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Abstract

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[1] Associate Professor of Finance, Stern School of Business. Any comments can be sent to the author at: Department of Finance, Stern School, NYU, 44 west 4th street, New York, NY 10012, (212)-998-0354. The paper was originally entitled: "Estimation of Time-varying Factor Premiums in an APT Model With Observable and Unobservable Factors". I would like to thank Edwin Elton and Martin Gruber for motivating and encouraging me to pursue this topic. I am also grateful to John Ammer, Pierluigi Balduzzi, Silverio Foresi, Hedi Kallal, and Larry Lang for helpful discussion, Ferson and French for providing me with some of the data, and William Y. Zhou for able research assistence. I acknowledge financial support from the New York University Summer Research Grant.

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Abstract

This paper develops a BMW^{TV} approach to the estimation of factor premiums by integrating the APT model of Burmeister and McElroy (1988) with time-varying risk premiums. It provides premium estimates for macro-factors over time under a unified APT framework which allows for both observable and latent factors. We find significant negative risk premiums for the market factor and the size factor during the sample period. We discover that risk premium and sensitivity estimates for the observable factors are quite sensitive to omitted latent factors, suggesting the importance of accounting for missing latent factors in conditional multi-factor models. We also find the mispricings under the APT model and the CAPM model are relatively small, but the results are quite sensitive to omitted factors. Our study shows that the variation of the size premium appears to be related to business cycles.

Price series are essential statistics in our understanding of markets because they contain information about economic behavior of market participants. For example, given the price and the marginal cost of pork bellies, we can make statement such as the pork belly market is inefficient if we observe that the pork belly prices have been consistently below marginal costs. Given the importance of price information and the dominant role of macroeconomic risks in the capital markets, the prices (risk premiums) of macroeconomic risks certainly have not received enough attention from financial research.

This paper develops a new approach to the estimation of risk premiums by integrating the APT model of Burmeister and McElroy (1988) with time-varying risk premiums (BMW^{TV}). A major advantage of this approach is that, by incorporating multiple latent "priced" factors in the economy, the estimation of risk prices will not be contaminated by missing factors.

The BMW^{TV} approach also has several other distinctive advantages. First, we obtain a confidence bound on the macro-factor premiums, thus providing a simple alternative to the Boudoukh, Richardson, and Smith (1993) procedure for testing nonnegative pricing restrictions implied by conditional asset pricing models. Second, we develop a simple procedure to measure the impact of errors-in-variables associated with macro factor estimates. Third, we nest the conditional CAPM into the APT model framework, thus making it possible to test both the conditional CAPM and the conditional APT restrictions in a single regression framework.

The BMW^{TV} approach is built on a large literature on time-varying risk premiums. Previous studies have used the latent-variable model to estimate time-varying risk premiums (see, for instance, Gibbons and Ferson (1985), and Campbell (1987)). Unfortunately, most of these studies include only unobserved factors thus it is difficult to give economic interpretation to the estimated factor premiums. This is an serious handicap for the application of the APT model since one often needs to know the APT prices associated with known macroeconomic factors.

One exception is the Chen, Roll, and Ross (1986) paper in which they introduce macroeconomic factors directly into a multi-factor model and study the risk premiums paid on these factors.[1] Ferson and Harvey (1991) extend the above study by allowing the risk premiums on the macro-factors to vary over time. However, these studies have a common caveat, which is the assumption that no systematic factors have been omitted from the study. Given the large number of macro-factors that could affect the securities market, the assumption seems quite implausible. Moreover, if the missing factors are priced by the market, then the prices of macro-factors will very likely be mis-measured, since these prices tend to pick up the slack left out by omitted factors. Realizing this problem, Brown and Otsuki (1992) and Ferson (1990) take into account the possibility that one important pricing factor could be excluded from the model, thus they include not only the observable factors but also a residual market factor in their study.

This paper takes a step further. We study the prices of macro-risks by allowing for the possibility that more than one pricing factor could be omitted. We derive an impossibility theorem which demonstrates that, unlike in a static (unconditional) APT model, multiple latent factors can not be resolved into a single latent factor in conditional APT models. Instead, they play important roles in asset pricing. Our empirical work confirms that missing latent factors could affect risk premium and factor loading estimates and could lead to asset mispricing. We design a simple statistical procedure to test how many factors are in the APT model and we also use mean pricing errors to measure the impact of missing factors.

The paper is organized as follows. Section I presents the multi-factor model with both observable and latent factors. Section II discuss the model specification and data. Section III provides estimates for the macro-factor premiums and the empirical tests of the APT and the CAPM, while Section IV concludes the study.

I. The Linear Factor Model

Following Burmeister and McElroy (1988), Brown and Otsuki (1992), and Ferson (1990), we assume that asset returns are generated by the following K-factor model:

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[1] See also Shanken and Weinstein (1990).

$$r_t = E_t + \beta f_t + \pi g_t + \varepsilon_t \tag{1}$$

where $r_t = (r_{1t},...,r_{Nt})'$ is the vector of returns in period t, in excess of the risk-free (one-month treasury bill) rate. E_t is the vector of expected excess returns in period t, conditional on information known to investors at the end of time t-1. f_t's are the J latent factors and the g_t's are the K-J observable factors. β and π are factor loadings towards the latent and observable factors respectively. We assume the loadings to be constant during the sample period. There is also an idiosyncratic error term, ε_t. Here, E_t is Nx1, β is NxJ, f_t is Jx1, π is Nx(K-J), g_t is (K-J)x1, ε_t is Nx1. We assume that $E_{t-1}[f_t,g_t] = 0$, $E_{t-1}[\varepsilon_t] = 0$, and $E[\varepsilon_t|f_t,g_t] = 0$.

Under the above assumptions, if we substitute the latent factors, f_t, with a set of reference portfolios, R_t,[2] and assume that the factor premiums are linear functions of state variables, X_{t-1}, then we obtain the following regression,

$$r_t = \Gamma X_{t-1} + \beta R_t + \gamma g_t + \eta_t. \tag{2}$$

We can show that the APT places restrictions on coefficients of the unrestricted linear model (2):[3]

$$\Gamma = \gamma \Theta_K. \tag{3}$$

Here is Γ a NxL matrix, γ is a Nx(K-J) matrix and Θ_K is a (K-J)xL matrix. Thus, we have the following restricted regression under APT:

$$r_t = \gamma \Theta X_{t-1} + \beta R_t + \gamma g_t + \eta_t. \tag{4}$$

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[2]By using the reference portfolios instead of the "mimicking" portfolios as in Huberman, Kandel, and Stambaugh (1987), we avoid making the assumption $E(\eta_t|R_t) = 0$.

[3]It is worth noting here that equation (3) is similar to the results of Brown and Otsuki (1992) and Ferson (1990), if there is only one unobservable factor (J=1). Equation (3) also collapses to the latent variable models of Gibbons and Ferson (1985), Campbell (1987), and Ferson (1989) if there are no observable factors (J=K). The derivation of (3) is provided in the Appendix.

Following Burmeister and McElroy (1988), we can nest a conditional CAPM model empirically into the APT model which is in turn nested within the linear regression model of (2). The conditional CAPM could impose the following testable restrictions on the coefficients of (4):

$$\Theta_K^j = c^j \Theta_K^1, \quad j=1,\ldots,K\text{-}J, \tag{5}$$

where Θ_K^1 is the vector of coefficients associated with the market risk premium and Θ_K^j ($j \neq 1$) is the vector of coefficients associated with other observable factors (see (A16) in the appendix). Following Burmeister and McElroy (1988), we will estimate (2) with and without the APT restriction, and further estimate equation (4) by imposing the CAPM restriction.

II. Model Specification, Data, and Estimation Procedure

We use monthly excess returns on twelve industrial portfolios, ten size portfolios, and four bond portfolios to conduct our study. The industry portfolios are value-weighted portfolios constructed using two-digit SIC codes. The size portfolios are ten value-weighted portfolios based on size deciles using the market value of equity outstanding at the beginning of each year.[4] The bond portfolios are a long-term corporate bond portfolio, a intermediate-term corporate bond portfolio, a junk bond portfolio and a one-year treasury bond portfolio. Returns on the bond portfolios are derived from Ibbotson Associates (1990).

Based on previous research, the economic state variables included in this study are the following: excess return on the market, dividend yield, inflation, relative treasury bill rate, and industrial production growth. Our measure of the market return is the return on the Standard and Poor's 500 index. The relative rate is calculated as the one-month treasury bill rate minus its twelve-month moving average. A similar variable is used in Fama and Schwart (1977), Campbell (1991), and Campbell and Ammer (1993) to capture changes in short-term interest rates and to forecast excess return on stocks and bonds. The dividend yield on this index is calculated by

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[4] We are grateful to Wayne Ferson for providing us with these industry and size portfolio returns. For more detail on the construction of these portfolios, see Ferson and Harvey (1991).

taking total dividends paid over the last twelve months relative to the current stock price. The one-month treasury bill rate and the inflation rate are obtained from Ibbotson Associates data series provided by the Center for Research in Security Prices (CRSP). The seasonally-adjusted monthly real industrial production index is taken from the Citibase tape. Our sample covers the time period from 1948:1 to 1987:12.

For the observable factors, we take residuals from a VAR regression whose elements are the above five economic state variables. There are three reasons why we choose innovations in economic state variables to be the systematic factors. First, they are major macroeconomic risks which people are interested in their market pricing. Second, based on recent research work by Campbell (1991), Campbell and Ammer (1993), and Campbell and Mei (1993), variation in expected returns are the major driving force of asset returns. Thus, using innovations of variables which help predict expected returns should allow us to capture most of the variation in asset returns. Third, by construction, the forecasting variables will be orthogonal to the factors. Thus, we have the forecasting variables determining asset expected excess returns and the factors driving asset unexpected excess returns according to equation (2).

To be more specific about the VAR process, the approach involves defining a vector X_{t+1} which has five elements.

$$X_{t+1} = AX_t + w_{t+1}, \qquad (6)$$

Higher-order VAR models could be stacked into a VAR(1) model in the same manner as discussed in Campbell and Shiller (1988). In equation (6), the matrix A is known as the companion matrix of the VAR. We use w_{t+1} as the observable factors in equation (2) and (4). Since the residuals on the market excess return and on the dividend yield are highly correlated, we dropped the dividend yield residuals from the factor list thus we only use four observable factors in our study: the market, the relative interest rate, the inflation, and the industrial production growth. We also included a constant term in the forecasting variables X_t. Based on the Schwarz criterion, we find that higher order VARs offer little efficiency gain over the one-lag VAR system thus we will use the parsimonious one-lag VAR in our empirical study.

A generalized method of moments (GMM) approach is employed to estimate equation (2), (4) and equation (4) with the CAPM restriction (5) imposed. Equation (3) and (5) are cross-equation restrictions, thus, they must be estimated simultaneously across a number of assets for appropriately testing the restrictions. The GMM approach is used to adjust for possible heteroskedasticity.

The testing of the number of factors in the economy is straightforward, since additional latent factors can be replaced by additional reference portfolio returns. The procedure is to first estimate the *unrestricted* ($K+1$ factor) model, then the *restricted* (K) model, and calculate the difference in their weighted sum of squared residuals, Q_U and Q_R. If the restricted model is true, we expect the difference ($L= Q_R - Q_U$) to be small. Under the null hypothesis, L follows a χ^2 distribution with degrees of freedom equal to the difference between the number of coefficients to be estimated in the two systems. If the restriction does not hold, the difference will be large, indicating a rejection of the restricted model. The same method could also be used to test the APT linear pricing restriction (3) and the CAPM restriction imposed on the multi-factor model (2) .

III. Empirical Results

A. Estimation of Regression (2) and Test of Minimum Number of Total Factors

Using excess returns of the decile size portfolios constructed from the CRSP file, the unrestricted multi-factor model of (2) is estimated for the time periods of 1948-1987.[5] For any given year t, we use excess returns to the Petroleum, Finance, and Durable Goods industrial portfolios as the "reference portfolios" returns to substitute out the latent factors. The orthogonality conditions used are : $E[\eta_t X_{t-1}] = 0$, $E[\eta_t g_t] = 0$, and $E[\eta_t b_t] = 0$, where b_t is a vector of excess

[5] We also use the industrial portfolios excess returns data sets constructed from the CRSP file to estimate a same set of regressions. In this paper, we will present our results mainly based on the size portfolios. The results on the industry portfolio could be obtained from the author upon request.

returns to four bond portfolios: a long-term corporate bond portfolio, a intermediate-term corporate bond portfolio, a junk bond portfolio and a one-year treasury bond portfolio.[6] To estimate the restricted model under APT and CAPM, we will use the same orthogonality conditions while further imposing restriction (3) and (5) on the parameters.

Table 1 presents the test results of how many total factors, K, we should use in the multi-factor model (2) for the time period of 1948-1987. The first column of Table 1 specifies the number of total factors, K, in the regression. The second column gives the sum of squared residuals, Q, corresponding to each specification. The third column provides the difference in the sum of squared residuals between its corresponding specification and the specification above it, which has one more latent factor. It measures the reduction in the sum of squared residuals by the introduction of one more latent factor into the regression. For example, $L_{(K=6)} = Q_{(K=6)} - Q_{(K=7)} =$ 20.93-14.61 = 6.32. If the specification is true, we expect L to be small. Under the null hypothesis, L has a χ^2 distribution with degrees of freedom (DF(L)) given by the fourth column. The degrees of freedom for K=7 (DF=10) is determined by the difference between the number of instruments used in the regression and the number of parameters estimated, while the degrees of freedom for K<7 (DF=10) is determined by the difference between the number of parameters estimated in regressions with K factors and K+1 factors. The last column gives the significance level by which the specification can be rejected.

Based on our initial results with four observable factors (K=4), we start by estimating (2) with K=7. As Table 1 shows, we can not reject the hypothesis that equation (2) might have 6 or 7 total number of factors. For example, the L value for K=6 (6.32) tells us that we can not reject K=6 at any significance level less than 75% (P=0.787). But the specification of K=5 is rejected at the 10% level (P=0.058). This result indicates there are at least six systematic factors at work in the market during this time period but the contribution of additional factors to the explanation of time variation in excess returns is statistically insignificant.

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[6] Here we are essentially assuming that the idiosyncratic risks on the bond portfolios are uncorrelated with the idiosyncratic risks on the size and the reference stock portfolios.

To guard against a possible bias of finding too many spurious factors, as pointed out by Conway and Reinganum (1988), we will compute mean pricing errors for various portfolios under different factor specifications. If additional factors make a difference in asset pricing, then they are not spurious. The intuition behind this test is that spurious factors are not priced and they should have little impact on asset pricing. The empirical results are presented in part F.

At this point, some readers may argue that why should we pay so much attention to omitted latent factors. Isn't it true that multiple latent factors could always be resolved into a single latent factor, and the single latent factor will do as good a job in asset pricing as multi-factors? Our response is that, while the above argument is valid for a static APT model (constant beta, constant risk premium) , it can not be applied to a conditional APT model such as model (1) used in the study. The reason is given by the following impossibility theorem:

The Impossibility Theorem:

In a conditional multi-factor model given by (1), there generally does not exist a transformation matrix which could re-normalize the multiple priced latent factors so that only one latent factor is priced.[7]

We give an intuitive example in Appendix to illustrate the theorem and a formal proof is available upon request. Our empirical results later will also show that omitted factors do matter in our factor premium estimation and asset pricing.

B. Test of the APT Linear Pricing Restriction (3) and Estimation of Factor Prices

Table 2 presents the test statistics for the APT linear pricing restriction (3). Since we can not tell exactly how many total risk factors there are in the market, we will test restriction (3) based on several specifications of K. The second column of Table 2 gives the weighted sum of squared

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[7] As a matter of fact, one needs a series of tranformation matrices to rotate the factors over time so that only one latent factor is priced. But this is going to violate the constant beta assumption.

residuals, Q, of regressions (2) with restriction (3) imposed. The third column gives the difference in the sum of squared residuals between the restricted and the unrestricted model (2). The sum of squared residuals of the unrestricted model, Q, is given in Table 1. Thus, we calculate $L_{(K=7, \text{table } 2)} = Q_{(K=7, \text{ table } 2)} - Q_{(K=7, \text{ table } 1)} = 25.35-14.61=10.74$, etc. The degrees of freedom (DF(L)) is determined by the difference between the number of parameters estimated with and without the restriction. From Table 2, we can see that we fail to reject the linear pricing restriction (3) for all specifications.

Figure 1a-1b provide plots of conditional risk premiums on the two observable factors with their conditional confidence bounds. The upper bound is calculated by adding 1.96 times the conditional standard deviation to the risk premiums and the lower bound by subtracting 1.96 times the conditional standard deviation. The conditional standard errors are calculated by observing that $\lambda_K = \Theta_K X_{t-1}$, thus $\text{var}(\lambda_K | X_{t-1}) = X_{t-1}'\text{var}(\Theta_K)X_{t-1}$, while the variance-covariance matrix of Θ_K is obtained by estimating equation (4). Since $\text{var}(\lambda_K | X_{t-1})$ is conditional on information at time t-1, we can see that the conditional standard errors (and confidence bounds) for the risk premium will vary over time according to economic state variables.

The bounds for the market risk premium are very tight, while the bounds for inflation premium are generally much looser.[8] This suggests that the estimation errors are quite small for the market risk premium while fairly large for the inflation premium. The risk premiums with their confidence bounds provide an easy test of factor pricing at any *ex ante* given time.[9] If the confidence bounds for any given factor embrace the zero-axis, then we can not reject the hypothesis that the factor is not priced at the time. One advantage of having the confidence bounds is that we can easily see that the market risk is priced most of the time while there are no significant pricings for inflation during the same sample period. The results from Figure 1 can be confirmed

[8] The results for the relative rate and IP growth are similar to those of inflation and can be obtained from the author upon request.

[9] The test may not be valid if it is used ex post. For example, the test can not be applied to the minimum of factor premiums. This is because the bounds are estimated with error and they hold true with 95% confidence level. Extreme events like the market crash of 1987 or extremely large or small premium estimates could be the 5% "abnormal" cases which happen even when the null hypothesis is true.

by table 3, which provides parameter estimates (Θ_K) for conditional risk premium with standard errors. The last column of table 3 provides the unconditional risk premiums per unit risk per month with their standard errors. Only the unconditional market premium is statistically significant.

In Table 4, we provide a simple frequency test about the significance of conditional factor premiums. The market premium's lower bound lies above zero for 290 months, which accounts for 60.7% of the total sample. To see whether the frequency of these occurrences is significantly different from the 5% level of occurrences allowed under the null hypothesis of zero factor premium, we provide a t-statistic which shows a value of 24.76, suggesting that there are times when the market premium is significantly positive.[10] The lower bounds for the real rate, inflation and IP growth never lie above zero during the sample period, suggesting no evidence of positive risk premiums.

A more interesting question is whether there are times when factor premiums are significantly negative. A casual observation of Figure 1 suggests that there are some months in which the two factor premiums appear to be negative. The market premium's upper bound falls below zero for 119 months, accounting for 24.9% of the total sample. To test whether the frequency of these occurrences is significantly different from the 5% level, we compute a t-statistic which shows a value of 23.266, suggesting that there are times when the market premium is significantly negative. The upper bounds for the real rate, inflation and IP growth never fall below zero during the sample period, suggesting no evidence of negative risk premiums.

Given the confidence bounds in figure 1a and the test results in Table 4, we have strong evidence indicating that the price for market risk is significantly negative during part of the sample period. This is interesting given the fact that most previous studies document the time-variation of risk premiums but find little evidence of significant negative risk premiums (see Fama (1991),

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[10] $T = \sqrt{n}\,(p\text{-}5\%)/\sqrt{p(1\text{-}p)}$, where n is the number of observations, p is the frequency of the lower bound lying above zero, 5% is the level of occurances allowed for under the null hypothesis of zero factor premium. It is worth noting that the frequency test implicitly assumes that the observations are independent. Given the small serial correlation in the factor premiums, This may not be a bad assumption. Moreover, we could also use the Newey-West procedure to design a more precise test by adjusting for the serial correlations.

Fama and French (1988)). In fact, most researchers argue that negative risk premiums are most likely sampling errors.[11] A negative premium suggests that either there is a mispricing of the market factor or the market factor sometimes provides hedge against certain economic risks, thus a negative premium is acceptable to investors. To gauge the economic significance of negative market premium, we break the sample into two periods, one of positive premium and the other of negative premium. The mean excess market return during the positive period is 1.30% per month with a volatility of 3.89% per month. The mean excess market return during the negative period is -0.97% per month with a volatility of 4.42% per month. There are 322 positive premium months and 156 negative premium months. A simple t-test gives us a t-statistic of 21.43, suggesting a very significant difference in mean excess returns between the positive months and the negative months! Our results here is consistent with those of Lo and MacKinlay (1992), in which they find it is possible to generate huge trading profits by employing market timing strategies based on market premium estimates.

It is worth noting here that the frequency test developed in the paper provides a simple and intuitive test of the hypothesis that risk premiums should be non-negative for certain macro-factors. It adds a new tool to empirical researchers' arsenal for testing non-negativity restrictions, which currently consists of the non-parametric approach developed by Boudoukh, Richardson, and Smith (1993) for conditional asset pricing models. Our approach complements theirs in a nice way. Their approach focuses on the testing of the restrictions over the whole sample period while our approach concentrates on the testing of the restrictions at any given point in time.

C. Robustness of the BMW^{TV} procedure

Figure 2 examines the robustness of market premium estimates under different model specifications. Figure 2a provide plots of market risk premium estimated under different factor

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[11] See Fama and Schwert (1977). One exception is the Boudoukh, Richardson, and Smith (1993) study, in which they reject the hypothesis that the conditional risk premium on the market is always nonnegative, using annual data and a non-parametric testing approach.

specifications. As we can see, the market risk premium is quite robust to different factor specifications. However, the factor premiums on relative rate, inflation and IPGrowth are quite sensitive to factor specifications, suggesting factor premium estimates are sensitive to missing latent factors.[12] Figure 2b compares three different market risk premium estimates: The market (RP) is estimated with a six-factor model of (1). The market (RP, lag) is estimated with the same model, while assuming we observe inflation and industrial production growth with a one-month lag. The market (RP, reg) is estimated with a single regression of market returns on conditional variables without imposing any asset pricing restrictions. As we can see from Figure 2b that the market premium is quite robust to these different model specifications.

Table 4 studies the robustness of the factor loading estimates on the observable factors. We provide the π estimates in equation (1) for the multi-factor model under K=4 and K=7. As can be seen, the beta estimates on the market factor and the IP-Growth factor are more or less robust to the two different factor specifications. The correlations between the beta estimates under the two specifications are 0.945 and 0.920 for the two factors. But the beta estimates on the relative rate and inflation are very sensitive to the different specifications, confirming our theoretical results that factor loading estimates are sensitive to missing latent factors. The results for K=5, 6 are quite similar. The last two rows of table 4 provide average beta estimates and corresponding unconditional risk compensations ($\lambda\beta$) for an equally-weighted portfolio of industrial stocks.

D. The Conditional Risk Premium for Size and Book-to-Market Equity

In Fama and French (1992), they discover that firm size and book-to-market equity could explain the cross-section of stock returns. In Fama and French (1993), they further discover that size and book-to-market equity are related to common factors in the economy. They help explain the cross-section of average stock return in a way that is consistent with multi-factor asset pricing model. However, since Fama and French (1992, 1993) use unconditional asset pricing models, we do not know: 1) whether size and book-to-market equity help explain the time-variation of

[12] These results can be obtained from the author upon request.

factor premiums over time and 2) whether size and book-to-market equity are "priced" in a conditional asset pricing model like (1).

When we use the same set of state variables as in Fama and French (1993) and regress excess returns on lagged values of these variables and their innovations as given by equation (2),[13] we discover that not only all the lagged variables are significant but the innovations are also significant, especially for size and book-to-market equity. This suggests that size and book-to-market equity help explain the time-variation of factor premiums over time. Since the innovations in equation (2) are systematic factors, we confirm and extend the result of Fama and French (1993) that size and book-to-market equity are related to common factors in the economy, which help explain the cross-section of average stock return in a way that is consistent with conditional multi-factor asset pricing models.

Figure 3a and 3b provide the conditional factor premiums associated with size and book-to-market equity. We can see that both factor premiums vary over time and the size premium tends to peak around the trough of NBER business cycles. At the bottom of Table 4, we provide some tests about the significance of size and book-to-market premiums. The size premium's lower bound lies above zero for 116 months and the book-to-market for 19 month, which accounts for 39.7% and 6.5% of the total sample. To see whether the frequencies of these occurrences are significantly different from the 5% level of occurrences allowed under the null hypothesis of zero factor premium, we compute t-statistics which show a value of 24.76 for the size premium and 4.21 for the book-to-market premium, suggesting that there are times when both factor premiums are significantly positive.

An interesting empirical question is whether there are times when both factor premiums are significantly negative. Since the factor loadings on the two factors are positive, a negative factor premium indicates either a possible mispricing of the two factors or they provide hedge against certain economic risks. A casual observation of Figure 3a and 3b suggests that there are some

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[13]The state variables include market excess returns, term premium, default premium, size, and book-to-market equity.

months in which the two factor premiums appear to be negative. The size premium's upper bound falls below zero for 32 months, accounting for 10.9% of the total sample. To see whether the frequency of these occurrences is significantly different from the 5% level, we compute a t-statistic which shows a value of 10.381 for the size premium, suggesting that there are times when the size premiums are significantly negative.[14] The book-to-market premium's upper bound lies below zero for only 11 months (3.8% of the sample), which is within the 5% level of occurrences allowed under the null hypothesis of zero premium.

E. Test of the CAPM Linear Pricing Restriction (5)

Table 6 presents the test statistics for the CAPM linear pricing restriction (5). The second column of Table 6 gives the sum of squared residuals, Q, of regressions (4) with the CAPM restriction (5) imposed. The third column gives the difference in the sum of squared residuals between model (4) with and without the CAPM restriction. The sum of squared residuals of model (4) is given in Table 2. Thus, we calculate $L_{(K=7,\ \text{Table 6})} = Q_{(K=7,\ \text{Table 6})} - Q_{(K=7,\ \text{table 2})} = 59.19-25.35 = 33.84$, etc. The degrees of freedom (DF(L)) is determined by the difference between the number of parameters estimated in two models. From Table 6, we can see that the CAPM restriction (5) is rejected for $K \geq 5$ but not for $K = 4$ ($P = 0.306, 0.021, 0.000, 0.004$; $K = 4, 5, 6, 7$).

F. Mispricing under the CAPM and the APT model

In table 2 and 5, we have tested the linear pricing relationship under the APT and the CAPM. Our tests generally reject the CAPM but not the APT model. As is well known, a failure to reject a hypothesis could be due to a lack of statistical power. In other words, although our test do not reject the APT model, the model could still have poor pricing for individual portfolios. Thus, it would be very helpful to have a measure of mispricing to gauge the economic significance of the

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[14]It is intriguing to observe that the *ex ante* size premium is quite large in absolute value but negative for the month of October 1987, indicating a possible presence of market mispricing.

tests and to compare the pricing performance of the APT and the CAPM. The mispricing measure could also be used to gauge the significance of latent factors in asset pricing, thus helping us eliminating those latent factors which contribute little to asset pricing. In this paper, we use the unconditional mean η_t, $E(\eta_t)$, as the measure for mispricing, where η_t is defined as $\eta_t = r_t - \gamma\Theta_K X_{t-1} - \beta R_t - \gamma g_t$. Since, in general, $\eta_t = (\Gamma - \gamma\Theta_K)X_{t-1} + \varepsilon_t - \beta\varepsilon_{Jt}$, it is easy to see that $E(\eta_t)=0$ if there is no mispricing in the APT model, i.e., $\Gamma = \gamma\Theta_K$. By the same token, we could use $E(\eta_t)$ as the measure for mispricing of the CAPM model by imposing restriction (5) on the Θ_K parameters.

Figure 4 provides a plot for the mispricing of the CAPM (K = 7) and the APT model under six and seven factors (K = 6, 7). The mispricing is given in percentage per month. We can see that the mispricing for both the CAPM and APT under K = 7 are quite small, since the absolute mispricings are less than 6 basis points per month for all size portfolios. However, the mispricing estimates are quite sensitive to different factor specifications. We found the mispricing to be much larger for certain portfolios when K = 6. For instance, the mispricing for the smallest decile portfolio could be as much as 2 percentage point per year.

This implies that omitted factors could affect asset pricing in an APT model with time-varying risk premiums. It suggests that, even if one has included the market portfolio as a reference portfolio (see, for instance, Ferson (1990) and Brown and Otsuki (1992)), one still needs to consider the effects of omitted factors for asset pricing.

G. Measurement Errors Associated with the First-Step VAR Factor Estimates.

The macro factors used in most APT studies are derived from estimates of some statistical models.[15] This implies that these factors carry measurement errors. Since the factors are used in the next step APT tests, there is a potential errors-in-variables problem in these tests. Most previous studies choose to ignore this problem. In this paper, using an approach similar to

[15] See, for example, Chen, Roll and Ross (1986), Ferson and Harvey (1991), and Shanken and Weinstein (1990). In this paper, the factors are derived from residuals of a VAR process.

Campbell and Mei (1993), we provide a simple statistic procedure to assess the impact of measurement errors. We will use the mispricing test as an example, but the same procedure can be easily applied to other tests as well.

To gauge the impact of measurement error on the mispricing results, I calculate the standard deviations for the mispricing estimates, which are written as a nonlinear function of the parameters of the VAR process.[16] Defining these parameters as γ and their covariance matrix as V, the standard deviations of the mispricing statistics, which is a nonlinear function $f(\gamma)$, can be calculated as $\sqrt{f_\gamma(\gamma)' V f_\gamma(\gamma)}$. Based on the standard deviations, we can compute the 95% confidence bounds for the mispricing estimates. The results are provided in Figure 5. The confidence bounds for the mispricing are fairly tight (less than 2 base point per month), suggesting that the "true" mispricing is quite small for the APT model used in the study. Thus, our basic conclusion about the capability of APT in explaining asset returns is unaffected by the measurement errors resulted from the first-step VAR estimation.

VI. Conclusion

This paper develops a BMW^{TV} approach to the estimation of risk premiums by integrating the APT model of Burmeister and McElroy (1988) with time-varying risk premiums. It provides premium estimates for macro-factors over time under a unified APT framework which allows for both observable and latent factors. We find significant negative risk premiums for the market factor and the size factor during the sample period. We discover that the risk premium and factor loading estimates for the observable factors are quite sensitive to omitted factors, suggesting the importance of accounting for latent factors in conditional multi-factor models. We also test the conditional CAPM and the APT restrictions imposed on asset excess returns. We find that the data generally do not reject the APT model while the CAPM model is strongly rejected. We also find that the mispricings under the APT model and the CAPM model are relatively small, but the results are quite sensitive to omitted factors.

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[16] Note, $g_{t+1} = w_{t+1} = X_{t+1} - A\ X_t$.

We also extend the result of Fama and French (1993) by showing that size and book-to-market equity are related to common factors in the economy, which help explain the cross-section of average stock return in a way that is consistent with <u>conditional</u> multi-factor asset pricing models. We also discover that the variation of the size premium appears to be related to business cycles.

There are several advantages associated with the BMW^{TV} approach developed in the paper. First, by using both observable and latent factors, we can study the time-variation of risk pricing on some interesting macroeconomic factors without worrying about the possible contamination by omitted factors. Second, a new frequency test procedure is developed for testing negative risk premiums. Third, the approach simultaneously estimates risk premiums and factor loadings via a GMM approach. Thus, it is immune to the errors-in-variables problem associated with the two-pass regression approach used by Fama and MacBeth (1973) and other studies.[17] Fourth, it is relatively easy to use the approach to compute the pricing errors under the APT model. This makes the approach attractive to portfolio managers who are interested in finding mispriced assets by using the APT model.

However, there are a few limitations associated with the approach developed in this paper. First, by jointly estimating risk premiums and factor loadings, the number of parameters increase very rapidly with the number of assets studied.[18] As is well known, the GMM approach does not guarantee convergence when it solves a large non-linear estimation problem.[19] Another limitation of the paper is that it assumes the factor loadings to be constant during the sample period. Even though Ferson and Harvey (1991) discover that factor loading variations only explain a small

[17] However, our procedure does suffer from a different kind of errors-in-variables problem as a result of obtaining factor estimates from a VAR process. Our empirical study suggests that the impact of this problem on our tests appear to be small.

[18] There are 94 parameters (=4x6+7x10) in the APT model of (4) with four observable factors, three unobservable factors, six economic state variables, and ten size portfolios.

[19] A simple remedy to this limitation is to use a revised version of the Fama and MacBeth (1973) two-pass approach, by which one estimates the linear regression model of (2) in the first pass, and solves the risk premium coefficients using equation (3) in the second pass. However, this two-pass approach is subject to the errors-in-variables problem as we mentioned earlier.

percentage of asset expected returns, we think it might be important to take the variation into account since certain mispricing under the constant beta model could come from time-variation in factor loadings. We leave this problem for future research.

Appendix

A. Derivation of the APT and CAPM Restrictions

Following Burmeister and McElroy (1988), we assume that (1) also holds for a set of reference asset $R_t = (r_{N+1,t},\ldots,r_{N+J,t})'$:

$$R_t = E_{Jt} + \beta_J f_t + \pi_J g_t + \varepsilon_{Jt} \tag{A1}$$

where E_{Jt} is Jx1, β_J is JxJ, π_J is Jx(K-J), ε_{Jt} is Jx1. β_J and π_J are assumed to be full column rank.

Without loss of generality, we could set $\beta_J = I_J$ and solve for the latent factors in (A1):

$$f_t = R_t - E_{Jt} - \pi_J g_t - \varepsilon_{Jt} \tag{A2}$$

Substituting (A2) into eq. (1) in the text:

$$r_t = \beta_{0t} + \beta R_t + \gamma g_t + \eta_t \tag{A3}$$

where:

$$\beta_{0t} = E_t - \beta E_{Jt},\ \gamma = \pi - \beta\, \pi_J,\ and\ \eta_t = \varepsilon_t - \beta\, \varepsilon_{Jt}, \tag{A4}$$

with $E_{t-1}[\eta_t] = 0$, $E[\eta_t | g_t] = 0$, and $E[\eta_t | R_t] \neq 0$. Here β_{0t} is Nx1, β is NxJ, γ is Nx(K-J), and η_t is Nx1.

Given the linear factor model in equation (1), the APT imposes the well known linear pricing restriction:

$$E_{t-1}[r_{it}] = E_{it} = \sum_{k=1}^{K} b_{ik}\lambda_{kt}, \tag{A5}$$

where λ_{kt} is the "market price of risk" for the k'th factor at time t. Denoting $\lambda_J = (\lambda_{1t},\ldots,\lambda_{Jt})'$ and $\lambda_K = (\lambda_{J+1,t},\ldots,\lambda_{Kt})'$, we write (A5) as

$$E_{t-1}\,[r_t] = E_t \;=\; \beta\lambda_J + \pi\lambda_K \tag{A6}$$

$$E_{t-1}\,[R_t] = E_{Jt} \;=\; \lambda_J + \pi_J\lambda_J \tag{A7}$$

Taking conditional expectation of (A3), we have

$$E_{t-1}[r_t] = \beta_{0t} + \beta E_{t-1}[R_t]. \tag{A8}$$

Substituting (A6) and (A7) into (A8) and using (A4), we have,

$$\beta_{0t} = \gamma\lambda_K. \tag{A9}$$

Now suppose that the information set at the end of time t-1 consists of a vector of L (L > K) economic or forecasting variables $X_{p,t-1}$, p=1...L, and that conditional expectations are linear in those variables. Then we can write λ_{kt} as:

$$\lambda_{kt} = \sum_{p=1}^{L} \theta_{kp}X_{p,t-1}, \text{ or simply, } \lambda_J = \Theta_J X_{t-1},\ \lambda_K = \Theta_K X_{t-1}, \tag{A10}$$

where Θ_J is JxL and Θ_K is (K-J)xL. Substituting (A10) into (A9), we have

$$\beta_{0t} = \gamma\Theta_K X_{t-1}. \tag{A11}$$

Equation (A11) is a generalization of the Burmeister and McElroy (1988) result. It differ from theirs in that it allows β_{0t} to vary over time according to the economic state variables X_t. Further substituting (A11) into (A3) gives the APT specification under time-varying risk premiums:

$$r_t = \gamma\Theta_K X_{t-1} + \beta R_t + \gamma g_t + \eta_t, \tag{A13}$$

Comparing equations (2) and (A13), we derive the APT restriction imposed on regression (2): $\Gamma = \gamma\Theta$.

In some cases, we may be interested in the sensitivities (π) of assets towards the observable factors. To identify π, we could use the following regression:

$$R_t = \Phi X_{t-1} + \pi_J g_t + u_{Jt}, \tag{A14}$$

where $u_{Jt} = f_t + e_{Jt}$. Here Φ is JxL and u_{Jt} is Jx1. Without loss of generality, we could also renormalize f_t such that it is orthogonal to g_t. In that case, we have $E(u_t|g_t)=0$. To identify π in equation (1), we need to estimate (2) and (A14) jointly and π can be identified from using equation (A4): $\pi = \gamma + \beta \pi_J$,

To derive the parametric constraints imposed on equation (1) by the conditional CAPM, We note that the conditional CAPM requires that

$$E_{t-1}(r_{it}) = \beta_i \lambda_{mt}, \text{ and } \lambda_{mt} = E_{t-1}(r_{mt}). \tag{A15}$$

where r_{mt} is the excess return on the market. Since factor premiums can be treated as conditional expected excess returns on mimicking factor portfolios, we must also have:

$$\lambda_{jt} = c^j \lambda_{mt}, \; j=1,\dots,K. \tag{A16}$$

Assuming the market is one of the observable factors and combining (A16) with (A10), we can obtain the following testable restriction imposed on the coefficients of equation (4) by the CAPM:

$$\Theta_K^j = c^j \Theta_K^1, \; j=1,\dots,K\text{-}J, \tag{5}$$

where Θ_K^1 is the conditioning parameters associated with the market premium in equation (A10).

B. A Simple Example about the Impossibility of Transformation

To see that there usually does not exist a tranformation matrix which could re-normalize the factors so that only one latent factor is priced, we will use the following two factor model with time-varying risk premiums: $r_t = E_t + \beta_1 f_{1t} + \beta_2 f_{2t} + \eta_t$, where $E_t = \lambda_{1t}\beta_2 + \lambda_{2t}\beta_2$. We assume

$\lambda_{1t} = X_{1t}$ and $\lambda_{2t} = X_{2t}$. The question is whether we can find a 2x2 transformation matrix T, such that:

$$T'T = I, \text{ and } T\begin{pmatrix} \lambda_{1t} \\ \lambda_{2t} \end{pmatrix} = \begin{pmatrix} \lambda_{1t}' \\ 0 \end{pmatrix}, \text{ for all } \lambda_{1t} \text{ and } \lambda_{2t}. \qquad \text{(A17)}$$

To satisfy the second part of (A17) for all λ_{1t} and λ_{2t}, it is easy to see that the second row of the T matrix has to be all zero. This violates the first part of (A17), which implies that T is a full rank matrix.

References

Ammer, John, 1992, Macroeconomic risk and asset pricing: Estimating the APT with observable factors, mimeo, Board of Governors.

Boudoukh, Jacob, Matthew Richardson, and Tom Smith, 1993, Testing inequality restrictions implied from conditional asset pricing models, *Journal of Financial Economics* , 34, 387-408.

Brown, Stephen and Mark Weinstein, 1983, A new approach to testing asset pricing models: The bilinear paradigm, *Journal of Finance* 38, 711-743.

Brown, Stephen and Toshiyuki Otsuki, 1992, Exchange rate volatility and equity returns, Working Paper, New York University.

Burmeister, Edwin and Marjorie McElroy, 1988, Joint test of factor sensitivities and risk premia for the arbitrage pricing theory, *Journal of Finance* 43, 721-733.

Campbell, John, 1987, Stock returns and the term structure, *Journal of Financial Economics* , 18, 373-399.

Campbell, John and John Ammer 1993, What moves the stock and bond markets? A variance decomposition for long term asset returns,*Journal of Finance* 48, 3-37.

Campbell, John Y. and Jianping Mei, 1993, Where Do Betas Come From? Asset Pricing Dynamics and the Sources of Systematic Risk, *Review of Financial Studies*, 6, 567-592.

Chan, K.C., Nai-fu Chen, and David Hsieh, 1985, An exploratory investigation of the firm size effect, *Journal of Financial Economics* 14, 451-471.

Chen, Nai-fu, Richard Roll, and Stephen Ross, 1986, Economic forces and the stock market, *Journal of Business* 59, 386-403.

Connor, Gregory and Robert Korajczyk, 1988, Risk and return in an equilibrium APT: Application of a new test methodology, *Journal of Financial Economics* 21, 255-289.

Conway, Delores and Marc Reiganum, Stable factors in security returns: Identification using cross-validation, *Journal of Business and Economic Statistics*, 6, 1-28

Fama, Eugene, 1991, Efficient capital markets: II, *Journal of Finance* 46, 1575-1619.

Fama, Eugene and Kenneth French, 1988, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3-25.

Fama, Eugene and Kenneth French, 1992, The cross-section of expected returns, *Journal of Finance*, 47, 427-465

Fama, Eugene and Kenneth French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

Fama, Eugene and James MacBeth, 1973, Risk, Return, and Equilibrium: Empirical Tests, *Journal of Political Economy*, 71, 607-636.

Ferson, Wayne, 1990, Are the latent variables in time-varying expected returns compensation for composition risk? *Journal of Finance* 45, 397-492.

Ferson, Wayne and Campbell Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385-415.

Gibbons, Michael and Wayne Ferson, 1985, Tests of Asset Pricing Models with Changing Expectations and an Unobservable Market Portfolio, *Journal of Financial Economics* 14, 217-236.

Grinblatt, Mark and Shridan Titman, 1983, Factor pricing in a finite economy, *Journal of Financial Economics* 12, 497-507.

Gultekin, Mustafa and Bulent Gultekin, 1987, Stock return anomalies and tests of the APT, *Journal of Finance* 42, 1213-1224.

Huberman, Gur, Shmuel Kandel, and Robert Stambaugh, 1987, Mimicking portfolios and exact arbitrage pricing, *Journal of Finance* 42, 1-9.

Keim, Donald and Robert Stambaugh, 1986, Predicting returns in the stock and bond markets, *Journal of Financial Economics* 17, 357-390.

Lehmann, Bruce and David Modest, 1988, The empirical foundations of the arbitrage pricing theory, *Journal of Financial Economics* 21, 222-254.

Jobson, J.D., 1982, A multivariate linear regression test for the arbitrage pricing theory, *Journal of Finance* 37, 1037-1042.

Lo and MacKinlay, 1992, Maximizing predictability in the stock and bond markets, MIT, working paper #3450-92-EFA.

Ross, Stephen, 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 13, 341-360.

Shanken, Jay, 1992, On the estimation of beta-pricing models, *Review of Financial Studies* 5, 1-33.

Shanken, Jay and Mark Weinstein, 1990, Macroeconomic variables and asset pricing: Further results, Working paper, University of Rochester.

Shukla, Ravi and Charles Trzcinka, 1990, Sequential tests of the arbitrage pricing theory: A comparison of principle components and maximum likelihood factors, *Journal of Finance* 45, 1541-1564.

Stulz, Rene, 1986, Asset Pricing and expected inflation, *Journal of Finance* 41, 209-244.

Trzcinka, Charles, 1986, On the number of factors in the arbitrage pricing model, *Journal of Finance* 41, 347-368.

Table 1

Test of Number of Factors in the Multi-factor Model (2)

Total Factors	Q	L	DF(L)	P
(1) K=7	14.61	–	10	0.147
(2) K=6	20.93	6.32	10	0.787
(3) K=5	38.73	17.80	10	0.058
(4) K=4	74.11	35.38	10	0.005

Note: Q is the weighted sum of squared residuals, corresponding to each specification of K in equation (2). L is the difference in the sum of squared residuals between its corresponding specification and the specification above it, which has one more latent factor. If the specification is correct, L has a χ^2 distribution with degrees of freedom (DF(L)) given by the fourth column. P gives the significance level by which the specification can be rejected. We use excess returns of the decile size portfolios to estimate (2) for the time periods of 1948-1987. The state variables used are the following: a constant, excess return on the market, dividend yield, inflation, relative treasury bill rate, and industrial production growth. We use excess returns to the Petroleum, Finance, and Durable Goods industrial portfolios as reference portfolios returns to substitute out the unobservable factors. We use four observable factors in our study: the market, the relative interest rate, the inflation, and the industrial production growth. They are derived from residuals of a VAR process.

Table 2

Test of the APT Linear Pricing Restriction (3)

Total Factors	Q	L	DF(L)	P
(1) K=7	25.35	10.74	36	0.999
(2) K=6	37.78	16.85	36	0.997
(3) K=5	69.19	30.46	36	0.729
(4) K=4	102.15	28.04	36	0.825

Note: Q is the weighted sum of squared residuals, corresponding to each specification of K in equation (2), with restriction (3) imposed. L is the difference in the sum of squared residuals between the Q values in table 1 and table 2 with the same K factors. If the APT restriction holds, L has a χ^2 distribution with degrees of freedom (DF(L)) given by the fourth column. P gives the significance level by which the APT restriction can be rejected. The variables and instruments used in estimating (3) are the same as those used in estimating (2), with restriction (3) imposed.

Table 3

Conditional Risk Premiums Estimates Θ_K in equation (3)

Size	cons	$Market_{t-1}$	DY_{t-1}	$R\text{-}Rate_{t-1}$	$Infn_{t-1}$	IPG_{t-1}	Un. Mean
Market	0.047	-0.001	0.042	-0.005	-0.001	0.000	0.560
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.05)
Interest	0.019	0.021	-0.061	0.003	-0.001	0.000	0.019
	(0.04)	(0.02)	(0.04)	(0.00)	(0.00)	(0.00)	(0.04)
Inflation	0.135	-0.106	-0.043	-0.036	-0.002	0.001	0.135
	(0.21)	(0.10)	(0.26)	(0.02)	(0.00)	(0.00)	(0.21)
IP Growth	0.682	0.534	-0.972	0.148	-0.019	0.007	0.682
	(0.81)	(0.44)	(0.99)	(0.08)	(0.02)	(0.00)	(0.81)

Note: Conditional Risk Premiums Estimates Θ_K for the size portfolios when K=7. Standard errors are provided in the parentheses. Un. Mean is the unconditional risk premiums per unit risk/month during the sample period. The unit for each variable is % per month for the market excess return, % per annum for the dividend yield (DY), % per annum for R-Rate, % per annum for infn (inflation), % per annum for IPG (industrial production growth, seasonally adjusted).

Table 4

Binomial Test of Negative Factor Premium

Factors	lower > 0	t-statistic	upper <0	t-statistic	#obs
Market	60.7%	51.04	24.9%	23.266	478
Interest	0%	-----	0%	-----	478
Inflation	0%	-----	0%	-----	478
IP Growth	0%	-----	0%	-----	478
SMB*	39.7%	24.76	10.9%	10.381	292
HML*	6.5%	4.21	3.8%	-----	292

Note: The second column provides the percentage of risk premium observations whose lower bounds lie above zero. The third column provides the t-statistics of a binomial test, that the percentage of observations whose lower risk premium bounds lie above zero would not exceed 5%. The fourth column provides the percentage of risk premium observations whose upper bounds fall below zero. The fifth column provides the t-statistics of a binomial test, that the percentage of observations whose upper risk premium bounds fall below zero would not exceed 5%. The last column gives the total number of observations in the sample.
*SMB (small minus big) and HML (high minus low) are the size and book-to-market equity factors used in Fama and French (1993).

Table 5

The Robustness of Beta Estimates (π) Under Different Specifications

Industry	Market	R-Rate	Infn	IPG	Market	R-Rate	Infn	IPG
	K=4, J=0				K=7,J=3			
Petroleum	0.977	5.133	1.253	-0.080	1.024	4.112	1.047	-0.027
Finance	0.946	-7.139	0.051	0.057	1.041	2.970	0.557	-0.019
Durables	1.142	2.654	-0.617	0.141	1.062	-1.694	-1.001	0.150
Basic Industry	1.095	1.346	-0.274	-0.014	1.050	-3.687	-0.420	0.038
Food/Tobacco	0.820	-5.073	-0.278	-0.113	0.842	-2.341	0.170	-0.146
Construction	1.154	-0.058	-0.561	-0.130	1.068	-1.043	-0.840	-0.100
Capital Goods	1.094	3.572	-0.316	-0.105	1.043	-2.342	-0.353	-0.086
Transportation	1.151	-5.428	0.313	0.199	1.066	-5.480	0.403	0.283
Utilities	0.575	-13.36	-0.528	0.033	0.671	2.568	0.079	-0.044
Textile/Trade	1.014	-2.552	-0.294	-0.023	0.946	-2.247	-0.432	-0.034
Service	1.108	-2.408	-0.533	-0.099	1.110	-3.054	0.081	-0.115
Leisure	1.149	0.246	-0.991	-0.113	1.115	-0.033	-0.408	-0.143
Average β	1.019	-1.923	-0.231	-0.021	1.003	-1.023	-0.093	-0.020
Average $\lambda\beta$	0.649	0.092	-0.130	0.058	0.554	-0.019	-0.012	-0.014

Note: Factor loadings estimates for the industry portfolios when K=4, 7.

Table 6

Test of the Conditional CAPM Restriction (5)

Total Factors	Q	L	DF(L)	P
(1) K=7	59.19	33.84	15	0.004
(2) K=6	83.67	45.89	15	0.000
(3) K=5	97.24	28.05	15	0.021
(4) K=4	119.36	17.12	15	0.306

Note: Q is the weighted sum of squared residuals, corresponding to each specification of K in equation (4) with the CAPM restriction (5) imposed. L is the difference in the sum of squared residuals between the Q values in table 6 and table 2 with the same K factors. If the CAPM restriction holds, L has a χ^2 distribution with degrees of freedom (DF(L)) given by the fourth column. P gives the significance level by which the CAPM restriction can be rejected. The variables and instruments used in estimating (4) under the CAPM are the same as those used in estimating (2).

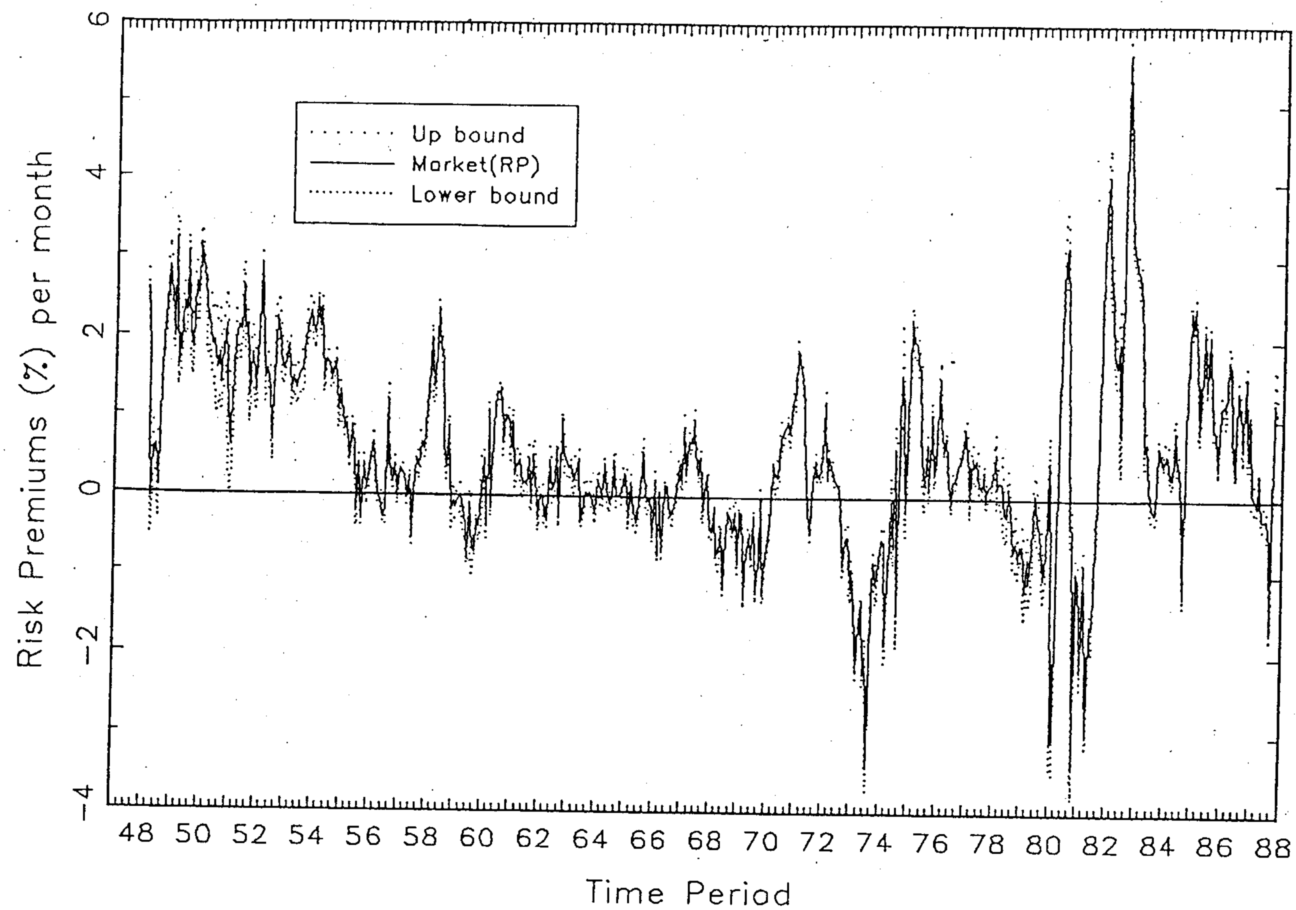

Figure 1a. Risk premium on the market for the APT model with six factors (four observable and two latent (K=6, J=2)). The risk premium is estimated by a nonlinear GMM approach using the decile size portfolios. 95% confidence bounds are provided for the risk premium. The upper bound is calculated by adding 1.96 times the conditional standard deviation to the risk premium and the lower bound calculated by subtracting 1.96 times the conditional standard deviation from the risk premium. The conditional standard deviations are taken from the variance-covariance matrix of the parameter estimates and state variables.

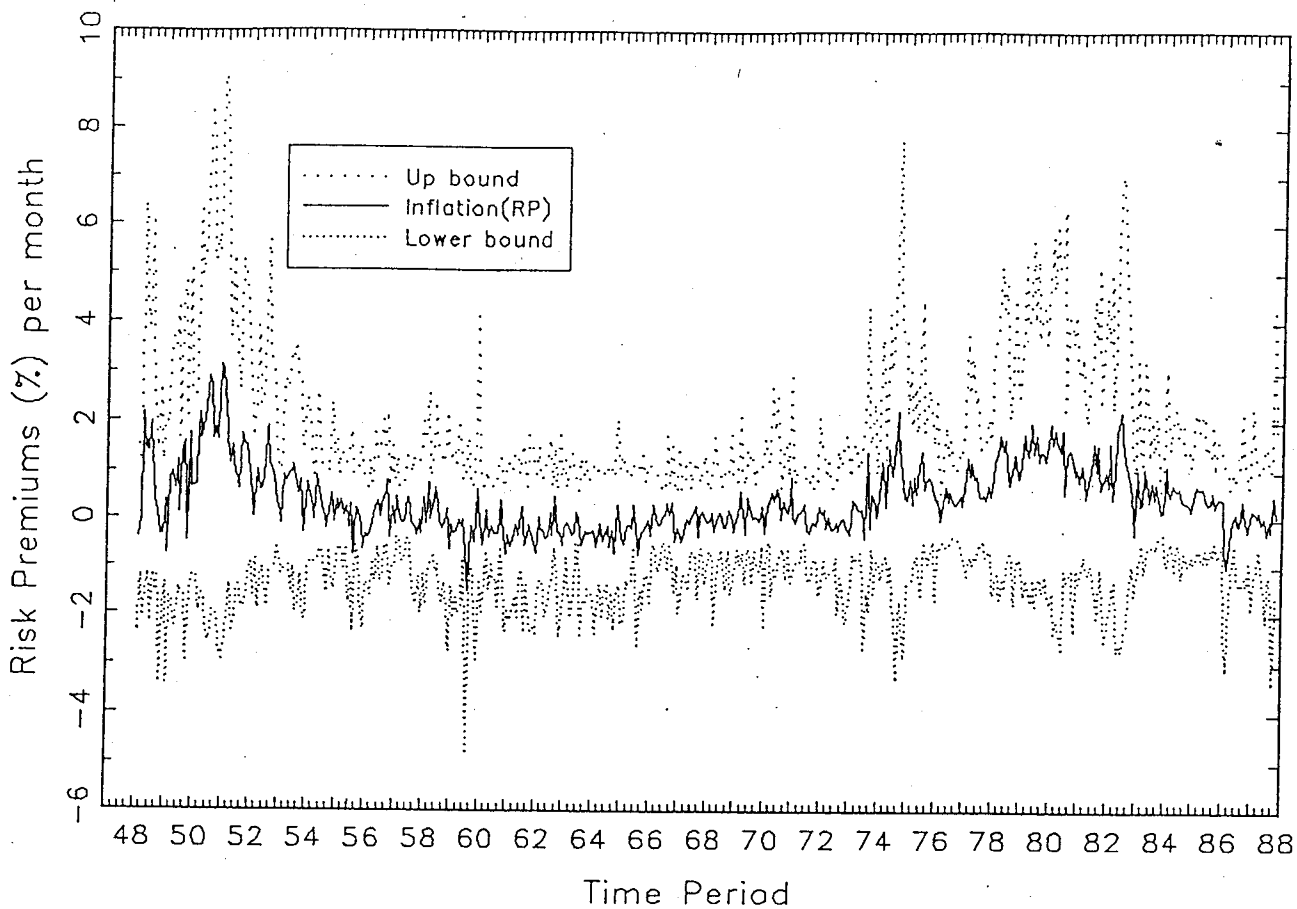

Figure 1b. Risk premium on inflation for the APT model with six factors (four observable and two latent (K=6, J=2)). The risk premium is estimated by a nonlinear GMM approach using the decile size portfolios. 95% confidence bounds are provided for the risk premium. The upper bound is calculated by adding 1.96 times the conditional standard deviation to the risk premium and the lower bound calculated by subtracting 1.96 times the conditional standard deviation from the risk premium. The conditional standard deviations are taken from the variance-covariance matrix of the parameter estimates and state variables.

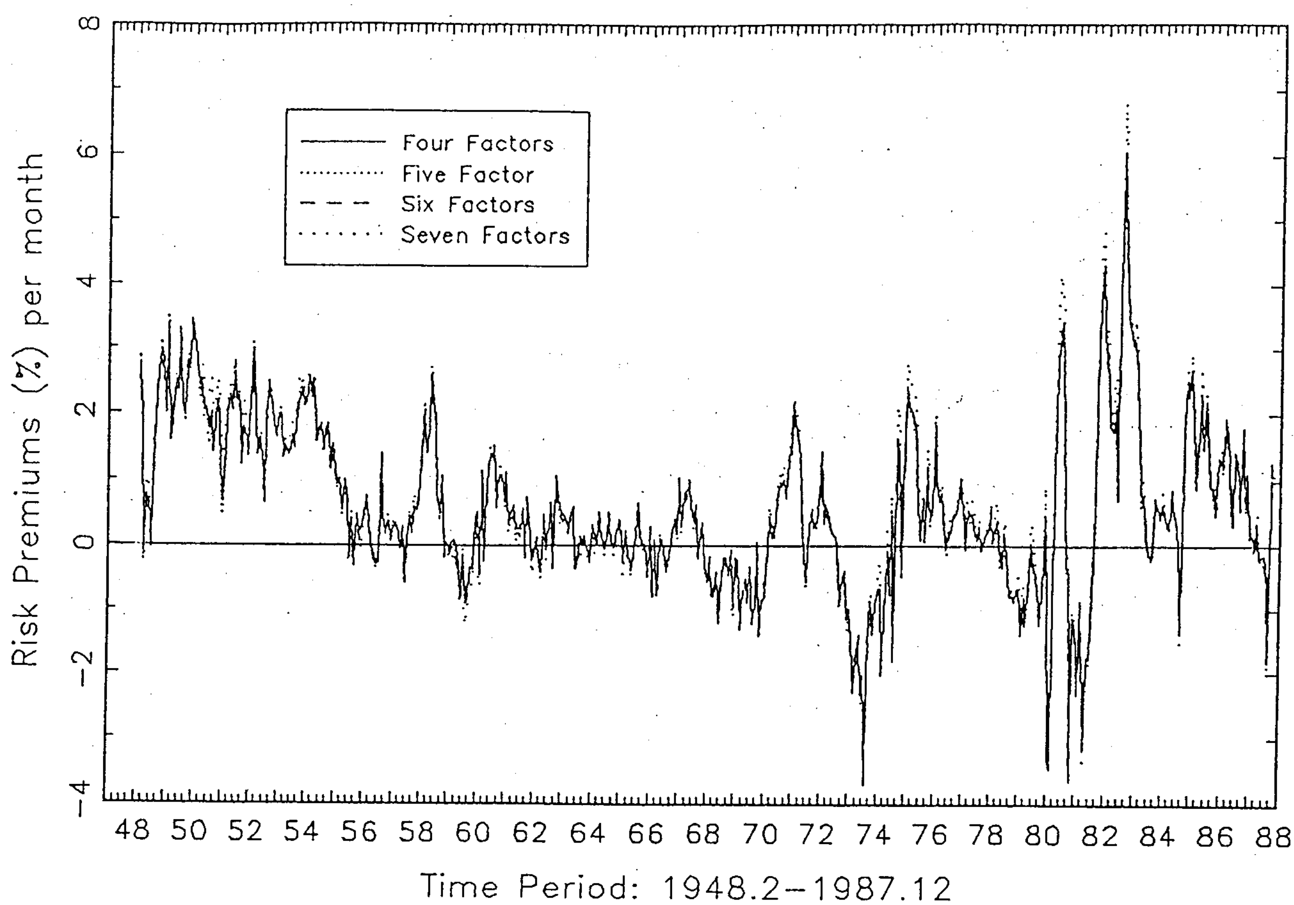

Figure 2a. Risk premium on the market under different factor specification. The risk premium is estimated by a nonlinear GMM approach using the decile size portfolios. The four-factor model includes four observable factors. The five-factor model includes four observable factors plus one unobservable factor. The six-factor model includes four observable factors plus two unobservable factors. The seven-factor model includes four observable factors plus three unobservable factors.

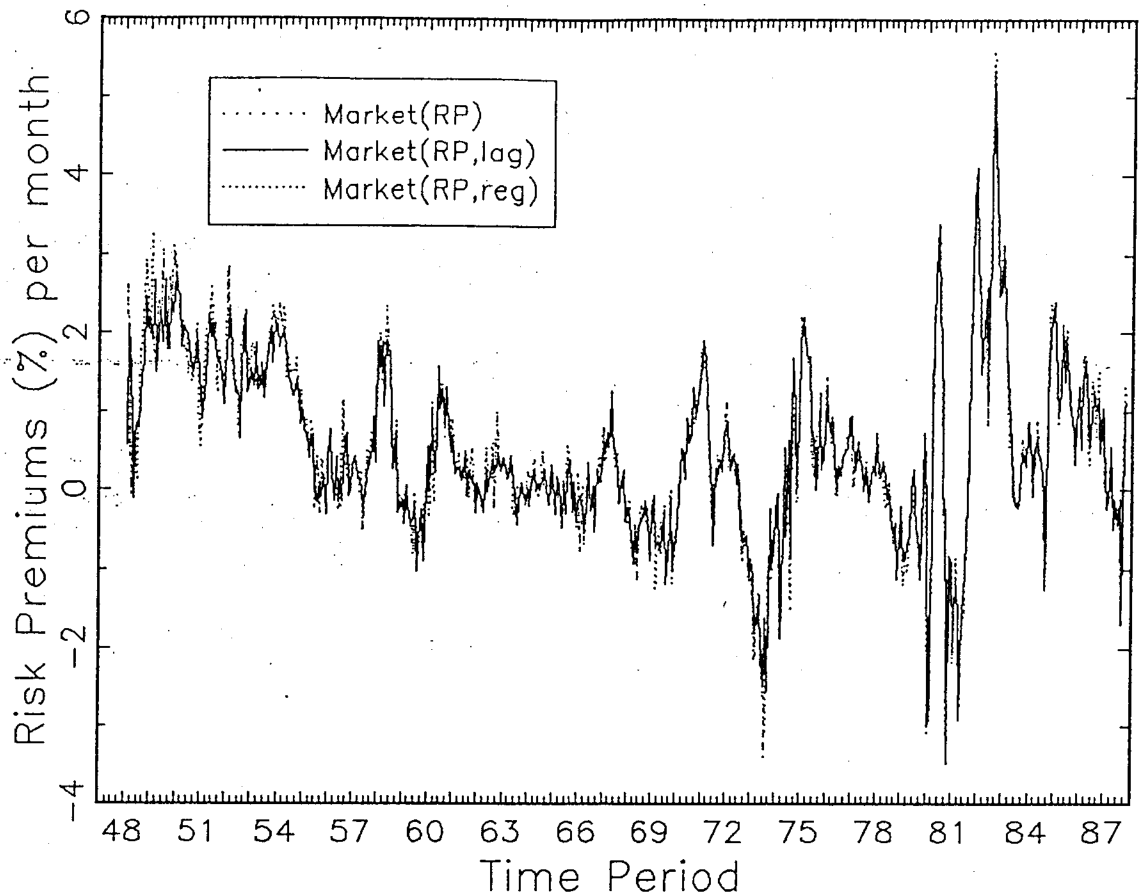

Figure 2b. Risk premium on the market under different model specification. The market (RP) is estimated with a six-factor model. The market (RP, lag) is estimated with the same model, while assuming we observe inflation and industrial production growth with a one-month lag. The market (RP, reg) is estimated with a single regression of market returns on conditional variables without imposing any asset pricing restrictions.

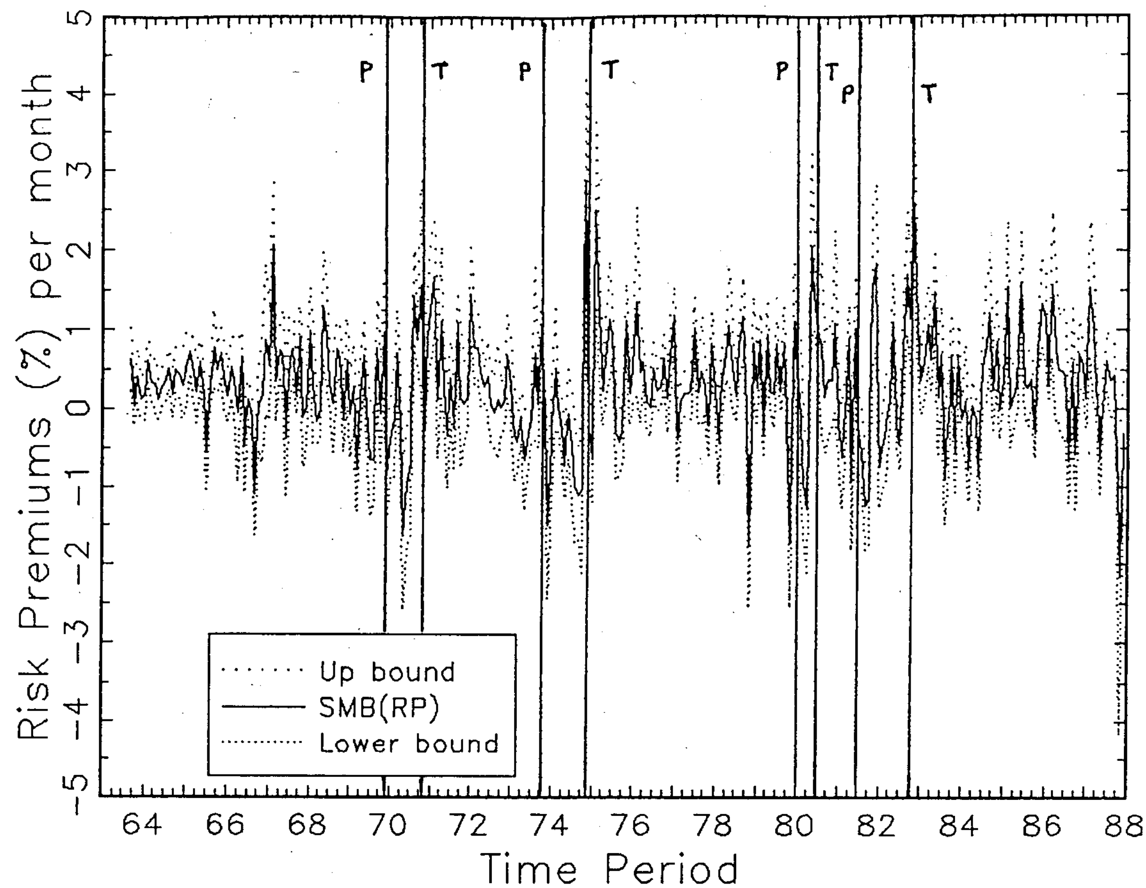

Figure 3a. Risk premium on the SMB (small minus big) size factor for the APT model with six factors (four observable and two latent (K=6, J=2)). The risk premium is estimated by a nonlinear GMM approach using the decile size portfolios. 95% confidence bounds are provided for the risk premium. The upper bound is calculated by adding 1.96 times the conditional standard deviation to the risk premium and the lower bound calculated by subtracting 1.96 times the conditional standard deviation from the risk premium. The conditional standard deviations are taken from the variance-covariance matrix of the parameter estimates and state variables.

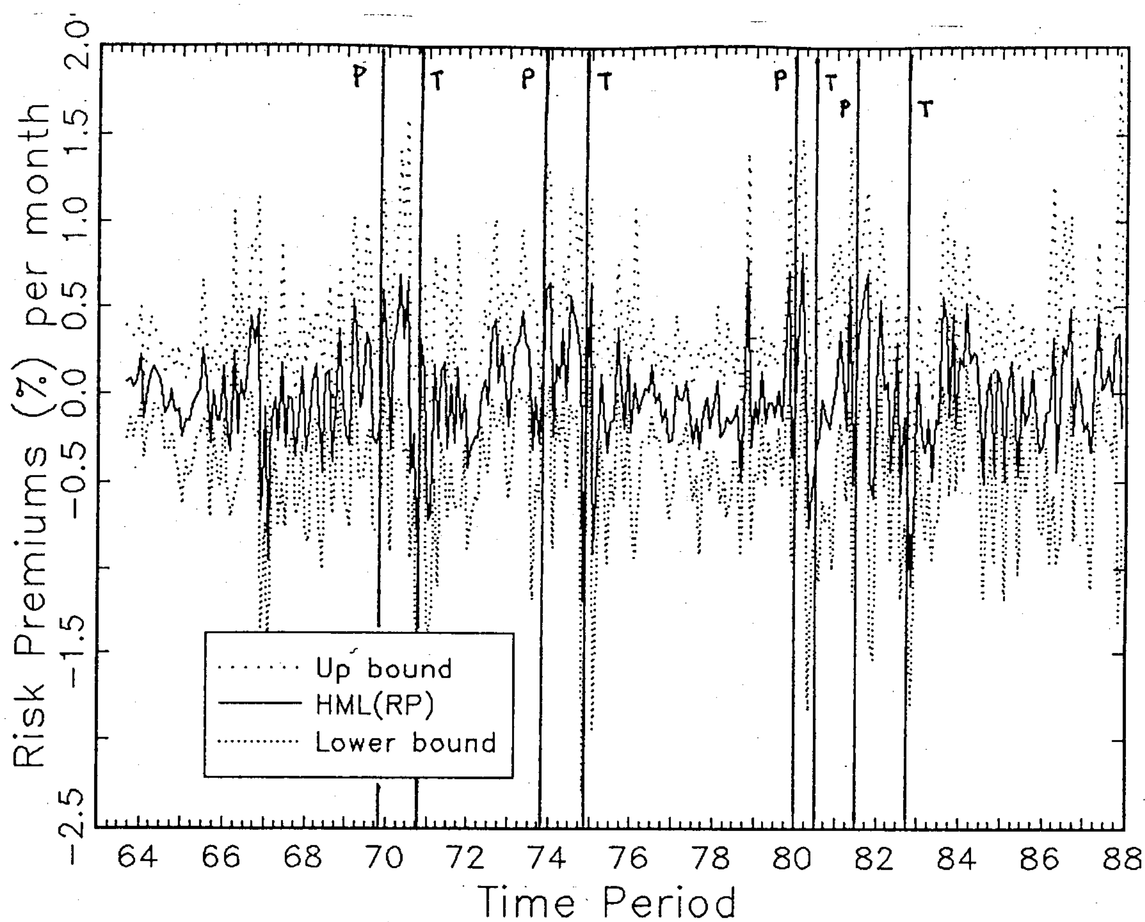

Figure 3b. Risk premium on HML (high minus low) book-to-market factor for the APT model with six factors (four observable and two latent (K=6, J=2)). The risk premium is estimated by a nonlinear GMM approach using the decile size portfolios. 95% confidence bounds are provided for the risk premium. The upper bound is calculated by adding 1.96 times the conditional standard deviation to the risk premium and the lower bound calculated by subtracting 1.96 times the conditional standard deviation from the risk premium. The conditional standard deviations are taken from the variance-covariance matrix of the parameter estimates and state variables.

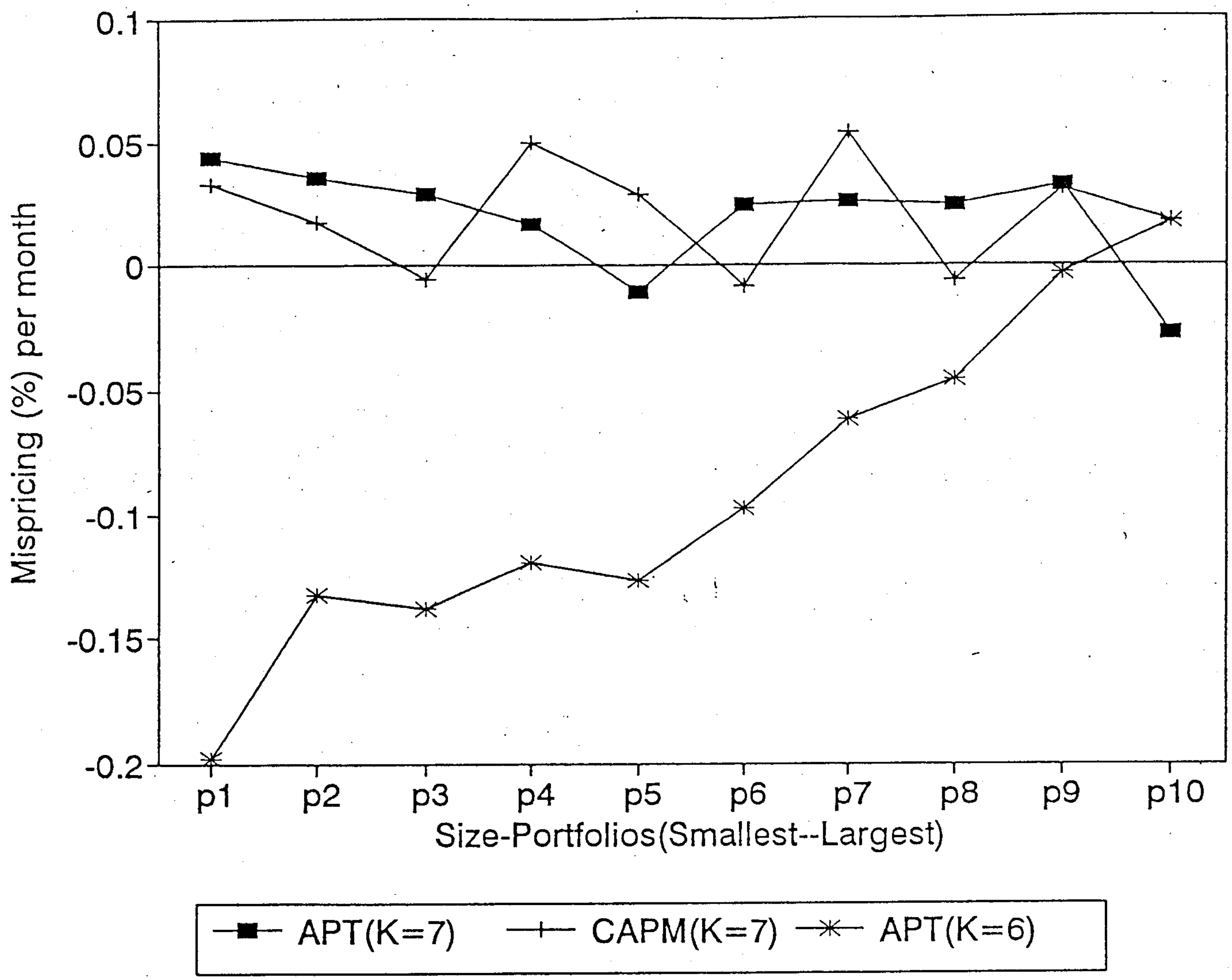

Figure 4. Mispricing under the APT (K = 6, 7) and CAPM (K = 7), in percentage per month, for decile portfolios formed by size. Portfolio 1 represents the portfolio of smallest firms while portfolio 10 represents the portfolio of largest firms. Mispricing is defined as the mean of excess return subtracting the risk premium as estimated by the APT or the CAPM, and subtracting the systematic risks.

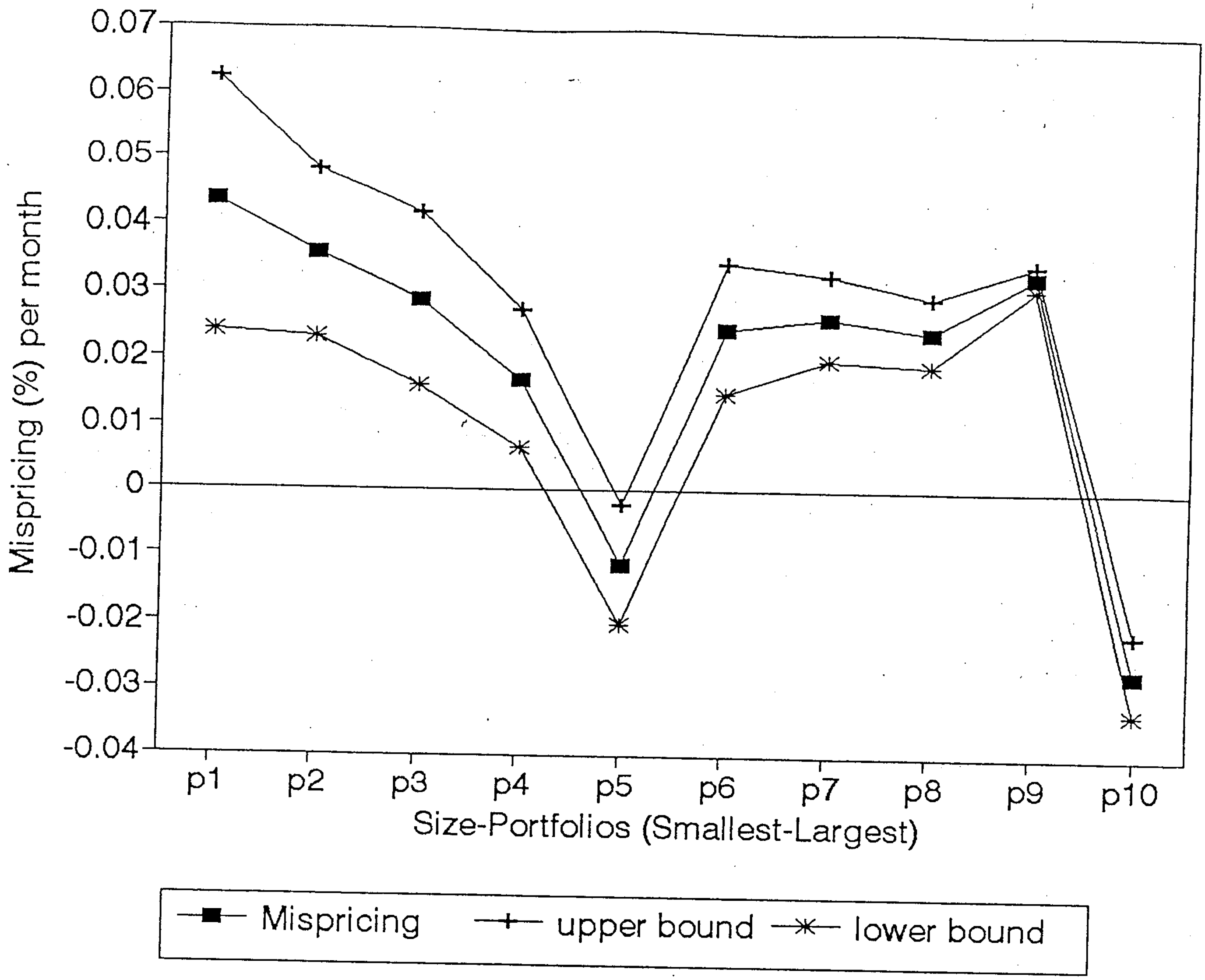

Figure 5. Mispricing under APT and the confidence bounds for adjusting error-in-variables. Mispricing are in percentage per month, for decile portfolios formed by size. Portfolio 1 represents the portfolio of smallest firms while portfolio 10 represents the portfolio of largest firms. The APT mispricing is estimated using a linear seven-factor model with four observable factors and three unobservable factors. The upper bound is calculated by adding 1.96 times standard deviation to the risk premium and the lower bound calculated by subtracting 1.96 times standard deviation from the risk premium. The standard deviations are derived from the variance-covariance matrix of the first-step VAR parameter estimates.