Valuation in Over-the-Counter Markets*

Darrell Duffie†
Nicolae Gârleanu‡
Lasse Heje Pedersen§

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Abstract

We provide the impact on asset prices of search-and-bargaining frictions in over-the-counter markets. Under natural conditions, prices are lower and illiquidity discounts higher when counterparties are harder to find, when sellers have less bargaining power, when the fraction of qualified owners is smaller, or when risk aversion, volatility, or hedging demand are larger. If agents face risk limits, then higher volatility leads to greater difficulty locating unconstrained buyers, resulting in lower prices. Information can fail to be revealed through trading when search is difficult. We discuss a variety of financial applications and testable implications.

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†Graduate School of Business, Stanford University, Stanford, CA 94305-5015, email: duffie@stanford.edu.

‡Wharton School, University of Pennsylvania, 3620 Locust Walk, Philadelphia, PA 19104-6367, email garleanu@wharton.upenn.edu.

§Stern School of Business, New York University, 44 West Fourth Street, Suite 9-190, New York, NY 10012-1126, email: lpederse@stern.nyu.edu.

Many assets, such as mortgage-backed securities, corporate bonds, emerging-market debt, bank loans, over-the-counter (OTC) derivatives, private equity, and real estate, are traded in OTC markets. Traders in OTC markets must search for counterparties, incurring opportunity or other costs. When two counterparties meet, their bilateral relationship is strategic; prices are set through a bargaining process that reflects each investor's alternatives to immediate trade.

We provide a theory and applications of dynamic asset pricing in OTC markets, one that explicitly treats search and bargaining. We show how the explicitly calculated equilibrium allocations and prices depend on investors' search abilities, bargaining powers, risk limits, and risk aversion, and discuss a variety of financial applications and testable implications.

Under natural conditions, illiquidity discounts are lower, and prices are higher, if investors can find each other more easily, if sellers have more bargaining power, if the fraction of qualified owners is greater, if volatility is lower, or if risk aversion is lower. If agents face risk limits, then higher volatility leads to greater difficulty locating unconstrained buyers, resulting in higher illiquidity discounts and lower prices. Finally, information can fail to be revealed through trade when search is difficult.

We first model the pricing of a consol bond traded by risk-neutral agents, a special case of the model by Duffie, Gârleanu, and Pedersen (2003). Investors contact one another randomly at some mean rate λ, a parameter reflecting search ability. When two agents meet, they bargain over the terms of trade based on endogenously determined outside options. Gains from trade arise from heterogeneous costs or benefits of holding assets.

We then extend this model to OTC markets with risky securities. This allows us to incorporate the effects of risk aversion, risk limits, and private information.

In our OTC market model, a risk-averse asset owner searches for a potential buyer when the asset ceases to be a relatively good hedge of his endowment. We show how asset prices are affected by search frictions and demonstrate how they could magnify the effective risk premium due to incomplete risk sharing, beyond that of a liquid but incomplete-markets setting such as Constantinides and Duffie (1996).

Our result complements the literature that studies the price effect of exogenously specified trading cost (Amihud and Mendelson (1986), Constantinides (1986), Vayanos (1998), and Huang (2003)) by endogenizing the trading cost in the context of OTC markets. Krainer and LeRoy (2002) study housing prices in a different search framework.

With the imposition of risk limits, we show how search-based liquidity frictions become endogenously dependent on volatility, in that higher volatility leads to smaller equilibrium holdings, resulting in more sellers and fewer available buyers. For sellers, this leads to longer search times and a relatively unfavorable bargaining position, which in turn implies higher illiquidity price discounts.

Introducing asymmetric information, we provide an example in which investors are sufficiently anxious about the threat of search delays that they offer "pooling prices," revealing no information. Hence, search frictions may not only slow the dissemination of information, but prevent it altogether.[1] The endogenous impact of asymmetric information on trading costs and asset prices has been addressed by Kyle (1985), Wang (1993), and Gârleanu and Pedersen (2000), among others.

Weill (2002) and Vayanos and Wang (2002) have extended our model in order to treat multiple assets in the same economy, obtaining cross-sectional restrictions on asset returns. In Duffie, Gârleanu, and Pedersen (2003), which focuses on the role of marketmakers, we show that search frictions have different implications for bid-ask spreads than do information frictions. Miao (2004) provides a variant of this model. Weill (2003) studies the implications of search frictions in an extension of our model in which marketmakers' inventories "lean against" the outside order flow. Newman and Rierson (2003) present a model in which supply shocks temporarily depress prices across correlated assets, as providers of liquidity search for long-term investors, supported by empirical evidence of issuance impacts across the European telecommunications bond market. In Duffie, Gârleanu, and Pedersen (2002), we use the modeling framework introduced here to characterize the impact on asset prices and securities lending fees of the common institution by which would-be shortsellers must locate lenders of securities before being able to sell short. Difficulties in locating lenders of shares can allow for dramatic price imperfections, as, for example, in the case of the spinoff of Palm, Incorporated, documented by Mitchell, Pulvino, and Stafford (2002) and Lamont and Thaler (2003). Further discussion of implications for over-the-counter asset pricing provided in Section 6.

[1] Wolinsky (1990) constructs a steady-state partially-revealing equilibrium in a search model with asymmetric information, while rational-expectations equilibria in frictionless markets are studied by Grossman (1981), Grossman and Stiglitz (1980), and others. See also Serrano and Yosha (1993) and Serrano and Yosha (1996).

Search models have been studied extensively in the context of labor economics, starting with the "coconuts" model of Diamond (1982), and in the context of monetary economics. Our search-and-bargaining structure, in particular is similar to that of the monetary model of Trejos and Wright (1995), but our objectives and results are different.

Section 1 lays out the basic model and results, using risk-neutral agents. Section 2 treats hedging motives for trade under risk aversion, and Section 3 provides a numerical example. Section 4 characterizes the implications of risk limits on prices and trades. Section 5 provides an illustration of how search frictions impede the dissemination of information through trade or prices. Further implications and financial applications are discussed in Section 6. Proofs and supplementary results are relegated to appendices.

1 Basic Search Model of Asset Prices

This section introduces an over-the-counter market, that is, a setting in which agents can trade only when they meet each other, and in which transaction prices are bargained.[2]

Agents are risk-neutral and infinitely lived, with a constant time-preference rate $\beta > 0$ for consumption of a single non-storable numeraire good.[3]

An agent can invest in a bank account — which can also be interpreted as a "liquid" security — with a risk-free interest rate of r. The liquid wealth W_t must be bounded below, and $r = \beta$ in the context of this section, since agents are risk neutral. Further, agents may trade a long-lived asset in an over-the-counter market. The asset can be traded only bilaterally, when in contact with a counterparty. We begin for simplicity by taking the illiquid asset to be a consol, which pays one unit of consumption per unit of time. Later, when introducing the effects of risk limits, or risk aversion, or asymmetric information regarding dividends, we generalize to random dividend processes.

An agent is characterized by an intrinsic preference for asset ownership that is "high" or "low." A low-type agent, when owning the asset, has a

[2]The model of this section is a special case of the one introduced by Duffie, Gârleanu, and Pedersen (2003), as are the solution technique and results until, but not including, Propostion 1.

[3]Specifically, an agent's preferences among adpated finite-variation cumulative consumption processes are represented by the utility $E\left(\int_0^\infty e^{-\beta t}\, dC_t\right)$ for a cumulative consumption process C, whenever the integral is well defined.

holding cost of δ per time unit. A high-type agent has no such holding cost. We could imagine this holding cost to be a shadow price for ownership by low-type agents, due, for example, to *(i)* low personal liquidity, that is, a need for cash, *(ii)* high financing costs, *(iii)* adverse correlation of asset returns with endowments (formalized in Section 2), *(iv)* a relative tax disadvantage, as studied by Dai and Rydqvist (2003) in an empirical analysis of search-and-bargaining effects in the context of tax trading,[4] or *(v)* a relatively low personal use for the asset, as may happen, for example, for certain durable consumption goods such as homes. The agent's intrinsic type is a Markov chain, switching from low to high with intensity λ_u, and back with intensity λ_d. The intrinsic-type processes of any two agents are independent.[5]

A fraction s of agents are initially endowed with one unit of the asset. Investors can hold at most one unit of the asset and cannot shortsell. Because agents have linear utility, it is without much loss of generality that we restrict attention to equilibria in which, at any given time and state of the world, an agent holds either 0 or 1 unit of the asset. Hence, the full set of agent types is $\mathcal{T} = \{ho, hn, lo, ln\}$, with the letters "$h$" and "$l$" designating the agent's current intrinsic liquidity state as high or low, respectively, and with "o" or "n" indicating whether the agent currently owns the asset or not, respectively.

We suppose that there is a "continuum" (a non-atomic finite measure space) of agents, and let $\mu_\sigma(t)$ denote the fraction at time t of agents of type $\sigma \in \mathcal{T}$. Normalizing the total number of agents to 1 implies that

$$1 \;=\; \mu_{ho}(t) + \mu_{hn}(t) + \mu_{lo}(t) + \mu_{ln}(t). \tag{1}$$

Equating the supply s with the total number of owners gives

$$s \;=\; \mu_{ho}(t) + \mu_{lo}(t). \tag{2}$$

An agent finds a counterparty with an intensity λ, where λ reflects the efficiency of the search technology. We assume randomly matched counterparties, so the probability that the counterparty is, say, an lo investor is μ_{lo}.

[4]Dai and Rydqvist (2003) study tax trading between a small group of foreign investors and a larger group of domestic investors. They find that investors from the "long side of the market" get part of the gains from trade, under certain conditions, which they interpret as evidence of a search-and-bargaining equilibrium.

[5]All random variables are defined on a probability space $(\Omega, \mathcal{F}, Pr)$ with corresponding filtration $\{\mathcal{F}_t : t \geq 0\}$ of sub-σ-algebras of $\mathcal{F}$ satisfying the usual conditions, as defined by Protter (1990). The filtration represents the resolution over time of information commonly available to investors. Asymmetric information is considered in Section 5.

Thus, the intensity of finding an *lo* investor is $\lambda\mu_{lo}$. Hence, assuming the law of large numbers applies,[6] *hn* investors contact *lo* investors at a total (almost sure) rate of $\lambda\mu_{lo}\mu_{hn}$ and, since *lo* investors contact *hn* investors at the same total rate, the total rate of such counterparty matchings is $2\lambda\mu_{lo}\mu_{hn}$.

To solve the model, we proceed in two steps. First, we use the insight that the only form of encounter that provides gains from trade is one in which low-type owners sell to high-type non-owners. From bargaining theory, we know (see Appendix A) that at these encounters, trade occurs immediately. We can therefore determine the steady-state asset allocations without reference to prices. Given the steady-state masses μ, we consider investors lifetime utility depending on their type, the bargaining problem, and the resulting price.

In equilibrium, the rates of change of the fractions of the respective investor types are

$$\begin{aligned} \dot{\mu}_{lo}(t) &= -2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t) \\ \dot{\mu}_{hn}(t) &= -2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_d\mu_{hn}(t) + \lambda_u\mu_{ln}(t) \\ \dot{\mu}_{ho}(t) &= 2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_d\mu_{ho}(t) + \lambda_u\mu_{lo}(t) \\ \dot{\mu}_{ln}(t) &= 2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_u\mu_{ln}(t) + \lambda_d\mu_{hn}(t). \end{aligned} \tag{3}$$

The intuition for, say, the first equation in (3) is straightforward: Whenever an *lo* agent meets an *hn* investor, he sells his asset and is no longer an *lo* agent. This (together with the law of large numbers) explains the first term on the right hand side of (3). The second term is due to type changes in which *lo* investors become *ho* investors, and the last term is due to type changes from *ho* to *lo* investors.

Duffie, Gârleanu, and Pedersen (2003) show that there is a unique stable steady-state solution, that is, a constant solution with $\dot{\mu}(t) = 0$. The steady state is computed by using (1)–(2) and the fact that $\mu_{lo} + \mu_{ln} = \lambda_d/(\lambda_u + \lambda_d)$ to write the first equation in (3) as a quadratic equation in μ_{lo}, given as Appendix equation (C.1).

Having determined the steady-state fractions of investor types, we compute the investors' equilibrium intensities of finding counterparties of each type and, hence, their utilities for remaining lifetime consumption, as well as the bargained price P. For a particular agent, his utility depends on his current type, $\sigma(t) \in \mathcal{T}$, and the wealth $W(t)$ in his bank account. Specifically,

[6] Duffie and Sun (2004) provide a measure-theoretic framework in which the law of large numbers indeed applies with search and random matching.

lifetime utility is $W(t) + V_{\sigma(t)}$, where, for each investor type σ in $\mathcal{T}$, V_σ is a constant to be determined.

In steady state, the expected growth of any agent's utility must be the discount rate r, which yields the steady-state equations

$$\begin{aligned} 0 &= rV_{lo} - \lambda_u(V_{ho} - V_{lo}) - 2\lambda\mu_{hn}(P - V_{lo} + V_{ln}) - (1-\delta) \\ 0 &= rV_{ln} - \lambda_u(V_{hn} - V_{ln}) \\ 0 &= rV_{ho} + \lambda_d(V_{ho} - V_{lo}) - 1 \\ 0 &= rV_{hn} + \lambda_d(V_{hn} - V_{ln}) - 2\lambda\mu_{lo}(V_{ho} - V_{hn} - P). \end{aligned} \tag{4}$$

The price is determined through bilateral bargaining. A high-type non-owner will pay at most his reservation value $\Delta V_h = V_{ho} - V_{hn}$, while a low-type owner requires a price of at least $\Delta V_l = V_{lo} - V_{ln}$. Nash bargaining, or the Rubinstein-type game considered in Appendix A, implies that the bargaining process results in the price

$$P = \Delta V_l (1-q) + \Delta V_h \, q\,, \tag{5}$$

where $q \in [0,1]$ is the bargaining power of the seller. The linear system of Equations (4)-(5) has a unique solution, with

$$P = \frac{1}{r} - \frac{\delta}{r} \frac{r(1-q) + \lambda_d + 2\lambda\mu_{lo}(1-q)}{r + \lambda_d + 2\lambda\mu_{lo}(1-q) + \lambda_u + 2\lambda\mu_{hn}q}\,. \tag{6}$$

This price (6) is the present value, $1/r$, of dividends, reduced by an illiquidity discount. The price is lower and the discount is larger, *ceteris paribus*, if the distressed owner has less hope of switching type (lower λ_u), if it is more difficult for the owner to find other buyers (lower μ_{hn}), if the buyer may more suddenly need liquidity himself (higher λ_d), if it is easier for the buyer to find other sellers (higher μ_{lo}), or if the seller has less bargaining power (lower q).

These intuitive results are based on partial derivatives of the right-hand side of (6) — in other words, they hold when a parameter changes without influencing any of the others. We note, however, that the steady-state type fractions μ themselves depend on λ_d, λ_u, and λ. The following proposition offers a characterization of the equilibrium steady-state effect of changing each parameter.

Proposition 1 *The steady-state equilibrium price P is decreasing in δ, s, and λ_d, and is increasing in λ_u and q. Further, if $s < \lambda_u/(\lambda_u + \lambda_d)$, then $P \to 1/r$ as $\lambda \to \infty$, and P is increasing in λ for all $\lambda \geq \bar{\lambda}$, for a constant $\bar{\lambda}$ depending on the other parameters of the model.*

The condition that $s < \lambda_u/(\lambda_u + \lambda_d)$ means that, in steady state, there is less than one unit of asset per agent of high intrinsic type.

It can be checked that the above results extend to risky dividends, for instance in the following ways: *(i)* If the cumulative dividend is risky with constant drift ν, then the equilibrium is that for a consol bond with dividend rate of ν; *(ii)* if the dividend rate and illiquidity cost are proportional to a process X with $E_t X(t+u) = X(t)e^{\nu u}$, for some constant growth rate ν, then the price and value functions are also proportional to X, with factors of proportionality given as above, with r replaced by $r - \nu$; *(iii)* if the dividend-rate process X satisfies $E_t X(t+u) = X(t) + mu$ for a constant drift m (and if illiquidity costs are constant), then the continuation values are of the form $X(t)/r + v_\sigma$ for owners and v_σ for non-owners, where the constants v_σ are computed in a similar manner.

Next, we model risky dividends using cases *(i)* and *(iii)* above, in the context of risk aversion and risk limits.

2 Risk-Aversion

This section provides a version of the asset-pricing model with risk aversion, in which the motive for trade between two agents is the different extent to which they derive hedging benefits from owning the asset. We provide a sense in which this economy can be interpreted in terms of the basic economy of Section 1.

Agents have constant-absolute-risk-averse (CARA) additive utility, with a coefficient γ of absolute risk aversion and with time preference at rate β. An asset has a cumulative dividend process D satisfying

$$dD(t) = m_D\,dt + \sigma_D\,dB(t), \tag{7}$$

where m_D and σ_D are constants, and B is a standard Brownian motion with respect to the given probability space and filtration $(\mathcal{F}_t)$. Agent i has a cumulative endowment process η^i, with

$$d\eta^i(t) = m_\eta\,dt + \sigma_\eta\,dB^i(t), \tag{8}$$

where the standard Brownian motion B^i is defined by

$$dB^i(t) = \rho^i(t)\,dB(t) + \sqrt{1-\rho^i(t)^2}\,dZ^i(t), \tag{9}$$

for a standard Brownian motion Z^i independent of B, and where $\rho^i(t)$ is the "instantaneous correlation" between the asset dividend and the endowment of agent i. We model ρ^i as a two-state Markov chain with states ρ_h and $\rho_l > \rho_h$. The intrinsic type of an agent is identified with this correlation parameter. An agent i whose intrinsic type is currently high (that is, with $\rho^i(t) = \rho_h$) values the asset more highly than does a low-intrinsic-type agent, because the increments of the high-type endowment have lower conditional correlation with the asset's dividends. As in the basic model of Section 1, agents' intrinsic types are pairwise-independent Markov chains, switching from l to h with intensity λ_u, and from h to l with intensity λ_d. An agent owns either θ_n or θ_o units of the asset, where $\theta_n < \theta_o$. For simplicity, no other positions are permitted, which entails a loss in generality. Agents can trade the OTC security only when they meet, as previously. The agent type space is $\mathcal{T} = \{lo, ln, ho, hn\}$. In this case, the symbols 'o' and 'n' indicate large and small owners, respectively. Given a total supply Θ of shares per investor, market clearing requires that

$$(\mu_{lo} + \mu_{ho})\theta_o + (\mu_{ln} + \mu_{hn})\theta_n = \Theta, \tag{10}$$

which, using (1), implies that the fraction of large owners is

$$\mu_{lo} + \mu_{ho} = s \equiv \frac{\Theta - \theta_n}{\theta_o - \theta_n}. \tag{11}$$

We consider a particular agent whose type process is σ, and let θ denote the associated asset-position process (that is, $\theta(t) = \theta_o$ whenever $\sigma(t) \in \{ho, lo\}$ and otherwise $\theta(t) = \theta_n$). We suppose that there is a perfectly liquid "money-market" asset with constant risk-free rate of return r, which, for simplicity, is assumed to be determined outside of the model (as is typical in the literature treating asset-pricing models based on CARA utility). The agent's money-market wealth process W satisfies

$$dW(t) = (rW(t) - c(t))\,dt + \theta(t)\,dD(t) + d\eta(t) - P\,d\theta(t),$$

where c is the agent's consumption process, η is the agent's cumulative endowment process, P is the asset price per share (which is constant in the equilibria that we examine). The last term thus captures payments in connection with trade. The consumption process is required to satisfy measurability, integrability, and transversality conditions stated in Appendix C.

We consider a steady-state equilibrium, and let $J(w,\sigma)$ denote the indirect utility of an agent of type $\sigma \in \{lo, ln, ho, hn\}$ with current wealth w. Assuming sufficient differentiability, the Hamilton-Jacobi-Bellman (HJB) equation for an agent of current type lo is

$$\begin{aligned} 0 &= \sup_{\overline{c}\in\mathbb{R}} \{-e^{-\gamma\overline{c}} + J_w(w,lo)(rw - \overline{c} + \theta_o m_D + m_\eta) \\ &\quad + \frac{1}{2}J_{ww}(w,lo)(\theta_o^2\sigma_D^2 + \sigma_\eta^2 + 2\rho_l\theta_o\sigma_D\sigma_\eta) - \beta J(w,lo) \\ &\quad + \lambda_u[J(w,ho) - J(w,lo)] + 2\lambda\mu_{hn}[J(w+P\overline{\theta},ln) - J(w,lo)]\}, \end{aligned}$$

where $\overline{\theta} = \theta_o - \theta_n$. The HJB equations for the other agent types are similar. Under technical regularity conditions found in Appendix C, we verify that

$$J(w,\sigma) = -e^{-r\gamma(w+a_\sigma+\bar{a})}, \tag{12}$$

where

$$\bar{a} = \frac{1}{r}\left(\frac{\log r}{\gamma} + m_\eta - \frac{1}{2}r\gamma\sigma_\eta^2 - \frac{r-\beta}{r\gamma}\right), \tag{13}$$

and where, for each σ, the constant a_σ is determined as follows. The first-order conditions of the HJB equation of an agent of type σ imply an optimal consumption rate of

$$\overline{c} = -\frac{\log(r)}{\gamma} + r(w + a_\sigma + \bar{a}). \tag{14}$$

Inserting this solution for $\overline{c}$ into the respective HJB equations yields a system characterizing the coefficients a_σ.

The price P is determined using Nash bargaining with seller bargaining power q, similar in spirit to the basic model of Section 1. Given the reservation values of buyer and seller implied by $J(w,\sigma)$, the bargaining price satisfies $a_{lo} - a_{ln} \leq P\overline{\theta} \leq a_{ho} - a_{hn}$. The following result obtains.

Proposition 2 *In equilibrium, an agent's consumption is given by (14), the* *value function is given by (12), and* $(a_{lo}, a_{ln}, a_{ho}, a_{hn}, P) \in \mathbb{R}^5$ *solve*

$$\begin{aligned}
0 &= ra_{lo} + \lambda_u \frac{e^{-r\gamma(a_{ho}-a_{lo})} - 1}{r\gamma} + 2\lambda\mu_{hn} \frac{e^{-r\gamma(P\overline{\theta}+a_{ln}-a_{lo})} - 1}{r\gamma} - (\kappa(\theta_o) - \theta_o\bar{\delta}) \\
0 &= ra_{ln} + \lambda_u \frac{e^{-r\gamma(a_{hn}-a_{ln})} - 1}{r\gamma} - (\kappa(\theta_n) - \theta_n\bar{\delta}) \qquad (15) \\
0 &= ra_{ho} + \lambda_d \frac{e^{-r\gamma(a_{lo}-a_{ho})} - 1}{r\gamma} - \kappa(\theta_o) \\
0 &= ra_{hn} + \lambda_d \frac{e^{-r\gamma(a_{ln}-a_{hn})} - 1}{r\gamma} + 2\lambda\mu_{lo} \frac{e^{-r\gamma(-P\overline{\theta}+a_{ho}-a_{hn})} - 1}{r\gamma} - \kappa(\theta_n),
\end{aligned}$$

with

$$\kappa(\theta) = \theta m_D - \frac{1}{2} r\gamma \left(\theta^2 \sigma_D^2 + 2\rho_h \theta \sigma_D \sigma_\eta \right) \qquad (16)$$

$$\bar{\delta} = r\gamma(\rho_l - \rho_h)\sigma_D\sigma_\eta \; > \; 0, \qquad (17)$$

as well as the Nash bargaining equation,

$$q\left(1 - e^{r\gamma(P\overline{\theta}-(a_{lo}-a_{ln}))}\right) = (1-q)\left(1 - e^{r\gamma(-P\overline{\theta}+a_{ho}-a_{hn})}\right). \qquad (18)$$

A natural benchmark is the price with vanishing search frictions:

Proposition 3 *If* $s < \mu_{hn} + \mu_{ho}$*, then, as* $\lambda \to \infty$*,*

$$P \to \frac{\kappa(\theta_o) - \kappa(\theta_n)}{r\overline{\theta}}. \qquad (19)$$

In order to compare the equilibrium for this model to that of the basic model, we use the linearization $e^z - 1 \approx z$, which leads to

$$\begin{aligned}
0 &\approx ra_{lo} - \lambda_u(a_{ho} - a_{lo}) - 2\lambda\mu_{hn}(P\overline{\theta} - a_{lo} + a_{ln}) - (\kappa(\theta_o) - \theta_o\bar{\delta}) \\
0 &\approx ra_{ln} - \lambda_u(a_{hn} - a_{ln}) - (\kappa(\theta_n) - \theta_n\bar{\delta}) \\
0 &\approx ra_{ho} - \lambda_d(a_{lo} - a_{ho}) - \kappa(\theta_o) \\
0 &\approx ra_{hn} - \lambda_d(a_{ln} - a_{hn}) - 2\lambda\mu_{lo}(a_{ho} - a_{hn} - P\overline{\theta}) - \kappa(\theta_n) \\
P\overline{\theta} &\approx (1-q)(a_{lo} - a_{ln}) + q(a_{ho} - a_{hn}).
\end{aligned}$$

These equations are of the same form as those in Section 1 for the indirect utilities and asset price in an economy with risk-neutral agents, with dividends at rate $\kappa(\theta_o)$ for large owners and dividends at rate $\kappa(\theta_n)$ for small

λ	λ_u	λ_d	s	r	β	q	δ
60	1.0	0.1	0.80	0.05	0.05	0.5	0.875

Table 1: Base-case parameters for basic model.

owners, and with illiquidity costs given by $\bar{\delta}$. In this sense, we can view the basic model as a risk-neutral approximation of the effect of search illiquidity in a model with risk aversion. The approximation error goes to zero for small risk aversion γ or small agent heterogeneity (that is, small $\rho_l - \rho_h$). Solving specifically for the price P in the associated linear model, we have

$$P = \frac{\kappa(\theta_o) - \kappa(\theta_n)}{r\bar{\theta}} - \frac{\bar{\delta}}{r} \frac{r(1-q) + \lambda_d + 2\lambda\mu_{lo}(1-q)}{r + \lambda_d + 2\lambda\mu_{lo}(1-q) + \lambda_u + 2\lambda\mu_{hn}q}. \quad (20)$$

We see that the price is the perfect-market price from Proposition 3, less an illiquidity discount. The expression (17) for $\bar{\delta}$ shows that the illiquidity cost in the basic model can be interpreted as a hedging-based incentive to trade. This incentive is increasing in the risk aversion γ, the endowment-correlation difference $\rho_l - \rho_h$, and the volatilities of dividends and endowments.

3 Illustrative Example

We consider an example that serves to illustrate both the basic model and the model with risk aversion, and how the latter can be well approximated by the former.

Table 1 contains the exogenous parameters for the base-case risk-neutral model. With the tabulated switching intensities for intrinsic types, agents are in a high intrinsic state for an average of 10 years out of every 11, that is, $\lambda_u/(\lambda_u+\lambda_d)$. The search and switching intensities shown imply the stationary fractions of each type that are listed in Table 2. We see that only a small fraction of the asset, 0.0054/0.8 or about 0.67% of the total supply, is misallocated through search frictions to low intrinsic types. The equilibrium asset price, 19.05, however, is substantially below the perfect market price of $r^{-1} = 20$, reflecting a significant impact of illiquidity on the price, despite the relatively small impact on the asset allocation.

Our base-case version of the risk-aversion model is specified by the basic-model parameters of Table 1 as well as the parameters of Table 3. The

μ_{ho}	μ_{hn}	μ_{lo}	μ_{ln}	P
0.7946	0.1145	0.0054	0.0855	19.05

Table 2: Steady-state masses and asset price, basic model.

γ	ρ_h	ρ_l	μ_η	σ_η	μ_D	σ_D	Θ	θ_o	θ_n
0.0035	-0.5	0.5	10000	10000	1	0.5	16000	20000	0

Table 3: Additional base-case parameters with risk-aversion.

parameters of these tables are consistent in the following sense. First, the "illiquidity cost" $\delta = \overline{\delta} = 0.875$ of low-intrinsic-type is that implied by (17) from the hedging costs of the risk-aversion model. Second, the total amount Θ of shares and the investor positions, θ_o and θ_n, imply the same fraction $s = 0.8$ of the population holding large positions, using (11). The investor positions that we adopt for this illustrative example are realistic in light of the positions adopted by high and low type investors in the associated Walrasian (perfect) market with unconstrained trade sizes, which, as shown in Appendix B, has an equilibrium large-owner position size of 17,818 shares and a small-owner position size of $-2,182$ shares. Third, the certainty-equivalent dividend-rate per share, $(\kappa(\theta_o) - \kappa(\theta_n))/(\theta_o - \theta_n) = 1$, is the same as that of the base-case model.

Figure 1 shows how prices are reduced by illiquidity to a degree that depends on the search intensity λ. (Note that λ does not affect $\overline{\delta}$ or $\kappa(\,\cdot\,)$, so the risk-neutral model is the same for all values of λ.) The figure reflects the fact that as the search intensity λ becomes large, the allocation and price become equal to the perfect-market ones (Propositions 1 and 3).

Figures 2 and 3 show how prices are discounted for illquidity, relative to the perfect-markets price, by an amount that depends on risk aversion and volatility. As we vary the parameters in these figures, we compute both the equilibrium solution of the risk-aversion model and the solution of the associated basic risk-neutral model that is obtained by the linearization (20), taking $\overline{\delta}$ from (17) case by case.

We see that the illiquidity discount increases with risk aversion and volatility and that both effects are large for our benchmark parameters. The illiquidity discount ranges between 1% and 16% depending on the risk and

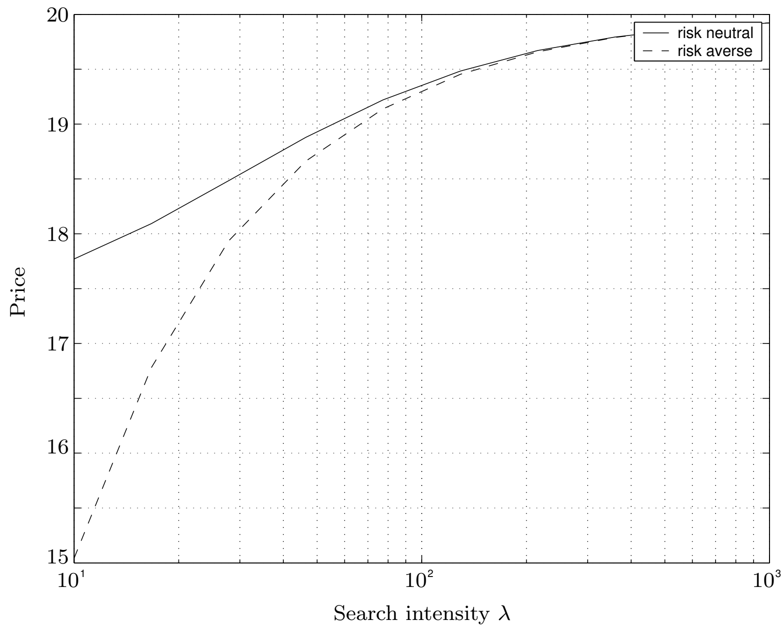

Figure 1: Price response to search intensity.

risk aversion.

Also, these figures show that the equilibrium price of the OTC market model with risk aversion is generally well approximated by our closed-form expression (20).

4 Risk Limits and Endogenous Position Size

In this section, we consider the impact of risk limits and illiquidity on prices and on the equilibrium allocation of risky assets. Specifically, we consider explicit limits on the volatilities of agents' positions, an idealization of risk limits imposed in practice,[7] such as bounds on volatility or value at risk (VaR).

Consider the following variant of the basic model of Section 1. Agents have the same preferences, including intrinsic-type processes, and the same search technology of the basic model. Rather than an asset paying a constant

[7]In practice, risk limits reflect agency costs, financial distress costs, and other costs that we do not model here.

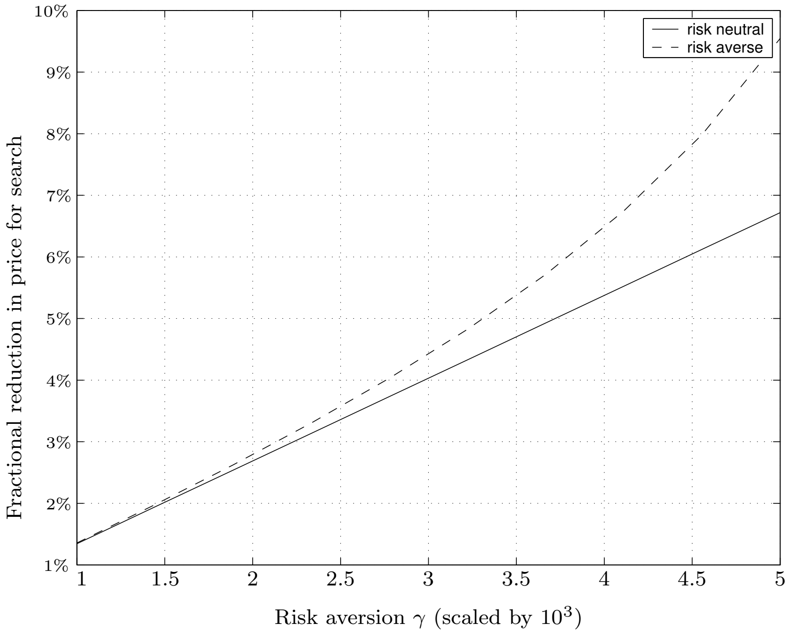

Figure 2: The response of search discount to risk aversion.

dividend rate, however, we suppose that the illiquid asset has a dividend-rate process X that is Lévy, meaning that it has independent and identically distributed increments over non-overlapping time periods of equal lengths. Examples include Brownian motions, simple and compound Poisson processes, and sums of these. Assuming that $X(t)$ has a finite second moment, it follows, for any times t and $t+u$, that

$$E_t\left[X(t+u) - X(t)\right] = mu, \tag{21}$$

for a constant drift m, and that, letting $\text{var}_t(\,\cdot\,)$ denote $\mathcal{F}_t$-conditional variance,

$$\text{var}_t\left(X(t+u) - X(t)\right) = \sigma_X^2 u, \tag{22}$$

for a constant volatility parameter $\sigma_X > 0$.

We will consider economies in which counterparties choose to trade at a price $P(X(t))$ at time t, for some Lipschitz function $P(\,\cdot\,)$ that we shall calculate in equilibrium. The total gain in market value associated with

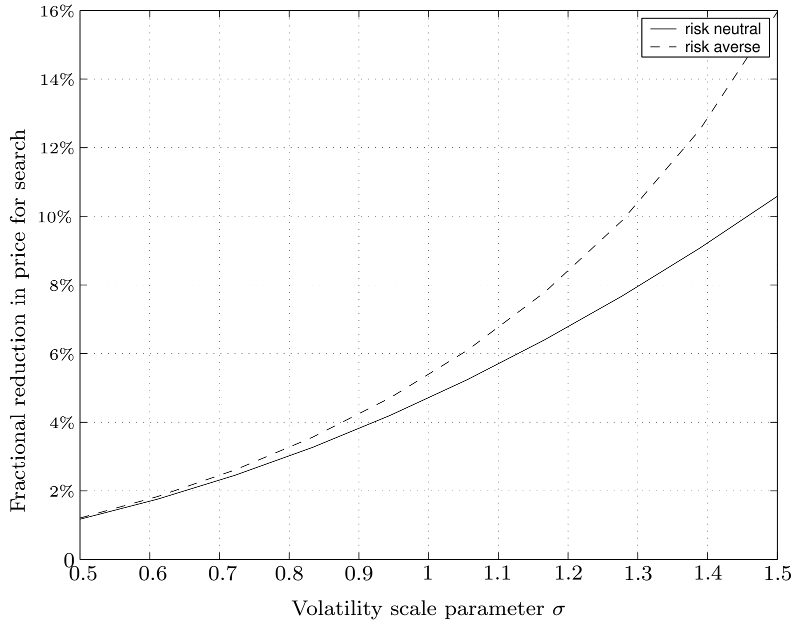

Figure 3: The response of search discount volatility, through scaling σ_η and σ_D by σ.

holding one unit of the asset between times t and $t+u$ is[8]

$$G_{t,u} = P(X(t+u)) - P(X(t)) + \int_t^{t+u} X(s)\, ds. \tag{23}$$

Agents are restricted to asset positions with a volatility limit $\overline{\sigma}$, in the sense that an agent is permitted to hold a position at any time t of size θ, long or short, only if it the associated mark-to-market volatility is no greater than a policy limit $\overline{\sigma}$, in that[9]

$$\overline{\lim}_{u \downarrow 0^+} \frac{1}{u} \mathrm{var}_t\left(\theta G_{t,u}\right) \leq \bar{\sigma}^2, \tag{24}$$

[8]The dividend process X is integrable with respect to t over compact time intevals since, without loss of generality, a Lévy process may be taken to be a right-continuous left-limits process.

[9]Because $\int_t^{t+u} X(s)\, ds$ is absolutely continuous with respect to u, this instantaneous volatility measure is determined by the limiting variance of $[P(X(t+u)) - P(X(t))]/u$, and the dividend part of the gain plays no role in this restriction.

replacing the position limits of 0 and 1 used in the basic model.

With only these adjustments of the basic model, namely the introduction of risky dividends and risk limits on positions, we anticipate an equilibrium asset price per share of the form

$$P(X(t)) = \frac{X(t)}{r} + p, \tag{25}$$

for a constant p to be determined. The portion $X(t)/r$ of the price that depends on X is the same as that in an economy with no liquidity effects, because illiquidity losses do not depend on $X(t)$.

The conjectured price process of (25) has a constant volatility, so we conjecture an equilibrium in which agents are either long or short by a fixed position size θ to be determined. These holdings are determined so that a high-type agent holds as large a long (positive) position as the risk limits allow, while a low-type agent holds as large a short position as allowed. (The model remains tractable if one also imposes a short-selling restriction or cost.) The total supply of shares per investor is some constant Θ.

The masses of the four types of agents evolve according to (3). Equation (1) continues to hold, and market clearing implies that

$$(\mu_{lo} + \mu_{ho} - \mu_{ln} - \mu_{hn})\theta = \Theta, \tag{26}$$

that is,

$$\mu_{lo} + \mu_{ho} = s \equiv \frac{\Theta}{2\theta} + \frac{1}{2}, \tag{27}$$

where we have used (1). Hence, one can solve for the equilibrium masses by exploiting the solution obtained for the basic model of Section 1, but, in this case, the fraction s of long position holders is endogenous.

The steady-state equilibrium price is of the conjectured form (25), and the indirect utility of an investor of type σ is of the form

$$V(X(t), \sigma) = \theta_\sigma \frac{X(t)}{r} + \theta v_\sigma, \tag{28}$$

where θ_σ is either θ or $-\theta$, depending on the type, and where the type-dependent utility coefficient v_σ is to be determined. The coefficients for the price and value functions are solved similarly to (4) and (5), in that

$$\begin{aligned}
0 &= rv_{lo} - \lambda_u(v_{ho} - v_{lo}) - 2\lambda\mu_{hn}(2p - v_{lo} + v_{ln}) - \left(\frac{m}{r} - \delta\right) \\
0 &= rv_{ln} - \lambda_u(v_{hn} - v_{ln}) + \left(\frac{m}{r} - \delta\right) \\
0 &= rv_{ho} - \lambda_d(v_{lo} - v_{ho}) - \frac{m}{r} \\
0 &= rv_{hn} - \lambda_d(v_{ln} - v_{hn}) - 2\lambda\mu_{ho}(v_{ho} - v_{hn} - 2p) + \frac{m}{r} \\
2p &= (v_{lo} - v_{ln})(1-q) + (v_{ho} - v_{hn})q.
\end{aligned} \tag{29}$$

In particular,

$$P(x) = \frac{1}{r}x + \frac{m}{r^2} - \frac{\delta}{r}\frac{r(1-q) + \lambda_d + 2\lambda\mu_{lo}(1-q)}{r + \lambda_d + 2\lambda\mu_{lo}(1-q) + \lambda_u + 2\lambda\mu_{hn}q}.$$

Thus, the volatility of the price is σ_X/r, so the largest admissible security position size is

$$\theta = \frac{r\bar{\sigma}}{\sigma_X}. \tag{30}$$

A notable impact of search is that the equilibrium position size θ decreases with the volatility of the asset, which in turn implies the following impact of search with risk limits on the asset price.

Proposition 4 *For a given bargaining power q, fix the unique equilibria associated with two economies that differ only with respect to the dividend volatility coefficient, σ_X. The larger dividend volatility is associated with longer expected search times for selling, and a lower asset price.*

This inverse dependence of the price on the volatility of the asset is a liquidity effect, brought about by a reduction in the risk-taking capacity of an investor relative to the total risk to be held. A larger volatility thus implies a smaller quantity of agents whose risk capacity qualifies them to buy the asset (that is, fewer liquid investors who do not already own the asset).

5 Asymmetric Information

It is natural that information about future dividends held privately by agents may be transmitted through trading. One might expect that the speed with which private information is spread increases with search intensities. We show, however, that this need *not* be the case. If meeting intensities are low, agents are eager to trade when they meet since they know that failure to trade is costly. This can lead to pooling equilibria in which *no* information is revealed through trading. We show that such pooling equilibria exist only for sufficiently small search intensities, though. We do not study equilibria in which information is disseminated through bargaining interaction, as did Wolinsky (1990), although this would also be interesting.

We model asymmetric information as follows. A Lévy dividend-rate process X has a constant jump-arrival intensity λ_J. At each successive jump time τ, the dividend jump size $X(\tau) - X(\tau-)$ is, with some probability $1-\zeta$, of mean J_0, and with probability ζ of mean $J_1 > J_0$. The unconditional mean jump size is therefore $J_m = \zeta J_1 + (1-\zeta)J_0$.

At each jump time, in the event that the next jump is to be drawn with the high conditional mean, a proportion $\nu \in [0,1]$ of the agents, independently selected, are immediately informed of this fact. The remaining agents are not. The allocation of this information is independent of agents' intrinsic liquidity types. In the event that the jump is to be drawn with the low conditional mean, nobody receives information regarding this fact. Thus, each agent is informed with probability $\gamma\nu$, and, conditional on not having received private information after the last jump, has a conditional mean next-jump size of

$$J^u = \frac{\zeta(1-\nu)J_1 + (1-\zeta)J_0}{1-\zeta\nu}.$$

Other than risky dividends and private information of this character, the assumptions of the basic model of Section 1 apply.

In order to keep our analysis relatively simple, we assume that, once two agents meet, one of them is drawn randomly to make a take-it-or-leave-it offer. We use the notation q_σ for the probability that an agent of type σ is the quoting agent. We are looking for conditions under which there is a pooling equilibrium in which sellers quote a price at which both informed and uninformed buyers are willing to buy, rather than quoting a more aggressive price at which uninformed buyers would decline to trade. Likewise, buyers quote pooling prices. Before we determine these pooling prices, we point out that our pooling equilibrium also requires that agents with no gains from trade must not reveal information by trading with each other. This is, however, consistent with optimal behavior. For instance, an uninformed owner of low intrinsic type does not sell to an informed agent with low discount rate, since there are no gains from trade between the two. If such a trade were to take place, then the uninformed would become informed, but the expected utility of these agents would remain unchanged.[10] Such trades are ruled out, for instance, if there is an arbitrarily small non-zero cost of making an offer.

We now turn to the determination of the value functions and pooling prices. The indirect utility of an informed agent of type σ is, in equilibrium, of the form

$$\theta_\sigma \frac{X(t)}{r} + v_{\sigma i},$$

where θ_σ is 0 or 1 depending on whether the type is an owner, and where $v_{\sigma i}$ is, for each σ, a coefficient to be calculated, and where the subscript i denotes "informed." Similarly, the equilibrium indirect utility of uniformed agents of type σ is

$$\theta_\sigma \frac{X(t)}{r} + v_{\sigma u}.$$

We define the reservation-value coefficients for each of the four cases as follows: $\Delta v_{lu} = v_{lou} - v_{lnu}$, $\Delta v_{hu} = v_{hou} - v_{hnu}$, $\Delta v_{li} = v_{hoi} - v_{hni}$, and $\Delta v_{hi} = v_{hoi} - v_{hni}$. We look for equilibria in which, naturally, informed agents have higher reservation values than those of uninformed agents, and all efficient trades can potentially happen, that is,

$$\Delta v_{hi} \geq \Delta v_{hu} \geq \Delta v_{li} \geq \Delta v_{lu}. \tag{31}$$

The only comparison that is not immediate, $\Delta v_{hu} \geq \Delta v_{li}$, is ensured by choosing the "informational advantage," $\lambda_J(J_1 - J^u)$, small enough relative to the liquidity disadvantage, determined by δ. Proposition 6 in Appendix C makes this statement precise. Appendix C also provides a complete analysis.

Here, we give only a flavor of the analysis required. In particular, pooling requires that certain incentive-compatibility constraints be met. For instance, an informed low-type owner must prefer to quote a price accepted by all high-type non-owners, rather than quoting a more aggressive price, which would be accepted only by informed non-owners. That is,

$$\Delta v_{hu} + v_{lni} \quad \geq \quad Pr(i \,|\, i)\, (\Delta v_{hi} + v_{lni}) + (1 - Pr(i \,|\, i)) v_{loi}\,, \tag{32}$$

where $Pr(i \,|\, i)$ is the probability of the buyer being informed given that the seller is informed. There are three other such constraints, but two of the four

[10]We note, however, that in a partially revealing equilibrium, in which being informed would be valuable for future behavior, there could exist strictly positive gains from such a trade.

conditions, in total, suffice, since these two imply the other two. The analysis in Appendix C shows that these incentive-compatibility constraints guarantee the existence of a pooling equilibrium. Below, we provide an example in which a pooling equilibrium exists for an open set of parameters. A pooling equilibrium exists, however, only if the meeting intensity λ is sufficiently low.[11] If λ is high, then uninformed high-valuation non-owners, for instance, find it profitable not to offer a price that reflects good information, but, rather, one that is only accepted by uninformed sellers. The intuition behind the result is simple: Failure to trade at any given opportunity is less costly when meeting other agents is easy. We summarize with the following result.

Proposition 5 *For any set of parameters for which $s \neq \lambda_u/(\lambda_u + \lambda_d)$, there exists a search intensity $\bar{\lambda}$ such that, for all $\lambda > \bar{\lambda}$, a pooling equilibrium cannot exist.*

When search is less intense, however, pooling equilibria may exist for an open set of parameters. Figure 4 provides an illustrative numerical example.

We use the parameters of Table 1 and take $J_0 = 1$, $J_1 = 1.1$, $\lambda_J = 0.2$, and $\zeta = 0.5$. We compute, for a range of contact intensities (λ), the minimal and maximal proportions of informed agents, ν, consistent with a pooling equilibrium. We see that, as λ increases, ν is confined to a smaller and smaller interval, depicted as the shaded region of Figure 4, until the two sufficient incentive-compatibility conditions can no longer be satisfied simultaneously. One can see that the seller's incentive constraint for pooling is more sensitive to λ than is the buyer's, because the buy side of the market is larger than the sell side. Hence, as λ increases, a seller's meeting intensity converges to infinity, which makes it tempting for the seller to quote aggressive prices. The buyer's meeting intensity, on the other hand, is bounded as λ increases.

6 Market Implications

Our framework has several asset pricing implications for over-the-counter (OTC) markets, that is, markets characterized by bilateral negotiation, de-

[11] There is one knife-edge parameter configuration, namely $s = \lambda_u/(\lambda_u+\lambda_d)$, under which a high λ need not destroy the pooling equilibrium. That should come as no surprise, since in this knife-edge case even a competitive market equilibrium is supported by a range of prices bounded by the proposed pooling prices.

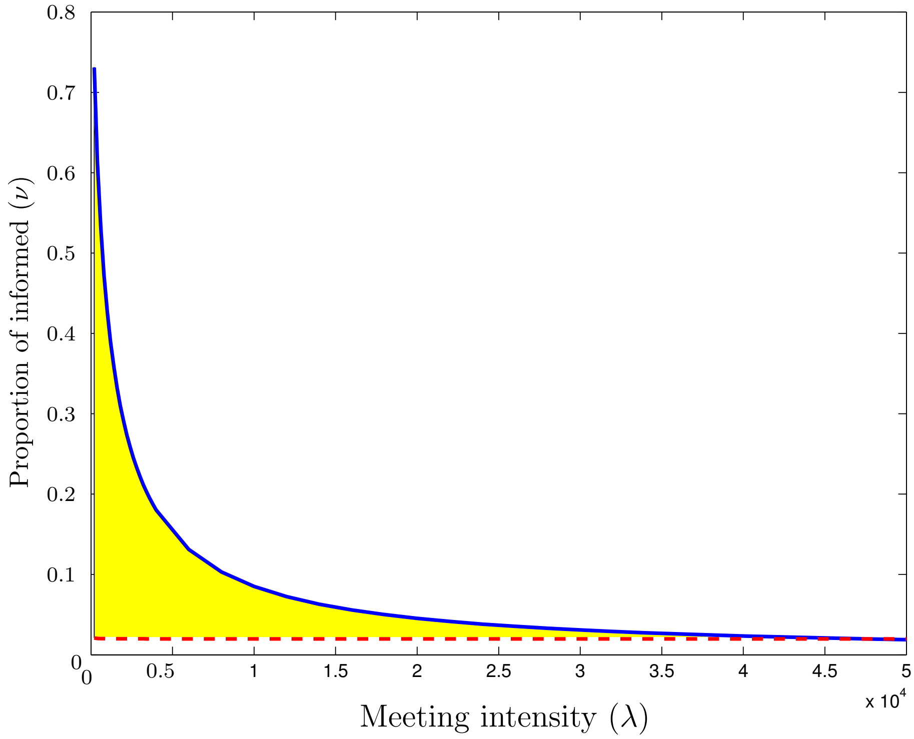

Figure 4: The shaded area is the set of (λ, ν), fixing other parameters, for which a pooling equilibrium exists. The lower (broken) line shows the lowest fraction ν of informed investors consistent with the pooling (incentive compatibility) condition for quotation by uninformed buyers. The upper (solid) line shows the highest value of ν consistent with the pooling condition for quotation by informed sellers. The other parameters adopted for this illustrative example are $\lambda_u = 1$, $\lambda_d = 0.1$, $s = 0.8$, $r = 0.05$, $\delta = 1$, $\lambda_J = 0.2$, $J_0 = 1$, $J_1 = 1.1$, and $\zeta = 0.5$.

layed by search for suitable counterparties. These price effects may be relevant for private equity, real estate, and OTC-traded financial products such as interest-rate swaps and other OTC derivatives, mortgage-backed securities, corporate bonds, government bonds, emerging-market debt, and bank loans. Exemplifying the imperfect ability to match buyers and sellers in OTC markets, traders in the market for European corporate loans have ironically described[12] trade in that market as "by appointment."

Even in the most liquid OTC markets, the relatively small price effects arising from search frictions receive significant attention by economists. For

[12]See *The Financial Times*, November 19, 2003.

example, the market for U.S. Treasury securities, an over-the-counter market considered to be a benchmark for high liquidity, is subject to widely noted illiquidity effects that differentiate the yields of on-the-run (latest-issue) securities from those of off-the-run securities. Positions in on-the-run securities are normally available in large amounts from relatively easily found traders such as hedge funds and government-bond dealers. Because on-the-run issues can be more quickly located by short-term investors such as hedgers and speculators, they command a price premium, even over a package of off-the-run securities of identical cash flows. Ironically, the importance ascribed to this relatively small premium is explained by the exceptionally high volume of trade in this market, and also by the importance of disentangling the illiquidity impact on measured Treasury interest rates for informational purposes elsewhere in the economy. Longstaff (2002) measures relatively larger illiquidity effects on government security prices during "flights to liquidity," which he characterizes as periods during which a large demand for quick access to a safe haven causes Treasury prices to temporarily achieve markedly higher prices than equally safe government securities that are not as easily found.

Part of the price impact represented by the spread between on-the-run and off-the-run treasuries is conveyed by shortsellers who are willing to pay a lending premium to owners of relatively easily located securities. A search-based theory of securities lending is developed in Duffie, Gârleanu, and Pedersen (2002). Empirical evidence of the impact on treasury prices and securities-lending premia ("repo specials") can be found in Duffie (1996), Jordan and Jordan (1997), and Krishnamurthy (2002). Fleming and Garbade (2003) document a new U.S. Governnment program to improve liquidity in treasury markets by allowing alternative types of treasury securities to be deliverable in settlement of a given repurchase agreement, mitigating the costs of search for a particular issue. Related effects in equity markets are measured by Geczy, Musto, and Reed (2002), D'Avolio (2002), and Jones and Lamont (2002). Difficulties in locating lenders of shares sometimes cause dramatic price imperfections, as was the case with the spinoff of Palm, Incorporated, one of a number of such cases documented by Mitchell, Pulvino, and Stafford (2002).

The potential for much larger price impacts in relatively less liquid OTC markets is exemplified in a study of Chinese equity prices by Chen and Xiong (2001). Certain Chinese companies have two classes of shares, one exchange traded, the other consisting of "restricted institutional shares" (RIS), which can be traded only privately. The two classes of shares are identical in every other respect, including their cash flows. Chen and Xiong (2001) find that RIS shares trade at an average discount of about 80% to the corresponding exchange-traded shares. Similarly, in a study involving U.S. equities, Silber (1991) compares the prices of "restricted stock" — which, for two years, can be traded only in private among a restriced class of sophisticated investors — with the prices of unrestricted shares of the same companies. Silber (1991) finds that restricted stocks trade at an average discount of 30%, and that the discount for restricted stock is increasing in the relative size of the issue. These price discounts can be captured in our search framework, but would be difficult to explain using standard models based on asymmetric information, given that the two classes of shares are claims to the same dividend streams, and given that the publicly-traded share prices are easily observable.

Our model can be used to predict the implications of a widespread shock to the abilities or incentives of traders to take asset positions. Such a "wealth shock" can be captured in our model by a simultaneous move by many (a non-zero mass of) investors to the low intrinsic state, leading to a sudden increase in the number of sellers (μ_{lo} rises) and reduction in the number of buyers (μ_{hn} falls). As a result, the price drops. A similar effect would occur with an upward shock to the transition intensity λ_d with which investors migrate to the low intrinsic state. The price drop is caused in part by a higher fraction of assets held by distressed traders, but, importantly, also by the worsened bargaining position of sellers. The effect is temporary if the transition intensities λ_u and λ_d are unaffected by the shock, and can otherwise be long-lived. As we have shown in Sections 2–4, if agents are risk averse or have risk limits, an increase in the risk of the asset has similar implications. Higher risk (in the form of higher dividend volatility or higher correlation between the dividends and the endowment processes) leads to larger utility losses for distressed agents. Agents can compensate for the increased risk by reducing their position limits, but then a larger fraction of the agents must hold the risky asset, and liquidity is further reduced because finding a buyer becomes more difficult. Hence, shocks to volatility can lead to liquidity problems and price drops, especially if risk-management practices imply a simultaneous tightening of position limits.

Search frictions also help explain how the relative size of an asset in the economy may affect its price (or price-dividend ratio). Proposition 1 shows that if a higher fraction (s) of the agents must hold the asset, then the price must fall. This resembles the usual effect of a downward-sloping demand curve. When comparing stocks in the cross section, however, there is the additional effect that more investors typically participate in the market for larger stocks, which also usually have a greater presence by marketmakers. (Duffie, Gârleanu, and Pedersen (2003) study marketmakers and endogenize their search intensity.) If, for instance, the number of investors participating in the market for a firm's shares is proportional to the size of the company, then this corresponds in our model to a higher[13] search intensity λ, leading to a more liquid market with a higher price-dividend ratio. (This result also holds if we assume that non-owners switch markets in a manner implying that the value V_{ln} of being a non-owner is equal to some constant for all markets.)

Such cross-sectional asset-pricing results are studied more directly by Weill (2002), who extends our model to the case of multiple assets and shows, among other things, that securities with a larger free float (shares available for trade) are more liquid and have lower expected returns. Vayanos and Wang (2002) also extend our model so as to explain concentrations of trade in a favored security, explaining for example the price difference between on-the-run and off-the-run Treasury bonds.

A different set of implications for financial markets is obtained in Duffie, Gârleanu, and Pedersen (2003), which studies marketmakers.[14] Outside investors remain able to find other investors with intensity λ, but in addition find marketmakers with some intensity ρ. This framework captures the feature that investors bargain sequentially with marketmakers.

The price negotiation between a marketmaker and an investor reflects the investor's outside options, including in particular the investor's ability to meet and trade with other investors or marketmakers. It is shown that the marketmaker's bid-ask spread is lower if the investor can find other investors on his own more easily. Further, the spread is lower if an investor can approach other marketmakers more easily. In other words, more "sophisticated" investors get *tighter* spreads from the marketmaker. Examples can be found in the typical hub-and-spoke structure of contact among marketmakers and their customers in OTC derivative markets. This distinguishes our search theory from traditional information-based theories that predict that more sophisticated (in this setting, more informed) investors get *wider*

[13]A higher total mass, μ, of participating agents leads to higher search intensities $\lambda\mu$, so if we re-normalize the mass to $\mu = 1$, we must simultaneously increase λ.

[14]Other search-based models of intermediation include Rubinstein and Wolinsky (1987), Bhattacharya and Hagerty (1987), Moresi (1991), Gehrig (1993), and Yavaş (1996).

spreads from marketmakers (Glosten and Milgrom (1985)).

The search-and-bargaining framework is a reasonable stylization of broker-dealer behavior in OTC markets for fixed-income derivatives. In these markets, a "sales trader" and an outside customer negotiate a price, implicitly including a dealer profit margin, that is based in large part on the customer's (perceived) outside option. In this setting, the risk that customers have superior information about future interest rates is often regarded as small. The customer's outside option depends on how easily he can find a counterparty himself (proxied by λ in the model) and how easily he can access other dealers (proxied by ρ in the model). As explained by Commissioner of Internal Revenue (2001) (page 13) in recent litigation regarding the portion of dealer margins on interest-rate swaps that can be ascribed to profit, dealers typically negotiate prices with outside customers that reflect the customer's relative lack of access to other market participants. In order to trade OTC derivatives with a bank, for example, a customer must have, among other arrangements, an account and a credit clearance. Smaller customers often have an account with only one, or perhaps a few, banks, and therefore have fewer search options. Hence, a testable implication of our search framework is that (small) investors with fewer search options receive less competitive prices. We note that these investors are less likely to be informed, so that models based on informational asymmetries alone would reach the opposite prediction.

Our results have been extended to illustrate that temporary external supply imbalances may have much bigger impacts on prices than would be the case with perfectly liquid markets, and that the degree of these price impacts can be mitigated by providers of liquidity such as underwriters, hedge funds, and marketmakers. Weill (2003) uses our approach to characterize the optimal behavior of marketmakers in absorbing supply shocks in order to mitigate search frictions by "leaning against" the outside order flow. Newman and Rierson (2003) use our approach in a search-based model of corporate bond pricing, in which large issues of credit-risky bonds temporarily raise credit spreads throughout the issuer's sector, because providers of liquidity such as underwriters and hedge finds bear extra risk as they search for long-term investors. They provide empirical evidence of temporary bulges in credit spreads across the European Telecom debt market during 1999-2002 in response to large issues by individual firms in this sector. Studying a different set of markets, Mikkelson and Partch (1985) find empirical support for "the notion that underwriting spreads are in part compensation for the selling effort." In particular, they find that underwriting spreads are positively related to the size of the offering.

Appendices

A Explicit Bargaining Game

The setting considered here is that of Section 1, with two exceptions. First, agents can interact only at discrete moments in time, Δ_t apart. Later, we return to continuous time by letting Δ_t go to zero. Second, the bargaining game is modeled explicitly.

We follow Rubinstein and Wolinsky (1985) and others in modeling an alternating-offers bargaining game, making the adjustments required by the specifics of our setup. When two agents are matched, one of them is chosen randomly — the seller with probability $\hat{q}$, the buyer with probability $1 - \hat{q}$ — to suggest a trading price. The other either rejects or accepts the offer, immediately. If the offer is rejected, the owner receives the dividend from the asset during the current period. At the next period, Δ_t later, one of the two agents is chosen at random, independently, to make a new offer. The bargaining may, however, break down before a counteroffer is made. A breakdown may occur because either of the agents changes valuation type, whence there are no longer gains from trade. A breakdown may also occur if one of the agents meets yet another agent, and leaves his current trading partner. The latter reason for breakdown is only relevant if agents are allowed to search while engaged in negotiation.

We consider first the case in which agents can search while bargaining. We assume that, given contact with an alternative partner, they leave the present partner in order to negotiate with the newly found one. The offerer suggests the price that leaves the other agent indifferent between accepting and rejecting it. In the unique subgame perfect equilibrium, the offer is accepted immediately (Rubinstein (1982)). The value from rejecting is associated with the equilibrium strategies being played from then ownards. Letting P_σ be the price suggested by the agent of type σ with $\sigma \in \{lo, hn\}$, letting $\bar{P} = \hat{q}P_{lo} + (1 - \hat{q})P_{hn}$, and making use of the motion laws of V_{lo} and

V_{hn}, we have

$$\begin{aligned} P_{hn} - \Delta V_l &= e^{-(r+\lambda_d+\lambda_u+2\lambda\mu_{lo}+2\lambda\mu_{hn})\Delta_t}(\bar{P} - \Delta V_l) + O(\Delta_t^2) \\ -P_{lo} + \Delta V_h &= e^{-(r+\lambda_d+\lambda_u+2\lambda\mu_{lo}+2\lambda\mu_{hn})\Delta_t}(-\bar{P} + \Delta V_h) + O(\Delta_t^2)\,. \end{aligned}$$

These prices, P_{hn} and P_{lo}, have the same limit $P = \lim_{\Delta_t \to 0} P_{hn} = \lim_{\Delta_t \to 0} P_{lo}$. The two equations above readily imply that the limit price and limit value functions satisfy

$$P = \Delta V_l\,(1-q) + \Delta V_h\,q, \tag{A.1}$$

with

$$q = \hat{q}. \tag{A.2}$$

This result is interesting because it shows that the seller's bargaining power, q, does not depend on the parameters — only on the likelihood that the seller is chosen to make an offer. In particular, an agent's intensity of meeting other trading partners does not influence q. This is because one's own ability to meet an alternative trading partner: *(i)* makes oneself more impatient, and *(ii)* also increases the partner's risk of breakdown, and these two effects cancel out.

This analysis shows that the bargaining outcome used in our model can be justified by an explicit bargaining procedure. We note, however, that other bargaining procedures lead to other outcomes. For instance, if agents cannot search for alternative trading partners during negotiations, then the same price formula (A.1) applies with

$$q = \frac{\hat{q}(r + \lambda_u + \lambda_d + 2\lambda\mu_{lo})}{\hat{q}(r + \lambda_u + \lambda_d + 2\lambda\mu_{lo}) + (1-\hat{q})(r + \lambda_u + \lambda_d + 2\lambda\mu_{hn})}. \tag{A.3}$$

This bargaining outcome would lead to a similar solution for prices, but the comparative-static results would change, since the bargaining power q would depend on the other parameters.

B Walrasian Equilibrium with Risk Aversion

This section derives the competitive equilibrium with risk averse agents (as in Section 2) who can immediately trade *any* number of risky securities. We note that this is different from a competitive market with fixed exogenous position sizes, that is, it is different from the limit considered in Proposition 3.

Suppose that the Walrasian price is constant at P, that is, agents can trade instantly at this price. An agent's total wealth — cash plus the value of his position in risky assets — is denoted by $\bar{W}$. If an agent chooses to hold $\theta(t)$ shares at any time t, then the wealth-dynamics equation is

$$d\bar{W}_t = (r\bar{W}_t - r\theta_t P - c_t)\,dt + \theta_t\,dD_t + d\eta_t.$$

The HJB equation for an agent of intrinsic type $\sigma \in \{h, l\}$ is

$$\begin{aligned} 0 \;=\; \sup_{\bar{c},\theta} \; & \{J_w(w,\sigma)(rw - \bar{c} + \theta(m_D - rP) + m_\eta) \\ & + \frac{1}{2} J_{ww}(w,\sigma)(\theta^2\sigma_D^2 + \sigma_\eta^2 + 2\rho_\sigma\theta\sigma_D\sigma_\eta) \\ & + \lambda(\sigma,\sigma')[J(w,\sigma) - J(w,\sigma')] - e^{-\gamma\bar{c}} - \beta J(w,\sigma)\}, \end{aligned}$$

where $\lambda(\sigma,\sigma')$ is the intensity of change of intrinsic type from σ to σ'. Conjecturing the value function $J(w,\sigma) = -e^{-r\gamma(w + a_\sigma + \bar{a})}$, optimization over θ yields

$$\theta_\sigma \;=\; \frac{m_D - rP - r\gamma\rho_\sigma\sigma_D\sigma_\eta}{r\gamma\sigma_D^2}. \tag{B.1}$$

Market clearing requires

$$\mu_h\theta_h + \mu_l\theta_l = \Theta,$$

with $\mu_h = 1 - \mu_l = \lambda_u/(\lambda_u + \lambda_d)$, which gives the price

$$P = \frac{m_D}{r} - \gamma\left(\Theta\sigma_D^2 + \frac{\sigma_D\sigma_\eta\,[\rho_l\lambda_d + \rho_h\lambda_u]}{\lambda_u + \lambda_d}\right). \tag{B.2}$$

Inserting this price into (B.1) gives the quantity choices

$$\theta_h \;=\; \Theta + \frac{\sigma_\eta\lambda_d\,[\rho_l - \rho_h]}{\sigma_D(\lambda_u + \lambda_d)} \tag{B.3}$$

$$\theta_l \;=\; \Theta - \frac{\sigma_\eta\lambda_u\,[\rho_l - \rho_h]}{\sigma_D(\lambda_u + \lambda_d)}. \tag{B.4}$$

C Proofs

Proof of Proposition 1: The dependence on δ and q is seen immediately, given that no other variable entering Equation (6) depends on either δ or q.

Viewing P and μ_σ as functions of the parameters λ_d and s, a simple differentiation exercise shows that the derivative of the price P with respect to λ_d is a positive multiple of

$$
\begin{aligned}
&(rq + \lambda_u + 2\lambda\mu_{hn}q)\left(1 + 2\lambda\frac{\partial\mu_{lo}}{\partial\lambda_d}(1-q)\right) \\
&- \quad (r(1-q) + \lambda_d + 2\lambda\mu_{lo}(1-q))\left(2\lambda\frac{\partial\mu_{hn}}{\partial\lambda_d}q\right),
\end{aligned}
$$

which is positive if $\frac{\partial\mu_{lo}}{\partial\lambda_d}$ is positive and $\frac{\partial\mu_{hn}}{\partial\lambda_d}$ is negative.

These two facts are seen as follows. From Equations (1)-(3) and the fact that $\mu_{lo} + \mu_{ln} = \lambda_d(\lambda_d + \lambda_u)^{-1} = 1 - y$, where

$$y = \frac{\lambda_u}{\lambda_u + \lambda_d},$$

it follows that μ_{lo} solves the equation

$$2\lambda\mu_{lo}^2 + (2\lambda(y-s) + \lambda_u + \lambda_d)\mu_{lo} - \lambda_d s = 0. \tag{C.1}$$

This quadratic equation has a negative root and a root in the interval $(0,1)$, and this latter root is μ_{lo}.

Differentiating (C.1) with respect to λ_d, one finds that

$$\frac{\partial\mu_{lo}}{\partial\lambda_d} \quad = \quad \frac{s - \mu_{lo} - 2\lambda\frac{\partial y}{\partial\lambda_d}\mu_{lo}}{2\lambda\mu_{lo} + 2\lambda(y-s) + \lambda_u + \lambda_d} > 0,$$

since $\frac{\partial y}{\partial\lambda_d} < 0$. Similar calculations show that

$$\frac{\partial\mu_{hn}}{\partial\lambda_d} \quad = \quad \frac{-\lambda_d + 2\lambda\frac{\partial y}{\partial\lambda_d}\mu_{hn}}{2\lambda\mu_{lo} + \lambda_u + \lambda_d} < 0,$$

which ends the proof of the claim that the price decreases with λ_d. Like arguments can be used to show that $\frac{\partial\mu_{lo}}{\partial\lambda_u} < 0$ and that $\frac{\partial\mu_{hn}}{\partial\lambda_u} > 0$, which implies that P increases with λ_u.

Finally,

$$\frac{\partial \mu_{lo}}{\partial s} = \frac{\lambda_d + 2\lambda\mu_{lo}}{2\lambda\mu_{lo} + 2\lambda(y-s) + \lambda_u + \lambda_d} > 0$$

and

$$\frac{\partial \mu_{hn}}{\partial s} = \frac{-\lambda_u - 2\lambda\mu_{hn}}{2\lambda\mu_{lo} + \lambda_u + \lambda_d} < 0,$$

showing that the price decreases with the supply s.

In order to prove that the price increases with λ for λ large enough, it is sufficient to show that the derivative of the price with respect to λ changes sign at most a finite number of times, and that the price tends to its upper bound, $1/r$, as λ tends to infinity. The first statement is obvious, while the second one follows from Equation (6), given that, under the assumption $s < \lambda_u/(\lambda_u + \lambda_d)$, $\lambda\mu_{lo}$ stays bounded and $\lambda\mu_{hn}$ goes to infinity with λ.

□

Proof of Proposition 2.

We impose on investors' choices of consumption and trading strategies the transversality condition that, for any initial agent type σ_0, $e^{-\beta T}E_0[J(W_T, \sigma_t)] \to 0$ as T goes to infinity. Intuitively, the condition means that agents cannot consume large amounts forever by increasing their debt without restriction. We must show that our candidate optimal consumption and trading strategy satisfies that condition.

We conjecture that, for our candidate optimal strategy, $E_0[J(W_T, \sigma_T)] = e^{(\beta-r)T}J(W_0, \sigma_0)$. Clearly, this implies that the transversality condition is satisfied since $e^{-\beta T}E_0[J(W_T, \sigma_T)] = e^{-rT}J(W_0, \sigma_0) \to 0$. This conjecture is based on the insights that *(i)* the marginal utility, $u'(c_0)$, of time-0 consumption must be equal to the marginal utility, $e^{(r-\beta)T}E_0[u'(c_T)]$, of time T consumption; and *(ii)* the marginal utility is proportional to the value function in our (CARA) framework. (See Wang (2002) for a similar result.)

To prove our conjecture, we consider, for our candidate optimal policy, the wealth dynamics

$$\begin{aligned} dW &= \left(\frac{\log r}{\gamma} - ra_\sigma - r\bar{a} + \theta_\sigma m_D + m_\eta\right) dt + \theta_\sigma \sigma_D \, dB + \sigma_\eta \, dB^i - P \, d\theta_\sigma \\ &= \left(-ra_\sigma + \theta_\sigma m_D + \frac{1}{2} r\gamma\sigma_\eta^2 + \frac{r-\beta}{r\gamma}\right) dt + \theta_\sigma \sigma_D \, dB + \sigma_\eta \, dB^i - P \, d\theta_\sigma \\ &= M(\sigma)\, dt + \sqrt{\Sigma(\sigma)}\, d\hat{B} - P \, d\theta_\sigma, \end{aligned}$$

where M, Σ and the standard Brownian motion $\hat{B}$ are defined by the last equation.

Define f by

$$f(W_t, \sigma_t, t) = E_t[J(W_T, \sigma_T)] = -E_t[e^{-r\gamma(W_T + a_{\sigma_T} + \bar{a})}].$$

Then, by Ito's Formula,

$$\begin{aligned} 0 &= f_t + f_w M(\sigma) + \frac{1}{2} f_{ww} \Sigma(\sigma) \\ &\quad + \sum_{\{\sigma' : \sigma' \neq \sigma\}} \lambda(\sigma, \sigma') \left(f(w + z(\sigma, \sigma') P, \sigma', t) - f(w, \sigma, t) \right), \end{aligned} \tag{C.2}$$

where $\lambda(\sigma, \sigma')$ is the intensity of transition from σ to σ' and $z(\sigma, \sigma')$ is -1, 1, or 0, depending on whether the transition is, respectively, a buy, a sell, or an intrinsic-type change. The boundary condition is $f(w, \sigma, T) = -e^{-r\gamma(w + a_\sigma + \bar{a})}$.

The fact that $f(w, \sigma, t) = e^{(\beta - r)(T - t)} J(w, \sigma)$ now follows from the facts that *(i)* this function clearly satisfies the boundary condition, and *(ii)* it solves (C.2), which is confirmed directly using (15) for a_σ.

□

Proof of Proposition 3.

This result follows from Equations (15)–(18) as well as the fact that $\lambda \mu_{hn} \to \infty$ and $\lambda \mu_{lo}$ is bounded.

□

Proof of Proposition 4: As stated formally by Equation (30), the position θ decreases with the volatility σ_X. As a consequence, the equilibrium agent masses change with an increase in σ_X in the same way as when the supply of the asset increases. That means, in particular, that μ_{hn} decreases, which translates into longer search times for a seller (type lo). Proposition 1 establishes that the price decreases with the supply, whence also with the volatility σ_X of the dividends .

□

Analysis of pooling equilibria with asymmetric information: We work under condition (31), which means that prices are set by the reservation values of the informed seller and uninformed buyer, and that the bid is higher than the ask. Let μ denote the non-jump part of the drift of X. That is, $E_s[X_t - X_s] = (\mu + \nu J_m)(t-s)$. Under these conditions, the coefficients of the value-functions and price satisfy

$$\begin{aligned}
v_{loi} &= \frac{\lambda_u v_{hoi} + 2\lambda\mu_{hn}(p + v_{lni}) + \lambda_J(\zeta\nu v_{loi} + (1-\zeta\nu)v_{lou}) + \lambda_J J_1 + r^{-1}\mu - \delta}{r + \lambda_u + 2\lambda\mu_{hn} + \lambda_J} \\
v_{lni} &= \frac{\lambda_u v_{hni} + \lambda_J(\zeta\nu v_{lni} + (1-\zeta\nu)v_{lnu})}{r + \lambda_u + \lambda_J} \\
v_{hoi} &= \frac{\lambda_d v_{loi} + \lambda_J(\zeta\nu v_{hoi} + (1-\zeta\nu)v_{hou}) + \lambda_J J_1 + r^{-1}\mu}{r + \lambda_d + \lambda_J} \\
v_{hni} &= \frac{\lambda_d v_{lni} + 2\lambda\mu_{lo}(v_{hoi} - p) + \lambda_J(\zeta\nu v_{hni} + (1-\zeta\nu)v_{hnu})}{r + \lambda_d + 2\lambda\mu_{lo} + \lambda_J} \\
v_{lou} &= \frac{\lambda_u v_{hou} + 2\lambda\mu_{hn}(p + v_{lni}) + \lambda_J(\zeta\nu v_{loi} + (1-\zeta\nu)v_{lou}) + \lambda_J J^u + r^{-1}\mu - \delta}{r + \lambda_u + 2\lambda\mu_{hn} + \lambda_J} \\
v_{lnu} &= \frac{\lambda_u v_{hnu} + \lambda_J(\zeta\nu v_{lni} + (1-\zeta\nu)v_{lnu})}{r + \lambda_u + \lambda_J} \\
v_{hou} &= \frac{\lambda_d v_{lou} + \lambda_J(\zeta\nu v_{hoi} + (1-\zeta\nu)v_{hou}) + \lambda_J J^u + r^{-1}\mu}{r + \lambda_d + \lambda_J} \\
v_{hnu} &= \frac{\lambda_d v_{lnu} + 2\lambda\mu_{lo}(v_{hou} - p) + \lambda_J(\zeta\nu v_{hni} + (1-\zeta\nu)v_{hnu})}{r + \lambda_d + 2\lambda\mu_{lo} + \lambda_J} \\
p &= (v_{loi} - v_{lni})(1-q) + (v_{hou} - v_{hnu})q\,.
\end{aligned} \tag{C.3}$$

We may view p as an expected-price coefficient; the realized-price coefficient is $v_{loi} - v_{lni}$ or $v_{hou} - v_{hnu}$, depending on who makes the offer.

In order for a pooling equilibrium to obtain, no agent should be willing to deviate from proposing the pooling prices. First, a low-valuation owner, whether informed or not, must prefer to quote a price that is accepted by all liquid non-owners, rather than quoting a more aggressive price, which would be accepted only by informed non-owners. That is,

$$\Delta v_{hu} + v_{lni} \geq Pr(i\,|\,i)\,(\Delta v_{hi} + v_{lni}) + (1 - Pr(i\,|\,i))v_{loi} \tag{C.4}$$

$$\Delta v_{hu} + v_{lnu} \geq Pr(i\,|\,u)\,(\Delta v_{hi} + v_{lnu}) + (1 - Pr(i\,|\,u))v_{lou}, \tag{C.5}$$

where $Pr(i\,|\,\xi)$ is the probability of the buyer being informed given that the seller has information status $\xi \in \{i, u\}$. The left-hand side of (C.4) is the value Δv_{hu} to an informed low-type owner of quoting the pooling price (given that there are gains from trade with this counterparty). The right-hand side is the value Δv_{hi} of asking for the most aggressive price, namely the reservation value of an informed non-owner (again, given that there are gains from trade with this counterparty). Similarly, (C.5) states that an uninformed low-discount-rate owner prefers to quote the pooling price. We note that (C.4)–(C.5) are based implicitly on an assumption about the uninformed investors' out-of-equilibrium beliefs. In particular, these beliefs must be consistent with the assumption that investors are not willing to pay more than their reservation values. One possible choice of out-of-equilibrium beliefs is that conditional on any out-of-equilibrium price offer, the expected jump of an uninformed remains J^u. While other beliefs are possible, there is no other pooling equilibrium in terms of prices and allocations.

Also, a high-type non-owner, whether informed or not, must prefer to buy at the pooling price with certainty rather than buying at a lower price only from uninformed sellers, that is,

$$v_{hoi} - \Delta v_{li} \geq Pr(u \,|\, i)\,(v_{hoi} - \Delta v_{lu}) + (1 - Pr(u \,|\, i))\, v_{hni} \quad \text{(C.6)}$$

$$v_{hou} - \Delta v_{li} \geq Pr(u \,|\, u)\,(v_{hou} - \Delta v_{lu}) + (1 - Pr(u \,|\, u))\, v_{hnu}. \quad \text{(C.7)}$$

It turns out that only the optimality conditions of the informed seller (C.4), and of the uninformed buyer (C.7) need to be checked. If these two conditions are satisfied, the other two optimality conditions follow automatically. (Proposition 6 below formalizes this claim.)

For a given set of parameters, either of the necessary and sufficient optimality conditions, (C.4) and (C.7), may or may not hold. Intuitively, the first condition fails when, keeping all other parameters fixed, there are "so many" informed agents (ν is sufficiently high) that an (informed) seller would benefit by quoting an aggressive price and risking the loss of a trade with an uninformed agent. Similarly, the second condition fails when, keeping all other parameters fixed, an uninformed buyer perceives the proportion of uninformed agents as too large given his own lack of information (ν is sufficiently small or large).

Proposition 6 *(i) The solution to the linear system (C.3) satisfies* $\Delta v_{li} \geq \Delta v_{lu}$ *and* $\Delta v_{hi} \geq \Delta v_{hu}$. *(ii) Fix all the parameters with the exception of* λ, J_0, *and* J_1. *Then there exists* $\epsilon > 0$ *such that, whenever* $(J_1 - J^u) < \epsilon$, $\Delta v_{hu} \geq \Delta v_{li}$ *for all* $\lambda > 0$. *(iii) If the solution to the linear system (C.3) satisfies* $\Delta v_{hi} \geq \Delta v_{hu} \geq \Delta v_{li} \geq \Delta v_{lu}$, *then conditions (C.4) and (C.7) ensure that this solution defines a pooling equilibrium.*

Proof: Let $\phi_h = \Delta v_{hi} - \Delta v_{hu}$ and $\phi_l = \Delta v_{li} - \Delta v_{lu}$. Appropriate linear combinations of Equations (C.3) yield

$$\begin{bmatrix} r+\lambda_u+2\lambda\mu_{hn}+\lambda_J(1-\nu\zeta) & -\lambda_u \\ -\lambda_d & r+\lambda_d+2\lambda\mu_{lo}+\lambda_J(1-\nu\zeta) \end{bmatrix} \begin{bmatrix} \phi_l \\ \phi_h \end{bmatrix}$$
$$= \lambda_J(J_1 - J^u) \begin{bmatrix} 1 \\ 1 \end{bmatrix} ,$$

which is immediately checked to have a positive solution.

For part *(ii)*, note that, when $J_1 = J_0 = J^u$, for the same reasons as in the main model, $\Delta v_{hi} = \Delta v_{hu} > \Delta v_{li} = \Delta v_{lu}$. Since the difference $\Delta v_{hu} - \Delta v_{li}$ is of the form

$$\frac{\alpha_0}{\lambda+\beta_0} - \frac{\alpha_0' + \alpha_1'\lambda}{\beta_0' + \beta_1'\lambda + \lambda^2}(J_1 - J^u),$$

with all the coefficients bounded uniformly in λ and independent of the jump sizes, and $\alpha_0 > 0$ and $\beta_0 > 0$, the claim follows.

Let us now turn to part *(iii)* of the proposition. Consider a seller with information status $\pi \in \{i, u\}$. The seller's bargaining power does not matter, since we assume that it is captured by an independent random draw that determines which side makes the "take-it-or-leave-it" offer. Our analysis first conditions on the event that the seller makes the offer. Equations (C.4) and (C.5) can be written as

$$\Delta v_{hu} \geq \Delta v_{hi} Pr(i \,|\, \pi) + \Delta v_{l\pi} \left(1 - Pr(i \,|\, \pi)\right).$$

In order to show that the constraint for $\pi = i$ is stronger than the constraint for $\pi = u$, it suffices to show that

$$\Delta v_{hi} Pr(i \,|\, i) + \Delta v_{li} Pr(u \,|\, i) \geq \Delta v_{hi} Pr(i \,|\, u) + \Delta v_{lu} Pr(u \,|\, u),$$

which is equivalent to

$$(\Delta v_{hi} - \Delta v_{li}) Pr(u \,|\, i) \leq (\Delta v_{hi} - \Delta v_{lu}) Pr(u \,|\, u),$$

which in turn holds because $\Delta v_{li} \geq \Delta v_{lu}$ and $Pr(u \,|\, i) \leq Pr(u \,|\, u)$.

Analogously, one deduces that the uninformed-buyer condition is stronger than the informed-buyer condition. Consequently, if (C.4) and (C.7) hold, then (C.5) and (C.6) also do, whence quoting pooling prices is optimal for all agents, given that everybody else does the same. This proves that the solution to (C.3) defines a pooling equilibrium.

□

Proof of Proposition 5: Assume first that $s < \lambda_u/(\lambda_u + \lambda_d)$. Consider, for each pair consisting of an owner and a non-owner of a given type, the difference of the equations in the system (C.3) corresponding to their value functions. Since $\lambda\mu_{hn}$ goes to infinity with λ, while $\lambda\mu_{lo}$ is bounded, one shows that

$$\lim_{\lambda\to\infty} \Delta v_{lu} = \lim_{\lambda\to\infty} \Delta v_{li} = \lim_{\lambda\to\infty} \Delta v_{hu} < \lim_{\lambda\to\infty} \Delta v_{hi}.$$

This conclusion is inconsistent with inequalities (C.4) and (C.5), which means that a pooling equilibrium cannot obtain for high λ. The intuition for the result is that an increase in λ increases without bound the ability to find an informed buyer, who is willing to pay strictly more than the pooling price. ate with λ, making pooling unattractive to the seller.

Analogously, when $s > \lambda_u/(\lambda_u+\lambda_d)$, one shows that the reservation-value coefficient of the uninformed seller, Δv_{lu}, does not converge (from below) to the common limit as the search ability for sellers converges to infinity, making it worthwhile to a buyer to quote aggressively.

□

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