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Why Do Security Prices Change? A Transaction-Level Analysis of NYSE Stocks

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Abstract

This paper develops a structural model of intraday price formation that embodies both public information shocks and microstructure effects in an internally consistent, unified setting. The model allows us to better understand the observed intra-day patterns in bid-ask spreads, price volatility, transaction costs, as well as the autocorrelations of transaction returns and quote revisions. For example, the model simultaneously sheds light on why, over the day, (i) the variance of transaction price changes is U-shaped while the variance of ask price changes is declining, (ii) the bid-ask spread is U-shaped although information asymmetry and uncertainty over fundamentals is decreasing, and (iii) the autocorrelations of transaction price changes are large and negative, yet the autocorrelations of ask price changes are small and negative. In addition, the model's parameters also provide a natural metric of price discovery and effective trading costs, which may prove useful in future studies.

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1. Introduction

Why do security prices change?

In the classical model of an efficient security market, prices move in response to new public information that causes traders to simultaneously revise their beliefs. Alternatively, the process of trading itself may generate price movements because of various market imperfections and frictions. A realistic description of intraday security price movements must capture both these elements.

Understanding the process of intraday price formation is important for several reasons. First, intraday prices and quotes exhibit several patterns that are not well understood. For example, it is well-known (see, for example, Harris (1986), Jain and Joh (1988) and McInish and Wood (1992)) that quoted bid-ask spreads and volume exhibit U-shaped patterns over the day. This finding is difficult to reconcile with evidence from theoretical models (Easley and O'Hara (1992), Madhavan (1992)) and laboratory experiments (Bloomfield (1996), Bloomfield and O'Hara (1996)) where information asymmetry and uncertainty over fundamentals, and hence bid-ask spreads, decline monotonically over the day as market participants learn from the trading process. Second, an examination of how much new public information flows or particular market frictions contribute to intraday price volatility is of great interest from a public policy viewpoint. Third, a better understanding of intraday price formation may also shed light on the magnitude, determinants, and composition of execution costs, a topic of considerable interest to portfolio managers, exchange officials, and traders.

Previous research has examined various microstructure phenomena (such as intraday bid-ask spreads, execution costs, and autocorrelation and volatility patterns of transaction prices and quotes) either individually or through generalized reduced-form investigations. By contrast, this paper develops and estimates a structural model of price formation which captures many of these frictions in a unified setting. Our model incorporates both public information shocks and microstructure effects. The microstructure phenomena modeled include the possibility of crosses within the quoted bid-ask spread, autocorrelation of order flow, as well as trading frictions arising from asymmetric information, dealer costs, and price discreteness.

The structural model described in this paper links two important areas of the literature. The first area examines the sources of intraday price volatility. While theoretical and laboratory experiments indicate that the trading process generates information which reduces pricing errors, the empirical evidence suggests otherwise. For example, French and Roll (1986) find that return volatility is significantly higher during trading hours than during non-trading hours, and attribute this to noise generated by the trading process. More recently, Hasbrouck (1991b) uses time-series techniques to decompose the random-walk component of returns using a VAR approach, while Hasbrouck (1993) decomposes the variance of the stationary component of returns. Like Hasbrouck, we decompose intraday volatility into components attributable to public information shocks and trading frictions, but our work differs in that our model is structural by nature. Thus, we can relate the decomposition one-to-one with the underlying economic parameters of the model, although we lose the generality of Hasbrouck's approach.

The second related area concerns the measurement of the components of the bid-ask spread. An extensive theoretical literature shows that the costs of trading, as represented by the effective bid-ask spread, consist of three components: asymmetric information, inventory carrying costs, and order processing costs. The relative importance of these components is a topic of considerable practical and academic interest, especially since it may provide insights into the why execution costs vary across markets and stocks.[1] More recently, Huang and Stoll (1996) develop a general model to investigate the components of the bid-ask spreads. Although their model (which was independently derived) is closely related to ours, it decomposes the non-information part of the spread into the inventory and order processing components. By contrast, our focus is more on explaining the effect of information flows on stock prices over the day, i.e., is complementary. In this sense, our work is also related to Huang and Stoll (1994), who examine the predictability of returns over very short horizons while controlling for various microstructure effects.

We estimate the model using transaction-level data for a number of stocks listed on the New York Stock Exchange (NYSE). Because the model is so simple, the parameters are estimated in a setting which provides a high comfort level in terms of estimation, though not

[1]See, e.g., Roll (1984), Glosten (1987), Glosten and Harris (1988), Choi, Salandro, and Shastri (1988), Stoll (1989), George, Kaul, and Nimalendran (1991), and Huang and Stoll (1994).

necessarily model, error, i.e., OLS normal equations plus some augmented direct estimates of order flow behavior and crossing probabilities. There are several interesting features of the model estimation. First, though the model captures many interesting microstructure effects, only transactions data is required for estimation purposes. Second, other than the model structure itself, very weak assumptions are placed on the transactions price generating process in estimating the parameters. Third, we cam interpret our model parameters in terms of quote data (e.g., bid-ask spreads, volatility and autocorrelation) and transaction price changes-based moment restrictions not used in estimation (e.g., autocorrelations).

Several interesting results emerge from our analysis:

- Both information flows and trading frictions are important factors in explaining intra-day price volatility in individual stocks.

- Information asymmetry decreases steadily throughout the day, consistent with theoretical models (Schreiber and Schwartz (1985), Handa and Schwartz (1991), and Madhavan (1992)) where market makers learn from order flow, as well as with evidence from experimental markets (Bloomfield (1996), Bloomfield and O'Hara (1996)). However, dealer costs increase over the day (possibly reflecting the costs of carrying inventory overnight), so that bid-ask spreads exhibit the U-shaped pattern noted in previous research.

- We provide an estimator of execution costs that takes into account the possibility that orders may execute within the bid-ask spread as well as information and inventory effects. The cost of transacting is significantly smaller than the bid-ask spread once the probability of executing within the quotes is considered. In contrast to the bid-ask spread, this measure of execution costs increases over the day. This result is consistent with concentrated trading at the open by discretionary liquidity traders who can selectively time their trades.

- The model provides insights into the determinants of the autocorrelations of quotes and returns, as well as other moments, such as the variance of quote changes. In particular, the autocorrelation of quote returns implied by our model may be positive or negative, even though beliefs follow a martingale and there are no lagged inventory effects.

Additionally, the pattern in return and quote autocorrelations implied by our model closely resemble the actual autocorrelations of the data. For example, in our sample, autocorrelations of price changes average approximately -0.219 on a transactions level basis, while our model implies an average of -0.211.

The paper proceeds as follows: In section 2, we develop a structural model of price formation, and describe the procedure for estimating this model. Section 3 provides a brief description of the data and estimates of the model's underlying parameters over the day. Section 4 discusses the economic implications of the model's parameters for execution costs, price discovery, and the autocorrelation of price changes and quote revisions. Section 5 provides a discussion of the model's limitations, suggestions for extensions to correct these failures, and Section 6 summarizes.

2. A Structural Model of Price Formation

2.1. Bid-Ask Quotes and Transaction Prices

We begin by examining a prototypical microstructure model of the quote and return generating mechanism.[2] Consider the market for a risky security whose fundamental value (which can be thought of as the present value of future dividends) evolves through time. The security is traded in an auction-dealer mechanism where liquidity providers (who may be traders using limit orders or designated market makers such as NYSE specialists) quote bid and ask prices at which they are willing to trade. An order also may be executed within the quotes.[3]

Let p_t denote the transaction price of the security at time t. Denote by x_t an indicator variable for trade initiation, where $x_t = +1$ if trade t is buyer-initiated and -1 if the trade is seller-initiated. Some trades (such as pre-negotiated crosses within the prevailing bid-ask spread) can be viewed as both buyer- and seller-initiated, and in this case $x_t = 0$. Let λ

[2]The model incorporates a variety of microstructure effects discussed individually. In particular, the models of Garbade and Silber (1979), Roll (1984), Glosten and Milgrom (1985), Choi, Salandro, and Shastri (1988), and Stoll (1989) can be viewed as special cases of our model.

[3]On the NYSE, such price improvement may occur because of limit orders or through the actions of floor traders or the specialist. In this paper we do not explicitly incorporate the limit order book. However, we can interpret a limit order trader as another market maker, and to this extent the bid and ask quotes may be thought of as arising not necessarily from the exchange floor. Such an approach is pursued by Greene (1996), who extends our model in this direction.

denote the unconditional probability the transaction occurs within the quoted spread, i.e., $\lambda = \Pr[x_t = 0]$. We assume that buys and sells are (unconditionally) equally likely, so that $E[x_t] = 0$ and $\text{Var}[x_t] = (1 - \lambda)$.

Before describing how quotes and transaction prices are determined, we first discuss the evolution of public beliefs. Changes in beliefs arise from two sources: (i) New public information announcements which are not associated with trading, and (ii) Order flow, which may provide a noisy signal about future asset values.

Public news announcements may cause revisions in beliefs without any trading volumes. Denote by ϵ_t the innovation in beliefs between times $t-1$ and t due to new public information. We assume that ϵ_t is an independent and identically distributed random variable with mean zero and variance σ_ϵ^2. In addition, if market makers' believe that some traders may possess private information about fundamental asset value, a buy (sell) order is associated with an upward (downward) revision of beliefs. We assume, following Glosten and Milgrom (1985), that the revision in beliefs is positively correlated with the *innovation* in the order flow.[4] Formally, the change in beliefs due to order flow is $\theta(x_t - E[x_t|x_{t-1}])$, where $(x_t - E[x_t|x_{t-1}])$ is the surprise in order flow and $\theta \geq 0$ measures the degree of information asymmetry or the so-called permanent impact of the order flow innovation. Higher values of θ indicate larger revisions for a given innovation in order flow; in the absence of information asymmetry, the parameter $\theta = 0$.

Our assumption of a fixed order size is consistent with much of the previous literature.[5] Alternatively, following Glosten and Harris (1988) and Madhavan and Smidt (1991), the revision in beliefs could be modeled as proportional to the net order imbalance in a particular period. Although it is possible to extend the model to incorporate such volume effects (as discussed below), there are several arguments in favor of a constant order size model. First, and most importantly, the simplifying assumptions regarding volume permit us to estimate a parsimonious model and compute closed-form solutions for the estimators of interest. Second, the assumption facilitates comparisons of our results with much of the previous literature on

[4]In Glosten and Milgrom (1985), the revision in beliefs is directly proportional to the actual order flow because it is implicitly assumed that order flow is uncorrelated.

[5]See, e.g., Roll (1984), Glosten and Milgrom (1985) Stoll (1989), Choi, Salandro, and Shastri (1988), George, Kaul, and Nimalendran (1991), Huang and Stoll (1994), and Huang and Stoll (1996).

intra-day price movements. Third, previous empirical studies find that the effect of order size is economically small relative to the indicator variables, suggesting that the gain in efficiency from modeling volume may be relatively modest.[6] Extensions to a volume-based model are discussed in more detail in Section 5.

Let μ_t denote the post-trade expected value of the stock conditional upon public information and the trade initiation variable. The revision in beliefs is the sum of the change in beliefs due to new public information and order flow innovations, so that

$$\mu_t = \mu_{t-1} + \theta(x_t - E[x_t|x_{t-1}]) + \epsilon_t. \tag{1}$$

Market maker bid and ask quotations are *ex post* rational (see, e.g., Glosten and Milgrom (1985)) so that the ask (bid) price is conditioned on a trade being buyer-initiated (seller-initiated). Let p_t^a denote the (pre-trade) ask price at time t and similarly define the bid price, p_t^b. Market maker quotations also reflect their compensation for their service in providing liquidity on demand. Let $\phi \geq 0$ represent market makers' cost per share for supplying liquidity. We can interpret ϕ as dealers' compensation for transaction costs, inventory costs, risk bearing, and possibly the return to their unique position.[7] It follows that the ask price (i.e., the price if $x_t = +1$) is $p_t^a = \mu_{t-1} + \theta(1 - E[x_t|x_{t-1}]) + \phi + \epsilon_t$. Similarly, the bid price is $p_t^b = \mu_{t-1} - \theta(1 + E[x_t|x_{t-1}]) - \phi + \epsilon_t$. Thus, ϕ captures the temporary (or transitory) effect of order flow on prices.

Not all orders are executed at the quoted bid or ask prices. For example, floor brokers or upstairs market makers may negotiate trades within the bid-ask spread. In this case, transactions are assumed to execute at the midquote, m_t, where $m_t = (p_t^a + p_t^b)/2 = \mu_{t-1} + \epsilon_t - \theta E[x_t|x_{t-1}]$. The transaction price can be expressed as the *ex post* belief plus (minus) the dealer cost for a buy (sell) order. Formally, we write $p_t = \mu_t + \phi x_t + \xi_t$, where ξ_t is an independent and identically distributed random variable with mean zero that captures the

[6]The small coefficients on volume may be explained by the fact that large-block trades often originate in the upstairs market. In the upstairs market, intermediaries or block brokers search for counter-parties to the trade, and typically arrange a transaction at or within the prevailing bid-ask spread. Further, if the floor market is truly anonymous, informed traders will break-up their large trades, so that trade direction may have more explanatory power than trade size.

[7]Demsetz (1968) examines the nature of transaction costs in determining bid-ask spreads; Amihud and Mendelson (1980) and Ho and Stoll (1983) develop models of inventory control and show that the bid-ask spread may reflect the dealers' carrying costs and risk aversion.

effect of stochastic rounding errors induced by price discreteness or possibly time-varying returns.[8] Observe that any systematic deviation from zero in the rounding error (arising perhaps from a tendency to round up on buys and down on sells) is captured in the dealer cost component ϕ.

Using equation (1), we obtain

$$p_t = \mu_{t-1} + \theta(x_t - E[x_t|x_{t-1}]) + \phi x_t + \epsilon_t + \xi_t. \tag{2}$$

To estimate equation (2), we must describe the temporal behavior of order flow. We assume a general Markov process for the trade initiation variable. Let γ denote the probability that a transaction at the ask (bid) follows a transaction at the ask (bid), i.e., $\gamma = \Pr[x_t = x_{t-1}|x_{t-1} \neq 0]$. As large traders typically breakup their orders into smaller components for easier execution, continuations are more likely than reversals, so that $\gamma > \frac{1}{2}$.[9]

Let ρ denote the first-order autocorrelation of the trade initiation variable, i.e., $\rho = \frac{E[x_t x_{t-1}]}{\mathrm{Var}[x_{t-1}]}$. It is straightforward to prove that $\rho = 2\gamma - (1-\lambda)$, so that the autocorrelation of order flow is an increasing function of the probability of a continuation, γ, and the probability of a midquote execution, λ. Observe that when there is no possibility of transacting within the quotes (i.e., $\lambda = 0$) and order flow is independent (i.e., $\gamma = \frac{1}{2}$), order flow is serially uncorrelated and $\rho = 0$.[10]

Next, we need to compute the conditional expectation of the trade initiation variable given public information. Observe that if $x_{t-1} = 0$, $E[x_t|x_{t-1}] = 0$. If $x_{t-1} = 1$, $E[x_t|x_{t-1} = 1] = \Pr[x_t = 1|x_{t-1} = 1] - \Pr[x_t = -1|x_{t-1} = 1] = \gamma - (1 - \gamma - \lambda) = \rho$. Similarly, If $x_{t-1} = -1$, $E[x_t|x_{t-1} = -1] = -\rho$. Thus, the conditional expectation $E[x_t|x_{t-1}] = \rho x_{t-1}$.

To transform equation (2) into a testable equation, we need to substitute out the unobservable prior belief, μ_{t-1}. Using the fact that $\mu_{t-1} = p_{t-1} - \phi x_{t-1} - \xi_{t-1}$ and $E[x_t|x_{t-1}] = \rho x_{t-1}$, we can express equation (2) as

$$p_t - p_{t-1} = (\phi + \theta)x_t - (\phi + \rho\theta)x_{t-1} + \epsilon_t + \xi_t - \xi_{t-1}. \tag{3}$$

[8]The assumption that the rounding error is serially uncorrelated is made for simplicity; since the rounding error is, on average, one-sixteenth of a dollar (i.e., half the minimum variation), the effects on our estimates of any serial correlation are unlikely to be significant.

[9]This may also occur because of price continuity rules, trade reporting practices, and other institutional factors.

[10]In our model, midquote transactions are informative because these transactions lead to innovations in market maker beliefs when order flow is autocorrelated.

Equation (3) forms the basis for our investigation of intraday price movements. In the absence of market frictions, the model reduces to the classical description of an efficient market where prices follow a random walk. However, in the presence of frictions (i.e., transaction costs and information asymmetries), transaction price movements reflect order flow and noise induced by price discreteness, as well as public information shocks.

2.2. *Model Estimation*

The four parameters governing the behavior of transaction prices and quotes are: (i) θ, the asymmetric information parameter, (ii) ϕ, the cost of supplying liquidity, (iii) λ, the probability a transaction takes place inside the spread, and (iv) ρ, the autocorrelation of the order flow. Let $\beta = (\theta, \phi, \lambda, \rho)$ denote the vector of price and quote parameters.

Equation (3) expresses transaction price changes as a linear function of contemporaneous and past order flows. Thus, with adjustments to the standard errors for serial covariance of the errors induced by price discreteness (i.e., $\xi_t - \xi_{t-1}$), equation (3) can be estimated via ordinary least squares. Unfortunately, not all of the parameters in the vector β can be identified this way. However, using a time-series of T observations on transaction price changes and trade initiation, the model's parameters can be estimated using maximum likelihood or a similar nonlinear estimation procedure. The drawback to this approach is that it requires strong distributional assumptions on the processes generating public information which may be far from reality.

We adopt an alternative estimation procedure based on the Generalized Method of Moments (GMM) procedure of Hansen (1982). GMM is a natural approach to estimate jointly the system of equations governing the transaction and quoted price processes. This technique is particularly appropriate here because it does not require strong assumptions for the stochastic process generating the data. Indeed, Huang and Stoll (1994, 1996) use GMM to test their models which closely resemble ours. Unlike maximum likelihood, the GMM procedure yields consistent parameter estimates of the nonlinear model without specific distributional assumptions. Further, given the growing literature on non-normality of stock returns (manifested in the conditional variance of returns), we can adjust for conditional heteroskedasticity using the results of Newey and West (1987). Below, we describe briefly the GMM approach to estimating the unknown parameters of the model.

The GMM procedure consists of choosing parameter values for β that minimize a criterion function based on the orthogonality restrictions (or moment conditions) implied by the model. Because of the model's implied linearity between the observable variables, the moment conditions correspond to the normal equations of OLS, with some additional restrictions which help decompose the OLS coefficients. Specifically, let $u_t = p_t - p_{t-1} - (\phi + \theta)x_t + (\phi + \rho\theta)x_{t-1}$. Then, the following population moments implied by the model exactly identify the parameter vector β and a constant (drift) α:

$$E \begin{pmatrix} x_t x_{t-1} - x_t^2 \rho \\ |x_t| - (1 - \lambda) \\ u_t - \alpha \\ (u_t - \alpha)x_t \\ (u_t - \alpha)x_{t-1} \end{pmatrix} = 0.$$

The first equation is simply the definition of the autocorrelation in trade initiation, the second equation defines the crossing probability, the third equation defines the drift term as the average pricing error, and the last two equations are the OLS normal equations.

The idea behind GMM is to choose parameter values for β such that the sample moments, denoted by $g_T(\beta)$, closely approximate these underlying population moments. Hansen (1982) proves that the GMM estimates, $\hat{\beta}$, are consistent and asymptotically normally distributed. In particular, the variance-covariance matrix of $\hat{\beta}$ equals $V_{\hat{\beta}} = [D_0' S_0^{-1} D_0]^{-1}$, where $D_0 = E\left[\frac{\partial g_T(\beta)}{\partial \beta}\right]$ and $S_0 = \sum_{l=-\infty}^{l=+\infty} E[f_t f_{t-l}]$, where f_t is the vector of arguments in the expectation that defines the moment conditions.[11]

3. Empirical Results

3.1. *Data Sources and Methods*

The data are drawn from a file of bid and ask quotations, transaction prices and volumes for equities in 1990, obtained from the Institute for the Study of Securities Markets (ISSM). Our initial sample is based on the first 750 stocks in the file. From the initial sample, we

[11]In practice, we estimate S_0 using the Newey-West procedure to obtain a heteroskedasticity consistent covariance matrix.

include only NYSE-listed common stocks trading in eighths which did not have any stock splits in the calendar year 1990.

As our objective is to understand the process by which security prices impound information, it is important to examine the evolution of the information parameters over the trading day.[12] Following Hasbrouck (1991b), we estimate the model for five intervals of the day: 9:30-10:00, 10:00-11:30, 11:30-2:00, 2:00-3:30, and 3:30-4:00. To ensure there are sufficient observations for model estimation, we consider only those stocks for which there are at least 250 observations per interval over the year 1990. These criteria reduce the sample to 274 stocks.

On a transaction basis, we impose filters on the data to eliminate outliers or recording errors.[13] For the opening period (9:30-10:00), overnight returns are eliminated since recent evidence (see, e.g., Amihud and Mendelson (1987)) indicates that they are likely to come from a different distribution. The opening transaction (which, in active stocks, is usually arranged in a batch or auction market), is eliminated.

To sign the trade initiation variable we must determine the quote prevailing at the time of the transaction. Although transactions and quotations in the ISSM data are time-stamped to the second, this presents a problem because there are often delays in the reporting of transactions resulting in a misaligned sequence of quotes and transactions. Thus, it is possible for a quote with the same time stamp as a transaction to represent the quote after the transaction. Accordingly, it has been common to use only those quotes which have been in existence for some time prior to the transaction as the quotes referring to the transaction. Lee and Ready (1991) suggest identifying a quote as prevailing at the time of the transaction if it was the latest quote for the stock and was at least five seconds old. More recent research by Blume and Goldstein (1992) suggests a 16 second lag may be more appropriate, and we adopt their procedure to determine the times of quotes.

Another consideration is whether to measure quote revisions in calendar or transaction

[12]This is especially relevant given empirical evidence (e.g., Harris (1989)) documenting temporal patterns in intraday returns and volatility.

[13]The filters are as follows: any trades below $1 or above $200 are excluded; and bid (ask) quote or transaction more than 50 percent away from the previous bid (ask) or transaction is eliminated; trades more than $5 from the midquote are eliminated; and for stocks trading above $10, any quote with a percentage spread above 20 percent is eliminated while for stocks below $10 any quote implying spreads over $2 is eliminated.

time. In general, because dealers may provide multiple quotations even if the security's price is unchanged, we measure quote revisions in transaction time. Thus, the revision in the ask quotes is defined as the change in the ask prices computed using the ask quotation just prior to a transaction.

3.2. Descriptive Statistics

Table 1 presents descriptive statistics for the sample of stocks. Panel A of the table provides details on the variance of transaction price and quote changes, number of transactions per day, market capitalization at year-end 1989, and stock price for the 274 stocks in the sample. The stocks are actively traded (the median number of transactions per day is 66), and have a relatively high level of market capitalization (the median size is $2.2 billion which corresponds roughly to the average size of NYSE- listed firms). Nonetheless, the stocks exhibit a wide range in terms of size and activity, and all variables are skewed to the right.

Panel B of Table 1 provides a breakdown of price volatility, transaction frequency, bid-ask spreads, and share volume in five intraday time intervals, for 1990. On an hourly basis, it is clear that share volume and trading frequency exhibit the U-shaped pattern documented by previous research. For example, while the number of transactions per hour equal approximately 17 during the opening and closing hours of trading, only 11 transactions occur during midday. Interestingly, for this sample, the volume per transaction is not U-shaped. Instead, the volume per trade is approximately 2,200 plus shares for the first two hours of trading, 1,900 plus shares over the next four hours, and then 1,600 during the last hour. Finally, the bid-ask spread displays the usual U-shaped pattern, ranging from 22.8 cents at the open to 20.4 cents midday back to 21 cents at the close.

More interesting, while the volatility of transaction price changes follows a U-shaped pattern, the variance of ask price changes declines throughout the day. This is especially true during the opening interval in which the standard deviation of quote changes drops from 7.8 cents to 6.9 cents.

3.3. Parameter Estimates

Table 2 presents summary statistics on the individual parameter estimates governing the stochastic process for transaction price changes and quote revisions across the 274 stocks. The table presents the mean coefficient estimate, mean standard error, the standard deviation of the estimates and the median estimates of the parameter vector β for the 274 stocks in each of the five intraday trading intervals. The parameter estimates have, in general, economically reasonable values.[14]

The extent of information asymmetry (i.e., θ) is the main parameter of interest. From Table 2, it is clear that the degree of this asymmetry drops sharply after the opening half-hour interval. The mean value of θ falls by over a third from the opening to the middle of the day (from 4.15 to 2.75 cents) and remains at this level until the final period where it increases slightly. To gauge how reliable these estimates are, Table 2 also provides the average standard error across the 274 stocks. The standard errors are quite tight, and range from 0.19 cents to 0.57 cents at different times of the day.

The decline in θ has a clear economic interpretation. Recall that θ represents the magnitude of the revision in the market maker's beliefs concerning the security's value induced by order flow. A decline in θ, therefore, represents less reliance on the signal content of order flow. The greater reliance on prior beliefs is consistent with either (i) market makers learning about fundamental asset values (i.e., price discovery) through the trading process, or (ii) a larger percentage of liquidity traders (or equivalently, less information asymmetry) at the end of the day. However, the monotonicity in the parameter estimates suggests to us that the former interpretation is more reasonable. From a theoretical viewpoint, it is also unlikely that the percentage of uninformed traders may be systematically higher at certain points of the day, since this would attract informed traders who seek to disguise their trades as in Admati and Pfleiderer (1988).

The transaction cost element ϕ is approximately 3.4 cents in the first half hour and increases steadily over the day to 4.6 cents in the final half hour interval, a rise of about 30 percent. Similar to the estimates of θ, the standard errors around these values tend to be relatively small, ranging from 0.17 cents to 0.53 cents at various times during the day. Taking

[14]Since the estimated drift α is essentially zero, its estimates are not reported in the table.

these results as given, the increase in ϕ over the day is consistent with inventory control models of market making. In our model, ϕ represents the economic costs of market making, and the increase in this parameter over the day may reflect the increasing risks associated with carrying inventory overnight. Previous studies of inventory control by market makers (see, e.g., Hasbrouck (1988), Madhavan and Smidt (1991), and Hasbrouck and Sofianos (1993b)) find relatively weak evidence of intraday inventory effects, although Madhavan and Smidt (1993) suggest that these effects may be apparent at lower frequencies. If inventory effects are manifested towards the end of the day, the conclusions of these studies may be worth reinvestigating.

The autocorrelation of order flow, ρ, also lies in a fairly narrow range. From Table 2, the mean value of the autocorrelation lies between 0.367 and 0.407. While the pattern is mildly U- shaped, the average standard errors and lack of supporting theory suggest there is no systematic pattern in the parameter over the day.[15] Nevertheless, as expected, the mean estimate is positive and the size of the implied autocorrelation suggests that ignoring this effect may have importance economic consequences.

The probability that a trade occurs within the quotes, λ, declines monotonically over the day. The mean estimates drop from 34 percent in the opening interval to 28 percent at the close. With average standard errors of around 1-2%, the drop is arguably large. Moreover, from a theoretical viewpoint, the steady decline in λ over the day is consistent with increased incentives by liquidity providers to place limit orders when spreads are wide and a higher probability of a cross in intervals of high activity.

4. Economic Implications of the Model

4.1. Assessing the Cost of Trading

4.1.1. The Bid-Ask Spread

Recent allegations of collusion among Nasdaq dealers in setting bid-ask quotes (see, e.g., Christie and Schultz (1994) and Christie, Harris, and Schultz (1994)) have focused renewed

[15]This figure is higher than those reported in previous studies because we explicitly model mid-quote executions; these occurrences are relatively high in our sample of actively traded stocks.

attention on the accurate measurement of transaction costs. We show in this section that the bid-ask spread is a misleading measure of the true costs of trading, and use the model to provide more accurate estimates.

The implied bid-ask spread at time t (i.e., $p_t^a - p_t^b$) is a random variable with mean $2(\theta + \phi)$. Let s denote the expected implied bid-ask spread. As s is a function of identifiable parameters, and the GMM estimators of these parameters have well-known asymptotic distributions, s can be estimated in a straightforward manner from the data. Specifically, it can be shown that the estimator of s, $2(\hat{\phi} + \hat{\theta})$, is consistent and asymptotically normal with variance

$$[2,\ 2] V_{\hat{\theta},\hat{\phi}} \begin{bmatrix} 2 \\ 2 \end{bmatrix},$$

where $\hat{\theta}$ and $\hat{\phi}$ denote the sample estimates of the parameters θ and ϕ and $V_{\hat{\theta},\hat{\phi}}$ is the estimated covariance matrix of the GMM estimators.[16]

Table 3 presents the mean coefficient estimate, mean standard error, the standard deviation of the coefficient estimate, and the median estimate of the bid-ask spread measure for 274 stocks in each of the five time intervals. Note that the spread is a function of the underlying parameters of the structural econometric model described in 2.1. Thus, its patterns can be interpreted economically within the model's framework.

The implied spread exhibits the familiar U-shaped pattern over the course of the day (see, for example, Table 1). In particular, the mean spread in the initial period is 15.2 cents, drops to 14.3 cents during the day and rises to 14.7 cents at the end of the day. However, the difference between the spread at the beginning (end) of the day and the spread at the middle of the day is small because our sample consists of actively traded stocks for whom the U-shaped pattern is less pronounced. In terms of the reliability of these results, note that the average standard error varies from 0.24 cents to 0.66 cents over the day.

Nevertheless, standard errors aside, it is important to point out that our model provides an explanation for this U-shaped pattern. In particular, the intraday patterns of the asymmetric information cost, θ, and the dealer cost, ϕ, govern the spread's behavior. At the start of the day, information asymmetries are large, so that the spread is wide. Over the

[16]In our discussion below, we use this notation to describe all parameter estimates and associated covariance matrices.

day, asymmetries are resolved through price discovery causing spreads to narrow. Toward the end of the day, the asymmetric information component is small but high transaction costs (possibly reflecting the risks of carrying inventory overnight) cause the spread to widen again.

On the positive side, these structural estimates of the spread do not rely on bid quotations data but are inferred from the autocovariance and other moment conditions of transaction price changes. Thus, one advantage of our structural framework is that, given the transactions data, the U-shaped pattern is implied by the model. On the negative side, however, because we do not use quotations data directly in estimation, the model's spread need not equal the actual (sample) quoted spread. Indeed, our estimates of the bid- ask spread are lower than the actual quoted spread. Part of the difference may reflect the increased probability of a midquote transaction when the spread is wide.[17] This systematic tendency could explain the deviation between the spreads implied by our model and the actual sample spreads. This is a topic for further research, and is discussed more in Section 5 below.

For further evidence of the relation between the model's structural parameters and the bid-ask spread, we can also estimate the fraction of the implied spread attributable to asymmetric information. Define by r the ratio of the information component of the spread (i.e., 2θ) to the total implied spread. If $r = 0$, the spread is entirely attributable to the costs of supplying liquidity, and if $r = 1$, direct liquidity costs are negligible and adverse selection costs constitute the entire bid-ask spread. Then, the estimate of r has mean

$$\frac{\theta}{\phi + \theta},$$

and asymptotic variance

$$\left[\frac{\theta + \phi - 1}{(\theta + \phi)^2}, \frac{-\theta}{(\theta + \phi)^2}\right] V_{\hat{\theta},\hat{\phi}} \left[\begin{array}{c} \frac{\theta+\phi-1}{(\theta+\phi)^2} \\ \frac{-\theta}{(\theta+\phi)^2} \end{array}\right].$$

An examination of the proportion of the spread due to asymmetric information r over the course of the day supports this hypothesis for the U-shaped pattern in spreads. As shown in Table 3, r is 51 percent in the initial period, and falls steadily to 36 percent in the

[17]We do indeed find that spreads are wide when midquote trades occur. Moreover, using NYSE reported statistics, about 30% of NYSE transactions occur at the midquote, but in transactions where the spread is larger than $\$\frac{1}{8}$, the figure increases to 70%. These stylized facts are consistent with our results.

third period and remains at about this level for the rest of the day.[18] Note that the average standard errors range between 1%-3%. Thus, these results over the day suggest a fairly large drop both economically and statistically.

4.1.2. The Effective Cost of Trading

An alternative measure of trading costs is the *effective bid-ask spread* measured by the expected price difference between a notional purchase at time t and a notional sale at some future time $t+k$. Recognizing the potential for a cross at either time, the potential changes are from ask to bid, ask to the midpoint, midpoint to bid, and midpoint to midpoint. If the notional sale takes place several transactions after the notional purchase (i.e., if k is sufficiently large), we can ignore the effect of the autocorrelation of order flow (which is of the order ρ^k).

Accordingly, we assume that, at the time of the notional sale, market makers, on average, expect buys and sells to be equally likely. In this case, the round-trip expected costs associated with each of the four possibilities are $2(\phi+\theta)$, ϕ, $(\phi+\theta)$, and 0, respectively. Under our assumptions, the conditional probability of executing at the midquote given the trade is buyer-initiated is λ, so the probabilities associated with each of the four possible price paths are, respectively, $(1-\lambda)^2$, $(1-\lambda)\lambda$, $(1-\lambda)\lambda$, and λ^2. It follows that the effective spread, denoted by s_t^E, takes the form $s^E = (1-\lambda)(2\phi+\theta)$.

The effective spread, s^E, can be consistently estimated via $(1-\hat{\lambda})(2\hat{\phi}+\hat{\theta})$, with corresponding asymptotic variance given by

$$[-(2\phi+\theta), 2(1-\lambda), 1-\lambda]\, V_{\hat{\lambda},\hat{\theta},\hat{\phi}} \begin{bmatrix} -(2\phi+\theta) \\ 2(1-\lambda) \\ 1-\lambda \end{bmatrix}.$$

The solution for the effective spread shows that the bid-ask spread overstates the true cost of trading for two reasons. First, a transaction on the NYSE may execute between the quoted bid and ask prices (see, e.g., Lee and Ready (1991) and Blume and Goldstein (1992)), so that the higher the probability of mid-quote execution λ, the lower the cost of trading. Second,

[18]By contrast, Huang and Stoll (1996) find that the majority of the bid-ask spread is due to dealer costs. The differences in our results may partly be explained by the sample stocks, since they focuse on the Major Market Index stocks which are heavily traded.

the bid-ask spread overstates the cost of a round-trip transaction because prices tend to rise following a purchase and fall after a sell, as noted by Stoll (1985) and Glosten and Harris (1988).

To see how our estimates of s^E compare to the implied bid-ask spread we compute the ratio of the effective spread to the implied spread. The ratio r^E depends both on the probability of executing within the quotes and the relative magnitudes of the information and cost components of the spread. The statistic r^E can be estimated by

$$\hat{r}^E = \frac{(1-\hat{\lambda})(2\hat{\phi}+\hat{\theta})}{2(\hat{\phi}+\hat{\theta})},$$

with the following asymptotic variance,

$$\left[-\frac{(2\phi+\theta)}{2(\theta+\phi)}, \frac{(1-\lambda)(4\phi+3\theta)}{2(\theta+\phi)^2}, \frac{(1-\lambda)(3\phi+2\theta)}{2(\theta+\phi)^2}\right] V_{\hat{\lambda},\hat{\theta},\hat{\phi}} \begin{bmatrix} -\frac{(2\phi+\theta)}{2(\theta+\phi)} \\ \frac{(1-\lambda)(4\phi+3\theta)}{2(\theta+\phi)^2} \\ \frac{(1-\lambda)(3\phi+2\theta)}{2(\theta+\phi)^2} \end{bmatrix}.$$

Table 3 shows that the estimates of the effective spread, s_E, are substantially smaller than the implied spread, s. In particular, the ratio of the effective spread to the implied spread (r^E) is, on average, 50 percent at the beginning of the day and increases monotonically to 60 percent at the end of the day. Moreover, while this pattern is less reliable as measured by the average standard error estimates of 2-4.7%, there is a consistent theoretical explanation for the monotonicity result. Specifically, unlike the implied spread, the effective spread does not exhibit a U-shaped pattern. Indeed, the effective spread increases monotonically over the day, from 7.3 cents in the opening interval to 8.6 cents at the close, with average standard errors ranging from 0.19 to 0.50 cents at various times.

Thus, there is some evidence to suggest that the U-shaped pattern in implied spreads does not carry through to the effective spread measure. In fact, our result seems surprising because the implied spread S is actually highest in the opening period. This result reflects two factors. First, the effective spread takes into account the probability of execution within the quotes, and this probability decreases over the day (see Table 2). Second, the asymmetric information parameter is largest in the opening period, and this parameter has more impact on the implied spread than on the effective spread. This is because the effective spread takes into account the systematic tendency for prices to rise (fall) following a transaction at the ask (bid). In contrast, the market maker's transaction costs increase over the day, and this has relatively more impact on the effective spread.

The fact that the effective spread is smallest at the open has interesting implications for theoretical models (e.g., Admati and Pfleiderer (1988)) which predict that trading should concentrate at certain periods of the day. Our results suggest a natural explanation for why this concentration should occur at the beginning of the day rather than at other times. Intuitively, discretionary liquidity traders will migrate to periods where their effective costs of trading are lowest. Discretionary traders find it cheapest to trade at the start of the day, even though the degree of information asymmetry is highest; the high value of θ is balanced against the market maker's transaction costs which increase over the course of the day. In turn, the concentration of trading by such traders increases the probability that a transaction occurs within the spread, which also sustains a small effective spread.

4.2. The Determinants of Price Volatility

The model can be used to decompose transaction price volatility into its components. Using equation (3), the variance of stock price changes is

$$\mathrm{Var}[\Delta p_t] = \sigma_\epsilon^2 + 2\sigma_\xi^2 + (1-\lambda)[(\theta+\phi)^2 + (\theta\rho+\phi)^2 - 2(\theta+\phi)(\theta\rho+\phi)\rho]. \tag{4}$$

Volatility reflects the variance in public news shocks uncorrelated with trading activity. In equation (4), the portion of volatility arising only from news shocks is measured by σ_ϵ^2. In addition, volatility reflects various microstructure induced noise.

Microstructure noise, in turn, arises from several sources including price discreteness (measured by the term $2\sigma_\xi^2$), and terms involving the asymmetric information and cost components and their interaction. The portion of price volatility attributable to asymmetric information, denoted by A, is measured by the terms in equation (4) involving only the parameter θ, i.e., $(1-\lambda)(1-\rho^2)\theta^2$. Similarly, the portion of volatility arising from transaction costs alone is $B = 2(1-\lambda)(1-\rho)\phi^2$. Finally, the variance of prices in equation (4) also includes an interaction term, $C = 2\phi\theta(1-\lambda)(1-\rho^2)$. This decomposition shows that, other things being equal, asymmetric information and transaction costs increase volatility. However, the magnitude of their effects is inversely related to the autocorrelation of order flow. Intuitively, the bid-ask bounce is a source of transaction price volatility, and this effect is strongest when order flow is uncorrelated. A similar result carries through for the effect of changes in the market maker's economic rents (i.e., his execution and liquidity costs) measured by ϕ. Interestingly, there is an interaction between the asymmetric information and cost components; these two elements reinforce each other.

Hasbrouck (1991a, 1991b) uses a vector autoregression of quotes and returns to infer the proportion of price volatility attributable to trading. We build on this idea to distinguish the relative importance of public information and various market frictions on price volatility. The fraction of variance attributable to trading frictions is π, where

$$\pi = \frac{2\sigma_\xi^2 + (1-\lambda)[(\theta+\phi)^2 + (\theta\rho+\phi)^2 - 2(\theta+\phi)(\theta\rho+\phi)\rho]}{\sigma_\epsilon^2 + 2\sigma_\xi^2 + (1-\lambda)[(\theta+\phi)^2 + (\theta\rho+\phi)^2 - 2(\theta+\phi)(\theta\rho+\phi)\rho]}. \tag{5}$$

To estimate π we need to identify two additional model parameters governing the volatility of prices, σ_ϵ^2 and σ_ξ^2. To do this, we need two additional moment conditions. One condition is suggested by equation (4) which places restrictions on the variance of transaction price changes. In addition, the serial covariance between successive pricing errors in equation (3) is $-\sigma_\xi^2$, providing a second orthogonality restriction. Then, we can estimate the additional parameters using the moment conditions discussed in Section 2.2 together with the following moments

$$E\begin{pmatrix} (u_t-\alpha)^2 - (\sigma_\epsilon^2 + 2\sigma_\xi^2) \\ (u_t-\alpha)(u_{t-1}-\alpha) + \sigma_\xi^2 \end{pmatrix} = 0,$$

where u_t is defined as $\Delta p_t - (\theta+\phi)x_t + (\theta\rho+\phi)x_{t-1}$ and α is the estimated drift term.[19]

We can further decompose π into four parts: (i) the effect of price discreteness, (ii) the asymmetric information effect, (iii) the trading cost effect, and (iv) the interaction between these effects, as measured by the ratio of the terms $2\sigma_\xi^2$, A, B, and C, respectively, to the variance of price changes. These measures allow us to assess the relative contributions of the microstructure frictions to price volatility.

Table 4 provides summary statistics on the individual parameter estimates and the percentage of the variance in price changes attributable to (i) public information, (ii) price discreteness, (iii) asymmetric information, (iv) trading costs, and (v) the interaction between the asymmetric information and cost components.

[19]The estimates of the other parameters are unaffected by the addition of these moment conditions.

For each component, the table displays the average proportional contribution to total price volatility, the mean coefficient estimate of this component, its standard deviation, and its median estimate over the 274 stocks, by time of day.

The public information component of volatility, σ_ϵ^2, declines by about a third over the day. The decline is monotonic except for the last half hour interval where there is a small increase in the variance. As the variance decreases, the fraction of variance attributable to market frictions (i.e., π) increases steadily from 54 percent at the open to 65 percent at the close. This result is consistent with evidence provided by French and Roll (1986) who find that prices are more variable during trading hours than during non-trading hours. This can be explained if public information events are more likely to occur during business hours or if the process of trading creates volatility. Our estimates suggest that both public information shocks and noise generated by the trading process are important sources of intraday volatility, but that the relative importance of public information declines over the day.

The decline in σ_ϵ^2 over the day may reflect more frequent occurrences of public information events (such as corporate earnings or dividend announcements) early in the day or the overnight accumulation of news. The result is also consistent with price discovery. In this interpretation, the high variance of public information shocks at the open reflects investor disagreement about the interpretation of public news events whose 'fundamental' volatility may actually be constant over time. As market participants learn about market clearing prices through the process of price discovery, a consensus emerges that narrows the dispersion in beliefs and hence volatility.

Table 4 provides a detailed breakdown of the contribution of market frictions to volatility for various subperiods of the day. The variance of the rounding error, σ_ξ^2, is, as expected, very small in magnitude and in many cases is not significantly different from zero. Interestingly, this term tends to increase over the day, possibly because it captures a variety of microstructure noise, and the variance of these omitted noise terms is positively related to the magnitude of dealer costs, ϕ.

The impact of asymmetric information on volatility is small and actually declines over the day. In the 9:30-10:00 a.m. period, asymmetric information (measured by A) captures, across the 274 stocks, 13.5 percent of the price change volatility. By the end of trading, however, this component has been reduced to 7.6 percent.

In contrast, the part of the bid-ask bounce due to market maker trading costs, B, is large and increases steadily over the day. As a result, the effect of dealer costs on volatility also increases from 22 percent at the open to 35 percent at the close. The interaction effect, C, is also relatively important, and accounts for approximately 17 to 19 percent of the volatility.

To summarize, the volatility attributable to public information shocks and asymmetric information declines over the day, but this decline is offset by steady increases in the portion of volatility attributable to transaction costs and price discreteness. The net effect is that volatility is highest at the open and close and is smallest at midday.

4.3. Autocorrelation of Price Changes and Quote Revisions

In the absence of market frictions, our model implies that prices follow a random walk and thus transaction returns will not be autocorrelated. However, in the presence of frictions (which in fact exist), this will no longer be the case. Given the estimated frictions (i.e., $\hat{\theta}$, $\hat{\phi}$,...), what are the implications for time variation in transaction returns and quote revisions?

4.3.1. Transaction Prices

Using equation (3), we obtain

$$\text{Cov}(\Delta p_t, \Delta p_{t-1}) = -\sigma_\xi^2 + \rho(1-\lambda)[(\theta+\phi)^2 + (\theta\rho+\phi)^2] - (1-\lambda)(\theta\rho+\phi)(\theta+\phi)(1+\rho^2). \quad (6)$$

Simplifying this expression, we can show that $\text{Cov}(\Delta p_t, \Delta p_{t-1}) < 0$. Stock price changes are negatively autocorrelated if there are costs to providing liquidity (i.e., $\phi > 0$) or if there are rounding errors induced by price discreteness (i.e., σ_ξ^2), as these frictions generate bid-ask bounce. Larger frictions and greater information asymmetry increase the absolute magnitude of the serial covariance term. The absolute size of the covariance term is a decreasing function of the probability of executing within the spread, λ, because this mitigates the bid-ask bounce. The autocorrelation of order flow, however, has an ambiguous effect on the serial covariance term.

The above theoretical results provide explicit representations for the serial covariances of transaction price changes. We can, therefore, use the parameter estimates of Table 2 to generate implied autocorrelations of transaction price changes. Of some interest, these autocorrelations are using estimates of (θ, ϕ, ρ, λ, σ_ξ^2) derived from different moment conditions. Table 5 presents, for the five intraday time intervals, the mean implied autocorrelation (from the model and estimated parameters) for the 274 stocks in the sample. Note that these implied estimates are computed on a stock-by-stock basis using equations (4) and (6). The average implied autocorrelations are -.0972,-.2026,-.2501,-.2588, and -.2525 respectively over the five trading intervals. The two most important factors for explaining this autocorrelation pattern are the increase in the costs to providing liquidity over the day and the general decline in the level of the volatility of the public information flow between 9:30AM and 2:00PM.

It is interesting to relate the implied correlations with the actual correlations present in the data. If these implied autocorrelations capture some of the characteristics of the data, the structural model may provide clues as to what drives the short-horizon time-variation of returns. Including the results above, Table 5 also presents the mean sample autocorrelation of transaction price changes, the standard deviation of these estimates and the standard deviation of the difference between the actual and implied estimates across the 274 stocks.

There is a close correspondence between the actual and implied autocorrelation of transaction price changes, especially after the 9:30-10:00 period. For example, the sample autocorrelations after this period equal (-.2166,-.2433,-.2484,-.2197) respectively which matches the average implied autocorrelations pattern and magnitude described above. This is especially interesting because the autocorrelation moments were not used to estimate the underlying parameters of the model. Thus, the similarity between the actual and implied estimates suggests that the structural model does have information for the source of the autocorrelation of transaction returns. However, the implied autocovariances are more dispersed than the actual sample estimates, possibly because the parameters are estimated with noise. As predicted, the actual and implied autocovariances are negative (and fairly substantially so) in all time periods.

4.3.2. Quote Revisions

The autocorrelation between successive ask revisions implied by our model may be positive or negative.[20] To show this, note that the definition of the ask price implies that

$$p_t^a - p_{t-1}^a = \theta(1-\rho)x_{t-1} + \epsilon_t + \xi_t^a - \xi_{t-1}^a, \tag{7}$$

where ξ_t^a is the rounding error on the ask side. A similar equation holds for the revision in bid prices. Equation (7) shows that the change in the ask price is related to the previous trade, but the greater the autocorrelation in order flow, the less the revision in beliefs. Intuitively, if order flow is highly correlated, successive transactions at the ask are more likely than a reversal from ask to bid, and the revision in beliefs reflects this fact.

Using equation (7), we obtain

$$\text{Cov}(\Delta p_t^a, \Delta p_{t-1}^a) = -\sigma_{\xi^a}^2 + \theta^2(1-\lambda)\rho(1-\rho)^2, \tag{8}$$

where we assume that ξ_t^a is distributed independently with mean zero and variance $\sigma_{\xi^a}^2$. This expression is not generally equal to zero, even though (taking expectations in equation (1)), market makers' beliefs follow a martingale. Indeed, the autocorrelation of ask revisions is zero only in the special case where: (i) there are no rounding errors arising from price discreteness (i.e., $\sigma_{\xi^a}^2 = 0$), and (ii) there is no information asymmetry (i.e., $\theta = 0$) or the autocorrelation of order flow is zero (i.e., $\rho = 0$) or the trivial case where all trades occur at the midquote. The covariance of ask price revisions does not depend on the cost parameter ϕ, because this parameter affects transaction prices, not ask prices or the midquote.

While price discreteness induces negative covariance in ask revisions, the overall covariance in equation (8) may be positive or negative depending on the sign of the autocorrelation order flow and the relative magnitudes of the parameters.[21] Intuitively, if order flow is positively correlated, successive transactions at the bid or the ask are more likely than reversals. Market makers take this effect into account in forming their beliefs, so that the expected

[20]Models of inventory control (Madhavan and Smidt (1993), Huang and Stoll (1994), and Huang and Stoll (1996)) allow quote revisions to be positively autocorrelated, but we do not explicitly model inventory considerations here.

[21]This is consistent with recent empirical studies which find ask-to-ask returns and mid-quote returns do not follow martingales. See, e.g., Hasbrouck and Ho (1987) and Handa (1991). The autocorrelation may also represent time-varying expected returns as in Conrad, Kaul, and Nimalendran (1991).

revision in beliefs is zero. However, successive transactions at the bid or the ask will still lead to quote revisions unless they are fully anticipated, and this creates positive serial correlation in ask price revisions.[22] The larger the information asymmetry component and the lower the probability of a cross within the quotes, the stronger this effect. By contrast, if the autocorrelation in order flow is negative, the covariance term in equation (8) is unambiguously negative, because reversals are more likely than continuations and quotes are revised in the direction of order flow. Thus, ask price changes may contain important information about the underlying determinants of security price movements.

Table 5 shows that the actual serial correlation of successive ask price revisions is negative (as in Huang and Stoll (1994)), but is small in magnitude. The model's implied serial correlation of quote revisions is of similar magnitude. However, our implied estimates are, on average, closer to zero than the actual estimates and become more negative over the day, so that the deviations become steadily smaller over the day. Recall that the theoretical autocovariance of ask price revisions is negative if the variance in the rounding error, σ_ξ^2 is sufficiently large. The fact that our implied estimates are, on average, slightly larger than the actual estimates, suggests that the variance of the rounding error due to price discreteness is underestimated, especially in the early part of the day. Alternatively, this finding may reflect possible autocorrelation of these errors.

Table 5 also provides a comparison between the intraday patterns in the sample variance of ask price changes and the implied variance of ask price changes (from the model). Of particular interest, the actual variances and the implied variances both drop after the beginning of the day, and level off around midday. For example, the implied variances drop from .0042 to .0035, and then level off at .0031. Similarly, the sample variances drop from .0061 to .0048 and then level off at .0040. To the extent that the implied variances of ask price changes are calculated using parameters estimated from transactions data, the similarity in patterns provides us with a potential explanation for the true pattern in the variance of ask price changes.[23] In particular, the fall in the variance of ask price changes is due to the com-

[22]This may also occur because of the presence of stale limit orders. In our model, quotes are rational, so this factor is omitted.

[23]While the patterns are similar, it should be pointed out, that like the spreads, the implied variance of ask price changes is of a lower magnitude than actual estimates. We hope to explore the relation between this result and the likewise result for bid-ask spreads in future research.

bination of (i) price discovery, and (ii) less public information arriving to the market. With quote revisions, transaction costs (i.e., ϕ) are not relevant, so the increase in ϕ throughout the day does not lead to a corresponding increase in the variance.

It is important to point out that these implied estimates of variances and autocorrelations of quote revisions are derived from the structural parameters of the model, $\beta = (\theta, \phi, \rho, \lambda)$. These parameters are estimated using transactions data within the model's framework, and do not rely on direct estimation of moments of bid/ask price changes.

5. Discussion

In this section, we discuss the theoretical and empirical limitations of our approach and some avenues for future research.

5.1. Limitations of the Model

There are several dimensions on which the model does not perform well that require further discussion. First, the difference between the implied bid-ask spread and actual spreads is troubling. The expected spread is underestimated by approximately one-third systematically throughout the day. One explanation for this result is that some quotes are not representative of the price level at which trades might take place. For example, midquote transactions are far more common when quoted spreads are large. While our model does not address this issue, one could imagine placing additional structure on the model, such as a provision for *market* liquidity to have some role in determining spreads though not transaction prices. This more complex model might also address the model's failure to capture a second characteristic of the data, namely the level of quote revision volatility. If bid-ask spreads are subject to a stochastic, *market* liquidity factor, then this factor would also lead to an increase in quote revision volatility, which is not captured by our model.

Interestingly, the intraday patterns of both of these characteristics are explained by the structural model. This suggests that our explanation for intraday patterns may still be reasonable. The final characteristic not satisfactorily explained by the model are the autocorrelations of transaction price changes during the opening period. The two most likely explanations are that we either overestimate the volatility of the public information component during this period, or ignore the overall higher levels of trading volume at the opening. Below, we discuss some natural extensions of the model, which might help address some of these and other related issues.

5.2. Extensions

5.2.1. Volume

In terms of the structural nature of the model, it would be interesting to extend the model to incorporate volume. While this issue is perhaps less relevant for the frequently traded stocks in our sample where almost all the trades take place either at the quotes or within the quotes, it will be especially important for series involving inactive securities. Indeed, Huang and Stoll (1996) pursue such an extension for their model and find evidence of volume effects. Even within our sample, however, there is anecdotal evidence that volume may play a role. Table 1 shows that volume per transaction gradually decreases throughout the day which could explain the intradaily decrease in the asymmetric information parameter, θ. A comparison with Table 2, however, shows that the drop in θ is largest from the first half hour to the next period which is not consistent with the volume behavior described in Table 1.

Nevertheless, incorporating volume into this setting would be an important contribution. One approach to including volume, while still maintaining the structural framework, is data driven. Specifically, we can model θ as a function $\theta = \theta(q_t)$, where q_t denotes the (absolute) trade size. Theoretical models (e.g., Kyle (1985)) suggest linear functional forms of the type $\theta(q_t) = \theta_0 + \theta_1 q_t$, while empirical studies (e.g., Hasbrouck (1991a), Keim and Madhavan (1996)) show that a concave relation of the form $\theta(q_t) = \theta_0\sqrt{q_t}$. Using a Taylor series expansion, for example, the researcher can avoid making these parametric assumptions and instead write

$$\theta(q_t) = \alpha + \beta_1 q_t + \beta_2 q_t^2 + \dots,$$

where $\theta(q_t)$ is substituted into equation (3), and the model estimation of Section 2.2 continues as before, albeit with many more explanatory variables (x_t, x_{t-1},$x_t q_t$, $x_t q_t^2$, $x_{t-1} q_t$, $x_{t-1} q_t^2$, $\dots$).

Alternatively, this general model could be approximated by estimating equation (3) for different order size ranges. That is, one could specify multiple indicator variables for various order size ranges. If, say, we have k different size ranges, we could extend the model to allow for k different trade indicators corresponding to a buy or sell in each size range and have k different θ's. Allowing for ϕ to also vary with trade size would increase the number of moment equations by $2k - 2$, but would otherwise be straightforward. This approach is taken by Huang and Stoll (1996).

There are several problems with introducing volume via an empirical approach. First, care must be taken so that the results are not biased by very large trades that occur outside the specialist-auction system. In particular, large block trades originating in the so-called "upstairs" market may significantly bias the estimation. Previous research (see, e.g., Seppi (1990), Madhavan and Cheng (1996), and Keim and Madhavan (1996)) demonstrates that upstairs trades are likely to be associated with significantly lower information asymmetry because they originate in a non-anonymous trading mechanism. From an empirical viewpoint, this would suggest truncating trade size above a fixed order size (not necessarily 10,000 shares) as in Hausman, Lo, and MacKinlay (1992) or using the observed distribution of trade size to create a limit that varies from stock to stock, as in Madhavan and Smidt (1991). Second, in the empirical setting described above, volume is treated as an exogenous variable. Clearly, if the hypothesis is that volume has important information about the degree of asymmetric information, then volume must be treated as an endogenous variable. Thus, the dynamic relation between intraday price changes and trading volume needs to be jointly modeled. Recent research (Brock and Kleidon (1992), Foster and Vishwanathan (1993a)) provides a first pass at developing a theory of volume, but it remains an open question how models of this type can be integrated into the structural setting of this paper.

5.2.2. Time-Consistency of Parameters

Implicitly in the results of Section 3 is the notion that the parameters change as a function of time. For example, the asymmetric information parameter, θ, goes from 4.15 cents on average during the opening half hour to 3.18 cents in the next one and a half hour period. Taken as given, the model implies that at precisely 10:00 AM there is a discrete jump in the parameter value. Clearly, even if the model is correctly specified, this jump is just an approximation of reality. Another structural extension is to explicitly model the intraday evolution of the parameters as a nonlinear function of elapsed time. Similar to the empirical approach with volume described above, one could estimate $\theta = \alpha + \beta_1 \tau + \beta_2 \tau^2 + \ldots$ (where τ is the time elapsed from the start of the appropriate trading horizon, either the open or some event day of interest), and approximate the true functional form using this expansion.

However, similar to volume, this approach also ignores the underlying theoretical structure of the model. For example, *ceteris paribus*, do we believe that it is the time of day that determines the level of the parameters, or some underlying structure (such as information releases due to overnight news) that tends to coincide with the time of day? This is an important distinction because, on days in which there is no overnight accumulation of information, the parameter values will erroneously imply a relation between the parameters and time. Of course, developing a model in such an environment is not a straightforward task. The hope, however, is that the simple structure of our model can be built upon to address some of these more complex issues.

A related extension is to estimate the model over intervals other than a trading day. We expect the average daily level of θ may decline over the week, since traders would learn about fundamental values over the course of trading. Indeed, Foster and Vishwanathan (1993b) suggest that declines in adverse selection costs over the week can explain negative abnormal returns on Mondays. Similarly, Cao and Choe (1995) find evidence from a number of markets that transitory volatility declines over the week. A similar conjecture is that ϕ should increase over the week when the model is estimated using daily (rather than intraday) intervals, reaching a maximum on Friday just before the weekend. The idea here is that if ϕ partly reflects the cost of holding overnight inventory then this should be greatest immediately before the three nights of an upcoming weekend. There may also be more natural sample periods defined by economic events such as earnings or dividend announcements. Indeed there is considerable discussion in the accounting literature about the extent of asymmetric information around announcement dates. Our model provides a starting point to developing a method to perform an " event study" to answer questions of this sort.

5.2.3. The Order Flow and Price Processes

In this paper, we model a very simple process for order flow. There are two natural extensions to our model. The first is to allow order flow to follow a more general order Markov process. While we allow only one lag, other researchers, most notably Hasbrouck (1991), allow for a much richer lag structure. Moreover, Hasbrouck finds some evidence that these lags have additional information about the underlying price process. This extension takes on special importance for applications involving longer horizons. For example, if additional lags are important, price volatility due to transitory effects may be more or less persistent than implied by the first order Markov model of order flow embedded in equation (3).

The second extension is more complex, and relates to the theoretical extensions described in Sections 5.2.1 and 5.2.2 above. In theory, the specialist can elicit order flow of a given sign through his placement of quotes. If in fact this strategy is employed, then the sign of order flow may depend on the magnitude of price changes. Thus, a theory of order flow would need to be outlined and then implemented within our structural setting. For our data this consideration would seem less important because the stocks in our sample are actively traded and intraday inventory effects of the type discussed above are likely to be economically small (See, e.g., Madhavan and Smidt (1991), Hasbrouck and Sofianos (1993a)). While the development of a theory for order flow may be necessary to apply the structural model generally across all stocks, there have been some empirical investigations which address the endogeneity issue (see Hasbrouck (1993) and Huang and Stoll (1994)). Perhaps, the approaches in these papers, with some underlying theoretical justification, can lead to an extension of the model.

As a related issue, it may also be important to provide a more detailed description of the price process itself. Three possible applications are (i) differentiating the types of orders that go to the market, (ii) taking account of firm specific versus market-wide information, and, similarly, (iii) breaking down firm-specific trades into information-based events. With respect to (i), for example, Greene (1996) extends our model to allow for limit orders and finds evidence that such trades occur within the posted spread. For an application of (ii), one could build in the effects of observed market movements currently captured by ϵ_t by allowing this to be a function of movements in a major market index such as the S&P 500 index.

5.2.4. Price Discreteness

An additional extension of the model in this paper would be to formally model price discreteness. On the one hand, the results in this paper suggest discreteness will not have substantive effects. For example, the autocorrelation of the residual of price changes (i.e., transaction price changes minus its structural form in equation (3)) is close to zero. On the other hand, there are many applications in which this will not be the case. For example, the extension to price discreteness is especially important for our model if it is to be applied for low priced stocks. In this paper, we treat the rounding error of prices as some unspecified stochastic random variable which is i.i.d. Strictly speaking, given the structural model of equation (3), this treatment of price discreteness introduces a misspecification as compared to the actual data. Few papers have addressed this issue directly; a notable exception, Hasbrouck (1996) models the rounding down (up) to the bid (ask) formally in the context of a market microstructure model. Perhaps, the research design of that paper can be integrated into the structural framework of this paper, and the extensions of 5.2.1–5.2.3, to produce a more universal model of intraday price movements.

6. Conclusions

Security prices change because of new public information and through information revealed in the trading process. This paper develops a model to explain the evolution of prices over the day that embodies these two features. The paper's contribution is threefold. First, with a relatively simple structural model, many patterns in intraday bid-ask spreads, execution costs, price and quote volatility, and autocorrelations of price changes and quote revisions can be jointly explained. The model provides a unified framework that sheds light on why, over the day, (i) the variance of transaction price changes is U-shaped while the variance of ask price changes is declining, (ii) the bid-ask spread is U-shaped although information asymmetry and uncertainty over fundamentals is decreasing, and (iii) the autocorrelations of transaction price changes are large and negative, yet the autocorrelations of ask price changes are small and negative.

Second, because the model is so simple, the structural parameters can be estimated in a setting which provides a high comfort level in terms of estimation error. Indeed, the parameters can be estimated using transactions data alone, and can then be used to infer characteristics of transaction price changes and quote revisions. Further, the model's simplicity allows for natural extensions to incorporate variable order size, time-varying parameters, price discreteness, and more detailed descriptions of the order flow and information arrival processes. Integrating these extensions and incorporating some of the generality of the reduced-form approach may yield a more universal model of price formation.

Third, although our focus is on the evolution of intraday prices, a model of this type can be applied to many other issues of interest. A partial list of such topics includes: (i) an analysis of limit order execution probability and its impact on execution costs; (ii) the extent to which market structure or firm characteristics affects the speed of price discovery (measured by the rate of decrease of the information asymmetry parameter); (iii) the dynamic relation between price volatility and order flow; and (iv) an inter-market analysis of the components of the bid-ask spreads. These, however, are topics for future research.

Table 1

Descriptive Statistics for the Sample of Stocks (1990)

Panel A provides summary statistics on the variance of transaction price changes, variance of ask price changes, average number of transactions per day, market capitalization, and price for 274 NYSE-listed stocks in 1990. Panel B provides mean estimates of the variance of transaction and ask price changes, number of transactions per hour, the mean hourly volume (in round lots of 100 shares), the volume per transaction (in round lots of 100 shares) and the dollar spread for five time intervals during the day.

Panel A	Mean	Std.Dev.	75%	Median	25%
Variance of ΔP	.0067	.0025	.0079	.0062	.0051
Variance of ΔP^{ask}	.0044	.0030	.0059	.0037	.0025
Transactions/Day	95	86	107	66	44
Market Cap. ($ bn.)	4.36	6.95	4.42	2.21	1.02
Price ($)	38.85	21.82	49.13	36.63	22.25

Panel B	9:30-10:00	10:00-11:30	11:30-2:00	2:00-3:30	3:30-4:00
Variance of ΔP	.0073	.0068	.0065	.0066	.0070
Variance of ΔP^{ask}	.0061	.0048	.0042	.0040	.0040
Transactions/hour	17	16	12	13	17
Volume/hour (100s)	385.8	357.4	235.6	252.3	272.5
Volume/transaction	22.7	22.3	19.6	19.4	16.0
Spread ($)	.228	.211	.204	.205	.210

Table 2
Summary Statistics of GMM Model Parameter Estimates

Table 2 presents summary statistics of the GMM model estimates of the parameters for the 274 NYSE-listed stocks in the 1990 sample period over five intraday trading intervals. The table presents the mean coefficient estimate across the stocks, the mean standard error of the mean estimates, the standard deviation of the estimates across the 274 stocks, and the median estimate for the four main parameters of interest: θ, the asymmetric information component; ϕ, the transaction cost component; ρ, the autocorrelation coefficient of the order flow; and λ, the probability a trade takes place between the quotes.

	9:30-10:00	10:00-11:30	11:30-2:00	2:00-3:30	3:30-4:00
θ					
Mean	0.0415	0.0318	0.0275	0.0274	0.0287
(Av. Std.Er.)	(0.0057)	(0.0023)	(0.0019)	(0.0022)	(0.0038)
Std. Dev.	0.0277	0.0212	0.0190	0.0190	0.0200
Median	0.0355	0.0274	0.0234	0.0236	0.0241
ϕ					
Mean	0.0344	0.0402	0.0437	0.0450	0.0461
(Av. Std.Er.)	(0.0053)	(0.0021)	(0.0017)	(0.0021)	(0.0036)
Std. Dev.	0.0166	0.0125	0.0109	0.0111	0.0119
Median	0.0368	0.0419	0.0450	0.0469	0.0485
ρ					
Mean	0.4073	0.3676	0.3684	0.3789	0.3847
(Av. Std.Er.)	(0.0370)	(0.0184)	(0.0166)	(0.0203)	(0.0330)
Std. Dev.	0.0724	0.0657	0.0720	0.0763	0.0884
Median	0.4021	0.3663	0.3700	0.3838	0.3871
λ					
Mean	0.3360	0.3086	0.2893	0.2874	0.2825
(Av. Std.Er.)	(0.0218)	(0.0108)	(0.0097)	(0.0118)	(0.0184)
Std. Dev.	0.0984	0.0971	0.0984	0.0949	0.0920
Median	0.3411	0.3105	0.2888	0.2898	0.2886

Table 3
Summary Statistics of Estimated Trading Costs

Table 3 presents summary statistics of estimates of trading costs for 274 NYSE-listed stocks in the 1990 sample period over five intraday trading intervals. Specifically, the mean coefficient estimate across the stocks, the mean standard error of the mean estimates, the standard deviation of the estimates across the 274 stocks, and the median estimate are provided for various parameters of interest: s, the implied spread; r, the fraction of the implied spread attributable to asymmetric information; s^E, the effective bid-ask spread; and r^E, the ratio of the effective to the implied spread.

	9:30-10:00	10:00-11:30	11:30-2:00	2:00-3:30	3:30-4:00
s					
Mean	0.1518	0.1440	0.1425	0.1448	0.1496
(Av. Std.Er.)	(0.0066)	(0.0027)	(0.0024)	(0.0029)	(0.0048)
Std. Dev.	0.0331	0.0252	0.0233	0.0238	0.0246
Median	0.1467	0.1389	0.1380	0.1419	0.1461
s^E					
Mean	0.0728	0.0773	0.0814	0.0834	0.0864
(Av. Std.Er.)	(0.0050)	(0.0022)	(0.0019)	(0.0024)	(0.0040)
Std. Dev.	0.0142	0.0129	0.0125	0.0125	0.0123
Median	0.0735	0.0768	0.0808	0.0838	0.0863
r					
Mean	0.5107	0.4149	0.3630	0.3553	0.3601
(Av. Std.Er.)	(0.0378)	(0.0167)	(0.0138)	(0.0165)	(0.0270)
Std. Dev.	0.2527	0.2153	0.1977	0.1943	0.1994
Median	0.4812	0.3923	0.3345	0.3302	0.3210
r^E					
Mean	0.5019	0.5552	0.5888	0.5927	0.5947
(Av. Std.Er.)	(0.0469)	(0.0220)	(0.0196)	(0.0240)	(0.0381)
Std. Dev.	0.1435	0.1414	0.1409	0.1375	0.1360
Median	0.4975	0.5392	0.5747	0.5842	0.5899

Table 4
The Components of the Volatility of Transaction Price Movements

Table 4 presents summary statistics of estimates of the components of the volatility of transaction price changes σ_p^2 for 274 NYSE-listed stocks in the 1990 sample period over five intraday trading intervals. Specifically, the average proportional contribution of the particular component of volatility, the mean coefficient estimate of this component across the stocks, the standard deviation of the estimates across the 274 stocks, and the median estimate are provided for the various components: σ_ϵ^2, the variance of public information; $2\sigma_\xi^2$, the variance of the price discreteness variable; σ_θ^2, the variance of price changes due to asymmetric information; σ_ϕ^2, the variance of price changes due to transaction costs; and $\sigma_{\theta\phi}^2$, the variance of price changes due to the interaction of asymmetric information and transaction costs. The estimates of these components are based on the GMM coefficient estimates described in Table 2.

	9:30-10:00	10:00-11:30	11:30-2:00	2:00-3:30	3:30-4:00
σ_p^2	.007	.0068	.0065	.0066	.0070
σ_ϵ^2					
Prop of ΔP	0.4626	0.4064	0.3679	0.3565	0.3526
Mean	0.00363	0.00302	0.00264	0.00259	0.00267
Std. Dev.	0.00260	0.00227	0.00203	0.00204	0.00221
Median	0.00300	0.00238	0.00203	0.00202	0.00211
$2\sigma_\psi^2$					
Prop of ΔP	0.0161	0.0272	0.0346	0.0380	0.0357
Mean	0.00004	0.00015	0.00002	0.00022	0.00024
Std. Dev.	0.00065	0.00040	0.00039	0.00048	0.00086
Median	0.00013	0.00016	0.00020	0.00022	0.00023
σ_θ^2					
Prop of ΔP	0.1345	0.0916	0.0740	0.0726	0.0762
Mean	0.00122	0.00077	0.00060	0.00059	0.00065
Std. Dev.	0.00168	0.00103	0.00081	0.00089	0.00100
Median	0.00068	0.00042	0.00032	0.00033	0.00034
σ_ϕ^2					
Prop of ΔP	0.2158	0.2928	0.3416	0.3488	0.3484
Mean	0.00122	0.00165	0.00193	0.00200	0.00211
Std. Dev.	0.00092	0.00109	0.00112	0.00111	0.00114
Median	0.00104	0.00148	0.00178	0.00188	0.00204
$\sigma_{\theta\phi}^2$					
Prop of ΔP	0.1711	0.1820	0.1819	0.1841	0.1872
Mean	0.00112	0.00120	0.00119	0.00122	0.00130
Std. Dev.	0.00081	0.00053	0.00049	0.00056	0.00068
Median	0.00119	0.00122	0.00117	0.00123	0.00130

Table 5
Actual and Implied Moments of Price Changes and Quote Revisions

Table 5 presents summary statistics for the difference between actual and implied moments of price changes and quote revisions for the 274 NYSE-listed stocks in the 1990 sample period over five intraday trading intervals. The table presents the mean estimate of both the implied moment (from the model) and corresponding sample moment, the standard deviation of the sample moment across the 274 stocks, and the standard deviation of the difference between the implied and sample moment across the stocks. The particular moments computed are: $corr(\Delta P_t, \Delta P_{t-1})$, the autocorrelation of price changes; $corr(\Delta P_t^{ask}, \Delta P_{t-1}^{ask})$, the autocorrelation of ask price changes; and $var(\Delta P_t^{ask})$, the variance of ask price changes.

	9:30-10:00	10:00-11:30	11:30-2:00	2:00-3:30	3:30-4:00
$corr(\Delta P_t, \Delta P_{t-1})$					
Implied	-0.0972	-0.2026	-0.2501	-0.2588	-0.2525
Sample	-0.1719	-0.2166	-0.2433	-0.2484	-0.2197
Std. Dev.	0.1168	0.1114	0.1069	0.1054	0.1139
Std. Dev. of Diff.	0.1831	0.0975	0.0875	0.0971	0.1278
$corr(\Delta P_t^{ask}, \Delta P_{t-1}^{ask})$					
Implied	0.0117	-0.0123	-0.0285	-0.0333	-0.0315
Sample	-0.0358	-0.0573	-0.0626	-0.0597	-0.0458
Std. Dev.	0.0659	0.0464	0.0401	0.0428	0.0542
Std. Dev. of Diff.	0.0648	0.0512	0.0444	0.0513	0.0842
$\mathrm{var}(\Delta P_t^{ask})$					
Implied	0.0042	0.0035	0.0031	0.0031	0.0032
Sample	0.0061	0.0048	0.0042	0.0040	0.0040
Std. Dev.	0.0028	0.0025	0.0022	0.0022	0.0022
Std. Dev. of Diff.	0.0017	0.0020	0.0022	0.0022	0.0022

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