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R&D? A Small Contribution to Productivity Growth

by

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Abstract

In this paper I calibrate the contribution of R&D investments to productivity growth. The basis for the analysis is the free entry condition. This yields a relationship between the resources devoted to R&D and the growth rate of technology. Since innovators are small, this relationship is not directly affected by the size of the R&D externalities, the presence of scale effects or diminishing returns in R&D after controlling for the growth rate of output and the interest rate. The resulting contribution of R&D to productivity growth in the US is smaller than three to five tenths of one percentage point. Interestingly, this constitutes an upper bound for the case where innovators internalize the consequences of their R&D investments on the cost of conducting future innovations. From a normative perepective, this analysis implies that, if the innovation technology takes the form assumed in the literature, the actual US R&D intensity may be the socially optimal.

Keywords: Research and development, productivity growth, total factor productivity.

JEL Classification: O40, E10.

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1 Introduction

What is the contribution of R&D to the growth of advanced economies? Is it the main factor or does it play just a minor role?

This question has been answered by computing the social return to R&D in a simple econometric framework. Typically, the endogenous variable is the Solow residual and the explanatory variables are the firm's or industry's own R&D intensity and the used R&D from other firms or industries. The estimated return to own R&D ranges from .2 to .5, while for the used R&D the estimate ranges from .4 to .8 with a total social return to R&D of about 70 to 100 percent.[1] These numbers are very large. Indeed, they imply that the Solow residual is fully accounted by R&D alone.

Before reaching this conclusion, we should have in mind an important caveat to this econometric approach. Namely, that there are many factors omitted in the typical regression that affect simultaneously TFP growth and the parties incentives to invest in R&D. The most obvious candidates are anything that enhances disembodied productivity, like the managerial and organizational practices, learning by doing,... All these elements have a clear effect on TFP and at the same time induce firms to invest in R&D. Some evidence in favor of the potential importance of this bias comes from the fact that, after including fixed effects in the regression, the effect of R&D on TFP growth almost disappears (Jones and Williams [1998]).

To overcome this omitted variable bias, I depart from the econometric framework. Instead, I use a model with endogenous development of new technologies to assess the importance of R&D for growth. From a methodological point of view, I do not attempt to calibrate directly the social return to R&D to figure out its role on growth. My route is more indirect because it decomposes the problem into two parts. First, I compute the effect of the amount of resources devoted to R&D on the output of the R&D sector (that is the growth rate of R&D driven technologies). Then, I use simple growth accounting to compute the effect of the growth of technology on productivity growth.

One possible way to establish the first relationship (i.e. between the resources devoted to R&D and the growth rate of technology) is to calibrate the production function of technology. Note that this approach entails probably even more challenges than the traditional productivity approach because in addition to measuring the externalities involved in R&D, we have to specify an R&D production function. I discuss this further below in the context of a specific empirical test.

[1]See Griliches [1992], Jones and Williams [1998] and Nadiri [1992] for references.

The approach I propose in this paper, instead, uses the no arbitrage condition that implies that in equilibrium no innovator is willing to invest more in R&D. This means that the effect of the share of resources devoted to R&D and the growth rate of technology is proportional to the inverse of the market value of innovations. The advantage of using a free entry condition as opposed to the production function is that, since innovators are small, they don't take into account the effect of their investment decisions on the aggregate variables when computing the value of an innovation. Therefore, I can use these observable aggregate variables to establish the effect of the R&D investments on the growth rate of technology without having to take any stand on the size of the R&D spillovers.

The results I obtain are quite striking given the existing consensus about the importance of R&D for growth.[2] The average annual growth rate of productivity in the US during the post-war period has been 2.2 percentage points. Less than 3 to 5 tenths of 1 percentage point are due to R&D.

The intuition for this small contribution is quite simple. The few resources devoted to R&D (about 2 percent of GDP) clearly signal a small private value of the innovations. But, as the bulk of the productivity literature has argued, there may be significant externalities that lead to large productivity gains even with few R&D investments. These externalities can appear in the production of final output or in the R&D process.

Production externalities arise because the improvement of technology has an effect on labor productivity beyond its contribution to the quality adjusted capital stock (i.e. it affects the Solow residual). Innovators can only appropriate the value of their innovation to the extent that it enhances the quality adjusted capital stock. Since innovations are embodied in capital goods, innovations with higher (social) value will be more demanded. Therefore, *ceteris paribus*, the market value of the innovations will be correlated with their social value. In terms of my two-step approach, this means that a large production externality raises the effect of the growth of technology on productivity growth but reduces the growth of technology associated with a given R&D intensity. As a result, the R&D contribution to productivity growth is not very sensitive to the size of the externalities in production.

[2]The only exception to this consensus is the BLS who reports a R&D contribution to Total factor productivity growth of 0.2 percentage points. The difference in the BLS estimates arises primarily from two sources. First, the BLS does not include federally-funded research in the measure of R&D. This is an important omission because from 1953 to 1994, the average share of the nondefense federally funded R&D in total nondefense R&D was 35 percent). Second, the BLS assumes a depreciation rate of 8.8 percent on R&D capital.

R&D externalities associate past R&D investments with a reduction in the cost of developing future innovations. To show the inconsistency of large R&D externalities and a low R&D intensity in steady state, suppose for a moment that the R&D externalities were large and that the economy is in steady state. Then, a small R&D intensity today, can generate a large growth rate of technology that in turn generate a large reduction in the costs of developing innovations tomorrow. As a result, tomorrow, agents want to devote a large share of resources into R&D; but this is inconsistent with the fact that the share of resources devoted to R&D is constant in steady state. Therefore, the observed low R&D intensity indicates that R&D externalities cannot be very large.

The rest of the paper is structured as follows. Section 2 sketches the basic argument. In section 3, I conduct the baseline calibration based on the model presented in Jones and Williams [2000]. This model is quite general and can accommodate idea-based models both with quality ladders and increasing variety of intermediate inputs. One clear goal of this paper is to show that the magnitude of the calibrated R&D contribution to productivity growth is very robust. Section 4 tries to show this. To accomplish this goal, I investigate elements that affect the relationship between the share of resources devoted to R&D and the growth rate of technology. I also consider more general production functions to analyze the effect of technology on productivity growth. In section 4.3, I move out of the steady state and consider how the calibrations would change had the US economy been in transition to the steady state. In section 5, I draw the welfare implications of the previous analysis. Specifically, the free entry condition establishes a relationship between the R&D intensity and the growth rate of technology that can be used to calibrate the size of the R&D externalities. Once this is done, we can solve the social planner's problem. This entails determining how much she would invest in R&D with the calibrated production structure. Then we can compare this socially optimal R&D intensity with the actual intensity and draw the appropriate policy prescriptions. The resulting picture after this journey is that R&D plays an small role in US productivity growth - definitely much smaller than what we thought- and that the observed R&D intensity may not be as far as previous work concluded from the socially optimal intensity.

2 The basic argument

Let's denote by A the level of technology associated with R&D investments. In the terminology of Romer [1990] or Grossman and Helpman [1991, ch. 3], this is the number of capital varieties though I will show later that this framework can accommodate other interpretations. To compute the R&D contribution to productivity growth, I start by investigating the relationship between the amount of resources devoted to R&D (expressed in units of final output, which is the numeraire), R, and the growth rate of technology. Then I use a production function to relate the growth rate of A to the growth rate of labor productivity. Throughout the paper I use $\dot{X}$ to designate the time derivative of variable X, and γ_X to denote the growth rate of variable X.

Let P_A denote the market price of a firm that has earned a patent to produce one of these varieties. The free entry condition implies that innovators make zero profits in equilibrium, therefore the cost incurred to develop the patent (R) is equal to the market value of the flow of new technologies $(P_A\dot{A})$.

$$P_A\dot{A} = R. \tag{Free Entry}$$

The free entry condition can be rewritten as in equation (1), where Y denotes the economy-wide output, s denotes the share of resources devoted to R&D (i.e. $s \equiv \frac{R}{Y}$).

$$P_A\ \gamma_A = s\frac{Y}{A} \tag{1}$$

Successful innovators earn patents that entitle them to charge a markup (η) above the marginal cost of production. As we shall see, the static operating profits earned by an innovator are

$$\pi = \frac{\eta-1}{\eta}\alpha\frac{Y}{A}. \tag{2}$$

To close the first step in the argument, we just have to derive the market price of an innovation. Suppose for simplicity that patents do not expire and that innovators are not taken over by new innovators with more sophisticated capital goods. Then the value of an innovation, P_A, must satisfy the following asset equation:

$$rP_A = \pi + \dot{P}_A, \tag{3}$$

where r is the relevant discount factor.

In steady state, all variables grow at constant rates. From equation (1), this implies that

$$\gamma_{P_A} = \gamma_Y - \gamma_A. \tag{4}$$

Substituting expressions (4), (2) and (1) into equation (3) and isolating γ_A we obtain the following expression for the growth rate of technology in terms of s :

$$\gamma_A = \frac{r-\gamma_Y}{\frac{\eta-1}{\eta}\frac{\alpha}{s}-1} \tag{5}$$

There are two important observations from this expression. First, γ_A in expression (5) does not depend directly on the size of the externalities in R&D or on the degree of the diminishing returns to aggregate R&D investments; note that I have not even specified the production function for technologies. This is the case, because we have mimicked the calculations made by small innovators that want to figure out the market price of their innovations (P_A) and do not take into account the effect of their investment decisions on aggregate variables like the interest rate or the growth rate of output. Since the externalities appear through these aggregate variables, we do not need to calibrate them once we control for γ_Y and r. Second, the quantitative result of the paper comes from the fact that γ_A is increasing in s. The link between these two variables does not come from a production function for technology; it follows from the positive relationship that the free entry condition (1) defines between the two. Quite crucially for my results, since the observed value of s is small (approximately 2 percent), γ_A is also going to be small.

The second step in the computation of the R&D contribution to productivity growth consists in calibrating the effect of the growth rate of R&D driven technology (γ_A) on the growth rate of productivity. For this we need to specify a production function of the form

$$Y = F(Z, A, K, L)$$

where Z is the level of disembodied productivity and K and L denote capital and labor respectively. From here, the contribution of R&D to productivity is

$$\left[\alpha_A + \alpha \frac{\partial K}{\partial A}\right] \gamma_A,$$

where α is the capital share and α_A is the elasticity of Y with respect to A.

It is useful to assign some tentative values to these parameters to make some back to the envelope calculations about the R&D contribution to productivity growth. A conservative value for η is 1.2, α is about 1/3, r is around 0.07, γ_Y has been 0.034 in the post-war period, s is approximately 0.02 and the elasticity of productivity growth with respect to R&D-driven technology growth $\left(i.e.\ \alpha_A + \alpha \frac{\partial K}{\partial A}\right)$ is about 0.1. This implies that γ_A is about 0.02, and $\left[\alpha_A + \alpha \frac{\partial K}{\partial A}\right] \gamma_A$ is about two tenths of one percentage point.

From this analysis, it is quite transparent that my calibration of the size of the R&D contribution to productivity growth is affected by elements that influence the pricing of an innovation, and the elasticity of output with respect to technology. As we shall see in section 3, the markup η contains information about the set of possible values of α_A and $\frac{\partial K}{\partial A}$. Moreover, in the more general production functions studied in section 4.2, I show that there is a trade off between the effect of γ_A on the growth rate of productivity and the effect of s on γ_A. This trade off allows me to extend the results to production functions where the production externality (α_A) is larger without altering the R&D contribution to productivity growth.

The market price of an innovation depends on several variables like the expected life of the patent, the market structure, how innovative the new capital product is, what is the gestation lag before the product is commercialized, the cost advantage obtained by the incumbent in the development of subsequent innovations, etc. In section 4.1, I study all these aspects of the R&D process and set an upper bound for the R&D contribution to productivity growth.

3 Jones and Williams

The baseline model I calibrate is presented in Jones and Williams [2000]. It is a generalization of Romer [1990] and Grossman and Helpman [1991, ch.3]. As in these papers, growth comes from the development of new varieties of intermediate goods. Final output is produced out of labor and intermediate goods. In particular, I assume the functional form in equation (6). This specification, introduces a wedge between the capital share and the elasticity of substitution across different varieties which is equal to $\alpha\rho$.

$$Y_t = Z_t L_t^{1-\alpha} \left(\sum_{i=1}^{A_t} x_{it}^{\alpha\rho} \right)^{\frac{1}{\rho}} \tag{6}$$

Standard profit maximization implies the following inverse demand curve for intermediate goods

$$p_{it} = \alpha L_t^{1-\alpha} \left(\sum_{i=1}^{A_t} x_{it}^{\alpha\rho} \right)^{\frac{1}{\rho}-1} x_{it}^{\alpha\rho-1}.$$

As is commonly assumed in the literature of endogenous technological change, successful developers of intermediate goods are granted infinitely lived patents that allow them to charge a markup (η) over and above the marginal cost of production (r_t).[3] The size of this mark up depends on the specific assumptions made about the degree of substitutability between different intermediate goods. Following Jones and Williams [2000], I introduce the concept of innovation clusters to model the

[3] The calibrated effect of R&D on growth is independent of the rate of transformation between final output and intermediate goods. This parameter that here is normalized to 1 just cancels out.

idea that there is some overlap between new and existing innovations that causes the obsolescence of the latter. Say, out of every $(\psi_N + \psi)$ intermediate goods developed, only ψ_N are completely new. The rest are just new versions of existing intermediate goods that are necessary to use the new intermediate good. These new versions are otherwise identical to the existing ones.[4] Note that this mechanism is similar to the Schumpeterian process of *creative destruction* emphasized by Aghion and Howitt [1992] and Grossman and Helpman [1991, chapter 4] and therefore limits the expected life span of the innovation.

When a new technological cluster is developed, the incumbents try to prevent the diffusion of the new technological cluster by reducing their prices. Two scenarios are possible here. It may be the case that these reactions do not constrain the pricing decisions of the innovators. Then the innovator charges the monopolist price $p = \eta^m r_t$, where $\eta^m = (\alpha\rho)^{-1}$. Alternatively, the limit pricing rule may be binding. Since in reality we observe that new products are developed and adopted I focus on the equilibrium where new intermediate goods are immediately adopted.[5] After imposing this restriction we can derive the limit pricing rule that is consistent with the adoption of new varieties. Jones and Williams [2000], show that the markup under limit pricing is

$$\eta^L = \left(\frac{\psi_N}{\psi} + 1\right)^{\frac{1}{\alpha\rho}-1}.$$

Intuitively, the higher the ratio of the number of new goods to the number of complementary goods that must be changed to use the new innovation ($\frac{\psi_N}{\psi}$), the lower is the limit markup because more incumbents are willing to reduce their prices to prevent adoption. Quite naturally, the limit price is also decreasing in the elasticity of substitution across intermediate goods. The resulting price for intermediate goods is $p_{it} = \eta r_t$ where the markup is the minimum of η^m and η^L.

$$\eta = \min \left\{ \overbrace{\frac{1}{\alpha\rho}}^{\text{monopolistic markup}}, \overbrace{\left(1 + \frac{\psi_N}{\psi}\right)^{\frac{1}{\alpha\rho}-1}}^{\text{limit pricing markup}} \right\}$$

[4]A simple example that illustrates this concept is a CD writer. Before the CD writer was developed, we just had a CD reader and a software for this to work. Now with the CD writer, we must modify the CD reader's software to make possible the interaction between the two drives. In this case, $\psi = 1$ (the software) and $\psi_N = 1$ (the CD writer).

[5]If this is not the case, there is no reason to undertake R&D investments.

Given this pricing behavior, the instantaneous profits of an innovator are:

$$\pi = \left(\frac{\eta - 1}{\eta}\right) \alpha \frac{Y}{A} \tag{7}$$

3.1 R&D technology

The R&D sector uses final output to produce new designs for intermediate goods. The production of designs considered here captures three interesting elements. First, because of the innovation clusters defined above, only a fraction $\frac{\psi_N}{\psi_N + \psi}$ of the designs corresponds to new varieties. Second, either by randomness or because of patent races, there may be a duplication of R&D effort. This *stepping on toes effect* is captured by $\lambda \in (0, 1]$ in equation (8). Finally, there are some spillovers from past innovators to the current ones. On the one hand, new varieties are easier to develop because their designs take advantage of the knowledge created by previous researchers (*standing on the shoulders effect*). On the other, there may be diminishing technological opportunities that make harder to develop successive varieties (*congestion effect*). If the standing on shoulders effects dominates, $\phi > 0$, otherwise $\phi < 0$. From the point of view of the atomistic researchers, there are constant returns to the resources devoted to R&D (R). This means that they perceive a marginal product equal to the average product over all the R&D firms. This is represented by $\tilde{\delta}$ in equation (8).

$$\frac{(\psi_N + \psi)}{\psi_N} \dot{A} = \tilde{\delta} R \equiv \delta R^{\lambda} A^{\phi} \tag{8}$$

Note that this specification accommodates both deterministic and stochastic technologies for the production of new varieties. Indeed, equation (8) is isomorphic to a quality ladder model where $\tilde{\delta}$ is interpreted as the probability of being successful and ψ_N is the size of the step in a quality ladder model á la Aghion and Howitt [1992] and Grossman and Helpman [1991, chapter 4].

Equation (8) can be rewritten as

$$\frac{(\psi_N + \psi)}{\psi_N} \gamma_A A = \tilde{\delta} s Y \equiv \delta \left(sY\right)^{\lambda} A^{\phi}. \tag{9}$$

From the R&D technology (9) it follows that the effect of the intensity of R&D investment (s) on γ_A depends on the size of intertemporal spillovers in R&D (ϕ) and on the degree of diminishing returns in the production of varieties (λ). One way to assess the role of R&D in productivity growth consists in calibrating the effect of s on γ_A from (9), and then use the production function (6) to relate γ_A and productivity growth. The main problems with this approach are that it is sensitive to the particular functional form assumed in (9) and that it is difficult to assess the magnitudes of λ and ϕ.[6] Therefore, it is convenient to find an alternative route that avoids the calibration of λ and ϕ. This shortcut comes from the free entry condition.

3.2 Free entry

If innovators are large, they internalize the intertemporal effects of their current R&D investments and the aggregate (static) diminishing returns to R&D. When innovators are small, they take as given the cost of developing a new product and neglect any externality from their investment. In this scenario, free entry brings down the value of innovations to the up front cost of development. Let's denote by P_A the market value of an innovation. Then, the equilibrium level of resources devoted to R&D is given by equation (10).

$$P_A \frac{\psi_N + \psi}{\psi_N} \dot{A} = R \tag{10}$$

Since innovations are priced in the market, P_A must satisfy an asset equation. This means that any difference between the opportunity cost of an innovation and the sum of its profit flow plus the capital gain must be arbitraged away. More formally,

$$\begin{aligned} \overbrace{rP_A}^{\text{opportunity cost}} &= \overbrace{\pi}^{\text{profit flow}} + \overbrace{\dot{P}_A - \frac{\psi}{\psi_N}\gamma_A P_A}^{\text{capital gain}} \\ r &= \frac{\pi}{P_A} + \gamma_{P_A} - \frac{\psi}{\psi_N}\gamma_A \end{aligned} \tag{11}$$

where r is the interest rate faced by innovators, $\dot{P}_A$ is the increase in the market value of the design and $\frac{\psi}{\psi_N}\gamma_A$ is the expected loss from being replaced by another innovator.

Equation (10) can be rewritten as:

$$P_A = \frac{s\psi_N}{\gamma_A (\psi_N + \psi)} \frac{Y}{A} \tag{12}$$

Using equations (7) and (12) we solve for the profit rate, and from equation (12), we can derive an expression for the growth rate of P_A in steady state.

$$\frac{\pi}{P_A} = \frac{\left(\frac{\eta-1}{\eta}\right) \alpha (\psi_N + \psi) \gamma_A}{s\ \psi_N}$$

[6] This is precisely one of the main problems with the econometric attempts to compute the social return to R&D.

$$\gamma_{P_A} = \gamma_Y - \gamma_A$$

Plugging this back into (11) we can solve for the growth rate of varieties (γ_A).

$$r = \overbrace{\frac{\left(\frac{\eta-1}{\eta}\right)\alpha\left(\psi_N+\psi\right)}{s\;\psi_N}\gamma_A}^{\text{profit rate}} + \overbrace{\gamma_Y - (1+\frac{\psi}{\psi_N})\gamma_A}^{\text{capital gain rate}} \tag{13}$$

$$\gamma_A = \frac{r-\gamma_Y}{\left(\frac{\left(\frac{\eta-1}{\eta}\right)\alpha}{s}-1\right)\left(1+\frac{\psi}{\psi_N}\right)} \tag{14}$$

Now we can easily solve for the growth rate of productivity and use this expression to figure out the contribution of R&D to productivity growth. From the production function,

$$\gamma_{Y/L} \equiv \gamma_Y - \gamma_L = \gamma_Z + \frac{1}{\rho}\gamma_A + \alpha\left(\gamma_x - \gamma_L\right) \tag{15}$$

To solve for γ_x, I take advantage of the symmetry of the intermediate goods in production. From the pricing rule and the inverse demand function, it follows then that

$$x_i = L\left(\alpha Z\frac{A^{\frac{1-\rho}{\rho}}}{\eta r}\right)^{\frac{1}{1-\alpha}}.$$

In steady state r is constant and therefore $\gamma_x - \gamma_L = \frac{1-\rho}{\rho(1-\alpha)}\gamma_A$. Plugging this back into (15), we obtain the following expression for the growth rate of productivity:

$$\gamma_{Y/L} = \frac{1}{1-\alpha}\gamma_Z + \overbrace{\frac{1}{\rho}\left(\frac{1-\alpha\rho}{1-\alpha}\right)\gamma_A}^{\text{Contribution of R\&D to productivity growth}} \tag{16}$$

Before calibrating the R&D contribution to productivity growth it is worthwhile making a few remarks. The specification of the production function for new technologies does not affect the calibration of the role of R&D on productivity growth. In particular, the size of the intertemporal externalities, ϕ, and the returns to scale in the production of varieties, λ, do affect the relationship between s and γ_A after controlling for r and γ_Y. This is the case because the agents are small and do not internalize the effect of their investment decisions on these aggregate variables. However, expression (13) restricts the values of ϕ and λ. To see this, note that in steady state, equation (9) implies that

$$\gamma_A = \frac{\lambda}{1-\phi}\gamma_Y.$$

Therefore, a low γ_A implies that neither λ nor ϕ cannot be very large. In section 5, I take advantage of this observation to compute the socially optimal R&D intensity and explore whether there is room for a more active R&D policy.

If $\phi = 1$,[7] this economy displays scale effects (i.e. the growth rate of output per capita is increasing in the size of the economy). Again, this is something that small innovators do not incorporate in their investing decisions, therefore the presence or not of scale effects does not affect the calibration results.

Note also that, since the baseline model is isomorphic to a quality ladder model, the R&D contribution to productivity growth that I compute next is independent on the actual structure of the R&D processes.

3.3 Quantitative Analysis

To assess the role of R&D in productivity growth we must calibrate seven parameters.

Table 1: Parameters

r	0.07
γ_Y	0.034
α	0.33
s	0.02
η	[1.1, 1.5]

r is calibrated to the average real stock return in the US post-war period from Mehra and Prescott [1985].[8] Pakes and Schankerman [1984] provide evidence that this is approximately the private rate of return to R&D once we take into account the obsolescence of patents and the gestation lags that I incorporate in the next section. The average real growth rate of output (γ_Y) between 1950 and 1999 in the US reported by the BLS is 0.034. The capital share (α) is calibrated to $\frac{1}{3}$. The markup (η) is calibrated with values in the interval [1.2, 1.5] that lies in the intervals given by Basu [1996] and Norrbin [1993] for the average markup in the economy. The lower values of this interval are slightly higher than the lower bound in Basu [1996] and Norrbin [1993] because I want

[7]This is completely ruled out both by the previous comment and by the existence of a steady state.

[8]The average real return in corporate bonds with Baa rating since 1976 is 0.066.

to calibrate the markup charged by an innovator. This is probably higher than the average markup in the economy because of the monopolistic power conferred by the patent system and because the higher ratio of the up front fix cost to the marginal cost of production for technological goods than for non-technological goods or services.

R&D is comprised of basic research, applied research, and development. Raw data on R&D expenditures derive from four NSF/SRS surveys: Research and Development in Industry, Academic Research and Development Expenditures, Federal Funds for Research and Development, and Survey of R&D Funding & Performance by Nonprofit Organizations. Funds used for R&D refer to current operating costs. These costs consist of both direct and indirect costs. They include not only salaries, but also fringe benefits, materials, supplies, and overhead. The R&D costs also include the depreciation of the capital stock employed in R&D activities. Figure 1 plots the evolution of the share of US non-defense R&D expenditures in the US GDP. In my calibrations, I use a value for s of 0.02, which is an upper bound for the average share of resources devoted to non-defense R&D during the post-war period in the US.[9]

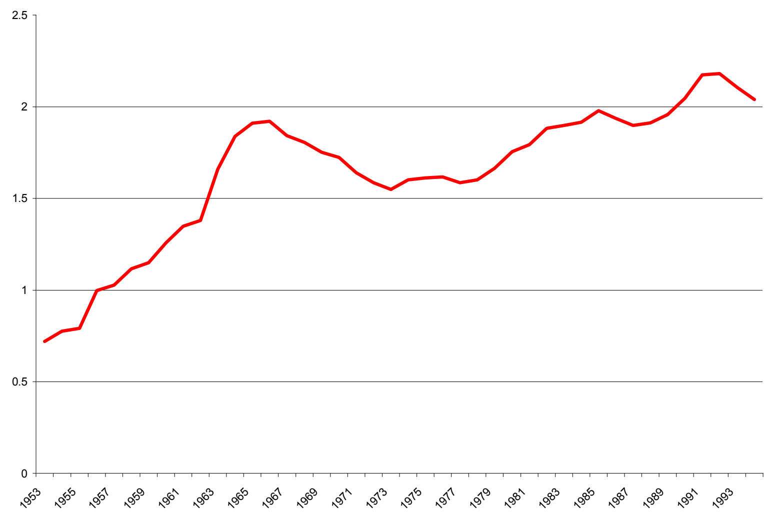

Figure 1: US share of non-defense R&D expenditures in GDP in percentage points. Source: NSF.

I do not attempt to calibrate $\frac{\psi}{\psi_N}$, though this can be done by using data on the average life span of a patent or, when more sophisticated concepts of firms are introduced, on the average life span of a firm or on the average number of patents held by an innovator. To calibrate ρ, I exploit the relationship between these variables and the markup. There are two cases depending on whether

[9]In the next section I consider the possibility of international spillovers and then I take into account that some of the R&D resources invested in other countries may generate growth in the US.

the innovator can charge the monopolistic markup or whether she is forced to use the limit pricing rule.

Case 1: If $\eta = \frac{1}{\alpha\rho}$ then $\frac{1}{\rho} = \eta\alpha$. Figure 2 plots the contribution of R&D to productivity growth for different values of η and $\frac{\psi}{\psi_N}$.

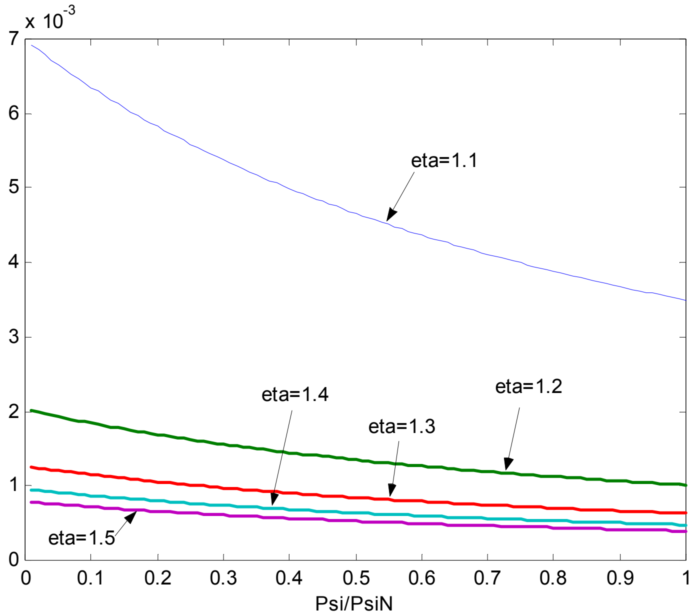

Figure 2: Contribution of R&D to productivity growth, monopolistic pricing.

In the figure we can see that under monopolistic pricing the contribution of R&D to growth is smaller than two tenths of one percentage point for the reasonable markups.[10]

Case 2: If $\eta = \left(1 + \frac{\psi_N}{\psi}\right)^{\frac{1}{\rho\alpha}-1}$ then

$$\rho = \left[\alpha\left(1 + \frac{\ln(\eta)}{\ln\left(1+\frac{\psi_N}{\psi}\right)}\right)\right]^{-1} \tag{17}$$

[10]For smaller markups like $\eta = 1.1$, the contribution of R&D to productivity growth is bounded above by seven tenths of a percentage point.

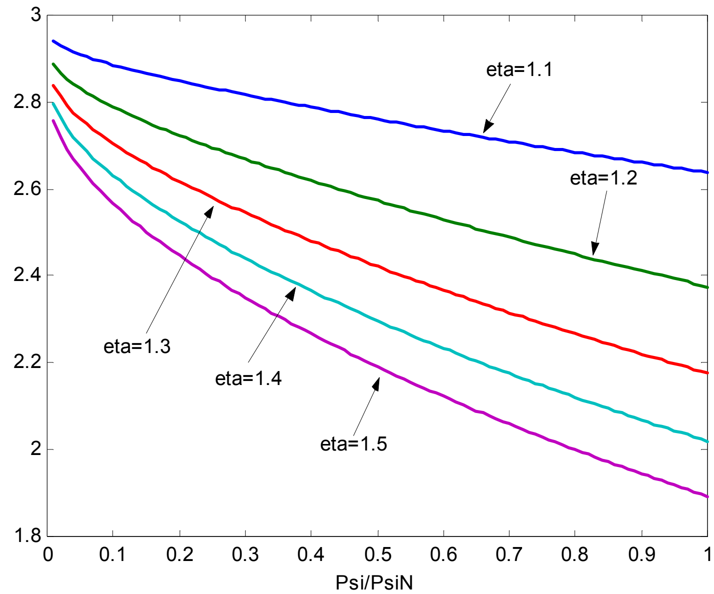

Figure 3: $\rho(\psi/\psi_N|\eta)$

Figure 3, plots the relationship between ρ and ψ/ψ_N implied by the limit pricing rule for several values of η. Figure 4, plots the contribution of R&D to productivity growth as a function of ψ/ψ_N under limit pricing for different markups.

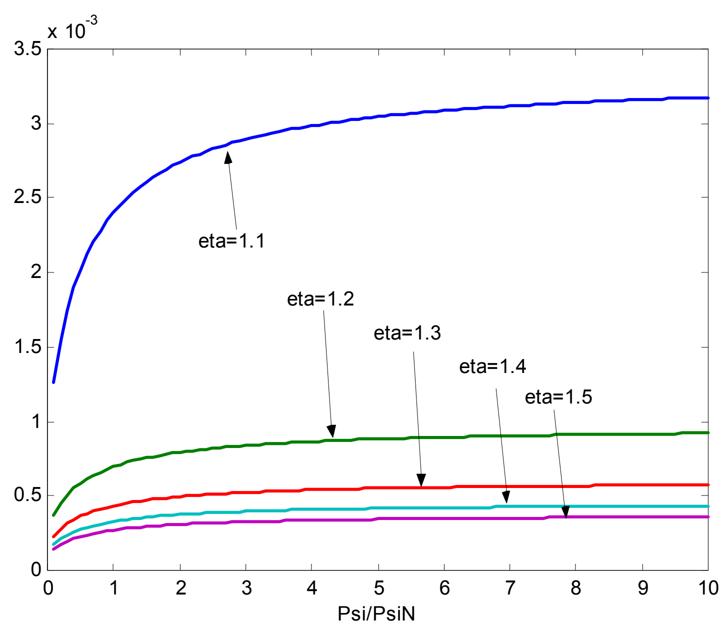

Figure 4: R&D contribution to productivity growth under limit pricing.

In this case, the upper bound of the contribution of R&D to productivity growth is one tenth of a percentage point for the more reasonable markups.[11] This necessarily implies that the externalities in the R&D process are not as large as the productivity literature has estimated. Note that this is something that one could start wondering even before starting the calibration exercise, just by looking at the small fraction of resources devoted to R&D. In the presence of externalities, current innovators can affect future's innovators incentives to undertake R&D. If these are very large, future R&D investments should be very large. But, in steady state, the share of R&D investments in GDP is constant, therefore the current R&D share in GDP should also be large in the presence of very significant externalities.

Before enriching the basic model to incorporate other important aspects of the R&D process and more general production functions, it is interesting to assess the robustness of the computed R&D contribution to productivity to the calibration of the interest rate (r) and the R&D intensity (s). For this I calibrate these parameters to levels substantially higher than the ones I consider most reasonable. In particular, I increase the real interest rate by three additional percentage points (until ten percent) and the R&D intensity by fifty percent (until three percent of GDP). Both under monopolistic (figure 5) and limit pricing (figure 6), we can see that the R&D contribution is still quite small for most of the possible values of ψ/ψ_N and for most of the markups.

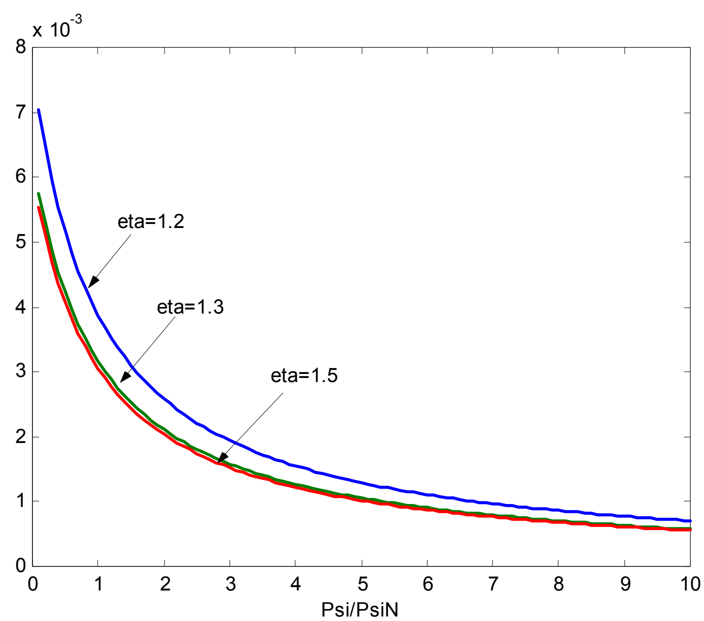

Figure 5: R&D contribution under monopolistic pricing, $r = 0.1$, $s = 0.03$.

[11]For $\eta = 1.1$, the upper bound is about three tenths of a percentage point.

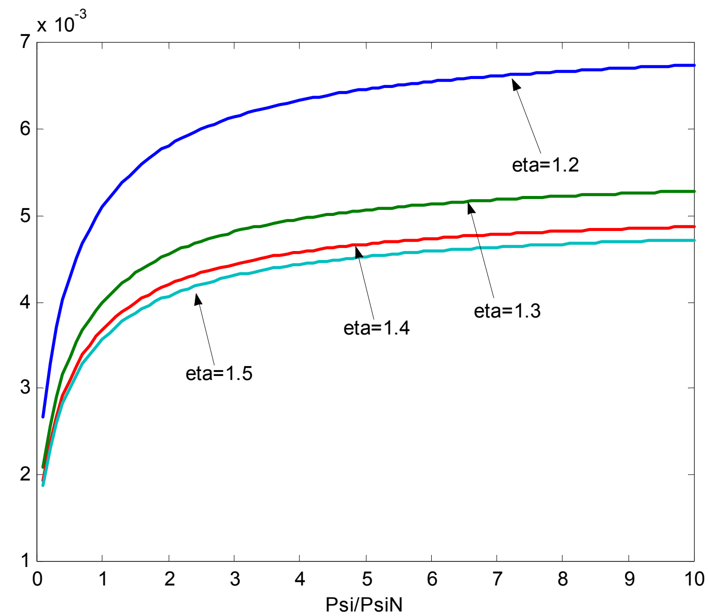

Figure 6: R&D contribution under limit pricing, $r = 0.1$, $s = 0.03$.

4 Extensions

Now, I extend the baseline model along several dimensions to show that the size of the R&D contribution to productivity growth is robust. The first group of extensions deals with considerations that affect the private value of an innovation. These include the presence of international spillovers in R&D investments, R&D lags, more drastic obsolescence processes and the possibility of successive R&D for incumbent firms. The second type of extensions generalizes the production function to capture more general externalities in the production of final output. Then, I relax the assumption that the economy is in steady state and compute the R&D contribution to productivity growth if the US economy had bee in transition during the post-war period.

4.1 The value of innovations

In the free entry condition, equation (12), we can see that the relationship between the share of resources devoted to R&D and the growth rate of technology is mediated by the value of innovations. Therefore, a good way to start analyzing the robustness of the results is by enriching the model with new dimensions of the R&D process that affect the value of innovations.

4.1.1 International technology flows

Intermediate goods flow internationally. The new technologies developed in Japan can be purchased in the US and used in the production of final output. This observation has two implications for the baseline analysis. On the one hand, I should use the R&D investments conducted in the whole world, and not just in the US, to calibrate the R&D intensity. On the other, a US innovator now can sell her innovation to the whole world, and therefore I should take into account the effect of this larger market size on the value of innovations. In terms of our calibration, the first effect implies that now the free entry condition is

$$P_{A_w} \frac{(\psi + \psi_N)}{\psi_N} \dot{A} = R_w, \tag{18}$$

where R_w represents the R&D in the world. Following the same logic as above, P_{A_w} can be expressed as:

$$P_{A_w} = \frac{\psi_N s_w}{(\psi + \psi_N)\gamma_A} \frac{Y_w}{A}, \tag{19}$$

where Y_w and s_w denote respectively the world level of output and the share of R&D in the world's output.

The second effect implies that the profits of a successful innovator are a function of the output of the countries where she can sell her innovations. Since innovations can be sold internationally, the new profit flow from a new variety is:

$$\pi_w = \left(\frac{\eta - 1}{\eta} \right) \alpha \frac{Y_w}{A}$$

As before, the value of an innovation is determined in the market and must satisfy an asset equation.

$$r = \frac{\pi_w}{P_{A_w}} + \gamma_{P_{A_w}} - \frac{\psi}{\psi_N} \gamma_A$$

The last term on the right hand side is the same as in the closed economy case. In steady state, equation (19) implies that $\gamma_{P_{A_w}}$ is equal to $\gamma_{Y_w} - \gamma_A$. But the interesting action takes place in the profit rate. There we can see that the two consequences from the internationalization of the economy exactly cancel out. More specifically,

$$\frac{\pi_w}{P_{A_w}} = \frac{\left(\frac{\eta-1}{\eta}\right) \alpha \left(\psi_N + \psi\right)}{s_w \; \psi_N} \gamma_A$$

Intuitively, the international flow of intermediate goods raises the resources devoted to develop the varieties that are ultimately used in the production of US output. The flip side of the coin is that US' (and any other country's) innovators can sell their goods to a larger market. Since both forces are proportional to Y_w, they cancel out.

Plugging these expression into the asset equation (11), we obtain the following growth rate of innovations:

$$\gamma_A = \frac{r - \gamma_{Yw}}{\left(\frac{\left(\frac{\eta-1}{\eta}\right)\alpha}{s_w} - 1\right)\left(1 + \frac{\psi}{\psi_N}\right)} \tag{20}$$

This exercise yields some interesting observations. Note that I have not specified any production function for R&D goods. As we have seen above, that is not necessary to calibrate the R&D contribution to productivity growth. In particular, the following general form is perfectly consistent with expression (20):

$$\frac{(\psi + \psi_N)}{\psi}\dot{A}_c = f_c\left(R_c, \{R_{-c}\}, A_c, \{A_{-c}\}\right),$$

where c indexes country c, $-c$ the sequence of other countries different from c and the only restriction on (the possibly country specific) function f_c is that there are diminishing returns in R_c. Note that this function captures all sorts of international spillovers in R&D.

Coming back to the calibration of the R&D contribution to productivity in the presence of international spillovers, it is easy to see that the figures obtained cannot be larger than the ones obtained in the previous section. Note from expression (20) that γ_A is increasing in s_w and decreasing in γ_{Y_w}. In the post-war period, the growth rate of output in the OECD has been higher than in the US, and the share of R&D in GDP is higher in the US than in the OECD. Therefore I keep the previous section's results as upper bounds for the R&D contribution to productivity growth.

4.1.2 R&D lags

In reality there is a lag between the outlay of the R&D investment and the beginning of the associated revenue stream. This lag corresponds both to the lag between project inception and conception (the gestation lag), and the time from project completion to commercial application (the application lag). Rapoport [1971] presents detailed data on the distribution of costs and gestation time for forty-nine commercialized innovations and the total innovation time for a subset of sixteen of them in three product groups- chemicals, machinery, and electronics. The total lag reported by Rapoport ranges from 1.2 to 2.4 years.

Wagner [1968] gathered survey data on the average R&D and application lags for process and product oriented R&D from thirty-six firms with long R&D experience in a variety of durable and nondurable goods industries. Unlike Rapaport's data, Wagner's takes into account expenditures on both technically successful and unsuccessful projects. She reports an average total lag of 2.17 years for nondurables and 2.62 for durables.

But, how does the inclusion of gestation lags affects the previous analysis?

The free entry condition now reads:

$$e^{-rl} P_{At+l} \frac{\psi_N + \psi}{\psi_N} \dot{A}_{t+l} = R_t,$$

where l is the gestation lag, t the time index and r the interest rate. From here

$$P_{A_{t+l}} = \frac{s_t \ \psi_N}{\gamma_{A_{t+l}} (\psi_N + \psi) \ e^{-(r-g_Y)l}} \frac{Y_{t+l}}{A_{t+l}}$$

Then the resulting steady state growth rate of varieties is:

$$\gamma_A = \frac{r - \gamma_Y}{\left(\frac{\left(\frac{\eta-1}{\eta}\right) \alpha e^{-(r-g_Y)l}}{s} - 1 \right) \left(1 + \frac{\psi}{\psi_N}\right)}$$

From the Rapoport and Wagner studies $l \leq 2.5$, therefore $e^{-(r-g_Y)l} \geq 0.914$. This has a very small effect on the previous calculations.

4.1.3 Correlated shocks

In the baseline model we have assumed that the obsolescence shocks are independent across the different intermediate goods in a common innovation cluster. This is probably not a very realistic assumption. When a new technological cluster is developed, there is a chance that it drives out of the market a large number of intermediate goods of an older cluster. In this scenario, the shocks faced by the intermediate goods in a given cluster are highly correlated. Next, I model this idea in the simple case where each new technological cluster makes completely obsolete an older cluster.[12,13]

[12] This is precisely the structure of the simple quality ladder models.

[13] I could generalize this obsolescence structure by allowing a fraction of the new clusters to have only a gradual effect on the existing clusters while the rest completely substitute old clusters. The resulting R&D contribution on productivity would then be somewhere between the values derived in section 3 and those obtained here.

Let's denote by V the value of a new cluster.[14] The number of new clusters created every instant is $\frac{\dot{A}}{\psi_N}$, therefore the free entry condition reads:

$$e^{-rl} V \frac{\dot{A}}{\psi_N} = R$$

V satisfies the asset equation

$$rV = \pi(\psi + \psi_N) + \dot{V} - \overbrace{\frac{\dot{A}}{\psi_N}}^{\#\text{ of new innovation clusters}} \overbrace{\frac{\psi + \psi_N}{A}}^{1/\#\text{ of existing clusters}} V,$$

where the only differences with respect to the baseline case are that now an asset is composed of $(\psi + \psi_N)$ innovations and therefore the profit flow must be multiplied by $(\psi + \psi_N)$, and that the probability of being driven out of the market is $\frac{(\psi+\psi_N)}{\psi}\gamma_A$ and in this event the value of the cluster completely depreciates.

From these two equations it follows that

$$\gamma_A = \frac{r - \gamma_Y}{\frac{\left(\frac{\eta-1}{\eta}\right)\alpha e^{-(r-g_Y)l}}{s}\left(1 + \frac{\psi}{\psi_N}\right) - \left(2 + \frac{\psi}{\psi_N}\right)} \tag{21}$$

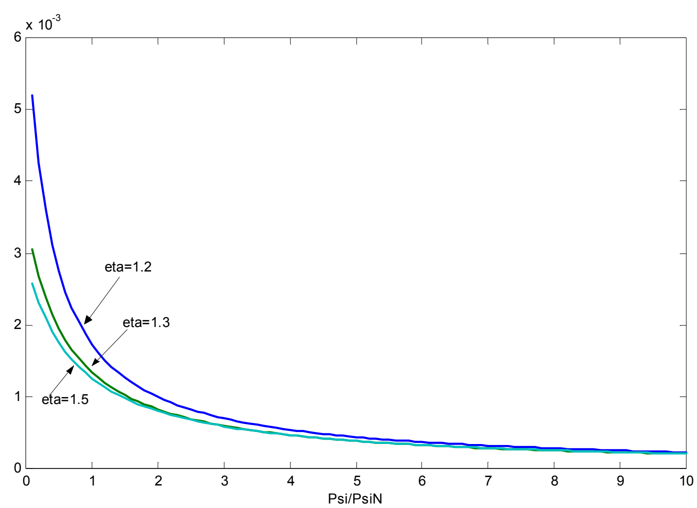

Figure 7: Contribution of R&D under monopolistic pricing and correlated obsolescence.

[14] It is easy to check that, in this setting, $V = (\psi + \psi_N)P_A$.

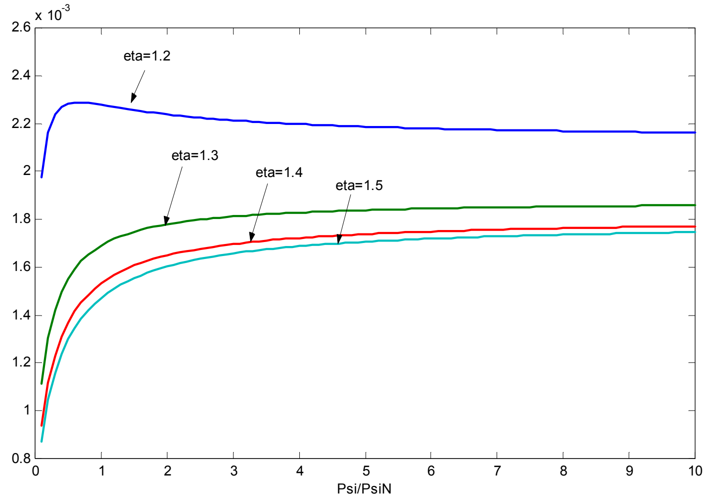

Figure 8: Contribution of R&D under limit pricing and correlated obsolescence shocks.

From figures 7 and 8, we can see that even after introducing correlated shocks and R&D lags, the contribution of R&D to productivity growth for the more reasonable markups is bounded above by 3 to 5 tenths of one percentage point.

4.1.4 Subsequent R&D cost advantage

Up to now, a firm has been characterized by the set of varieties that form an innovation cluster. All of these intermediate goods are developed simultaneously and once they become obsolete the firm vanishes. However, the evidence tells us that a large fraction of innovations are developed by firms that have already developed some other innovation clusters. This can be due to the fact that the costs of innovation decline with the number of varieties developed.[15] If this is the case, innovations are more valuable than what we have computed so far. Investing in R&D not only grants the right to the future revenues from the new innovation cluster but also the option to develop more clusters in the future at a lower cost. To reconcile the higher value of innovations with the observed low s, the free entry condition now dictates a lower growth rate of varieties and a smaller contribution of R&D to productivity growth.

[15] Or alternatively, that the probability of being successful in the R&D investments increases with the firm's experience in R&D activities.

This additional complexity is useful to generalize the argument made above to large firms. These internalize part of the positive consequences of their investment decisions. By taking advantage of part of the externalities, the value of the R&D firm increases, and the growth rate of varieties induced by a given share of R&D is lower than when firms are small. Hence, the benchmark contribution computed above gives an upper bound for the role of R&D in productivity growth when firms are allowed to grow.

To show this more formally, let's suppose that an incumbent firm j with $i > 0$ active innovation clusters has the ability to develop up to i new innovation clusters every instant. Let's r_{ij} denote the amount of R&D this firm conducts for each of the i projects. Success at each of the projects arrives with an independent Poisson rate $\delta(r_{ij}, \{R_c\}_{c=1}^{\infty}, \{A_c\}_{c=1}^{\infty})$, where R_i and A_i denote respectively the total R&D investments and the total number of varieties available in the market from the firms with exactly $i-1$ active clusters. The only restrictions I impose on δ are that it is increasing and concave in r_{ij}. Let N_{i-1} be the number of firms with exactly $i-1$ active technological clusters. The framework described so far implies that the total number of clusters developed every instant by firms with already $i-1 \geq 0$ active technological clusters is

$$\frac{\dot{A}_i}{\psi_N} = N_{i-1}\; i\delta(r_{ij}, \{R_c\}_{c=1}^{\infty}, \{A_c\}_{c=1}^{\infty}). \tag{22}$$

One important modification introduced with this setup is that the firm now internalizes part of the intertemporal consequences of its R&D investments because it is aware that succeeding in developing the next technological cluster increases the chances of developing new clusters in the future. However, for simplicity, I still assume that the number of firms in each size group (N_i) is large and therefore that the effects of r_{ij} on R_i, and of A_{ij} on A_i are negligible.[16] This means that incumbents select r_{ij} to maximize $i\,[\delta(r_{ij}, \{R_c\}_{c=1}^{\infty}, \{A_c\}_{c=1}^{\infty})\,(V_{i+1} - V_i) - r_{ij}]$ taking as given $\{R_c\}_{c=1}^{\infty}$ and $\{A_c\}_{c=1}^{\infty}$.

The optimal level of r_{ij} (denoted by r^*) does not depend directly on i, and satisfies the following first order condition:

$$\frac{\partial\delta(r^*, \{R_c\}_{c=1}^{\infty}, \{A_c\}_{c=1}^{\infty})}{\partial r_{ij}}\,(V_{i+1} - V_i) = 1, \text{ for } i > 0 \tag{23}$$

[16] This seems to me the most reasonable scenario: one where firms internalize the cost advantage of subsequent innovation but do not internalize the aggregate diminishing returns to R&D or the aggregate intertemporal externalities.

From this framework, it follows that

$$\frac{1}{R_i}\frac{\dot{A}_i}{\psi_N} = \frac{\delta(r^*, \{R_c\}_{c=1}^{\infty}, \{A_c\}_{c=1}^{\infty})}{r^*} > \frac{\partial\delta(r^*, \{R_c\}_{c=1}^{\infty}, \{A_c\}_{c=1}^{\infty})}{\partial r_{ij}} = \frac{1}{V_{i+1} - V_i}, \text{ for } i > 0$$

where the first equality comes from the production function for R&D (22), the inequality is a consequence of the concavity of δ on r_{ij}, and the second equality follows from the first order condition (23). Rewriting this, we observe that incumbent R&D firms make positive profits on average from successive innovations.

$$(V_{i+1} - V_i)\,\frac{\dot{A}_i}{\psi_N} > R_i \tag{24}$$

For new R&D firms, however, free entry brings down the expected value of a firm with exactly one cluster to the cost of developing the first innovation cluster.

$$V_1\frac{\dot{A}_1}{\psi_N} = R_1 \tag{25}$$

As before, we can derive the relationship between s and γ_A by pricing the R&D firms. The value of a firm with exactly $i \geq 1$ active clusters satisfies the following asset equation:

$$rV_i = i\pi(\psi + \psi_N) + \dot{V}_i - i\frac{\dot{A}}{\psi_N}\frac{(\psi+\psi_N)}{A}\,(V_i - V_{i-1}) + i\max_{r_i}\delta(r_i, \{R_c\}_{c=1}^{\infty}, \{A_c\}_{c=1}^{\infty})\,(V_{i+1} - V_i) - r_i$$

Using the definition of r^* and dividing by V_i, we obtain:

$$r = i\frac{\pi}{V_i}(\psi + \psi_N) + \frac{\dot{V}_i}{V_i} - i\frac{\dot{A}}{\psi_N}\frac{(\psi+\psi_N)}{A}\left(\frac{V_i - V_{i-1}}{V_i}\right) + i\delta(r^*, \{R_c\}_{c=1}^{\infty}, \{A_c\}_{c=1}^{\infty})\left(\frac{V_{i+1} - V_i}{V_i}\right) - i\frac{r^*}{V_i}$$

At this point, we can make a very useful observation. If $V_i = iV_1$, the RHS of this equation is independent of i. This means that we can find a solution to this system of difference equations by just solving the one for V_1. Using this shortcut we can reduce the system to:

$$r = \frac{\pi}{V_1}(\psi + \psi_N) + \frac{\dot{V}_1}{V_1} - \frac{\dot{A}}{\psi_N}\frac{(\psi+\psi_N)}{A} + \overbrace{\delta(r^*, \{R_i\}_{i=1}^{\infty}, \{A_i\}_{i=1}^{\infty}) - \frac{r^*}{V_1}}^{\text{profit rate from subsequent R\&D}} \tag{26}$$

The free entry condition for new innovators (25) implies that:

$$V_1 = \frac{s_1\psi_N}{\frac{\dot{A}_1}{A_1}\frac{A_1}{A}}\frac{Y}{A},$$

where s_1 is the share of R&D conducted by entrants in total output. Since in the steady state γ_{A_1}, s_1 and $\frac{A_1}{A}$ are constant, $\gamma_{V_1} = \gamma_Y - \gamma_A$.

From (25) and (24) it follows that

$$\begin{aligned}
\sum_{i=1}^{\infty}(V_i - V_{i-1})\frac{\dot{A}_i}{\psi_N} &> \sum_{i=1}^{\infty} R_i \\
V_1\frac{\dot{A}}{\psi_N} &> R \\
V_1 &> \frac{s\psi_N}{\gamma_A}\frac{Y}{A}, \qquad (27)
\end{aligned}$$

where the second inequality takes advantage of the fact that $V_i = iV_1$.

Plugging (27) into the asset equation (26) we obtain the following inequality:

$$r > \frac{\left(\frac{\eta-1}{\eta}\right)\alpha(\psi+\psi_N)}{s\psi_N}\gamma_A + \gamma_Y - \gamma_A - \frac{(\psi+\psi_N)}{\psi_N}\gamma_A + \overbrace{\delta(r^*, \{R_i\}_{i=1}^{\infty}, \{A_i\}_{i=1}^{\infty}) - \frac{r^*}{V_1}}^{>0}$$

And from here,

$$\gamma_A < \frac{r - \gamma_Y - \overbrace{\left(\delta(r^*, \{R_i\}_{i=1}^{\infty}, \{A_i\}_{i=1}^{\infty}) - \frac{r^*}{V_1}\right)}^{>0}}{\frac{\left(\frac{\eta-1}{\eta}\right)\alpha e^{-(r-g_Y)l}}{s}\left(1+\frac{\psi}{\psi_N}\right) - \left(2+\frac{\psi}{\psi_N}\right)}$$

To relate this expression with the growth rate of varieties when innovators are small (21), recall that incumbents make positive profits from subsequent R&D. This means that the new term in the numerator is strictly positive and that instead of an equality, now we have an strict inequality. As a result, the γ_A implied by s when we allow firms to partially internalize the future cost advantages of their current R&D (i.e. when they are large) is lower than when they are small.

4.2 Externalities in production

So far, I have shown the robustness of the claim that a small R&D intensity implies a small growth rate of R&D-driven technology. In this section, I analyze the robustness of the small R&D contribution to productivity growth. One possible route to show this is by arguing that the elasticity of productivity growth with respect to γ_A is small. In the baseline model, this elasticity is equal to $\frac{1-\alpha\rho}{\rho} + \frac{\alpha(1-\alpha\rho)}{\rho(1-\alpha)}$, where the first term corresponds to the static externality of A on output and the second to the capital deepening driven by the development of new technologies. One might argue that with a more general production function we could parameterize the externality in production to explain an arbitrarily large growth rate of productivity just with R&D investments.

However, this is not the case because most of the R&D-driven innovations are embodied in capital goods (Solow [1959]). This means that innovations with higher (social) value are more demanded and their innovators enjoy larger profit rates. Therefore, the market value of the innovations is correlated to their value for the producer of final output. In terms of my two-step approach, this means that a large production externality raises the effect of the growth of technology on productivity growth but also reduces the growth of technology associated with a given R&D intensity. As a result, the R&D contribution to productivity growth is not very sensitive to the size of the externalities in production.

To see this more formally, consider an environment that is exactly the same as in the baseline model but with the following aggregate production function:

$$Y = ZL^{1-\alpha}\left[\int_0^A a_i x_i^{\alpha\rho} di\right]^{\frac{1}{\rho}},$$

where now the level of R&D-driven technology is a continuous variable and the capital varieties have different efficiencies a_i. To introduce some flexibility on the size of the production externality, I set $a_i = bi^{\sigma-1}$, where b is any positive constant that, without loss of generality, I normalize to 1, $\sigma > \alpha\rho$ and i is a technology index. When $\sigma > 1$, newer innovations are more efficient than older innovations. Note also that the size of the externality in production is increasing in σ. To see this, suppose for a moment that the demand is the same for all the capital varieties, then the production function could be rewritten as

$$Y = ZL^{1-\alpha}A^{\frac{\sigma-\alpha\rho}{\rho}}K^{\alpha},$$

where as before $K \equiv Ax$.[17] If this was the resulting production function, the size of the production externality would be $\frac{\sigma-\alpha\rho}{\rho}$. Of course, this is not exact because the demand for each capital variety (x_i) varies with a_i. In particular, the inverse demand for a particular variety i, is

$$p_i = Z\alpha L^{1-\alpha}\left(\int_0^A a_i x_i^{\alpha\rho} di\right)^{\frac{1}{\rho}-1} a_i x_i^{\alpha\rho-1}.$$

[17] Admittedly, I restrict the analysis to Cobb-Douglas production functions. This however seems a good description of the data. It is easy to show that within the class of constant elasticity of substitution production functions, any other function yields a time varying labor share. That is inconsistent with the US evidence in the post-war period.

Due to the isoelastic nature of the demand, innovators set a price equal to a constant markup η times the marginal cost of production r. Doing some tedious algebra, we can easily find that when the state of the art technology has index A, the level of output is given by expression (28) and the profits for an innovator that developed a variety with index $i \leq A$ are given by (29).[18]

$$Y = LZ^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}(\eta r)^{\frac{-\alpha}{1-\alpha}}A^{\frac{\sigma-\alpha\rho}{\rho(1-\alpha)}} \tag{28}$$

$$\pi_{iA} = \frac{\eta-1}{\eta}\alpha\underbrace{\frac{\sigma-\alpha\rho}{1-\alpha\rho}}_{\text{Level effect}}\frac{Y}{A}\underbrace{\left(\frac{A}{i}\right)^{\frac{\sigma-1}{1-\alpha\rho}}}_{\text{Gradual substitution}} \tag{29}$$

In this last expression, we can appreciate two effects of σ on the profits of an innovator. The higher curvature in the efficiency of capital vintages (σ), the higher the initial level of profits, but also the faster final good producers gradually substitute towards the new, more efficient, varieties.

Expression (29) can be rewritten in terms of the vintage of the capital variety sold by the innovator. More specifically, let v_i be the vintage of the capital good with index i. In steady state, A grows at the constant rate γ_A. Without loosing much perspective, let's suppose that the economy started in the balanced growth path. Then the profits at time t of an innovator that developed a vintage v_i capital good are:

$$\pi_{v_it} = \frac{\eta-1}{\eta}\alpha\frac{Y}{A}\frac{\sigma-\alpha\rho}{1-\alpha\rho}e^{-\left(\frac{\sigma-1}{1-\alpha\rho}\right)(t-v_i)\gamma_A}. \tag{30}$$

When $\sigma > 1$, the innovations embodied in newer capital varieties are more profitable than those embodied in older vintages. The converse is true when $\sigma < 1$. Consequently, the market value of an innovation generically varies in the cross-section. Let $P_{At,v}$ denote the price at time t of an innovation embodied in a vintage v capital good. From free entry, we know that

$$P_{Att} = \frac{s\psi_N}{\gamma_A(\psi_N+\psi)}\frac{Y}{A}. \tag{31}$$

Since P_{Atv} is determined in the market, it satisfies the following differential equation:

$$rP_{Atv} = \pi_{vt} + \dot{P}_{Atv} - \frac{\psi}{\psi_N}\gamma_A \tag{32}$$

[18]It can also be shown that

$$K = \int_0^A a_i x_i di = \chi_K Z^{\frac{1}{1-\alpha}} L A^{\frac{(\sigma-\alpha\rho)(1-\rho)+\rho(1-\alpha)(2\sigma-1-\sigma\alpha\rho)}{\rho(1-\alpha)(1-\alpha\rho)}},$$

where χ_K is a positive constant; and that

$$Y = \chi_Y Z A^{\tilde{\sigma}} K^{\alpha} L^{1-\alpha},$$

where χ_Y is another positive constant and $\tilde{\sigma} = \frac{\sigma(1-\alpha\rho)}{\rho}$.

It is easy to see that $P_{Atv} = P_{Att}\ e^{-\gamma_A\left(\frac{\sigma-1}{1-\alpha\rho}\right)(t-v)}$.[19] This implies that $\gamma_{P_{Atv}} = \gamma_Y - \gamma_A\left(\frac{\sigma-\alpha\rho}{1-\alpha\rho}\right)$. Dividing both sides of equation (32) by P_{Avt} and plugging (30) and this expression for $\gamma_{P_{Atv}}$, we can derive expression (33).

$$\gamma_A = \frac{r-\gamma_Y}{\left(\frac{\left(\frac{\eta-1}{\eta}\right)\alpha}{s}\left(1+\frac{\psi}{\psi_N}\right)-1\right)\left(\frac{\sigma-\alpha\rho}{1-\alpha\rho}\right)-\frac{\psi}{\psi_N}} \tag{33}$$

This expression differs from the growth rate of technology in the baseline model (13) in two respects. First, the profit rate of innovations increases with σ. Second, a higher σ implies a higher expected capital loss due to the depreciation of the market value of the innovations. The first effect raises the current value of an innovation while the second reduces it. However, in expression (33) it is clear that the first force dominates the second, and the higher is the externality in production (σ) the lower is the growth rate of technology associated with a given R&D intensity.

To complete the calculation we just have to derive the growth rate of productivity from expression (28).

$$\gamma_{Y/L} = \frac{1}{1-\alpha}\gamma_Z + \frac{\sigma-\alpha\rho}{\rho\,(1-\alpha)}\gamma_A$$

Note that, for a given γ_A, the R&D contribution to productivity growth is increasing in σ. However, doing some simple algebra we can check that, after taking into account the effect of σ on γ_A, the R&D contribution to productivity growth is decreasing in σ. To assess the quantitative importance of these effects, I plot the R&D contribution for several values of σ and ψ/ψ_N when the markup is equal to 1.2, figure 9, and 1.3, figure 10. For simplicity I restrict myself to the case of monopolistic markups (i.e. $\eta = (\alpha\rho)^{-1}$).

[19] For this you can solve the differential equation (32) plugging in (30) and using the initial condition (31).

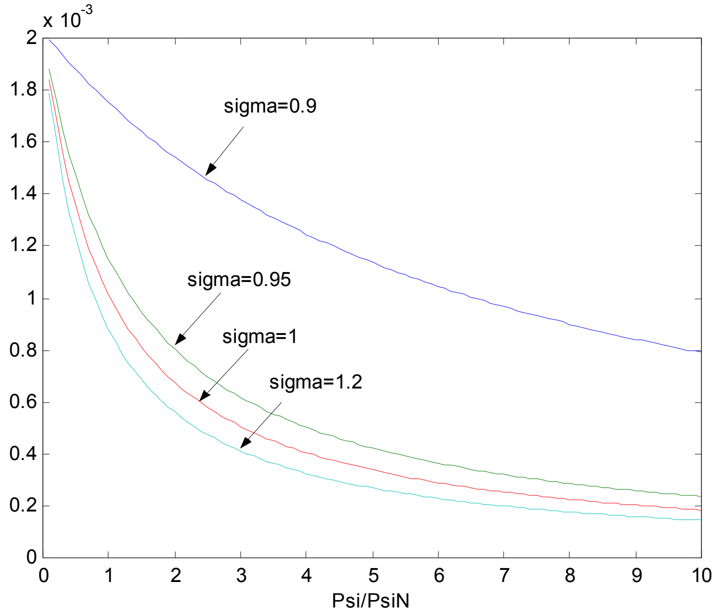

Figure 9: R&D contribution to productivity growth with $\eta = 1.2$ for several $\sigma's$

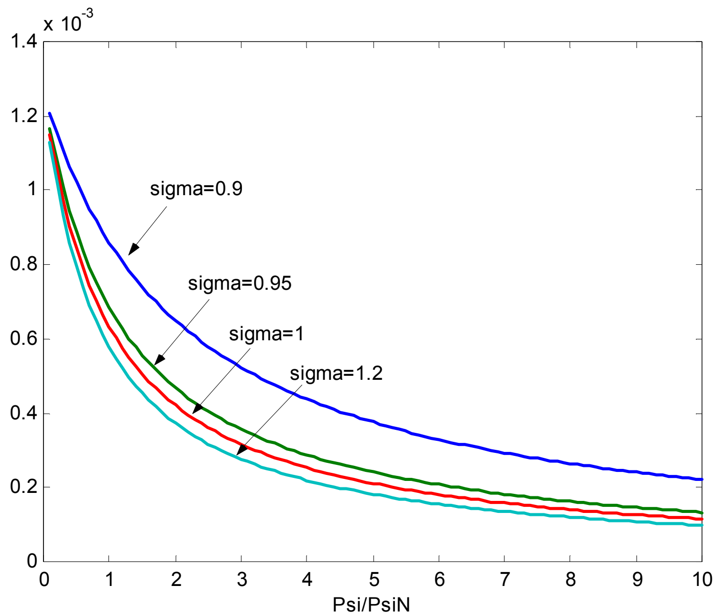

Figure 10: R&D contribution to productivity growth with $\eta = 1.3$, and several $\sigma's$.

From these figures, we can see that the small R&D contribution to productivity growth is quite robust to the size of the production externality.

4.3 Transition and relation to Jones [2001]

Jones [2001] has argued that the US economy has been in a transition to the steady state during the post-war period. In this section I extend the previous analysis to the transition and relate my findings to Jones [2001].

When deriving the relationship between s and γ_A, I have only assumed that the economy is in steady state to compute the price appreciation of the innovations. Remember that free entry implies that

$$P_A = \frac{s\psi_N}{\gamma_A\left(\psi_N + \psi\right)}\frac{Y}{A}.$$

If the economy is in the transition, then

$$\gamma_{P_A} = \overbrace{\gamma_Y - \gamma_A}^{\text{Steady State}} + \overbrace{\gamma_s - \gamma_{\gamma_A}}^{\text{Transition}}.$$

The asset pricing equation holds at every instant, but now we have to recognize the new expression for the price appreciation of an innovation. This yields the following differential equation for γ_A:

$$\frac{d\left(\ln(\gamma_A)\right)}{dt} - \left[\frac{\eta-1}{\eta}\frac{\alpha\left(\psi_N+\psi\right)}{s\psi_N} - \left(1+\frac{\psi}{\psi_N}\right)\right]\gamma_A = -\left(r-\gamma_Y-\gamma_s\right) \tag{34}$$

Since s is time varying this differential equation does not have a close form solution. To approximate the average growth rate of A, we can solve this differential equation calibrating s to the average R&D intensity during the transition. Then the solution to this equation takes the form

$$\gamma_A(t) = \left[C_1 e^{\int_0^t (r(v)-\gamma_Y(v)-\gamma_s(v))\,dv} - \left[\frac{\eta-1}{\eta}\frac{\alpha\left(\psi_N+\psi\right)}{s\psi_N} - \left(1+\frac{\psi}{\psi_N}\right)\right]\int_0^t e^{\int_v^t (r(\tau)-\gamma_Y(\tau)-\gamma_s(\tau))\,d\tau} d\tau\right]^{-1}$$

For illustrative purposes, suppose that the term $(r(v)-\gamma_Y(v)-\gamma_s(v))$ is constant. Then this expression is equal to

$$\gamma_A(t) = \left[\tilde{C}e^{(r-\gamma_Y-\gamma_s)\,t} + \frac{\left[\frac{\eta-1}{\eta}\frac{\alpha(\psi_N+\psi)}{s\psi_N} - \left(1+\frac{\psi}{\psi_N}\right)\right]}{(r-\gamma_Y-\gamma_s)}\right]^{-1}$$

In the long run $(r-\gamma_Y-\gamma_s) > 0$, therefore for a steady state to exist, it is necessary that $\tilde{C}=0$. This implies that

$$\gamma_A = \frac{r-\gamma_Y-\gamma_s}{\left[\frac{\eta-1}{\eta}\frac{\alpha(\psi_N+\psi)}{s\psi_N} - \left(1+\frac{\psi}{\psi_N}\right)\right]}. \tag{35}$$

In figure 1, we can appreciate an upward trend in s for the post-war period. This positive growth in s, yields a lower γ_A in expression (35) than if it had remained constant.[20] Intuitively, a (temporary) upward trend in the share of resources devoted to R&D is due to an expected appreciation in the value of innovations. Therefore the current market price of innovations is higher and, from free entry, the associated growth rate of R&D driven-technology must be lower.

4.3.1 Relationship to Jones [2001]

In a recent paper, Jones [2001] also analyzes the sources of growth in the US post-war experience and concludes that most of the growth in TFP can be accounted for by R&D. Interestingly, the basic models underlying Jones's and my analysis are the same. The difference in our conclusions resides in the different approaches followed in the quantitative analysis of the model. Instead of exploiting the free entry condition, Jones [2001] poses a production function for new technologies that he estimates to figure out how much growth can be attributed to R&D. In particular, he considers the specification reproduced in equation (36) where H_A is the number of workers in the R&D sector, and both λ and ϕ are smaller than 1.

$$\frac{\dot{A}}{A} = \hat{\delta} H_A^{\lambda} A^{\phi-1}. \tag{36}$$

$$Y = A^{\sigma} K^{\alpha} L^{1-\alpha} \tag{37}$$

Expression (36) coupled with the Cobb-Douglas production function for final output presented in expression (37) imply an average contribution of R&D to TFP growth that is approximately equal to $\vartheta \gamma_{H_A}$, where $\vartheta = \frac{\sigma}{1-\alpha}\frac{\lambda}{1-\phi}$. It is transparent in this expression that Jones's conclusions follow from the specification used for the R&D technology and from the calibration of ϑ. Jones calibrates ϑ by estimating a log-linear approximation of (36) where A is imperfectly measured by the total factor productivity (B) as shown in equation (38).

$$\log B_t = \log A_t + \epsilon_t \tag{38}$$

[20] From expression (15), it follows that, when we take into account the transition of the US during the post-war period, the resulting R&D contribution to productivity growth is also lower than if we assume that the US economy was in steady state.

More specifically, Jones estimates the following equation

$$\Delta \log B_{t+1} = \beta_0 + \lambda\gamma_B \left[\log H_{At} - \frac{1}{\vartheta}\log B_t\right] + \varepsilon_{t+1}, \tag{39}$$

where $\varepsilon_{t+1} \equiv \triangle\epsilon_{t+1} + \frac{\lambda\gamma_B}{\vartheta}\epsilon_t$ is a serially correlated error term.

As Jones points out, the estimation of equation (39) creates as many difficulties as the regressions in the productivity literature. In particular, the estimate of $\frac{1}{\vartheta}$ is likely to be biased for at least two reasons. First, business cycle fluctuations in R&D expenditures imply that the regressor is endogenous. Second, the measurement error in A (and the potential misspecification of the R&D production equation) also generate a correlation between the error term (ε_{t+1}) and the regressor ($\log B_t$). However, Jones appeals to the possible cointegration between $\log H_{At}$ and $\log B_t$ which imply that the OLS estimate of $\frac{1}{\vartheta}$ is super consistent (Hamilton [1994]).

An important practical issue is whether this asymptotic result can be invoked in a finite sample application like Jones's. Campbell and Perron [1991] study this question using Monte Carlo analysis and conclude that a useful rule of thumb is that asymptotic results can be exploited in samples of the size encountered in empirical applications when we can reject the null of no cointegration using the asymptotic critical values.

Table 2 presents results of an augmented Dickey-Fuller test on the residuals of the following regression:[21]

$$\log H_{At} = \alpha_0 + \alpha_1 \log B_t + u_t$$

Table 2: Cointegration regression

Dependent Variable	$\hat{u}_t$
$\hat{\beta}_0$	0.0031
	(0.0089)
$\hat{\rho}$	0.804
	(0.085)
R^2	0.66
N	43
Dickey-Fuller t-statistic: $(\hat{\rho}-1)/\hat{\sigma}_{\hat{\rho}}$	-2.3

[21] I have experimented also with several lag structures in the first differences of the residuals but these were never significant. Even when including them, the results of the test were the same.

Source: http://emlab.berkeley.edu/users/chad/Sources50.asc

Robust standard errors in parenthesis.

The critical value for this statistic at the 5 percent significance level is -3.42, which is lower than the Dickey-Fuller t-statistic. Therefore we cannot reject the null that there is no cointegration between $\log H_{At}$ and $\log B_t$. While this statistical tests does not altogether rule out the possibility that $\log H_{At}$ and $\log B_t$ share a common trend, they do suggest that it may be difficult to exploit the asymptotic properties of cointegration systems in samples of the size we currently have in order to calibrate ϑ, and that exploring alternative approaches may be useful. This paper has presented one such alternative which implies a value of ϑ around 0.05. This value is the lower bound used by Jones [2001] in his analysis.

5 Welfare

So far I have conducted a positive analysis of the contribution of R&D to productivity growth. However, the previous findings can be used to conduct a normative analysis. In particular, we can proceed in the following three steps. First, specify a production function for innovations; second, use the computed growth rate of R&D-driven technology (γ_A) to quantify the size of the externalities in the production of new technologies. Finally, solve the social planner's problem and determine the socially optimal R&D intensity (s^*).

Note that in contrast to the positive analysis, now it is necessary to specify a production function for new technologies, therefore our results will depend on the particular functional form specified. In this sense, this section just intends to compare our approach to previous ones. To this end, we adopt the R&D technology used by Jones and Williams [2000] which generalizes the innovation technology posed in Stokey [1995]. Specifically, they assume that

$$\dot{A}_t = \delta \frac{1}{1+\psi/\psi_N} R_t^{\lambda} A_t^{\phi}, \tag{40}$$

where both λ and ϕ are bounded above by 1. In steady state,γ_A is constant, therefore

$$\frac{\gamma_A}{\gamma_Y} = \frac{\lambda}{1-\phi}. \tag{41}$$

Expression (41) relates the size of the R&D externalities to the actual growth rate of the US economy and to the growth rate of R&D-driven technology that I have already quantified in section 4.2.

Following Jones and Williams [2000], I consider a production function for final output that displays static externalities. As in section 4.2, I relate the size of this externality to the elasticity of substitution across different varieties and to the importance of embodied productivity growth. In particular, output is produced according to expression (42).

$$Y_t = A_t^{\tilde{\sigma}} Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{42}$$

From section 4.2, we know that, in this context, the growth rate of R&D technology (γ_A) is given by expression (43) where $\sigma = \rho\tilde{\sigma}/\left(1-\alpha\rho\right)$.

$$\gamma_A = \frac{r-\gamma_Y}{\left(\frac{\left(\frac{\eta-1}{\eta}\right)\alpha}{s}\left(1+\frac{\psi}{\psi_N}\right)-1\right)\left(\frac{\sigma-\alpha\rho}{1-\alpha\rho}\right)-\frac{\psi}{\psi_N}} \tag{43}$$

Expressions (41) and (43) define a relationship between λ and ϕ for given (η, α, s, r, γ_Y, ψ/ψ_N, ρ, σ), where these parameters can be calibrated in the decentralized economy. Table 3 summarizes this calibration.

Table 3: Parameters

r	0.07
γ_Y	0.034
α	0.33
s	0.02
ψ/ψ_N	0.25
ρ	$1/(\eta\alpha)$
σ	$\{0.9, 0.95, 1, 1.05\}$
η	1.2

There are a few remarks worthwhile making about this calibration. First, note that the markup η is calibrated conservatively, and that I impose monopolistic pricing of the innovations in order to calibrate the elasticity of substitution across different varieties. Finally, note that in each technological cluster there are four times as many completely new goods as new versions of old goods.

Figure 11 plots the size of the R&D externality (ϕ) associated with various levels of the static externality (σ) and with the size of the stepping on the toes effect $(1-\lambda)$. This relationship is quite robust to the variation of the rest of the parameters and in this benchmark I have chosen values of η, ρ, ψ/ψ_N and θ that yield a higher schedule for ϕ.

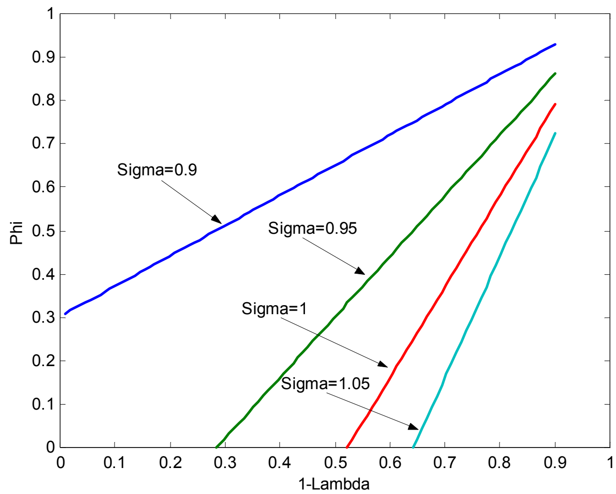

Figure 11: $\phi(1-\lambda)$

Now that we have bounded the R&D technology using actual US data, we can solve the Social planner's problem to determine the optimal R&D intensity (s^*).

Her problem can be formalized as follows:

$$\max_{\{c_t, R_t\}} \int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta}\, e^{-\varsigma t} dt$$

subject to:

$$\begin{aligned}
c_t L_t + I_t + R_t &= Y_t = A_t^{\tilde{\sigma}} Z_t K_t^{\alpha} L_t^{1-\alpha} \\
\dot{K}_t &= I_t, K_0 > 0 \\
\dot{A}_t &= \delta \frac{1}{1+\psi/\psi_N} R_t^{\lambda} A_t^{\phi}, \;\; A_0 > 0 \\
\frac{\dot{Z}}{Z} &= \gamma_Z; \;\; Z_0 > 0 \\
\frac{\dot{L}}{L} &= n; \;\; L_0 > 0
\end{aligned}$$

After setting up the Hamiltonian and deriving the first order conditions it is easy to see that in steady state, the social planner devotes a share of output s^* to R&D investments, and this results in a growth rate γ_A^* of R&D-driven technology, where the expressions for these two variables are as follows:

$$\begin{aligned} s^* &= \tilde{\sigma}\lambda\left[\frac{(1-\phi)}{\lambda}\left[\theta+\phi-1\right]+\frac{\varsigma-n\left(\theta-1\right)}{\gamma_A^*}\right]^{-1} \\ \gamma_A^* &= \frac{(1-\alpha)}{\frac{1-\phi}{\lambda}(1-\alpha)-\tilde{\sigma}}\left[n+\frac{\gamma_Z}{1-\alpha}\right] \end{aligned}$$

To compute s^* I just have to calibrate some parameters that I have not quantified yet. These are γ_Z, ς, θ. In this model, Z is exogenous. Therefore it is reasonable to assume that γ_Z is the same in the decentralized and in the planned economy. The production function implies that in steady state, $\gamma_Z = (1-\alpha)(\gamma_Y - n) - \tilde{\sigma}\gamma_A$.

ς and θ determine the consumer preferences. The optimal consumption path for the representative consumer in the decentralized economy must satisfy the following Euler equation.

$$\gamma_c = \frac{1}{\theta}\left[r-n-\varsigma\right] \qquad \text{(Euler Equation)}$$

The growth rate of the labor force in the US in the post-war period (n) has been equal to 0.0144 and the growth rate of consumption per capita (γ_c) has been 0.021. Therefore if we calibrate the discount rate (ς) to 0.04, the Euler equation implies an inverse of the elasticity of intertemporal substitution (θ) between 1 and 2.

Figure 12 plots the resulting optimal R&D intensities for several values of σ and for the $\lambda's$ that yield a ϕ in the interval [0,1]. The most striking fact from this figure is that the optimal R&D intensities are not much higher than the actual ones. This finding is robust to alternative parameterizations of θ, ς, σ, and of the parameters that determine γ_A.

Kortum [1993] has estimated λ to be between 0.1 and 0.6. Interestingly, for this range of λ, the actual R&D intensity roughly coincides with the intensity that the social planner prescribes.

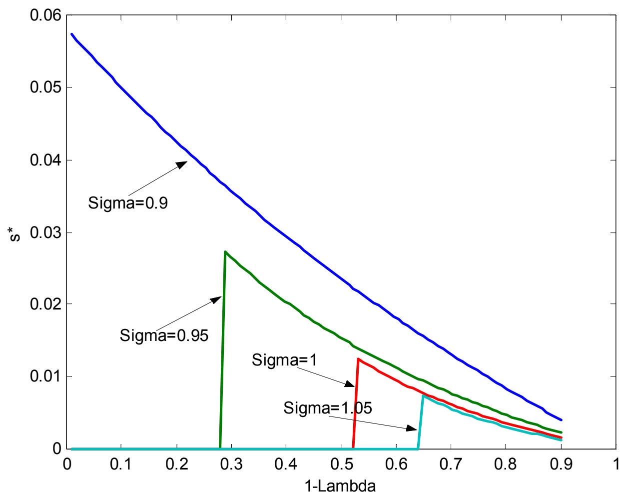

Figure 12: $s^*(1-\lambda)$

This conclusion contrasts with Jones and Williams [2000] who find that "the decentralized economy typically underinvests in R&D relative to what is socially optimal". The reason for the divergence in our findings is that we pursue different strategies to quantify γ_A. While Jones and Williams assume that all TFP growth can be explained by R&D-driven investments, this paper allows the free entry condition to determine the magnitude of γ_A.

6 Where does this leave us?

Productivity increases because we learn how to use our factors more efficiently. This learning may be a by-product of other activities not directed at increasing the productivity of resources or the result of investment efforts directed towards the improvement of productivity. In this paper I have shown that one of these investments, R&D, is not responsible for a large share of productivity growth in the US. Since the US is the world leader in R&D, this conclusion can be made extensive to the other nations.

Our prior was that R&D is the main source of long run growth. The immediate question that emerges from this analysis is "then, what is the driving force of productivity growth?". This question should be placed at the top of the research agenda.

From a normative perspective, this analysis implies that the decentralized economy may not be underinvesting in R&D.

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