
Consumption-based (γ = 15, ρ = .999)	0.00	0.0148	12.16%	1%	-0.0003	23.33%	5.30%	1%	1.0140
Consumption-based (γ = 20, ρ = .999)	0.00	0.0147	12.12%	1%	-0.0004	29.05%	5.30%	1%	1.0188
Consumption-based (γ = 25, ρ = .999)	0.00	0.0146	12.09%	1%	-0.0005	34.49%	5.30%	1%	1.0233
Consumption-based (γ = 30, ρ= .999)	0.00	0.0145	12.06%	1%	-0.0006	39.65%	5.30%	1%	1.0276

This table reports comparisons of option-based and consumption-based unconditional risk-neutral S&P500 return variance. The consumption-based nominal pricing kernel is defined as $M_{t+1} = \rho[C_{t+1}/C_t]^{-\gamma}I_{t+1}^{-1}$ where C_t is the level of consumption and I_{t+1} is the gross inflation rate. The tested consumption-based pricing kernels are estimated at a range of levels of utility function concavity (γ = 1, 5, 10, 15, 20, 25, 30) and a fixed rate of time preference (ρ = .999).

The risk-neutral standard deviation is the square root of the risk-neutral variance. All estimates are averages over the period 1988:01 – 1997:12 and are reported in annualized terms. The statistical significance of the unconditional risk-neutral variance difference (and unconditional riskless interest rate difference) is obtained using confidence intervals constructed using the bootstrap percentile method with 10,000 simulation replications. A value of 1%, 5%, or 10% represents rejection of equality at the specified confidence level.

The estimated option-based unconditional risk-neutral variance risk-premium is equal to the difference between the estimated option-based unconditional risk-neutral variance and the estimated unconditional S&P500 squared excess return. The estimated consumption-based unconditional risk-neutral variance risk-premium is equal to the difference between the estimated unconditional consumption-based risk-neutral variance and the estimated unconditional S&P500 squared excess return. Over the sample period, the estimated unconditional S&P500 squared excess return is estimated as a sample average and has an annualized value of .0151. The estimated unconditional risk-neutral variance risk-premium is annualized by multiplying the estimated monthly value by 12.

The unconditional consumption-based riskless rate is estimated as $RF_{c,monthly}$ = 1/average(M_t) – 1. The unconditional riskless rate is estimated as 1/average(B_t) – 1 where B_t is the one-month Treasury Bill price derived from the one-month Treasury Bill rate reported in the CRSP Risk-Free Rates file. Both interest rates are annualized using the formula $rf_{annual}= (1 + rf_{monthly})^{12} - 1$. The required ρ to replicate the unconditional riskless interest rate is equal to (model ρ) / (pricing kernel mean * (1 + estimated unconditional riskless rate)).

Table 6

Option-based and consumption-based unconditional risk-neutral S&P500 return variance comparison

General pricing kernel specification, 1988:01 - 1997:12

	(1) Delta	(2) Estimated unconditional risk-neutral variance	(3) Estimated unconditional risk-neutral standard deviation	(4) Significance of unconditional risk-neutral variance difference	(5) Estimated unconditional risk-neutral variance risk premium	(6) Estimated unconditional consumption-based riskless rate	(7) Estimated unconditional riskless rate	(8) Signficance of unconditional riskless rate difference	(9) Required ρ to replicate estimated unconditional riskless rate
Option-based		0.0278	16.67%		0.0127				
Consumption-based (γ = 1, ρ = .999)	0.99	0.0149	12.22%	1%	-0.0002	-2.37%	5.30%	>10%	0.9927
Consumption-based (γ = 5, ρ = .999)	0.99	0.0162	12.71%	1%	0.0010	-76.66%	5.30%	5%	0.8812
Consumption-based (γ = 10, ρ = .999)	0.99	0.0222	14.89%	>10%	0.0071	-99.83%	5.30%	1%	0.5849
Consumption-based (γ = 15, ρ = .999)	0.98	0.0277	16.66%	>10%	0.0126	-99.99%	5.30%	1%	0.4411
Consumption-based (γ = 20, ρ = .999)	0.97	0.0278	16.67%	>10%	0.0127	-99.99%	5.30%	1%	0.4554
Consumption-based (γ = 25, ρ = .999)	0.96	0.0278	16.67%	>10%	0.0127	-99.99%	5.30%	1%	0.4678
Consumption-based (γ = 30, ρ = .999)	0.95	0.0278	16.67%	>10%	0.0127	-99.99%	5.30%	1%	0.4766

This table reports comparisons of option-based and consumption-based unconditional risk-neutral S&P500 return variance. The consumption-based nominal pricing kernel is defined as $M_{t+1} = \rho[(C_{t+1} - X_{t+1})/(C_t - X_t)]^{-\gamma} I_{t+1}^{-1}$ where C_t is the level of consumption, X_t is the level of habit, and I_{t+1} is the gross inflation rate. The habit is defined as the five-year moving average of historical consumption scaled by δ. The pricing kernel is compatible with habit formation ($\delta > 0$), consumption durability ($\delta < 0$), or constant relative risk aversion ($\delta = 0$).

The tested consumption-based pricing kernels are estimated at a range of levels of utility function concavity (γ = 1, 5, 10, 15, 20, 25, 30), a fixed rate of time preference (ρ = .999), and a level of habit (δ) set equal to the "implied delta." The implied delta is the delta within the range [-.99,.99] that minimizes the distance between the consumption-based and option-based unconditional risk-neutral variance for fixed levels of γ and ρ.

The risk-neutral standard deviation is the square root of the risk-neutral variance. All estimates are averages over the period 1988:01 – 1997:12 and are reported in annualized terms. The statistical significance of the unconditional risk-neutral variance difference (and unconditional riskless interest rate difference) is obtained using confidence intervals constructed using the bootstrap percentile method with 10,000 simulation replications. A value of 1%, 5%, or 10% represents rejection of equality at the specified confidence level.

The estimated option-based unconditional risk-neutral variance risk-premium is equal to the difference between the estimated option-based unconditional risk-neutral variance and the estimated unconditional S&P500 squared excess return. The estimated consumption-based unconditional risk-neutral variance risk-premium is equal to the difference between the estimated unconditional consumption-based risk-neutral variance and the estimated unconditional S&P500 squared excess return. Over the sample period, the estimated unconditional S&P500 squared excess return is estimated as a sample average and has an annualized value of .0151. The estimated unconditional risk-neutral variance risk-premium is annualized by multiplying the estimated monthly value by 12.

The unconditional consumption-based riskless rate is estimated as $RF_{c,monthly} = 1/\text{average}(M_t) - 1$. The unconditional riskless rate is estimated as $1/\text{average}(B_t) - 1$ where B_t is the one-month Treasury Bill price derived from the one-month Treasury Bill rate reported in the CRSP Risk-Free Rates file. Both interest rates are annualized using the formula $rf_{annual} = (1 + rf_{monthly})^{12} - 1$. The required ρ to replicate the unconditional riskless interest rate is equal to (model ρ) / (pricing kernel mean * (1 + estimated unconditional riskless rate)).

Table 7

Option-based and consumption-based unconditional risk-neutral variance risk-premium comparison

Panel A. Option-based variance risk-premium (1988:01 - 1997:12), Consumption-based variance risk-premium (1964:01 - 1997:12)
Constant relative risk aversion pricing kernel specification

	(1) Delta	(2) Estimated unconditional risk-neutral variance risk premium	(3) Estimated unconditional consumption-based riskless rate	(4) Estimated unconditional riskless rate	(5) Signficance of unconditional riskless rate difference	(6) Required ρ to replicate estimated unconditional riskless rate
Option-based		0.0064				
Consumption-based (γ = 1, ρ = .999)	0.00	0.0000	8.46%	6.35%	1%	1.0006
Consumption-based (γ = 5, ρ = .999)	0.00	-0.0001	17.57%	6.35%	1%	1.0074
Consumption-based (γ = 10, ρ = .999)	0.00	-0.0002	29.54%	6.35%	1%	1.0156
Consumption-based (γ = 15, ρ = .999)	0.00	-0.0003	42.10%	6.35%	1%	1.0234
Consumption-based (γ = 20, ρ = .999)	0.00	-0.0004	55.20%	6.35%	1%	1.0310
Consumption-based (γ = 25, ρ = .999)	0.00	-0.0005	68.77%	6.35%	1%	1.0382
Consumption-based (γ = 30, ρ = .999)	0.00	-0.0006	82.73%	6.35%	1%	1.0451

Panel B. Option-based variance risk-premium (1988:01 - 1997:12), consumption-based variance risk-premium (1964:01 - 1997:12)
General pricing kernel specification

	(1) Implied delta	(2) Estimated unconditional risk-neutral variance risk premium	(3) Estimated unconditional consumption-based riskless rate	(4) Estimated unconditional riskless rate	(5) Signficance of unconditional riskless rate difference	(6) Required ρ to replicate estimated unconditional riskless rate
Option-based		0.0064				
Consumption-based (γ = 1, ρ = .999)	0.99	0.0002	3.91%	6.35%	>10%	0.9970
Consumption-based (γ = 5, ρ = .999)	0.99	0.0030	-58.46%	6.35%	>10%	0.9237
Consumption-based (γ = 10, ρ = .999)	0.98	0.0064	-84.95%	6.35%	10%	0.8488
Consumption-based (γ = 15, ρ = .999)	0.94	0.0064	-82.60%	6.35%	10%	0.8591
Consumption-based (γ = 20, ρ = .999)	0.91	0.0064	-81.73%	6.35%	10%	0.8626
Consumption-based (γ = 25, ρ = .999)	0.88	0.0064	-81.22%	6.35%	10%	0.8646
Consumption-based (γ = 30, ρ = .999)	0.86	0.0064	-80.78%	6.35%	10%	0.8662

This table reports comparisons of option-based and consumption-based unconditional risk-neutral S&P500 return variance risk premia using the full consumption history. The consumption-based nominal pricing kernel is defined as $M_{t+1} = \rho[(C_{t+1} - X_{t+1})/(C_t - X_t)]^{-\gamma} I_{t+1}^{-1}$ where C_t is the level of consumption, X_t is the level of habit, and I_{t+1} is the gross inflation rate. The habit is defined as the five-year moving average of historical consumption scaled by δ. The pricing kernel is compatible with habit formation (δ > 0), consumption durability (δ < 0), or constant relative risk aversion (δ = 0).

The tested consumption-based pricing kernels are estimated at a range of levels of utility function concavity (γ = 1, 5, 10, 15, 20, 25, 30), a fixed rate of time preference (ρ = .999), and a level of habit (δ) set equal to zero (for the constant relative risk-aversion pricing kernel) or set equal to the "implied delta." The implied delta is the delta within the range [-.99,.99] that minimizes the distance between the consumption-based and option-based unconditional risk-neutral variance risk premium for fixed levels of γ and ρ.

The estimated option-based unconditional risk-neutral variance risk-premium is equal to the difference between the estimated option-based unconditional risk-neutral variance and the estimated unconditional S&P500 squared excess return over the period from 1988:01 – 1997:12. The estimated consumption-based unconditional risk-neutral variance risk-premium is equal to the difference between the estimated unconditional consumption-based risk-neutral variance and the estimated unconditional S&P500 squared excess return over the period from 1964:01 – 1997:12. Over the full consumption history, the estimated unconditional S&P500 squared excess return is estimated as a sample average and has an annualized value of .0214. Unconditional risk-premia are reported in annualized terms.

The unconditional consumption-based riskless rate is estimated as $RF_{c,monthly} = 1/\text{average}(M_t) - 1$. The unconditional riskless rate is estimated as $1/\text{average}(B_t) - 1$ where B_t is the one-month Treasury Bill price derived from the one-month Treasury Bill rate reported in the CRSP Risk-Free Rates file. Both interest rates are annualized using the formula $RF_{annual} = (1 + RF_{monthly})^{12} - 1$. The required ρ to replicate the unconditional riskless interest rate is equal to (model ρ) / (pricing kernel mean * (1 + estimated unconditional riskless rate)). The significance of the estimated unconditional riskless rate difference is determined using the bootstrap percentile method with 10,000 simulation replications.

Figure 1
Monthly consumption and habit levels
Annualized, 1964:01-1997:12

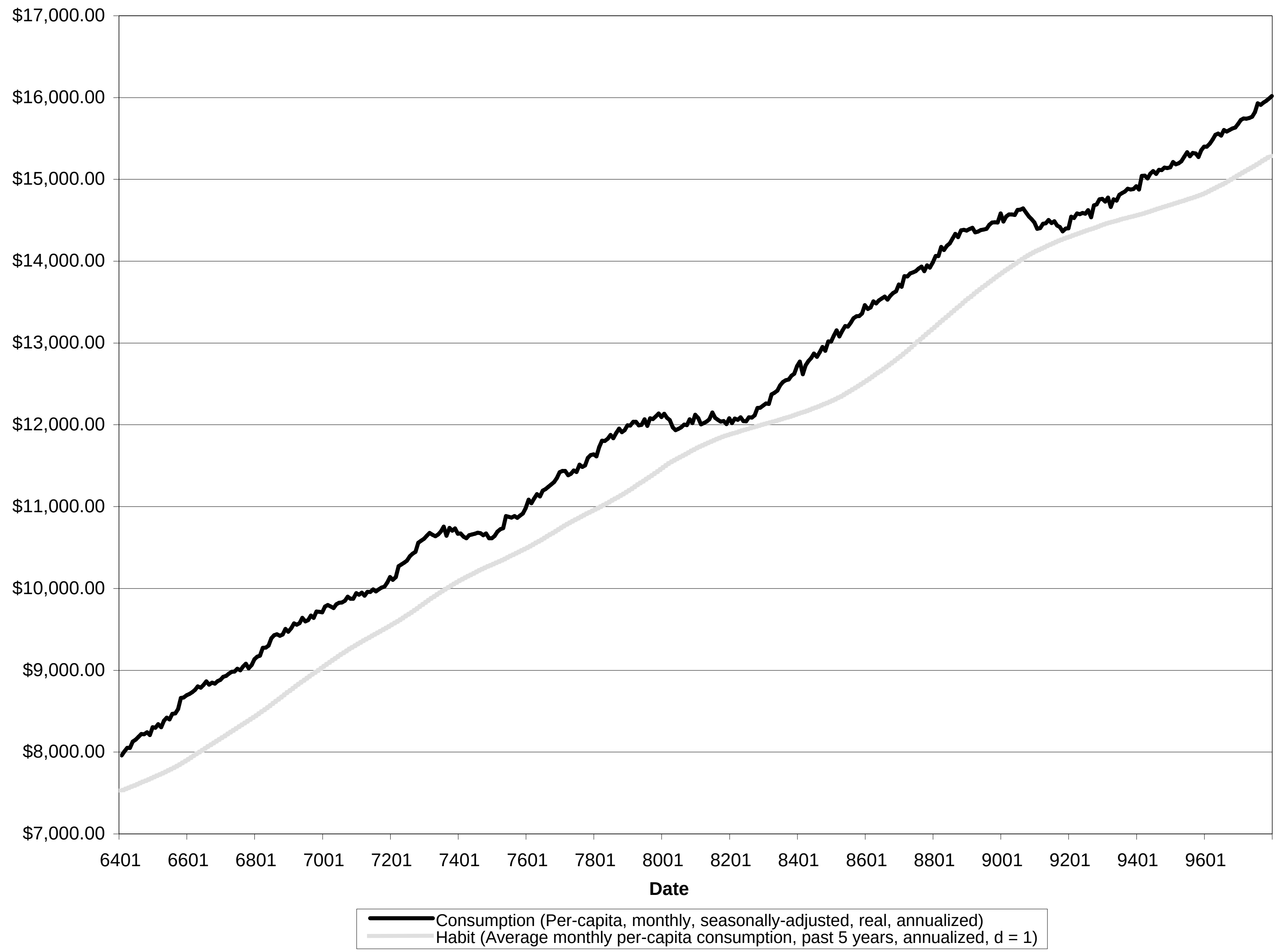

Figure 2
Nonparametric estimates of
S&P500 one-month implied volatility functions

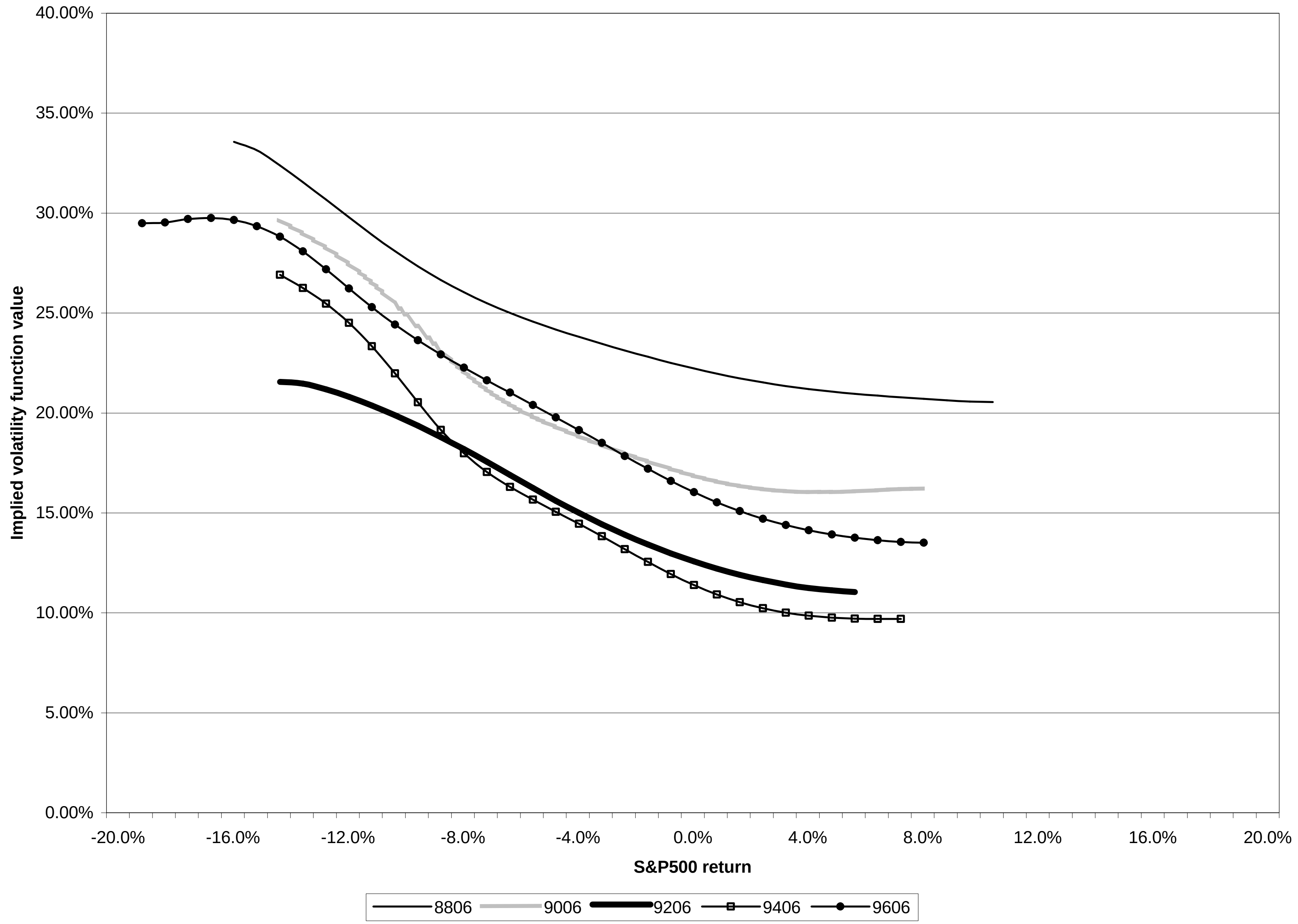

Figure 3
Semi-parametric estimates of
risk-neutral S&P500 one-month return densities

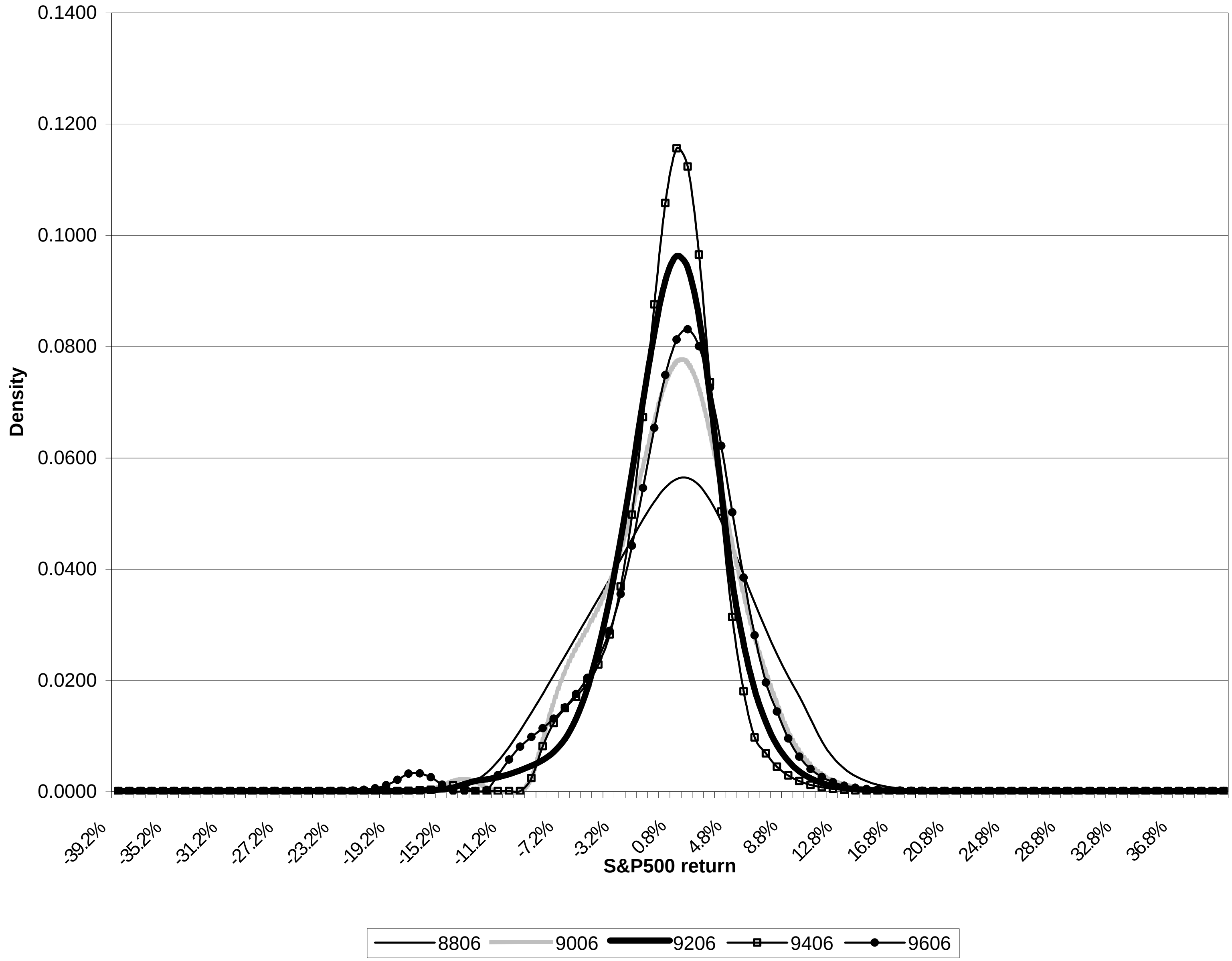

Figure 4
Semi-parametric estimates of
option-based conditional risk-neutral
S&P500 one-month return standard deviation
Annualized, 1988:01-1997:12