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Abstract

This paper examines the convexity bias introduced by pricing interest rate swaps off the Eurocurrency futures curve and the market's adjustment of this bias in prices over time. The convexity bias arises because of the difference between a futures versus a forward contract on interest rates, since the payoff to the latter is non-linear in interest rates. Using daily data from 1987-1996, the differences between market swap rates and the swap rates implied from Eurocurrency futures prices are studied for the four major interest rate swap markets - \$, £, *DM* and ¥. The evidence suggests that swaps were being priced off the futures curve (i.e. by ignoring the convexity adjustment) during the earlier years of the study, after which the market swap rates drifted below the rates implied by futures prices. The empirical analysis shows that this spread between the market and futures-implied swap rates cannot be explained by default risk differences, term structure effects, liquidity differences or information asymmetries between the swap and the futures markets. Using alternative term structure models (Vasicek, Cox-Ingersoll and Ross, Hull and White, and Black and Karasinski), the theoretical value of the convexity bias is found to be related to the empirically observed swap-futures differential. This is evidence of mispricing of swap rates during the earlier years of the study, with a gradual elimination of that mispricing by incorporation of a convexity adjustment in swap pricing over time.

1 Introduction

Does the market price interest rate swaps correctly? This is a pertinent question for a market that has grown tremendously over the last 10 years, to reach a stage where the notional principal amount outstanding exceeds $25.5 trillion[1]. A pricing error of even a few basis points in this market translates to large amounts - a 6 basis point spread on a $200 million, 10 year swap, is worth about $1 million. Despite the size of this market and the consequent importance of accurate pricing, there is relatively sparse theoretical and empirical research on the pricing of interest rate swaps.

This paper examines the bias introduced by pricing interest rate swaps off the Eurocurrency (Eurodollar, Euro DM, etc.) futures curve. The Eurocurrency futures markets and the interest rate swap markets are intricately linked to each other, due to the fact that almost invariably, traders hedge their swap positions with Eurocurrency futures (e.g. short swaps can be hedged with short positions in a strip of appropriate Eurocurrency futures contracts). The close relationship has prompted the use of interest rates implied by Eurocurrency futures prices in pricing swaps. However, this causes an upward bias in implied swap rates, because the forward rates that should be used to price swaps are lower than the rates implied by Eurocurrency futures prices, due to the highly negative correlation between overnight interest rates and futures prices. This bias, known as convexity bias, arises because of the negative convexity exhibited by (pay fixed) interest rate swaps, whilst Eurocurrency futures contracts have no convexity since their payoffs are linear in the interest rates. Hence, futures prices should be corrected for convexity before using them for computing swap rates. This paper looks at whether this convexity correction has been incorporated efficiently into interest rate swap pricing over time in the four major international currencies - \$, £, *DM* and ¥.

Prior research in this area has primarily focused on comparing futures and forward prices and explaining the differences between the two prices. One of the earliest theoretical studies, by Cox, Ingersoll and Ross (CIR, 1981), shows that contractual distinctions between futures and forward contracts would create a price divergence even in an efficient market with no transactions costs[2].

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[1] Recent estimates by the International Swaps and Derivatives Association (ISDA) indicate that the notional outstanding volume of transactions of privately negotiated (over-the-counter) derivatives at the end of 1996 amounted to over $25.5 trillion, of which interest rate swaps amounted over $19.2 trillion. New business activity during 1996 amounted to about $17.9 trillion for all derivatives and about $13.7 trillion for interest rate swaps. Source: International Swaps and Derivatives Association, News Release, October 15, 1997.

[2] See also the related papers by Jarrow and Oldfield (1981) and Richard and Sundaresan (1981).

The argument is based on the marking-to-market (daily resettlement) feature of futures contracts which affects the timing of the cash flows between the two counterparties to the contract. On a discounted basis, this difference in payment streams between futures and forward contracts creates a price divergence. They use arbitrage arguments to show that the price differences increase with the covariance of the futures price changes with the riskless bond price changes, and if this covariance is positive, then the futures price would be less than the forward price (i.e., the implied forward rate would be lower).

An implication of this proposition is that the CIR marking-to-market effect would be small when the covariance is negligible. This is also confirmed by a study of foreign exchange futures by Cornell and Reinganum (1981), where the covariance is very small (of the order of 10^{-7} or less). Muelbroek (1992) tests the CIR proposition in the Eurodollar futures and forwards markets (where the covariance is found to be positive and significant) and confirms the CIR result that the marking-to-market feature explains the price differences between futures and forwards. The study also supports the CIR prediction that specific covariances determine the sign and magnitude of this price difference.

There have been several studies in the U.S. Treasury bill market[3] that examine the futures-forward price differences to market inefficiency. In the Eurodollar market, Sundaresan (1991) argues that the "settlement to yield" feature of the Eurodollar futures contract (as opposed to settlement to prices) implies that the forward prices would differ from futures prices, even in the absence of marking-to-market. Kamara (1988) asserts that differences in liquidity give rise to the futures-forwards price differences.

A recent study by Grinblatt and Jegadeesh (1996) presents evidence that the observed spreads between futures and forwards Eurodollar yields cannot be explained by the futures contract's marking-to-market feature. They derive closed-form solutions for the yield spread and show that, theoretically, it should be small. They also show that liquidity, taxation and default risk cannot account for the large spreads observed, and that the spreads are attributable to the mispricing of futures contracts relative to the forward rates. However, they only analyze relatively short term contracts with maturities upto one year, whereas the spread due to the marking-to-market feature is expected to be more pronounced only in longer term contracts.

[3] Rendleman and Carabini (1979), Elton, Gruber and Rentzler (1984), Park and Chen (1985) and Kolb and Gray (1985).

Overall, the empirical research on pricing futures versus forward contracts has examined the data for relatively short-dated contracts. Since long-dated swap contracts involving forward commitments of five years and longer are commonplace in the financial markets, it is worth examining how large the differences are for long-dated contracts.

In spite of this literature on the futures-forwards yield differences, there has been no systematic study investigating the impact of these differences in the interest rate swaps market. Studies on swap pricing have generally focused on the impact of counterparty default risk on swap rates. One of the first theoretical studies in this area, by Cooper and Mello (1991), derives the relationship between swap market default spreads and debt market default spreads. They analyze three possible treatments in default and their wealth transfer effects, to obtain equilibrium swap rates. Duffie and Huang (1996) formalize a model of swap pricing with two-sided default, by extending the approach used in Duffie and Singleton (1996). They also study the impact of "netting" i.e., the offsetting of cash flows across different contracts entered into by two counter-parties, on the value of a swap portfolio when two-sided credit risk is involved. Baz and Pascutti (1996) price swap contracts and their associated credit risk under alternative swap covenant structures.

On the empirical side, there are very few studies in this area. Koticha (1993) and Mozumdar (1996) examine the impact of the slope of the term structure on the default risk in swaps. Koticha (1993) uses data from five currencies, \$, £, *DM*, ¥, and *FF* to test the effect of the slope of the term structure on swap rates. By regressing the swap spreads on the slope of the term structure and a credit risk proxy, Koticha finds that the coefficient of the slope term is negative and significant. Mozumdar (1996) uses a non-linear specification for swap pricing and estimates a parameter that proxies for the fraction of the promised cash flows that are not received in the event of default. Since the distribution of errors from this estimation is difficult to specify, he uses Generalized Method of Moments to estimate the default risk parameter, using two alternative sets of data from dollar and DM swaps during the period 1990 to 1996. His broad conclusion is that the default risk parameter is positive and statistically significant, in the case of dollar swaps, but not for DM swaps. Sun, Sundaresan and Wang (1993) examine the effect of dealer's credit reputations on swap quotations and bid-offer spreads. They observe that bid-offer spreads of swap dealers are sensitive to their credit quality, with the lower credit quality dealer's swap rates being bracketed by the higher credit quality dealer's quotes. The other issue that they investigate is the proposition that while the default risk of a counterparty increases both the coupon rate on a par bond (its yield based on LIBOR rates) as well as the swap rate, the effect is more pronounced on the bond yields than on the swap rates. Eom, Subrahmanyam and Uno (1997) investigate the pricing of Japanese yen interest rate swaps. They find that the swap spread in the yen market displays an inverted U-shape, in contrast to the dollar market where it rises monotonically, and attribute this behavior to liquidity effects and market segmentation in the JGB market. Their empirical results show that the swap spread is negatively related to the level and slope of the term structure and positively related to the curvature, as well as to proxies for the short and long term credit risk factors.

A recent study by Minton (1997) concludes that swap valuation models based on replicating portfolios of non-callable corporate par bonds or on replicating portfolios of Eurodollar futures contracts perform well, though they are not completely consistent with the implications of differential counterparty default. However, she does not consider the convexity adjustment while pricing swaps off the Eurodollar futures curve. She reports that OTC swap rates do not move one-for-one with equivalent swap rates derived from Eurodollar futures prices, and attributes this lack of equivalence between OTC swaps and the corresponding Eurodollar strips to the absence of counterparty default risk in the futures market. However, as we show in this paper, the primary reason for the difference between OTC swap rates and swap rates derived from Eurodollar futures prices is the convexity differential between swaps and futures. Proxies for counterparty default risk are unable to explain the variation in the spread (between OTC swap rates and swap rates implied from Eurodollar futures prices) to any significant extent.

Our paper studies the impact of the convexity bias on the pricing of interest rate swaps, and the incorporation of this correction into swaps pricing over time. We find that swaps were priced right on the Eurocurrency futures curve during the earlier years of the study (1987-1990). This suggests that the market effectively ignored the convexity adjustment that is required to be made. After that period, market swap rates are found to drift lower than the swap rates implied from Eurocurrency futures prices. This spread between market and implied swap rates is not explained by default risk differences, term structure effects, liquidity differences or information asymmetries between the swaps and the Eurocurrency futures markets. Using the one factor Vasicek, CIR, Hull and White (HW), and Black and Karasinski (BK) models, the theoretical bias due to convexity is found to be comparable to the spread observed in the data. This is consistent with the hypothesis that there was mispricing in the swap rates during the earlier years of the study, and that this mispricing has been eliminated over time by incorporating a correction for convexity in pricing swaps.

The remainder of the paper is organized as follows. In Section 2, we present a theoretical framework for valuing swaps and relate the convexity of swaps to the differences between futures and forward contracts. Section 3 theoretically derives the convexity correction in swaps and futures using alternative term structure models (Vasicek, CIR, HW, and BK). The data sample is described in Section 4, along with a brief overview of the issues involved in constructing the yield curve using futures prices. In Section 5, we present empirical evidence of mispricing of swap rates due to convexity bias, and show that the mispricing cannot be explained by other factors like counterparty default risk, information asymmetry, term structure effects or liquidity. In section 6, the four term structure models are empirically estimated, and then used to calculate the magnitude of the convexity bias for pricing swaps of varying maturities. Tests are conducted to examine whether the theoretically computed convexity corrections are significantly related to the empirically observed spread between OTC swap rates and swap rates computed from Eurocurrency futures prices. Section 7 concludes.

2 The pricing of interest rate swaps

A plain vanilla fixed-for-floating swap is an agreement in which one side agrees to pay a fixed rate of interest in exchange for receiving a variable/floating rate of interest during the tenor (maturity) of the swap. The other counterparty to the swap agrees to pay floating and receive fixed. The two interest rates are applied to the swap's notional principal amount. The fixed rate, also called the swap rate, is set against a floating reference rate (which is often the London Inter-Bank Offered Rate). The floating rate is reset several time over the swap's life - usually every 3-6 months.

In the terminology of the swap market, the fixed rate payer is known as the buyer of the swap, the floating rate payer is known as the seller and the fixed rate of interest is known as the price of the swap. Since the interest payments are computed on the same notional principal for both the counterparties, there is no exchange of principal between them at maturity. The fixed rate in the swap is fixed at initiation for the entire life of the swap.

Interest rate swaps of all types and maturities are traded in the Over-The-Counter (OTC) markets. The most common version of the fixed-for-floating plain vanilla swap is one where payments are made semi-annually and the floating rate is the 6-month US$ LIBOR. At the initiation date, the market value of the swap is set to zero. Then, on each payment date, the difference between interest payments based on the fixed swap rate and the 6-month US$ LIBOR prevailing six months ago is exchanged between the two counterparties.

Conceptually, in the absence of credit risk, a long position in a swap contract can be thought of as a long position in a floating rate note and a short position in a fixed rate note. Since the floating rate is set equal to the market rate for the entire term of the floater, it is assumed that the hypothetical floater would trade at par on a reset date. Hence, the fixed rate on the swap must be set so that the hypothetical fixed rate note would also trade at par, making the net present value of the swap zero. This leads to the following basic swap pricing proposition stated by Cooper and Mello (1991) and several others:

In the absence of default risk, the arbitrage-free fixed swap rate should equal the yield on a par coupon bond that makes fixed payments on the same dates as the floating leg of the swap.

Traditional swap valuation methodology equates the discounted value of cash inflows and cash outflows for both the counterparties, so that the net present value of the contract is zero at initiation. To illustrate this valuation framework for a generic, default-free swap, the following notation is used in the paper:

N = total number of swap cash flows/payment dates,

c = fixed swap coupon rate,

L_i = floating (LIBOR) interest rate for the i-th cash flow ($i = 1,2,\ldots,N$),

t_i = time corresponding to the i-th cash flow,

V_t^j = values of the swap at time t to the j-th counterparty, $j=c,L$, referring to the fixed and the floating rate payers respectively,

d_t = appropriate pure discount factor (spot rate) for time t.

The arrival of information is modeled by the filtration $\Im_t$ corresponding to the probability space $(\Omega,\Im,P)$.

Consider a swap with a notional principal of \$1. The value of the swap (at initiation) to the fixed rate payer is

$$V_0^c = E_Q[\sum_{i=1}^{N} d_{t_i}(L_{i-1} - c)|\Im_0] = 0 \tag{1}$$

where the expectation operator E_Q is defined with respect to the risk-neutral distribution of L_i conditional on the information available at the time the swap is initiated (date 0). The discount factor is given by

$$d_t = \exp\left[-\int_0^t r_s ds\right] \tag{2}$$

where r_s is the riskfree short rate at time t. Similarly, the value of the swap to the floating rate payer is given by

$$V_0^L = E_Q[\sum_{i=1}^{N} d_{t_i}(c - L_{i-1})|\Im_0] = 0 \tag{3}$$

Either of these two relationships can be used to price a swap, given the observed term structure of interest rates at date *0*. Under the risk-neutral expectations operator, the forward prices of the zero-coupon bonds equal the expectation of the spot prices that will prevail in the future; hence the risk-neutral expectation of L_i can be obtained from the observed zero-coupon term structure. Let B_s^t be the date s price of a zero-coupon bond with unit face value, maturing at date t. Then,

$$E_Q[L_i|\Im_0] = \frac{1}{t_{i+1} - t_i}\left[\frac{B_0^{t_i}}{B_0^{t_{i+1}}} - 1\right] \tag{4}$$

These can be used to solve for the value of the fixed swap rate c that satisfies (1).

2.1 *Interest rate swap as a portfolio of forward contracts*

An alternative method of valuing a swap contract is to treat it as the sum of a series of forward rate agreements (FRA's).[4] As shown by Smith et al. (1988), this follows because the cash flows of a default-free par swap can be replicated by the cash flows of a portfolio of FRA's maturing at consecutive settlement dates. At each swap payment date, the gain or loss in the currently maturing implicit forward contract is realized. However, since FRA rates are not always readily quoted, they may, in turn, be imputed from prices of Eurocurrency futures contracts.

Consider a plain-vanilla fixed-for-floating swap with N payment dates. Each of these N legs of the swap can be thought of as a FRA, with a forward rate equal to the swap coupon (c). For (any) one leg of the swap, on the payment date t_i, the swap counterparties exchange a "difference check" D_i, received by the counterparty for whom the swap is in-the-money. The value of the difference check to the fixed rate payer (the long side in the swap) is given by

$$D_i = (t_i - t_{i-1})[L_{i-1} - c] \tag{5}$$

[4] A third method for valuing a swap is by considering a reversal of the swap at the market swap rate. This would yield an annuity equal to the difference between the fixed payments at the original swap rate and the new swap rate, which can be easily valued, since the cash flows are known.

where L_{i-1} is the LIBOR rate set at time t_{i-1} for the time period (t_{i-1}, t_i), and the notional principal amount for the swap is $1. If $L_{i-1} > c$, then the swap is in-the-money for the fixed rate payer, and thus, she receives the amount D_i at date t_i from the floating rate payer. The converse is true if $L_{i-1} < c$. Thus, the value of the swap (to the fixed rate payer) at any swap payment date t_k is the discounted value of all the remaining difference checks under the risk neutral expectations operator, given by

$$V_k^c = E_Q\left[\sum_{i=k+1}^{N} \exp[-\int_{t_k}^{t_i} r_s ds] D_i \middle| \Im_0\right] = E_Q\left[\sum_{i=k+1}^{N} \exp[-\int_{t_k}^{t_i} r_s ds].(t_i - t_{i-1})(L_{i-1} - c) \middle| \Im_0\right] \tag{6}$$

Now consider a Forward Rate Agreement (FRA) based on LIBOR, maturing at date t_i, with a forward price c, on a principal amount of $1 (same as the swap notional principal). The buyer of the FRA commits to paying the fixed rate c in exchange for receiving LIBOR on the underlying loan principal, for the time period (t_i, t_{i+1}). Since the settlement of the FRA contract is made in discounted form, at date t_i, the value of the contract (say F_i) to the fixed rate payer is given by

$$F_i = \exp[-\int_{t_i}^{t_{i+1}} r_s ds].(t_{i+1} - t_i)[L_i - c] \tag{7}$$

Thus, the value of a portfolio of FRAs maturing at successive dates (to the long side, say FRA_k) at date t_k is the discounted value of all the F_i under the risk neutral expectations operator, given by

$$FRA_k = E_Q\left[\sum_{i=k+1}^{N} \exp[-\int_{t_k}^{t_{i-1}} r_s ds].F_{i-1} \middle| \Im_0\right] = E_Q\left[\sum_{i=k+1}^{N} \exp[-\int_{t_k}^{t_i} r_s ds].(t_i - t_{i-1})(L_{i-1} - c) \middle| \Im_0\right] \tag{8}$$

Assuming that the swap payment dates occur after equal intervals, the value of one swap leg to the fixed rate payer, discounted by one time period, is identical to the value of a FRA to the long side (the swap leg has to be discounted by one period to be comparable to the FRA, because of the discounted settlement feature of the FRA contract). Since the swap is composed of N such payment legs, the cash flow characteristics of the swap are identical to a portfolio of N FRAs (after adjusting for the differential one-period discounting), each maturing at successive swap payment dates. This is also evident from the functions for the value of the swap (V_k^c) (equation (6)) and that for the portfolio of FRAs (FRA_k) (equation (8)), which are identical. *Hence a long swap position is similar to a portfolio of long positions in N successive FRAs.*

2.2 *The relationship between forward and futures contracts on interest rates*

Although prices of LIBOR forward contracts (FRA's) are available, their liquidity is small, especially for long-dated contracts. However, since Eurocurrency futures prices are based on

LIBOR rates[5], they can be used to calculate the relevant forward rates. The basic difference between forward and futures contracts is the daily marking-to-market feature in futures contracts. Consider a FRA maturing at date t_i. This contract settles to the LIBOR rate L_i prevailing at the maturity date. Assuming the dates to be semi-annual (which is the most common case for interest rate swaps), the LIBOR rate L_i refers to the rate during the time period (t_i , t_{i+1}). Since Eurocurrency futures contracts are written on 3-month time deposits, the futures contract would settle to the 3-month LIBOR rate for the time period $(t_i , t_{i+1/2})$. As an illustration, consider a short position in a 6 month FRA on the 6-month LIBOR. This contract matures in 6 months, and locks in the investing rate (receive fixed) for six months, from month 6 to month 12. Now, consider a long position in a 6-month and a 9-month Eurocurrency futures contract on the 3-month LIBOR. This portfolio of two futures contracts also locks in the investing rate from month 6 to month 12, by rolling over the 3-month deposit at the end of month 9. To preclude any arbitrage, the 6-month forward rate at month 6 must equal the compounded 6 month rate using the individual 3-month rates at the end of month 6 and month 9. Hence, the investment rate locked in using the short FRA must equal the investment rate locked in using two long Eurocurrency futures contracts (this is ignoring, for the time being, convexity and other differences between futures and forward rates). Therefore, two successive Eurocurrency futures contracts, maturing at t_i and $t_{i+1/2}$, would be similar to a single FRA maturing at t_i. Due to this reason, *a short position in a FRA on the 6-month LIBOR can be replicated by a portfolio of long positions in 2 successive Eurocurrency futures contracts.*

2.3 *Futures-forwards yield differences and swap convexity*

Although they are both driven by the same kinds of interest rates, interest rate swaps and Eurocurrency futures contracts differ in one key respect. In a swap cash is exchanged only once for each leg of the swap, whereas in Eurocurrency futures, gains and losses are settled every day. This affects the relative valuation of these two derivative instruments.

In the case of Eurocurrency futures, the only source of risk is the change in the forward or futures rate, since all gains/losses are settled right away. On the other hand, swaps, while being exposed to changes in the forward rate, are also exposed to changes in the term rate. The nominal gain on a short swap (conventionally, the short side in a swap receives fixed) when the forward rate falls is equal to the nominal loss on the position when the forward rate rises. However, the present values of the gain and the loss on the swap are not the same. This is because the price of a zero coupon bond rises as forward rates fall and falls as forward rates rise. Since the discount factor

[5] For a description of the Eurodollar futures markets, see Appendix A.

behaves asymmetrically to increases and decrease in forward rates, the gain on the short swap is worth more when the forward rates fall because the discount rate used to present value the gain also falls. Conversely, the loss on the swap when forward rates rise is worth less because the discount rate for valuing this loss is higher. This is graphically depicted in Figure 1. The price-yield relationship for the short swap position exhibits positive convexity, i.e., the price increases more when yields falls than the price falls when the yields rise. Eurocurrency futures, on the other hand, exhibit no convexity at all. Because of this difference in the convexities of the two instruments, a short swap hedged with a short position in Eurocurrency futures benefits from changes in the level of interest rates.

The value of this convexity difference depends on a few factors. Firstly, higher volatility of interest rates implies a greater value of the convexity difference. Secondly, the correlation between changes in forward rates and changes in term rates determines whether the value of the convexity difference will be positive or negative. A positive correlation between the two interest rates implies a positive value for this difference, which is usually the case as forward interest rates and zero coupon rates tend to be highly correlated. Thirdly, the convexity difference is more pronounced for longer maturity swaps, as the impact of zero coupon rates on the discounted present value of gains and losses is higher[6].

The basic reason for this convexity difference between swaps and futures can be traced back to the convexity difference between forwards and futures. In valuing a swap, we need to use forward rates to discount cash flows. Futures rates are used as proxies for forward rates (as forwards are not actively traded), which is technically incorrect, because there are significant pricing differences between futures and forwards interest rates. Effective forward rates need to be implied from the equivalent futures before they can be used for pricing purposes. The differences in market structures[7] theoretically imply a difference in the yields between futures and forward

[6] The difference also depends on the tenor of the underlying interest rate index, say 3 months or 6 months, but this effect is small.

[7] Futures and forward contracts are similar - both are agreements between two parties to trade a specific good or an asset at a future date for a pre-determined price. However, they are settled quite differently. Futures contracts are standardized contracts, traded on organized exchanges. The futures exchange virtually eliminates default risk by assuming the opposite side of each trade, guaranteeing payment. Positions are marked-to-market everyday to reduce default, as gains/losses are settled on a daily basis. On the other hand, forward contracts, traded mostly in the OTC markets, are exposed to default risk. They are not standardized, with all payments made at the maturity of the contract. Moreover, forward positions can be synthetically replicated by taking positions in the cash market. For example, a trader can create a forward position in a 3 month time deposit starting 3 months in the future by going long a LIBOR deposit with 6 month maturity and going short the LIBOR deposit with 3 month maturity.

contracts[8].

The difference in convexity between forward and futures contracts can be theoretically decomposed into two components - the first component, due to the maturity of the contract (the *term effect*); and the second component, due to the fact that there is a time lag between the time when the interest rate is observed and the time when the corresponding payoff occurs (the *tenor effect*). For a forward contract ($f(t,T,T+m)$) and a futures contract ($F(t,T,T+m)$)[9], the term effect corresponds to the convexity differential due to the stochastic discount factor between time t and time T, while the tenor effect refers to the convexity introduced by the stochastic discount factor, between time T and time $T+m$ (corresponding to the tenor of the underlying interest rate). As the maturity of the contract increases, the term effect accounts for a larger proportion of the total convexity differential. The tenor effect shows less variability, and is affected only by the level and volatility of the interest rates, not by the maturity of the contracts. In this paper, hereafter, the convexity differential refers to the *total* convexity bias, which is a combination of the term and tenor effects[10].

Empirically, it has been shown that a significant yield difference exists between these two contracts, with forward yields being lower than the corresponding futures yields they are tied to, since the correlation between the overnight interest rate and the price of the futures contract is negative. Since forward rates should be used to value swaps, the use of futures rates without correcting for convexity would impart an upward bias to swap yields. In that case, the spread between market swap yields and the implied swap yields (computed directly off the Eurocurrency futures curve without correcting for convexity) would randomly fluctuate around zero. If the swap market recognizes this mispricing and corrects futures rates for convexity fully, this spread should be negative (as convexity correction would lower the swap yield, thereby making the market yield lower than the raw, unadjusted implied swap yield). It follows, therefore, that a zero spread between market and unadjusted swap yields is an indication of market inefficiency.

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[8] A theoretical derivation of the difference between forward and futures prices using general contingent claims valuation principles is presented in Appendix B.

[9] $f(t,T,T+m)$ refers to the price of a forward contract at time t, expiring at time T, on an interest rate of tenor m, i.e. between time T and time $T+m$. The notation is similar for the price of a futures contract ($F(t,T,T+m)$).

[10] We considered separating the two convexity effects in the empirical work that follows. However, since the variability of the tenor effect is likely to be very small, virtually all of the variation would come from the term effect. Hence, we decided to focus on the total convexity effect in our empirical work.

3 Theoretical derivation of the convexity adjustment

The differences between futures and forward rates, which lead to the convexity bias in interest rate swaps, are potentially attributable to the marking-to-market feature of futures contracts. This section characterizes these differences using four different term structure models, to examine the impact of different assumptions on the underlying interest rates, on convexity adjustment. Closed- form solutions are computed for the Eurocurrency futures-forwards rate differences using the Vasicek (1977) and Cox, Ingersoll and Ross (CIR, 1985) models, while the trinomial tree approach of Hull and White (1994) is used to numerically estimate the convexity bias for the no-arbitrage models of Hull and White (1990, HW) and Black and Karasinski (1991, BK).

In order to *price* convexity, a model of interest rates is needed to provide an unambiguous description of the evolution of rates. The model needs to be rich enough to allow for a wide variety of future yield environments while at the same time constraining the yield movements within reasonable bounds. There is also a trade-off between analytic tractability and numerical accuracy and consistency. The models of Vasicek and CIR allow for analytic solutions; however, they do not fit the current yield curve exactly, thereby introducing a mispricing in the underlying zero-coupon bonds themselves. The no-arbitrage models of HW and BK are numerically consistent with the current prices of zero-coupon bonds, but they are computationally intensive. The BK model is also analytically intractable because of the lognormal assumption of interest rates. This section examines the modeling and estimation of these models and their implications for pricing the convexity bias in swaps. Since correlations of yield movements along the yield curve have minimal impact on the convexity adjustment, the discussion is confined to one-factor models.[11]

3.1 *Derivation of convexity adjustment using Vasicek/CIR models*

The Vasicek as well as the CIR models have the following general form

$$dr = \kappa(\mu - r)dt + \sigma r^{\beta} dz \tag{9}$$

where

dr = change in the instantaneous short rate,

κ = speed of adjustment (mean-reversion),

[11] The use of multi-factor models may be important for pricing instruments where these correlations are important, for example, swaptions, mortgage-backed securities or bond options.

μ = long run mean of the short rate,

σ = instantaneous short-rate volatility parameter, and

dz = standard Wiener process increment.

In the context of this study, the state variable r can be interpreted as the instantaneous LIBOR interest rate. The exponent β contributes to the determination of the distributional properties of the short rate; $\beta=0$ implies normal rates (Vasicek), while $\beta=1/2$ gives the CIR model (the conditional distribution of the short rate is noncentral chi-square)[12]. The choice of β is dictated by a compromise between analytic tractability ($\beta=0$ and $\beta=1/2$) and reasonableness of the resulting distribution. The Vasicek specification permits interest rates to become negative.

Under the Vasicek model ($\beta=0$), the short rate follows the Ornstein-Uhlenbeck process and the date s price of a pure discount bond with unit face value maturing at date t is given by

$$B_s^t = A^v(t-s)\exp[-B^v(t-s)r_s] \tag{10}$$

where the functions $A^v(x)$ and $B^v(x)$ are defined as

$$\begin{aligned} A^v(x) &= \exp\left[[B^v(x)-x](\mu - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{2\kappa^2}) - \frac{\sigma^2}{4\kappa} B^v(x)^2 \right], \\ B^v(x) &= \frac{1}{\kappa}[1-\exp(-\kappa x)], \end{aligned} \tag{11}$$

and λ is the market price of risk per unit of σ.

Given this solution, the forward rate $f(t_i, t_{i+1})$ at date s under the Vasicek model can be computed as follows

$$f(t_i, t_{i+1}) = \frac{1}{t_{i+1}-t_i}\left[\frac{B_s^{t_i}}{B_s^{t_{i+1}}} - 1\right] \tag{12}$$

Under the CIR model ($\beta=1/2$), the short rate follows a square root process, with the discount bond prices given by

$$B_s^t = A(t-s)\exp[-B(t-s)r_s] \tag{13}$$

where the functions $A(x)$ and $B(x)$ are defined as

[12] $\beta=1$ yields lognormal interest rates.

$$A(x)=\left[\frac{2\gamma\exp\left[(\kappa+\gamma+\lambda)\frac{x}{2}\right]}{2\gamma+(\kappa+\gamma+\lambda)\left[\exp(\gamma x)-1\right]}\right]^{\frac{2\kappa\mu}{\sigma^2}}$$

$$B(x)=\frac{2\left[\exp(\gamma x)-1\right]}{2\gamma+(\kappa+\gamma+\lambda)\left[\exp(\gamma x)-1\right]} \tag{14}$$

$$\gamma=\sqrt{(\kappa+\lambda)^2+2\sigma^2}$$

and λ is the market price of risk. The forward rates can be computed from discount bond prices in a similar manner.

The futures rate F can be shown to satisfy the following partial differential equation[13]

$$F_t+F_r[\kappa(\mu-r)-r\lambda]+\frac{1}{2}F_{rr}r\sigma^2=0 \tag{15}$$

subject to the boundary condition that at maturity, the futures rate must equal the LIBOR at that date

$$F_s=L_s \tag{16}$$

Under the Vasicek model, the futures rate $F(t_i,t_{i+1})$ at date s is given by:

$$F(t_i,t_{i+1})=\frac{1}{t_{i+1}-t_i}\left[\frac{1}{A^v(t_{i+1}-t_i)}E\left[\exp[B^v(t_{i+1}-t_i).r_{t_i}]\right]-1\right] \tag{17}$$

where $A^v(x)$ and $B^v(x)$ are the functions defined for Vasicek bond prices above, and

$$E\left[\exp[B^v(t_{i+1}-t_i).r_{t_i}]\right]=\exp\left[B^v(t_{i+1}-t_i)E[r_{t_i}]+\frac{1}{2}B^v(t_{i+1}-t_i)^2\operatorname{var}[r_{t_i}]\right],$$

$$E[r_{t_i}]=\exp(-\kappa t_i).r_s+[1-\exp(-\kappa t_i)].(\mu-\frac{\lambda\sigma}{\kappa}), \tag{18}$$

$$\operatorname{var}[r_{t_i}]=\frac{\sigma^2}{2\kappa}[1-\exp(-2\kappa t_i)].$$

For the CIR model, the solution of the partial differential equation gives the futures rate $F(t_i,t_{i+1})$ at date s as follows:

$$F(t_i,t_{i+1})=\frac{1}{t_{i+1}-t_i}\left[\frac{1}{A(t_{i+1}-t_i)}E\left[\exp[B(t_{i+1}-t_i).r_{t_i}]\right]-1\right] \tag{19}$$

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[13] See Grinblatt and Jegadeesh (1996), for example.

where $A(x)$ and $B(x)$ are the functions defined for the CIR bond prices, and

$$E\left[\exp[B(t_{i+1}-t_i).r_{t_i}]\right] = \frac{\exp\left[\dfrac{B(t_{i+1}-t_i).\exp[-(\kappa+\lambda)t_i].r_s}{1-B(t_{i+1}-t_i).\dfrac{\sigma^2}{2}.C(t_i)}\right]}{\left[1-B(t_{i+1}-t_i).\dfrac{\sigma^2}{2}.C(t_i)\right]^{\frac{2\kappa\mu}{\sigma^2}}}, \tag{20}$$

$$C(t_i) = \frac{1}{\kappa+\lambda}\left[1-\exp[-(\kappa+\lambda)t_i]\right].$$

The theoretical difference between the futures and the forward rates is computed using the expressions obtained above. Eurocurrency futures prices are adjusted for convexity by subtracting this difference from the futures rates to arrive at the correct implied forward rates. These rates are then used to price the swap - the swap rate thus obtained is the convexity-adjusted swap rate.

3.2 *Modeling convexity using no-arbitrage models of HW and BK*

The no-arbitrage models of term structure take the current term structure as an input rather than as output, thus making the yield curve consistent with the prices of observed zero-coupon bonds. A generalized one-factor model that includes mean reversion has the form

$$df(r) = \left[\theta(t) - a(t)f(r)\right]dt + \sigma(t)dz \tag{21}$$

where

$f(r)$ = some function f of the short rate r,

$\theta(t)$ = a function of time chosen so that the model provides an exact fit to the initial term structure, usually interpreted as a time-varying mean,

$a(t)$ = reversion parameter (can be modeled as time-varying),

$\sigma(t)$ = volatility parameter (can be modeled as time-varying).

In practice, the reversion and volatility parameters are assumed to be time-independent, otherwise the over-parameterization of the model can result in unacceptable assumptions about the future evolution of volatilities. The time-varying mean-parameter ($\theta(t)$) is used to fit the initial term structure exactly. When $f(r)=r$, the resultant model is the HW model (also referred to as the extended-Vasicek model)

$$dr = \left[\theta(t) - ar\right]dt + \sigma dz \tag{22}$$

$f(r)=ln(r)$ leads to the BK model

$$d\ln r = [\theta(t) - a\ln r]dt + \sigma dz \tag{23}$$

The volatility parameter, σ, determines the overall level of volatility, while the reversion parameter, a, determines the relative volatilities of long and short rates. The probability distribution of short rate is Gaussian in the HW model[14] and lognormal in the BK model.

The HW model is analytically tractable, with the time t price of a discount bond maturing at time T given by

$$P(t,T) = A(t,T)\exp(-B(t,T)r) \tag{24}$$

where

$$B(t,T) = \frac{1}{a}[1-\exp(-a(T-t))] \tag{25}$$

and

$$\ln A(t,T) = \ln\frac{P(0,T)}{P(0,t)} + B(t,T)f(0,t) - \frac{\sigma^2}{4a}(1-\exp(-2at))B(t,T)^2 \tag{26}$$

As derived in Appendix B, the futures price of an interest rate equals its expected future price in a risk-neutral world, i.e.

$$F(t,T_1,T_2) = \widetilde{E}_t[R(T_1,T_2)] \tag{27}$$

Now

$$R(T_1,T_2) = -\frac{1}{T_2-T_1}\ln P(T_1,T_2) = -\frac{1}{T_2-T_1}\ln A(T_1,T_2) + \frac{1}{T_2-T_1}B(T_1,T_2)r(t) \tag{28}$$

For the HW model

$$\widetilde{E}[r(t)] = f(0,t) + \frac{\sigma^2 B(0,t)^2}{2} \tag{29}$$

Substituting for $lnA(t,T)$ and $E[r(t)]$ yields

$$\widetilde{E}[R(T_1,T_2)] = f(0,T_1,T_2) + \frac{B(T_1,T_2)}{T_2-T_1}\left[B(T_1,T_2)\left(1-e^{-2aT_1}\right) + 2aB(0,T_1)^2\right]\frac{\sigma^2}{4a} \tag{30}$$

where the second term represents the convexity differential between futures and forwards.

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[14] The Gaussian rate assumption in the HW model admits the possibility of negative interest rates, but the resulting tractability allows the analytic derivation of convexity adjustment. Moreover, the probability of negative interest rates is sufficiently small for reasonable values of the volatility parameter, and the convexity adjustment is not sensitive to the possibility of negative rates.

In the case of the BK model, the convexity bias is estimated numerically by pricing the futures off the interest rate tree.

4 Data

The data for this study consists of daily Eurocurrency futures and interest rate swap rates over 10 years (1987-1996) for the four major markets - US Dollar (USD), British Pound Sterling (GBP), German Deutschemark (DEM), and Japanese Yen (JPY). All the rates and prices are closing, midmarket quotes (average of bid and ask quotes). The swap rates and Eurocurrency futures prices were extracted from DataStream. To check the quality of data, Eurocurrency futures prices were also obtained from the Futures Industry Association, and no significant differences or outliers were found in the two data sets. The swap rates were randomly checked with historical quotes maintained by Bloomberg Financial Markets, and again there were no significant differences between the two[15]. The fixed-for-floating interest rate swaps studied consist of maturities of two, three, four and five years for USD, and maturity of two years for GBP, DEM and JPY[16]. Daily spot one- to three-month LIBOR rates are also taken from DataStream. The methodology for computing swap rates from Eurocurrency futures prices and the swap pricing conventions for the four currencies are described in Appendix C.

4.1 *Futures yield curve construction*

As described in Appendix C, zero coupon bond prices implied from Eurocurrency futures prices are needed to price swaps of arbitrary maturities[17]. Thus, it is essential to construct a Eurocurrency yield curve to price interest rate swaps. The construction of a complete yield curve is also necessitated by the fact that the time periods spanned by Eurocurrency futures contracts do not exactly coincide with the tenor of the swap that has to be priced. Futures contracts mature

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[15] Sun, Sundaresan and Wang (1993) have documented that swap rates obtained from DataStream and Data Resources, Inc. (DRI) are not economically different from each other.

[16] Swaps longer than two year maturity could not be studied for GBP, DEM and JPY due to the poor liquidity of the respective Eurocurrency futures contracts for longer maturities.

[17] The discount functions to value swaps should ideally be obtained using the swap yield curve. However, lack of sufficient data from liquid contracts prevents that from being done. The only publicly available market data are for swap coupons on new, plain-vanilla swaps with maturities of 2, 3, 4, 5, 7 and 10 years. Market values of swaps with odd maturities, non-standard coupons or exotic features are not publicly available. Hence, the prices of swap contracts have to be imputed from the prices of some other actively traded financial instruments for which the prices are competitively determined and publicly available. For this purpose, it is common practice to use 3-month Eurocurrency futures prices to derive the LIBOR term structure, the market for which is deep for maturities going out to at least 5 years.

on fixed dates, once every three months, whereas the spot swap quotations are for contracts maturing on dates which are, in general, different from the futures maturity dates. Thus, it is necessary to interpolate between futures rates. Furthermore, to construct the yield curve out to the expiration date of the first futures contract (i.e. in the 0-3 months maturity range), data from the cash market for Eurocurrency deposits have to be used.

One of the main problems in accurately and concisely representing the Eurocurrency yield curve is the quality and density of interest rate data in the cash and futures markets. The cash market provides good quality data for short term maturities (upto 6 months). The futures market provides longer term data (out to 10 years), but only in intervals of 3 months. Moreover, cash and futures yields on Eurocurrency deposits spanning common maturities do not always coincide. This, coupled with the relative sparsity of futures data, sometimes makes close approximation of the sharp curvature at the front end of the curve difficult.

In this paper, a single yield curve is estimated using data from the cash market out to the first futures expiration date (0-3 months maturity, depending on the date), and then data from the futures market out to 10 years. The high density of cash market data for short term maturities helps define the shape of the Eurocurrency yield curve at the extreme front end. The cubic spline interpolation method[18] is used to define the complete shape of the yield curve.

5 Empirical evidence

Table 1 and figure 2a present the descriptive statistics for market swap rates and the differential between these swap rates and the implied rates calculated from Eurodollar strip rates (referred to, hereafter, as the *swap-futures differential*), for US dollar swaps. In general, the implied swap rates are found to be lower than the market quotes. For example, the average implied swap rate for 5 year USD swaps is lower than the average market rates by 9.31 bp. However, the difference is seen more clearly when the data are segmented into three time periods - 1987-90, 1991-93, and 1994-96. This segmentation is primarily based on the interest rate environment during these years, which is shown in figure 2. During the first sub-segment, i.e., 1987-90, interest rates were relatively high, but stable. In this period, it can be seen that the term premium on swaps was relatively low, with a remarkably low volatility in swap rates (0.5-0.6 bp for all swap maturities). On the other hand, the volatility in the swap-futures differential measured by its standard

[18] Cubic spline interpolation has been extensively used in term structure modeling, starting with McCulloch (1971).

deviation is the highest in this sub-period for two-year swaps (4 bp), and is relatively high for three year swaps (2.8 bp). The very low value of the differential in this period (on average less than 1 bp) suggests, *prima facie*, the pricing of swaps right on the Eurodollar futures curve (thus implying mispricing due to convexity bias). The second sub-period (1991-93) corresponds to an era of declining interest rates. Swap term-premiums are very high (sometimes over 120 basis points between a 2-year and a 5-year swap), and so is the volatility of swap rates (1.2-1.3 bp). The value of the swap-futures differential is the highest for this sub-period (averaging 8-10 basis points for swaps of various maturities), indicating a trend towards recognition and incorporation of convexity correction in the pricing of swaps (with market quotes being lower than the corresponding rates implied from Eurodollar futures prices). The volatility of the swap-futures differential is also lower during this period. In the third sub-period (1994-96), there is again an increase in interest rates, along with a narrowing of the swap term premium, though the volatility of swap rates is relatively high (about 0.8 bp, primarily due to the high volatility of interest rates in this period). The swap-futures differential is significantly negative (at the 1% significance level) for this sub-period for all maturities (the average differential varying between 6 bp for 2 year swaps to 10 bp for 10 year swaps).

From Table 1, it can be seen that swap rates derived from Eurodollar futures prices are greater than market swap rates for all maturities as well as in every sub-period. However, the differential between these two rates is insignificantly different from zero for the first sub-period (1987-90), while it is significantly negative for the second sub-period (1991-93). During the third sub-period (1994-96), the swap-futures differential is negative and significant at the 1% level for all swap maturities. The mean differential is increasing in swap maturity, which is consistent with the hypothesis that longer dated swaps exhibit greater convexity. The evidence is also consistent with the hypothesis of swaps being priced right off the Eurodollar futures curve during the earlier years, and then being adjusted for convexity in later years. The absolute amount of convexity adjustment, being a function of term structure parameters such as volatility, varies over time. The volatility of the swap-futures differential is uniformly decreasing in the swap maturity, implying higher significance levels for longer dated swaps. This is also consistent with the fact that convexity increases with maturity.

Figures 3-6 present the time series plots of the swap-futures differential for USD swaps of 2, 3, 4 and 5 years maturity. These figures clearly indicate pricing of swaps right on the Eurodollar futures curve until 1991, and then a gradual incorporation of the convexity adjustment (indicated by the negative drift in the swap-futures differential after 1991, followed by stabilization of the market swap rate *below* the rate implied from Eurodollar futures).

Table 2 and figure 2b present similar descriptive statistics for swaps in the pound sterling market. The swap rates and swap-futures differential exhibit a behavior similar to the US dollar market. A lower volatility in swap rates in accompanied by a higher volatility in the differential. Also, the differential in the first two sub-periods is either positive or insignificantly different from zero, while it is significantly negative during 1994-96, indicating incorporation of the convexity adjustment during this period. Figure 7 presents a time series plot of the swap-futures differential for the GBP market, which clearly illustrates mispricing upto 1992, and then incorporation of the convexity adjustment (by way of a negative drift in the differential).

Table 2 and figures 2c, 2d, 8 and 9 present descriptive statistics for swaps in the Deutschemark and Japanese Yen markets[19]. The results for these two markets are similar - there is strong evidence of mispricing of swaps upto 1994, after which there is evidence of incorporation of convexity adjustment in swaps pricing.

This swap-futures differential is too large to be explained by difference in the two markets in terms of bid-ask spreads (which are typically less than 3-4 basis points in the swaps market) or measurement error due to asynchronocity in the exact timings of these quotes (both swaps and futures prices are closing day quotes). Moreover, there is no reason to expect these two factors to bias the differential in any one direction. Before making an adjustment for convexity, we will first attempt to explore alternative explanations for the differential, in order to examine if there are other factors which can potentially explain it.

5.1 *Counterparty default risk*

Swaps, being traded in the over-the-counter market, are exposed to counterparty default risk, while Eurocurrency futures, which are exchange-traded, are virtually free of default risk because of the implicit guarantee of the contracts provided by the clearinghouse and backup margins/collateral. If there is a significantly higher default risk in swaps, then the pricing of swaps should incorporate it, and the market swap rate should be higher. However, as first observed by Litzenberger (1992), the default risk of interest rate swaps is significantly mitigated due to the following reasons:

[19] The analysis for DEM and JPY swaps is from 1991 onwards, due to lack of Eurocurrency contracts of 2 yr maturity prior to 1991, which prevents swap replication using Eurocurrency futures.

1. Since a swap involves the exchange of only *net* interest payments, the amount of cash flow at risk is much lower than for a bond of comparable maturity.
2. No principal payments are exchanged so that potential default is confined to differential coupon payments.
3. Many swaps have netting provisions, which stipulate that in the event of default, the counterparties settle all contracted liabilities (even though they may not be due yet). This reduces the cash flow at risk even further. Furthermore, a firm's probability of defaulting on a swap is much lower than that on a bond, as the former probability reflects the joint probability of the firm being financially distressed and the swap having negative value to the firm.
4. Swaps with lower credit quality counterparties are often collateralized, so that the potential loss in the event of default is reduced.

Hence, the default risk of swaps is significantly lower that that of a bond of comparable maturity. However, it is still marginally higher than the default risk in futures contracts, since the latter are marked-to-market each trading day (and are, therefore, subject to the guarantee of the clearing house and the posting of margins). To examine if this differential default risk can explain some part of the variation in the swap-futures differential, we present the estimates of five regression models:

$$\text{I.} \quad Swap\ Diff_t = a + b(TED\ Spread)_t + \varepsilon_t, \tag{31a}$$

$$\text{II.} \quad \Delta\ Swap\ Diff_t = a + b(\Delta\ TED\ Spread)_t + \varepsilon_t, \tag{31b}$$

$$\text{III.} \quad \Delta\ Swap\ Diff_t = a + b(\Delta\ AAA\text{-}Govt)_t + \varepsilon_t, \tag{31c}$$

$$\text{IV.} \quad \Delta\ Swap\ Diff_t = a + b(\Delta\ BBB\text{-}AAA)_t + \varepsilon_t, \tag{31d}$$

$$\text{V.} \quad \Delta\ Swap\ Diff_t = a + b(TED\ Spread)_t + c(\Delta\ AAA\text{-}Govt)_t + d(\Delta\ BBB\text{-}AAA)_t + \varepsilon_t, \tag{31e}$$

In the first two models, the TED spread (Treasury Eurodollar spread - measured as the difference between 3-month Eurodollar time deposit rates and 3-month treasury rates) is used as an explanatory variable to explain the variation in the swap-futures differential (defined as the difference between market swap rates and the implied swap rates computed from Eurodollar futures prices). Treasury-bills are free from default risk, but Eurodollar deposits are based on the LIBOR rates. When the banking industry as a whole does poorly, LIBOR rates increase significantly relative to treasury-bills, thereby increasing Eurodollar deposit rates. Thus, TED spread tends to widen in times of financial crisis and tighten in periods of stability, and is widely regarded as a measure of aggregate default risk in the economy, at the *short-end* of the term structure.

We examine next the impact measures of default risk at the *long-end* of the term structure. In regression model *III*, the *AAA-Govt* spread (defined as the spread between monthly averages of the AAA bond yields and the long term government bond yields reported by Standard & Poor's) is used as the explanatory variable, while in regression model *IV*, the *BBB-AAA* spread (defined as the spread between monthly averages of the BBB and the AAA bond yields reported by Standard & Poor's) is the explanatory variable. Both of these variables are aggregate measures of corporate default risk at the long-end of the term structure. The *AAA-Govt* spread is a measure of "one-sided" corporate default risk because Government bonds (in the home currency) are default-free. The *BBB-AAA* spread is used as a measure of "two-sided" default risk in the corporate sector. These spreads tend to increase when the corporate sector default risk goes up. Hence these spreads are presumed to capture the corporate default risk effects. Model *V* is estimated as a multivariate regression with first differences of the three variables (*TED Spread, AAA-Govt,* and *BBB-AAA*) used to explain the variation in the swap-futures differential. This multivariate test controls for the interaction between the independent variables in order to examine their combined effect, if any, on the swap-futures differential.

In Table 3, Panel A, the significantly positive slope coefficients, *b*, are consistent with the default risk hypothesis - a widening TED spread would imply greater risk of default, thereby implying higher values for the swap-futures differential, as defined. The time series for the swap-futures differential and the TED spread are then tested for non-stationarity. The null hypothesis of non-stationarity fails to be rejected for the TED spread series and for the swap-futures differential time series over all maturities. Hence, because both of these time series are nearly integrated over time, the second regression model is estimated using first differences. It can be seen from table 3, Panel B that, although significant, the absolute values of the slope coefficients are less than 10% (for regression model II), which implies that a 10 basis point increase in TED spread corresponds to a less than 1 basis point increase in the swap-futures differential. More importantly, the adjusted R^2 values are less than 5%, implying that the regression model explains only a very small part of the day to day variation in the swap-futures differential. Therefore, while the regression models are statistically significant, they are of low economic significance.

Similarly, the results of regression models *III* and *IV* are reported in table 3, Panels C and D respectively. Due to non-stationarity of the time-series, these two models are also estimated using first differences. The results for model *III* are insignificant for all swap maturities, indicating that unilateral corporate default risk does not explain any variation in the swap-futures differential. The results for model *IV* are statistically significant (except for swaps of 3 year maturity), but the slope coefficients are less than 5% and the adjusted R^2 values around 4%. The results for the multivariate test (presented in table 3, Panel E) suggest some significance for the coefficient of the *BBB-AAA* spread, but the adjusted R^2 values are even lower than those for the univariate test. Hence corporate default risk proxies (individually, as well as in combination with an aggregate economy-wide default risk proxy) also fail to explain the variation in the swap-futures differential.

Therefore, default risk does not explain the differential between market and futures-implied swap rates.

5.2 *Information asymmetries in swap and Eurocurrency futures markets*

Eurocurrency futures are contracts traded on organized exchanges. Due to very high trading volumes and open interest, these markets are closely scrutinized by traders and investors. Hence, these markets could be assumed to be reasonable efficient, in terms of current information being reflected in prices. The swap market, on the other hand, is an OTC market with lower volumes (especially for longer dated swaps) and less standardization. Hence, it is conceivable that the swap markets may not be as informationally efficient as the Eurocurrency futures markets. Therefore, timing differences in information flow across the two markets may explain the differential between market swap rates and swap rates implied from Eurocurrency futures prices. If changes in one rate can predict future changes in the other rate, then the information relevant for future interest rates is not being simultaneously incorporated in the swap and futures markets.

In the first part of the analysis, we examine whether changes in the swap rate implied from Eurocurrency futures prices lead the changes in market swap rates. If this were true, it would imply that Eurocurrency futures markets are informationally more efficient than swap markets. Table 4 presents estimates of the following univariate regressions:

$$\Delta_{\tau+1,\tau} Mkt\ Swap = a_\delta + b_\delta \Delta_{\tau,\tau-\delta} Euro\ Swap + \varepsilon_{\tau,\delta} \quad \delta = 0,1,2,5,10,20 \qquad (32)$$

In this model, daily changes in the market swap rate are regressed on lagged daily changes in implied swap rate. To take care of all possible lagged effects, the regressions are done for lags of 1, 2, 5, 10 and 20 days. Since these days are actual trading days, a lag of 5 days corresponds to a week, while a lag of 20 days corresponds to approximately 1 month. The results in Table 4 for the US dollar markets show that almost all of the information is simultaneously incorporated in the swap and the futures markets. Lag 1 and lag 2 regressions have significant coefficients for all swap maturities, but with much lower slope values (20% and 10% respectively), and very low R^2 values (4% and 2% respectively). Therefore, the predictability in futures rates is economically insignificant. Beyond lag 2, many of the coefficients are not significant, with R^2 values below 1%. Similar inferences can be drawn for the British pound sterling, German Deutschemark, and Japanese yen markets from the results presented in Table 4.

To further test the predictability of market swap rates from implied swap rates, the following multivariate regression model is estimated:

$$\Delta_{\tau+1,\tau} MS = \alpha + \beta_0\, \Delta_{\tau+1,\tau} ES + \beta_1\, \Delta_{\tau,\tau-1} ES + \beta_2\, \Delta_{\tau-1,\tau-2} ES + \beta_3\, \Delta_{\tau-2,\tau-3} ES + \beta_4\, \Delta_{\tau-3,\tau-4} ES + \beta_5\, \Delta_{\tau-4,\tau-5} ES + \varepsilon_\tau \qquad (33)$$

In this model, daily changes in market swap rates are regressed on lagged daily changes (upto a lag of 5 days) in the implied swap rates in a multivariate regression. Table 5 presents the results of this regression for US dollar swaps of 2, 3, 4 and 5 years maturity. It can be seen that the contemporaneous change in the implied swap rate explains nearly all of the variation in the market swap rate. The contemporaneous change has a coefficient close to 1. The lag 1 change in the implied swap rate has a much lower coefficient of 8% - 9% (depending on the maturity of the swap), though it is statistically significant, implying some predictive power in the previous day's change in implied swap rate. However, beyond lag 1, virtually all of the slope coefficients are insignificantly different from zero. These results further reinforce the conclusion that there are virtually no delays in the flow of information from the futures to the swaps markets. Table 5 presents similar multivariate regression results for the other 3 markets (Pound Sterling, Deutschemark, and Yen) as well. In these markets also, the results indicate that there is no significant delay in information flow from futures to the swap markets.

In some markets (especially the Japanese Yen), the swap markets are more liquid and active than the Eurocurrency futures markets. Therefore, there is a possibility that the swap markets may be more informationally efficient than the Eurocurrency futures markets in these currencies. If this is the case, then there would again be a divergence between the market and the implied swap rate.

If there is a significant delay in the flow of information from the swap to the futures markets, then lagged changes in the market swap rate should predict changes in the swap rate implied from futures prices. To test this hypothesis in the US dollar markets, the following multivariate regression model is estimated for swaps of 2, 3, 4 and 5 years maturity:

$$\Delta_{\tau+1,\tau} ES = \alpha + \beta_0 \Delta_{\tau+1,\tau} MS + \beta_1 \Delta_{\tau,\tau-1} MS + \beta_2 \Delta_{\tau-1,\tau-2} MS + \beta_3 \Delta_{\tau-2,\tau-3} MS + \beta_4 \Delta_{\tau-3,\tau-4} MS + \beta_5 \Delta_{\tau-4,\tau-5} MS + \varepsilon_\tau \qquad (34)$$

In this regression, daily changes in the implied swap rate are regressed on lagged daily changes in the market swap rate, going up to a lag of 5 days. The results in Table 6 show that the slope coefficient for the contemporaneous changes in the market swap rate is highly significant, while most of the other slope coefficients (for lagged changes in market swap rate) are not significantly different from zero. Therefore, in the US dollar markets, there is no reason to believe that the swap markets lead the Eurodollar futures markets in incorporating current information in prices. Table 6 also presents results for a similar multivariate regression model for the Pound Sterling, Deutschemark, and Yen markets. Again, there is no significant evidence of market swap rates being able to predict implied swap rates.

The results presented thus far indicate that there is virtually no information asymmetry between the swap and the Eurocurrency futures markets, for all the four currencies analyzed. Therefore, the swap-futures differential observed in these markets cannot be attributed to informational reasons.

5.3 *Term structure effects*

This section of the paper examines whether changes in term structure parameters affect swaps differently from futures, and if this effect can explain changes in the swap-futures differential. In the presence of stochastic interest rates, proxies for changes in the yield curve should be empirical determinants of swap rates. The underlying reasoning is that in an upward (downward) sloping term structure environment, the fixed (floating) rate payer bears more default risk, on average, and hence demands a risk premium through a lower (fixed) rate. Thus, a negative relationship is postulated between the slope of the term structure and the spread between the swap rate and the yield on a Treasury bond of comparable maturity. To examine this effect, the following regression model is estimated:

$$\Delta\ Swap\ Diff_t = \beta_0 + \beta_1\ \Delta\ Level_t + \beta_2\ \Delta\ Slope_t + \beta_3\ \Delta\ Vol_t + \varepsilon_t \quad (35)$$

Daily changes in the swap-futures differential are regressed on the daily changes in the level of interest rates, the slope of the term structure and short term interest rate volatility. 3-month T-bill rates are used for changes in the level of interest rates, the difference (in basis points) between the 5-year and 3-month[20] treasury rates is used as a proxy for the slope of the term structure, and the 60-day historical volatility[21] of the 3-month T-bill rate is used as a proxy for the short-term interest rate volatility. Table 7 presents the results of this regression for USD swaps of 2, 3, 4 and 5 years maturity. Even though the level and the slope coefficients are statistically significant, the very low R^2 values indicate that the regression model does not explain the variation in the swap-futures differential to any satisfactory extent. Hence, changes in term structure parameters cannot explain the observed differential between market and futures-implied swap rates.

5.4 *Liquidity*

The Eurocurrency futures market is a very active market characterized by very high trading volumes. The interest rate swap market is very liquid today, but was not very liquid during the end eighties and early nineties. Therefore, illiquidity in swaps could have increased the market swap rates, so that even if they were convexity-corrected, it is possible that the liquidity premium would raise the swap rate to (or above) the level of the rate implied from Eurocurrency futures prices. The gradual disappearance of this liquidity premium over time would then cause the swap-futures differential to drift below zero. If this was the cause for the observed differential, then a suitable proxy reflecting the liquidity in the interest rate swap market should explain the swap-futures differential.

To test this hypothesis, the following two regression models are estimated:

I. $Swap\ Diff_t = a + b(Bid\text{-}Ask\ Spread)_t + \varepsilon_t$, (36a)

II. $\Delta\ Swap\ Diff_t = a + b(\Delta\ Bid\text{-}Ask\ Spread)_t + \varepsilon_t$, (36b)

[20] The slope is computed only upto 5 year maturity because the swaps considered in this study are for a maximum maturity of 5 years.

[21] A 60 trading day period roughly corresponds to three calendar months. The regression model was re-estimated using different number of days in the time window for computing historical volatility, and the results were not significantly different from those obtained using a 60 day window.

where the bid-ask spread[22] in swap rates is used as a proxy for liquidity in the swap market (a higher bid-ask spread reflects lower liquidity and vice-versa). Since the time series for the swap-futures differential as well for the bid-ask spreads are nearly integrated over time, the second regression model is estimated using first differences. Table 8 presents the results of these regressions for USD swaps of 2, 3, 4 and 5 years maturity. It can be seen that the changes in bid-ask do not explain any variation in the swap-futures differential, when the regression model is estimated using first differences to correct for serial correlation. Therefore, the increasing liquidity of the swap market cannot be the cause for the observed behavior of the swap-futures differential.

The empirical analysis in this section indicates that several alternative explanations for the swap-futures differential - credit risk, information asymmetry, term structure effects and liquidity effects - do not explain much of the variation in the swap-futures differential, although some of the factors are statistically significant. In the following section, we investigate the impact of the convexity adjustment on the differential.

6. Empirical estimation of the convexity adjustment

6.1 *Vasicek and CIR models*

For the Vasicek (OU) process [equation (9) with $\beta=0$], the conditional density of instantaneous rate at any future date t_i, given today's level of instantaneous rate (r_s), is a normal distribution with mean and variance as follows:

$$E[r_{t_i}|r_s] = r_s \exp[-\kappa(t_i - s)] + \mu[1 - \exp[-\kappa(t_i - s)]],$$
$$Var[r_{t_i}|r_s] = \frac{\sigma^2[1 - \exp[-2\kappa(t_i - s)]]}{2\kappa}. \tag{37}$$

Hence, the stochastic differential equation for the Vasicek process can be written as a discrete-time AR(1) process:

$$r_{t_{i+1}} = \exp[-\kappa(t_{i+1} - t_i)]r_{t_i} + \mu[1 - \exp[-\kappa(t_{i+1} - t_i)]] + \varepsilon_{t_{i+1}} \tag{38}$$

where the error term ε is normally distributed with mean 0 and variance given by the equation above. This AR(1) model can be rewritten as the following regression model:

$$r_{t_{i+1}} = a + br_{t_i} + \varepsilon_{t_i} \tag{39}$$

[22] Bid-ask spreads were as high as 12-15 basis points in the swaps market in the 1980s, while they have now reduced to 3-4 basis points.

This equation is estimated using standard OLS procedures, from which the slope coefficient is used to estimate κ and then combined with the intercept to estimate μ. The σ parameter is estimated from the mean squared error of the regression. Throughout the estimation procedure, the market price of risk is assumed to be zero ($\lambda=0$), using the local expectations hypothesis.

For the CIR model, the conditional density of the instantaneous rate has a noncentral chi-square distribution with the mean and variance

$$E[r_{t_i}|r_s] = r_s \exp[-\kappa(t_i - s)] + \mu[1-\exp[-\kappa(t_i - s)]],$$

$$Var[r_{t_i}|r_s] = r_s\left[\frac{\sigma^2}{\kappa}\right][\exp[-\kappa(t_i - s)] - \exp[-2\kappa(t_i - s)]] + \mu\left[\frac{\sigma^2}{2\kappa}\right][1-\exp[-2\kappa(t_i - s)]]^2.$$

.....(40)

The mean of this distribution is the same as that of the OU process, but the variance is now a function of the state variable, and is, therefore, time dependent[23]. Due to this variance structure, the AR(1) estimation procedure has to be modified. The regression model is still the same as in (39), but the error term ε is no longer identically and independently distributed. Since it is a function of the state variable, ordinary least squares (OLS) does not apply. However, it can be viewed as a regression model with heteroskedasticity, which can be estimated using weighted least squares. In the first step, (39) is estimated using OLS, from which estimates of κ and μ are obtained. Since the conditional variance of the errors from this regression must be as specified by (40), a second regression is estimated using squared errors as follows:

$$\varepsilon_t^2 = \beta_0 + \beta_1 r_{t-1} + u_t \qquad (41)$$

The intercept as well as the slope can be used to obtain estimates of the σ parameter. The one that generates estimates more consistent with other studies is chosen between the two. As before, the market price of risk is assumed to be zero during this estimation.

Weekly observations of the 3-month LIBOR rate are used over the period 1975-1996 to estimate the parameters for USD, GBP and DEM. For JPY, the data used is for the 3-month LIBOR rates from 1978-1996. The parameter estimates are reported in Table 9 for the Vasicek and the CIR models. The estimates for USD are very close to those reported by other studies. For the other currencies, no such benchmarks are available, but the parameter values are within reasonable bounds.

[23] This can be observed from the differential equations also. The drift terms in both the equations are the same, but the square-root diffusion is a function of the state variable, while the OU diffusion term is constant.

6.2 *HW and BK models*

The empirical estimation of the HW and BK models is carried out by constructing a trinomial tree for interest rates.[24] LIBOR cash rates upto 1 year maturity and swap rates upto 10 years maturity are used to construct the LIBOR zero curve (upto 10 years maturity) by a bootstrapping procedure. This LIBOR zero curve is used to calibrate the interest rate tree to fit the initial term structure exactly. The volatility parameter σ and the mean-reversion parameter a (defined in equation (21)), are chosen so as to provide a 'best-fit' to the market prices of interest rate caps. The minimization of squared error is accomplished using a non-linear least squares estimation technique.

6.3 *Convexity bias estimation results*

Using these four models, the convexity differential between futures and forwards is presented in figure 10. The resultant impact of this convexity differential on the bias in swap pricing is presented in table 10. The magnitude of the mean spread after 1993 roughly corresponds to the magnitude of convexity adjustment for swaps of respective maturities, which further reinforces the evidence that the observed drift in the swap-futures differential is due to incorporation of convexity adjustment in swap rates. From figure 10, it is evident that different assumptions on the underlying interest rate process have a significant impact on the behavior of convexity adjustment as a function of maturity. The convexity curves are roughly similar for the HW and BK models, in magnitude as well as curvature. The Vasicek model estimates are very close to the HW estimates up to 3 year maturity contracts, but are significantly lower for longer-dated contracts. A closer look at the parameter estimates for these models provides a possible explanation for this behavior. The estimate of mean reversion for the Vasicek (and CIR) model (0.2731) is much higher than that for the HW (0.08) or the BK (0.06) models.[25] A very high mean-reversion would tend to decrease long rate volatility very significantly, which would lead to a much lower impact of maturity on convexity adjustment. The estimates of CIR model are also distorted due to this bias.

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[24] Details of the tree construction methodology can be obtained from Hull and White (1994).

[25] The HW and BK models have time-varying means, which implies that interest rates are modeled as reverting to a time-varying 'target', instead of a constant one. This reduces the dependence on the reversion speed, leading to lower, and possibly more realistic, estimates of the reversion parameter for these models as compared to those for Vasicek or CIR models. This is also a manifestation of the inability of the single factor Vasicek or CIR models to satisfactorily describe the curvature of the entire yield curve.

The results using any of the models suggest that the convexity adjustment can be very large for long dated contracts. For a 10-year futures contract, our calculations suggest that this adjustment can be of the order of 80-100 basis points, which translates to a convexity adjustment of about 35-40 basis points for a 10 year swap (which "averages out" the adjustment for the cash flows of various maturities upto 10 years). Even a conservative estimate of the bias for a 5 year USD swap is about 12-16 basis points, which is significantly higher than the bid-ask spread of 3-4 basis points observed in the swap market. More significantly, the bias is comparable to the observed swap-futures differential, and is in the right direction (i.e. market swap rates are lower than implied rates during the latter years of the study). This supports the hypothesis that the mispricing observed in swap rates during the earlier years of the study (1987-1990) was due to the convexity bias, and that this mispricing has been gradually corrected over time.

6.4 *The relationship between the convexity adjustment and market swap rates*

As a final check on the robustness of the conclusion that the observed spread between market and futures-implied swap rates is primarily a result of the convexity adjustment, the following regression model is estimated:

$$\Delta\ Swap\ Diff_t = (a_0+a_1D_t) + (b_0+b_1D_t)(\Delta\ CA)_t + \varepsilon_t \qquad (42)$$

The convexity adjustment (*CA*) is calculated using the HW model. D_t is a dummy variable that takes the value *0* during the time period 1987-91, and *1* during 1992-96. The dummy variable is used to separate the two distinct pricing regimes - the first one (1987-91) when there is apparent mispricing as convexity adjustments are ignored, and the second one (1992-96) when this mispricing appears to have been corrected. Given that the HW model properly adjusts for convexity, during the second sub-period, a 1 bp change in convexity correction should be reflected in an equivalent 1 bp change in the observed swap-futures differential. However, by the same token, there should not be any significant relationship between convexity changes and the observed differential in the first sub-period. Hence, the null hypothesis is that $b_1=1$ and $b_0=0$. Also, if changes in the convexity are the only factors that affect the changes in the swap-futures differential, the constant terms in equation (42) (a_0 and a_1) should be insignificantly different from zero. Non-zero constant terms that are statistically significant would indicate the presence of other factors that affect the observed swap-futures differential.

Table 11 presents the results of this regression. In Panel A, the model is estimated directly, using OLS. The estimates of the constant terms are significantly different from zero. Hence, this is indirect evidence that the convexity adjustment was not made in the first sub-period. However, estimates of a_0 and a_1 have opposite signs, and the combination of these two results in a constant term close to zero, indicating that, for the second sub-period, the convexity adjustment is fully reflected in the differential. The slope coefficient b_0 is significantly different from zero. The estimates of b_1 are close to 1 only for swaps of 2 years maturity. For swaps of 3, 4 and 5 years maturity, the slope coefficient b_1 is significantly less than 1 (between 0.29 and 0.44), while it should be insignificantly different from 1 as per the null hypothesis. This slope coefficient appears to be attenuated towards zero due to measurement error as well as missing variable bias. Hence the same regression model is estimated in Panel B using the instrumental variables technique, to correct for these biases. The instruments used for this estimation are term structure parameters that are correlated with the convexity adjustments, but uncorrelated with the errors of the original regression model. Using the volatility of interest rates (contemporaneous, lagged by one day, and lagged by two days), level of interest rates, and the slope of the term structure as instruments, the two-stage least squares method[26] is used to estimate the parameters of the model. The results in Panel B show that the IV estimation technique corrects for the biases to a large extent. The estimates of the constant terms (a_0 and a_1) are insignificantly different from zero for all swap maturities. The slope coefficient b_0 is also insignificantly different from zero (except for 2 year maturity swaps). More importantly, the estimates of the slope coefficient b_1 are higher and closer to 1.[27] The results in Panel B clearly show that changes in convexity bias influenced the changes in the swap-futures differential nearly *one-for-one* during the second sub-period (1992-96), while they had an insignificant impact on the differential during the first sub-period (1987-91).

The results indicate that the swap-futures differential observed during the second sub-period is an outcome of the incorporation of convexity adjustments in swap pricing. The broad conclusion of this regression is that the convexity adjustment appears to have been ignored in the earlier period, while in the latter period the adjustment seems to have been made in market swap rates.

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[26] It can be shown that this IV estimator is unbiased and consistent.

[27] The estimate of b_1 is still *significantly* less than 1 for swaps of 4 and 5 years maturity. However, this test is a *joint* test of the convexity estimation model (in this case HW) being correct and the convexity being the only parameter influencing the swap-futures differential. If convexity is generated by a multi-factor term structure model in the real world, this slope coefficient would be different from one even when convexity is the only parameter influencing the swap-futures differential.

7 Conclusion

This paper examines the incorporation of the convexity bias in the pricing of interest rate swaps from 1987-1996, for four major swaps markets - \$, £, *DM* and ¥. Empirical evidence suggests that swaps were being priced using raw futures prices, unadjusted for convexity, during the early part of the sample period. During the later part of the study, market swap rates drift below the swap rates implied from Eurocurrency futures prices. The spread between market and futures-implied swap rates is found to be comparable in magnitude to the theoretical value of the convexity bias estimated using the Vasicek, CIR, Hull and White, and Black and Karasinski term structure models. Alternative hypotheses for this observed swap-futures differential (and the changes in this differential over time) are evaluated using default risk differences, informational asymmetries, term structure effects and liquidity differences between the swap and the futures markets. However, none of these factors offers a satisfactory explanation for this differential. We conclude, therefore, that this is evidence of mispricing of swap rates during the earlier part of the study, with a gradual elimination of this mispricing by incorporation of convexity correction in swap pricing over time.

Our findings suggest that during the end eighties and the early nineties, there was a systematic advantage of hedging a short swap position with short Eurocurrency futures contracts. The reason for this arbitrage was the mispricing of swap rates due to non-incorporation of a convexity correction in the swap curve construction techniques. The arbitrage may have been limited due to market frictions as well as internal or external constraints on bank participation in futures markets. However, the persistence of significant mispricing for several years goes against the conventional wisdom that any pricing discrepancy between two markets would be quickly arbitraged away.

In future work, we plan to investigate further the linkages, if any, between the convexity bias and the liquidity and credit risk factors.

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Table 1

This table presents descriptive statistics for swap rates and the spread between these swap rates and the implied swap rates calculated using Eurodollar futures prices (*diff*), for maturities of two, three, four and five years. The swap rate is expressed in percentage terms. The swap-futures differential, expressed in basis points, is the market swap rate minus the calculated implied rate. Daily data is used from January 1987 through December 1996. Panel A presents figures for the entire time period, while Panels B, C, and D present figures for three economically relevant sub periods, 1987-90, 1991-93 and 1994-96, respectively. For the sub-period 1987-90, 4-year and 5-year differentials could not be calculated due to non-availability of Eurodollar futures data.

	Maturity of Swap							
	2 years		3 years		4 years		5 years	
	swap (%)	diff (bp)	swap (%)	diff (bp)	swap (%)	diff (bp)	swap (%)	diff (bp)
Panel A: Overall (1987-1996)								
Mean	6.97	-4.90	7.27	-6.98	7.50	-8.06*	7.67	-9.31*
Median	6.89	-6	7.16	-8	7.37	-8	7.58	-10
Standard Deviation	1.79	4.50	1.67	4.77	1.58	3.58	1.50	3.93
Minimum	3.92	-21	4.2	-27	4.52	-27	4.77	-28
Maximum	11.02	31	10.79	12	10.67	3.79	10.78	6.12
Panel B: 1987-90								
Mean	8.85	-0.73	9.04	0.62	9.18	-	9.26	-
Median	8.77	0	8.98	1	9.14	-	9.23	-
Standard Deviation	0.67	3.98	0.57	2.78	0.51	-	0.48	-
Minimum	7.1	-16	7.45	-15	7.7	-	7.88	-
Maximum	11.02	31	10.79	12	10.67	-	10.78	-
Panel C: 1991-93								
Mean	5.38	-8.33**	5.92	-9.56**	6.32	-7.95	6.64	-8.83
Median	5.01	-8	5.64	-9.25	6.12	-9	6.49	-10
Standard Deviation	1.28	2.72	1.25	2.49	1.21	4.35	1.18	5.10
Minimum	3.82	-21	4.2	-20	4.52	-27	4.77	-27.9
Maximum	7.97	0	8.26	-1	8.53	3.79	8.71	6.12
Panel D: 1994-96								
Mean	6.20	-6.21**	6.42	-8.21**	6.59	-8.17**	6.72	-9.78**
Median	6.14	-6	6.38	-8	6.57	-8	6.68	-10
Standard Deviation	0.80	2.19	0.78	3.09	0.75	2.57	0.73	2.11
Minimum	4.15	-20	4.55	-27	4.89	-26	5.16	-26
Maximum	8.22	7	8.28	4	8.29	2	8.31	1

*Statistically less than zero at the 5% significance level
**Statistically less than zero at the 1% significance level

Table 2

This table presents descriptive statistics for swap rates and their spread from implied rates calculated using Eurocurrency futures prices in the British Pound (GBP), German Deutschemark (DEM) and Japanese Yen (JPY) markets. The statistics are presented for swap maturities of two years only. The swap rate is expressed in percentage terms, while the swap-futures differential is in basis points. Daily data is used from January 1987 through December 1996 for GBP swaps, while the time period for DEM and JPY swaps is 1991-1996. Panel A presents figures for the entire time period, while Panels B, C and D present figures for three sub-periods.

	GBP Swaps		DEM Swaps		JPY Swaps	
	swap rate (%)	swap-futures differential (bp)	swap rate (%)	swap-futures differential (bp)	swap rate (%)	swap-futures differential (bp)
Panel A: Overall	*(1987-1996)*		*(1991-1996)*		*(1991-1996)*	
Mean	9.32	-4.20	6.51	-2.93	3.40	-6.84
Median	9.475	-5	6	-4	3	-9
Standard Deviation	2.53	4.94	1.98	3.99	1.94	5.38
Minimum	5.05	-15	4	-10	1	-16
Maximum	14.86	19	9.75	6	7.46	9
Panel B: 1987-90						
Mean	11.67	3.58**	-	-	-	-
Median	11.925	4	-	-	-	-
Standard Deviation	1.63	1.24	-	-	-	-
Minimum	8.95	-6.67	-	-	-	-
Maximum	14.86	6	-	-	-	-
Panel C: 1991-93						
Mean	8.60	-2.35	8.02	1.52	4.83	1.34
Median	9.6	-2	8.98	2	4.56	1
Standard Deviation	2.13	4.80	1.53	1.85	1.60	2.48
Minimum	5.05	-15	4.9	-4	1.82	-8
Maximum	12.305	19	9.75	6	7.46	9
Panel D: 1994-96						
Mean	7.10	-7.42**	5.00	-5.21*	1.97	-9.57
Median	7	-7.67	5.06	-6	1.7	-10
Standard Deviation	0.77	2.31	0.97	2.63	0.92	2.60
Minimum	5.195	-15	3.57	-10	0.63	-16
Maximum	8.58	-1.67	6.77	5	3.53	0

*Statistically less than zero at the 5% significance level
**Statistically less than zero at the 1% significance level

Table 3

This table presents estimates of the slope coefficients of the following five regressions:

I. $Swap\ Diff_t = a + b(TED\ Spread)_t + \varepsilon_t$,

II. $\Delta\ Swap\ Diff_t = a + b(\Delta\ TED\ Spread)_t + \varepsilon_t$,

III. $\Delta\ Swap\ Diff_t = a + b(\Delta\ AAA\text{-}Govt)_t + \varepsilon_t$,

IV. $\Delta\ Swap\ Diff_t = a + b(\Delta\ BBB\text{-}AAA)_t + \varepsilon_t$,

V. $\Delta\ Swap\ Diff_t = a + b(\Delta\ TED\ Spread)_t + c(\Delta\ AAA\text{-}Govt)_t + d(\Delta\ BBB\text{-}AAA)_t + \varepsilon_t$,

where *Swap Diff* refers to the difference between market swap rates and the implied swap rates computed from Eurodollar futures prices, *TED Spread* refers to the Treasury Eurodollar spread, computed as the difference between 3 month Eurodollar time deposit rates and the corresponding 3 month T-bill rate, *AAA-Govt* refers to the spread between monthly averages of the AAA corporate bond yields and the long-term Government bond yields reported by Standard & Poor's, and *BBB-AAA* refers to the spread between monthly averages of the BBB and the AAA corporate bond yields reported by Standard & Poor's. Daily data is used to estimate model I. First differences of the same daily data are used to estimate model II, where *Δ Swap Diff* refers to the one-day change in the swap spread and *Δ TED Spread* refers to the one-day change in the TED spread. Models III and IV are estimated using first differences of monthly data. The multiple regression model V is also estimated using first differences of monthly data. Figures in parenthesis are the t-statistics for the estimated slope coefficients.

	Maturity of Swap			
	2 year	3 year	4 year	5 year
Panel A: Regression Model I				
Slope	0.0726	0.1321	0.0941	0.0996
	(19.17)	(22.16)	(8.89)	(10.92)
Adj. R^2	12.9%	20.0%	6.2%	12.1%
Panel B: Regression Model II				
Slope	0.0998	0.0914	0.0829	0.0749
	(9.84)	(10.57)	(7.27)	(6.09)
Adj. R^2	3.7%	5.3%	4.2%	4.0%
Panel C: Regression Model III				
Slope	0.0012	-0.0093	-0.0197	0.0173
	(0.04)	(-0.19)	(-0.39)	(0.32)
Adj. R^2	0	0	0	0
Panel D: Regression Model IV				
Slope	0.0378	0.0247	0.0527	0.0549
	(2.31)	(1.01)	(2.03)	(2.01)
Adj. R^2	3.8%	0.1%	4.2%	4.1%
Panel E: Regression Model V				
TED Spread	0.0239	0.0277	0.0060	-0.0029
	(2.02)	(1.31)	(0.23)	(-0.10)
AAA-Govt	0.0048	-0.0113	-0.0192	0.0172
	(0.16)	(-0.23)	(-0.38)	(0.33)
BBB-AAA	0.0303	0.0199	0.0519	0.0553
	(1.83)	(0.79)	(1.96)	(1.98)
Adj. R^2	5.5%	0	1.7%	1.5%

Table 4

This table presents estimates of the slope coefficient in the following regression model, for the USD, GBP, DEM and the JPY markets:

$$\Delta_{\tau+1,\tau} Mkt\ Swap = a_\delta + b_\delta \Delta_{\tau,\tau-\delta} Euro\ Swap + \varepsilon_{\tau\delta} \qquad \delta = 0,1,2,5,10,20.$$

where Mkt Swap refers to the market swap rate and Euro Swap is the implied swap rate computed from Eurodollar futures prices. $\Delta_{\tau+1,\tau}$ *Mkt Swap* refers to the change in the market swap rate from day τ to day $\tau+1$ and $\Delta_{\tau,\tau-\delta}$ *Euro Swap* is the change in the implied swap rate from day $\tau-\delta$ to day τ. The model has been estimated for swaps of maturities from 2 to 5 years for USD swaps, and 2 years for GBP, DEM and JPY swaps. The data used is daily data, and the sample period is 1987-1996 for USD swaps, 1990-1996 for GBP swaps, and 1991-1996 for DEM and JPY swaps. The Euro Swap rates are lagged by δ days (trading days), hence $\delta=5$ corresponds to a weekly change (in most cases), while $\delta=20$ approximately corresponds to a monthly change. Figures in parenthesis are the t-statistics for the estimated slope coefficients.

	USD Swaps				GBP Swaps	DEM Swaps	JPY Swaps
Lag (δ)	2 year	3 year	4 year	5 year	2 year	2 year	2 year
0	1.0025	0.9844	0.9728	0.9831	0.9726	0.9609	0.9244
	(86.46)	(104.91)	(86.26)	(75.32)	(194.46)	(193.22)	(114.10)
Adj. R^2	75.4%	84.9%	86.5%	87.1%	95.7%	97.0%	94.3%
1	0.2354	0.2332	0.2145	0.2113	0.1094	0.0447	0.0703
	(10.27)	(9.87)	(7.12)	(5.92)	(4.54)	(1.56)	(2.07)
Adj. R^2	4.1%	4.7%	4.1%	3.9%	1.1%	0.1%	0.4%
2	0.0984	0.1080	0.0986	0.1052	0.0465	0.0689	0.0106
	(6.47)	(6.84)	(4.88)	(4.39)	(2.85)	(3.47)	(0.46)
Adj. R^2	1.6%	2.3%	1.9%	2.1%	0.4%	0.9%	0%
5	0.0449	0.0275	0.0176	0.0102	0.0434	0.0234	0.0117
	(4.82)	(2.81)	(1.38)	(0.68)	(4.45)	(1.92)	(0.86)
Adj. R^2	0.9%	0.3%	0.1%	0	1.1%	0.2%	0%
10	0.0224	0.0169	0.0138	0.0098	0.0128	0.0114	0.0115
	(3.44)	(2.43)	(1.47)	(0.88)	(1.89)	(1.35)	(1.15)
Adj. R^2	0.4%	0.2%	0.1%	0	0.2%	0.1%	0%
20	0.0181	0.0040	0.0203	0.0171	0.0137	0.0138	0.0102
	(4.14)	(1.82)	(3.31)	(2.31)	(3.02)	(2.52)	(1.51)
Adj. R^2	0.6%	0.1%	0.8%	0.5%	0.5%	0.5%	0.1%

Table 5

This table presents multiple regression estimates of daily changes in the market swap rate on lagged daily changes in the implied swap rate computed from Eurocurrency futures prices, with lags of 0, 1, 2, 3, 4, and 5 days, for the USD, GBP, DEM and JPY markets. The model estimated is:

$$\Delta_{\tau+1,\tau} MS = \alpha + \beta_0\, \Delta_{\tau+1,\tau} ES + \beta_1\, \Delta_{\tau,\tau-1} ES + \beta_2\, \Delta_{\tau-1,\tau-2} ES + \beta_3\, \Delta_{\tau-2,\tau-3} ES + \beta_4\, \Delta_{\tau-3,\tau-4} ES + \beta_5\, \Delta_{\tau-4,\tau-5} ES + \varepsilon_\tau$$

where MS refers to the market swap rate and ES refers to Euro Swap rates, i.e., swap rates implied from Eurocurrency futures prices. $\Delta_{\tau+1,\tau} MS$ refers to the change in MS from day τ to day $\tau+1$ (and similarly for all the ES variables). Thus the current change in the market swap rate is being regressed on the current as well as lagged changes in the Euro swap rate, going upto a lag of 5 days. The model has been estimated for swaps of maturities of 2, 3, 4 and 5 years for USD markets, and for swaps of 2 years maturity for GBP, DEM and JPY markets. The data used is daily data, and the sample period is 1987-1996 for USD, 1990-1996 for GBP, and 1991-1996 for DEM and JPY markets. Figures in parenthesis are the t-statistics for the respective coefficients.

	USD Swaps				GBP Swaps	DEM Swaps	JPY Swaps
Indep. Var.	2 year	3 year	4 year	5 year	2 year	2 year	2 year
$\Delta_{\tau+1,\tau}$ *Euro Swap*	0.9883 (85.5)	0.9721 (104.56)	0.9636 (87.55)	0.9718 (75.93)	0.9703 (193.54)	0.9585 (194.75)	0.9238 (114.88)
$\Delta_{\tau,\tau-1}$ *Euro Swap*	0.0751 (6.45)	0.0895 (9.51)	0.0828 (7.45)	0.0859 (6.67)	0.0162 (3.21)	0.0177 (3.59)	-0.0085 (-1.05)
$\Delta_{\tau-1,\tau-2}$ *Euro Swap*	-0.0192 (-1.63)	-0.0050 (-0.53)	0.0041 (0.37)	0.0028 (0.22)	0.0052 (1.03)	0.0199 (4.04)	-0.0033 (-0.42)
$\Delta_{\tau-2,\tau-3}$ *Euro Swap*	0.0758 (6.45)	0.0014 (0.15)	-0.0015 (-0.14)	0.0025 (0.19)	-0.0008 (-0.16)	-0.0041 (-0.83)	0.0085 (1.05)
$\Delta_{\tau-3,\tau-4}$ *Euro Swap*	-0.0037 (-0.31)	0.0098 (1.05)	0.0109 (0.99)	0.0109 (0.84)	-0.0056 (-0.91)	0.0007 (0.14)	0.0147 (1.83)
$\Delta_{\tau-4,\tau-5}$ *Euro Swap*	-0.0102 (-0.88)	0.0027 (0.29)	0.0072 (0.65)	-0.0019 (-0.15)	0.0072 (1.44)	-0.0081 (-1.65)	-0.0041 (-0.51)
Adjusted R^2	76.2%	85.6%	87.3%	87.7%	95.8%	97.0%	94.4%

Table 6

This table presents multiple regression estimates of daily changes in the implied swap rate computed from Eurocurrency futures prices on lagged daily changes in the market swap rate, with lags of 0, 1, 2, 3, 4, and 5 days, for the USD, GBP, DEM and JPY markets. The model estimated is:

$$\Delta_{\tau+1,\tau} ES = \alpha + \beta_0 \Delta_{\tau+1,\tau} MS + \beta_1 \Delta_{\tau,\tau-1} MS + \beta_2 \Delta_{\tau-1,\tau-2} MS + \beta_3 \Delta_{\tau-2,\tau-3} MS + \beta_4 \Delta_{\tau-3,\tau-4} MS + \beta_5 \Delta_{\tau-4,\tau-5} MS + \varepsilon_\tau$$

where ES refers to Euro Swap rates, i.e., swap rates implied from Eurocurrency futures prices and MS refers to the market swap rate. $\Delta_{\tau+1,\tau} ES$ refers to the change in ES from day τ to day $\tau+1$ (and similarly for all the MS variables). Thus the current change in the Euro swap rate is being regressed on the current as well as lagged changes in the market swap rate, going upto a lag of 5 days. The model has been estimated for swaps of maturities of 2, 3, 4 and 5 years for USD markets, and for swaps of 2 years maturity for GBP, DEM and JPY markets. The data used is daily data, and the sample period is 1987-1996 for USD, 1990-1996 for GBP, and 1991-1996 for DEM and JPY markets. Figures in parenthesis are the t-statistics for the respective coefficients.

	USD Swaps				GBP Swaps	DEM Swaps	JPY Swaps
Indep. Var.	2 year	3 year	4 year	5 year	2 year	2 year	2 year
$\Delta_{\tau+1,\tau}$ *Euro Swap*	0.7529 (86.95)	0.8609 (104.57)	0.8901 (87.43)	0.8851 (75.29)	0.9843 (193.49)	1.0119 (193.63)	1.0192 (116.89)
$\Delta_{\tau,\tau-1}$ *Euro Swap*	0.0365 (4.21)	-0.0004 (0.04)	-0.0051 (-0.49)	-0.0123 (-1.05)	0.0031 (0.62)	-0.0065 (-1.24)	0.0393 (4.50)
$\Delta_{\tau-1,\tau-2}$ *Euro Swap*	0.0014 (0.16)	0.0004 (0.05)	-0.0064 (-0.63)	-0.0025 (-0.21)	-0.0044 (-0.86)	-0.0185 (-3.52)	0.0038 (0.43)
$\Delta_{\tau-2,\tau-3}$ *Euro Swap*	-0.0249 (-2.87)	0.0005 (0.06)	-0.0034 (-0.33)	-0.0069 (-0.59)	0.0046 (0.91)	0.0033 (0.63)	-0.0110 (-1.27)
$\Delta_{\tau-3,\tau-4}$ *Euro Swap*	-0.0054 (-0.62)	-0.0177 (-2.15)	-0.0168 (-1.65)	-0.0154 (-1.24)	0.0047 (0.92)	0.0003 (0.06)	-0.0108 (-1.24)
$\Delta_{\tau-4,\tau-5}$ *Euro Swap*	0.0088 (1.01)	0.0012 (0.15)	-0.0039 (-0.38)	-0.0055 (-0.47)	-0.0058 (-1.15)	0.0079 (1.52)	-0.0026 (-0.29)
Adjusted R^2	75.6%	84.9%	86.7%	87.0%	95.7%	97.0%	94.5%

Table 7

This table presents multiple regression estimates of daily changes in the swap-futures differential (defined as the difference between market swap rates and the implied rates computed using Eurodollar futures prices) on daily changes in the level of interest rates, the slope of the term structure and short term interest rate volatility. The model estimated is:

$$\Delta\, Swap\ Diff_t = \beta_0 + \beta_1\, \Delta\, Level_t + \beta_2\, \Delta\, Slope_t + \beta_3\, \Delta\, Vol_t + \varepsilon_t$$

where *level* refers to the level of 3-month T-bill rates (changes given in basis points), *slope* is the term structure slope defined as the difference between the 5 year and 3-month treasury rates (changes in basis points), and *Vol* refers to the volatility of short term interest rates (changes in percentage terms), given by the 60-day historical volatility of the 3-month T-bill rate (60 trading day period roughly corresponds to three calendar months). The data used is daily data, and the sample period is 1987-1996. Figures in parenthesis are the t-statistics for the respective coefficients.

	Maturity of swap			
Independent variable	2 year	3 year	4 year	5 year
$\Delta\, Level_t$	-0.3231 (-11.39)	-0.1798 (-6.18)	-0.1475 (-4.89)	-0.1718 (-7.22)
$\Delta\, Slope_t$	-0.2369 (-8.76)	-0.1511 (-6.15)	-0.1556 (-6.72)	-0.1624 (-9.02)
$\Delta\, Vol_t$	42.8301 (1.60)	-33.3686 (-1.33)	-18.905 (-0.78)	-40.2145 (-1.64)
Adjusted R^2	5.9%	2.9%	4.6%	11.8%

Table 8

This table presents estimates of the slope coefficients of the following two regressions:

I. $\quad Swap\ Diff_t = a + b(Bid\text{-}Ask\ Spread)_t + \varepsilon_t$,

II. $\quad \Delta\ Swap\ Diff_t = a + b(\Delta\ Bid\text{-}Ask\ Spread)_t + \varepsilon_t$,

where *Swap Diff* refers to the difference between market swap rates and the implied swap rates computed from Eurodollar futures prices, and *Bid-Ask Spread* refers to the spread between Bid and Offer rates for swaps. Daily data is used to estimate model I. First differences of the same daily data are used to estimate model II, where *Δ Swap Diff* refers to the one-day change in the swap-futures differential and *Δ Bid-Ask Spread* refers to the one-day change in the Bid-Ask Spread. Figures in parenthesis are the t-statistics for the estimated slope coefficients.

	Maturity of Swap			
	2 year	3 year	4 year	5 year
Panel A: Regression Model I				
Slope	1.3309	2.3152	2.7558	1.9929
	(15.74)	(24.01)	(13.14)	(10.19)
Adj. R^2	9.1%	22.7%	12.7%	10.7%
Panel B: Regression Model II				
Slope	0.2599	-0.0764	-0.1841	-0.2407
	(3.58)	(-1.44)	(-0.95)	(-1.45)
Adj. R^2	0.5%	0.1%	0%	0.1%

Table 9

This table presents parameter estimates of the Vasicek and CIR models

$$dr = \kappa(\mu - r)dt + \sigma^{\beta} dz$$

where $\beta=0$ implies the Vasicek model, and $\beta=½$ implies the CIR model. The Vasicek model is estimated using the following discretized AR(1) process:

$$r_{t_{i+1}} = \exp[-\kappa(t_{i+1} - t_i)]r_{t_i} + \mu[1 - \exp[-\kappa(t_{i+1} - t_i)]] + \varepsilon_{t_{i+1}}$$

The CIR model is estimated using a two step weighted least squares approach. In the first step, the discretized AR(1) process defined above is estimated. In the second step, squared errors from the first step are regressed on lagged values of the spot rate to estimate the σ parameter as follows:

$$\varepsilon_t^2 = \beta_0 + \beta_1 r_{t-1} + u_t$$

where

$$\beta_0 = \mu\left[\frac{\sigma^2}{2\kappa}\right][1 - \exp[-2\kappa\Delta t]]^2,$$

$$\beta_2 = \left[\frac{\sigma^2}{\kappa}\right][\exp[-\kappa\Delta t] - \exp[-2\kappa\Delta t]].$$

The models are estimated for all the four currencies (USD, GBP, DEM and JPY). The data consists of weekly observations of the 3-month LIBOR rate (taken from DataStream), from 1975-1996 for USD, GBP and DEM, and from 1978-1996 for JPY. Using the local expectations hypothesis, the market price of risk is assumed to be zero ($\lambda=0$).

	Parameter			
	κ	μ	σ	
			Vasicek	CIR
USD	0.2731	0.0738	0.0265	0.1756
GBP	0.5546	0.0981	0.0312	0.1528
DEM	0.1726	0.0508	0.0146	0.0907
JPY	0.2456	0.0460	0.0218	0.1394

Table 10

This table presents indicative estimates of convexity adjustments in USD swaps of various maturities, using four different term structure specification - Vasicek, CIR, Hull & White (one-factor) and Black-Karasinski. The convexity adjustment is expressed in basis points, and is estimated for one particular (representative) term structure environment. The actual convexity adjustment would vary depending upon the volatility and other factors.

Maturity Of Swaps (yrs)	Convexity Adjustments			
	Term Structure Model			
	Vasicek	CIR	Hull-White	Black-Karasinski
2	3.7	9.2	3.7	1.8
3	6.6	16.6	7.1	3.6
4	9.6	24.1	11.3	6.0
5	12.5	31.3	16.0	8.8

Table 11

This table presents estimates of the slope coefficients of the following regression model for the US dollar markets:

$$\Delta\, Swap\ Diff_t = (a_0 + a_1 D_t) + (b_0 + b_1 D_t)(\Delta\, CA)_t + \varepsilon_t\,,$$

where *Swap Diff* refers to the difference between market swap rates and the implied swap rates computed from Eurodollar futures prices, *D* is a dummy variable which takes the value *0* for the period 1987-1991 and the value *1* for the period 1992-96, and *CA* refers to the estimated values of convexity adjustment for various swap maturities using the one-factor Hull and White model. Daily data is used to estimate the model, using first differences. Panel A presents the results for the given regression model. Panel B presents the results for the given model estimated by the instrumental variables technique, using the term structure parameters (level, volatility - contemporaneous, lag 1& lag 2, and slope) as instrumental variables. Figures in parenthesis are the t-statistics for the estimated slope coefficients.

	Maturity of Swap			
	2 year	3 year	4 year	5 year
Panel A				
a_0	1.0783	0.9767	1.9328	0.9081
	(4.99)	(2.80)	(2.68)	(0.43)
a_1	-1.3262	0.7166	-1.6127	-2.1473
	(-4.07)	(1.56)	(-2.14)	(-0.99)
b_0	-0.2815	-0.1403	0.6325	-0.6063
	(-4.03)	(4.84)	(6.71)	(-2.91)
b_1	1.0576	0.4432	0.3294	0.2943
	(9.01)	(4.84)	(3.35)	(1.39)
p-value (b_1=1)	0.623	0	0	0
Adj R^2	51.9%	59.8%	59.0%	55.7%
Panel B: Instrumental Variables Estimation				
a_0	0.1726	0.3001	2.3089	4.0364
	(0.51)	(0.49)	(1.41)	(1.77)
a_1	-0.7427	-1.6523	-0.7858	-2.1631
	(-1.75)	(-2.41)	(-0.47)	(-0.93)
b_0	-0.4862	-0.1158	-0.2467	-0.3282
	(-4.43)	(-1.01)	(-1.24)	(-1.65)
b_1	1.4688	1.1126	0.8279	0.8567
	(9.54)	(8.27)	(4.09)	(4.21)
p-value (b_1=1)	0.002	0.402	0.395	0.481
Adj R^2	50.9%	55.7%	57.3%	55.1%

Figure 1

This figure presents the profit and loss on a swap position, Eurodollar futures position as well as the net profit/loss on a combined position for changes in the forward rate.

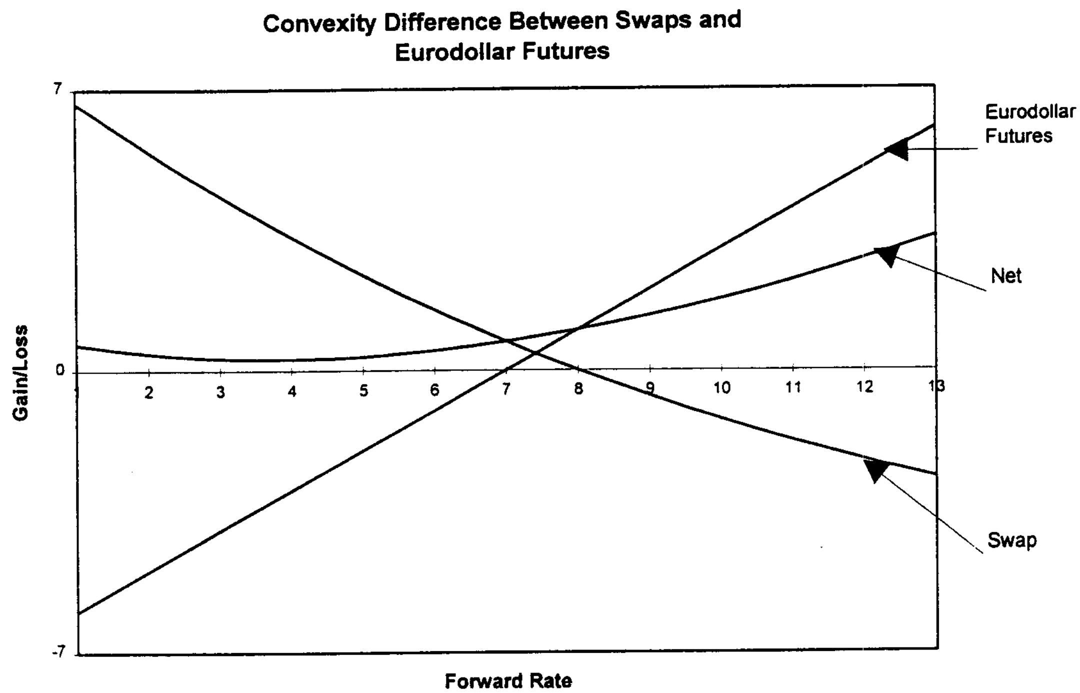


Figure 2a

This figure presents 2, 3, 4 and 5 year USD swap rates. The data used are daily data from 1987-1996.

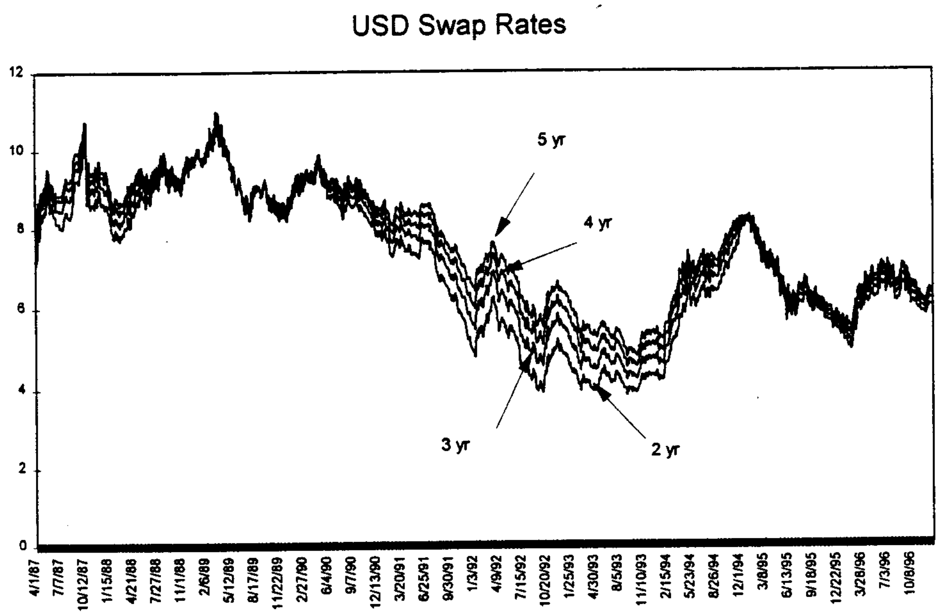


Figure 2b

This figure presents 2 year GBP swap rates. The data used are daily data from 1987-1996.

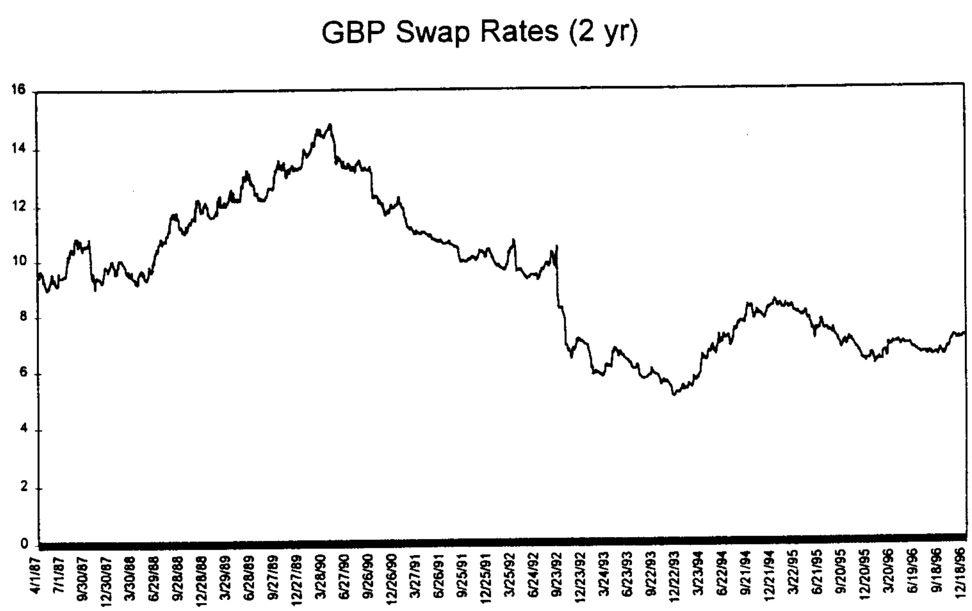


Figure 2c

This figure presents 2 year DEM swap rates. The data used are daily data from 1991-1996.

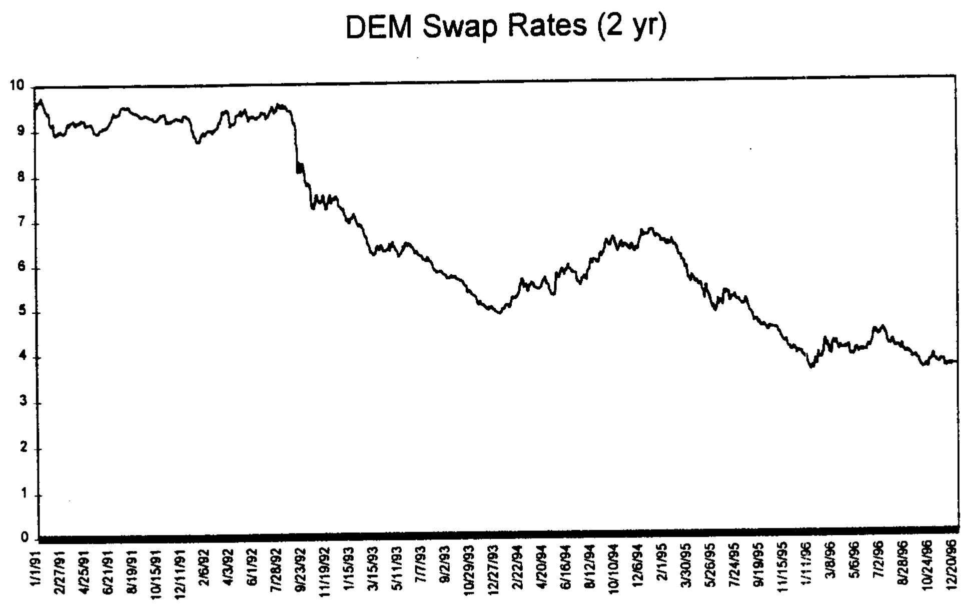


Figure 2d

This figure presents 2 year JPY swap rates. The data used are daily data from 1991-1996.

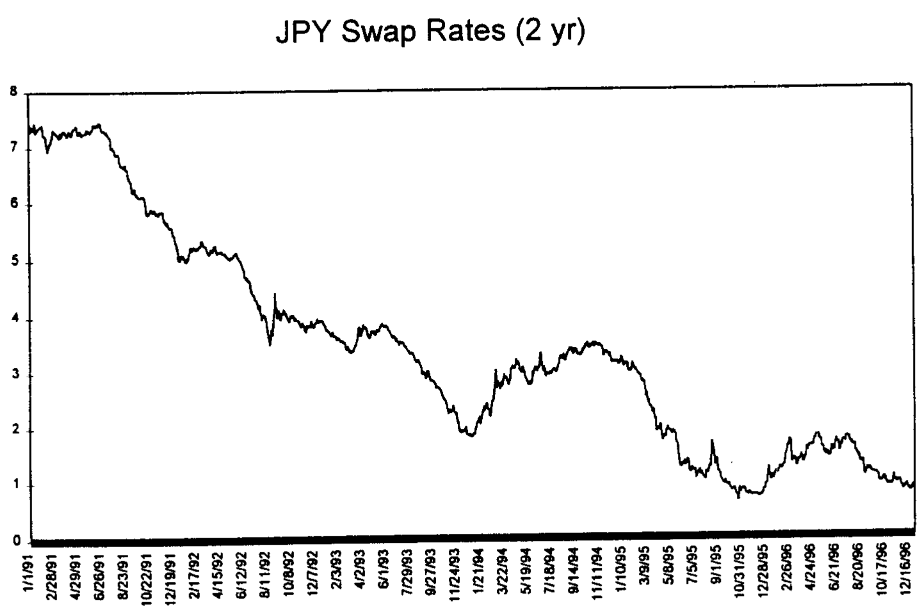


Figure 3

This figure presents the spread between market swap rates and implied swap rates computed from Eurodollar futures prices, for 2 year USD swaps. Daily data is used from 1987-1996.

USD Swaps (2 yr)

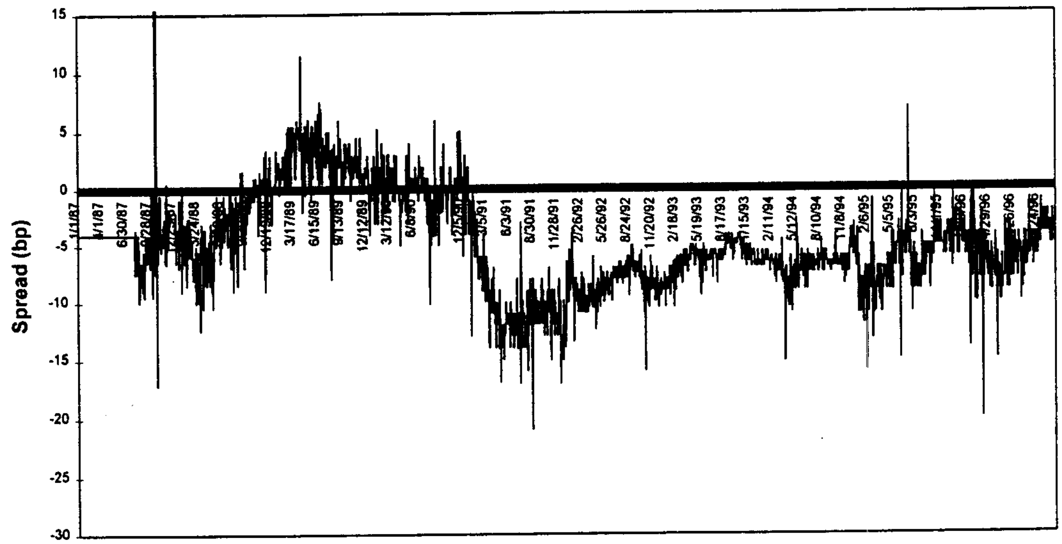


Figure 4

This figure presents the spread between market swap rates and implied swap rates computed from Eurodollar futures prices, for 3 year USD swaps. Daily data is used from 1987-1996.

USD Swaps (3 yr)

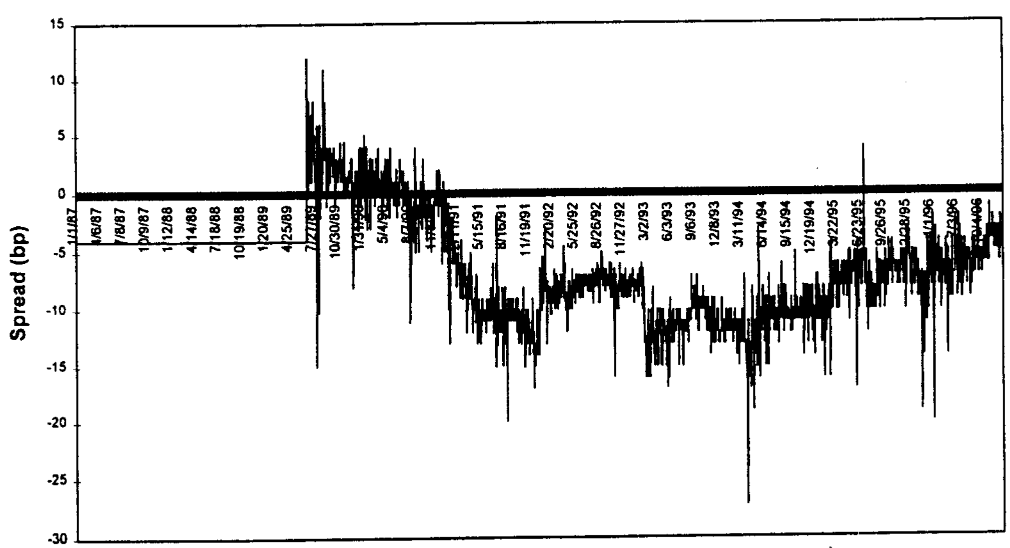


Figure 5

This figure presents the spread between market swap rates and implied swap rates computed from Eurodollar futures prices, for 4 year USD swaps. Daily data is used from 1987-1996.

USD Swaps (4 yr)

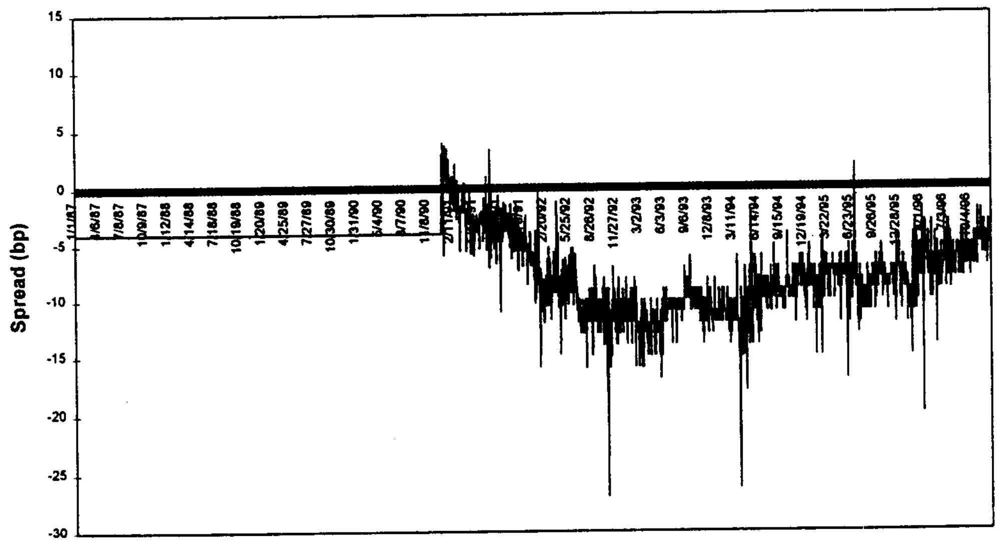


Figure 6

This figure presents the spread between market swap rates and implied swap rates computed from Eurodollar futures prices, for 5 year USD swaps. Daily data is used from 1987-1996.

USD Swaps (5 yr)

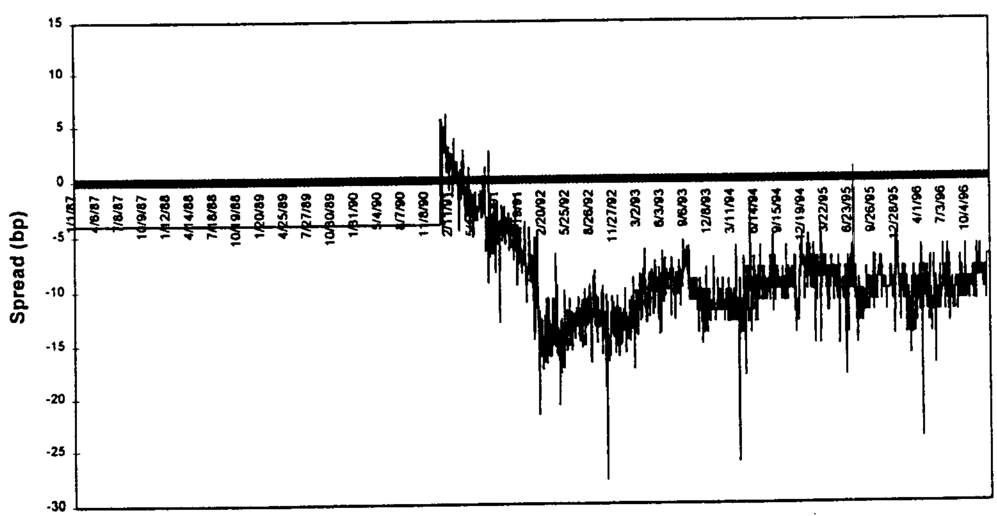


Figure 7

This figure presents the spread between market swap rates and implied swap rates computed from Eurocurrency futures prices, for 2 year GBP swaps. Daily data is used from 1987-1996.

GBP Swaps (2 yr)

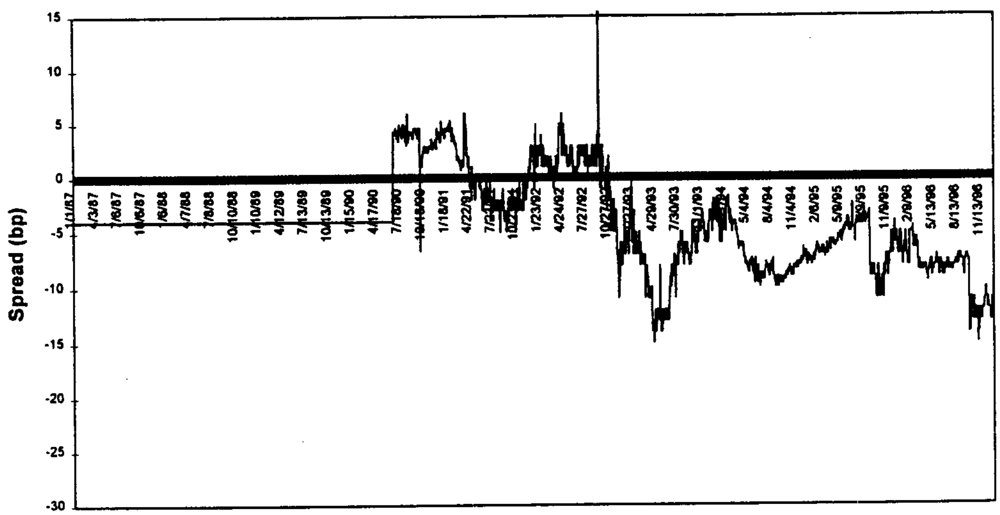


Figure 8

This figure presents the spread between market swap rates and implied swap rates computed from Eurocurrency futures prices, for 2 year DEM swaps. Daily data is used from 1991-1996.

DEM Swaps (2 yr)

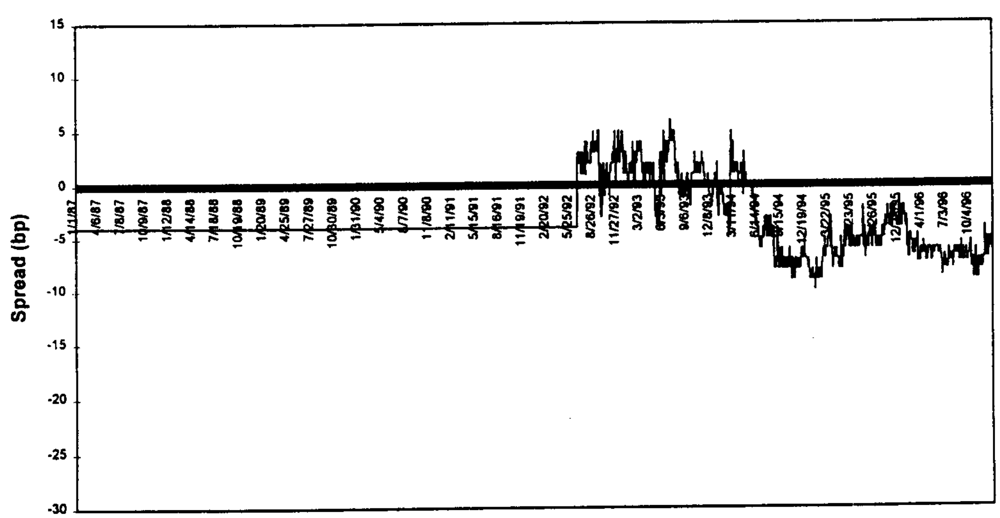


Figure 9

This figure presents the spread between market swap rates and implied swap rates computed from Eurocurrency futures prices, for 2 year JPY swaps. Daily data is used from 1991-1996.

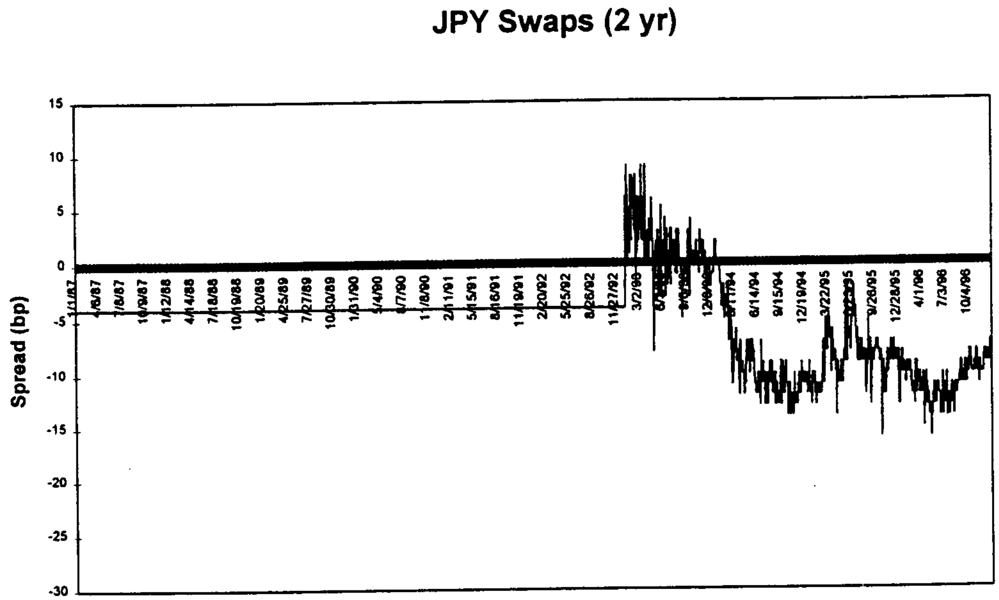
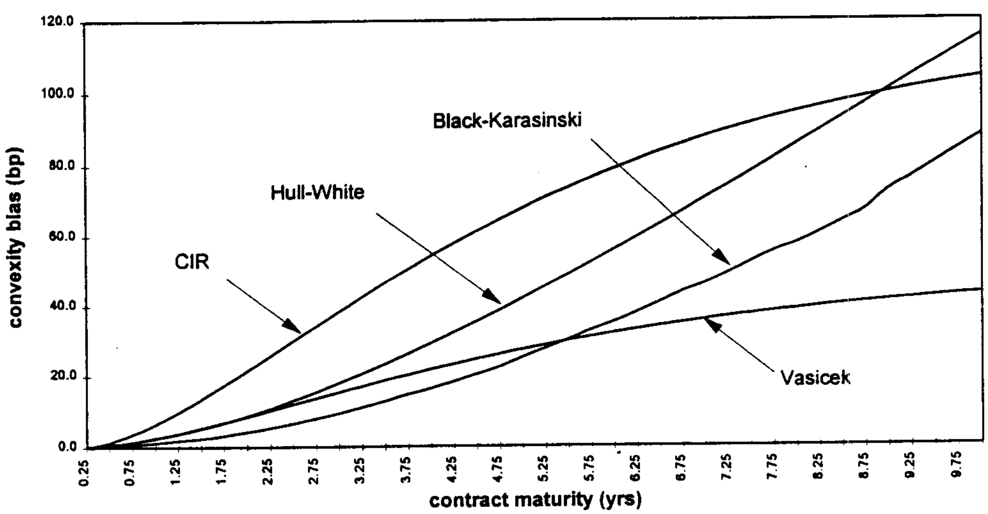


Figure 10

This figure presents estimates of convexity differential between futures and forward contracts upto maturity of 10 years, using four different term structure models - Vasicek, CIR, Hull and White, and Black and Karasinski.

Convexity Bias using different term structure models



Appendix A

Structure of the Eurodollar Futures Market

Eurodollar futures contracts are futures written on 3-month Eurodollar time deposits. The contracts settle to the 3 month LIBOR, which is the yield implied by the 3 month Eurodollar time deposit (the underlying asset). The market for Eurodollar futures is one of the largest futures markets for a financial instrument. Eurodollar futures are the most liquid futures contracts ever to be traded in the United States. Due to this reason, they are regarded as the best hedge against changes in future interest rates. They are traded at the International Monetary Market (IMM) at Chicago, London International Financial Futures Exchange (LIFFE), Singapore International Monetary Exchange (SIMEX) and the Tokyo International Financial Futures Exchange (TIFFE).

Four different Eurodollar futures contracts are traded for each calendar year, with settlement dates being the third Wednesday of the maturity month, which can be March, June, September of December. The maturity date of the underlying Eurodollar time deposit is 3 calendar months after the settlement date. The futures contract settles by cash, without any timing or delivery options to either the short or the long side. They provide for daily mark-to-market repricing. The maturity of the futures contracts extends upto 10 years for Eurodollar futures traded on the IMM. Each contract is for a deposit with a principal value of $ 1 million. The price of each contract is quoted as a number under 100 with two digits after the decimal point, and it equals 100 minus the 3 month LIBOR rate.

Eurodollar futures are quoted on a 360 day basis. The minimum price movement is a basis point ($ 0.01), which is equivalent to a $ 25 change in the value of the futures contract ($ 1 million * 0.0001 * 90/360) irrespective of its maturity.

Appendix B

Theoretical difference between forward and futures prices

General contingent claims valuation theory can be used to show that forward and futures prices are different under stochastic interest rates. The following notation is used to theoretically derive the difference between forward and futures prices:

$f(t,T_1,T_2)$	= price at time t of a forward contract on a zero-coupon bond maturing at time T_2 for delivery at T_1,
$F(t,T_1,T_2)$	= price at time t of a futures contract on a zero-coupon bond maturing at time T_2 with expiration date T_1,
$P(t,T)$	= price at time t of a zero-coupon bond maturing at time T,
$r(t)$	= instantaneous riskless rate at time t,
$B(t)$	= value of a money market account at time t.

Assuming complete markets and no-arbitrage, the forward price is given by

$$f(t,T_1,T_2) = \frac{P(t,T_2)}{P(t,T_1)}$$

The forward price can be interpreted as the future value at time T_1 of a T_2 maturity bond.

The futures price can be evaluated using risk-neutral valuation in a backward inductive manner. At time T_1-1, the value of the futures contract is given by

$$\tilde{E}_{T_1-1}\left(\frac{P(T_1,T_2) - F(T_1-1,T_1,T_2)}{B(T_1)}\right)B(T_1-1) = 0$$

by the definition of the futures contract.

Since the value of the money market account $B(T_1)$ is known at time T_1-1 and the futures price at expiration is the spot price, i.e.

$$F(T_1,T_1,T_2) = P(T_1,T_2)$$

we get

$$F(T_1-1,T_1,T_2) = \tilde{E}_{T_1-1}\left(F(T_1,T_1,T_2)\right)$$

Hence the time T_1-1 futures price is the time T_1-1 expectation of its value at time T_1. Next, at time T_1-2, we get

$$\widetilde{E}_{T_1-2}\left(\frac{F(T_1-1,T_1,T_2)-F(T_1-2,T_1,T_2)}{B(T_1-1)}\right)B(T_1-2)=0$$

which yields the expression

$$F(T_1-2,T_1,T_2)=\widetilde{E}_{T_1-2}\left(F(T_1-1,T_1,T_2)\right)$$

Continuing the same argument inductively backwards in time gives the final result

$$F(t,T_1,T_2)=\widetilde{E}_t\left(F(t+1,T_1,T_2)\right)$$

Therefore, futures prices are martingales under the risk-neutral measure. Using the law of iterated expectations, this martingale property implies that

$$F(t,T_1,T_2)=\widetilde{E}_t\left(P(T_1,T_2)\right)$$

i.e. the futures price is the time t expectation of the underlying T_2 maturity zero-coupon bond's price at time T_1.

We can now relate forward and futures prices. The forward prices can be alternatively written as

$$f(t,T_1,T_2)=\frac{P(t,T_2)}{P(t,T_1)}=\widetilde{E}_t\left(\frac{P(T_1,T_2)}{B(T_1)}\right)\frac{B(t)}{P(t,T_1)}$$

therefore

$$f(t,T_1,T_2)=\widetilde{E}_t\left(P(T_1,T_2)\right)\widetilde{E}_t\left(\frac{1}{B(T_1)}\right)\frac{B(t)}{P(t,T_1)}+\operatorname{cov}_t\left(P(T_1,T_2),\frac{1}{B(T_1)}\right)\frac{B(t)}{P(t,T_1)}$$

which yields the final expression

$$f(t,T_1,T_2)=F(t,T_1,T_2)+\operatorname{cov}_t\left(P(T_1,T_2),\frac{1}{r(t)....r(T_1-1)}\right)\frac{1}{P(t,T_1)}$$

The forward price equals the futures price plus a convexity adjustment term. This additional term reflects the covariance between the T_2 maturity zero-coupon bond's price and the spot rates over the time period $[t,T_1]$. When interest rates are deterministic, or when the future spot price is known with certainty, this covariance term is zero, and hence forward and futures prices are identical.

Appendix C

Calculating Implied Swap Rates from Eurocurrency Futures Strip Rates

A Eurocurrency strip is a series of futures contracts with successive expiration dates. The buyer of a strip locks in a rate of return for a term equal to the length of the strip. For example, a 1-year strip would consist of contracts with four successive expirations, locking in a one year term rate. Thus, strip rates can be used to compute zero-coupon bond prices. As an illustration, the 6-month zero-coupon price on futures expiration date would be given by:

$$B_6 = \frac{1}{(1+\frac{F_1 d_1}{360})(1+\frac{F_2 d_2}{360})}$$

where

B_k is the zero-coupon bond that matures in 'k' months

F_i is the futures rate (100 - futures price) in the Eurocurrency strip, i=1,2,.....,n

d_i is the actual number of days in the 'i'th period, i=1,2,......,n

In general, there would be a need to price swaps on any date, which would normally not fall on a futures expiration date. In this case, zero-coupon bond prices are computed using an initial spot rate and the Eurocurrency strip as follows:

$$B_T = \frac{1}{(1+\frac{R_0 d_0}{360})(1+\frac{F_1 d_1}{360})*.......*(1+\frac{F_T d_T}{360})}$$

where

B_T is the price of the zero-coupon bond maturing at date T

R_0 is the spot LIBOR to the first futures expiration date

d_0 is the number of days to the first futures expiration

Once zero-coupon bond prices are computed, pricing a swap is a straightforward exercise. Considering a swap as a long position in a floating rate note and a short position in a fixed rate note, and assuming the hypothetical floater to trade at par, the fixed rate on the swap is set so that the hypothetical fixed rate note would also trade at par. For a swap of maturity 'n' years with semi-annual reset, the coupon would be calculated as:

$$\frac{C}{2} * B_{½} + \frac{C}{2} * B_1 + ...+(\frac{C}{2}+100) * B_n = 100$$

$$\Rightarrow \qquad C = \frac{2 * 100 * (1 - B_n)}{\sum_{k=1}^{2n} B_{k/2}}$$

where

C is the swap coupon

B_i are the zero-coupon bond prices for maturity of 'I' years

Swap Pricing Conventions

	Currency			
	USD	**GBP**	**DEM**	**JPY**
Day Count	30/360	Actual/365	30/360	Actual/365
Payment Frequency	semi-annual	semi-annual	annual	semi-annual
Floating Leg	6-month USD LIBOR	6-month Sterling LIBOR	6-month DEM LIBOR	6-month Euroyen LIBOR