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Key words: nonlinear cross-equation restriction, asset pricing, bond returns.

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Abstract

This paper examines asset pricing theories for treasury bonds using longer maturities than previous studies and employing a simple multi-factor model. We allow bond factor loadings to vary over time according to term structure variables. The model examines not only the time variation in the *expected* returns of bonds but also their *unexpected* returns. This allows us to explicitly test some asset pricing restrictions which are difficult to study under existing frameworks. We confirm that the pure expectation theory of the term structure of interest rates is rejected by the data. Our empirical study of a two-factor model finds substantial evidence of time-varying term-premiums and factor loadings. The fact that factor loadings vary with long term interest rates and yield spreads suggest that bond return volatilities are sensitive to interest rate levels.

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One of the major topics in empirical finance has been the testing of asset pricing theories. The vast majority of this literature is concerned with explaining expected returns on common equities. (For recent examples see Fama and French (1989) and Campbell and Mei(1993)) However, recently a number of authors have examined asset pricing theories for bonds. These studies can be placed in two categories.

One type of research is concerned with explaining changing risk premiums over time. Campbell (1987), Campbell and Hamao (1991), Fama and French (1989), Keim and Stambaugh (1986), and Stambaugh (1988), all related returns on bond indexes to other economic variables to explain changing risk premiums over time. A second type of research has involved tests of the Cox, Ingersoll and Ross (1985) valuation model and its generalizations. (See Brown and Dybvig (1986), Longstaff and Schwartz (1992), and Pearson & Sun (1989)).

In this paper, we will use a multi-factor model to explain bond returns. Tests of single factor models have generally found that one factor provides a poor explanation of bond prices. Most researchers have found at least two factors as useful in explaining the structure of bond returns (See Elton, Gruber and Michaely (1990), Brennan and Schwartz (1983), Nelson and Schaefer (1983), or Longstaff and Schwartz (1992)). Our model extends the work of Stambaugh (1988) and other authors cited above by modeling not only the time-variation in expected returns but also by characterizing the movements in unexpected returns. We believe a good bond model should be able to describe both types of movements in bond prices.

We will examine our model for government bonds using a maturity range longer than that employed in previous studies. The data are McCulloch's estimates of spot and forward rates. The advantage of this choice is that we can examine a broader range of maturities.[1] We find that some of the characteristics of risk premiums change when longer maturities are utilized.

The paper is divided into four sections. In the first section we specify the multi-factor model and discuss the test procedures that are used. In the second section we discuss the data. In the third section we discuss the results of our tests. The fourth section summarizes the results.

I. Modeling Rates of Return

In the spirit of much of the recent empirical works on bond valuation, we start by assuming that the price of all default free bonds can be modeled in terms of a small set of state variables.[2] We will illustrate the models we use in terms of the assumption of two state variables although the methodology we employ later in the paper allows us to explicitly test this assumption against other alternatives (either a larger number or a smaller number of

[1] Researchers using pure discount bonds to estimate spot rates have generally focused on spot rates up to 6 months (See Fama (1984)) or occasionally 12 months (See Stambaugh (1988)). The exceptions to this are Fama and Bliss (1987) and Jorion and Mishkin (1991), who did examine spot rates with up to 5 years to maturity.

[2] See Brennan and Schwartz (1983), Cox, Ingersoll and Ross (1981), Nelson and Schaefer (1983) for examples of this approach.

relevant state variables).[3] The two variables we use are the long rate and the spread between the long rate and the short rate. These can be viewed as the two state variables determining bond prices. Alternatively, as Cox, Ingersoll and Ross (1981) have shown, if prices are deterministic functions of two unknown state variables, such that its possible to invert the system then two interest rates can be used as proxies for the unknown state variables. We will refer to our two interest rates as state variables though we understand they may in fact be proxies for two other unobservable state variables.[4] In this paper we will restrict our attention to zero coupon bonds. This is not unnecessarily restrictive as any coupon paying bond can have its price expressed as a weighted average of a set of zero coupon bonds. To represent the price of a zero coupon bond as a function of the state variables, let us define

y_L = the spot rate on a long bond

y_S = the spot rate on a long bond minus the spot rate on a short bond (term spread)

t = the time a bond is valued

i = the time at which the bond matures

[3]The assumption of two state variables in bond returns is made by Brennan and Schwartz (1983), Nelson and Schaefer (1983), and Longstaff and Schwartz (1991). Elton, Gruber and Michaely (1990)provide empirical evidence that support this assumption.

[4] We use the long rate and the spread rather than the long rate and the short rate because as Nelson and Schaefer (1983) have shown the long rate and the spread are nearly uncorrelated. We have repeated much of the analysis sing the long rate and the short rate rather than the long rate and spread. The results are almost identical and the conclusion unchanged.

$P(y_L, y_S, t, i)$ = the price of the bond which is a function of the four variables in parenthesis

$y(y_L, y_S, t, i)$ = the interest rate on a zero coupon bond of maturity i.

The price of any zero coupon bond can be expressed as

$$P(y_L, y_S, t, i) = e^{-(i-t)y(y_L, y_S, t, i)} \qquad (1)$$

To value a bond we need to model the process driving the relevant state variables. We assume that both the long rate and the spread follow a Weiner process which exhibits mean reversion and that the long rate has a standard deviation which is proportional to the square root of the long rate itself.

Thus,

$$dy_L = K_L(u_L - y_L)dt + \sqrt{y_L}\sigma_L dz_L \qquad (2)$$

$$dy_S = K_S(u_S - y_S)dt + \sigma_S dz_S \qquad (3)$$

Where

1. u_L and u_S are the mean long rate and spread respectively.
2. σ_L and σ_S are the instantaneous standard deviations.
3. K_L and K_S are reversion parameters.
4. dz_L and dz_S are standard Weiner processes.

While both the long rate and the spread are assumed to revert to their respective means, the proportionality assumed in the stochastic term for the long rate precludes the possibility of long rates turning negative.[5] Spreads while reverting to the mean can in fact be negative.

Employing Ito's Lemma we find that[6]

$$\frac{dP}{P}=\frac{\partial P}{\partial y_L}dy_L+\frac{\partial P}{\partial y_S}dy_S+\frac{dP}{\partial t}dt+1/2\frac{\partial^2 P}{\partial y_L^2}y_L\sigma_L^2dt+1/2\frac{\partial^2 P}{\partial y_S^2}\sigma_S^2dt \qquad (4)$$

Employing equation (2), (3) and (4) we can express the rate of return on any bond as

$$\frac{dP}{P}=u_i dt-(i-t)\,\sigma_L\sqrt{y_L}\frac{dy}{dy_L}dz_L-(i-t)\,\sigma_S\frac{dy}{dy_S}dz_S \qquad (5)$$

Where $u_i = f(y_L, y_S, t, i)$. While this expresses the return on any bond at any point in time we can learn more about the return process by using relative pricing relationships and the no-arbitrage condition. More specifically at any moment in time we can write the above equation as

$$R_i = \bar{R_i}(t, y_L, y_S) + B_{iL}(t, y_L, y_S)dz_L + B_{iS}(t, y_L, y_S)dz_S \qquad (6)$$

Notice that expected values for any bond are not fixed but are functions of at least t, y_L, and y_S. Furthermore, from (5), the

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[5]Other powers of the long rate in the stochastic process will also preclude negative rates. Thus in the empirical section we will examine the sensitivity of our analysis to including the long rate raised to powers other than one half.

[6]The term which contains the covariance of $z_L z_S$ is omitted because of the assumption, based on previous empirical work, that z_L and z_S are uncorrelated and thus the term is zero.

sensitivity coefficient of any bond to dz_L and dz_S are functions of at least t, the long rate and the spread (both directly in the coefficients and indirectly through the derivatives). This equation must hold exactly for any bond. If we examine three arbitrary government bohds A, B, and C and let X_i, stand for the fraction of funds invested in bond i, we can always form a portfolio for which

$$X_A B_{AL} + X_B B_{BL} + (1 - X_A - X_A) B_{CL} = 0$$

and

$$X_A B_{AS} + X_B B_{BS} + (1 - X_A - X_A) B_{CS} = 0$$

Since these bonds have zero default risk, a portfolio with zero sensitivity to the long and short rate must return the riskless rate or

$$X_A \overline{R_A} + X_B \overline{R_B} + (1 - X_A - X_A) \overline{R_C} = R_F$$

for there to be no arbitrage available in the market. Thus, we must have,

$$\overline{R}_i - R_{Ft} = \lambda_{Lt} B_{iLt} + \lambda_{St} B_{iSt}, \qquad (7)$$

where the λ's represent the price of the systematic risk associated with the long rate and the spread and where the λ's and B's are conditional values depending on a set of information which is available at the time expectation are formed. As pointed out above, B_{iL}, B_{is} and λ's are functions of the long rate and the spread at the time expectation are formed. Restricting the information set to contain only the long rate and the spread and assuming the conditional risk premiums (and B's) are linear in the state variables, we have

$$\begin{aligned} \lambda_L &= c_1+c_2y_L+c_3y_s \\ \lambda_s &= c_4+c_5y_L+c_6y_s \\ B_{iL} &= \beta_{i1}+\beta_{i2}y_L+\beta_{i3}y_s \\ B_{is} &= \beta_{i4}+\beta_{i5}y_L+\beta_{i6}y_s \end{aligned} \qquad (8)$$

The linear form we have assumed is a simple alternative to constant risk premiums and betas.[7] It follows naturally from the linear form used for conditional expected returns by Campbell (1987), Ferson (1990), and others. It nests constant risk premiums and betas as special cases. It can be thought of as a Taylor approximation to some nonlinear relationships of risk premiums and betas with the state variables.

Combining equation (6), (7), and (8), we have

$$r_{it} = b_{i0}+b_{i1}y_{Lt}+b_{i2}y_{st}+b_{i3}y_{Lt}y_{st}+b_{i4}y_{Lt}^2+b_{i5}y_{st}^2 + \tilde{\epsilon}_{it} \qquad (9)$$

where $r_{it} = R_{it} - R_{Ft}$ is the excess return on a bond and $\tilde{\epsilon}_{it}$ is the unexpected excess return: $\tilde{\epsilon}_{it} = B_{iL}dz_L + B_{is}dz_s$.

[7]For conditions under which risk premiums are linear in state variables see Duffie and Kam (1993).

Given that (9) is derived from (7) and (8), the coefficients matrix of (9) should satisfy the following relationship:

$$(b_{i0}, b_{i1}, b_{i2}, b_{i3}, b_{i4}, b_{i5}) = (ß_{i1}, ß_{i2}, ß_{i3}, ß_{i4}, ß_{i5}, ß_{i6}) \begin{vmatrix} c_1 & c_2 & c_3 & 0 & 0 & 0 \\ 0 & c_1 & 0 & c_3 & c_2 & 0 \\ 0 & 0 & c_1 & 0 & c_2 & c_3 \\ c_4 & c_5 & c_6 & 0 & 0 & 0 \\ 0 & c_4 & 0 & c_6 & c_5 & 0 \\ 0 & 0 & c_4 & c_5 & 0 & c_6 \end{vmatrix}$$

The above restrictions hold for each of the N maturities. Thus the constraint becomes

$$\tilde{b} = ß\,\tilde{\theta} \qquad (10)$$

where $\tilde{b}$ and $ß$ are matrices with 6 columns and N rows and $\tilde{\theta}$ is 6 by 6. It is easy to see from the above equation that

$$\text{rank}\ (\tilde{b}) \leq \min\ [\text{rank}\ (ß),\ \text{rank}\ (\tilde{\theta})] \qquad (11)$$

By employing a latent variable test in the spirit of Campbell (1987) and Ferson (1989), we can examine the rank of $\tilde{b}$. Gaining insight into the rank of $\tilde{b}$ may give us added insight into the structure of the model. For example, a rank of $\tilde{b}$ greater than zero implies that some coefficients in θ are not zero, which rejects the pure expectation theory of term structure (no term premiums). As a second example, if the rank of $\tilde{b}$ is greater than three, then we might have a two-factor model with time-varying betas or a three-factor model with constant betas.[8]

The rank restriction test in the latent-variable framework does not exploit the unique structure of the θ matrix. Recognizing in equation (9), $\tilde{\varepsilon}_{it} = B_{iL}dz_L + B_{is}dz_s$, substituting in the expression for B's contained in equation (8) and using the symbol Δz rather than dz for the random component since we approximate a continuous series with discreet data, we arrive at the following equation

$$r_{it} = b_{i0} + b_{i1}y_{Lt} + b_{i2}y_{st} + b_{i3}y_{Lt}y_{st} + b_{i4}y_{Lt}^2 + b_{i5}y_{st}^2 + \beta_{i1}\Delta z_L + \beta_{i2}y_{Lt}\Delta z_L + \beta_{i3}y_{st}\Delta z_L + \beta_{i4}\Delta z_s + \beta_{i5}y_{Lt}\Delta z_s + \beta_{i6}y_{st}\Delta z_s + e_{it}, \quad (12)$$

where e_{it} is the approximation error. Equation (12) can be utilized to estimate the beta coefficients explicitly and to test the restrictions given by (10). Equation (10) can be viewed as cross-equation restrictions with some parameters unknown.

Equation (12) is similar in spirit to the multi-factor model of Ferson (1990), in which he uses residuals from regressions to proxy for the factors. However, we assume that the yield and spread variable follow discreetized Weiner processes. Ferson estimates his model in a single step, while we estimate equation (12) in two-steps. First, we estimate the innovations in the discreetized Weiner processes(the Δz_L and Δz_s), and then we use the

[8]We can not distinguish the two models unless we specify what the factors are.

innovations as factors in estimating equation (12). Following Ferson (1990), we also test the cross-sectional asset pricing restrictions imposed on the parameters of (12) by equation (10).

To test the cross-sectional asset pricing restrictions, we estimate the unrestricted model (12) jointly for all bonds and obtain a variance-covariance matrix for all the parameter estimates. Then, we solve for the θ matrix in equation (10) using the parameter estimates. θ is a nonlinear function of the parameter estimates. The exact procedure is explained in detail in the appendix. It is easy to see from equation (10) that the elements in θ are restricted in certain ways, namely, some elements are restricted to be zero or the same as others. We will test these restrictions using a Wald-test given the θ estimates and its variance-covariance matrix. Unlike the chi-square test of Hansen (1982), the Wald-test only require us to estimate the unrestricted model, which can be performed with a simple seemingly unrelated regressions (SUR) procedure. Thus, (10) can be tested without using a nonlinear optimization procedure. Because of the large number of parameters and orthogonality conditions involved in a GMM estimation, we think a simple linear regression approach is preferable.[9] In the study, we also use the Wald-test to test various simplifications of the θ matrix. In particular, we test whether the risk premiums, λ_{Lt} and λ_{St}, are constant over time, as

[9]We need to solve a nonlinear optimization problem for 48 parameters if we use the GMM approach to estimate equation (12) for seven bonds. As is well known, GMM estimation of large number of parameters with nonlinear restrictions often result in lack of convergence.

well as the hypotheses that the sensitivities, B_{iL} and B_{iS}, are constant over time.

To obtain estimates of innovations in the long rate and the spread, Δz_L (and Δz_S), a simplified version of the three-step Ferson and Foerster (1994) approach is used to estimate residuals from the following discreet form of (2) and (3),

$$\Delta y_{Lt} = a + b\ y_{Lt} + \sqrt{y_L}\sigma_L \Delta z_L \tag{13}$$

$$\Delta y_{St} = c + d\ y_{St} + \sigma_S \Delta z_S \tag{14}$$

In the first step, an OLS regression is used to estimate the a and b parameters in equation (13). In the second step, a WLS regression is used to re-estimate (13) with the weights being the absolute values of the residuals calculated from the OLS regression. Then in the last step, we construct the Δz_L series by taking the residuals from that WLS regression and deflating them by $\sqrt{y_L}$.[10]

II. Data

The data we use are the McCulloch estimates of spot and instantaneous forward rates. The exact procedure used to derive these estimates is discussed in McCulloch (1987) and Shiller (1990). The McCulloch estimates have become the standard for

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[10] Ideally, we should deflate them by $\sqrt{y_L}\sigma_L$, but the constant σ_L will not affect our study, since it only affects the scales of the betas in regression (12) but not the cross-equation restriction (10). Δz_S is obtained by a similar procedure.

extracting estimates of spot rates from a full set of yield data.[11] McCulloch utilizes a full data set which makes it possible to average out errors introduced by bid ask spreads, non synchronous trading, and random pricing errors. Campbell and Shiller (1991) compare the McCulloch data to the Fama and Bliss data and find the estimates are very similar over the maturities included in the Fama and Bliss data. However, the McCulloch data includes bonds of much longer maturities. Among the techniques that utilize the full data set, the McCulloch estimates are widely accepted as the best, even by researchers who developed alternative procedures (See Nelson and Schaefer (1983)).

The data supplied to us were continuously compounded spot and forward rates for thirty-one maturities of 1, 2, 3,..., 18 months, 21 months, 2 years, 3 years,..., 13 years.[12] The estimates were provided at a monthly interval over the 40 year period January 1947 to March 1987, a total of 483 observations for each maturity.

To estimate (12), we needed to convert spot rates to rates of return. For maturities up to eighteen months the procedure was straight forward since we have spot rates (and hence bond prices) for all relevant maturities. For maturities greater than 18 months, monthly spot rates are not in our data set for all required maturities and thus must be estimated. How this estimation is

[11] McCulloch assumes a constant tax rate in his estimation. The issue of bond clienteles (and therefore multiple tax rate) has been a subject of recent research (see Ronn and Shin (1992)).

[12] There are some missing data for bonds with maturities longer than 13 years. Since we require data in each month, 13 years is the maximum maturity we can include.

accomplished is best explained by an example. To estimates the 23 month continuously compounded spot rate, we subtracted the instantaneous forward rate associated with the 24-month bond from the 24 month rate times 24 and dividing the resulting number by 23. Tests of this procedure over maturities where both actual and estimated yields were available showed a very close fit.

IV. Empirical Results

Summary statistics for selected treasury bonds are presented in Table 1. The excess returns are calculated as continuously compounded monthly holding period returns in excess of the one-month treasury bill rate. They are given in percent per month. The first row of panel A presents the mean excess returns for bonds with different maturities, which first increase and then decrease with maturity. The second row is the standard deviation of the monthly returns. The standard deviation displays a monotonic increasing relationship with maturity. Panel B of Table 1 gives correlations of excess return across different maturities. Excess returns on bonds with similar maturities are highly correlated while the correlation is much smaller for bonds with different maturities.

Figure 1 provides the time series plot of the long rate y_L and the spread y_S. We utilized the yield to maturity on a ten year zero-coupon bond as the long rate and the spread between the yield to maturity on a ten year bond and a three month bill as the spread.[13] As one can see, the long rate and spread are a lot more volatile during the late 70's and early 80's, which reflect the change of monetary policy during that time period.

Table 2 presents the results of the regression of excess returns on the state variables and their quadratic terms, which are known at the beginning of the period. The t statistics have been corrected for heteroscedasticity based on the White (1980) procedure.

The explanatory power is similar to that reported by Campbell (1987), and Stambaugh (1988). This is true even though we use a different set of lagged variables and we examine the relationship not just for maturities less than one year, but rather for maturities up to 13 years.[14] Note that from the F-test we can reject at very high level of statistical significance, the hypothesis that all slope coefficients in equation (9) are jointly zero. Thus, we could rule out the pure expectation theory of the term structure of interest rates which states that the risk premiums should be zero. We also perform the test of $b_{i3}=b_{i4}=b_{i5}=0$ for each bonds. The significance level for the F-tests are 0.012,

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[13] We tried a number of other combinations. The results are described below.

[14] We tried alternative state variables and obtained similar results. These alternatives include the yield on a 10-year pure discount instrument and the spread in yield between a 10-year and 6-month, a 7-year yield and the spread between a 7-year and a 3-month yield, a 4-year yield and the spread between a 4-year yield and 8-month yield and a 1-year yield and the spread between a one year and a three month yield. Using very short rates, 1-year and 3-months did produce a slightly higher R^2 in equation (9), but produced a lower R^2 for equation (12).

0.038, 0.019, 0.011, 0.033, 0.030, and 0.019 respectively. Thus, we reject the hypothesis of $b_{i3}=b_{i4}=b_{i5}=0$.

Table 3 presents tests of the Rank restrictions described earlier. This methodology is the same as that employed by Stambaugh (1988). From Table 3 we can begin to draw conclusions about the rank of the $\tilde{b}$ matrix. From Table 3 we can see that there is mixed evidence concerning rejection of the hypothesis that the rank ($\tilde{b}$) = 1. While on the basis of the overall sample, we can not reject the rank one at the 10% level, we could reject it at slightly higher levels. When we split our sample into two subperiods we find that rank one is rejected at the 5% level in one of the two subsamples.[15] This means that we could rule out the one factor model with constant sensitivity or a constant premium model with time-varying sensitivity proportional to the state variables.[16] However, Table 3 shows that we are unable to differentiate between a rank of 2 or greater for $\tilde{b}$.

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[15] Similar results are obtained with the choice of other pairs of interest rates as instrumental variables. An alternative test based on the Wald-test also rejects H_0: rank($\tilde{b}$)=1 at the 5% level.

[16] Stambaugh (1988) found that this procedure provided significant insight into pricing assets. His study differed from ours in both the maturity of the assets studied (he didn't examine maturities beyond 12 months) and the choice of predetermined state variables. He used the forward premiums from two-months to six-months as state variables. We also replicated Stambaugh's latent variable-model test with a full range of maturities for both excess returns and forward rates. Our results were in general consistent with those of Stambaugh. For the constant beta latent-variable model, we found strong evidence against the one-, two- and three-factor models. We also found that using forward rates with a wide range of maturities improves the statistical power of the latent-variable model tests. These results could be obtained from the authors upon request.

While the rank restriction test allows us to reject the simplest form of the return model, it doesn't answer all questions about our model, such as whether we have constant λ's or B's. By estimating equation (12) without the restrictions imposed by equation (10), and then testing various restrictions we can learn more about the structure of the model.

Examining the top panel of Table 4 we see that the single factor model in which there is only one state variable either y_L or y_S is rejected by the data. The data does not reject the model in which B_{iL} is only a function of y_L and the model in which B_{iS} is only a function of y_S. The model with either the λ's forced to be constant or the B's forced to be constant is rejected. The rejection of constant betas implies that the conditional bond return volatilities are closely associated with long-term bond yield and spread.

We also find from Table 4 that the linear asset pricing restriction of (10) is rejected by the data. One possible explanation of the rejection is due to the power of the tests. Equation (10) imposes thirty restrictions on the eighty-four parameters estimated from regression (12). The Wald-test is a joint test of all thirty restrictions. The test will pick up some small but statistically significant deviation from the equation.

This claim is partly supported by panel B of table 4, where we provide R^2 for fitted values of bond excess returns using both the restricted and unrestricted model. The panel shows that R^2 for both models are quite similar with a range of .85 to almost 1.0. In general, the methodology employing cross-equation restrictions and systematic factors (Δz_L and Δz_s) is much more powerful than the latent variable methodology in allowing us to choose among alternative specifications.[17] This claim is partly supported by the fact that the restriction is not rejected by the Schwartz (1978) test, which in this context is that the statistic should be greater than the logarithm of sample size, which is 6.18, if the restriction is rejected (see Deaton (1988)). We think the Schwartz (1978) test to be a more appropriate test here because it takes into consideration the dimension of the restriction and sample size. Nevertheless we report results using the Wald test as well as the Schwartz test for the Wald test has become standard in the literature.

We can learn more about the model from examining the time series performance of the sensitivities and risk premiums. This is done in Figures 2, 3, and 4. Examining Figure 2 shows that the long risk premium (λ_L) was generally negative and stable over the early years but varied somewhat in the late 70's to early 80's. Examing Figure 3 shows that the sensitivities to the long rate are also negative. The negative risk premiums and sensitivities result in positive expected returns. The risk premium for spread (λ_S) was positive over most of the period and volatile throughout the period. Examing Figure 4 shows that the sensitivities to the

[17] Another possible rejection of equation (10) could be due to either some kind of mispricing in the bond market, or approximation errors in equation (8) and return computations, or errors-in-variables because of the multi-step approach for estimating innovations in the long rate and the spread.

spread were positive for all but the longer maturity bond. Comparing Figures 3 and 4 and noting the scale shows that the sensitivity to the spread for short maturity bonds was fairly stable over time, but for other bonds the sensitivity varied quite a bit. The sensitivity on long bonds was fairly stable in early years, but varied considerably from the 70's on. This clearly indicates that bond return volatilities are closely associated with long-term bond yield and spread, which is consistent with our test that the constant beta model is rejected.

It is interesting to examine the conditional term premium that arises from our model. The conditional term premium is equal to the prediction of the excess return (over the riskless rate) which is equal to the sum of the product of the betas and λ's examined in Figures 2, 3, and 4. This is equivalent to fitting the restricted form of our model.

Figure 5 shows the conditional term premium for a 10-year bond, 1-year bond, and a 6-month bond. This figure confirms results from previous studies that the term premiums are much more variable for long term bonds than for short term bonds. In fact, the low variance of the predictable part of the term premium for short term bonds makes it clear that studies which only analyze the behavior of short term debt instruments may reach misleading conclusions about the stability of the term structure of interest rates.

Figure 5 presents inconclusive evidence about which theory of term premium is correct. We can say that predictable term premiums appear to exist. However, we can not state that the term premium are always positive or that they increase with maturity. On the other hand the premiums on the one year and 6 month bond are always positive and the one year term premium is always higher than the 6 month premium. However, There are long periods of time where the predicted term premium on the 10-year bond is negative and there are long periods where the 10-year bond has a lower term premium than the 1 year or 6-month bond. Thus, the results are consistent with a more complex explanation of term premiums such as preferred habitat.[18] Our study complements a recent study by Boudoukh, Richardson, and Smith (1993), where they also find some inconclusive evidence against the liquidity preference theory, using treasury bills from one to eleven month in maturities.

All of the above results involved assumptions about the process driving the long interest rate and the spread and the particular choice of maturities used for the state variables. It is worthwhile briefly discussing the robustness of our results under alternative assumptions.

V. Robustness

In the Stochastic Model of long rates (equation (8)) we assumed that the coefficient of the Δz term contained the long rate to the one half power. The reason for utilizing the long rate in this term is that the stochastic process can not lead to a negative

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[18] Similar results were obtained for the unconstrained version of the model.

long rate. However, utilizing the long rate to any power would accomplish this goal. An important empirical question is which power is most consistent with observed data. We examined four different values for the power on the interest rate variable-zero, one half, one and three halves. While we recognize that zero has the undesirable property of allowing the stochastic process to become negative, an important question is does utilizing the long rate to a positive power in the stochastic process significantly affect how well the model describes the data.

When alternative powers of y_L were tried in the model we found that[19]

1) The performance of the model was unchanged whether the square root of y_L was used in the estimation of dz or it was omitted from the term

2) Including y_L to powers higher than 1/2 resulted in a poor fit of the model.

Thus, utilizing the long rate to the one half power in the stochastic process has the desirable property of generating long rates which can not take negative values while maintaining the same explanatory power as a model without this property. Utilizing the long rate to the power of one-half works better than alternative values of the power.

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[19] Varying the power on the long rate affects the estimate of dz in equation (8). Recall that equation (8) was estimated using a weighted least square procedure. The estimate of dz was obtained by first estimating it for the long rate to a power of zero and then adjusting it by dividing this estimate by r to the various powers.

The second choice that we made was the specific rates to use as state variables. Figure 6 shows the explanatory power of the unconstrained model for a number of alternative choices. It is easy to see that all choices of state variables capture a great deal of the variation in excess returns across maturities. The same result was true when we tested the expected return process (equation 10). Our choice once again generally explained the greatest amount of expected return with the exception of very short rates which was discussed earlier. Finally, we examined the tests of our model for alternative choices of state variables. These results are shown in Panel B to D of Table 4. Comparing these results to Panel A shows the conclusions are the same. Our model cannot be rejected by the Schwartz test in most cases, but constant λ's or B's can be rejected. Thus, for all of our tests the results are robust across choice of state variables.

We have also performed two additional regressions to see if our model has captured inflation risk in bond pricing. In the first regression, we add lagged inflation to equation (4). We find no evidence of inflation offering any extra explanatory power over the time variation of risk premium. In the second regression, we added unexpected inflation to equation (12) (computed from residuals of a VAR process including the original state variables plus inflation). We also see no evidence of the inflation factor offering any significant explanatory power over bond excess returns. Thus, inflation can explain neither the expected nor the unexpected returns in our model, which suggest that inflation risk is spanned by our factors.[20]

Conclusion

In this paper we examined the ability of a two-state return generating process to explain both the expected and unexpected returns. While others have examined this type of model for asset pricing using short maturities (generally under 6 months), this is the first paper to test such models for bonds with maturities over 5 years. For a wide range of choices concerning state variables, we are able to reject both constant risk premiums and constant sensitivities. Both the restricted and the unrestricted models are found to be capable of capturing most of the variation in bond excess returns. The predicted risk premiums change over time with long bonds sometimes having a greater and sometimes smaller risk premium than short bonds. The paper has also made an methodology contribution by accounting for both the expected and unexpected bond returns. This allows us to explicitly test some asset pricing restrictions which are difficult to study under existing frameworks.

[20]These results can be obtained from the authors upon request.

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Appendix

To test the rank restriction, we first partition the excess return matrix $R = (R_1, R_2)$, where R_1 is a TxK matrix of excess returns of the first K assets and R_2 is a Tx(N-K) matrix of excess returns on the rest of the assets. Using equations (9), we can perform the following regression analysis:

$$\text{(a1)} \qquad R_1 = X\Theta + \mu_1$$
$$R_2 = X\alpha + \mu_2$$

where X is a TxL matrix of the state variables, Θ and α are regression coefficents. If the pricing relationship in (7) and linear expectation in (8) hold, they imply that the data should not be able to reject the null hypothesis H_0: $\alpha = \Theta B$, where B is a matrix of Kx(N-K) elements. The regression system of equation (a1) given the restriction in equation (10) can be estimated and tested using Hansen's (1982) Generalized Method of Moments (GMM), which allows for conditional heteroskedasticity and serial correlation in the error terms. It is easy to see from equation (9) that the error term in system (a1) has conditional mean zero given the instruments $X_{pt}=(1, y_L, y_s, y_L y_s, y_L^2, y_s^2)$. Following Hansen, we begin by constructing a N x L sample mean matrix: $G_T = U'X / T$. Next, we stack the column vectors on top of each other to obtain a NLx1 vector of g_T. A two-step algorithm is then used to find an optimal solution for $g_T' W^{-1} g_T$ by minimizing over the parameter space of (Θ, α). In the first step, the identity matrix is used as the weighting matrix W. After obtaining the initial solution of Θ_o and α_o, we next calculate the residuals μ_1 and μ_2 from the system of equations in (a1) and construct the following weighting matrix:

$$\text{(a2)} \qquad W = \frac{1}{T} \sum_t \left(u_t u_t'\right) \otimes \left(Z_t Z_t'\right),$$

where $\otimes$ is the Kronecker product. Then we use the weighting matrix as given by (a2) to resolve the optimization problem of minimizing $g_T' W^{-1} g_T$ over the choice of (Θ, α). It can be shown (see Hansen (1982)) that under the null hypothesis, T times the weighted sum of squares of the residuals, $g_T' W^{-1} g_T$, is asymptotically chi-square distributed, with the degrees of freedom equal to the difference between the number of orthogonality conditions and the number of parameters estimated: (N-K) (L-K), where N is the number of assets studied, K is the rank, and L is the number of columns in X_{pt}. After obtaining the weighted sum of squared residuals, we can perform a chi-square test to determine if the data rejects the restricted regression system (a1).

To test restriction (10) directly, we estimate (12) jointly for all bonds. After obtaining the $\tilde{b}$ and $\tilde{\beta}$ estimates, we solve for Θ as:

(a3) $$\Theta = (\beta'\beta)^{-1}\beta'\tilde{b}$$

To calculate the variance-covariance matrix associated with the estimation error for Θ, we first let γ and **V** represent the entire set of parameters and the variance-covariance matrix respectively. Next, we write Θ as a nonlinear function $f(\gamma)$ of the parameter vector γ. The variance-covariance matrix for the Θ estimates is then estimated as $[f_\gamma(\gamma)' V f_\gamma(\gamma)]$. After obtaining the variance-covariance matrix for $\tilde{b}$, $\tilde{\beta}$, and Θ, it is straightforward to perform the Wald-type test based on various restrictions imposed on the parameters. More specifically, if we want to test the restriction H_0: $R\Theta - b=0$, we can construct the following test statistic:

(a4) $$\text{Wald} = (R\Theta - b)'(R\Omega R')^{-1}(R\Theta - b)/m,$$

where Ω is the variance-covariance matrix for Θ and m is the number of rows in matrix R. Under the null hypothesis, the Wald-statistic asymptotically has a F-distribution, with the degrees of freedom equal to m and the number of observations minus the number of regressors in equation (12).

Table 1

Summary Statisticas

A. Mean and Standard Deviation for Selected Bond Returns in Excess of One-Month T-bill

	3-month	6-month	9-month	1-year	5-year	10-year	13-year
Mean	0.042	0.064	0.063	0.066	0.082	0.053	0.042
S.D.	0.100	0.243	0.386	0.507	1.820	2.930	3.660

B. Correlation Matrix for Selected Bond Excess Returns

	3-month	6-month	9-month	1-year	5-year	10-year	13-year
3-month	1	0.935	0.882	0.838	0.627	0.526	0.504
6-month		1	0.976	0.945	0.757	0.658	0.639
9-month			1	0.989	0.822	0.715	0.693
1-year				1	0.859	0.747	0.716
5-year					1	0.927	0.869
10year						1	0.971
13-year							1

Notes: Excess Returns are calculated in excess of one-month t-bills. The sample period for this table is 1947:1-1987:3, with 483 observations. Units on excess returns are percentage per month.

Table 2

Regression of excess returns on forecasting variables from 1947:1-1987:3. The t-statistics are adjusted for heteroskedasticity. The following are the parameter estimates for equation (9). The y_{Lt} is the yield on a 10-year zero coupon bond. The y_{st} is spread between the yield on a 10-year zero coupon bond and the yield on a 3-month zero coupon bond.

$$r_{it} = b_0 + b_1 y_{Lt} + b_2 y_{st} + b_3 y_{Lt}^2 + b_4 y_{st}^2 + b_5 y_{Lt} y_{st} + e_{it}$$

Maturity	b_0	b_1	b_2	b_3	b_4	b_5	F-test	$\bar{R}^2$	DW
3m	0.072	-0.011	-0.034	0.002	0.001	0.006	0.001	0.084	1.67
	(3.420)	(-1.650)	(-1.670)	(1.080)	(1.540)	(0.933)			
6m	0.138	-0.027	-0.074	0.005	0.002	0.016	0.018	0.054	1.62
	(2.720)	(-1.530)	(-1.520)	(1.070)	(1.230)	(0.868)			
9m	0.217	-0.052	-0.116	0.009	0.003	0.028	0.023	0.044	1.66
	(2.720)	(-1.900)	(-1.570)	(1.290)	(1.300)	(0.976)			
1y	0.281	-0.073	-0.151	0.014	0.004	0.040	0.008	0.048	1.66
	(2.670)	(-2.070)	(-1.590)	(1.490)	(1.390)	(1.070)			
5y	0.754	-0.248	-0.389	0.049	0.014	0.115	0.016	0.048	1.88
	(1.880)	(-1.870)	(-1.110)	(1.450)	(1.200)	(1.020)			
10y	1.100	-0.407	-0.557	0.086	0.022	0.178	0.009	0.060	1.92
	(1.630)	(-1.910)	(-0.969)	(1.520)	(1.240)	(1.040)			
13y	1.470	-0.554	-0.684	0.107	0.031	0.212	0.005	0.061	1.91
	(1.690)	(-1.970)	(-0.950)	(1.500)	(1.290)	(0.942)			

Note: t-statistics are in parenthesis. The number shown under F-test gives the p-value for the joint test of $b_{i1} = b_{i2} = b_{i3} = b_{i4} = b_{i5} = 0$.

Table3

Chi-square statistic for test of rank restrictions in expected excess returns of zero-coupon bonds with maturities up to 13 years. We test the rank restrictions on the coefficients of regression of excess returns on a set of instruments by the latent variable specification. The p-value (in parentheses) is the significance level at which the rank restricition is rejected. The tests use the generalized methods of moments and allow for heteroscedasticity.

	Rank (degrees of freedom)			
Sample Periods	1(30)	2(20)	3(12)	4(6)
47:1-87:3	39.85 (0.107)	13.93 (0.834)	2.40 (0.998)	0.45 (0.998)
47:1-70:1	47.21 (0.023)	21.54 (0.365)	3.84 (0.986)	0.32 (0.999)
70:2-87:3	39.19 (0.121)	11.82 (0.921)	3.13 (0.994)	0.56 (0.997)

Note: Excess return on bonds with seven different maturities are used in the rank restriction test. They are bonds with 3-months, 6-months, 9-months, 1-year, 5-years, 10-years, 13-years to maturity. We use a constant, long rate, spread and their quadratic terms as the state variables in the test. The long rate, y_L, is the yield on a 10-year zero coupon bond. The spread, y_S, is the spread between the yield on a 10-year zero coupon bond and the yield on a 3-month treasury bill.

Table 4

A. F-statistics for tests of various restrictions.

	no y_L (DF=14)	no y_s (DF=14)	no y_L in β_s (DF=7)	no y_s in β_L (DF=7)	cons. β (DF=28)	cons λ (DF=12)	eq.(10) (DF=30)
Test A	384.0	3.925	1.423	1.266	311.2	2.074	4.312
	(0.00)	(0.00)	(0.19)	(0.28)	(0.00)	(0.02)	(0.00)
Test B	183.2	2.789	4.964	2.448	114.5	1.443	5.003
	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.14)	(0.00)
Test C	136.8	3.028	1.632	0.987	147.1	2.216	2.472
	(0.00)	(0.00)	(0.12)	(0.43)	(0.00)	(0.01)	(0.00)
Test D	117.2	8.485	2.411	3.828	101.3	3.151	8.536
	(0.00)	(0.00)	(0.02)	(0.00)	(0.00)	(0.02)	(0.00)

B. R^2 for bond excess returns

	3-months	6-months	9-months	1-years	5-years	10-years	13-years
Unrestricted	0.919	0.915	0.882	0.855	0.897	0.999	0.946
Restricted	0.885	0.910	0.885	0.847	0.872	0.977	0.930

Note: Excess return on bonds with seven different maturities are used in the test. They are bonds with 3-months, 6-months, 9-months, 1-year, 5-years, 10-years, 13-years to maturity. The p-value (in parentheses) is the significance level that the restricition is rejected. The y_L is the yield on a 10-year zero coupon bond, the y_s is the spread between the yield on a 10-year zero coupon bond and the yield on a 3-month treasury bill in Test A. The y_L is the yield on a 4-year zero coupon bond, the y_s is the spread between the yield on a 4-year zero coupon bond and the yield on a 8-month treasury bill in Test B. The y_L is the yield on a 1-year zero coupon bond, the y_s is the spread between the yield on a 1-year zero coupon bond and the yield on a 3-month treasury bill in Test C. The y_L is the yield on a 7-year zero coupon bond, the y_s is the spread between the yield on a 7-year zero coupon bond and the yield on a 3-month treasury bill in Test D. The y_L is the yield on a 10-year zero coupon bond, the y_s is the spread between the yield on a 10-year zero coupon bond and the yield on a 3-month treasury bill in Panel B.

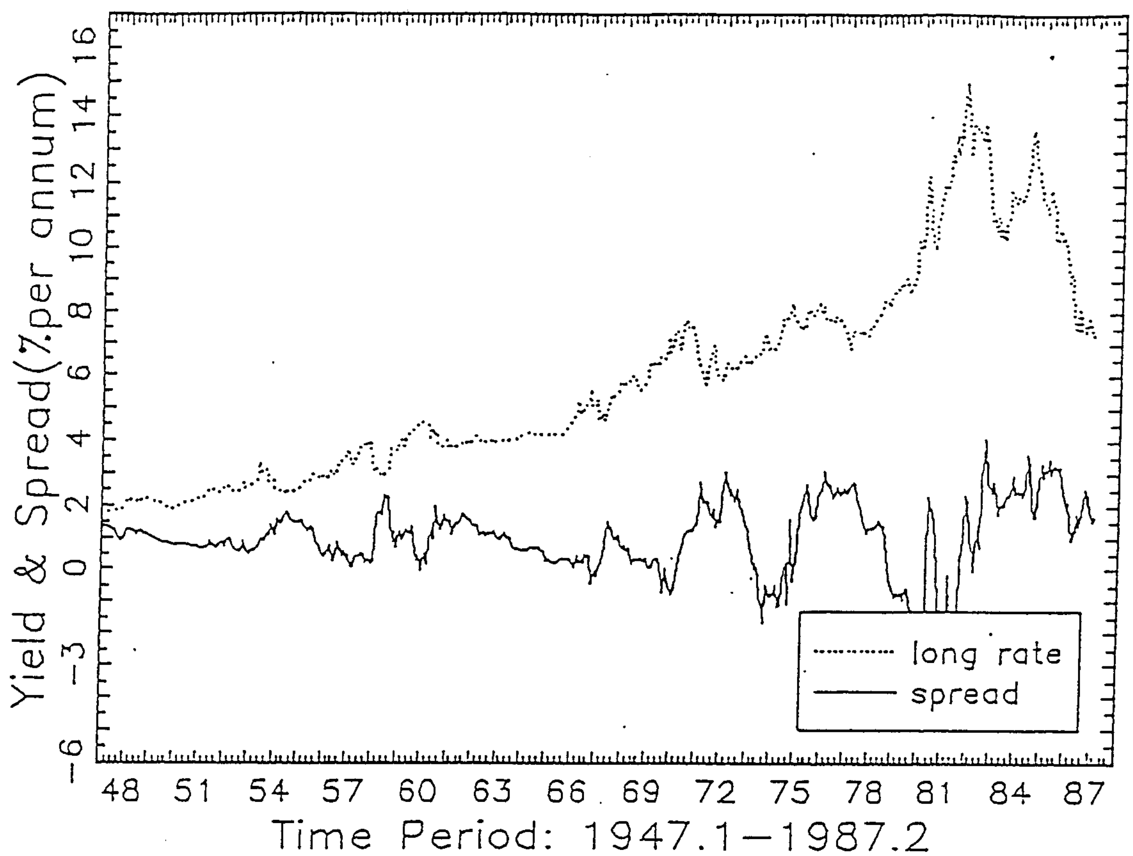

Figure 1. Plot of the long rate (y_L) and the spread (y_s). y_L is the yield on a 10-year zero coupon bond. y_s is the spread between the yield on a 10-year zero coupon bond and the yield on a 3-month treasury bill. The sample period for this figure is 1947.1-1987.3, with 483 observations.

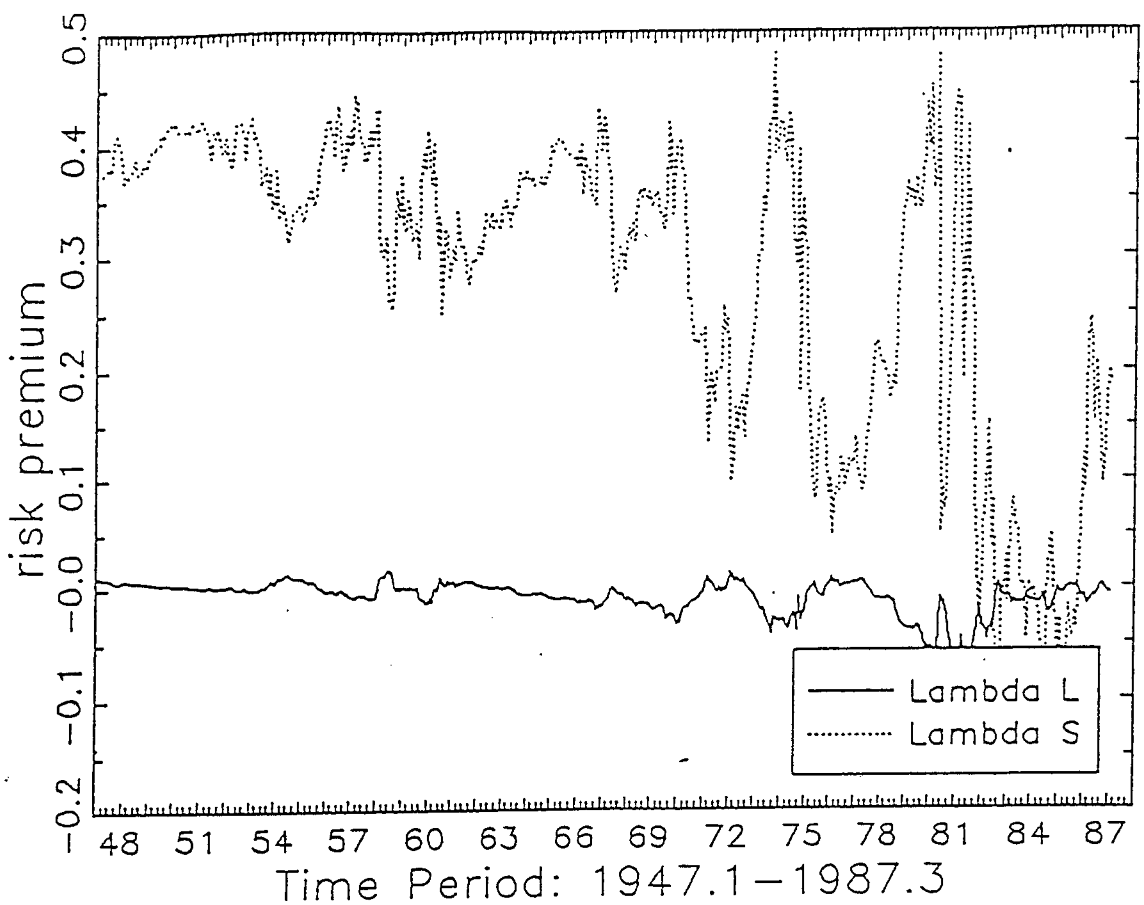

Figure 2. Plot of risk premium λ_{Lt} and λ_{st}. λ_{Lt} is the risk premium compensating for taking risk on unexpected changes in yield of a long term bond. λ_{st} is the risk premium compensating for taking risk on unexpected changes in the spread between yield on a long term bond and yield on a short term bond. The sample period for this figure is 1947.1-1987.3, with 483 observations.

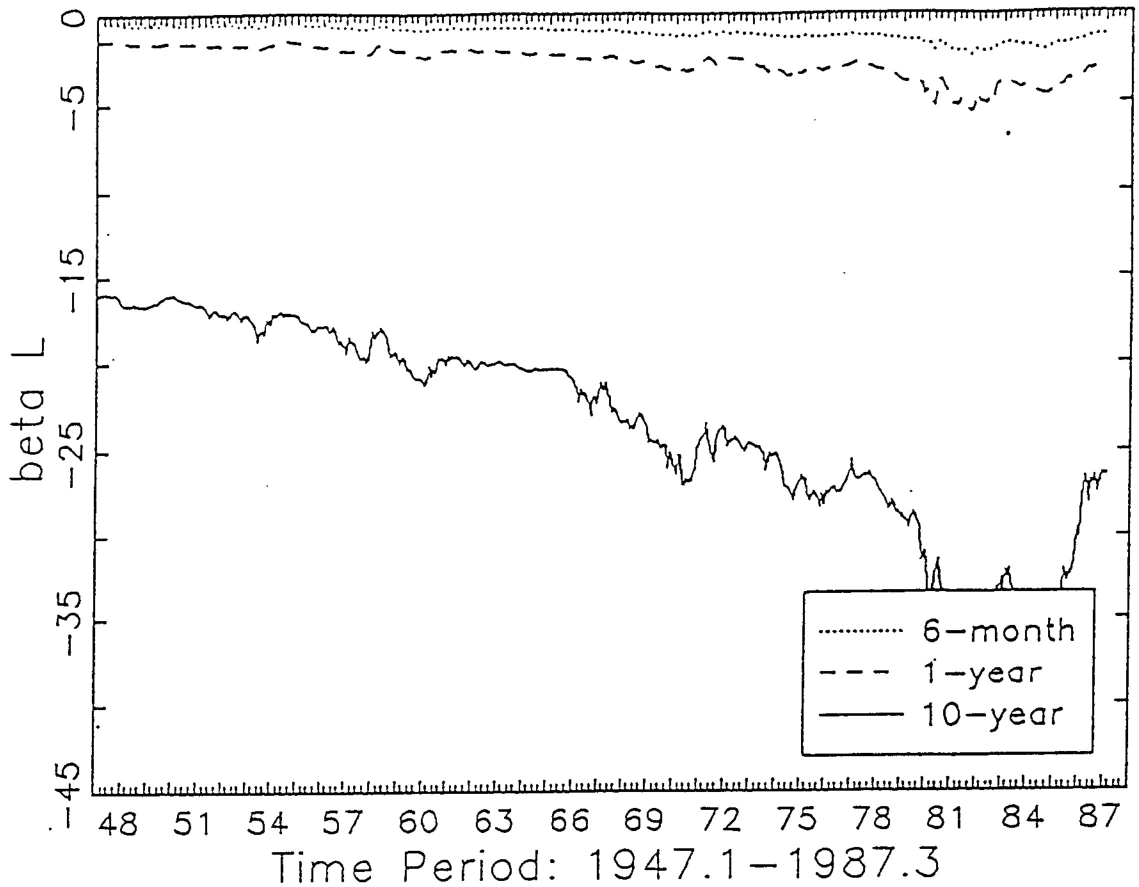

Figure 3. Plot of sensitivities, B_{iL}, towards unexpected changes in the long rate. The sensitivities are estimated for six-month treasury bills, one-year and ten-year treasury bonds. In general, the longer the maturity, the more sensitive the bond towards the unexpected changes in the long rate. The sample period for this figure is 1947.1-1987.3, with 483 observations.

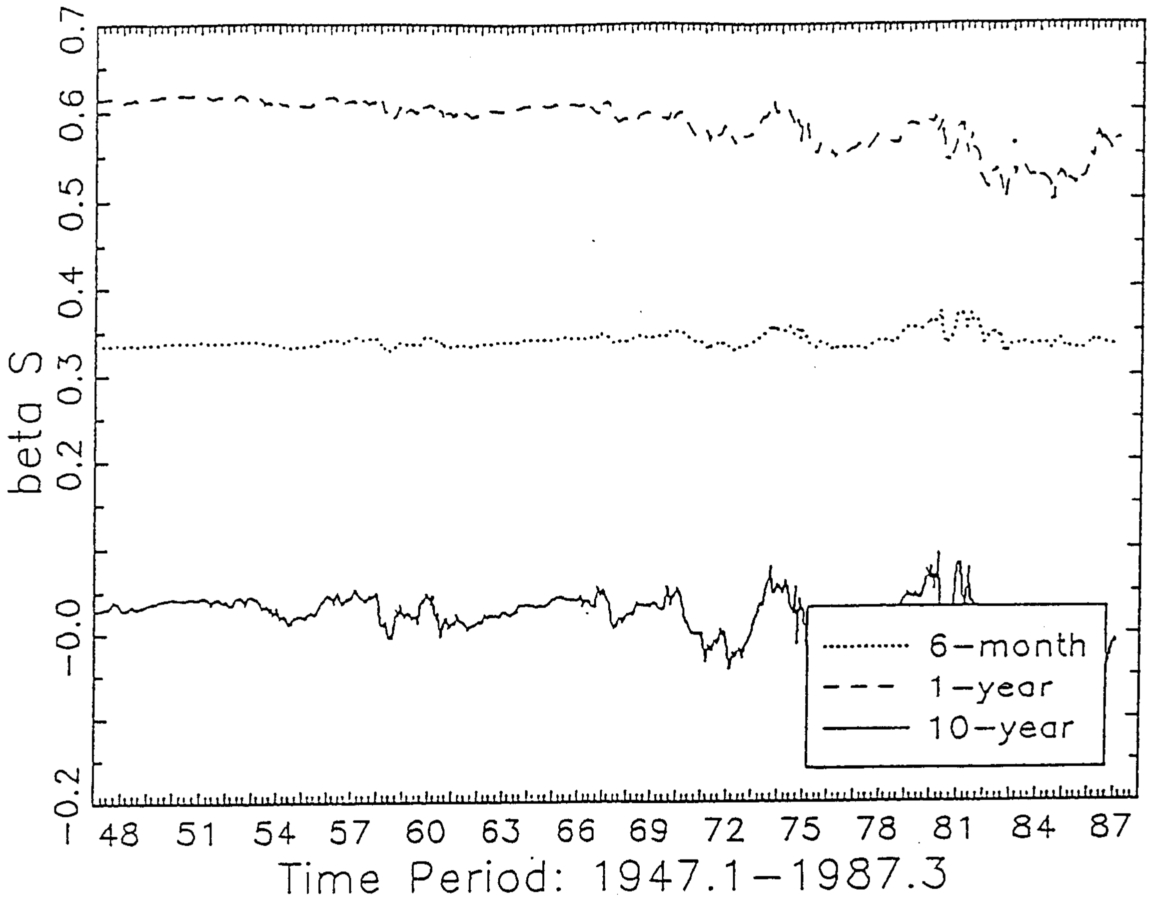

Figure 4. Plot of sensitivities, B_{is}, towards unexpected changes in the spread. The sensitivities are estimated for six-month treasury bills, one-year and ten-year treasury bonds. In general, the longer the maturity, the more sensitive the bond towards the unexpected changes in the spread. The sample period for this figure is 1947.1-1987.3, with 483 observations.

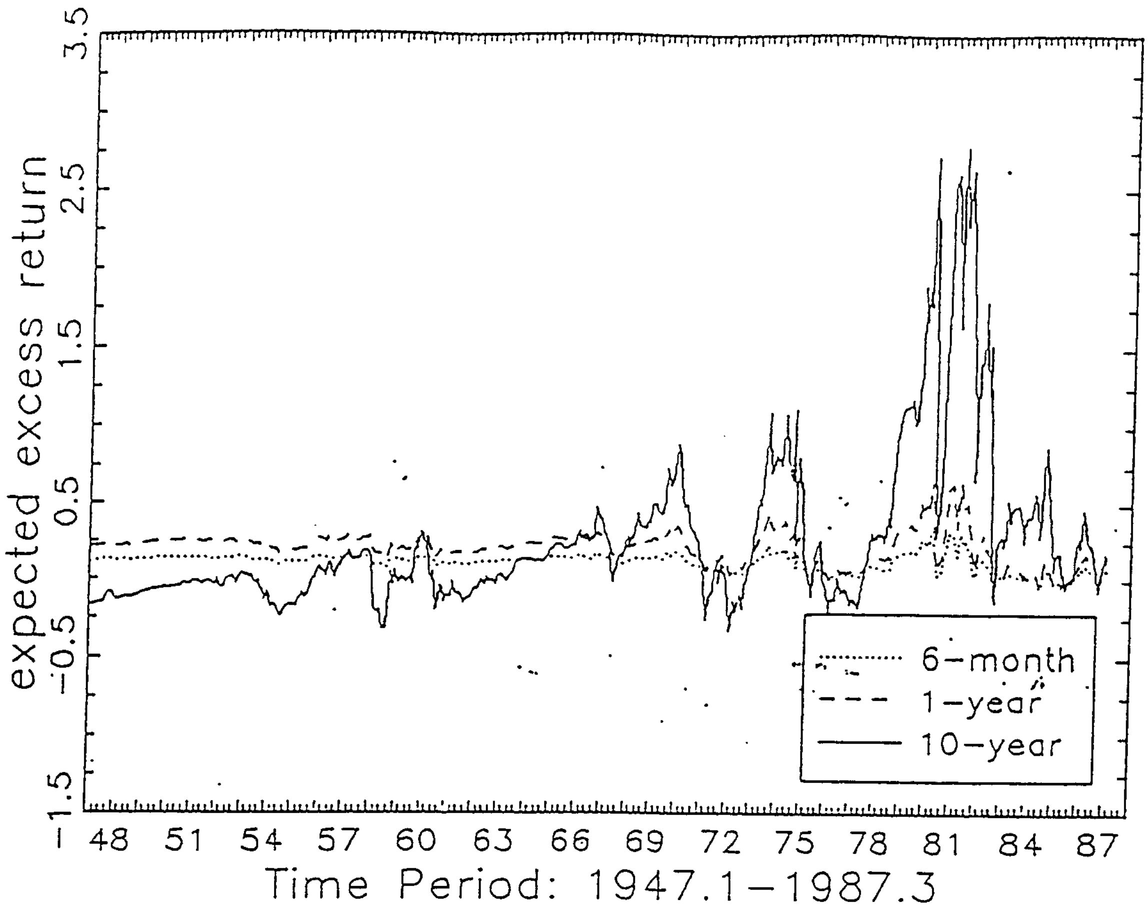

Figure 5. Conditional term premium for short and long term bonds. Conditional term premium is defined as the sum of the product of the β's and λ's in figure 3-5. y_L is the yield on a 10-year zero coupon bond. y_S is the spread between the yield on a 10-year zero coupon bond and the yield on a 3-month treasury bill. The sample period for this figure is 1947.1-1987.3, with 483 observations.

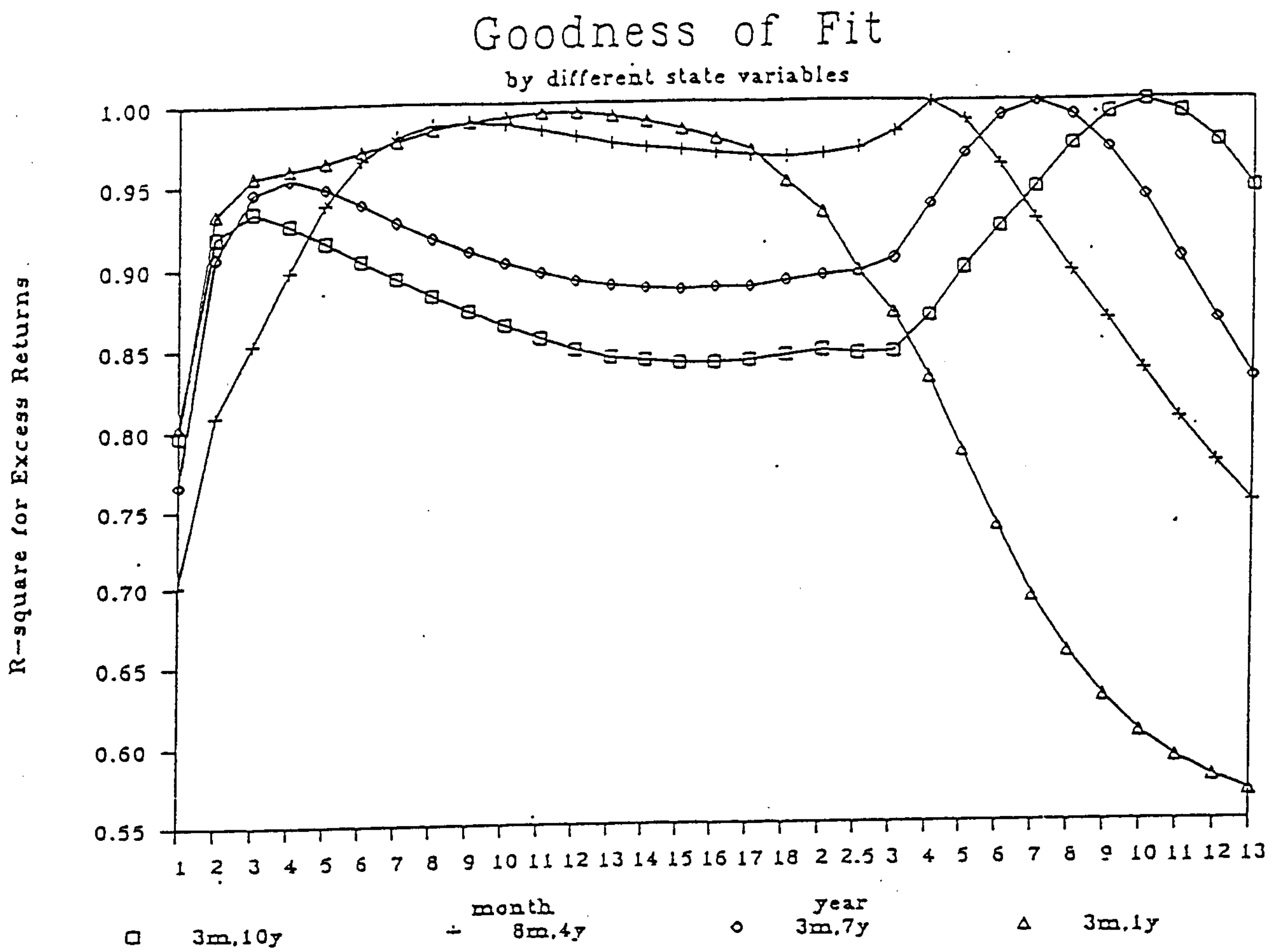

Figure 6. R^2 for the regression of equation (12) under four alternative specifications: a) The y_L is the yield on a 10-year zero coupon bond. The y_s is the spread between the yield on a 10-year zero coupon bond and the yield on a 3-month treasury bill. b) The y_L is the yield on a 4-year zero coupon bond. The y_s is the spread between the yield on a 4-year zero coupon bond and the yield on a 8-month treasury bill. c) The y_L is the yield on a 7-year zero coupon bond. The y_s is the spread between the yield on a 7-year zero coupon bond and the yield on a 3-month treasury bill. d) The y_L is the yield on a 1-year zero coupon note. The y_s is the spread between the yield on a 1-year zero coupon note and the yield on a 3-month treasury bill. Δz_L and Δz_s are innovations of y_L and y_s.