VALUATION AND GROWTH RATES MANIPULATION

Arieh Gavious
Department of Industrial Engineering and Management
Faculty of Engineering
Ben-Gurion University of the Negev
Beer-Sheva, Israel 84105
E-mail ariehg@bgumail.bgu.ac.il

Joshua Ronen
Department of Accounting, Taxation, and Business Law
Stern School of Business
New York University
313 Tisch-Hall
New York, NY 10012-1118
E-mail jronen@stern.nyu.edu

Varda (Lewinstein) Yaari
Department of Business administration
School of Management
Ben-Gurion University of the Negev
Beer-Sheva, Israel 84105
E-mail: yvarda@nihul.bgu.ac.il, alexgum21@hotmail.com

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Abstract

Valuation requires the prediction of future growth rate of persistent earnings, which depend on past and present internal, unobservable, investment decisions. In this study, we investigate the "management" of the series of growth rates in a multi-period principal-agent model with a moral hazard problem between owners (the principal) and the manager (the agent). We find that the manager's choice of efforts might yield a series of increasing expected growth rates, contrary to owners' preferences. Consequently, the extrapolation of expected future earnings of an owner-controlled firm should differ from that of a management-controlled firm.

Key words: Valuation, moral hazard, growth rates, smoothing

1. Introduction

It is now widely accepted that a major goal of the financial reports is to provide investors with information that enables then to predict future cash flows (see e.g., Statement of Financial Accounting Concept #4). That is, to provide an input for valuation. Two facts are undisputable: Companies manage earnings,[1] and accounting earnings are important for valuation. Degeorge, Patel, and Zeckhauser (1999, p.1) state:

> Analysts, investors, senior executives, and boards of directors consider the earnings signal the most important item in the financial report issued by publicly held firms.[2]

In this study, we analyze how moral hazard affects the characteristics of the series of the growth rates of persistent earnings and the consequent valuation. We model the firm as a multi-period principal-agent contract wherein the risk-averse, work-averse manager's periodical effort determines stochastically the firm's investment, which in turn, determines the growth rate of persistent income and valuation.

As a benchmark case, we analyze the equilibrium when the owners are also the managers. Since this case is free of moral hazard, and there is no asymmetry of information between the owners and management, we refer to it as the first-best case to distinguish it from the principal-agent scenario, henceforth referred to as the second-best. We find that the

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[1] An eagle view of the literature: For works that include reviews of the empirical evidence, see, for example, Ronen, Sadan, and Snow (1977), Ronen and Sadan (1981)], Schipper (1989), Healy and Wahlen (1999), Stolowy and Breton (2000), Beneish (2001), and Fields, Lys, and Vincent (2001)); for practitioners' views, see e.g., O'glove (1987), Pijper (1993), Schilit (1993), McBarnet and Whelan (1999), and Levitt (1998)); see also Scott (1997), and Belkaoui (1999).

[2] Similarly, Beaver (1998, P.38) states:

> No other figure in the financial statements receives more attention by the investment community than earnings per share. The relationship between accounting earnings and security prices is probably the single most important relationship in security analysis.

expected temporal choice of effort is characterized by exerting higher effort earlier, which leads to a series of diminishing expected marginal growth rates. Next, we analyze the second-best effort choice in a management-controlled firm. We find the opposite trend for some parameters: the manager-agent prefers to exert more effort later on, which results, in expectation, in a strictly increasing series of expected growth rates. The total value of the firm, as is common in second-best cases, is lower than in the first best scenario.

Our results are a contribution in three aspects. First, valuation requires the prediction of future earnings. Our analysis implies that extrapolation of future earnings in a management-controlled firm might be different from that of a firm managed by its owners, when all other relevant factors are the same. Specifically, the curve of the extrapolated future earnings growth , on the average, is likely to be strictly concave in the first-best case and strictly convex in the second-best case. The main implication for empirical studies is that studies using valuation models and the expectations of earnings growth rates should control for the ownership structure. Specifically, management controlled firms, wherein moral hazard could play a large role, should be expected to generate increasing growth rates. By contrast, owner controlled firms, with a lesser potential for moral hazard, should be expected to exhibit decreasing growth rates.

Second, as well known, there are two types of smoothing: real smoothing – management takes production/investment actions that deliberately reduce the variability of the series of actual earnings— and artificial smoothing -- once economic earnings are realized, the firm can alter the reported earnings through the choice of accounting treatment. While numerous studies analyzed artificial smoothing (e.g., Ronen and Sadan (1981), Penno (1987), Suh (1990), Yaari (1993), Sivaramakrishnan (1994), Chaney and Lewis (1995), Elitzur (1995), Elitzur and Yaari (1995), Evans and Sridhar (1996), Boylan and Villadsen (1997), Demski (1998), Maindiratta Ronen and Srinidhi (2000), and Sankar and

Subramanyam (2000)), only a few studies considered real smoothing. Because our study is concerned with a real cash flow effect, the management of the actual growth rate falls in the realm of real smoothing.

Third, our study reinforces the multi-period nature of earnings management in contrast to a number of studies that look at the correlation between unmanaged earnings and discretionary accruals in a single period context. For example, analytically, the first to link moral hazard to valuation are Ramakrishnan and Thakor (1982,1984). Empirically, Bartov, Tsui, and Gul (2000) conduct a cross-section analysis in order to compare the performance of different models of discretionary accruals management. These studies are moot on the behavior of the series of growth rates of persistent earnings.

To the best of our knowledge, Lambert (1984), Boylan and Villadsen (1997) and Maindiratta, Ronen, and Srinidhi (2000) are the only studies that analyze earnings management in an infinite horizon game, when the firm is modeled as a principal-agent contract. Lambert (1984) analyzes real smoothing as the equilibrium of a principal-agent game between the agent--- the smoother—and the principal, the receiver of the smoothed outcome signal. He studies a two-period game where the agent chooses production effort twice, in response to the contract and his observation of the history of outcomes. Trading-off disutility over effort with utility over monetary reward, the second-period effort is negatively correlated to first-period outcome. This dynamic has two implications: Because it produces a negative correlation between the first and second-period outcomes, it reduces the variability of the total outcome. That is, denoting outcome in period t by x, t=1,2, the total outcome is $y=x_1+x_2$, then, $Var(y)=var(x_1)+var(x_2)+2cov(x_1,x_2) < var(x_1)+var(x_2)$. On the other hand, a high (low) first-period outcome is likely to be followed by a lower (higher) outcome in the second period because of reduced (increased) second-period effort. Hence, the inter-period comparison of outcomes reveals an anti-smoothed path. The difference between our study and Lambert's is that we study the open loop equilibrium of the game (i.e., our agent commits to all decisions at the start of the game). Lambert's results indicate that had we analyzed the closed form equilibrium of this setting (the agent chooses the second-period effort <u>after</u> observing the first-period outcome), the agent's actions would introduce negative correlation into otherwise uncorrelated earnings, which is likely to enhance our Proposition 3.

Boylan and Villadsen (1997), henceforth, BV, study an infinite horizon model, wherein accrual accounting allows the risk-averse, work-averse agent to artificially smooth the accounting reports. Maindiratta, Ronen, and Srinidhi (2000), henceforth, MSR, study the optimal reporting strategy of a manager who cannot access the capital market freely, but can use his private knowledge of future outcome to smooth the consumption stream (in an infinite horizon model), through the artificially smoothed stream of reports. They establish that the manger smoothes the reports and interestingly, the manager's smoothing satisfies the GAAP requirements of consistency, unbiasedness and cash flow convergence.

There are a few differences between these works and ours. First, and foremost, the treatment of agent's actions with the attendant moral hazard is different. BV model the agent's actions as production decisions that affect only the outcome of the period in which they are taken, while we model effort as an investment decision with long-run consequences. In MSR the owners' wealth develops as follows: $x_{t+1}=x_t+cy_t+\varepsilon_{t+1}$, where x is cash flows before payment to the manager, y is a forward-looking value-relevant signal that is observed by the manager alone, and ε is white noise. In our study, y is determined by the manager's effort, which is influenced by incentives set by the principal. Hence, while they focus on the asymmetry of information between owners and the manager, we focus on the moral hazard aspect.

Another difference is that these studies are concerned with artificial smoothing of cash flows, which is driven by their assumption that the risk-averse agent does not have access to the capital market. The smoothing of the report renders the firm a substitute for the imperfect capital market [For some parameters, however, BV's model collapses to a series of one-shot games, where a summary statistic, such as retained earnings, contains all relevant past information; the equilibrium earnings management strategy is to inflate income (see, e.g., their numerical example #1).] Here, we focus solely on real smoothing of growth rates, which are important for valuation. Other differences are technical:

- We break the infinite horizon of the firm into a forecast period and a continuation period, analyzing the investment decisions in the forecast period (a pervasive approach to valuation, see e.g., Copland, Koller, and Murrin (1994)).
- We solve for the open loop of the game, while they solve the closed loop.

The paper proceeds as follows: Section 2 presents the model. Sections 3 and 4 analyze the growth rate manipulation in the first-best and the second-best case, respectively. Section 5 concludes and offers empirical implications.

2. The model

The firm is a going concern, modeled as a principal-agent contract between owners (the principal) and the manager (the agent). The firm generates outcomes that are defined in terms of an AR(1) process, with permanent and transitory components.

2.1. The firm

In each period the firm generates an outcome, x_t, which is either earnings or cash flows, which is comprised of the periodical permanent outcome, x_t^P, and noise,

$$x_t = x_t^P + \varepsilon_t, \qquad (1)$$

where e_t is white noise, normally distributed with zero mean and standard deviation, σ_X. The difference between the actual and the permanent outcome is explained by unavoidable occurrence of non-recurring events, such as a one-time lawsuit by a dissatisfied customer, or gains/losses from discontinued operations. [The assumption that the variance of e_t is independent of the period is innocuous.]

In each period, the firm makes net investment I_t in excess of replacement capital. The investment is a normally distributed random variable, whose mean is determined by the effort of the manager[3], e_t, i.e., $I_t \sim N(ne_t, \sigma_g^2)$, where n is a firm-specific parameter, which, for parsimony, is assumed not to vary with the period. The actual net investment might differ from the level originally planned (despite the manager's effort) because of intervening factors beyond the manager's control that are unknown at the time the investment planning takes place, such as unforeseen delays, unexpected price changes, macro-economic shocks, and so on.

Denoting the rate of return on an incremental investment by r, 0 < r < 1, the net investment in the (t-1)th period, I_{t-1}, increases the next period permanent outcome, x_t^p, by rI_{t-1}. That is,

$$x_t^p - x_{t-1}^p = rI_{t-1}. \qquad (2)$$

In what follows, for tractability and without loss of generality, we assume that r is a constant. Our results can be easily generalized to the case where r is stochastic.

The focus of the analysis is the effect of the manager's choice of effort on the period's growth rate in permanent outcome, g_t, defined as:

[3] Modeling effort as a factor that affects the mean, and not the variance, ensures that demand for smoothing is not driven trivially by the risk-averse player's reluctance to bear risk, as measured by variance. For one-shot games that allow effort to affect the variance of a performance measure, see Meth (1996) and Dye and Demski (1999).

$$g_t \equiv \frac{x_t^p - x_{t-1}^p}{x_{t-1}^p} = \frac{r\, I_{t-1}}{x_{t-1}^p}. \qquad (3)$$

Note that investment could be negative, reflecting the liquidation of previously accumulated assets that increase current-period free cash flows at the expense of future cash flows. For example, a firm may "waste" the goodwill of customers' who have learnt to trust the quality of its products by introducing a low quality good that increases current revenues but sacrifices future revenues.

Equations (1), (2) and (3) define the evolution of permanent outcome, $\{x_t^P\}$, periodical outcome, $\{x_t\}$, and the free cash flows that are distributed to shareholders as dividends, $\{F_t\}$, as follows:

$$x_t^P = x_{t-1}^P(1+g_t). \qquad (4)$$

$$x_t = x_{t-1}^P(1+g_t) + e_t. \qquad (5)$$

$$F_t = x_t - I_t = x_t^p\left(1-\frac{g_{t+1}}{r}\right) + e_t. \qquad (6)$$

2.2. Valuation

As is common in the valuation literature (see e.g., Copland, Koller, and Murrin (1994)), we divide the game's horizon into two periods: a 'forecast' phase and a 'continuation' phase. The former lasts two periods, t=T,T+1, and the latter starts at the beginning of period t=T+2.

Denote by $\hat{i}_t = \prod_{s=1}^{s=t}(1+i_s)$, where i_s is the risk-averse shareholders' discount rate in period s, s=1,2,…,t, adjusted for a premium for risk. Since risk varies from one period to another, we recognize that the discount rate is period-dependant. Then, the gross value of the firm (before deduction of the manager's compensation cost) at the beginning of period T, V_T, is:

$$V_T = E_{\boldsymbol{e}_t, I_t}\left[\sum_{t=T}^{t=T+1} \frac{F_t}{\hat{i}_t} + \sum_{t=T+2}^{\infty} \frac{F_t}{\hat{i}_t}\right]. \qquad (7)$$

The first term is the net present value of cash flows in the forecast phase; the second is the continuation value, discounted back to the beginning of period T.

In what follows, we state variables at their beginning-of-the-period value. Since cash flows are distributed at the end of the period immediately after the publicization of the financial reports, we discount them to the beginning of the period.

Upon substituting (6) into (7), we obtain:

$$V_T = E\left[\sum_{t=T}^{t=T+1} \frac{x_t^p(1-\frac{g_{t+1}}{r}) + \boldsymbol{e}_t}{\hat{i}_t} + x_{T+2}^p \frac{\bar{V}_{T+2}}{\hat{i}_{T+1}}\right], \qquad (8)$$

where $\bar{V}_{T+2}$ is the continuation period value of the firm per unit of the T+2[th] period's permanent income, x_{T+2}^p, discounted to the beginning of period T+2 [Denoting by $\bar{g}$ and i, the stationary long-run growth rate of firm's earnings and the owners' discount rate, by (6) and the formula for a sum of an infinite converging series, $\bar{V}_{T+2} = \frac{1-\frac{\bar{g}}{r}}{i-\bar{g}}$].

Substituting equation (4) and (6) into (8) and taking expectations with respect to $\boldsymbol{e}_t$, yields:

$$V_T = x_{T-1}^p(1+g_T)\, E\left[\frac{1-\frac{g_{T+1}}{r}}{1+i_T} + \frac{(1+g_{T+1})(1-\frac{g_{T+2}}{r})}{(1+i_T)(1+i_{T+1})} + \frac{(1+g_{T+1})(1+g_{T+2})}{(1+i_T)(1+i_{T+2})}\bar{V}_{T+2}\right]. \qquad (9)$$

We assume that the market learns the permanent outcome with a one-period lag. This component of earnings is revealed by supplemental explanations provided by the management, such as the Management Discussion & Analysis; analysts' evaluation of the income statement, and the whisper and winks game between management and selected followers (Levitt (1998)). This feature implies that equation (9) is well defined since the market observes both x^{P}_{T-1} and I_{T-1}, which determines g_T.

2.3. The Manager

The goal of this study is to analyze the manager's choice of effort, e_T and e_{T+1}, since these decisions affect g_{T+1} and g_{T+2}, respectively.

At the beginning of each sub-period in the forecast phase, the manager exerts unobservable investment-relevant effort. The manager is risk-averse and effort-averse. His utility function, U_t, is separable in von Neumann-Morgenstern utility function over monetary compensation, U, and in disutility, $C(e_t)$, over effort, $\hat{e} \quad e_t \quad 0$.

Denoting the periodical compensation schedule by S_t, $S_t \quad S_0 = 0$,[4] and the manager's discount factor by r, the manager derives utility over the total discounted stream of monetary payoffs starting in period T, $\sum_{t=T}^{\infty} \frac{S_t}{(1+r)^{t-T+1}}$. In what follows, we make the standard assumption that U is a continuous monotone increasing, strictly concave, function.

[4] The economic implication of the boundary condition, $S_t \quad S_0$ (presumably because the manager has limited liability), is that a first-best contract supported by a threat of penalties is not feasible. [For a paper that expands on the effect of limited liability on the shape of the principal-agent contract in a binary setting, consult Ronen and Yaari (2001).] We do not assume that owners enjoy limited liability, even though in reality, owners in incorporated companies do [see Fischer and Verrecchia (1997)]. Instead, we make the weaker assumption that the agent's reservation utility is sufficiently low so as to guarantee that the expected owners' residual share, i.e., the present value of the expected free cash flows net of the manager's compensation, is non-negative.

Effort generates periodical disutility of $C(e_t)$, where $C(\cdot)$ is twice continuously differentiable, increasing, convex function, normalized so that $C(0)=0$. To ensure an interior solution, we assume that $C'(0)=0$ and $C'(\hat{e})=\infty$.[5]

The manager's expected utility at the beginning of period T, is:

$$E[U_T] = E_{e_t, I_t}\left[U\left(\sum_{t=T}^{\infty}\frac{S_t}{(1+r)^{t-T+1}}\right)-\sum_{t=T}^{\infty}\frac{C(e_t)}{(1+r)^{t-T}}\right], \qquad (10)$$

In what follows, we assume that the manager is willing to contract with the owners because the expected utility as defined in (10) guarantees him at least his strictly positive reservation utility during his tenure.

Realistically, managers that are not performing well might be laid off. We assume that the manager's welfare is tied to the firm's stock performance even after he quits the firm, either because of the manager's golden parachute (his post-retirement benefit plan), or because he is not being laid off immediately so that he completes his tenure during a 'forecast period,', which is relatively short.

2.4. Technicalities

We make the following technical assumptions: (i) r > i_t, and (ii) $\bar{g}$ < I, where $\bar{g}$ is the long-run stationary growth rate in the continuation phase. The first condition implies that

[5] The principal-agent paradigm developed the first-order approach to solving the principals' program (Rogerson (1984), Jewitt (1991)). Namely, the incentive compatibility constraint in the principal's program (that states that the compensation schedule induces the manager to exert effort) is replaced by the first-order condition of the manager's objective function with respect to effort. We too use the first-order-approach. However, the normal distribution does not fulfill the conditions specified in Rogerson, and the conditions in Jewitt are rather tedious and involve restrictions on the compensation schedule. We legitimize the use of the first-order approach by the combination of uniqueness of the solution the principal's program (hence, the necessary conditions are sufficient) and our boundary conditions on the manager's effort (which ensure an interior solution, whereby the first-order conditions of the agent's utility function with respect to effort are indeed satisfied).

a positive investment is profitable from the owners' perspective, and the second condition ensures a finite firm's value in the continuation phase.

Our analysis focuses on the decisions made during the forecast period. In what follows, we assume that the manager commits to all decisions at the beginning of the game. That is, he chooses e_{T}, e_{T+1}, at the beginning of the Tth period, and shareholders sign a two-period contract with the manager.[6] Beyond that, the best estimate of the firm's value and growth rate is based on the combination of the industry-wide average and firm-specific ability to generate excess earnings. Hence, by (3), the growth rate in period t, g_t, given x^{p}_{t-1}, is a normally distributed stochastic variable conditional on x^{p}_{t-1} with mean, m_t, and variance, t^2_t, as follows:

$$m_t = E[\,g_t \,|\, x^{p}_{t-1}\,] = \frac{r}{x^{p}_{t-1}} E(I_{t-1}) = \frac{r\, n_{t-1} e_{t-1}}{x^{p}_{t-1}}, \qquad (11)$$

and variance,

$$\mathrm{var}[\,g_t \,|\, x^{p}_{t-1}\,] = t^2_t = \left(\frac{r}{x^{p}_{t-1}}\right)^2 \mathrm{var}(I_{t-1}) = \left(\frac{r}{x^{p}_{t-1}}\right)^2 ó^2_g \,. \qquad (12)$$

3. The Benchmark case: the owners-controlled firm.

In this section, we analyze a benchmark case, wherein there is no conflict of interest between the owners and the manager, because the shareholders manage the firm. [This case is realistic in companies where management buys out the firm and in companies whose

[6] The contract may include options with an exercise price that depends on the market price, a bonus that depends on accounting earnings, and a fixed salary. The one-shot studies by Bushman and Indjejikian (1993), and Kim and Suh (1993) show that the manager's contract should include an accounting-based bonus and options whose value depends on the market price; Baiman and Verrecchia (1995) find that it should include only options. These studies constitute a special case of Banker and Datar's (1989) result, which shows that, when a principal

officers are the entrepreneurs who established them.] The comparison of this case to the principal-agent setting highlights the implication of “management” of the growth rates.

Denoting by c the monetary equivalent of the manager’s personal cost of exerting effort, C(.), the choice of efforts during the forecast period maximizes the expected value of the firm at the beginning of period T less the cost of effort, $E[V_T] - \sum_{t=T}^{t=T+1} \frac{c(e_t)}{(1+i_o)^{t-T}}$, where i_o is the risk-free discount rate of owners. In what follows, without loss of generality, we set $i_o = 0$. Since in the continuation period, the optimal effort is uniquely determined by the long-run growth rate, we are concerned solely with the characterization of $\{e_T, e_{T+1}\}$, which stochastically determine $\{g_{T+1}, g_{T+2}\}$.

The efforts are chosen to maximize:

$$\max_{e_T, e_{T+1}} \; E[V_T] - c(e_T) - c(e_{T+1}) \qquad (13)$$

In the proof of Proposition 1, we develop (13). The first-order-conditions of the owner’s objective function with respect to the two effort levels, after rearranging, are:

$$e_T: \qquad \frac{r\,n}{(1+i_T)(1+i_{T+1})} + \frac{r\,n\,\bar{V}_{T+2}}{(1+i_T)(1+i_{T+1})} = c'(e_T) + \frac{n}{(1+i_T)}. \qquad (14)$$

$$e_{T+1}: \qquad \frac{r\,n\,\bar{V}_{T+2}}{(1+i_T)(1+i_{T+1})} = c'(e_{T+1}) + \frac{n}{(1+i_T)(1+i_{T+1})}. \qquad (15)$$

As expected, the owners equate the marginal benefit with the marginal cost. The cost is made of both the reduction in current free cash flows available for dividend payments, and the direct cost of expending effort.

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has two noisy signals, and neither is a sufficient statistic of the other, he would use both in the design of the contract.

Mathematically, we see that the benefit of e_T (the left-hand-side of (14)) includes the returns on the investment in the remainder of the forecast period plus the long run horizon, while the benefit of e_{T+1} (the left-hand side of (15)) affects only the long run value. On the other-hand, the cost of expending effort in period T+1 (on the right-hand-side of (15)) is discounted by the work-averse owner-manager relative to effort expended in period T (the right-hand side of 14)).

Proposition 1:

(a) The higher n, the higher the equilibrium effort in period T(T+1).

(b) The equilibrium effort in period T, e_T, increases with r and decreases with i_t, t=T,T+1. The equilibrium effort in period T+1, e_{T+1}, increases with r, and decreases with i_t, t=T,T+1.

Proof: See Appendix.

Proposition 1 provides the comparative statics of the equilibrium effort levels during the forecast period. The effort is chosen by balancing future benefit from the yields of the present investment against the cost, as discussed above. The higher r, the higher the marginal benefit, which renders expending effort more attractive. The higher n, and the lower i_t, the higher the marginal benefit and the marginal cost, but because of the long-run repercussions of the marginal increase in these variables, the net effect is an increase in the net benefit, and hence, the net effect induces higher effort.

The fact that in each period the effort's choice reflects the trade-off between cost and benefit (measured as marginal productivity) is important for understanding the temporal choice of effort. Specifically, while the first-period's effort impacts both periods' outcomes plus the continuation value, the second-period's effort impacts only the second period's outcome plus the continuation value. Hence, the marginal productivity of effort in the first period is higher and consequently, the first-period effort is likely to be higher (see Lemma 1 below), which, as we shall show below, translates to an increasing series of growth rates of the permanent outcome (see Proposition 2 below).

Lemma 1: Effort in the T^{th} period is higher than in the $T+1^{th}$ period.

Proof: See Appendix.

Lemma 1 characterizes the temporal choice of effort. It establishes that the owners-managers prefer early effort to be higher than the later effort. [The intuition is clear from the above discussion. At the risk of repetition, in the medium range of time, the distinction between early and late effort is meaningful, since an earlier exertion of effort adds one more period of expected investment productivity, which makes the owners wealthier, and hence is more desirable (under our assumption that $r > i$). In the long run, in contrast, there is no difference between the benefits derived from the two effort levels. A growing firm achieves its growth by augmenting assets over time; hence, early effort has multiplicative effects on future growth.

Proposition 2: The expected ratio of the expected growth rates in period T+1 and T+2, conditional on positive x_T^P, exceeds 1, i.e., $E\left[\frac{m_{T+1}}{m_{T+2}}\middle| x_T^P, x_T^P > 0\right] > 1$.

Proof: See Appendix.

Proposition 2 states that in expectation, the expected growth rate in period T is higher than in period T+1, i.e., the series of expected growth rates is expected to be strictly concave.

At first glance, it seems that Proposition 2 is a corollary to Lemma 1: since expected effort in the first period of the forecast period is higher than the second-period effort, so will be the expected difference in persistent income. However, this effect determines only the numerator of the growth rate expression (see equation (3)), not the denominator.

We emphasize that the result in Proposition 2 is restricted to the temporal behavior of the expected ratio of the expected growth rates, which implies that the realized ratio of the expected growth rates may behave in an opposite fashion.

It can be shown that in a world where the firm-specific parameter, n, is period-dependant and i_o is positive, our results hold when the future value of the firm-specific parameter, $n_{T+1}(1+i_o)$, is weakly lower than the present firm-specific parameter's value, n_T.

4. The management-controlled firm

4.1. *The principal-agent contract*

In this section, we analyze the behavior of the series of growth rates during the forecast period when the firm is controlled by management. The owners affect the decision-making of management by designing an incentive contract at the beginning of period T. The compensation scheme is a mechanism designed to overcome, at a cost, the problem that the owners cannot induce the manager to exert the unobservable effort by direct means, (see, e.g., Holmstrom (1979)). The basic premise of the literature on incentives is that the principal designs a contract that aligns the manager's incentives with his objectives, by, for example, endowing managers with stock options (see e.g., Scott Ch. 10). Since owners wish to maximize the expected value of the firm, the total compensation of the manager in the forecast period, $S = \frac{S_T}{1+r} + \frac{S_{T+1}}{(1+r)^2}$, is designed as a function of the firm's value and free cash flows. That is, S_T is a function of free cash flows and firm's value in period T, F_T and V_T (F_T

is defined in equation (6), V_T is defined in equations (7), (8), and (9)), while S_{T+1} is a function of the same variables for the period T and T+1, F_T, V_T, F_{T+1}, and V_{T+1}.

To present the owner's program, we denote variables that pertain to the continuation phase by upper bar (with the exception of the owners' discount rate, i). For example, $\bar{S}$ is the manager's compensation in the long run, $\bar{e}$ is the long-run effort, and $\bar{U}o$ is the discounted long-run reservation utility of the manager, which is a given parameter. At the beginning of period T, the owners design the contract that maximizes firm's value, net of the expected manager's compensation, V_T^N, by solving the following program:

$$\max_{\{S(x_t)\}_{t=T}^{t=T+1}} E[V_T - S]$$

s.t.

$$E_{e_t, I_t}[U(W)] - \sum_{t=T}^{T+1} \frac{C(e_t)}{(1+r)^{t-T}} \geq Ko, \qquad \text{(PC)}$$

$$\{e_t\}_{t=T}^{t=T+1} \in \arg\max E_{e_t, I_t}[U(W)] - \sum_{t=T}^{T+1} \frac{C(e_t)}{(1+r)^{t-T}}. \qquad \text{(IC)}$$

$$S_T \geq 0, S_{T+1} \geq 0.$$

where: $Ko = Uo + \bar{U}o + \sum_{t=T+2}^{\infty} \frac{(1+r)C(\bar{e})}{(1+r)^{t-T+1}}$.

$$E[V_T - S] =$$

$$E\left[\frac{F_T - S_T(V_T, F_T)}{(1+i_T)} + \frac{F_{T+1} - S_{T+1}(V_T, V_{T+1}, F_T, F_{T+1})}{(1+i_T)(1+i_{T+1})} + \sum_{t=T+2}^{t=\infty} \frac{F_t - \bar{S}}{(1+i_T)(1+i_{T+1})(1+i)^{t-T+1}} \middle| x_{T-1}^P\right].$$

$$W = \frac{S_T(V_T, F_T)}{1+r} + \frac{S_{T+1}(V_T, V_{T+1}, F_T, F_{T+1})}{(1+r)^2} + \sum_{t=T+2}^{\infty} \frac{\bar{S}}{(1+r)^{t-T+1}}.$$

The shareholders maximize the firm's value, net of the agent's compensation, subject to the (PC), (IC), and the non-negativity constraints. The (PC) constraint states that the expected compensation guarantees that the agent's expected utility equals the total of his reservation utility during the 'forecast phase, Uo, where Uo 0, and his reservation utility during the continuation phase, $\bar{\text{U}}$o where $\bar{\text{U}}$o 0, so that he is willing to participate in the contract. The (IC) constraint states that the agent chooses effort levels that maximize his expected utility.

Note the difference in the compensation contracts during the forecast and the continuation periods. In the short run, the owners must design an incentive scheme to motivate the effort-averse manager to exert effort. Beyond the forecast period, however, the shareholders can offer a 'bang-bang' contract: A constant payment, $\bar{S}$ that guarantees the manager's reservation utility, $\bar{\text{U}}$o, when he exerts effort that guarantees on the average $\bar{g}$, accompanied with the threat that if performance drops below a certain level, he is laid off. Because of the infinite horizon of the principal-agent relationship beyond the forecast period, a first- best can be achieved wherein the risk-averse agent is paid a constant salary (see Rubinstein and Yaari (1983), and Radner (1985)). Since the compensation in the continuation period is not affected by decisions made during the forecast period, in what follows, we suppress $\sum_{t=T+2}^{\infty} \frac{\bar{S}}{(1+r)^{t-T+1}}$ in the utility function of the manager.

Lemma 2:

Denote by U′ the marginal utility of the manager with respect to total compensation, and by ë, $ö_T$, and $ö_{T+1}$, the Lagrange multipliers of (PC) and (IC) in periods T and T+1, respectively.

(a) The equilibrium contract satisfies the following pointwise conditions:

For $S_t > 0$, t=T,T+1,[7]

$$S_T: \qquad \frac{1+\boldsymbol{r}}{(1+i_T)\,U'(S_T)} = \boldsymbol{l} \quad \boldsymbol{j}_T \frac{F_T - E[F_T]}{Var(F_T)} + [r\ \boldsymbol{j}_T - \boldsymbol{j}_{T+1}]\frac{F_{T+1} - E[F_{T+1}]}{Var(F_{T+1})} + r\,[\boldsymbol{j}_T + \boldsymbol{j}_{T+1}]\frac{F_{T+2} - E[F_{T+2}]}{Var(F_{T+2})} \qquad (16)$$

$$S_{T+1}: \quad \frac{(1+\boldsymbol{r})^2}{(1+i_T)(1+i_{T+1})\,U'(S_{T+1})} = \frac{1+\boldsymbol{r}}{(1+i_T)\,U'(S_T)} \qquad (17)$$

(b) The contract includes resettling. That is, the principal designs a single compensation schedule that specifies the total compensation in the forecast phase. Any payment in periods T and T+1 is an advance that is deducted from the total compensation paid in period T+2.

Proof: <u>See Appendix.</u>

As well known in the literature, the payment is a monotone function of outcomes. Note that since effort in period T decreases the expected cash flows in period T, we obtain a negative sign on the free cash flows in the forecast period. Also, because the total payment matters, rather than how it is divided between the two periods (as long as it included the required imputed discount rate), the T+1 schedule is proportional to the T-period schedule, reflecting the different attitudes of owners and manager towards discounting time.

The fact that the principal actually designs only one contract that settles the accounts with the agent ex-post is driven by the perfection of the capital market[8]: our assumption that the manager derives utility over total compensation, and not over the stream of compensation

[7] The following first-order-conditions are given for non-binding limited-liability constraints, which are: $\forall\ S_t$, $U(S_t)$ $U(S_0)=U(0)$, t=T,T+1.

[8] For more on this point, see the discussion in Yaari (1993). She studies the firm as a finite-horizon repeated principal-agent contract with no moral hazard between owners (the principal) and management (the agent) where the firm can borrow/lend at better terms than the manager (the agent). She also finds that the access to the capital market explains the principal designing a single schedule that settles the agent's payment at the end of the game, after all performance variables are realized.

payments, presumes a frictionless capital market that allows the manager to optimize his consumption trajectory over time by borrowing/lending in the capital market at the same rate as his subjective discount factor, *ñ*. The importance of the perfection of the capital market for artificial (cosmetic) smoothing is well recognized. [See, e.g., Maindiratta, Ronen, and Sindhi (2000).] Because the capital market is imperfect, in each period the agent must consume what he earns in that period. The agent's demand for consumption smoothing motivates the demand for smoothed reported outcomes (for further discussion of this issue, consult Demski (1988)). Our characterization of the manager's utility in this way guarantees that our results are not motivated by his demand for consumption smoothing. As a repeated principal-agent game, this feature distinguishes our study from Lambert (1983) and Rogerson (1985a), whose repeated principal-agent contract takes place in an imperfect capital market environment

4.2. The Equilibrium

In this subsection, we analyze the effect of moral-hazard on valuation, working out the characteristics of the solution in a parallel fashion to Section 3.

Lemma 3: By our assumption that the reservation utility of the agent in the forecast period is non-negative:

(a) Let i_t < ñ. Then, the manager exerts a lower level of effort in each period than owners-managers do if:

$$\sum_{t=T}^{t=T+1} \frac{E(F_t)}{(1+r)^{t-T+1}} - U\left(\sum_{t=T}^{t=T+1} \frac{E(F_t)}{(1+r)^{t-T+1}} \right) > 0.$$

(b) The manager exerts more effort in period T+1 than in period T.

Proof: See Appendix.

Part (a) of Lemma 3 states that the manager exerts less effort than owners-managers do if his utility function is sufficiently concave and he is less patient. This result is driven by a free-rider problem, lower patience, and the manager's risk aversion. Because (a) the risk-averse manager does not reap the full benefit of his effort (i.e., the free-riding problem inherent in principal-agent relationship, see, e.g., Holmstrom (1982)); and (b) is less tolerant to waiting for the benefit of the investment (i_t< ñ), his incentives to exert effort are weaker than those of the owners-managers, as studied in the previous section. Moreover, this effect is exacerbated by the risk aversion of the manager. That is, when his utility function is sufficiently concave, the marginal benefit from a dollar reward declines rapidly and thus decreases the benefit from exerting effort.

This result requires some conditions: first, the manager does not receive all the free cash flows. While the result that the principal and the agent share outcome was proved in a one-shot game by Rogerson (1985b), we are unaware of such a proof in a repeated game. It stands to reason that it applies here as well, or else, the owners would liquidate their equity. Second, the owners are more patient than the manager in waiting for the benefits from investment, i_t< ñ, and hence are more willing to wait to reap the fruit of current efforts; and third, the concavity condition. This last condition depends on the manager's limited-liability level, his non-negative reservation-utility level, and on the specification of his utility function. It holds for a large menu of utility functions, including: logarithmic utility functions, U(z)=ln(z), z> 0, the constant relative risk-aversion measure utility function, $U(z)=z^{1/2}$, z 0, and the constant absolute risk-aversion coefficient (CARA) utility functions, $U(z)=Ko - exp^{âz}$ with an intercept, Ko, that is not too large, when limited liability bars negative payments.

Part (b) of Lemma 3 contains a surprising result. The temporal choice of effort by the manager is contrary to the preferences of the owners. Because of the favorable wealth effect of the first-period effort in the forecast period relative to the second-period effort, the agent prefers to trade-off effort for monetary compensation and expends less effort in the first period. This is true regardless of the conditions specified in part (a). While this dynamic is not a surprise by itself, its consequences are a surprise. We expect a divergence between the owners' effort and the manger's chosen efforts, but not one as dramatic as we find: the inter-temporal pattern is contrary to the owner's wishes.

In what follows, we denote the effort when owners-managers choose effort by e_t^F, F for first-best, and the effort when the manager-agent chooses effort by e_t^S, S for second-best. Denoting the equilibrium levels by stars, it is immediate to see that Lemmas (1) and (2) imply: $e_T^{F*} > e_{T+1}^{F*} > e_{T+1}^{S*} > e_T^{S*}$.

To illustrate the difference between the first- and the second-best cases, we provide numerical examples. We assume that: $U(z)=\sqrt{z}$; ñ= 0.25; $i_T= i_{T+1}$ =0.2; i=0.1, $\bar{g}$ =3.06%; n=1 r=0.1; $C(e_t)=\frac{(e_t)^2}{2}$; $\bar{S}$ =0, all variances equal 0.1 and the permanent earnings in period T-1 equal 2 (the explicit expressions for all relevant variances are presented in Table 2 in the proof of Lemma 2).

In the principal-agent setting, we use Lemma 2 part (b). Without loss of generality, we solve the example when the agent is paid once, at the end of period T+1, i.e., S_T=0.

TABLE 1: NUMERICAL EXAMPLES

	First Best	**Second Best**
Effort in period T	9.40	2.7
Effort in period T+1	9.31	3.0

The numerical examples illustrate that effort is much higher in the first-best case than in the principal-agent game, and that the intertemporal patterns of effort are opposite to each other. While effort in period T is higher than the effort in period T+1 in the first-best game, it is lower than the effort in period T+1 in the second-best scenario.

Proposition 3: The expected ratio of expected growth rates in the principal-agent setting is lower than that in the first-best setting. That is,

$$E\left[\frac{m^F_{T+1}}{m^F_{T+2}} \middle| x^P_T, x^P_T > 0\right] > E\left[\frac{E(m^S_{T+1})}{E(m^S_{T+2})} \middle| x^P_T, x^P_T > 0\right].$$

Proof: See Appendix

Proposition (3) is similar to Proposition (2) and is a corollary of part (b) of Lemma 3. Had it not been for a denominator effect in the formula of the growth rate that reduces the expected ratio of the expected period T+2 growth rate relative to the expected period T+1 growth rate, the expected ratio of expected growth rates in the first (second) best would exceed (be lower than) one.

Our proposition has empirical implications. That is, the time series of growth rates of firms that are run by owners is different from those of managed-controlled firms. Hence, the difference between the two types of firms can be used to test indirectly for the efficiency of incentives. For example, are managers who hold a higher portion of the firm's shares or simply hold more shares, more likely to exhibit the same pattern as owners-controlled firms?

We now turn to the discussion of the variability of the temporal growth rates.

Observation:

Let the conditions in Lemma (3) part (a) hold. Then, the expected precision, i.e., the expected reciprocal of variance, of the periodical growth rates of an owners-controlled firm, $g^{\mathrm{F}}_{\mathrm{T+1}}, g^{\mathrm{F}}_{\mathrm{T+2}}$, is higher than the expected precision of a management-controlled firm's growth rates, $g^{\mathrm{S}}_{\mathrm{T+1}}, g^{\mathrm{S}}_{\mathrm{T+2}}$.

Proof:

By equation (12), the precision is positively proportional to $(x^{\mathrm{p}}_{\mathrm{t\text{-}1}})^2$. Since, by Lemma 3(a), the manager-agent exerts less effort in each period during the forecast period, the second-best expected permanent earnings are lower on the average; hence, the precision is lower at periods T+1, T+2. Taking expectations over the precision completes the proof. Q.E.D.

This observation shows that the expected precision (the reciprocal of variance) of the second-best growth rate in each period is lower than the first-best. [We cannot compare the variances of the two regimes; it is not possible to take expectations over the reciprocal of persistent income, $(x^{\mathrm{p}}_{\mathrm{t\text{-}1}})^2$ because when $x^{\mathrm{p}}_{\mathrm{t\text{-}1}}$=0, the integral of the Normal distribution does not converge.] Our result is an artifact of the fact that second-best effort is lower, which decreases expected permanent earnings. Since the growth rate is a pure number that is calibrated by the permanent earnings size, the precision of a growth rate is, by definition, an increasing function of the square of the permanent earnings size. Our finding reflects this size effect. If we were to divide precision by the square of the permanent earnings size, we would obtain no difference between the first-best and the second-best cases, which is consistent with our modeling of effort as affecting the mean investment only.

1. Summary and empirical implications

Valuation depends on the growth rates of persistent earnings and cash flows, which, in turn depends on the investment decisions of the firm. In this study, we model the firm as a multi-period principal-agent contract in order to investigate the management of the growth rate of persistent earnings by the manager (the agent), through his unobservable investment decisions. Our main findings and the conclusions for empirical research are:

- The expected absolute value of a management-controlled firm is lower than the expected value of an owner-controlled firm.
- The expected ratio of growth rates of an owners-controlled firm decreases over time, while the expected ratio of growth rates of a manager-controlled firm increases. Consequently, the extrapolation of expected future growth-rates is strictly concave for owners-controlled firms and strictly convex for management-controlled firms.
- Management-controlled firms decrease the precision of temporal growth rates. When the variability of growth rates is multiplied by the persistent earnings size, this effect disappears.

Our study has a number of empirical implications:

- The extrapolation of future earnings in management-controlled firm is strictly convex, on the average, in contrast, to the strictly concave extrapolation of the owners-controlled firms.
- Given that different industries are characterized by different life-cycles (Fine (1998)), we expect that the results of this study will be more pronounced in industries with longer cycles.

- given the current global economic crisis, our results predict that the current expectations for an economic slow-down will accelerate a decline in growth rates in owner-controlled firms.

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APPENDIX

Proof of Proposition 1:

Preliminary

The objective function of the owners is:

$$E[V_T]-c(e_T)-c(e_{T+1})=\frac{E[x_T-I_T]}{(1+i_T)}+\frac{E[x_{T+1}-I_{T+1}]}{(1+i_T)(1+i_{T+1})}+\frac{E[x_{T+2}]\bar{V}_{T+2}}{(1+i_T)(1+i_{T+1})}-c(e_T)-c(e_{T+1})=$$

$$\frac{x^P_{T-1}+\ r\,n\,e_{T-1}\ -n\,e_T}{(1+i_T)}+\frac{x^P_{T-1}+\ r\,ne_{T-1}+r\,n\,e_T-n\,e_{T+1}}{(1+i_T)(1+i_{T+1})}+$$

$$\frac{(x^P_{T-1}+r\,n\,e_{T-1}\ +r\,n\,e_T+r\,n\,e_{T+1})\bar{V}_{T+2}}{(1+i_T)(1+i_{T+1})}-c(e_T)-c(e_{T+1}).$$

The first-order conditions with respect to effort are given by (14) and (15).

Proof

The proof of parts (a) and (b) is based on the first-order-conditions with respect to effort levels. Rearranging (14) and (15), yields,

$$n\left(\frac{-1}{(1+i_T)}+\frac{r}{(1+i_T)(1+i_{T+1})}+\frac{r\bar{V}_{T+2}}{(1+i_T)(1+i_{T+1})}\right)=c'(e_T).$$

$$n\left(\frac{-1}{(1+i_T)(1+i_{T+1})}+\frac{r\bar{V}_{T+2}}{(1+i_T)(1+i_{T+1})}\right)=c'(e_{T+1}).$$

Note that our assumptions that the owners' long-run discount rate is larger than the long-term growth rate guarantees that the left-hand-side (l.h.s) of the first-order-condition is positive and hence, the T+1 period's effort yields a solvable equation.

The equilibrium effort level of each period equates the marginal benefit (written on the left-hand side) with the marginal cost (on the right-hand-side). The higher the effort, the higher the marginal benefit, and the higher the marginal cost, which, by the assumption on the convexity of the cost function over effort implies that the level of effort I higher. Since the marginal benefit for both effort levels increases in n, and decreases in i_t, t=T,T+1, the proof is complete. Q.E.D.

Proof of Lemma 1:

Multiply and divide the first term in (14) by $(1+i_{T+1})$. Subtract (15) from (14), to obtain:

$$\frac{n}{(1+i_T)(1+i_{T+1})}\{(1+i_{T+1})\ +r\ -1\} = c'(e_T) - c'(e_{T+1}).$$

Since, by assumption, $r > i_t$, $c'(e_T) > c'(e_{T+1})$, which by the convexity of the cost function over effort, implies that $e_T > e_{T+1}$. Q.E.D.

Proof of Proposition 2:

By definition of m_t, $m_t = r\,n\,\frac{e_{t-1}}{x^p_{t-1}}$, the ratio of expected growth rates is,

$$\frac{m_{T+1}}{m_{T+2}} = \frac{e_T}{e_{T+1}}\frac{x^p_{T+1}}{x^p_T} = \frac{e_T}{e_{T+1}}\frac{x^p_T + rI_T}{x^p_T}.$$

Hence, $E\left[\frac{m_{T+1}}{m_{T+2}}\right] = \frac{e_T}{e_{T+1}}\left[1+\frac{r\,n\,e_T}{x^p_T}\right] > 1+\frac{r\,n\,e_T}{x^p_T} > 1.$

The first inequality obtains by Lemma 1. Q.E.D.

Proof of Lemma 2:

Preliminaries:

Part (a):

The relevant distribution functions on Date T are: F_T, F_{T+1}, F_{T+2}, are normal distributions, whose parameters are summarized in the Table 2.

TABLE 2: Summary of the Relevant Distribution Functions

	Mean	Variance
F_T	$x^P_{T-1}(1+m(g_T \mid x_{T-1})) - n\, e_T$	$(x^P_{T-1})^2 s^2 + s_g^2 + s_e^2$
F_{T+1}	$x^P_{T-1}(1+m(g_T \mid x_{T-1})) + rn\, e_T - n\, e_{T+1}$	$(x^P_{T-1})^2 s^2 + (1+r^2) s_g^2 + s_e^2$
F_{T+2}	$x^P_{T-1}(1+m(g_T \mid x_{T-1})) + rn\, e_T + r\,n\, e_{T+1} - \overline{I}$	$(x^P_{T-1})^2 s^2 + 2r^2 s_g^2 + s_e^2$

<u>The (IC) constraints are</u>:

We denote by $f(F_t)$ the density function of F_t, and by $f_{e_t}(F_t)$ its derivative with respect to e_t. We derive the first-order condition of the manager's objective function with respect to e_T and e_{T+1}, to find the temporal choice of effort levels in period T, and T+1, respectively. Since we employ the first-order approach, (IC) are the first-order conditions of the manager's expected utility function with respect to effort as follows:

e_T :

$$\iiint U\left(\frac{S_T(V_T, F_T)}{1+r} + \frac{S_{T+1}(V_T, V_{T+1}, F_T, F_{T+1})}{(1+r)^2}\right) f'_{e_T}\, dF_T dF_{T+1} dF_{T+2} - C'(e_T) = 0,$$

where: $f'_{e_T} = f_{e_T}(F_T) f(F_{T+1}) f(F_{T+2}) + f(F_T) f_{e_T}(F_{T+1}) f(F_{T+2}) + f(F_T) f(F_{T+1}) f_{e_T}(F_{T+2})$.

e_{T+1} :

$$\iiint U\left(\frac{S_T(V_T, F_T)}{1+r} + \frac{S_{T+1}(V_T, V_{T+1}, F_T, F_{T+1})}{(1+r)^2}\right) f'_{e_{T+1}}\, dF_T dF_{T+1} dF_{T+2} - \frac{C'(e_{T+1})}{1+r} = 0,$$

where: $f'_{e_{T+1}} = f(F_T) f_{e_{T+1}}(F_{T+1}) f(F_{T+2}) + f(F_T) f(F_{T+1}) f_{e_{T+1}}(F_{T+2})$.

<u>Part (a)</u>:

Since the distribution functions of free cash flows are normal, their derivative with respect effort is:

$$f_{e_T} = \frac{\partial f(F_t)}{\partial e_t} = \left[\frac{F_t - E[F_t]}{\text{var}(F_t)}\right] \frac{\partial E[F_t]}{\partial e_t} f(F_t).$$

Substituting in (IC) and solving the Euler equation associated with the principal's program yields the required result.

Part (b):

The equilibrium conditions reveal that the payment for the performance at period T (resp. T+1) depends (pointwise) on the, yet unrealized, outcomes of period T+1 and T+2 (resp. T+2). Hence, the compensation package of the manager involves an advance in period T (resp. T+1) and a final settlement in the period T+2 (such as the case with options[9], bonus banks, and other long-term payments arrangements). De-facto, then, the principal designs one compensation schedule only that is based on the total performance by the beginning of the continuation phase. Q.E.D.

Proof of Lemma 3:

Preliminaries

Step 1:

By the preliminaries of Lemma 2, subtracting the first-order condition with respect to effort in period T+1 from the first-order condition with respect to effort in period T, yields:

[9] There are numerous arrangements: stock options, stock appreciation rights (similar to stock options except that upon exercise the holder receives cash rather than shares), restricted stock (common stock that carries restrictions on transferability for a stated period of time), and phantom stock (similar to restricted stock, except that at the expiration of the restrictions the manager receives the cash value of shares rather than stock).

$$\iiint \left[U\left(\frac{S_T(V_T, F_T)}{1+\boldsymbol{r}} + \frac{S_{T+1}(V_T, V_{T+1}, F_T, F_{T+1})}{(1+\boldsymbol{r})^2} \right) \right] \times$$

$$[f_{e_T}(F_T)f(F_{T+1})f(F_{T+2}) + f(F_T)f_{e_T}(F_{T+1})f(F_{T+2}) + f(F_T)f(F_{T+1})f_{e_T}(F_{T+2}) -$$

$$f(F_T)f_{e_{T+1}}(F_{T+1})f(F_{T+2}) + f(F_T)f(F_{T+1})f_{e_{T+1}}(F_{T+2})]dF_T dF_{T+1} dF_{T+2} - C'(e_T) + \frac{C'(e_{T+1})}{1+\boldsymbol{r}} = 0.$$

Since the distribution functions of free cash flows are normal, their derivative with respect effort is: $f_{e_T} = \frac{\partial f(F_t)}{\partial e_t} = \left[\frac{F_t - E[F_t]}{\text{var}(F_t)} \right] \frac{\partial E[F_t]}{\partial e_t} f(F_t)$. Specifically,

$\underline{e_T}$:
$$\left[\frac{F_T - E[F_T]}{\text{var}(F_T)} \right] \frac{\partial E[F_T]}{\partial e_T} = -n \frac{F_T - x^P_{T-1}(1+m(g_T | x_{T-1})) + n\, e_T}{(x^P_{T-1})^2 s^2 + s_g^2 + s_e^2}.$$

$$\frac{F_{T+1} - E[F_{T+1}]}{\text{var}(F_{T+1})} \frac{\partial E[F_{T+1}]}{\partial e_T} = r\,n \frac{F_{T+1} - x^P_{T-1}(1+m(g_T | x_{T-1})) - rn\; e_T + n\, e_{T+1}}{(x^P_{T-1})^2 s^2 + (1+r^2) s_g^2 + s_e^2}.$$

$$\frac{F_{T+2} - E[F_{T+2}]}{\text{var}(F_{T+2})} \frac{\partial E[F_{T+2}]}{\partial e_T} = r\,n \frac{F_{T+2} - x^P_{T-1}(1+m(g_T | x_{T-1})) - r\, e_T - n e_{T+1} + \bar{I}}{(x^P_{T-1})^2 s^2 + 2r^2 s_g^2 + s_e^2}.$$

$\underline{e_{T+1}}$:

$$\frac{F_{T+1} - E[F_{T+1}]}{\text{var}(F_{T+1})} \frac{\partial E[F_{T+1}]}{\partial e_{T+1}} = -n \frac{F_{T+1} - x^P_{T-1}(1+m(g_T | x_{T-1})) - rn\; e_T + n\, e_{T+1}}{(x^P_{T-1})^2 s^2 + (1+r^2) s_g^2 + s_e^2}.$$

$$\frac{F_{T+2} - E[F_{T+2}]}{\text{var}(F_{T+2})} \frac{\partial E[F_{T+2}]}{\partial e_{T+1}} = r\,n \frac{F_{T+2} - x^P_{T-1}(1+m(g_T | x_{T-1})) - rn\; e_T - n\, e_{T+1} + \bar{I}}{(x^P_{T-1})^2 s^2 + 2r^2 s_g^2 + s_e^2}.$$

Step 2:

By step 1, the comparison between the two effort levels can be written as:

$$\iiint\left[U\left(\frac{S_T(V_T, F_T))}{1+r}+\frac{S_{T+1}(V_T, V_{T+1}, F_T, F_{T+1})}{(1+r)^2}\right)\right]\times$$

$$\left[\frac{-F_T + x^P_{T-1}(1+m(g_T\,|x_{T-1}))-e_T}{A}+\frac{F_{T+1}-x^P_{T-1}(1+m(g_T\,|x_{T-1}))-\mathrm{r}\,e_T+e_{T+1}}{\dfrac{A+\mathrm{r}^2 s_g^2}{1+\mathrm{r}}}\right]\times$$

$$f(F_T)f(F_{T+1})f(F_{T+2})dF_T dF_{T+1} dF_{T+2} - C'(e_T)+\frac{C'(e_{T+1})}{1+r}=0.$$

where: $A=(x^P_{T-1})^2 s^2 + s_g^2 + s_e^2 > 0.$

<u>The proof of Part (b)</u>

We prove part (b) first. We proceed in a piecemeal fashion. First, suppose that, by contradiction, $e_T > e_{T+1}$. I.e.,

(a) By the convexity of the disutility over effort, $-C'(e_T)+\dfrac{C'(e_{T+1})}{1+r}<0.$

(b) Note that:

$$\left[\frac{x^P_{T-1}(1+m(g_T\,|x_{T-1}))-e_T}{A}-\frac{x^P_{T-1}(1+m(g_T\,|x_{T-1}))+\mathrm{r}\,e_T - e_{T+1}}{\dfrac{A+\mathrm{r}^2 s_g^2}{1+\mathrm{r}}}\right]\times$$

$$\iiint\left[U\left(\frac{S_T(V_T, F_T))}{1+r}+\frac{S_{T+1}(V_T, V_{T+1}, F_T, F_{T+1})}{(1+r)^2}\right)\right]f(F_T)f(F_{T+1})f(F_{T+2})dF_T dF_{T+1} dF_{T+2} < 0,$$

because: by our assumption that $e_T > e_{T+1}$,

$$x^P_{T-1}(1+m(g_T\,|x_{T-1}))-e_T < x^P_{T-1}(1+m(g_T\,|x_{T-1}))+\mathrm{r}\,e_T - e_{T+1}\text{, and, } A > \frac{A+\mathrm{r}^2 s_g^2}{1+\mathrm{r}},$$

because r < 100%, i.e., $(1+\mathrm{r})A - A - \mathrm{r}^2 s_g^2 = \mathrm{r}\,A - \mathrm{r}^2 s_g^2 = \mathrm{r}\,(x^P_{T-1})^2 s^2 + \mathrm{r}\,s_g^2 + \mathrm{r}\,s_e^2 - \mathrm{r}^2 s_g^2 >$

$r(x^P_{T-1})^2 s^2 + r\sigma_e^2 > 0.$

[By our assumption on the non-negativity of the reservation utility of the agent, the integral is positive because it is the expected utility of the agent over monetary income. If (PC) were not binding because the agent has limited liability, his expected utility would certainly be positive.]

(c) To complete the proof requires showing that:

$$\iiint \left[U\left(\frac{S_T(V_T, F_T))}{1+r} + \frac{S_{T+1}(V_T, V_{T+1}, F_T, F_{T+1})}{(1+r)^2} \right) \right] \left[\frac{-F_T}{A} + \frac{F_{T+1}}{\frac{A + r\sigma_g^2}{1+r}} \right] f(F_T) f(F_{T+1}) f(F_{T+2}) dF_T dF_{T+1} dF_{T+2}$$

$< 0.$ (A*)

To proceed, we need the following lemma:

Lemma: The compensation schedule is a monotone, continuous function of cash flows.

This is a corollary to Lemma 2. By virtue of our assumptions that the manager's utility function is continuous, and random variables are normal, the compensation schedule is continuous everywhere. [Note that the limited-liability function does not affect continuity, because when it is binding , the contract is a uniformly flat compensation schedule.] The fact that the contract is a monotone function is easily established by the same methods as in Holmstrom (1979) and Lambert (1983). Q.E.D.

By this Lemma 2, we can apply the Generalized Second Mean Value Theorem (Spiegel p. 82), and rewrite (A*) as:

$$U\left(\frac{\hat{S}_T}{1+r} + \frac{\hat{S}_{T+1}}{(1+r)^2} \right) \iiint \left[\frac{-F_T}{A} + \frac{F_{T+1}}{\frac{A + r\sigma_g^2}{1+r}} \right] f(F_T) f(F_{T+1}) f(F_{T+2}) dF_T dF_{T+1} dF_{T+2} < 0.$$

The negativity of the integral is explained by the same arguments explicated in (b).

(a)-(c) imply that the left-hand-side (l.h.s.) of the expression in step (2) is negative. That is, the required contradiction obtains. In equilibrium, then, $e_{\mathrm{T}} < e_{\mathrm{T+1}}$.

Part (a):

In what follows, we denote the effort when owners-managers choose effort by e_{t}^{F}, F for first-best, and his efforts when the manager-agent chooses effort by e_{t}^{S}, s for second-best. Denoting the equilibrium levels by stars, it is immediate to see that if the T+1th effort is higher in the first-best regime, then this result holds, because, by Lemma 1 and part (b) of Lemma 3, $e_{\mathrm{T}}^{\mathrm{F*}} > e_{\mathrm{T+1}}^{\mathrm{F*}} > e_{\mathrm{T+1}}^{\mathrm{S*}} > e_{\mathrm{T}}^{\mathrm{S*}}$. In words, by Lemma 1, the first-best effort in period T is higher than that in period T+1, and by Lemma 3 part (b), the second-best effort in period T+1 is higher than the second-best effort in period T. Hence, if the first-best effort in period T+1 is higher than the second-best level effort in period T+1, we obtain that the first-best effort is higher than the second-best effort in each period.

Step 1:

The first-best effort in period T+1 satisfies:

$$\iiint \left[\frac{\mathrm{F_T}}{1+\mathrm{i_T}} + \frac{\mathrm{F_{T+1}}}{(1+\mathrm{i_T})(1+\mathrm{i_{T+1}})} \right] \frac{\mathrm{df}(x_{\mathrm{T}}, x_{\mathrm{T+1}}, x_{\mathrm{T+2}})}{\mathrm{d}e_{\mathrm{T+1}}} - c'(e_{\mathrm{T+1}}^{\mathrm{F}}) = 0.$$

The second-best effort in period T+1 satisfies:

$$\iiint \mathrm{U} \left[\frac{S_{\mathrm{T}}}{1+\boldsymbol{r}} + \frac{S_{\mathrm{T+1}}}{(1+\boldsymbol{r})^2} \right] \frac{\mathrm{df}(x_{\mathrm{T}}, x_{\mathrm{T+1}}, x_{\mathrm{T+2}})}{\mathrm{d}e_{\mathrm{T+1}}} - \frac{c'(e_{\mathrm{T+1}}^{\mathrm{S}})}{1+\boldsymbol{r}} = 0.$$

Step 2:

Subtracting the first-order conditions from each other and rearranging, yields

$$(1)\quad \iiint\left[\frac{F_T}{1+i_T}+\frac{F_{T+1}}{(1+i_T)(1+i_{T+1})}-U\left(\frac{S_T}{1+\boldsymbol{r}}-\frac{S_{T+1}}{(1+\boldsymbol{r})^2}\right)\right]\frac{df(x_T, x_{T+1}, x_{T+2}\,|e^{F*}_{T+1})}{de_{T+1}}+$$

$$(2)\quad +\iiint U\left(\frac{S_T}{1+\boldsymbol{r}}-\frac{S_{T+1}}{(1+\boldsymbol{r})^2}\right)\left[\frac{df(x_T, x_{T+1}, x_{T+2}\,|e^{F*}_{T+1})}{de_{T+1}}-\frac{df(x_T, x_{T+1}, x_{T+2}\,|e^{S*}_{T+1})}{de_{T+1}}\right]-$$

$$(3)\quad c'(e^{F*}_{T+1})+\frac{c'(e^{S*}_{T+1})}{1+\boldsymbol{r}}=0.$$

Step 3:

The expression in (2) is positive by the assumption that the agent's compensation is lower than free cash flows and by the assumption that the manager is risk-averse. That is, by Jensen's inequality,

$$\iiint\left[\frac{F_T}{1+i_T}+\frac{F_{T+1}}{(1+i_T)(1+i_{T+1})}-U\left(\frac{S_T}{1+\boldsymbol{r}}-\frac{S_{T+1}}{(1+\boldsymbol{r})^2}\right)\right]\frac{df(x_T, x_{T+1}, x_{T+2}\,|e^{F*}_{T+1})}{de_{T+1}}>$$

$$\frac{E[F_T]}{1+i_T}+\frac{E[F_{T+1}]}{(1+i_T)(1+i_{T+1})}-U\left(\frac{E(S_T)}{1+\boldsymbol{r}}-\frac{E(S_{T+1})}{(1+\boldsymbol{r})^2}\right)>$$

By the fact that the manager's compensation falls short of free cash flows,

$$\frac{E[F_T]}{1+i_T}+\frac{E[F_{T+1}]}{(1+i_T)(1+i_{T+1})}-U\left(\frac{E(F_T)}{1+\boldsymbol{r}}-\frac{E(F_{T+1})}{(1+\boldsymbol{r})^2}\right)>$$

By the assumption that the manager is less patient than owners,

$$\sum_{t=T}^{t=T+1}\frac{E(F_t)}{(1+\boldsymbol{r})^{t-T+1}}-U\left(\sum_{t=T}^{t=T+1}\frac{E(F_t)}{(1+\boldsymbol{r})^{t-T+1}}\right)>0.$$

The last inequality obtains by assumption of the proof.

Hence, the sum of the second and the third are negative. If we prove that they are co-monotonic, (i.e., if (2) is negative so must be (3)), the proof is complete, because, by convexity of the cost function of effort, and our assumption on the relationship between the owners' discount factor and the manager's discount factor,

$$-c'(e^{F*}_{T+1}) + \frac{c'(e^{S*}_{T+1})}{1+\boldsymbol{r}} < 0 \text{ if } e^{F*}_{T+1} > e^{S*}_{T+1}.$$

<u>Step 4</u>:

We wish to show that when $e^{F*}_{T+1} > e^{S*}_{T+1}$, (2) is negative. To that end, we prove the following lemma.

<u>Lemma</u>: Let z be a normal stochastic variable, with a mean that is a linear increasing function of effort, *e*. Then, if (i) W(z) is a concave functional, and (ii) a> b,

$$\int W(z)\left[\frac{df(z|a)}{de} - \frac{df(z|b)}{de}\right] < 0.$$

The proof is based on the fact that

$$\frac{df(z|a)}{de} = \left[\frac{z - E[z]}{s_z^2} f(z)\right]\frac{\partial E[z]}{\partial e} dz = -\frac{\partial E[z]}{\partial e}\frac{df(z|a)}{dz}.$$

Since $\frac{\partial E[z]}{\partial e}$ is a positive scalar, integrating by parts yields:

$$\int W(z)\left[\frac{df(z|a)}{de} - \frac{df(z|b)}{de}\right] = -\frac{\partial E[z]}{\partial e}\int W'(z)\left[\int\left[\frac{\partial f(z|a)}{\partial e} - \frac{\partial f(z|b)}{\partial e}\right]dz\right]dz =$$

$$-\frac{\partial E[z]}{\partial e}\int(-W'(z))\left[f(z|a) - f(z|b)\right]dz < 0.$$

This inequality obtains because $-W'(z)$ is, by assumption, an increasing function, and because when a>b, the term in the integral is positive by first-order-stochastic dominance.

This lemma establishes that (2) in step (3) is indeed negative when $a = e^{F*}_{T+1} > e^{S*}_{T+1} = b$. Q.E.D.

Proof of Proposition 3:

The proof is immediate from Proposition 2 and Lemma 3, parts (a) and (b). Denoting by superscript F the first-best variable and by superscript S the second-best variable,

$$\frac{E\left[\dfrac{m_{T+1}^{F}}{m_{T+2}^{F}}\right]}{E\left[\dfrac{m_{T+1}^{S}}{m_{T+2}^{S}}\right]} = \left[\frac{\dfrac{e_{T}^{F}}{e_{T+1}^{F}}}{\dfrac{e_{T}^{S}}{e_{T+1}^{S}}}\right]\left[\frac{E[x_{T+1}^{pF}]}{E[x_{T+1}^{pS}]}\right] > \frac{E[x_{T+1}^{pF}]}{E[x_{T+1}^{pS}]} = \frac{E[x_{T}^{p} + r\, e_{T}^{F}]}{E[x_{T}^{p} + r\, e_{T}^{S}]} > 1. \quad \text{Q.E.D.}$$