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An Asset Allocation Puzzle: When is A Puzzle Not A Puzzle?

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AN ASSET ALLOCATION PUZZLE:

WHEN IS A PUZZLE NOT A PUZZLE?

by

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In a recent article in this journal, Canner, Mankiw and Weil (CMW) argue that in general popular investment advice, and in particular the investment advice of four investment advisors, is inconsistent with modern portfolio theory and is irrational. As CMW state, since portfolio theory is so well known and easy to implement, finding irrationality here has serious implications for the assumption of rationality throughout economics. As part of their analysis, CMW assert that investment advice can only be viewed as rational if the ratio of bonds to stocks either remains constant or increases as the investor seeks higher return (takes on more risk). As evidence of advisor irrationality, CMW point out that investment advisors frequently advocate decreases in the ratio of bonds to stocks to obtain higher returns. They state, "Although we cannot rule out the possibility that popular advice is consistent with some model of rational behavior, we have so far been unable to find such a model."[1]

In this comment we show that in the absence of a riskless asset the rationality test of CMW is not a result of the efficient set mathematics of Modern Portfolio Theory (MPT), but rather that their results require both data with particular properties and the assumption that unrestricted short sales are allowed. If short sales are allowed the efficient set mathematics must result in the ratio of bonds to stocks monotonically changing as risk increases, but the monotonic relationship can be either decreasing or increasing depending on the inputs used. If short sales are not allowed, the ratio of bonds to stock must be decreasing for the high range of expected returns as risk increases.

Furthermore, we show that under relevant assumptions the specific advice given by the investment advisors is consistent with modern portfolio theory (MPT). As part of their analysis CMW analyze the portfolio problem when short sales are forbidden but riskless lending and borrowing exists and when a riskless asset does not exist but short sales are allowed. We argue that the realistic problem involves no short sales and no riskless lending and borrowing (in real terms), that this is the problem the advisors are solving, and that their advice is rational in this

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[1] Canner et al (1997), page 181.

framework.

We proceed in three steps. We first review some of the basic tenets of MPT, discuss alternative formulations of the problem, and examine which of these alternatives is appropriate for the problem being analyzed. Then, using efficient set mathematics, we show that CMW's criteria to test for rationality do not hold. Finally we show that the specific asset allocations proposed by each of the investment advisors selected by CMW are consistent with MPT under realistic estimates of inputs to the portfolio optimization problem.

I. Modern Portfolio Theory

The simplest form of modern portfolio theory represents the investor's problem in mean return, standard deviation space as a choice among efficient portfolios (minimum standard deviation for any expected return). Depending on the investor's tolerance for risk, a different efficient portfolio will be selected from among those in the efficient set. The shape, composition and characteristics of the efficient frontier depend on assumptions about the existence of a riskless asset and whether investors can short sell risky assets or not. Variations in these assumptions lead to different models for determining the efficient frontier. We will briefly review these alternative models and the assumptions that make each one correct. We will then examine the advisors' recommendations to see which model is consistent with their recommendations.

If investors can risklessly lend and borrow at the same rate, then the separation theorem holds and the investor's problem is to find the optimal mix of the riskless asset and the optimal risky portfolio. Investors with different risk tolerances simply hold different percentages of the riskless asset and the risky portfolio and the relative proportions invested in each of the risky assets remains constant. The separation theorem holds whether short sales of risky assets are allowed or disallowed.

If a riskless asset does not exist, the important assumption affecting the characteristics of the efficient frontier is whether short sales are allowed or not. If short sales are allowed, the two fund theorem holds and all portfolios on the efficient frontier are a linear combination of any two other efficient portfolios.[2] Assume we observe three efficient portfolios. If short sales are allowed, then the proportions invested in each risky asset for any of the three portfolios is a constant linear combination of the proportions held in the other two. All assets are held in positive or negative proportions except that for each asset there is a maximum of one efficient portfolio where it is held with zero weight.

If short sales are not allowed, the nature of the efficient frontier changes. The two fund theorem no longer holds. Securities enter and leave the efficient frontier at different risk return tradeoffs. The points where they enter or leave are called corner portfolios. Securities may be held in zero weight for a range of risk tolerance and some assets are never held. Generally, the maximum return portfolio on the efficient frontier will consist of one asset and the minimum risk portfolio will consist of multiple assets. Efficient portfolios are linear combinations of two other efficient portfolios only if all three portfolios lie at or between adjacent corner portfolios.

In order to choose among the alternative models, we must first decide on whether the analysis is performed in terms of real or nominal returns and then decide on whether short sales are allowed or not. First consider the choice of real or nominal returns. The choice rests on which of these assumptions is most relevant for investment advice to an investor saving for expenditures in the future (e.g., retirement). It is clear that all of the investment advisors quoted by CMW are concerned with the recommended portfolios meeting needs in the far future. The

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[2] See Black (1972). The discussion of short sales allowed follows the definition of short sales used by CMW, namely that the proceeds are fully available for investment in other assets. A more realistic characterization of short sales that restricts their use (the Lintner definition; see Lintner (1965)) results in an efficient frontier with characteristics more like the one described for no short sales. Under Lintner's definition the two fund theorem no longer holds.

price of goods and services in the future is uncertain. The investor cares about how many loaves of bread he or she can consume, not how many dollars he or she will have. CMW recognize this and perform their analysis in real terms. If we accept that the analysis should be performed in real terms, then even T-bills become risky and the analysis must be performed in the absence of a riskless asset.[3]

This leaves us with a choice of whether short sales as described by CMW should be allowed or not. CMW discuss the investment advice of Fidelity, Merrill Lynch, Jane Bryant Quinn and the *New York Times.*[4] The investment advisors are concerned with the allocation across a stock portfolio, a bond portfolio and a money market portfolio. What are these portfolios? For Fidelity it is certainly the mutual funds of these types they offer. For Merrill Lynch the portfolios can be mutual funds or bond or stock accounts. Finally, Quinn explicitly discusses mutual funds as the investment vehicle. For open-end funds and managed accounts, short sales were not possible at the time of the advice.[5] Even if we assume mutual funds or managed accounts could be sold short, there is a second problem. The definition of short sales used by CMW assumes that the investors can short sell at no cost, and that the full proceeds are available for investment in other assets.[6] Since CMW use the T-bill rate as a proxy for a money market account, this requires investors to borrow at the same rate as the U.S. Government. For a

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[3] More recently, with the advent of inflation index-linked bonds, something approximating a real riskless asset does exist. However, such a bond did not exist at the time of the analysis in CMW.

[4] The Merrill Lynch recommendations are in Underwood and Brown (1993). The Fidelity recommendations are in Mark (1993).

[5] The only possibility of short sales of asset classes is for large closed end funds. For obvious reasons, closed end money market funds don't exist, and this is the primary asset class short sold in the CMW analysis.

[6] Lintner (1965) recognized this and reformulated portfolio theory under more realistic assumptions about the use of proceeds from short sales. While Lintner's definition of short sales leads to an alternative proof of the CAPM, two fund separation does not hold under his definition of short sales.

stock or bond managed account, CMW's analysis requires that investors short sell an entire asset category (which may be their whole managed account) at no cost, and have the entire proceeds available for investment. In short, CMW assume the investor, by employing short sales, can create portfolios of extreme risk and return through this costless leveraging of starting capital. But investors cannot do this. Individual investors pay a high fee for short sales, cannot short sell an entire managed portfolio, and the use of funds that arise from short sales of individual securities is restricted by brokerage firms.[7] Thus an assumption of no short sales is the most realistic assumption. The most relevant portfolio problem for investors saving for the long term and choosing among commingled funds involves formulating the problem in real returns with no riskless lending and borrowing and no short sales.

This leaves us with a question as to what the investment advisors cited by CMW are assuming. Both Merrill Lynch and Jane Bryant Quinn present long discussions of the fact that investors should be concerned with what their savings can purchase, not how large their savings are. Quinn either presents historic data in real terms, or when she presents data in nominal terms she includes the rate of inflation in the same table. All authors are concerned with long term savings policy. In addition, the analysts' recommendations are inconsistent with riskless lending and borrowing, since as CMW point out, the asset composition changes as the recommendation changes. Both the discussion and analysis of the investment advisors quoted by CMW suggest that they are analyzing the asset allocation problem in real terms.

We also believe that the investment advisors are dealing with the case where short sales are not allowed. None of the advisors show portfolios that involve the short sales of any assets. Furthermore, several of the financial advisors explicitly suggest that the assets be held in the form of managed accounts or mutual funds, neither of which can be sold short. In addition, for all but one of the four authors cited by CMW, one of the three portfolios is *not* a linear combination

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[7] Insofar as the assets are held in the form of pension funds, e.g., 401K plans, there are additional legal and self-imposed constraints on short sales.

of the other two. This can only happen if short sales are not allowed. Next, the recommendation of Merrill Lynch has 5% in cash for two different recommendations. This can only happen if short sales are not allowed. Finally, Quinn's recommendation for an aggressive investor involves 100% in common stock. There are no reasonable assumptions about cash and bonds (except they are identical) that would result in their both being held in zero amounts in a particular efficient portfolio if short sales are allowed. Thus Quinn's recommendations are only consistent with no short sales.

In summary, it seems reasonable to assume that the most appropriate model to use for investors allocating money across a money market fund, a bond fund and a stock fund and saving for the distant future, is one assuming no riskless asset and no short sales of risky assets. Furthermore, this is in fact the only form of the model that is consistent with the advice of the four investment advisors discussed by CMW.

II The CMW Rationality Test

CMW argue that as a test of rationality, the ratio of bonds to stocks should rise as an investor's risk tolerance increases. In this section we will examine the rationality criteria both for the framework CMW use (short sales allowed) and the framework we feel is the one used by the advisors. We show that a) when unrestricted short sales are allowed an increase is not a result of efficient set mathematics but is rather an artifact of the particular estimates of expected returns, variances and covariances which CMW use; and b) in the case of short sales not allowed it is impossible for a continuously rising bond to stock ratio to be rational over the entire range of risk tolerance.

A. Short Sales Allowed

If short sales are allowed, the two fund separation theorem holds and the proportion invested in any asset is linear when plotted against expected return (or risk). This means that the

ratio of bonds to stocks must be monotonic when plotted against the proportion invested in stock. However, as proved in the appendix, it can be monotonically increasing or decreasing. For the particular inputs CMW use, it is monotonically increasing as shown in Figure 1-A.[8]

While there are many sets of historic data and subjective estimates of the future for which the ratio is monotonically increasing, there are also many sets of historic data and plausible estimates of the future for which it is monotonically decreasing. For example, if CMW had used as estimates of the future the history of real monthly returns over the final five years of their sample (more recent data), they would have found that the ratio of bonds to stocks was monotonically decreasing. We present this input data in Table 1-A and the relationship between the ratio of debt to equity and the fraction of the portfolio in stock in Figure 1-B. In Section III we develop sets of input data that are consistent with the recommendation of Fidelity, Jane Bryant Quinn, and Merrill. These are shown in Tables 1-B to 1-D. In Figures 1-C, D and E we have graphed the relationship between the debt to equity ratio and the fraction of the portfolio in common stock for data consistent with the recommendations of these three advisors when short sales are allowed. Notice that for Fidelity and Jane Bryant Quinn the ratio of bonds to stocks is monotonically decreasing, while for Merrill it is monotonically increasing.

The fact that with short sales allowed the relationship between the ratio of bonds to stocks and the fraction of stock in the portfolio can be monotonically increasing or decreasing depending on the choice of input data means that the shape of this relationship cannot be used as a test for rationality when short sales are allowed.

B. Short Sales Not Allowed

When short sales are forbidden, the efficient set mathematics are more complicated.

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[8] CMW's analysis results in bonds being sold short at low risk levels and T-bills at high risk levels. In fact, at the high risk levels T-bills must be sold short in amounts in excess of the initial capital the investor has. The assumption here is that investors can borrow unlimited sums of money at the same rate and with the same risk as the U.S. government.

Efficient portfolios are only mixtures of adjacent efficient portfolios between corner portfolios. Thus only between corner portfolios need there be a monotonic relationship. Each corner portfolio is an inflection point and changes the slopes of the lines plotting asset proportions versus the expected return of the efficient portfolio.

However, there are some general statements we can make about the bond-to-stock ratio and the proportion in stocks. As long as stocks have the highest expected return, the ratio of bonds to stocks must decrease for high regions of expected return. This follows since if short sales are not allowed and stocks have the highest expected return, the highest return efficient portfolio is 100% in common stocks. If the curve is monotonic it must decrease over its entire length. The most common shapes for the relationship are humped (increasing ratio of bonds to stocks followed by a decreasing ratio as risk increases) and a continuously decreasing curve.

CMW, in their Table 2, present the set of data which they used to construct their efficient frontier. Employing this data to solve for the efficient frontier with no short sales and no riskless lending and borrowing leads to the portfolio composition graphed in Figure 2. We can see from this figure that with the data employed by CMW, an investor who, starting with a low return portfolio and seeking a higher return, first increases the ratio of bonds to stock as CMW state. But to obtain still higher returns, the investor substitutes stock for bonds, decreasing the ratio of bonds to stocks until the rate falls to zero. Thus, even using the data of CMW, the downward sloping ratio of bonds to stocks of the advisors is not evidence of inconsistencies with MPT as CMW assert. In particular, over the high risk segment which CMW discuss, it is downward sloping, and in fact it must be downward sloping for any input data where stocks offer the highest expected return. Using the input data consistent with the recommendations of the advisors shows that with no short sales the Merrill estimates result in a relationship like CMW while for Jane Bryant Quinn and Fidelity the ratio of bonds and stocks declines monotonically throughout. These relationships are shown in Table 2. Thus, for many plausible sets of inputs, there isn't any range of risk where the ratio of bonds to stocks increases.

III Consistency with Specific Recommendations

To examine the appropriateness of the specific recommendations of each of the advisers, we need estimates of mean returns, variances of returns and correlation coefficients. An examination of the reports of the advisors makes it clear they are concerned with the risk and return that will arise in the future from asset allocation across a diversified portfolio of assets and more specifically a money market fund, a stock fund and a bond fund.[9] CMW use as a proxy for the future return on these portfolios the return from 1926 to 1992 for the S&P Index, the long term bond index, and the 30-day Treasury bill index as constructed by Ibbotson. As we will show shortly, the use of Ibbotson data exactly reproduces the recommendations of one of the advisors quoted by CMW. However, the Ibbotson data starting in 1926 is not the only, or necessarily even the best, data to use for forecasts of the future behavior of capital markets. Forecasts can differ from the history presented in Ibbotson for several reasons. First, the portfolios being analyzed by the investment advisors may be poorly represented by the Ibbotson indexes. Second, forecasters may employ more recent market data to modify simple extrapolation of the past.

The fact that some of the investment advisers are using estimates other than those obtained from using Ibbotson indexes over the full time period is easily demonstrated. If they had all been using exactly the same estimates as input to the portfolio selection process, they would not have made recommendations that are inconsistent with each other.

We start by discussing why investment advisors may choose indexes other than the Ibbotson indexes employed by CMW to represent fund returns. While the S&P Index is the most commonly used measure of the return on large stock portfolios, several of the advisors recommend portfolios including smaller stocks and foreign stocks. The risk and return characteristics of these portfolios will differ from the S&P Index. Ibbotson long term bond index

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[9] For example, Mark Lary (1993).

is computed using the monthly return on a single bond with a maturity of approximately 20 years. Bond funds hold portfolios of bonds with a much shorter maturity than 20 years and this affects the distribution of returns. The shorter maturity and the diversification of a bond fund makes the risk of bond funds much less than the Ibbotson long bond index. If one wishes to use Ibbotson data to replicate the portfolio of a typical bond fund, a mixture of 80% of their intermediate government and 20% of their long term corporate bond index does a better job than the index employed by CMW (see Elton, Gruber, Das and Hlavka (1993)). The opposite is true for money market funds. The 30 day T-bill rate likely understates the risk on money market mutual funds. Actual money market funds generally invest longer and in more risky instruments than the 30 day T-bill in an attempt to outperform short-term government debt. Nevertheless, Ibbotson data is commonly used as a first step in aggregate asset allocation planning, though as discussed above other indexes are often used instead.

Keep in mind that whether using the Ibbotson data or other indexes of performance, historical estimates may be only a starting place. The analyst may well modify historical values to reflect beliefs about how current conditions differ from the past. If historical data is being used, it is necessary for an analyst to select an observation interval. The most commonly used interval to measure returns is monthly.

In inferring the data on which analysts based their recommendations, we have a serious problem. We can never determine exactly what a particular analyst used for estimating parameters because there are an infinite number of estimates that can produce a given set of recommendations.[10] Thus, we can never be sure we are using the same inputs the advisor used. What we can, and will, do is to show that there exist sets of reasonable inputs such that the

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[10] The input parameters that can be selected are mean returns (3), standard deviation of returns (3), correlations (3), and risk tolerances (3), or a total of 12 parameters. The output is the proportions for each recommendation. Since the proportions invested in each type of security add to one, this leaves six variables to solve for (two proportions for each of the three recommendations of the advisors). While the proportions must be positive, there are an infinite number of solutions.

advice of each financial analysts discussed by CMW is consistent with modern portfolio theory using real returns and no short sales.

The easiest recommendations to replicate are those presented by Merrill Lynch. Given the date of publication, Merrill Lynch most likely observed data through 1992.[11] To obtain the Merrill Lynch recommendations we assume that the historic monthly joint distribution of real returns (from 1926 to 1992) is predictive of future returns. As a first step in analyzing the Merrill Lynch recommendations we followed the procedure of CMW and assumed that the appropriate proxies were the Ibbotson, S&P Index, long term bond index and 30 day Treasury index. The input data computed from Ibbotson is shown in Panel B of Table 1.

Based on this data, we can solve for the efficient frontier. Since the Merrill Lynch guide discusses in great detail the importance of holding some cash in a portfolio, and since the recommendation showed no less than five percent in cash for the most aggressive portfolio, we solved for a set of efficient portfolios with the constraint that cash not fall below 5% of the portfolio.[12] The ratio of bonds to stocks as a function of the proportion in stocks is presented in Panel A of Table 2.

The three Merrill Lynch recommendations discussed by CMW are marked by x's in Table 2.[13] While the ratio of bonds to stocks rises and then falls for the input data consistent with Merrill Lynch for the risk return levels reported by Merrill Lynch, the ratio of bonds to stocks falls as the amount in stock increases. The Merrill Lynch recommendations are totally consistent

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[11] We also computed data ending in 1991 and 1993. The numbers are essentially the same.

[12] Merrill Lynch states that there are at least three reasons why an investor should always hold a portion of the portfolio in cash: in case of emergencies, to take advantage of opportunities, and to pay taxes.

[13] We, like the four analysts discussed by CMW, rounded investment weights to the nearest 5%. Our results replicate the recommendations when they are rounded to the nearest 5%.

with employing inputs calculated from the full history of Ibbotson and employing modern portfolio theory. The principal difference between our analysis and CMW is that we assume no short sales and they assume unrestricted short sales are allowed.

In examining the recommendations of Jane Bryant Quinn and Fidelity, it was necessary to make more sophisticated assumptions about the data they were using. After all, if they had simply used historic Ibbotson data from 1926 on they would have gotten exactly the same results as Merrill Lynch, and the same results as each other. Fortunately, the authors' discussion which accompanies their recommendations gives us an idea of what data the authors may have been using.

First consider the recommendations of Jane Bryant Quinn. In Panel C of Table 1 we present one set of inputs which are reasonable given the timing of Jane Bryant Quinn's book and which lead to her recommendations. Her book has a 1991 publication date. However, the data quoted in the book ends in 1989. Given production time and her data, it is likely that her analysis was done in early 1990.

Portfolio management requires the analyst's best estimate of future values. Analysts start with history and modify it. Jane Bryant Quinn examines Ibbotson data and in her book puts more emphasis on recent data. Analysts need to develop estimates of means, standard deviation, and correlation coefficients. However, the one type of input where they are most likely to accept historic data without modifying it is in estimating future correlation coefficients. Since Quinn relies on Ibbotson data, and given that Ibbotson recommends using twenty years of data to compute correlation coefficients, correlation coefficients were calculated from twenty years of Ibbotson data ending in December 1989. These are the numbers shown for correlation coefficients in Panel C of Table 1. The remaining numbers are one set of values of expected return and standard deviation of return which are consistent with Quinn's results. The expected stock return, the standard deviation of stock returns and the standard deviation of bond returns are at or close to their historical values. The mean return on bonds, the T-bill return and the T-bill standard deviation are further from historical values. Are the deviations from historical values for

these inputs reasonable in early 1990?

In early 1990, long-term government yields were about 9% and corporates yielded about 1% more.[14] Inflation for the year 1990 was 6.1%. If yields are used as a proxy for expected return, the estimate of 3% a year or .24% per month employed for bond funds in Table 1 seems quite reasonable. The expected real return for Treasury bills is significantly higher than the historic mean calculated from 1926 on. However, real returns on Treasury bills for the more recent years prior to Quinn's forecast were much higher. Whether one used 1, 5, 10 or 20-year holding periods, the historical mean return was close to or above the estimate used to produce Quinn's recommendations. For example, T-bills offered a real return of about twenty-five basis points per month over the ten years starting in 1980, and offered a real return of 14 basis points in the year in which Quinn prepared her forecast. The estimate of 12 basis points per month is reasonable given the long-term history of Treasury bill returns and their more recent performance.

The final number that is different from its historical value is the standard deviation of Treasury bills. The number we used is much higher than historic values. There are two reasons why this is reasonable. Money market funds do not invest exclusively in 30 day Treasury bills. Their actual portfolio is generally of longer duration and includes riskier assets. Second, compared to history, real returns on Treasury bills were more uncertain in 1990. Thus, assuming the standard deviation of real returns on money market funds was much higher than the historical standard deviation of 30 day Treasury bills was very rational in 1990. This data produces the results shown in Panel B of Table 2. The ratio of bonds to stocks falls throughout including the three recommendations discussed by CMW and marked by x's. We have shown that Quinn's recommendations are consistent with modern portfolio theory and a sensible set of estimates. Again, the key difference between our analysis and CMW is that we assume short sales are not

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[14] We fully recognize yields need not be expected returns. However, assuming that the expected price change on bonds is zero and bonds are at par (the assumptions needed for expected return to be equal to yield) is quite common.

allowed.

The third set of recommendations presented by CMW is that of Fidelity. Fidelity is making recommendations concerning proportions in three types of Fidelity funds (stock funds, bond funds and money market funds). The Ibbotson index chosen by CMW to represent bond funds is the Ibbotson Long Term Bond Index. Since Fidelity's recommendations are clearly concerning allocation across mutual funds, we used indexes that better represent returns on bond funds. As discussed earlier, an 80/20 mix of the Ibbotson Intermediate Term Government Bond Index and the Long Term Corporate Index is a good proxy for a typical bond mutual fund.

The date given for the Fidelity recommendations is Winter 1993. Therefore, to get an estimate of the correlation matrix we used monthly data from the beginning of available data through December 1992, where the bond index is an 80/20 mix. This is the data shown in Panel D of Table 1. Once again we modified mean returns and standard deviation of returns. We modified the historical data to reflect the nature of the market investment being recommended by Fidelity. The large changes involved the forecasts for T-bills. The mean return on T-bills is again well above the mean return using the full data set and close to the one used for Quinn. This is appropriate given the recent history of real returns on T-bills at the time of the forecast. The estimate of T-bill standard deviation consistent with Fidelity recommendations is higher than the historical average. This is also appropriate given the duration and security risk characteristics of money market funds relative to T-bills. The mean return on the bond proxy is increased slightly, 1 bp over historical levels (using an 80/20 mix). The mean return on stocks is increased by 20 bp. This increase reflects the higher return of non-S&P stocks and the tendency of mutual funds to hold non-S&P shares in 1992. Finally, the standard deviation for stocks is the historical standard deviation while for bonds it is only slightly higher than historic, given an 80/20 mix. With the data that produces the Fidelity recommendations the bond-stock ratio falls throughout (see Panel C, Table 2 marked by x's), and CMW's rationality test fails.[15] Once again, with a sensible set of

[15] We do not discuss the *New York Times* numbers for two reasons. First, the framework is unclear. Second, the recommendations seem to be a consensus of people who were

assumptions the recommendations of an advisor are obtained using standard mean variance analysis.[16]

CONCLUSION

CMW, in a recent article in the AER, declare that the advice given by the investment community in general, and a set of investment advisors in particular, is inconsistent with modern portfolio theory and is irrational. In particular they single out the fact that the financial community advocates a decrease in the ratio of bonds to stocks as investors exhibit less risk aversion as evidence of irrationality. Their test for rationality is not a result of the efficient set mathematics, but rather the particular data they use. If short sales are allowed, the ratio of bonds to stocks must either monotonically increase or monotonically decrease, depending on the estimates used. Thus, changes in the ratio of bonds to stocks is not a useful rationality test.

If short sales are forbidden, then using their data the bond-stock mix first rises and then falls, and in general must fall over higher risk portfolios. We argue that short sales not allowed is the relevant case for advisors making a recommendation for allocation across mutual funds or managed accounts. Furthermore, we show that there are reasonable sets of forecasts of security returns and risk for which the specific advice of the financial advisors examined is correct. Of most importance is that for most investors, CMW misled us. Moving to a high level of risk tolerance, representing the range of recommendations put forth by the advisors cited by CMW, investors should decrease the ratio of bonds to stocks in their optimum portfolios. While we can never prove the advisors were acting rationally, we provide evidence that is consistent with their acting rationally, while CMW provide no evidence that they were not.

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interviewed. Averaging across different recommendations does not produce a set of recommendations consistent with average expectations. Thus we do not expect to be able to find a consistent set of inputs using reasonable input data.

[16] We imposed one other constraint. Like Merrill Lynch, we believe Fidelity is recommending a minimum amount in money market funds. In solving the problem we imposed a minimum of 5% in the money market fund.

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The Distribution of Monthly Real Returns*

A. 5 Year Data

Asset	Mean return (percent)	Standard Deviation (percent)	Correlation with	
			Bonds	Bills
Stocks	0.960	3.930	0.581	0.429
Bonds	0.640	2.390	1.000	0.267
Bills	0.170	0.219	0.267	1.000

B. Merril Lynch

Asset	Mean return (percent)	Standard Deviation (percent)	Correlation with	
			Bonds	Bills
Stocks	0.732	5.840	0.195	0.083
Bonds	0.162	2.300	1.000	0.335
Bills	0.047	0.560	0.335	1.000

C. Quinn

Asset	Mean return (percent)	Standard Deviation (percent)	Correlation with	
			Bonds	Bills
Stocks	0.770	5.840	0.370	0.180
Bonds	0.240	2.200	1.000	0.320
Bills	0.120	1.240	0.320	1.000

D. Fidelity

Asset	Mean return (percent)	Standard Deviation (percent)	Correlation with	
			Bonds	Bills
Stocks	0.930	5.840	0.190	0.083
Bonds	0.190	1.650	1.000	0.466
Bills	0.110	1.000	0.466	1.000

* All estimates are presented as monthly data.

TABLE 1

ASSET PROPORTIONS FOR VARIOUS RISK LEVELS**

A. Merril Lynch

Bills	Bonds	Stocks	bond to stock	
70	10	20	0.50	
60	15	25	0.60	
50	20	30	0.67	
40	25	35	0.71	
30	30	40	0.75	
20	35	45	0.78	X
10	40	50	0.80	
5	40	55	0.73	X
5	30	65	0.46	
5	20	75	0.27	X
5	10	85	0.12	
5	5	90	0.06	
5	0	95	0.00	

B. Jane Bryant Quinn

Bills	Bonds	Stocks	bond to stock*	
80	15	5	3.00	
70	20	10	2.00	
60	25	15	1.67	
50	30	20	1.50	X
40	30	30	1.00	
30	35	35	1.00	
20	40	40	1.00	
10	40	50	0.80	X
0	40	60	0.67	
0	30	70	0.43	
0	20	80	0.25	
0	10	90	0.11	
0	0	100	0.00	X

C. Fidelity

Bills	Bonds	Stocks	bond to stock*	
80	15	5	3.00	
70	20	10	2.00	
60	25	15	1.67	
50	30	20	1.50	X
40	35	25	1.40	
30	35	35	1.00	
20	40	40	1.00	X
10	45	45	1.00	
5	40	55	0.73	
5	30	65	0.46	X
5	20	75	0.27	
5	10	85	0.12	
5	0	95	0.00	

Table 2

**All numbers rounded to nearest 5%

* Note that the unchanged bond stock ratio is do to rounding

X asset allocations recommended by financial advisors

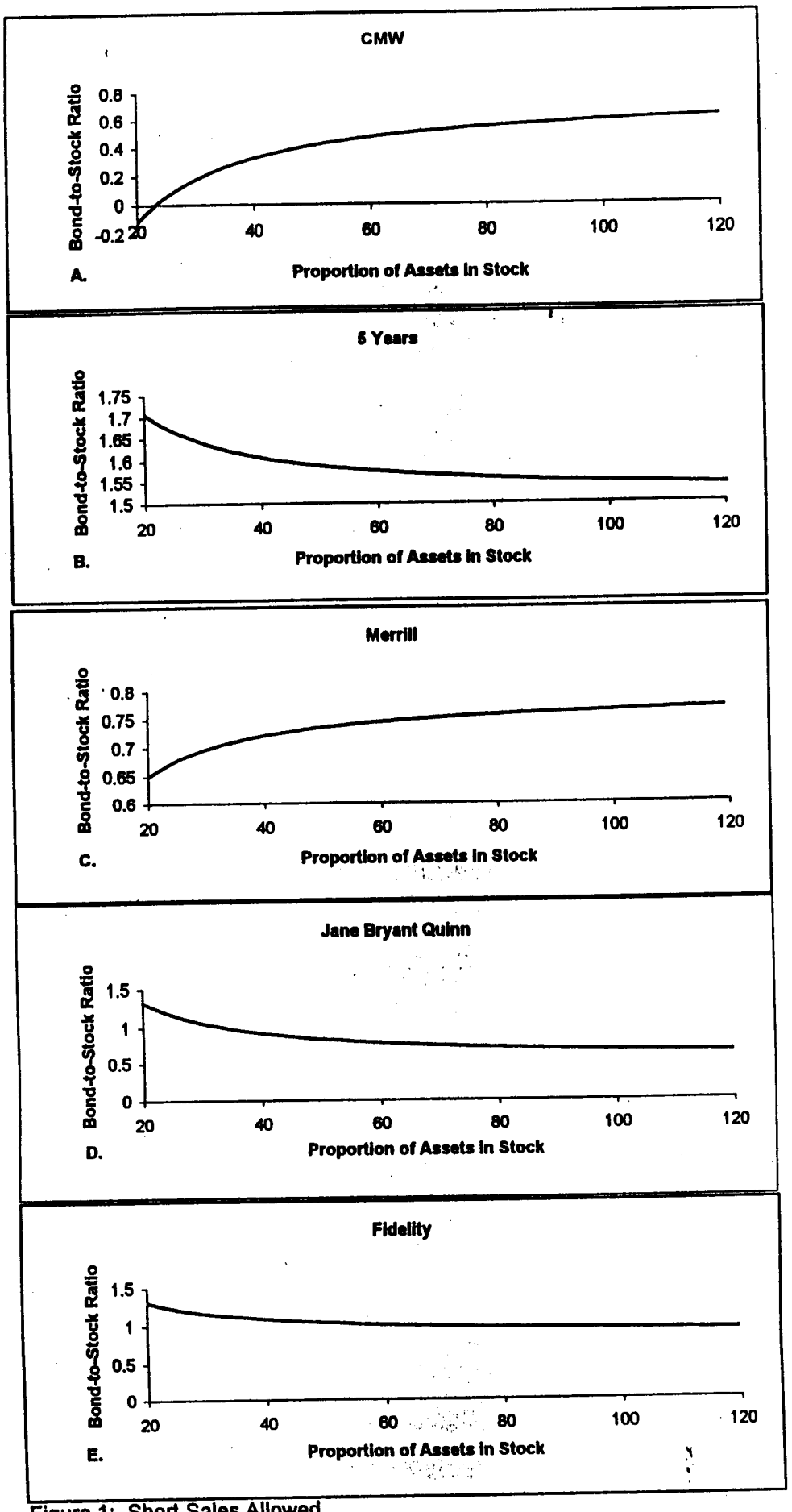

Figure 1: Short Sales Allowed

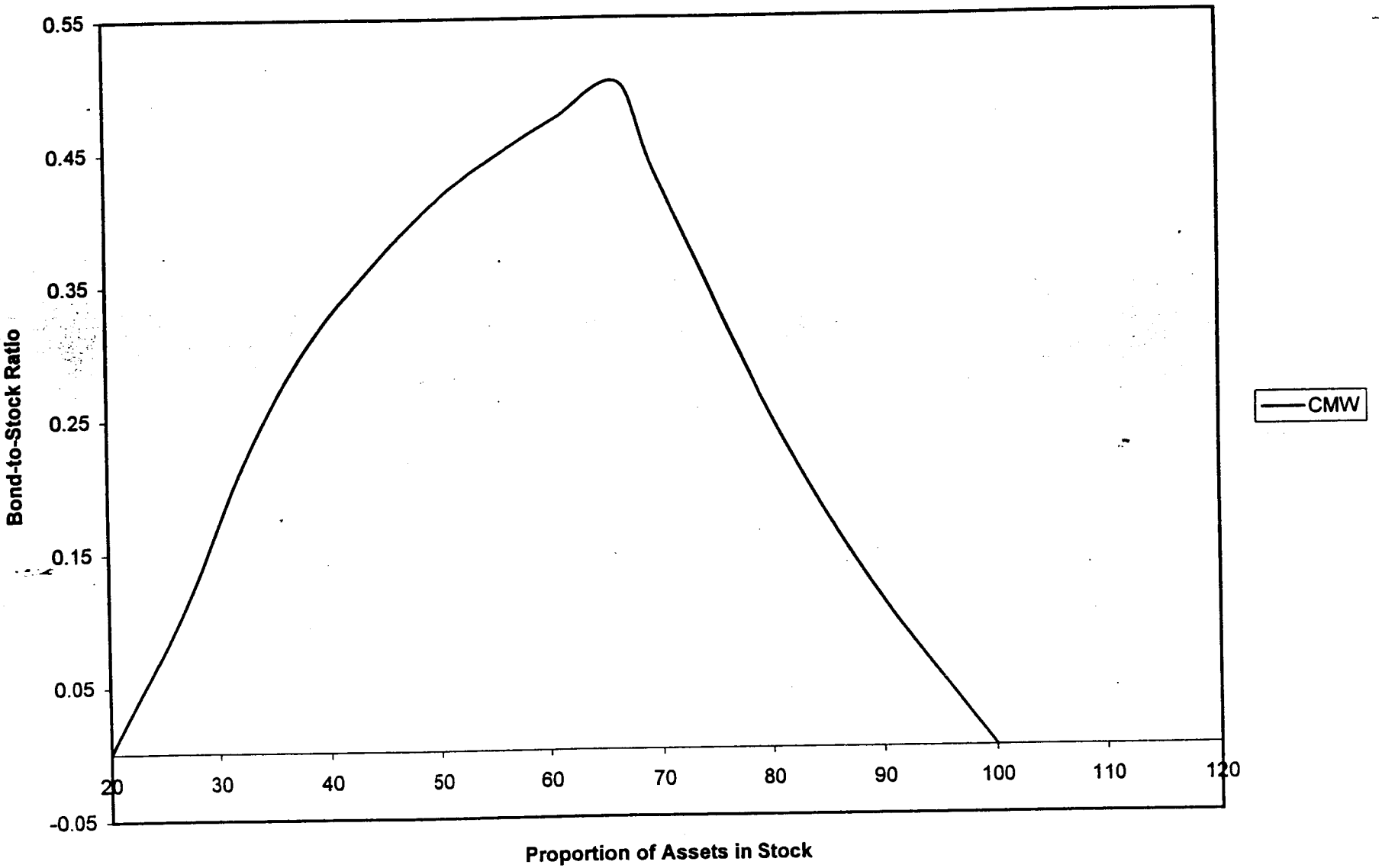

Figure 2: Short Sales Not Allowed

APPENDIX

The proof of montonicity and the condition for the ratio of bonds to stocks to be increasing can be derived from efficient set mathematics. From Roll (1977) Theorem 1,

$$X_S = C_0 + C_1 \overline{R}_P$$

$$X_B = C_2 + C_3 \overline{R}_P$$

Where:

1. X_B is the proportion in Bonds
2. X_S is the proportion in Stocks
3. $\overline{R}_P$ is the expected return on an efficient portfolio
4. C's are constants

Solving for $\overline{R}_P$ in the top equation eliminating it in the second and rearranging yields.

$$\frac{X_B}{X_S} = +\left[C_2 - \frac{C_0 C_3}{C_1}\right]\frac{1}{X_S} + \frac{C_3}{C_1}$$

Taking the derivative

$$\frac{d\left(\frac{X_B}{X_S}\right)}{dX_S} = -\left[C_2 - \frac{C_0 C_3}{C_1}\right]\frac{1}{X_S^2}$$

Since the term in the brackets is a constant monotonicity is proven. Define A and D as two efficient Portfolios with D having more stock. Than the condition for the bond stock ratio to be increasing as the fraction invested in stock increases is

$$\frac{X_B^D}{X_S^D} > \frac{X_B^A}{X_S^A}$$

Where the superscript refers to the Portfolio, let A be the Global Minimum Variance Portfolio and D be defined as in Roll (1977) equation $A.17$ (if D doesn't have more stock reverse the inequality) then the condition for the bond to stock ratio to be increasing for higher risk levels is

$$\frac{V_B^{-1}R}{V_S^{-1}R} > \frac{V_B^{-1}1}{V_S^{-1}1}$$

Where V^1 is the inverse of the variance covariance matrix, the subscript indicates a row, R is a vector of expected returns and 1 a vector of ones. If the inequality is reversed the ratio of stocks to bonds is monotonically decreasing.