Shakeouts and Market Crashes*

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September 30, 2003

Abstract

Stock-market crashes tend to follow run-ups in prices. These episodes look like bubbles that gradually inflate and then suddenly burst. We show that such bubbles can form in a Zeira-Rob type of model in which demand size is uncertain. Two conditions are sufficient for this to happen: A declining hazard rate in the prior distribution over market size and a convex cost of investment. For the period 1971-2001 we fit the model to the Telecom sector.

1 Introduction

Stock-market crashes tend to follow run-ups in prices. The NYSE index rose in the late 1920s, and then crashed in October 1929, and the Nasdaq rose steadily through the 80's and 90's and crashed after March 2000. These episodes therefore look like bubbles that gradually inflate and then suddenly burst.

In a learning model of the Zeira-Rob type we study the possibility of bubble-like behavior of stock prices, but driven by fundamentals. This model generates a crash when an irreversible creation of capacity overshoots demand. We add to Rob (1991) a convex adjustment cost for the growth of industry capacity and a declining hazard rate in the prior distribution over market size. Many of the ideas are also in Zeira (1987, 1999), Caplin and Leahy (1994) and in Horvath, Schivardi and Woywode (2001).

Figure 1 portrays the 30-year history of the Nasdaq index and its Telecom component. The first panel shows the financial side, the second also includes indexes of real activity. The two panels seem to be linked in that they both experience a sharp reduction in early 2000. We seek to explain this link – the slow but simultaneous rise in the stock market and the capital invested and then a sharp and sudden decline.

*We thank M. Ebell for comments and the NSF for support.

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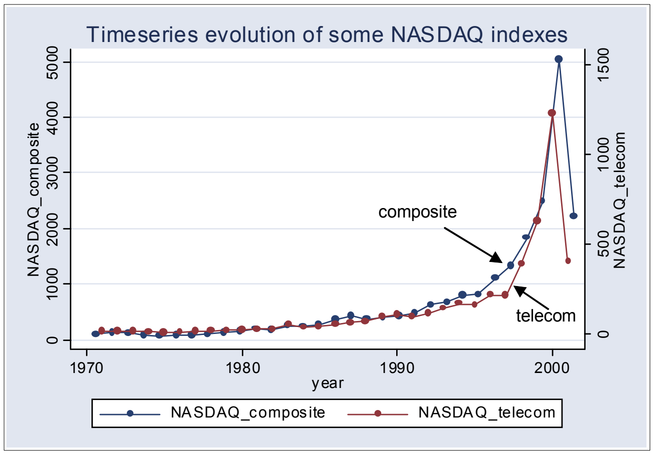


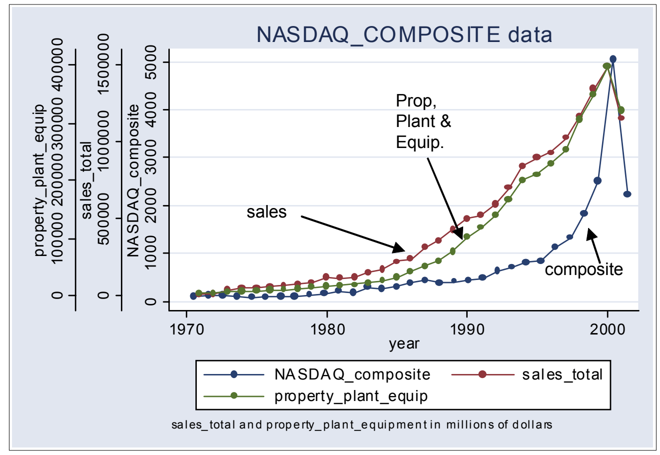


Figure 1: Evolution of Stock Prices and of Fundamentals

The link exists other sectors too, where value appears to have responded the fundamentals around the time of the 2001 crash. Figure 2 reports the March 2000-to-March 2001 changes in prices and in fundamentals. Sectors that experienced greater drops in value also experienced larger declines in sales.

Our explanation of the run-up is of the "Peso Problem" type. Krasker (1980) used this term to describe the rational response of agents to a major event that is not realized within the sample period. Before the year 2000, that was how things looked to agents in the market. Two conditions that jointly lead to such an outcome are: (i) A declining hazard rate in the prior distribution over market size and (ii) A convex cost of investment. The "decreasing hazard" assumption is the technical feature of the model ensuring that, as the market grows, further growth looks ever more feasible, and the likelihood of a crash looks ever more remote. In such a situation it is rational to be more and more optimistic about the market's growth potential as market size increases. If and when the crash does come, however, our model predicts that

- the more remote the possibility of a crash looks, and
- the steeper the adjustment cost of rapid creation of capacity,

the bigger the crash will be. We shall fit the model to the Telecom sector where a substantial crash decidedly did happen in early 2000.[1]

The welfare implication is rather surprising: Along the path to the crash, investment was not too fast but too *slow*. In other words, our model states that in light of what was known at the time, Telecom capacity should have expanded more rapidly

[1] An example is the collapse in the price of on-line advertising at sites such as Yahoo and AOL – by a factor of three or more – and those of their competitors (Angwin 2002).

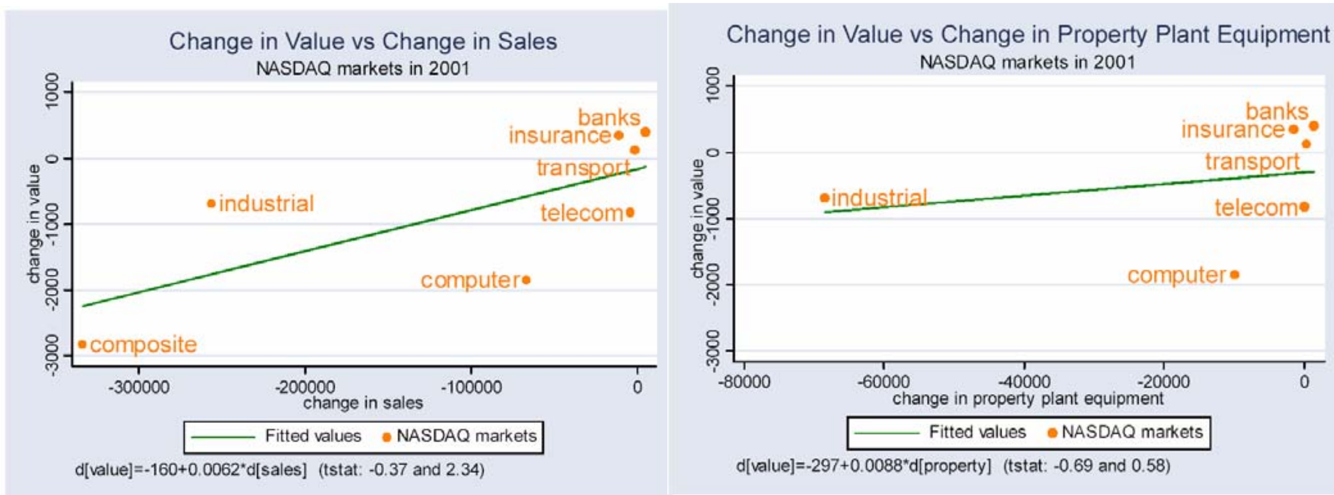


Figure 2: (March 2000-March 2001) Size of crash vs. changes in sales (panel 1) and changes in capacity (panel 2)

and its the crash should have happened earlier. This is because expanding capacity entails a positive *informational externality* that a competitive firm ignores when choosing how much to invest. Along the equilibrium path the crash hazard declines, so that older markets are more likely to survive.

Other work stressing the role of fundamentals in market crashes is Boldrin and Levine (2000), Greenwood and Jovanovic (1999), and Jovanovic and MacDonald (1994), who focus on the effect that arrival on a new technology has on devaluing the existing technology, and Mazzucato and Semmler (1999), who find that in the automobile industry stock prices were the most volatile during the period 1918-1928 when market shares were the most unstable.

Section 2 models a single industry in which producers take the prices of their product and inputs as given. Section 3 simulates the model. Section 4 offers preliminary evidence on the model. Section 5 estimates the prior distribution over market size on which everything hinges. Section 6 contains some extensions.

2 Model

The model is one of an industry that has a well-defined notion of market size. Investing in this market is risky because capacity may exceed market size. Firms continually update their beliefs about what market size and create new capacity based on these beliefs. When capacity outstrips demand, the price falls and further growth stops. We study the dynamics leading up to the crash.

Demand.—The market demand function expresses willingness to pay as a function

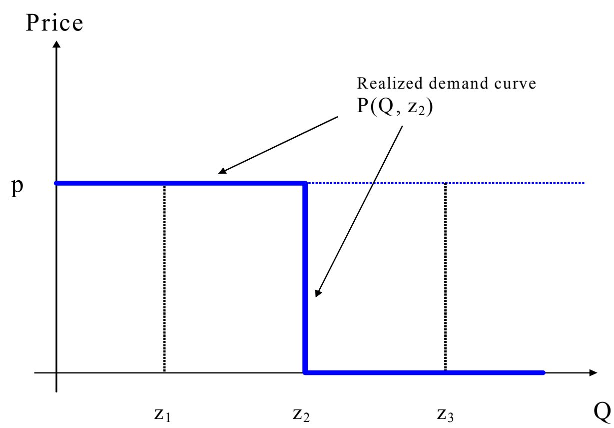


Figure 3: TYPICAL DEMAND CURVE $P(Q, z_2)$

of the quantity, Q, supplied to the market,

$$P(Q,z) = \begin{cases} p & if \;\; Q \leq z \\ 0 & if \;\; Q > z \end{cases}$$

Demand is uncertain. Willingness to pay, p, is known. What is unknown is the extend of demand, the true realization of which is denoted by z, which we may think of as the number of new consumers, each demanding one unit of good per unit time. The parameter z does not change over time. Its a random variable drawn at time $t = 0$ from a distribution $F(z)$ and never drawn again afterwards; 3 shows a family of demand curves indexed by various values of z, and highlights the demand curve that would occur if $z = z_2$. The distribution $F(z)$ is common knowledge among the potential entrants at $t = 0$. It is the common prior distribution which is updated in light of experience.

Production.—Firms are infinitessimal and of indeterminate size. Production cost is zero, and the salvage value of production capacity is negligible. As long the price is positive, industry output is the same as the industry's capacity to produce it. Let k denote the industry's capacity and let n denote new capacity, i.e., aggregate investment. Capacity does not depreciate, and, as long industry price is positive, it is not scrapped. Therefore, capacity evolves as follows

$$k' = k + n. \tag{1}$$

Initial capacity, $k_0 \geq 0$ is given.

Investment.—Adjustment costs of investment are rising at the industry level, but constant at the level of an individual firm. The unit cost, c, of adding capacity rises with *aggregate* investment:

$$c = C(n),$$

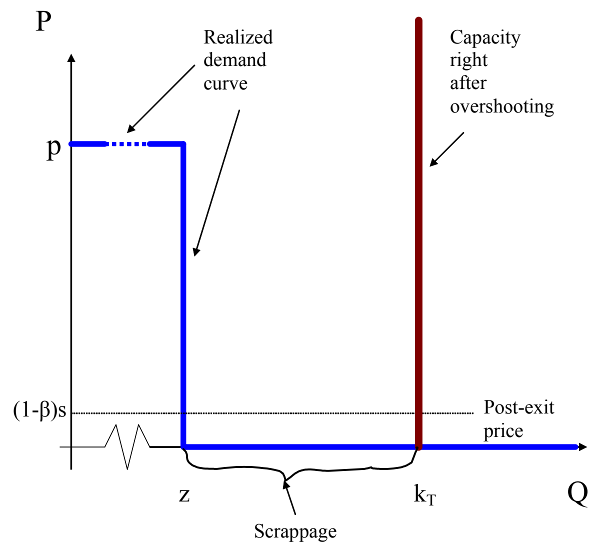


Figure 4: Scrappage at date T

where $C'(n) \geq 0$. An example is a scarce input into the construction of capacity the price of which rises with aggregate investment. to reflect a pecuniary or non-pecuniary congestion effect. Each firm sees $C(n)$ as independent of its of its own actions. Investment becomes productive capacity in the following period.

Industry viability.—To ensure that the industry can get off the ground in the first place, we assume that if no one else were to invest, it would be optimal to do so. The discounted proceeds that a unit of capacity would deliver if the market were never to crash would be $\beta p/(1-\beta)$. The cost of creating a unit of capacity if no one else is creating it would be $C(0)$. Hence our viability condition is

$$C(0) < \frac{\beta p}{1-\beta}. \tag{2}$$

Learning.—Firms share the common prior over z; the C.D.F. $F(z)$. All firms know the history of prices and industry outputs. Based on this they revise their opinion about z. "Overshooting" happens at date T when k exceeds z for the first time. Before date T, firms know only that $z \geq k$. At date T, we assume that firms learn z exactly.[2]

To recapitulate, there are three distinct epochs:

1. *Before date$T$.*—Agents know only that $k \leq z$.

[2] This is the simplest way to have a permanent decline in the value of existing capital at T. Other scenarios entailing more gradual learning about z after date T would also lead to a sharp decline in value at date T.

2. *At date T.*—The first time that $k > z$, firms learn z, perhaps because spare capacity $k_T - z$ becomes public information. This excess capacity is at once scrapped for a return of s and equilibrium product price is $(1-\beta)\,s$. This is illustrated in Figure 4.

3. After date T.—Product price remains for ever at s, and the stock price remains for ever at s. There is no further dynamics

To simplify the algebra we assume that $s = 0$.

The unit value of capacity.—Suppose that when information that firms have is only k and the knowledge of whether k. The value of a unit of capacity in an industry with the state

$$V\left(k,z\right)=\begin{cases} v(k) & \text{if} \quad k\le z \\ 0 & \text{if} \quad k>z. \end{cases}$$

We have, in other words, assumed that the value of all firms drops to zero if and when k exceeds z for the first time. The idea is that the salvage value of the capital is low and that the capital does not depreciate. Therefore, once industry capacity outstrips demand, the product price crashes to zero, and with it the market value of the firm. At that date z becomes publicly known and $k - z$ units are scrapped.

2.1 Equilibrium

All the dynamics in the model stop after date T. The dynamics that we now discuss is for dates $t = 1, 2, ..., T$, i.e., for periods during which $k_t < z$.

In this equilibrium strategies depend only on the industry state, k. All firms are of measure zero, so it does not matter whether new capacity is created by incumbents or by new entrants. We therefore do not distinguish between the two and, as a result, we do not have a theory of firm size; all we need is that firms be of measure zero.

Stock prices.—Investors are all rational.[3] The Bellman equation in the region $k < z$ is a function of the investment rule $n\left(k\right)$ which leads to next period capacity $k' = k + n\left(k\right)$, so that

$$v\left(k\right)=p+\beta v\left(k+n\left[k\right]\right)\frac{1-F\left(k+n\left[k\right]\right)}{1-F\left(k\right)} \tag{3}$$

Optimal investment.—We assume that $C\left(0\right) = 0$, and that $C\left(n\right)$ is unbounded. Then the marginal investment condition always holds with equality. In state k, it reads

$$C\left(n\left[k\right]\right)=\beta v\left(k+n\left[k\right]\right)\frac{1-F\left(k+n\left[k\right]\right)}{1-F\left(k\right)}. \tag{4}$$

[3] This distinguishes our model from many in the finance literature, e.g., Abreu and Brunnemeier (2002)

Definition 1 *Equilibrium is a pair of functions $v(k)$ and $n(k)$ defined for $k \in [0, z]$ that satisfy (3) and (4).*

Existence and uniqueness of the equilibrium.—We do not have a general proof. Instead, we can show that a unique equilibrium exists only in three special cases, two of which are not consistent with Figure 1. We now reduce the two equilibrium conditions (3) and (4) to a single (second-order) difference equation. Then, we shall prove the existence and uniqueness of the equilibrium by showing that a unique solution exists to this difference equation.

Substituting from (4) into (3), the relation is

$$v(k) = p + C(n[k]). \tag{5}$$

From (5) it is clear that if we can get n to rise over time, C will rise and, hence, so will v. That is, stock prices will rise with t until date T.

With (4) this implies

$$C(n) = \beta (p + C[n']) \frac{1 - F(k+n)}{1 - F(k)}. \tag{6}$$

or, rearranged,

$$\begin{aligned} C(n') &= -p + C(n) \frac{1}{\beta} \frac{1 - F(k)}{1 - F(k+n)} \\ &= -p + C(n) \frac{1}{\beta} \exp\left\{ \int_k^{k+n} h(s)\, ds \right\} \end{aligned} \tag{7}$$

where $h(z) = \frac{f(z)}{1-F(z)}$. Upon using (1) to eliminate n, we obtain an implicit second-order difference equation in k:

$$C(k'' - k') = -p + C(k' - k) \frac{1}{\beta} \frac{1 - F(k)}{1 - F(k')}$$

The initial condition k_0 is not sufficient to pin down a unique path.

2.2 Two special cases

We can easily calculate the unique equilibrium for two special cases of the model. However, both entail stock prices that remain constant until date T.

1. *Constant-hazard.*—Let $F(z) = 1 - e^{-\lambda z}$. Then $h(z) = \lambda$ and (7) collapses to

$$C(n') = -p + C(n) \beta^{-1} e^{\lambda n}$$

The only admissible solution is a constant n_t, which solves for n the equation

$$C(n) = \beta e^{-\lambda n} \frac{p}{1 - \beta e^{-\lambda n}} \text{ all } t. \tag{8}$$

The LHS of (8) is increasing in n while the RHS is decreasing therefore the solution is unique. (Moreover, any non-stationary solution (n_t) would be explosive because $\beta^{-1} e^{\lambda n} > 1$ for all $n \geq 0$). Since n is constant, (5) tells us that stock prices, $v(k)$, will be constant until date T, and iterations of (1) imply that $k_t = k_0 + nt$ for $t \leq T$.

2. *Constant* $C(n)$.—Let $C(n) = c$ for all n. This is Rob's (1991) case. Then (2) implies that $c < \frac{\beta p}{1-\beta}$, and (4) always holds a positive n. Then (5) implies that $v(k) = p + c$ for all k. I.e., stock prices are again constant. Finally, (6) reads

$$1 + \frac{p}{c} = \frac{1}{\beta} \frac{1 - F(k)}{1 - F(k+n)},$$

which, by $c < \frac{\beta p}{1-\beta}$, implies the existence of a unique investment function $n = \psi(k) > 0$. Together with (1), this gives the sequence k_t uniquely.

Evidently, we must relax both assumptions if we are to have any chance of reconciling this model with the rising stock prices in Figure 1. We now undertake that task.

3 The case of a decreasing h.

This section derives properties that an equilibrium must satisfy if $h' < 0$ and $C' > 0$. Then we shall show that a unique equilibrium exists when F is Pareto and when $C(n) = cn$. We start with the general case.[4]

Assumptions on F.—Suppose that the support of F is $[z_{\min}, \infty)$. Suppose furthermore that $h(z) > 0$ for all $z > z_{\min}$, and that $h'(z) < 0$. That is, F has a strictly decreasing hazard. Assume, moreover, that $\lim_{z\to\infty} h(z) = 0$. We relax these assumptions in Section 4.2 where we impose a finite bound on z.

Analysis.—Now define the scalar n_∞ implicitly as the solution to the equation

$$C(n_\infty) = \frac{\beta p}{1 - \beta}. \tag{9}$$

Lemma 2 *If $n_\infty \equiv \lim_{t\to\infty} n_t$ exists, then it is unique and satisfies (9)*

[4] Prat (2003) provides a survivorship-bias type of rationale for why the hazard rate may be declining, especially when the uncertainty over market types is large.

Proof. The RHS of (4) is at most equal to the RHS (9). Therefore

$$n_t < n_\infty.$$

Moreover, the conditional probability of the market surviving for another period is

$$\xi(k,n) \equiv \frac{1-F(k+n)}{1-F(k)} = \exp\left\{-\int_k^{k+n} h(z)\,dz\right\} \tag{10}$$

If h is decreasing, ξ is increasing in k and decreasing in n. Since k_t is an increasing sequence, we then must have

$$\xi(k_t,n_t) \geq \xi(k_0,n_\infty)$$

Therefore for any t, the return to a unit of incumbent capital is

$$\begin{aligned} v_t &= \sum_{j=t}^{\infty} \beta^{j-t} \prod_{\tau=t}^{j} \xi(k_\tau,n_\tau)\,p \\ &\geq p\sum_{j=t}^{\infty} [\beta\xi(k_0,n_\infty)]^{j-t} \\ &\geq \frac{p}{1-\beta\xi(k_0,n_\infty)} \end{aligned}$$

Therefore, for any t

$$n_t \geq n_{\min} > 0,$$

where $n_{\min}$ solves

$$C(n_{\min}) = \frac{\beta\xi(k_0,n_\infty)\,p}{1-\beta\xi(k_0,n_\infty)}$$

But then $k_t \geq tn_{\min}$, and so $\lim_{t\to\infty} k_t = \infty$. Therefore the RHS of (7) converges to $p + C(n)/\beta$. Rearranging, we get (9). ■

The next proposition contains the result we need. The algebra simplifies if we re-state the difference equation (7) as a difference equation in $c = C(n)$. Assume that C is a one-to one increasing map from $R_+ \to R_+$ and that its range is all of R_+ so that $n(c) \equiv C^{-1}[c]$ is uniquely defined for all $c \geq 0$. Then write (7) as

$$\begin{aligned} c' &= -p + \frac{c}{\beta}\exp\left\{\int_k^{k+n(c)} h(z)\,dz\right\} \\ &\equiv \phi(c,k). \end{aligned} \tag{11}$$

This difference equation is easier to work with

Proposition 3 *Before the crash (i.e., for $t < T$),*

$$n_{t+1} > n_t.$$

Proof. Since $n(c)$ is strictly increasing, $\phi(c,k)$ is strictly increasing in c. Since $h(z)$ is a decreasing function, $\phi(c,k)$ is strictly decreasing in k. Suppose, contrary to the claim, that $n_{t+1} \leq n_t$. Then $c_{t+1} \leq c_t$. Since $C(0) = 0$, we must have $n_{t+1} > 0$, or else (4) would be violated. But then $k_{t+1} > k_t$, and therefore

$$c_{t+2} = \phi(c_{t+1}, k_{t+1}) < \phi(c_t, k_t) = c_{t+1}.$$

Iterating this argument leads to the conclusion that

$$n_t \geq n_{t+1} \Longrightarrow n_{t+1} > n_{t+2} > .... \geq n_{\min}$$

And since the initial value $n_t < n_\infty$, we conclude that $\lim_t n_t < n_\infty$. But $k_t \geq tn_{\min}$ and once again $\lim_{t\to\infty} k_t = \infty$, and therefore (9) must hold, and this is a contradiction. ■

Since a bounded monotone sequence must have a limit, we conclude that (n_t) indeed does have a limit and that this unique limit solves (9).

Lemma 4 *If h is decreasing in z, ϕ is decreasing in k.*

Proof. Differentiating,

$$\frac{\partial \phi}{\partial k} = \left[h\left(k + C^{-1}[c]\right) - h(k)\right] \frac{c}{\beta} \exp\left\{\int_k^{k+C^{-1}(c)} h(z)\,dz\right\} < 0$$

because h is decreasing, whereas $C^{-1}[c] > 0$. ■

Since ϕ decreases with k, the mode of convergence will therefore be as shown in Figure 5.

Older markets are less likely to crash—An equilibrium of this sort will imply a rising probability of survival for the market along the equilibrium path. From (7) and (10) the conditional survival probability is

$$\xi(k_t, n_t) = \frac{1}{\beta}\left[\frac{p}{C(n_t)} + \frac{C(n_{t+1})}{C(n_t)}\right]^{-1}$$

By Proposition 3 C is increasing with t and $\frac{C(n_{t+1})}{C(n_t)} > 1$ on the transition path. The survival probability therefore rises overall from $\frac{1}{\beta}\left[\frac{p}{C(n_0)} + \frac{C(n_1)}{C(n_0)}\right]^{-1}$ to $\frac{1}{\beta}\left[\frac{p}{C(n_\infty)} + 1\right]^{-1}$, but we cannot show that the increase is monotonic.

3.1 Simulated example: Pareto F

We simulate the equilibrium of a special case of a decreasing hazard distribution, the Pareto distribution:

$$F(z) = 1 - \left(\frac{z}{z_{\min}}\right)^{-\rho}.$$

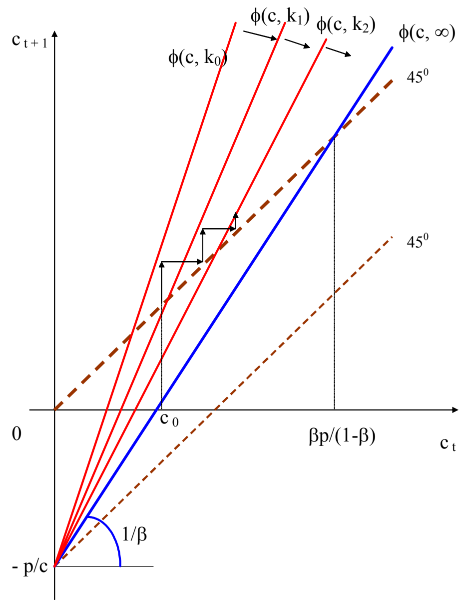


Figure 5: Behavior of the solution, c_t, to the difference equation (11).

We shall show that a unique equilibrium exists in this case. Its hazard rate is $\frac{\rho}{z}$. Now (6) reads

$$C(n) = \beta(p + C[n'])\left(\frac{k}{k'}\right)^{\rho}$$

Then, since $k' = k + n$, this equation reads

$$C(n) = \beta(p + C[n'])\left(\frac{1}{1+\frac{n}{k}}\right)^{\rho}$$

which we can rearrange this into the difference equation

$$C(n') = -p + C(n)\frac{1}{\beta}\left(1 + \frac{n}{k}\right)^{\rho} \tag{12}$$

This is the specific form depicted in Figure (5).

Assume that

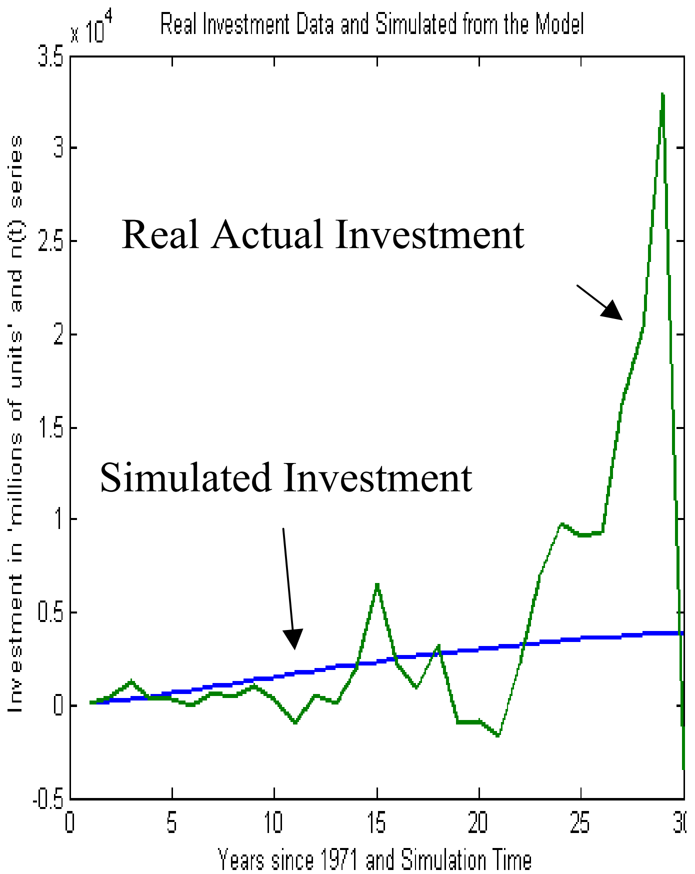
$$C(n) = cn$$

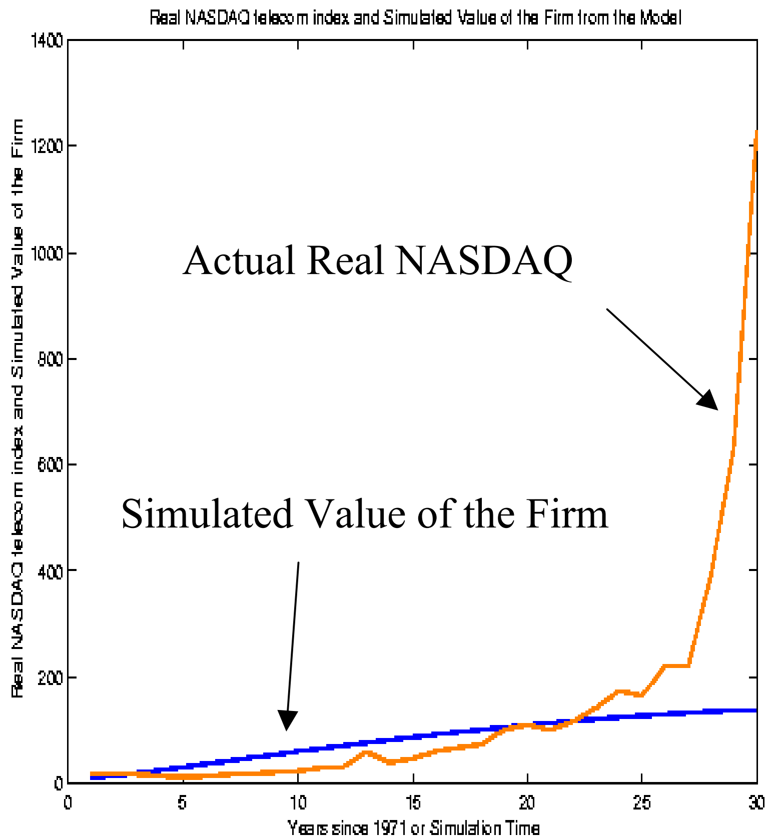
Then (12) reads

$$n' = -\frac{p}{c} + \frac{1}{\beta}n\left(1 + \frac{n}{k}\right)^{\rho} \tag{13}$$

The second difference equation is

$$k' = k + n \tag{14}$$


x 10^4
Real Investment Data and Simulated from the Model
Investment in 'millions of units' and n(t) series
Real Actual Investment
Simulated Investment
Years since 1971 and Simulation Time


Real NASDAQ telecom index and Simulated Value of the Firm from the Model
Real NASDAQ telecom index and Simulated Value of the Firm
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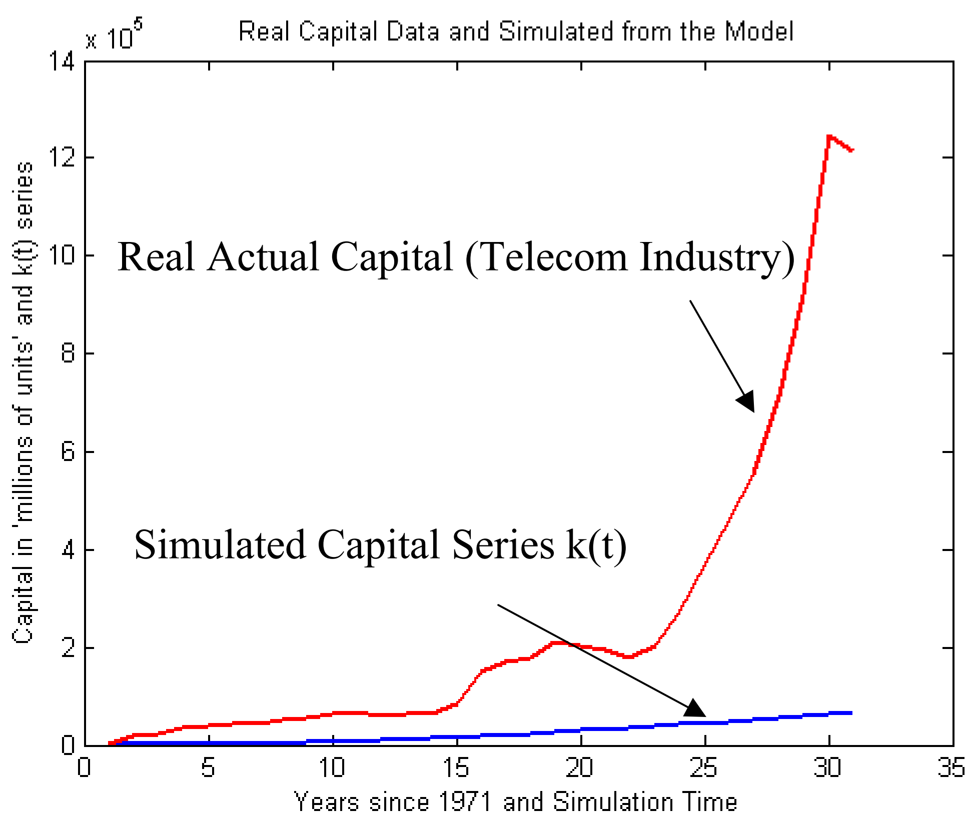

x 10^5
Real Capital Data and Simulated from the Model
Capital in 'millions of units' and k(t) series
Real Actual Capital (Telecom Industry)
Simulated Capital Series k(t)
Years since 1971 and Simulation Time

The Appendix proves the existence and uniqueness of the solution to (13), (14) for the sequence (k_t). It also describes in detail the algorithm used to find the simulated solution to the second-order difference equation implied by (13) and (14):

$$k_{t+2} = k_{t+1} + \frac{1}{\beta}\left(k_{t+1} - k_t\right)\left(\frac{k_{t+1}}{k_t}\right)^{\rho} - \frac{p}{c}.$$

The simulation in the next figure is based on the following parameter values:

parameter	p	c	ρ	β
calibrated value	7.83	0.033	1	1.1

The Figure portrays the simulated time paths of n_t and k_t against their observed proxies: the actual Telecom sales series used as the proxy for k and the fist difference of sales as the proxy for n_t.

4 Fundamentals and the crash

Before estimating the model formally, we argue that its basic assumptions and implications are plausible. We focus on four components of the model: (i) The role of Telecom-product prices, (ii) The behavior of price-earnings ratios, (iii) time to build, and (iv) the assumption of perfect competition.

4.1 Telecom-product prices

Let p_t denote the telecom index. The model predicts that $p_t = \bar{p}$ until date T (i.e., March 2000), and that at T, p_t should fall to $(1-\beta)\,s$ and that it should stay there. Figure 6 shows the product-price index. Clearly the decline is much more gradual but it is nevertheless substantial and happens at about the right time.

4.2 Price-earnings ratios of telecom firms

The model implies not only a rise in stock prices, but also a rise in a monotonic rise in the Price-earnings ratio.[5] This is driven by the monotonic rise in the market's survival probability (the mirror image of the declining hazard and the rising market-survival probability). Figure 7 plots the evolution of the average P-E ratio, and that of the various percentiles of the P-E distribution. The Figure shows that P-E indeed does rise slightly over the sample period. Its dispersion also rises, (which the model does not explain)

[5] We thank Raj Mehra for urging us to report this ratio.

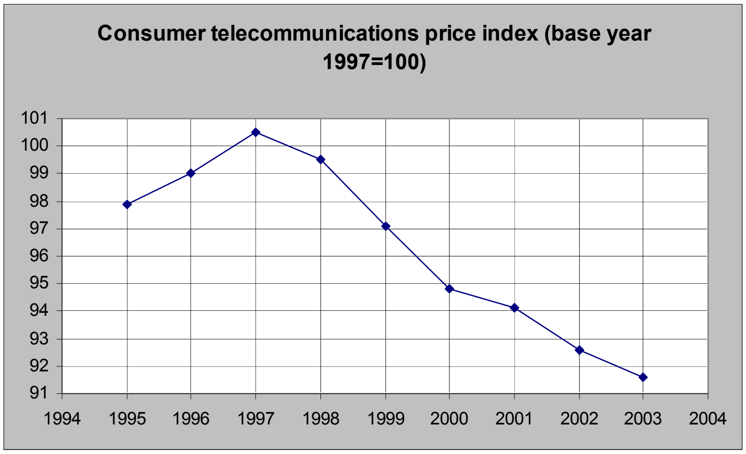


Figure 6: Price index for Telecom output

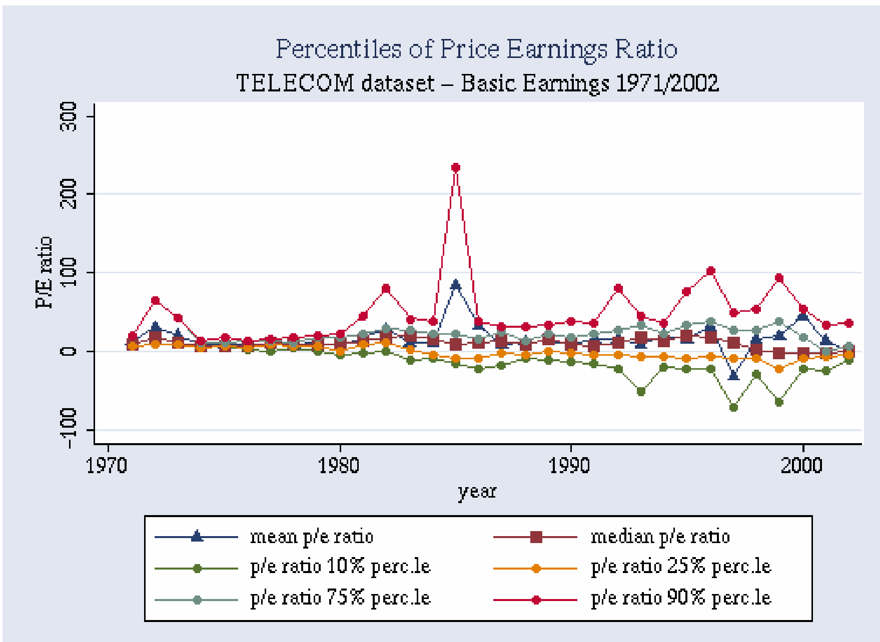


Figure 7: Evolution of the P/E ratio

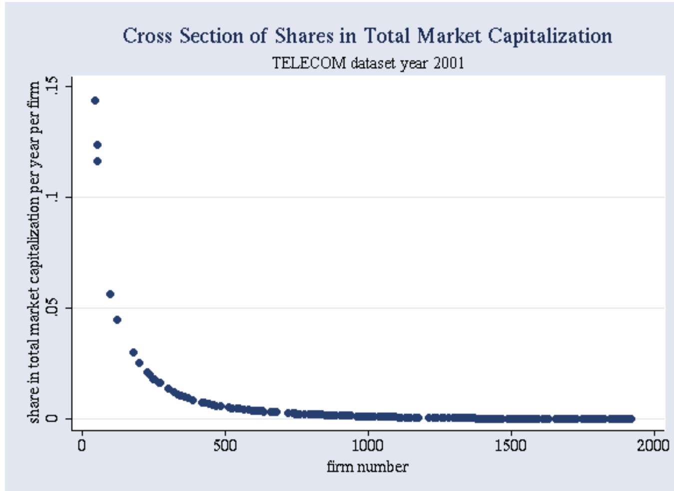


Figure 8: Value share by value rank

4.3 Time to build delays

The spirit of the model is that the crash happens when capacity overshoots demand. This could not happen to any great extent (i.e., k_T could not exceed z by much) if capacity creation was instantaneous. In that event prices would at once signal that further creation was unwarranted and serious spare capacity could not develop. For the mechanism to be credible, there must be significant time to build delays. Koeva (2000, Table 1) reports a 24-month time-to-build estimate for Telecom – roughly the economy-wide average.

A vivid historical example where time to build probably helped magnify a price collapse is the market for office space in New York City around the 1929 crash. Many office buildings were commissioned weeks or months before the crash of 1929, but were completed afterwards. The best known among these are the Chrysler Building, completed in the spring of 1930, and the Empire State building some months later. Both remained largely vacant until WW2, contributing to the collapse of office-space rents in the area.

4.4 Market shares of telecom firms

Firms are competitive in the model. Figure 8 plots the capitalizations of the leading Telecom companies on the Nasdaq, as a fraction of the total Telecom capitalization on the Nasdaq. Because some major players like Motorola do not list on the Nasdaq, these numbers overestimate the shares of these firms. The leading five Nasdaq firms accounted for just under 50% of Nasdaq Telecom's capitalization, but a sizably smaller share of NYSE + Nasdaq together.

5 Estimating h with panel data

To estimate h we need data on a set of markets. In our model a market crash happens only once in any market. Figure 2 confirms the model's prediction that market crashes are accompanied by reductions in sales. Thus when we move to larger data sets on which we have no stock-price information, we shall infer a crash where we see a large sales reduction. Where we find no such reductions, we shall assume no crash has taken place. We shall use the following two data sets:

1. Jorgenson's data from BLS and BEA for 35 industries defined at the 2-digit SIC level – a fairly aggregate level.[6].

2. Gort and Klepper (1982) ("GK") collected data on 46 fairly narrow products. For 16 of those industries, data on sales and prices time series are available continuously. See Appendix C for details.

5.1 Construction of the Measure of Demand Hazard

For both GK's and Jorgenson's data-sets we proceeded as follows (Appendix C also describes in detail the data that the names used below correspond to).

1. In our model, k is proportional to sales. As a proxy for k we shall therefore use

$$\texttt{real\_sales}_i \equiv \frac{sales_i}{CPI}$$

   to be the real sales of industry i.

2. We interpret a "consistent" decline in the `real_sales` time series (or its flattening out) after a continuous consecutive rise as a "failure" that is a realization of z.[7] We follow Stock's (2001) suggestion and model structurally our break in trend. Consequently we search for such a point in the `real_sales` series. We Re-phrase this search problem as a search of a structural break or a broken trend or change in regime in a time series. We need the unknown date of the break. When that date is unknown, usual Chow tests are problematic[8], and there is more than one way to proceed.– Stock (2001) has a survey. We chose a procedure to find the break in trend which exploits the SupWald statistic

[6] Dataset collection and decomposition methodology are described at `http://post.economics.harvard.edu/faculty/jorgenson/data/35klem.html`.

[7] Units sold increase until z for that particular industry is reached: after that units sold should be constant or decreasing (for one period) according to our model.

[8] They work fine when we know a priori that two samples come from different populations. If first we have to choose the break point and then run a Chow test, we have to change the test distributions to avoid pre-testing bias.

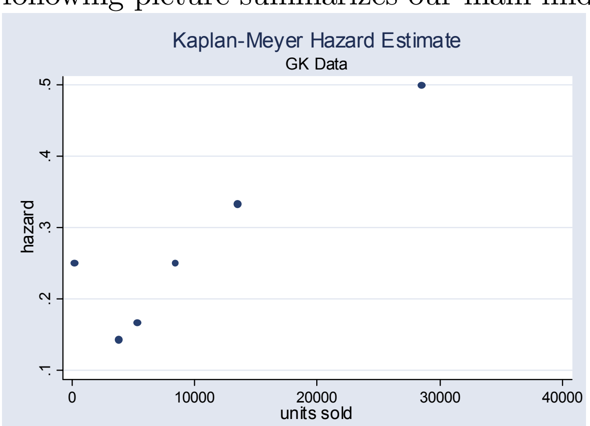
as introduced in [3]. Industries for which such a point exist record a value of $z = 1$ at such level of `real_sales`. Pictures in the appendix report the level in `ureal_sales` time series that discriminant analysis suggest being z for each industry. Of course it might be the case that such point does not exist for some industries: in this case our z-dummy for such industries will assume the value 0 everywhere (meaning industry is a censored observation).

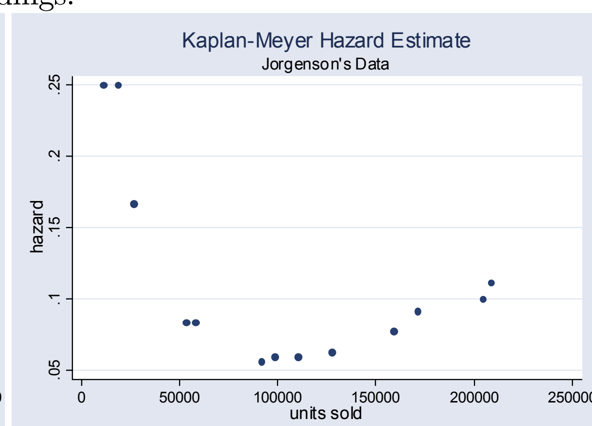
3. At this point we use survival analysis tools to estimate the empirical hazard implied by the {`z`,`real_sales`} series where `real_sales` replaces `time` in ordinary survival analysis while `z` is the ordinary failure dummy.

4. A third issue is **left censoring**. In the GK data, it is claimed that the number of firms in the industry is recorded since the very first introduction of the product. Still time series for units_sold and prices are only collected starting 1943. In Jorgenson's data-set time series are all collected starting 1957, clearly not the first date of introduction of many of the goods represented, so that left censoring is a serious problem. We simply assume that these industries started their life in 1957. We think that the data, used under this assumption, are still a useful way to test our hypothesis that the hazard rate decreases – it should decrease regardless of when (i.e., at what true age) we start estimating it.

5.2 Non-parametric estimates of the hazard

Demand Hazard is non-parametrically estimated using Kaplan-Meyer estimator. The following picture summarizes our main findings:





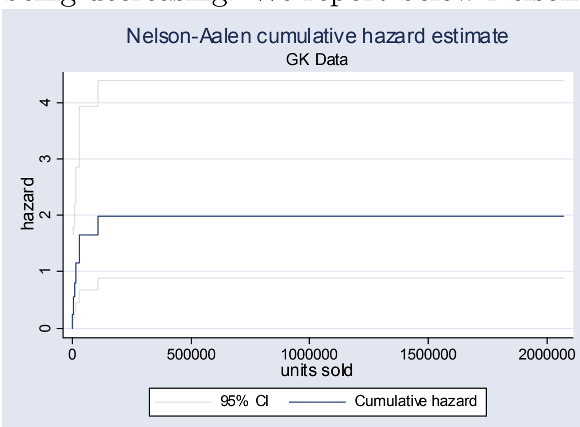
The picture reports Kaplan-Meyer empirical hazard estimate[9] on the entire sample of 16 industries for GK's data and on the entire sample of 35 industries for Jorgenson's

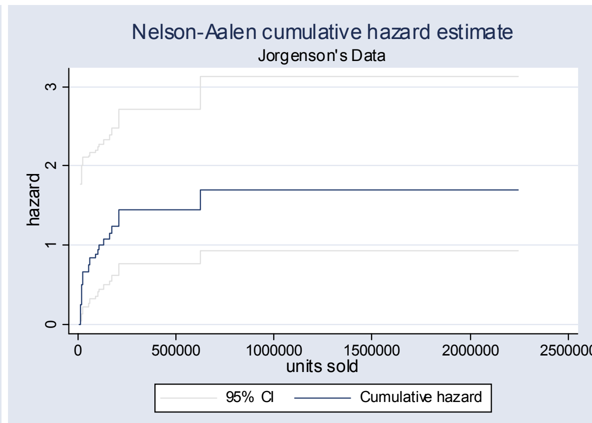
[9] The Nelson-Aalen estimator of the cumulative hazard is defined up to the largest observed time as:

$$\widehat{H}(t) = \sum_{j|t_j \leq t} \frac{d_j}{n_j}$$

where n_t is population alive at t and d_t is number of failures up to time t.

data (note that "analysis time" in duration analysis is `real_sales` according to our procedure). As it is clear from the picture we cannot accept the assumption of a decreasing hazard across the board. A strong non-monotonicity is apparent. The good news is that a consistent part of the hazard is initially decreasing: if the crash happens in this portion of the demand, the predictions of the model could be supported. It is sometimes hard to interpret variation in the hazard: the cumulated hazard is an alternative smoothed version of it. Its being concave suggest the hazard being decreasing. We report below Nelson-Aalen cumulative hazard estimate:





It displays a very "rough concavity", overall. The Jorgenson data reveal a similar hazard and smoothed hazard estimate. The pattern in these data is similar, but the hazard estimate decreases for a longer span and the cumulative hazard is more concave. This is more in line with our model.

6 Robustness and extensions

This section gathers some extensions and robustness checks

6.1 Non-monotone hazard

This subsection shows that if we start with the Pareto-distributed z of Section 3.1, and impose a finite limit, Z, on market size z, (thereby making the hazard rate non-monotonic, rising sharply at $z = Z$) the equilibrium does not change much as long as Z is large. Let

$$\theta \equiv F(Z),$$

The empirical hazard is:

$$h(t_j) \equiv H(t_j) - H(t_{j-1}) = \frac{d_{t_j}}{n_{t_j}}$$

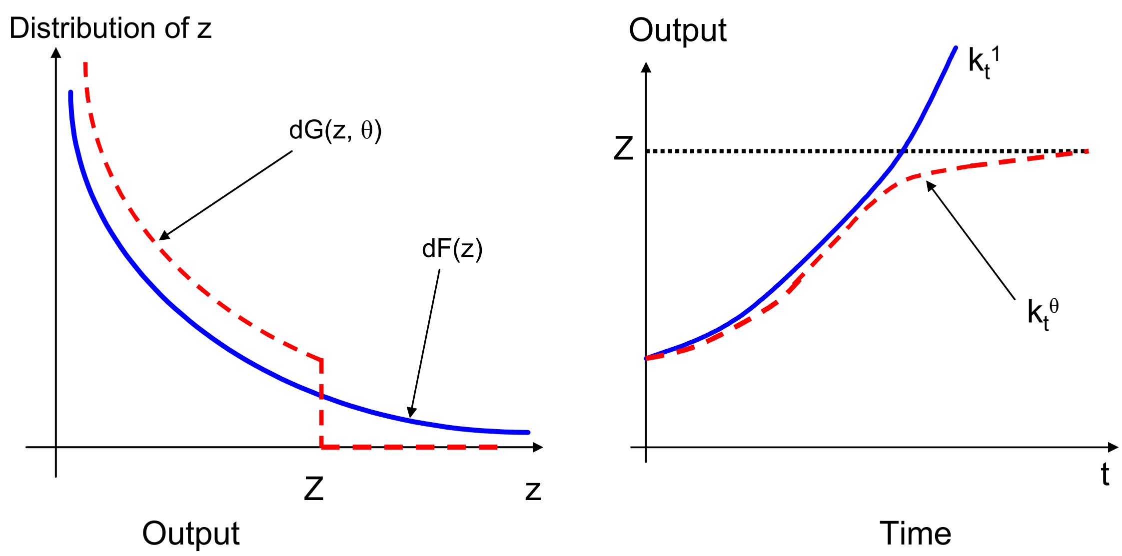


Figure 9: Finite (dashed) vs. infinite (solid) equilibrium

and define the CDF of the new truncated distribution of z as

$$G(z,\theta) = \begin{cases} \frac{1}{\theta} F(z) & \text{for } z \leq F^{-1}(\theta) \\ 1 & \text{for } z > F^{-1}(\theta) \end{cases}$$

Let $\left(k_t^\theta\right)_{t=0}^{\infty}$ be an equilibrium sequence for the distribution $G(z,\theta)$, so that $(k_t^1)_0^\infty$ is the equilibrium sequence of Section 2. In this subsection we shall show that for each t,

$$\lim_{\theta \to 1} k_t^\theta = k_t^1.$$

In this sense, a large finite world in which $z \leq Z$ approximates the infinite world of the Pareto prior over z. The general idea is portrayed in Figure 9.

Substituting G for F in (7), it reads

$$C(n') = -p + C(n) \frac{1}{\beta} \frac{1 - G(k,\theta)}{1 - G(k+n,\theta)}$$

When F is Pareto with $\rho = 1$,

$$G(z,\theta) = \frac{1}{\theta}\left[1 - \left(\frac{z}{z_{\min}}\right)^{-1}\right]$$

for $z < Z$, where

$$Z = \frac{z_{\min}}{1-\theta}.$$

Then (7) reads

$$k'' = k' + \frac{(k'-k)}{\beta}\left(\frac{\theta - 1 + \left(\frac{k}{z_{\min}}\right)^{-\rho}}{\theta - 1 + \left(\frac{k'}{z_{\min}}\right)^{-\rho}}\right) - \frac{p}{c} \equiv \psi\left(k', k, \theta\right) \tag{15}$$

Then if $\left(k_t^{\theta}\right)_{t=0}^{\infty}$ is an equilibrium sequence, it must be obtainable by iterating ψ from the pair $\left(k_0, k_1^{\theta}\right)$. Note that

$$\lim_{\theta \to 1} \psi\left(k', k, \theta\right) = k' + \frac{(k'-k)}{\beta}\left(\frac{k'}{k}\right)^{\rho} - \frac{p}{c} \tag{16}$$

Lemma 5 *For θ sufficiently close to unity,*

$$\frac{\partial}{\partial k'}\left(\frac{k''}{k'}\right) > 0.$$

Proof. At $\theta = 1$,

$$\frac{k''}{k'} = 1 + \frac{\left(1 - \frac{k}{k'}\right)}{\beta}\left(\frac{k'}{k}\right)^{\rho} - \frac{p}{k'c}$$

so that $\frac{\partial}{\partial k'}\left(\frac{k''}{k'}\right) > 0$ at $\theta = 1$. But $\partial^2\psi/\partial k'\partial\theta$ exists in the neighborhood of $\theta = 1$ and the claim follows. ■

Then if k' rises (thereby raising k''/k' as well), it turns out that k'''/k'' will rise too:

Lemma 6 *For θ sufficiently close to unity, $\frac{\partial}{\partial k'}\left(\frac{k''}{k'}\right) > 0$ implies that*

$$\frac{\partial}{\partial k'}\left(\frac{k'''}{k''}\right) > 0.$$

Proof. At $\theta = 1$,

$$\frac{k'''}{k''} = 1 + \frac{\left(1 - \frac{k'}{k''}\right)}{\beta}\left(\frac{k''}{k'}\right)^{\rho} - \frac{p}{k''c}$$

so that $\frac{\partial}{\partial k'}\left(\frac{k'''}{k''}\right) > 0$ at $\theta = 1$. Since the cross-partial derivatives exist in the neighborhood of $\theta = 1$, the claim follows. ■

Corollary 7 *For θ sufficiently close to unity,*

$$\frac{\partial k_t^{\theta}}{\partial k_1} > 0$$

Proof. For any t, Lemmas 5 and 6 imply that a simultaneous rise in k_1 will raise k_2/k_1 which, in turn, implies a rise in k_3/k_2 and so on. This proves the claim. ■

Proposition 8 *For each t,*

$$\lim_{\theta\to\infty} k_t^\theta = k_t^1.$$

Proof. The proof is in four steps.

(i) ψ is continuous on the set $A^\theta = \{(k,k') \mid k \le k' \text{ and } F(k') < \theta.\}$.

(ii) If (k,k') are continuous in θ, so is k''. This follows from (i)

(iii) If n_0 is continuous in θ, so is k_t for any t, as long $(k_{t-1},k_t) \in A^\theta$. This is because k_0 is fixed and in because we can iterate the result in (ii) using ψ.

(iv) n_0^θ is continuous in θ at $\theta = 1$. Suppose not. Then n_0^θ would jump at $\theta = 1$. Suppose the jump was positive. Then k_1/k_0 would jump up. Then by Lemmas 5 and 6 and Corollary 7, k_t would jump up for each t. But then (4) (which must hold at $\theta = 1$) would fail to hold at some $\theta < 1$. To see why re-write it as

$$cn_0^\theta = p\sum_{t=0}^{\infty}\beta^t\left(\frac{1-G\left(k_t^\theta,\theta\right)}{1-G\left(k_0^\theta,\theta\right)}\right) \tag{17}$$

The LHS of (17) would jump up. But the RHS is continuous in θ and, as the entire (k_t) sequence jumps up, the RHS would exhibit a downward jump, a contradiction. Similar logic works if n_0^θ has a negative jump.

Putting $(i)-(iv)$ together proves the claim. ■

Via (5), this result implies that stock prices, v_t, and industry output, k_t, also converge to the equilibrium we described in Section 3.1. Thus the main thrust of the results of Section 3 holds up in finite worlds in which the demand hazard eventually starts to rise.

6.2 Contagion upstream

We have so far modelled a single industry and applied it to the Telecom sector. The Telecom sector is a part of the Nasdaq, yet the entire Nasdaq crashed. This section shows how the crash in one sector can spread to an upstream sector. Assume that k is produced competitively by a fixed number μ of firms in the upstream industry. For a give capital-goods firm the cost of producing x units of capital is $g(x)$, where $g' > 0$ and $g'' > 0$. Consider the symmetric situation in which every capital-goods firm produces the same amount, x. Suppose that k is purchased only by one industry. That is, the only downstream buyers of k are in the industry we have modelled in the previous sections. Equilibrium then requires that

$$n = \mu x.$$

The price per unit of capital is $C(n)$, where n is investment in the downstream industry. The first-order condition for optimal production of k is

$$C(n) = g'\left(\frac{n}{\mu}\right)$$

Therefore the market value of each capital-good producer is

$$V_{k,t} = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[C(n_\tau) - g\left(\frac{n_\tau}{\mu}\right) \right].$$

and the value of the industry is $\mu V_{k,t}$. Now $C(n) - g\left(\frac{n}{\mu}\right) = g'\left(\frac{n}{\mu}\right) - g\left(\frac{n}{\mu}\right)$ is increasing in n.

Simultaneous crashes upstream and downstream.—As n rises, so does the value of the upstream producers. When the downstream industry crashes, so does the upstream industry. When n_t permanently falls to zero, $V_{k,t}$ does too. If the industry supplies capital to more than one final-good industry, then the the equilibrium condition changes; $V_{k,t}$ would still fall, but not to zero.

7 Conclusion

This paper has linked the phenomenon of market crashes and excess capacity. As Figure 2 shows, Telecom is not the only industry where stock prices and output both fell suddenly. Other markets, including the market for PC, unexpectedly became "saturated" making further investment unprofitable (Wall Street Journal 2000). Yet these were the very markets where, prior to the crash, there was a stock-price run-up.

The driving force behind the price run-up was, we argue, a rise in optimism. In our model, this rising optimism in the model is fully rational, provided that prior beliefs over market size have a decreasing hazard. Cross-industry evidence favoring such hazards, however, is not conclusive. For now, all we can claim is that we have explained how "saturation" can develop and cause a market to crash, and explored how to go about judging whether this mechanism is important.

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8 Appendix A: NASDAQ index formulas

The section explains how we arrived at Figures 1, 2, and so on. Some definitions first:

- The **market capitalization** is obtained by multiplying the number of shares in issue by the mid price.
- The **mid price** of a security is obtained by taking the average between the best bid price and the best offer price available on the market during normal business hours.
- The **number of shares outstanding** are used to calculate the market capitalization for each component of the index. These shares represent capital invested by the firm's shareholders and owners, and may be all or only a portion of the number of shares authorized[10].
- **Constituent** is any firm listed on NASDAQ

The Nasdaq Composite Index is weighted arithmetically where the weights are the market capitalizations of its constituents. The index is the summation of the market values (or capitalizations) of all companies within the index and each constituent company is weighted by its market value (shares in issue multiplied by the mid price). The formula used for calculating the index is straightforward. However, determining the capitalization of each constituent company and calculating the capitalization adjustments to the index are more complex. The index value itself is simply a number which represents the total market value of all companies within the index at a particular point in time compared to a comparable calculation at the starting point. The daily index value is calculated by dividing the total market value of all constituent companies by a number called the divisor. The divisor is then adjusted when changes in capitalization occur to the constituents of the index (see Revision of the Divisor) allowing the index value to remain comparable over time.

$$I_t = I_0 \frac{total\, market\, value_t}{divisor_t} = I_0 \frac{\sum_{i=1}^{N_t} P_{it} S_{it}}{D_t}$$

where t is the date at which we want to calculate the index I, $t = 0$ is a reference date or base date we start with (like February 1971 for the composite index which is

[10]Shares that have been issued and subsequently repurchased by the company for cancellation are called treasury shares, because they are held in the corporate treasury pending reissue or retirement. Treasury shares are legally issued but are not considered outstanding for purposes of voting, dividends, or earnings per share calculation. Shares authorised but not yet issued are called un-issued shares. Most companies show the amount of authorised, issued and outstanding, and treasury shares in the capital section of their annual reports. It is possible to back out the total number of outstanding shares of each company from the balance sheet. In COMPUSTAT it is possible to obtain market capitalization by using the following DATA items: (DATA6+DATA199*DATA25-DATA60)

set to 100) P_{it} is the price of a share of company i at date t, S_{it} is the total number of shares outstanding for company i at date t and D_t is a divisor, introduced to make the index comparable over time (basically keeps track of changing in the pool of firms or their share policies and allows the composite index only to track growth rates over periods) and defined below:

$$D_t = \sum_{i=1}^{N_0} P_{i0} S_{i0} + \sum_{j=1}^{t} G_{j-1} \frac{I_0}{I_{j-1}}$$

where G_{j-1} is net new money raised at time $j-1$ through the issue of new companies, new shares, rights issues, capital reorganizations or even capital repayments. This figure may be negative. If G is zero between periods the index boils down to:

$$I_t = \frac{total\, market\, value_t}{total\, market\, value_0} I_0 = I_0 \sum_{i=1}^{N_t} \frac{P_{it} S_{it}}{\sum_{i=1}^{N_0} P_{i0} S_{i0}}$$

9 Appendix B: Existence, uniqueness and simulation of solutions to (13), (14)

The pair of difference equations [(13), (1)] in (n, k) has no finite steady state for k. In order to be able to linearize around a steady state, we change variables from k, the level of capacity, to its rate of growth

$$x = \frac{n}{k}.$$

We shall now analyze the evolution of the pair (n, x). The change of variables transforms the pair (13) and (1) into the following pair of difference equations

$$n' = -\frac{p}{c} + n\frac{1}{\beta}(1+x)^{\rho}, \tag{18}$$

$$x' = \frac{x}{(1+x)}\left[-\frac{p}{cn} + \frac{1}{\beta}(1+x)^{\rho}\right]. \tag{19}$$

Lemma 9 *(13) and (1) are equivalent to (18) and (19).*

Proof. In the law of motion for k,

$$k' = k + n,$$

divide by n' to obtain

$$\begin{aligned}\frac{k'}{n'} &= \frac{k}{n'}+\frac{n}{n'} \\ &= \frac{k}{n}\frac{n}{n'}+\frac{n}{n'} \\ &= \left(1+\frac{k}{n}\right)\frac{n}{n'}.\end{aligned}$$

Inverting both sides,

$$\begin{aligned}\frac{n'}{k'} &= \frac{n'}{n}\frac{1}{\left(1+\frac{k}{n}\right)} \\ &= \frac{1}{\left(1+\frac{k}{n}\right)n}\left[-\frac{p}{c}+n\frac{1}{\beta}\left(1+\frac{n}{k}\right)^{\rho}\right] \\ &= \frac{1}{\left(1+\frac{k}{n}\right)}\left[-\frac{p}{cn}+\frac{1}{\beta}\left(1+\frac{n}{k}\right)^{\rho}\right] \\ &= \frac{x}{(1+x)}\left[-\frac{p}{cn}+\frac{1}{\beta}(1+x)^{\rho}\right]\end{aligned}$$

Therefore (13) and (1) are equivalent to (18) and (19). ■

These equations have the unique steady state. Now the steady state of the system is

$$\begin{pmatrix} n \\ x \end{pmatrix} = \begin{pmatrix} \frac{\beta p}{(1-\beta)c} \\ 0 \end{pmatrix}$$

So let us linearize around it. The Jacobian evaluated at the steady state is

$$\begin{bmatrix} \frac{1}{\beta} & \frac{p\rho}{(1-\beta)c} \\ 0 & 1 \end{bmatrix}$$

The characteristic roots are $\left(\frac{1}{\beta},1\right)$. As is standard we set $n'=n$ and $x'=x$ to find two curves crossing in the steady state

$$n'=n \Longrightarrow n=\frac{p}{c}\left(\frac{\beta}{[1+x]^{\rho}-\beta}\right)\equiv\Phi(x)$$

$$x'=x \Longrightarrow n=\frac{p\beta}{c}\left(\frac{1}{[1+x]^{\rho}-\beta(1+x)}\right)\equiv\Psi(x)$$

These are the two demarcation curves in the phase diagram, and they cross at the steady state. Both are downward sloping (at least if $\rho=1$) the ratio

$$\frac{\Psi(x)}{\Phi(x)}=\frac{[1+x]^{\rho}-\beta}{[1+x]^{\rho}-\beta-\beta x}=\begin{cases} >1 & \text{if } x>0 \\ =1 & \text{if } x=0 \\ <1 & \text{if } x<0 \end{cases}.$$

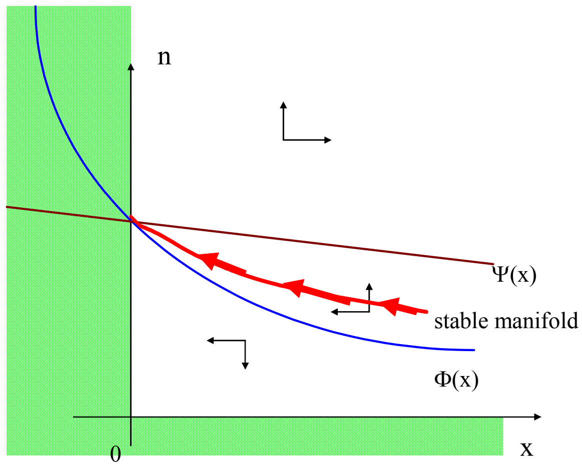


Figure 10: PHASE DIAGRAM FOR n AND $x = \frac{n}{k}$.

So, $\Psi(x)$ is steeper than $\Phi(x)$, and the two curves cross at the steady state, as shown in Figure[11] 10. The area where either $n < 0$ or $x < 0$ is not relevant for the pre-overshooting stage of the game, hence it is shaded.

To be able to draw the typical arrows on the phase diagram rewrite the system as

$$\begin{aligned} n' &= -\frac{p}{c} + n\frac{1}{\beta}(1+x)^{\rho} \equiv A(x,n), \\ x' &= \frac{x}{(1+x)}\left[-\frac{p}{cn} + \frac{1}{\beta}(1+x)^{\rho}\right] \equiv B(x,n). \end{aligned}$$

Then,

$$A(x, \Phi[x]) = n \tag{20}$$

and

$$B(x, \Psi[x]) = x \tag{21}$$

The vertical arrows.—First we show that if $n > \Phi(x)$, we move even higher. And the opposite if $n < \Phi(x)$. That is,

Claim 10

$$n \gtrless \Phi(x) \Longrightarrow A(x,n) \gtrless n.$$

Proof. The relevant portion of the phase diagram is that for $x \geq 0$. For all such x,

$$\frac{\partial A(x,n)}{\partial n} \geq \frac{1}{\beta} > 1$$

[11] Mathematica was used to draw the stable manifold.

Together with (20) the claim follows. ■

The horizontal arrows.—Next we show that if $n > \Psi(x)$, we move to the right. And if $n < \Psi(x)$, we move to the left. That is,

Claim 11

$$n \gtreqless \Psi(x) \Longrightarrow B(x, n) \gtreqless x.$$

Proof. We have

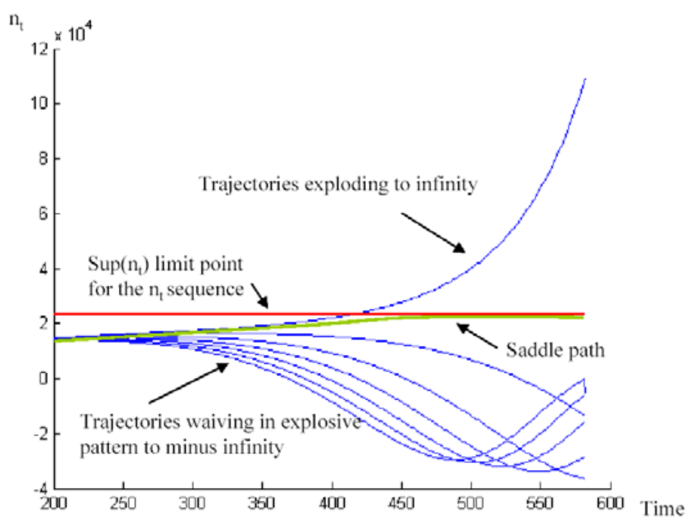
$$\frac{\partial B(x,n)}{\partial n} = \frac{x}{(1+x)} \frac{p}{cn^2} > 0$$

if $x > 0$. Together with (21) the claim follows. ■

These two claims pin down the arrows that are displayed in Figure 10 along with the saddle path. The evolution of n and x is valid only before the overshooting date. The shaded area is not admissible because x cannot be negative.

9.0.1 Simulating the Model

In the figure below we plot the time path for n. This figure shows in greater detail what would happen to the trajectory if it did not start on the saddle path.



Equilibrium time path for n_t as it approaches n_∞

We now describe the algorithm we used to simulate the model:

1. Choose an initial condition for k_0
2. Posit a rather coarse initial grid for initial values n_0
3. For each n_0 from the grid simulate the system equations to generate values for the sequences $\{n, k\}_t$ for many periods

4. Discard any n_0 for which the n_t sequence either explodes, or is not not monotone increasing

5. Once all the n_0 in the initial grid are tried we choose the n_0 with the longest number of observations (before explosion or violation of monotonicity cut off the simulation) and make up a *finer* grid around it

6. The process starts over at point 3 up to the point we can find an initial condition that can generate a monotone sequence for "enough" periods.

The simulations portrayed in Section 3.1 plot the time-paths simulated n_t and k_t against their observed proxies: the actual Telecom sales series used as a proxy for capital (`sales_total`) and the fist difference of `sales_total` (a proxy for n_t).

10 Appendix C: The Gort-Klepper data

Appendix C reports the names of the products whose time series were used in our estimations as well as their SIC codes. A "?" or xxxx next to a SIC code means that the SIC code for the corresponding industry is not sure or unknown. Unfortunately in GK's and Jorgenson's data set the SIC codes are not reported. The tables report what we could retrieve with confidence. "sub" followed by a 4-digit number, means that the product is below the finest 4-digit industry SIC classification, hence not available in Census data: data are from one or more products making up the corresponding 4-digit level of aggregation data (but not all of them). More than one "sub" implies that data for that product correspond to products making up more than one SIC 4-digit industry.

product_index	product name	SIC
6	computers	sub3571
7	crystal piezo	sub3679
8	ddt	sub2879
9	electrocardiographs	sub3845
10	electric blankets	sub3634
15	freezers home and farm	sub3632
18	lasers	sub3674?
21	missiles guided	sub3761
24	nylon	sub2821/sub2284/sub2824/sub2281/sub2282
26	penicillin	sub2834/sub2833
27	pens ball point	sub3951
40	styrene	sub2821/sub2911 /sub2869
43	tape recording	sub3651
45	television receivers monochrome	sub3651
48	transistors	sub3674
50	tubes cathode ray	sub3671

GK dataset products, codes (used in STATA analysis), and SIC category

In Section 4.1 the time series of the GK data-set are named as follows:

product abbreviation code: `product_index;`

units sold: `units_sold;`
sales: `total_sales`;
price series: `price;`
period of the real business cycle in which the observation is measured: `nber_rbc_code;`
dummy for war periods: `war.`