A New Strategy For Dynamically Hedging Mortgage-Backed Securities

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Abstract

This paper develops a new strategy for dynamically hedging mortgage-backed securities (MBSs). The approach involves estimating the joint distribution of returns on MBSs and T-note futures, conditional on current economic conditions. We show that our approach has a simple intuitive interpretation of forming a hedge ratio by differentially weighting past pairs of MBS and T-note futures returns. An out-of-sample hedging exercise is performed for 8%, 9% and 10% GNMAs over the 1990-1994 period for weekly and monthly return horizons. The dynamic approach is very successful at hedging out the interest rate risk inherent in all of the GNMAs. For example, in hedging weekly returns on 10% GNMAs, our dynamic method reduces the volatility of the GNMA return from 41 to 24 basis points, whereas a static method manages only 29 basis points of residual volatility. Moreover, only 1 basis point of the volatility of the dynamically hedged return can be attributed to risk associated with U.S. Treasuries, which is in contrast to 14 basis points of interest rate risk in the statically hedged return.

Introduction

Institutions hold significant positions in mortgage-backed securities (MBSs) for a variety of reasons. Whether these positions reflect trades on relative value or involve inventory holdings due to core businesses, hedging the interest rate risk of these securities is an important concern. This is especially true given the well-documented cases of huge monetary losses incurred by financial institutions and investment groups with respect to their MBS portfolios.

MBS valuation, and by extension hedging, is not a straightforward exercise. While fixed-rate MBSs issued by government agencies represent default-free claims to the interest and principal of the underlying mortgages, the timing of these cash flows depends on the prepayment behavior of the pool. In particular, as interest rates fall, individuals have an incentive to refinance existing mortgages at the new lower rates. Thus, fixed-rate MBS investors are implicitly writing a call option on a corresponding fixed-rate bond. Even though prepayments can occur for reasons not associated with interest rate movements, interest rates are the predominant factor in valuing MBSs. Because of this predominance, U.S. Treasury securities, or, more specifically, Treasury note (T-note) futures, are often used to hedge MBSs. The reasons are twofold: (i) T-note futures are very liquid derivatives, and (ii) the prices of these instruments are determined by the underlying term structure of interest rates and thus relate directly to the value of MBSs.

There are two common approaches to hedging MBSs using T-note futures. The first is purely empirical and involves the regression of past returns on MBSs against past returns on T-note futures. The resulting relation can then be used to hedge the interest rate risk of MBSs using the risk in T-notes. The advantage of this method is that it does not involve strong assumptions regarding the underlying model for the evolution of interest rates or prepayments. The disadvantage, however, is severe. This method is static in nature. It does not explicitly adjust the hedge ratio for changes in interest rates and mortgage prepayments.[1] That is, the observations used in the regression represent an average of the relation between MBSs and T-note futures only over the sample period, which may or may not representative of the current period.

As an alternative, the second approach is model-based. It involves specification of the interest rate process and a prepayment model. These assumptions then help map an MBS pricing functional to interest rates and possibly other factors.[2] The approach represents a dynamic method for determining comovements between MBS prices and T-note futures prices. Conditional on current values of the relevant economic variables and on particular parameter values, these comovements are completely specified. The problems with this approach are twofold. First, there is no consensus

regarding what is a reasonable specification of how the term structure moves through time, and how these movements relate to prepayment behavior. Any model price is going to be tied down closely to these possibly ad hoc assumptions.[3] Second, and more subtle, is the recognition that the parameter values themselves may often be "chosen" or estimated from a static viewpoint. For example, empirical prepayment models often reflect ad hoc projections of prepayment rates on sets of housing and interest-rate factors. Do the resulting coefficients, which represent an average of the relation in the past, have the same link to the variables describing the current period? Many of the well-documented MBS-hedging fiascoes would imply that this is not the case.

In this paper, we provide a first pass at bridging the gap between the regression and the model-based approaches. Our goal is to maintain some of the distribution-free properties of the purely empirical method, while recognizing the important dynamic properties of MBSs and other fixed income instruments. In particular, we propose a method based on estimating the conditional probability density of MBS returns, T-note futures returns, and relevant current information (such as the level, slope and curvature of the term structure, interest rate volatility, and prepayment history). This method allows us to hedge MBSs with T-note futures, conditional on current information. Thus, the hedge ratio is derived in a similar way to ones from existing empirical methods, but is dynamically adjusted depending on the current state of the economy.

Using data over the period 1987-1994, and performing an out-of-sample analysis, we find that this dynamic hedging method performs considerably better than the static regression method. For example, in hedging weekly returns on 10% GNMAs, our dynamic method reduces the volatility of the GNMA return from 41 to 24 basis points, whereas a static method manages only 29 basis points of residual volatility. Moreover, only 1 basis point of the volatility of the dynamically hedged return can be attributed to risk associated with U.S. Treasuries, which is in contrast to 14 basis points of interest rate risk in the statically hedged return.

Our analysis provides several additional insights. First, the analysis is straightforward to perform for any return horizon; thus, the hedge can be tailored to portfolio rebalancing over any interval, be it a day, a week, or a month. We investigate empirically this feature and comment on the differences between the hedge ratios for weekly and monthly returns. Second, we perform a detailed comparison between the static and dynamic empirical methods for hedging MBSs with T-note futures. Of particular interest, we show that the dynamic hedge is much more successful at reducing the risk associated with interest rate movements. Third, to the extent that some MBS return volatility remains after the T-note futures hedge, an empirical analysis of the possible de-

terminants of the unexplained variation is provided. The results suggest that systematic volatility does exist, but that it is not associated with movements in the term structure. Fourth, the MBS hedge is created under the assumption that the relation between MBS returns and T-note futures returns is linear. We evaluate this assumption nonparametrically, and comment on the potential need to adjust the hedge for movements in interest rates (and thus T-note returns) of varying magnitudes.

The paper is organized as follows. Section I describes the dynamic hedging method in some detail. An intuitive interpretation of the method is given vis a vis the empirical linear regression approach. Section II describes the data used in the study, and provides a preliminary analysis. Of particular interest, the relation between MBS returns and T-note futures returns is examined nonparametrically. Section III provides a detailed empirical analysis of the hedge ratios and hedged MBS returns. Section IV concludes the paper.

I. Dynamic Hedging of MBSs

In this section, we outline an approach for dynamically hedging MBSs using T-note futures. The basic idea is to estimate a conditional hedge ratio between returns on an MBS and returns on a T-note futures. The hedge ratio is conditional in the sense that we account for relevant current information. This is important for MBSs because, as interest rates change, expected future prepayments change, and thus the timing of the future cash flows also changes. In order to estimate this conditional hedge ratio, a structural model is usually required (as with model-based MBS valuation approaches). Unfortunately, this requirement involves making a number of assumptions on the underlying processes, which may or may not be reasonable.

In this paper, we take a different approach towards estimating the conditional hedge ratio. Using estimates of the joint and marginal probability densities of the return series and relevant variables, we estimate the conditional hedge ratio directly.[4] The advantage of this approach is that it does not require strong, model-specific assumptions. Of course, the disadvantage is that our approach introduces estimation error. Below, we describe the probability density estimation method, show how it is applied to calculating hedge ratios, and explain how it relates to the regression approach.

The Multivariate Density Estimation Method

In general, multivariate density estimation (MDE) is a method for estimating the joint density of a set of variables. Given the joint and marginal densities of these variables, the corresponding

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conditional distributions and conditional moments, such as the mean, can be calculated. Thus, one can imagine relating the expected return on an MBS to the return on a T-note futures, conditional on relevant information available at any point in time.

Suppose we are given T observations, $\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_T$, where each $\mathbf{z}_t$ is an m-dimensional vector. For our application, the vector $\mathbf{z}$ might include the MBS and T-note futures return, as well as several variables describing the state of the economy. One popular consistent measure of the joint density is the Parzen (fixed window width) density estimator. The density at any point $\mathbf{z}^*$ is estimated as the average of densities centered at the actual data points $\mathbf{z}_t$. The further a data point is away from the estimation point, the less it contributes to the estimated density. Consequently, the estimated density is highest near high concentrations of data points and lowest when observations are sparse.

More formally,

$$\hat{f}(\mathbf{z}^*) = \frac{1}{Th^m} \sum_{t=1}^{T} K\left(\frac{\mathbf{z}^* - \mathbf{z}_t}{h}\right),$$

where $K(\cdot)$ is called the kernel function (with the properties that it is symmetric and integrates to unity) and is often chosen to be a density function, h is the window width or smoothing parameter (which helps determine how tight the kernel function is), and $\hat{f}(\mathbf{z}^*)$ is the estimate of the probability density at $\mathbf{z}^*$.

The researcher has at his discretion the choice of kernel functions and bandwidths. For example, one popular class of kernel functions are the symmetric Beta family which includes the normal density, the Epanechnikov (1969) kernel and the biweight kernel as special cases. For the purposes of this study, we use the normal density,

$$K(\mathbf{w}) = (2\pi)^{-\frac{m}{2}} e^{-\frac{1}{2}\mathbf{w}'\mathbf{w}}.$$

With respect to the bandwidth, we use the following objective measure suggested by Scott (1992),

$$h_j = k_j \hat{\sigma}_j T^{-\frac{1}{m+4}},$$

where $\hat{\sigma}_j$ is the standard deviation of the variable z_j, and k_j represents an adjustment factor related to the sparseness of the data around the evaluation points.[5] Note that the choice of bandwidths can be quite general, e.g., different bandwidths for each data point (h_t) or each dimension $(h_j, j = 1, \ldots, m)$ as above.

Let $\mathbf{z}_t \equiv (R_{t,t+1}^{mbs}, R_{t,t+1}^{TN}, \mathbf{x}_t)$, where $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$ are returns on the MBS and T-note futures from t to $t+1$, respectively, and $\mathbf{x}_t$ is an $(m-2)$-dimensional vector of factors known at time t.

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We can then obtain the conditional mean, $\mathrm{E}[R_{t,t+1}^{mbs}|R_{t,t+1}^{TN},\mathbf{x}_t]$, i.e., the expected MBS return given movements in the T-note return, conditional on the current economic state as described by $\mathbf{x}_t$. Specifically, it is possible to show that

$$
\begin{aligned}
\mathrm{E}[R_{t,t+1}^{mbs}|R_{t,t+1}^{TN},\mathbf{x}_t] &= \int R_{t,t+1}^{mbs}\frac{f(R_{t,t+1}^{mbs},R_{t,t+1}^{TN},\mathbf{x}_t)}{f_1(R_{t,t+1}^{TN},\mathbf{x}_t)}dR^{mbs}\\
&= \frac{\sum_{i=1}^{t}R_{t-i,t+1-i}^{mbs}K_1^{t-i}(\cdot,\cdot)}{\sum_{i=1}^{t}K_1^{t-i}(\cdot,\cdot)}
\end{aligned}
\tag{1}
$$

where $K_1^{t-i}(\cdot,\cdot) \equiv K_1((R_{t-i,t+1-i}^{TN}-R_{t,t+1}^{TN})/h^{TN}),(\mathbf{x}_{t-i}-\mathbf{x}_t)/h)).$

Note that $K_1(\cdot,\cdot)$ is the marginal density, $\int K(z)dR^{mbs}$, which is also a multivariate normal density in our application (i.e., the joint density between the T-note futures return and the $m-2$ dimensional vector of variables describing the state of the economy).

Given $\mathrm{E}[R_{t,t+1}^{mbs}|R_{t,t+1}^{TN},\mathbf{x}_t]$, a hedge ratio can be formed by estimating how much the return on the MBS changes as a function of changes in the T-note futures return, conditional on currently available information $\mathbf{x}_t$. That is,

$$
\frac{\partial\mathrm{E}[R_{t,t+1}^{mbs}|R_{t,t+1}^{TN},\mathbf{x}_t]}{\partial R_{t,t+1}^{TN}} = \frac{\sum_{i=1}^{t}R_{t-i,t+1-i}^{mbs}\frac{\partial K_1^{t-i}(\cdot,\cdot)}{\partial R_{t,t+1}^{TN}}}{\sum_{i=1}^{t}K_1^{t-i}(\cdot,\cdot)} - \frac{\sum_{i=1}^{t}R_{t-i,t+1-i}^{mbs}K_1^{t-i}(\cdot,\cdot)\sum_{i=1}^{t}\frac{\partial K_1^{t-i}(\cdot,\cdot)}{\partial R_{t,t+1}^{TN}}}{\left[\sum_{i=1}^{t}K_1^{t-i}(\cdot,\cdot)\right]^2},
\tag{2}
$$

where it is possible to show that

$$
\frac{\partial K_1^{t-i}(\cdot,\cdot)}{\partial R_{t,t+1}^{TN}} = -[(R_{t-i,t+1-i}^{TN}-R_{t,t+1}^{TN})/(h^{TN})^2]K_1^{t-i}(\cdot,\cdot).
$$

Several comments are in order. First, equation (2) provides a formula for the hedge ratio between an investor's MBS position and T-note futures. For example, if $\frac{\partial\mathrm{E}[R_{t,t+1}^{mbs}|R_{t,t+1}^{TN},\mathbf{x}_t]}{\partial R_{t,t+1}^{TN}}$ equals .5, then for every \$1 of an MBS held the investor should short \$.50 worth of T-note futures. Second, this hedge ratio will change dynamically, depending on the current economic state described by $\mathbf{x}_t$. For example, suppose $\mathbf{x}_t$ is an $m-2$ vector of term structure variables. As these variables change, whether they are the level, slope, or curvature of the term structure, the hedge ratio may change in response. Thus, the appropriate position in T-note futures will vary over time. Third, the hedge ratio is a function of the unknown return on the T-note futures. If the conditional relation between MBS returns and T-note future returns is always linear, then the same hedge ratio will be appropriate, irrespective of how T-note futures move. If the relation, however, is not linear, then the investor must decide what type of T-note moves to hedge. For example, the investor might

want to form the MBS hedge in the neighborhood of the conditional mean of the T-note futures return, since many of the potential T-note futures returns will lie in that region. On the other hand, it may be the case that the investor is concerned about the tails of the distribution of T-note futures returns, and thus adjusts the hedge ratio to take account of potential extreme moves in interest rates and T-note futures. Fourth, the hedge ratio is horizon specific. In contrast to the instantaneous hedge ratio, our method's implied hedge ratio directly reflects the distribution of MBS returns over the relevant horizon. Thus, different hedge ratios may be appropriate for daily, weekly or monthly horizons.

Interpretation of the Dynamic Hedge Ratio

At first glance, equation (2) appears somewhat daunting. Its interpretation, however, is quite simple. To see this, first consider the hedging method of regressing MBS returns on T-note futures. The resulting regression coefficient, or hedge ratio, is given by

$$\beta = \frac{\sum_{i=1}^{t} R_{t-i,t+1-i}^{mbs} R_{t-i,t+1-i}^{TN} - T \mu_{mbs} \mu_{TN}}{\sum_{i=1}^{t} (R_{t-i,t+1-i}^{TN} - \mu_{TN})^2}, \tag{3}$$

where $\frac{1}{T} \sum_{i=1}^{t} R_{t-i,t+1-i}^{mbs} = \mu_{mbs}$ and $\frac{1}{T} \sum_{i=1}^{t} R_{t-i,t+1-i}^{TN} = \mu_{TN}$.

There is a clear interpretation of the hedge ratio implied by the β coefficient. That is, the hedge ratio is constructed by taking pairs of past MBS and T-note returns, and then equally weighting these pairs' comovements (in this case, by the variability of the T-note futures return). The problem with this approach is that all observations get equal weight. Thus, in estimating the hedge ratio today, comovements between MBS and T-note returns in high interest rate environments get the same weight as in low interest rate environments. A static hedge ratio, of course, is not appropriate for hedging MBSs.

The dynamic hedging strategy outlined above also has a clear interpretation. In fact, it is possible to rewrite equation (2) in the following form:

$$\frac{\partial E[R_{t,t+1}^{mbs} | R_{t,t+1}^{TN}, \mathbf{x}_t]}{\partial R_{t,t+1}^{TN}} = \sum_{i=1}^{t} R_{t-i,t+1-i}^{mbs} [(R_{t-i,t+1-i}^{TN} - R_{t,t+1}^{TN})/(h^{TN})^2] \omega_i(t) -$$

$$[\sum_{i=1}^{t} R_{t-i,t+1-i}^{mbs} \omega_i(t)][\sum_{i=1}^{t} [(R_{t-i,t+1-i}^{TN} - R_{t,t+1}^{TN})/(h^{TN})^2] \omega_i(t)], \tag{4}$$

where $\omega_i(t) = \frac{K_1^{t-i}(\cdot,\cdot)}{\sum_{i=1}^{t} K_1^{t-i}(\cdot,\cdot)}$.

Thus, the hedge ratio given in (4) is constructed by taking past pairs of MBS and T-note futures returns, and then differentially weighting these pairs' comovements by determining how

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"close" ($R^{TN}_{t-i,t+1-i}, \mathbf{x}_{t-i}$) pairs are to a chosen value of $R^{TN}_{t,t+1}$ and current information $\mathbf{x}_t$. Equation (4) then is similar in spirit to a regression hedge, except that the ω weights are no longer constant, but instead depend on current information. The nice idea behind density estimation is that these weights are not estimated in an ad hoc manner, but instead depend on the true (albeit estimated) underlying distribution of relevant variables. Thus, if the current information $\mathbf{x}_t$ is not close to $\mathbf{x}_{t-i}$ in a distributional sense, then ω_i puts little weight on the observation pair $R^{mbs}_{t-i,t+1-i} R^{TN}_{t-i,t+1-i}$.

Our approach has a clear advantage over the regression hedge given by (3). The hedge ratio in (4) explicitly takes into account the current economic state. For example, if interest rates are currently high, but the term structure is inverted, then more weight will be given to past comovements between MBSs and T-note futures in that type of interest rate environment. Thus, the hedge ratio adjusts to current economic conditions. In the remainder of the paper, we provide a detailed analysis of the performance of dynamic hedging strategies using data on Government National Mortgage Association (GNMA) MBSs. This analysis is performed out-of-sample, and thus avoids familiar overfitting problems that can arise within the estimation sample.

II. Data Description

Data Sources

In this paper, we employ three main data sources over the period January 1987 to May 1994: (i) mortgage-backed security prices from Bloomberg Financial Markets, (ii) 10-year T-note futures prices from Technical Tools Inc., and (iii) various term structure information, including the 3-month, 1-year, 5-year and 10-year yields from the Federal Reserve Board of Governors and Bloomberg Financial Markets.

With respect to MBS data, we collected weekly data on 30-year fixed-rate GNMA MBSs, with coupons of 8%, 9% 10%. The prices represent dealer-quoted bid prices on $X\%$ coupon-bearing GNMAs traded for forward delivery on a *to be announced* (TBA) basis.[6] Note that the TBA market is most commonly employed by mortgage originators who have a given set of mortgages that have not yet been pooled. However, trades can also involve existing pools, yet on an unspecified basis. This means that, at the time of the agreed-upon-transaction, the characteristics of the mortgage pool to be delivered (e.g., the age of the pool, its prepayment history, etc.) are at the discretion of the dealer.

With respect to the interest rate series, weekly data on the yield on the 3-month Treasury bill,

the 1-year Treasury bill, 5-year Treasury note and 10-year Treasury note were collected for the 1987-1994 sample period. From these series, we construct several potential interest rate factors to be used in the empirical section of the paper. Specifically, we relate unhedged and hedged GNMA returns to changes in the following factors:

- the 10-year yield (level),

- the spread between the 10-year yield and the 3-month yield (slope)

- the difference between one-half the sum of the 5-year and 3-month yields and the one-year yield (curvature).[7]

- interest rate volatility — where volatility is measured as an exponentially smoothed sum of past daily squared changes in the 3-month rate. We consider three different smoothing parameters — .99, .96 and .90. The exponentially smoothed measure is often preferred to historical volatility because it takes into account more recent information, and in fact has a representation much like the popular GARCH(1,1) process. As an aside, this volatility measure corresponds to JP Morgan's RiskMetrics ($\theta = .96$).

Data Analysis

Figure 1 graphs the 10-year yield (the conditioning variable used in our analysis) over the 1987-1994 sample period. Throughout the paper, we denote this yield as the *level of interest rates*. The figure also shows the point at which the out-of-sample analysis begins. An overall look at the figure suggests a decline in interest rate levels throughout most of the sample. However, it is also clear that there is substantial variation in the interest rate level over this period, which suggests it may have important dynamic information for pricing, and thus hedging, MBSs. Since the level of interest rates generally falls over the sample, the moneyness of the prepayment option of the MBSs comes into play. This effect is analyzed below.

In our discussion of the MDE method in Section I, we described the issue of how MBS returns can be estimated as a function of the unknown return on T-note futures. This is important because it tells us under what conditions the T-note futures will hedge all movements in MBS returns. As an illustration of the methodology, we focus on the MBS and T-note futures return relation, conditional on the level of interest rates. Specifically, over the entire sample period, Figures 2A and 2B graph the MDE-implied expected weekly return on the 10% and 8% GNMA respectively

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against contemporaneous movements in the T-note futures return, conditional on different levels of the 10-year yield.[8] Several interesting features of the relation are apparent.

First, conditioning on the current economic state does seem to matter — that is, the hedge ratio changes as a function of the level of interest rates. For example, consider the 10% GNMA in Figure 2A. At a 10-year yield of 6%, the line is quite flat, while at 9% it is relatively steep. When long-rates are at 6%, the prepayment option of the underlying mortgages of the 10% GNMA is deep in-the-money, and therefore MBS returns are relatively insensitive to interest rate changes. In contrast, when rates are 9%, the prepayment option is out-of-the-money, and MBSs behave much more like the corresponding fixed-rate bonds. When interest rates are high, MBS returns move inversely with interest rate changes, and therefore more closely with interest-rate sensitive securities such as T-note futures. This observation is even more apparent when we compare Figures 2A and 2B. The underlying mortgages of the 8% GNMA become in-the-money at much lower rates than the 10% GNMA. At each interest rate level, therefore, Figure 2B shows that the 8% GNMA return is more sensitive to T-note futures returns, with the slope being steeper at each point. Of some interest, the hedge ratios for the 8% GNMA are almost identical at interest rate levels of 9% and 7.5%. At these levels, small movements in interest rates (as described by the -1.5% to 1.5% weekly return on T-note futures) have similar effects on the MBS return because the prepayment options of the 8.5% mortgages are out-of-the-money.[9]

Second, over the range from -1.5% to 1.5% weekly return on T-note futures, the relation between conditional expected returns on GNMA TBAs and T-note futures is, for the most part, linear. Thus, the hedge ratio will not necessarily have to be adjusted for nonlinearity using T-note options or a similar interest rate derivative. This said, however, Figures 2A and 2B do display a small amount of concavity. For example, consider the 10% GNMA at an interest rate level of 9%. For a T-note futures return between -1.5% and 0%, the MBS return increases by 1%; in contrast, between 0% and 1.5%, the increase is only .8%. From a theoretical viewpoint, a small amount of concavity is to be expected. As interest rates fall, expected returns on MBSs become less sensitive to interest rate changes via the prepayment option. Figures 2A and 2B show that, for weekly MBS returns, this is not a first-order effect over the range of reasonable values of T-note futures returns.

III. Empirical Analysis

Because the relation between GNMA returns and T-note futures returns is for approximately linear, the hedging analysis to follow focuses on hedge ratios at the conditional mean of T-note futures

returns, conditional on the level of interest rates. To the extent there is any nonlinearity, the hedge ratio is still appropriate locally for realizations of T-note futures returns in the neighborhood of the mean of its conditional distribution. Since the level of interest rates is the predominant factor underlying both MBSs and T-notes, this variable provides a good starting point for a dynamic hedging analysis. The strategy is to compare the unhedged GNMA returns, the static linear hedged returns (equation (3)), and the dynamic MDE hedged returns (equation (4)), where these returns are calculated as follows:

1. The unhedged GNMA returns are calculated for horizons of one and four weeks over the sample period December 1989 to May 1994.

2. The static linear hedged return from t to $t+1$ is given by $R_{t,t+1}^{mbs} - \beta_t R_{t,t+1}^{TN}$, where β_t is the coefficient from regressing the MBS return on the contemporaneous T-note futures return over the past 150 weeks. β_t is reestimated every period via a rolling regression, and the monthly horizon hedge ratios are estimated using overlapping observations.

3. The dynamic MDE hedged return takes the same form, except β_t is now estimated using MDE in a rolling estimation over the past 150 weeks of data. In this analysis, we condition on the level of the 10-year yield; thus, β_t takes a measure of the past observations of R^{mbs} and R^{TN}, nonlinearly weighted by how "close" past interest rates are to the current level.

This section is organized as follows. First, we directly compare the hedge ratios implied by the two methods, both across different GNMA coupons and different return horizons. Second, a direct comparison of the hedging methods is provided, with an emphasis on documenting how much of the MBS return volatility remains, and whether this volatility is interest-rate dependent. Third, to the extent some volatility remains, we investigate the source of this volatility and whether the MBS hedge can be improved.

The Hedge Ratio

Figures 3A–3D provide the hedge ratio, β_t, for both the linear and MDE dynamic hedges for weekly returns on GNMA 10s, weekly returns on GNMA 8s, monthly returns on GNMA 10s and monthly returns on GNMA 8s, respectively. Over the 1990 to 1994 sample period, there is a general decline in the hedge ratio, irrespective of the method or horizon length. This is to be expected. Interest rates are declining during this period, and the MBS's prepayment option is becoming more in-the-

money. As a result, MBSs are less sensitive to interest rate movements — hence, the investor needs to hedge with smaller amounts of T-note futures.

At first glance, it may seem strange that the hedge ratio of the static linear method actually changes through time. Note, however, that the linear regressions are performed on a rolling basis. Thus, as observations enter and leave the estimation window, the estimated hedge ratio may change. Because interest rates are declining for most of the period, the hedge ratio also declines.

A comparison between the hedge ratios for the 10% GNMA (Figures 3A and 3C) and that of the 8% GNMA (Figures 3B and 3D) is especially revealing. While the overall pattern of the hedge ratios over time is similar, the magnitude of these ratios is quite different. In fact, the GNMA 8's hedge ratio is uniformly higher than the GNMA 10's. For example, consider the hedge ratio for weekly GNMA returns using the dynamic MDE method. In January 1990, every $1 at stake in an investment in a GNMA 8 TBA (GNMA 10 TBA) requires a $.85 ($.50) offsetting position in T-note futures, while in May 1994, a $.50 ($.20) offsetting position is required. The fact that the GNMA 8s require a larger position in T-note futures reflects their greater sensitivity to interest rates, not least the moneyness of the prepayment option.

For much of the sample, the dynamic MDE hedge requires a smaller position in T-note futures than does the linear hedge. This again should not be surprising. The MDE method conditions on current information; so, as interest rates fall, this fall is impounded immediately into the weights placed on prior pairs of MBS and T-note futures returns (equation. (4)). On the other hand, the rolling linear regression method introduces this information slowly, and with equal weight, as the estimation window changes. The speed at which information gets incorporated also explains why the MDE dynamic hedge ratios are much more variable than the linear static hedge ratio. Since the MDE-based hedge ratio adjusts immediately to information, the variability of the interest rate level (see Figure 1) induces variation in the dynamic hedge ratios. These points are very apparent over the last 70 weeks of the sample (January 1993 — May 1994). It is also a period in which investors suffered large losses in hedged portfolios due to overestimating the underlying interest rate sensitivity of MBSs. At least for the GNMA TBAs studied here, the dynamic method has made an appropriate adjustment during this latter period.

Finally, note that the MDE dynamic hedge positions change depending on whether the position is held for one week versus one month. For example, consider the GNMA 10 TBA. While the hedge positions for weekly and monthly returns generally decline over the entire sample, the hedge ratio for monthly returns is much more flat than for weekly returns during the 1993–1994 period.

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There are two possible explanations for this difference. Even though the full 150 weeks of data are used to form monthly returns, these returns are overlapping and thus contain less information than independent observations. Thus, the monthly hedge ratio may be less reliable than the weekly hedge ratio. Alternatively, the hedge ratio may just reflect the possibility that the level of interest rates implies different conditional changes in future interest rates over different horizons. Since it is commonly believed that interest rates are mean reverting, this explanation has some merit. It is an empirical question, however, whether this mean reversion implies enough differences between the conditional distribution of interest changes at weekly and monthly horizons.

Analysis of Hedged MBS Returns

Table 1 provides the mean, volatility and autocorrelation of the unhedged, linearly hedged, and MDE dynamically hedged 8%, 9% and 10% GNMA TBA returns for both 1- and 4-week horizons. Consider the mean returns of these hedged and unhedged positions for 1-week horizons. The weekly mean returns for the unhedged GNMAs are .078%, .077% and .069% respectively for the 8s, 9s and 10s, which translates to between 3.6% and 4.1% on an annual basis. Because TBAs are forward contracts, this mean return represents a risk premium for holding MBSs. Of course, we should note that this period is one in which interest rates generally fell, so these mean estimates may just reflect the particular sample period, rather than an ex ante risk premium per se. Nevertheless, since much of this premium may be compensation for bearing interest rate risk, it should not be surprising that the mean hedged returns drop to .005%, .020% and .027% for the 8s, 9s and 10s respectively (using MDE). This fall represents a decline in overall interest rate exposure due to the hedge itself.

The most interesting results from Table 1 are the standard deviations of the unhedged and hedged returns. First consider the weekly return horizons for the GNMA TBAs. The unhedged return volatility is 68.5, 53.1 and 41.4 basis points respectively for the 8s, 9s and 10s. The decline in the overall volatility for higher coupon GNMAs just reflects the effect of the prepayment option vis a vis interest rate movements. Linearly hedging MBSs using T-note futures in the static regression substantially reduces this volatility risk to 31.8, 30.9 and 28.6 basis points, respectively. Of particular interest, our dynamic method fares even better — the volatility of the hedged GNMAs is 28.4, 24.5 and 24.2 basis points, a 10%-20% drop relative to the static approach. With respect to using the MDE method, the largest gains are with the GNMA 9s and 10s, since the prepayment option is much more relevant for these securities given the level of interest rates over the sample

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period.

This same pattern in results carries through to the four week horizons. While the unhedged 8%, 9% and 10% GNMA returns face 136.4, 99.9 and 74.6 basis points of volatility respectively, the MDE (linear) hedge reduces this risk to 58.3 (59.9), 47.2 (57.3) and 44.0 (52.4) basis points. For example, suppose an institution with a large GNMA portfolio is faced with the possibility of a large negative realization (say 2 standard deviations) on their GNMA position. This means that, for a \$100 million one month exposure to MBSs, the MDE hedge reduces the value at risk from \$2.38 million to only \$1.04 million for the 8% GNMA.

Table 1 also gives the autocorrelation of the unhedged and hedged GNMA returns. Even if GNMA and T-note futures returns are not serially correlated (which is true for this sample), this does not imply that the hedged return will be serially uncorrelated. The reason is that the estimates of the hedge ratio, β_t, may be correlated through time either due to estimation error (via the rolling regression or MDE) or due to the conditioning variable (such as the interest rate level). Table 1 shows, however, that the serial dependency of β_t does not necessarily lead to serial correlation in the hedged position's returns. For example, for weekly returns, all of the autocorrelations are below 4%. With respect to monthly returns, the MDE hedged positions pick up minor autocorrelation — -9.6%, -8.3% and -8.5% for the 8s, 9s and 10s, respectively. The magnitudes are consistent with the hypothesis that these autocorrelations are not significantly different from zero.

So far, our discussion has centered around how much the hedging reduces the total volatility of the position in GNMA TBAs. While this is an important goal, it is only suggestive of how much interest rate volatility is removed. It is of interest to find out how much of the variation of the returns on the GNMAs is due to variation of T-note futures returns. Table 2 provides such information.

For example, consider the weekly return on the GNMA 8 (GNMA 10). Table 2 shows that 77% (51%) of the variation of the unhedged return on GNMA 8s (GNMA 10s) can be explained by variation in the contemporaneous T-note futures return. This translates to 60 (30) basis points of volatility due to T-note futures returns. This is in contrast to 10% (23%) explained variation in the static linear hedged return, which is equivalent to 10 (14) basis points of remaining volatility due to changes in T-note futures prices. The MDE dynamic hedge fares much better. Only .2% (.3%) of the remaining variation of the hedged return can be related to the T-note futures return — that is, only 1 basis point of T-note risk remains in either case. Clearly, if the goal of the hedging exercise is to remove risk associated with U.S. Treasuries, the MDE method is a clear winner on

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that score.[10]

As a final comment on the comparison between the static linear and dynamic MDE hedges, note that the correlations of the linear hedge with the weekly T-note futures return are negative (-32.1%, -50.1% and -48.2% for GNMA 8s, 9s and 10s, respectively). This means that the static method continually overhedges the GNMAs with T-note futures. This is because the linear method is slow to adjust to the decline in interest rates over the sample period. As a result, the hedge is mistakenly set-up based on data in relatively high interest rate periods, during which the prepayment option is out-of-the-money and MBSs are more sensitive to interest rate movements. Because the dynamic MDE hedge conditions on the current economic state, it does not suffer from this problem (recall also Figures 3A–3D).

If the MDE dynamic method for hedging MBSs succeeds in removing the primary source of interest rate risk, what is the source of the remaining volatility? As a first pass, Table 3 reports the contemporaneous correlations of the dynamically hedged and unhedged GNMA returns across coupons for both weekly and monthly returns. The contemporaneous correlations across coupons drops as the interest rate risk is hedged away; however, substantial cross-correlation still remains. For example, the correlation between unhedged weekly returns on GNMA 8s and 10s is 88%, while the correlation between their hedged counterparts is 66%.

Is this remaining 66% correlation all due to hedging error, or is there some systematic effect across GNMA TBAs? To check this, we also calculate the correlation between the unhedged and hedged GNMA returns. While the correlations are much smaller, they are not zero, ranging instead from 15% to 50%. As an example, consider weekly returns on GNMA 8s and 10s. 2.2% (11.8%) of the variation of hedged 8% GNMAs (10% GNMAs) can be explained by unhedged returns on 10% GNMAs (8% GNMAs). This is equivalent to 4.2 (8.3) basis points of volatility risk due to variation of MBS returns, not related to Treasury movements. This result suggests that measurement error cannot account entirely for the remaining volatility. Below, we investigate this point in more detail.

Sources of Variation of GNMA TBA Returns

While the above results demonstrated that the MDE dynamic hedge removes the risk associated with T-note returns, the question remains whether this hedge is sufficient to remove all types of fundamental interest rate risk. Tables 4A (weekly) and 4B (monthly) provide the R^2 and corresponding basis points risk of unhedged, linearly hedged and MDE dynamically hedged GNMA returns in relation to changes in the levels of various interest rate factors. These factors broadly

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span four different types of information: (i) the level of interest rates, (ii) the slope of the term structure, (iii) the curvature of the term structure, and (iv) interest rate volatility. (For more details, recall the description of the factor variables given in Section I).

Several observations are particularly interesting. Both the unhedged and the linearly hedged returns face considerable interest rate risk, which is consistent with the documented *ex post* correlation with T-note futures returns (see also Table 2). This is in stark contrast to the MDE hedged returns, which show little correlation with changes in any of the interest rate factors. That is, even though only T-note futures are used to hedge MBS returns, the dynamic hedge reduces volatility risk associated with the level, slope and curvature of the term structure. For example, consider the weekly return on the 10% GNMA. The MDE dynamic hedge leaves 1.3, 0.3 and 0.4 basis points of volatility due to these three factors respectively. In contrast, the unhedged (linearly hedged) returns face 28.6 (13.5), 18.5 (9.2) and 3.3 (2.5) basis points of volatility. To the extent that the volatility of the unhedged return is only 41.4 basis points, it is clear how successful the dynamic hedge is.

Similar results carry through to monthly returns (Table 4B). In particular, with respect to interest rate risk, the dynamic hedge is far superior to the static hedging method. However, there are two main differences relative to weekly returns. First, the GNMA returns hedged using the dynamic MDE hedge leave some residual risk which is correlated with interest rate changes. We cannot distinguish whether this is due to estimation error in the hedge ratios (see Figures 3B and 3D versus 3A and 3C), or whether it just reflects the difficulty in hedging longer horizon returns using only one instrument (T-note futures). Second, the curvature of the term structure now plays a larger role. For example, of the MDE hedged 8%, 9% and 10% GNMA return's volatility of 58.3, 47.2 and 44.0 basis points, respectively, changes in curvature from month to month explain 11.6, 13.0 and 11.1 basis points.

Tables 4A and 4B also show that regardless of the volatility measure used, contemporaneous changes in the volatility of interest rates have very little relation with either the unhedged or hedged GNMA returns series. Because the results are similar for all different values of the exponential smoothing parameter, Tables 4A and 4B report the volatility using only one measure, $\theta = .96$. While the lack of correlation could be due to estimation error with respect to our estimate of volatility, it should be noted that the measures used are quite standard in the literature (e.g., historical volatility and GARCH(1,1)).

The conclusions we reach from Tables 4A and 4B are twofold. First, the dynamic hedge removes

most, if not all, of the different types of interest rate risk inherent in MBSs. Of special interest, this hedge is performed using just one instrument (i.e., T-note futures) and conditions on just one economic variable (i.e., the level of interest rates). Second, this result means that the volatility of this hedged return is not related to interest rates. It also cannot be completely explained by errors due to either estimation or measurement of GNMA prices. This is because the hedged GNMA returns are correlated with unhedged GNMA returns (which are not estimated), and across different coupons (which makes systematic price errors less likely).

One possibility for explaining these correlations across GNMA returns is that interest rate changes affect these returns nonlinearly. Recall, from Figures 2A and 2B, that due to the prepayment option the relation between expected GNMA returns is concave (albeit slightly) in T-note returns. Hence, the T-note futures return may not be able to hedge away all MBS risk associated with large movements in interest rates. Since the correlations documented in Tables 2 and 5 may not detect this, it seems worthwhile exploring whether the dynamically hedged returns are related nonlinearly to interest rate movements. Given the concavity evident in Figures 2A and 2B, squared interest rate movements are a potentially good instrument for detecting this nonlinear relation, if it exists. The correlations of squared interest rate changes with the various GNMAs range from -10% to -20%, which is consistent with a concave relation. This amounts to approximately 4–5 basis points of nonlinear interest rate risk, not accounted for by the T-note futures hedge. One possible way to avoid this risk maybe to add another instrument which will allow us to capture these nonlinear movements, e.g., options on T-note futures.

Even if this nonlinear interest rate risk is accounted for, then the question remains: is the remaining volatility specific to particular GNMA securities (and thus more in the realm of estimation and GNMA price errors) or is it still systematic across all GNMAs?

If the remaining volatility is systematic, then one possible explanation is that GNMA returns are nonstationary due to pool seasoning, and that this nonstationarity induces correlated risk across coupons. To check this, we investigated the periods in which 8%, 9% and 10% GNMAs are originated. In these periods, given the declining interest rate environment, the newly originated GNMA pools are the most likely pools to be delivered in the TBA market. In contrast, when there are no originations, seasoned pools must be delivered. A cursory look at the behavior of the hedged GNMA returns vis a vis origination does not explain the remaining volatility. For example, from January 1990 to November 1992, a substantial number of 9% GNMAs were originated, while very few originations of GNMA 8s took place. The opposite situation occurred in the latter part of our

sample (12/92-5/94). If the remaining volatility is due to the seasoning of the mortgage pools, we would expect the correlation patterns of the 8% and 9% GNMAs to be different. Though there is a general decline in the magnitude of the correlations with unhedged GNMA returns over the sample, the correlation patterns of the 8% and 9% GNMAs are very similar.

Of course, the residual risk of a given GNMA could be hedged using other GNMAs. We investigated this issue by hedging GNMAs using GNMAs of nearby coupons (i.e., 10s with $9\frac{1}{2}$s, 9s with $8\frac{1}{2}$s, and 8 with $7\frac{1}{2}$s). As expected, the volatility of the hedged returns is considerably lower than that using T-note futures. For example, for weekly returns on the 10% GNMA, the volatility is 11.2 basis points. Of more interest, however, is the fact that this hedged return is -32.7% correlated with interest rate changes, leaving 3.7 basis points of residual interest rate risk (compared with only 1.3 for the T-note futures hedge). Moreover, there are a number of reasons why hedging GNMAs with other GNMAs may not be practical. First, the transaction costs are higher. While the TBA market is quite liquid (e.g., spreads of $\frac{1}{32}nd$ to $\frac{4}{32}nd$), T-note futures often exhibit lower spreads (e.g., between $\frac{1}{64}th$ and $\frac{1}{32}nd$). Second, this type of hedge may not be sensible if the institution's core business leads to positive holdings of GNMA of all coupons, or if an institution is trading on the price of MBSs relative to U.S. Treasuries.

If the institution/investor does not use other MBSs to hedge GNMAs, are there other securities which can serve this function? Without addressing this question specifically, it should be clear that the answer depends on the nature of the remaining volatility risk of GNMAs. If that risk is systematic across the economy (though not related to interest rates), then it is probably the case that some other security can hedge away this risk. It remains, however, an open question as to (i) what this risk is, and (ii) what securities can effectively hedge this risk. If the risk is systematic across only MBSs and thus idiosyncratic with respect to other securities, then the risk may not be relevant. That is, it can be diversified away within a portfolio of fixed income instruments and other assets.

IV. Concluding Remarks

This paper develops a new strategy for dynamically hedging mortgage-backed securities (MBSs). The approach involves estimating the joint distribution between returns on MBSs and T-note futures, conditional on current economic conditions. We show that our approach has a simple intuitive interpretation of forming a hedge ratio by differentially weighting past pairs of MBS and T-note futures returns. The weights are determined by how close, in a distributional sense, current

variables (like the level of interest rates) are to past values of these variables. As an application, an out-of-sample hedging exercise is performed for 8%, 9% and 10% GNMAs over the 1990-1994 period for weekly and monthly return horizons. The dynamic approach is very successful at hedging the interest rate risk inherent in all the GNMAs. For example, in hedging weekly returns on 8%, 9% and 10% GNMAs, our dynamic method reduces the risk of the GNMA returns associated with interest rate movements to only 1.2, 2.6 and 1.3 basis points, respectively. This is in contrast to 60.2, 42.0 and 29.5 basis points of interest rate risk if no hedge had been performed.

Clearly, our approach is general enough to be applied to a number of traded derivatives, not only MBSs. In fact, the only obstacle in practical applications is the need for past data on the derivative. Furthermore, the uses of MDE extend to the estimation of inputs which can be used in theoretical models. Therefore, when illiquid derivatives need to be priced and/or hedged, MDE, in conjunction with a theoretical model can be used. This point is discussed further in Boudoukh, Richardson and Whitelaw (1995), in the context of term structure-related inputs.

Endnotes

1. For a discussion of some of the problems associated with static hedges, see, for example, Breeden (1991) and Breeden and Giarla (1992). With respect to linear regression hedges in particular, Batlin (1987) discusses the effect of the prepayment option on the hedge ratio between MBSs and T-note futures.

2. Davidson and Herkowitz (1992) provide an analysis of the various theoretical methodologies for valuing MBSs in practice. In particular, the advantages and disadvantages of each approach are discussed in detail. With respect to the particular issue of hedging the interest rate risk of MBSs, Roberts (1987) gives an analysis, focusing primarily on model-based approaches to MBS valuation.

3. Chapter 15 of Hull (1993) provides a comparison and discussion of the major term structure models, with an emphasis on their underlying assumptions.

4. Boudoukh, Richardson, Stanton and Whitelaw (1994) develop a nonlinear, multifactor pricing model for MBSs using density estimation techniques. Their model can also be used to hedge MBSs. In particular, given the MBS price as a function of K factors, $K+1$ securities (including the MBS) can be used to hedge away all the relevant risks. Note, however, that within the framework of their model, they can provide only an instantaneously riskless position. They ignore some of the horizon-specific issues that arise in this paper.

5. One problem with multivariate density estimation is that the fixed window width density estimator produces (i) spurious peaks at points where the observations are sparse, and (ii) not enough resolution where the observations are dense. As discussed later in this section, much of the analysis of the relation between MBS and T-note futures returns is conducted at the mean T-note futures return and the level of the 10-year yield. Since many of the past return observations are centered around the mean, we let $k_{TN} = .5$ to produce greater resolution. In contrast, as the 10-year yield tends to lie in sparse regions relative to historical yields (due to the fall in interest rates over the sample), we let $k_{y_{10}} = 2$ in order to avoid spurious peaks. Note that these values are chosen ex ante, and are not necessarily the optimal ones given the data. Nevertheless, the choice should help alleviate some, if not most, of the spurious estimation problems.

6. Using the GNMA TBA price series, return series were calculated, with adjustments being

made once a month when the prices of the different contracts are spliced together. Note that the adjustment is made during the splice week using a version of the Cost of Carry model, modified for prepayments, known as the Dollar Roll Breakeven method (see Askin and Meyer (1986)).

7. These factors are commonly considered as the sources of interest rate risk, e.g., see Litterman and Scheinkman (1991).

8. The graph shows the relation over ranges of T-note futures returns and 10-year yields which occurred during the sample period. Within these ranges, the MDE approach interpolates between points. Problems can occur when it is necessary to extrapolate beyond the range of data in the sample.

9. Note that $X\%$ GNMAs are backed by $X + \frac{1}{2}\%$ underlying mortgages due to the servicing fee associated with MBSs.

10. Table 2 shows that similar conclusions are reached for hedges using monthly returns. For the MDE dynamic hedge, there is a little residual risk associated with T-note futures returns, but it is substantially less than both the unhedged and static linearly hedged GNMA returns.

TABLE 1

Comparison of Hedging Methods

	GNMA 8		GNMA 9		GNMA 10	
	1 wk.	4 wks.	1 wk.	4 wks.	1 wk.	4 wks.
Unhedged						
Mean (%)	.078	.326	.077	.316	.069	.286
Vol. (%)	.685	1.364	.531	.999	.414	.746
Autocorr.	.011	.138	-.019	.109	-.045	.082
Linear Hedge						
Mean (%)	.007	.041	.017	.086	.024	.126
Vol. (%)	.318	.599	.309	.573	.286	.524
Autocorr.	.015	-.107	.012	.021	.060	.039
MDE Hedge						
Mean (%)	.006	.123	.020	.178	.027	.189
Vol. (%)	.285	.583	.245	.472	.242	.440
Autocorr.	.011	-.096	.016	-.083	.036	-.085

Notes: Table 1 compares the mean, volatility and autocorrelation of unhedged returns on GNMA TBAs and hedged returns using two different approaches. The approaches involve hedging GNMAs with T-note futures, resulting in the hedged return, $R_{t,t+1}^{mbs} - \beta_t R_{t,t+1}^{TN}$, where $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$ are the out-of-sample returns on GNMAs and T-note futures respectively, and β_t, the hedge ratio, is estimated using the prior 150 weeks of data in one of two ways: (i) a linear hedge based on a regression of past $R_{t,t+1}^{mbs}$ on $R_{t,t+1}^{TN}$, and (ii) a MDE hedge using the distribution of $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$, conditional on the 10-year yield at time t. The estimation is performed on a rolling basis and covers the out-of-sample period, December 1989 to May 1994. Results are reported for both weekly and overlapping monthly returns.

TABLE 2

The Sensitivity of Unhedged/Hedged GNMA Returns to T-note Futures Returns

	GNMA 8		GNMA 9		GNMA 10	
	1 wk.	4 wks.	1 wk.	4 wks.	1 wk.	4 wks.
Unhedged						
Correlation	.878	.899	.792	.809	.711	.729
R^2	.770	.808	.627	.655	.505	.532
Vol. (basis pts)	60.2	122.6	42.0	80.8	29.5	54.4
Linear Hedge						
Correlation	-.321	-.367	-.507	-.552	-.482	-.509
R^2	.103	.134	.257	.304	.232	.259
Vol. (basis pts)	10.2	21.9	15.7	31.6	13.8	26.6
MDE Hedge						
Correlation	-.043	.198	-.107	-.046	-.056	-.158
R^2	.002	.039	.011	.002	.003	.025
Vol. (basis pts)	1.2	11.5	2.6	2.2	1.3	7.0

Notes: Table 2 reports the correlation of returns on hedged and unhedged GNMAs with contemporaneous T-note futures returns, the R^2 from the corresponding regression, and the magnitude of the variation of unhedged/hedged GNMA returns explained by T-note futures returns (measured by its volatility in basis points). GNMAs are hedged with T-note futures, resulting in the hedged return, $R_{t,t+1}^{mbs} - \beta_t R_{t,t+1}^{TN}$, where $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$ are the out-of-sample returns on GNMAs and T-note futures respectively, and β_t, the hedge ratio, is estimated using the prior 150 weeks of data in of two ways: (i) a linear hedge based on a regression of past $R_{t,t+1}^{mbs}$ on $R_{t,t+1}^{TN}$, and (ii) a MDE-based hedge using the distribution of $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$, conditional on the 10-year yield at time t. The estimation is performed on a rolling basis and covers the out-of-sample period, December 1989 to May 1994. Results are reported for both weekly and overlapping monthly returns.

TABLE 3

Contemporaneous Correlations of Unhedged/Hedged GNMA Returns

Upper Triangle (weekly)/ Lower Triangle (monthly)

		Unhedged GNMA			MDE Hedged GNMA		
		8%	9%	10%	8%	9%	10%
Unhedged	GNMA 8%	1.000	.951	.876	.341	.235	.149
	GNMA 9%	.954	1.000	.952	.317	.372	.241
	GNMA 10%	.875	.951	1.000	.343	.447	.438
MDE Hedged	GNMA 8%	.494	.423	.382	1.000	.824	.659
	GNMA 9%	.258	.330	.343	.840	1.000	.836
	GNMA 10%	.090	.177	.335	.640	.844	1.000

Notes: Table 3 reports the contemporaneous correlation between MDE dynamically hedged GNMA returns and unhedged GNMA returns. The GNMAs are hedged with T-note futures, resulting in the hedged return, $R_{t,t+1}^{mbs} - \beta_t R_{t,t+1}^{TN}$, where $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$ are the out-of-sample returns on GNMAs and T-note futures respectively, and β_t, the hedge ratio, is estimated using the prior 150 weeks of data. This MDE-based estimate uses the distribution of $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$, conditional on the 10-year yield at time t. The estimation is performed on a rolling basis and covers the out-of-sample period, December 1989 to May 1994. Results are reported for both weekly returns (upper triangular matrix) and overlapping monthly returns (lower triangular matrix).

TABLE 4A

The Sensitivity of Unhedged/Hedged GNMA Weekly Returns to Factor Changes

	GNMA 8			GNMA 9			GNMA 10		
	Unhdg	Linear	MDE	Unhdg	Linear	MDE	Unhdg	Linear	MDE
<u>Level</u>									
Correlation	-.863	.301	.023	-.775	.489	.092	-.691	.472	.053
R^2	.745	.090	.001	.601	.240	.009	.478	.223	.003
Vol. (basis pts)	59.1	9.6	.6	41.1	15.1	2.3	28.6	13.5	1.3
<u>Slope</u>									
Correlation	-.546	.231	-.001	-.487	.357	.048	-.447	.320	.012
R^2	.298	.053	.000	.237	.127	.002	.200	.103	.000
Vol. (basis pts)	37.4	7.3	0.0	25.8	11.0	1.2	18.5	9.2	0.3
<u>Curvature</u>									
Correlation	-.048	-.100	-.023	-.086	-.129	-.080	-.080	-.087	-.016
R^2	.002	.010	.001	.007	.0167	.006	.006	.008	.000
Vol. (basis pts)	3.3	3.2	0.7	4.6	4.0	2.0	3.3	2.5	0.4
<u>Volatility</u>									
Correlation	.016	-.028	-.039	.022	-.016	-.012	.033	.007	.029
R^2	.000	.001	.002	.001	.000	.000	.001	.000	.001
Vol. (basis pts)	1.1	0.9	1.1	1.1	0.5	0.3	1.3	0.2	0.7

Notes: Table 4A reports the correlation of returns on hedged and unhedged GNMAs with contemporaneous changes in various factors, the R^2 from the corresponding regression, and the magnitude of the variation of unhedged/hedged GNMA returns explained by these factor changes (measured by volatility in basis points). The factors include contemporaneous changes in the following interest rate variables : (i) 10-year yield (i.e., level), (ii) spread between 10-year and 3-month yields (i.e., slope), (iii) one-half the sum of the 5-year and 3-month yields minus the one-year yield (i.e., curvature), and (iv) interest rate volatility estimate based on an exponentially θ smoothed sum of past squared changes ($\theta = .96$). GNMAs are hedged with T-note futures, resulting in the hedged return, $R_{t,t+1}^{mbs} - \beta_t R_{t,t+1}^{TN}$, where $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$ are the out-of-sample returns on GNMAs and T-note futures respectively, and β_t, the hedge ratio, is estimated using the prior 150 weeks of data in one of two ways: (i) a linear hedge based on a regression of past $R_{t,t+1}^{mbs}$ on $R_{t,t+1}^{TN}$, and (ii) a MDE-based hedge using the distribution of $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$, conditional on the 10-year yield at time t. The estimation is performed on a rolling basis and covers the out-of-sample period, December 1989 to May 1994. Results are reported for weekly returns.

The Sensitivity of Unhedged/Hedged GNMA Monthly Returns to Factor Changes

	GNMA 8			GNMA 9			GNMA 10		
	Unhdg	Linear	MDE	Unhdg	Linear	MDE	Unhdg	Linear	MDE
<u>Level</u>									
correlation	-.888	.348	-.213	-.799	.523	.032	-.719	.488	.150
R^2	.789	.121	.046	.638	.280	.001	.518	.238	.023
Vol. (basis pts)	121.1	20.8	12.4	79.8	30.3	1.5	53.6	25.6	6.6
<u>Slope</u>									
Correlation	-.534	.319	-.053	-.437	.488	.182	-.386	.436	.252
R^2	.285	.102	.003	.191	.238	.033	.149	.190	.064
Vol. (basis pts)	72.8	19.1	3.1	43.7	28.0	8.6	28.8	22.8	11.1
<u>Curvature</u>									
Correlation	-.209	-.234	-.199	-.290	-.288	-.275	-.319	-.250	-.253
R^2	.044	.055	.039	.084	.083	.075	.102	.063	.064
Vol. (basis pts)	28.4	14.0	11.6	29.0	16.5	13.0	23.8	13.1	11.1
<u>Volatility</u>									
Correlation	-.002	.055	-.012	.005	.032	.002	-.020	-.024	.011
R^2	.000	.003	.000	.000	.001	.000	.000	.001	.000
Vol. (basis pts)	0.3	3.3	0.7	0.5	1.8	0.1	1.5	1.3	0.5

Notes: Table 4B reports the correlation of returns on hedged and unhedged GNMAs with contemporaneous changes in various factors, the R^2 from the corresponding regression, and the magnitude of the variation of unhedged/hedged GNMA returns explained by these factor changes (measured by volatility in basis points). The factors include contemporaneous changes in the following interest rate variables : (i) 10-year yield (i.e., level), (ii) spread between 10-year and 3-month yields (i.e., slope), (iii) one-half the sum of the 5-year and 3-month yields minus the one-year yield (i.e., curvature), and (iv) interest rate volatility estimate based on an exponentially θ smoothed sum of past squared changes ($\theta = .96$). GNMAs are hedged with T-note futures, resulting in the hedged return, $R_{t,t+1}^{mbs} - \beta_t R_{t,t+1}^{TN}$, where $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$ are the out-of-sample returns on GNMAs and T-note futures respectively, and β_t, the hedge ratio, is estimated using the prior 150 weeks of data in one of two ways: (i) a linear hedge based on a regression of past $R_{t,t+1}^{mbs}$ on $R_{t,t+1}^{TN}$, and (ii) a MDE-based hedge using the distribution of $R_{t,t+1}^{mbs}$ and $R_{t,t+1}^{TN}$, conditional on the 10-year yield at time t. The estimation is performed on a rolling basis and covers the out-of-sample period, December 1989 to May 1994. Results are reported for monthly overlapping returns.
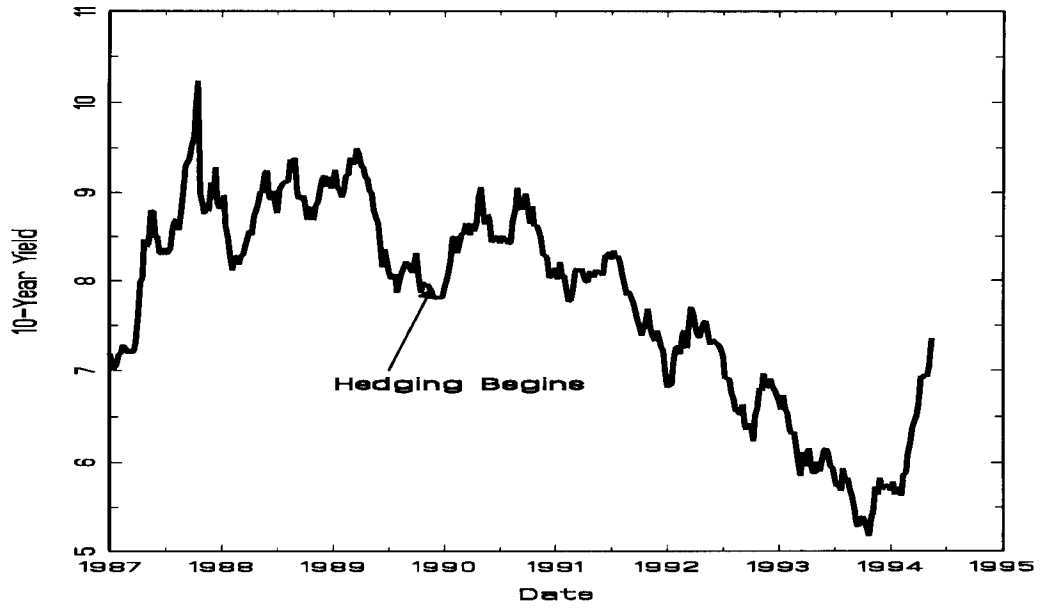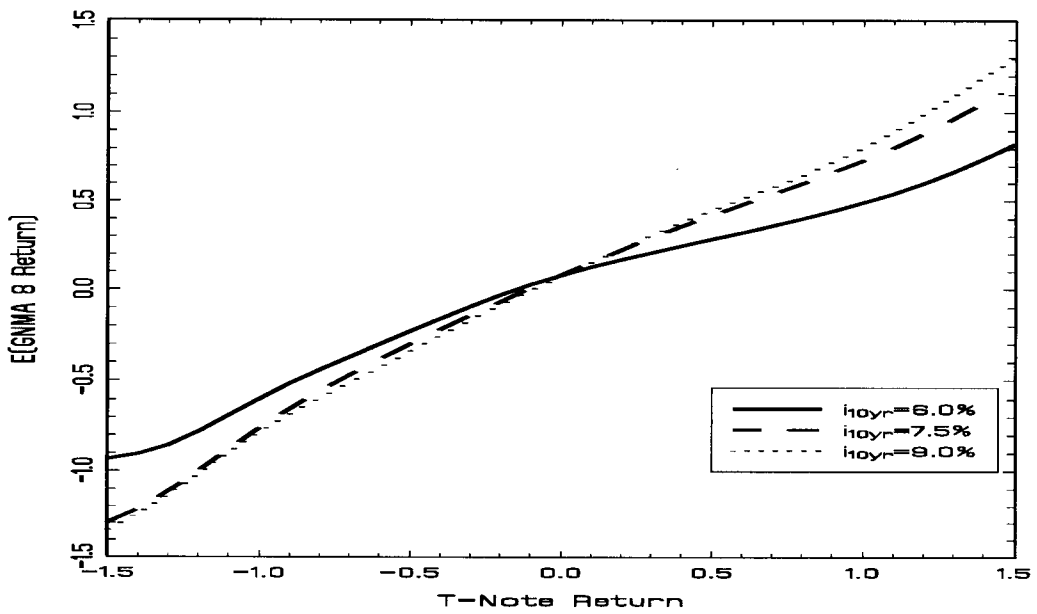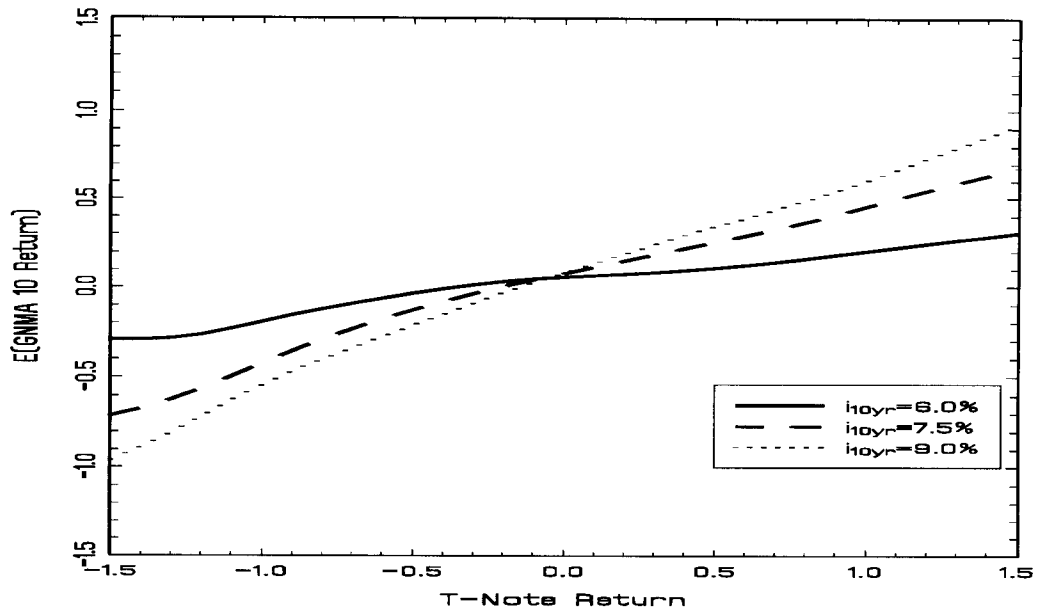
Figure 1: The yield on the 10-year Treasury note, weekly from January 1987 to May 1994.

Figures 2A-2B: The expected weekly return on a 10% (top) and an 8% (bottom) GNMA as a function of the contemporaneous 10-year T-note futures return, conditional on three different levels of the 10-year T-note yield. The relation is estimated using MDE over the period January 1987 to May 1994. Returns are in percent per week.
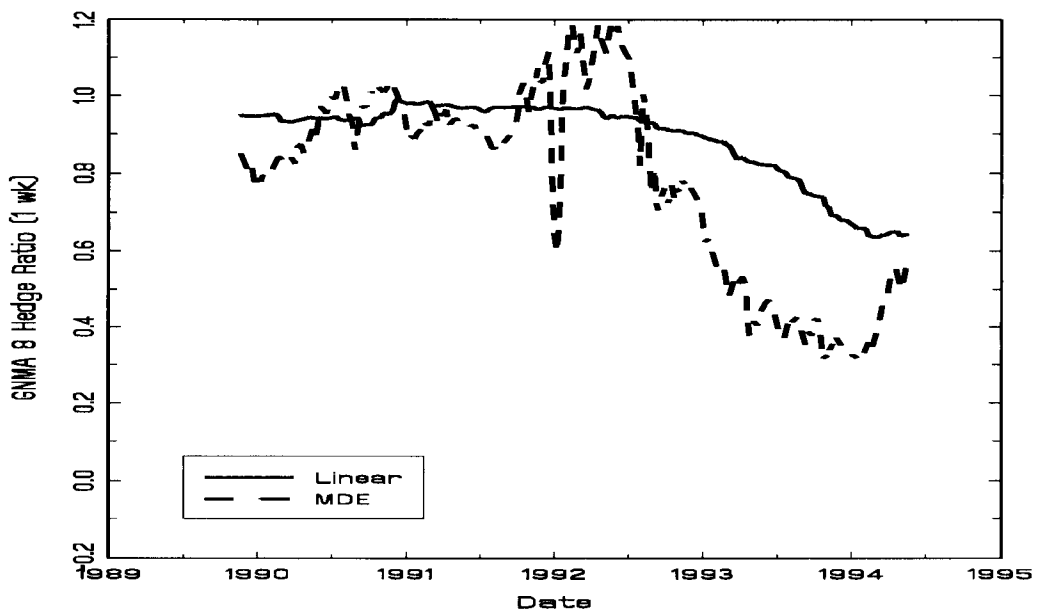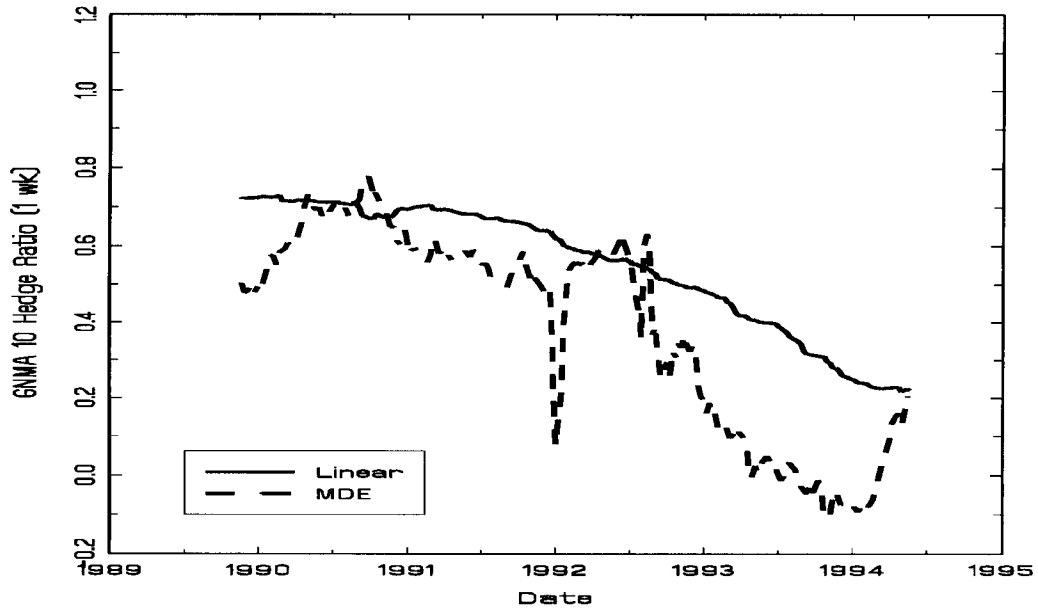
Figure 3A-3B: Hedge ratios for hedging weekly 10% (top) and 8% (bottom) GNMA returns using the 10-year T-note futures. Hedge ratios are estimated on a 150-week rolling basis using both a linear regression and MDE. The MDE hedge ratios condition on the level of the 10-year T-note yield.
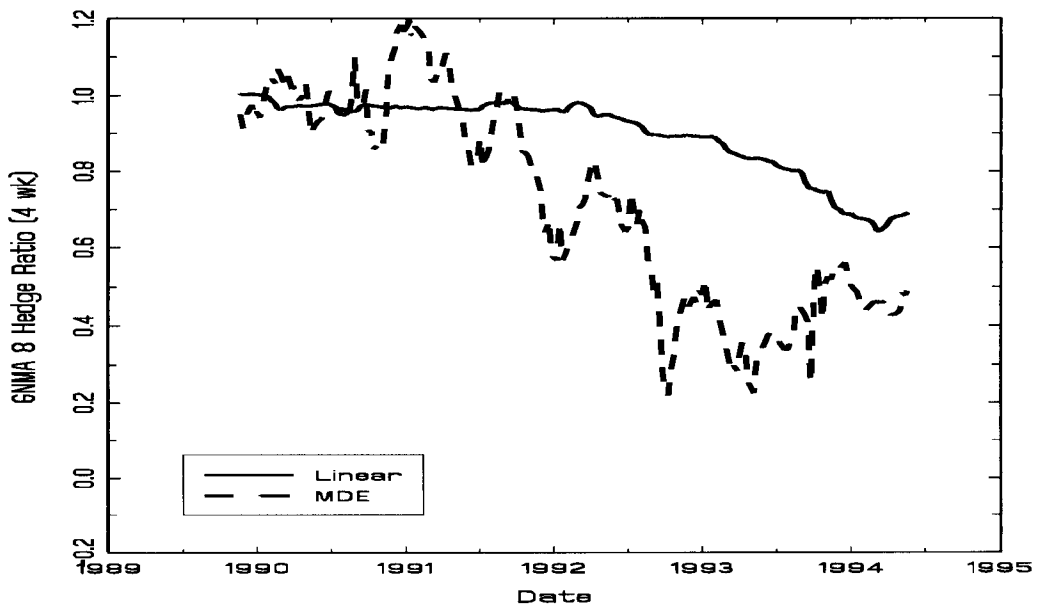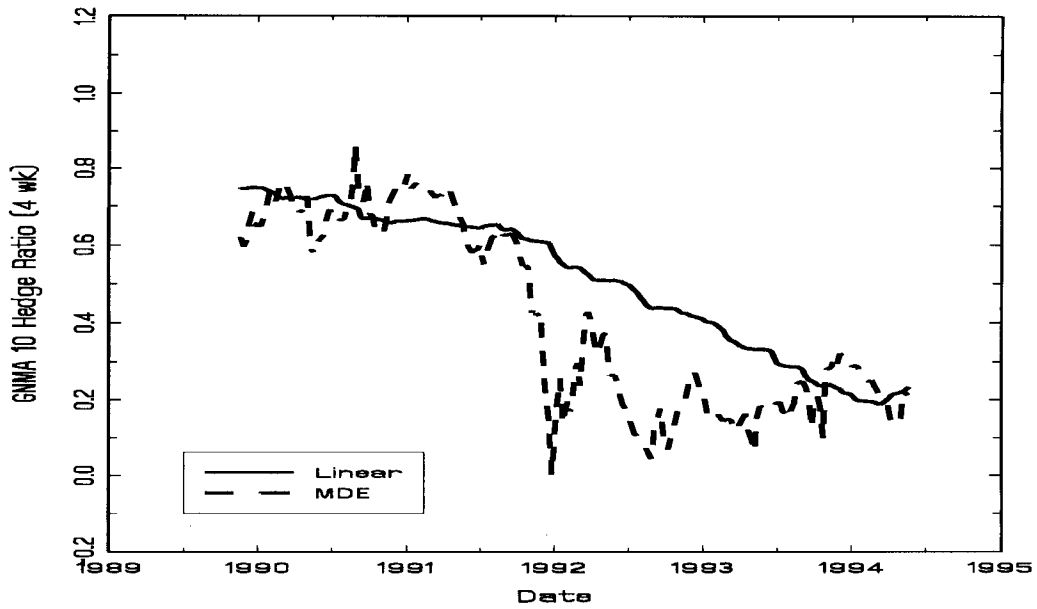
Figure 3C-3D: Hedge ratios for hedging monthly 10% (top) and 8% (bottom) GNMA returns using the 10-year T-note futures. Hedge ratios are estimated on a 150-week rolling basis using both a linear regression and MDE. The MDE hedge ratios condition on the level of the 10-year T-note yield.

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