Estimation of Employee Stock Option Exercise Rates and Firm Cost*

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Abstract

This paper is the first to perform a comprehensive estimation of employee stock option exercise behavior and option cost to firms. We develop a GMM-based methodology, robust to heteroskedasticity and correlation across exercises, for estimating the rate of voluntary option exercise as a function of the stock price path and of various firm and option holder characteristics. We use it to estimate an exercise function from a sample of 870,624 employee-option grants at 47 publicly-traded firms between 1980–2005, finding that volatility has a counterintuitive effect, and that men exercise faster than women. We also estimate the rate of employment termination, which determines forfeitures, cancellations, and forced exercises. We use the estimated exercise and termination functions in a simulation based valuation model to analyze the effect of different firm and option holder characteristics on option value, and show that the true value of these options can differ substantially from values calculated using the usual FASB approximation.

JEL classification: G14.

With the explosive growth of employee stock options in corporate compensation, investors, auditors, and regulators have become increasingly concerned about the cost of these options to shareholders. Recent regulation requiring firms to recognize option cost has intensified the demand for suitable valuation methods. The difficulty is that these are long-lived American options, so their value depends crucially on how employees exercise them. Yet, because employees face hedging constraints, standard option theory does not apply.

In standard theory for an ordinary American call, the holder can sell the option at any time, so it is reasonable to assume that he times the option exercise in order to maximize the option's present value. The present value-maximizing exercise policy and its implications for option value are well-researched (see, for example, Black and Scholes (1973), Merton (1973), or Kim (1990)). However, the holder of an employee stock option cannot sell the option and typically faces significant hedging constraints. The holder must bear at least part of the risk of the option payoff, so simply maximizing the option's present value is generally not optimal. Indeed, evidence indicates that employees systematically exercise options on non-dividend paying stocks well before expiration (see, for example, Huddart and Lang (1996), Bettis, Bizjak, and Lemmon (2005)).

As long as the exercise decision generates an option payoff that is subject only to hedgeable risks, such as stock price risk, and diversifiable risks, such as uncertainties that are idiosyncratic across employees, then unconstrained market participants can replicate those payoffs and so pricing by no arbitrage is still possible. The option valuation problem then reduces to accurately characterizing the option payoffs, that is, the exercise policies of executives. Until recently, however, full-blown estimation has not been possible because of insufficient data and inadequate methodology. Detailed employee option grant, exercise, and cancellation data are proprietary and very difficult to obtain for a large number of firms. In addition, traditional hazard rate models are not suitable for describing voluntary option exercises, where partial and repeated exercise of options from a given grant is the norm.

This paper is the first to perform a complete empirical estimation of employee stock option exercise behavior and option cost to firms. We develop a GMM based methodology for formally estimating the rate of voluntary option exercise as a function of the underlying stock price path, time remaining to expiration, and firm and option holder characteristics. This estimation methodology is robust to both heteroskedasticity and correlation across option exercises. Moreover, unlike regression-based approaches, it never predicts that the fraction of options exercised in any given period will be less than zero or more than 100%. We also estimate the rate of employment termination, which forces an option forfeiture if the option is unvested, a cancellation if it is vested but out of the money, or else an exercise.

Reliable estimation of any option exercise model requires a large sample that includes a wide variety of stock price paths. We estimate our model using a comprehensive sample of option exercise grant and exercise data for all employees at 47 firms from 1980 to 2005. The proprietary data were provided by corporate participants in a sponsored research project that was funded by the Society of Actuaries. The methodology presented in this paper is the first step in developing an actuarial science for valuing compensatory stock options, similar to that for pension liabilities.

In our estimation we find that the rate of voluntary option exercise is positively related to the level of the stock price and the imminence of a dividend payment, but positively related to volatility, in contrast to what standard theory would predict. In addition, the exercise rate is higher if the stock price is in the 90th percentile of its distribution over the past year or if the option has just vested. We also find that, holding all else equal, men are more likely to exercise their options than women.

The estimated exercise function, together with a model for involuntary terminations, can be combined with Monte Carlo simulation in the usual way to calculate the value of these options to shareholders, taking the employees' exercise and termination behavior into account. This approach is similar to the prepayment modeling and valuation methods developed for mortgage-backed securities (see, for example, Schwartz and Torous (1989)).

We compare the prices based on our estimation with the adjusted Black-Scholes method suggested as an approximate valuation technique by the FASB. We find that this FASB approximation exhibits both significant pricing errors and very different qualitative behavior, compared with the options' true value. For example, for the default parameter set for women, the FASB approximation tells us that the ESO value should decrease as the underlying stock's beta increases, while the true value actually *increases*.

1 Previous Literature

The principles of employee option stock valuation and the need to study exercise behavior are well-understood in the literature. One approach that has been taken is to model the exercise decision theoretically. The employee presumably chooses an option exercise policy as part of a greater utility maximization problem that includes other decisions, such as portfolio, consumption, and effort choice, and this typically leads to early exercise for the purpose of diversification. Papers that develop utility-maximizing models and then calculate the implied cost of options to shareholders include Huddart (1994), Detemple and Sundaresan (1999), and Ingersoll (2006).

Combining theory and data, papers such as Carpenter (1998) and Bettis, Bizjak, and Lemmon (2005) calibrate utility-maximizing models to mean exercise times and stock prices in the data, and then infer option value. However, these papers provide no formal estimation and the approach relies on the validity of the utility-maximizing models used. Huddart and Lang (1996) and Heath, Huddart, and Lang (1999) provide more flexible empirical descriptions of option exercise patterns, but do not go as far as option valuation.

A number of analytic methods for approximating executive stock option value have also been proposed in the literature. The FASB currently permits using the Black-Scholes formula with the expiration date replaced by the option's expected life. Jennergren and Näslund (1993), Carr and Linetsky (2000)), and Cvitanić, Wiener, and Zapatero (2004) derive analytic formulas for option value assuming exogenously specified exercise boundaries and stopping rates. Hull and White (2004) propose a model in which exercise occurs when the stock price reaches an exogenously specified multiple of the stock price and forfeiture occurs at an exogenous rate. However, until the accuracy of these methods can be determined, the usefulness of these methods cannot be assessed.

2 Modeling Exercise Behavior

2.1 Hazard Rates

At first sight, it seems natural to use hazard rates to model the exercise of employee stock options, since they have often been used in the finance literature to model outwardly similar events, such as mortgage prepayment (see Schwartz and Torous (1989)) and corporate bond default (see, for example, Duffie and Singleton (1999)).[1] However, whereas it makes sense to think of the prepayment of one mortgage as being independent of the prepayment of another (once we've controlled for factors like the level of interest rates), ESOs are typically exercised in blocks. As a result, the exercise of one option in a given grant held by an individual is extremely highly correlated with the exercise of another option in the same grant held by the same individual. It is also highly (though less so) correlated with the exercise of options in other grants held by the same individual. This high degree of correlation between options makes it difficult to use standard econometric techniques (which assume independence between events), to estimate hazard rates at the individual option level.[2]

One attempt to solve this problem was suggested by Armstrong, Jagolinzer, and Larcker

[1] A hazard rate is defined as the likelihood (per period) of an option's being exercised in the next instant, conditional on not having being exercised previously. For good introductions to hazard rate analysis, see Cox (1972) or Kalbfleisch and Prentice (1980).

[2] This issue also arises in modeling corporate bond default. One popular solution, when the number of firms involved is small, has been to use "copula functions", which explicitly model this correlation [See, for example, Li (2001)]. However, in our case the number of options (and hence the number of correlation coefficients) is too high to be feasible.

(2006). Instead of using a hazard rate to model the exercise of individual options, they use a hazard rate to model the exercise behavior of an entire grant of options held by an individual. Aggregating in this way gets around the problem of correlation between individual option exercises, but it introduces a new problem. Whereas a hazard rate describes an event with two states – either something has happened, or it has not – the proportion of an option grant that exercises in a given period is (almost) a continuous variable, which can take on any value between 0 and 1. Armstrong et al. (2006) therefore work with the discrete variable $\text{Exercise}_{i,k,t}$ which equals 1 if employee i exercises at least 25% of the vested and unexercised options in grant k on day t (and at least 10% of all options from the grant); otherwise it equals zero. This gets over most of the correlation-related problems described above, but introduces new problems of its own. First, unlike, say, death from a disease, this variable can equal 1 more than once, so standard hazard rate estimation techniques may not immediately apply. Second, important information is lost in this aggregation process. For example, consider two option holders who both have exactly the same likelihood of exercising on any given date; however, option holder 1 always exercises 25% of the remaining grant whenever he exercises, whereas option holder 2 always exercises 100% of his remaining options. The conditional probability of a given option's being exercised at any instant is four times as high for options held by option holder 2 versus option 1, so their options are likely to be worth very different amounts, yet the Exercise variable modeled by Armstrong et al. (2006) would behave exactly the same way for the two option holders.

2.2 Modeling Fractional Exercise

A solution to all of the problems above is not to use a hazard rate approach at all, but instead to model the *fraction* of each grant exercised each period. Heath et al. (1999) follow this approach, regressing the fraction of each grant exercised against various explanatory variables. However, their regression approach has some problems. In particular, it may generate expected exercise fractions that are negative or greater than 100%, both of which cause problems for valuation.[3] One possible solution is to transform the proportion exercised, such as by using a logistic transformation,

$$\log\left(\frac{y}{1-y}\right),$$

which can take on any value between $-\infty$ and $+\infty$, and use this on the right hand side of the regression. Unfortunately, by Jensen's inequality, the expected proportion exercising is not

[3] Attempting to remedy this, for example by truncating the variables, will lead to biases.

just the inverse transformation of the expected transformed proportion. More important, this approach cannot handle the (numerous) dates on which there are no options exercised at all. Heath et al. (1999) also aggregate across individuals, thus discarding potentially important information about the differences in exercise behavior across individuals.

Like Heath et al. (1999), we also model the fraction of each grant exercised by each holder each period, but we do so in a manner that always generates consistent estimates of expected exercise rates that are guaranteed always to be between 0% and 100%, while explicitly handling the correlation between option exercises within and between different grants held by the same individual. Our approach, based on the fractional logistic approach of Papke and Wooldridge (1996), also allows for arbitrary heteroskedasticity in the exercise rates.

Let y_{ijt} be the fraction exercised at time t of grant j held by individual i, and write

$$y_{ijt} = G(X_{ijt}\beta) + \epsilon_{ijt}, \tag{1}$$

where X_t is some set of covariates in I_t, the information set at date t, where G, the expected fraction exercised at date t, is a function satisfying $0 < G(z) < 1$, and where

$$\begin{aligned} E(\epsilon_{ijt} \mid I_t) &= 0, \\ E(\epsilon_{ijt}\, \epsilon_{i'j't'}) &= 0 \quad \text{if } i \neq i' \text{ or } t \neq t'. \end{aligned}$$

From now on, we shall use the logistic function,

$$G(X_{ijt}\beta) = \frac{\exp(X_{ijt}\beta)}{1 + \exp(X_{ijt}\beta)},$$

which is easily shown to take on only values between 0 and 1. Note that, while we are assuming the residuals ϵ_{ijt} are uncorrelated between individuals and across time periods, we are allowing for ϵ_{ijt} to be arbitrarily correlated between different grants held by the same individual at a given point in time, and we are not making any further assumptions about the exact distribution of ϵ_{ijt}, or even about its variance. In particular, unlike assuming a beta distribution for y_{ijt} (see Mullahy (1990) or Ferrari and Cribari-Neto (2004)), we are allowing a strictly positive probability that y_{ijt} takes on the extreme values zero or one.

As in Papke and Wooldridge (1996), we estimate the parameter vector β using quasi-maximum likelihood (see Gouriéroux, Monfort, and Trognon (1984)) with the Bernoulli log-likelihood function,

$$l_{ijt}(\beta) = y_{ijt} \log\left[G(X_{ijt}\beta)\right] + (1 - y_{ijt}) \log\left[1 - G(X_{ijt}\beta)\right]. \tag{2}$$

Estimation involves solving

$$\max_{\beta} \sum_{i,j,t} l_{ijt}(\beta).$$

The K first order conditions, corresponding to the K elements of β, are given by

$$\begin{aligned}
\sum_{i,j,t} \frac{dl_{ijt}(\beta)}{d\beta} &= \sum_{i,j,t} X_{ijt} \left[G'(X_{ijt}\beta) \left(\frac{y_{ijt}}{G(X_{ijt}\beta)} - \frac{1-y_{ijt}}{1-G(X_{ijt}\beta)} \right) \right] \\
&= \sum_{i,j,t} X_{ijt} \left(y_{ijt} - G(X_{ijt}\beta) \right) \qquad (3) \\
&= 0.
\end{aligned}$$

Equation (1) implies (using iterated expectations) that the population expectation of these first order conditions is zero, hence this QML estimator, $\widehat{\beta}$, is a (consistent) GMM estimator of β, with no assumptions other than Equation (1). Following the notation in Papke and Wooldridge (1996), define the residual

$$\widehat{u}_{ijt} \equiv y_{ijt} - G(X_{ijt}\widehat{\beta}),$$

and define

$$\begin{aligned}
\widehat{G}_{ijt} &\equiv G(X_{ijt}\widehat{\beta}) \equiv \widehat{y}_{ijt}, \\
\widehat{g}_{ijt} &\equiv G'(X_{ijt}\widehat{\beta}).
\end{aligned}$$

To allow for heteroskedasticity and for correlation between option grants held by a given individual, write

$$\text{var}(u) = \Omega = \begin{pmatrix} \Sigma_1 & & \dots & & 0 \\ & \ddots & & & \\ \vdots & & \Sigma_i & & \vdots \\ & & & \ddots & \\ 0 & & \dots & & \Sigma_I \end{pmatrix},$$

where each Σ block corresponds to all of the option grants held by a given individual on a particular date. Then the asymptotic covariance matrix of $\widehat{\beta}$ takes the "sandwich" form (see Gouriéroux et al. (1984)),

$$\text{var}\left(\widehat{\beta}\right) = \widehat{\mathbf{A}}^{-1} \widehat{\mathbf{B}} \widehat{\mathbf{A}}^{-1},$$

where

$$\widehat{\mathbf{A}} = \sum_{i,j,t} \frac{\partial^2 l_{ijt}(\widehat{\beta})}{\partial\beta\partial\beta'} = \sum_{i,j,t} \widehat{g}_{ijt} X_{ijt} X'_{ijt}, \tag{4}$$

and

$$\widehat{\mathbf{B}} = \mathbf{X}'\widehat{\Omega}\mathbf{X},$$

where $\mathbf{X}$ is a matrix containing all of the stacked X_{ijt} values, and $\widehat{\Omega}$ is a consistent estimator of Ω given by

$$\widehat{\Omega} = \begin{pmatrix} \widehat{\Sigma}_1 & & \ldots & & 0 \\ & \ddots & & & \\ \vdots & & \widehat{\Sigma}_i & & \vdots \\ & & & \ddots & \\ 0 & & \ldots & & \widehat{\Sigma}_I \end{pmatrix}$$

where

$$\widehat{\Sigma}_i = \widehat{u}_i \widehat{u}'_i.$$

This covariance matrix is robust both to arbitrary heteroskedasticity and to arbitrary correlation between the residuals in a given block.[4]

3 Data

As discussed above, our estimation strategy is carried out using a proprietary data set comprising complete histories of employee stock option grants, vesting structures, and option exercise and cancellation events for all employees who received options at forty seven publicly traded corporations between 1980 and 2005.[5] As shown in Table 1, there is considerable heterogeneity in the sample of firms both in terms of their industry type (measured at the one-digit SIC) and in terms of the firm sizes measured by market cap and numbers of employees. The largest firms are in the manufacturing sector (SIC 2 and 3) and the transportation and communications sector (SIC 4). Even within these sectors, however, the heavy manufacturing

[4]For further discussion of calculating standard errors in the presence of clustering, see Rogers (1993), Baum, Schaffer, and Stillman (2003), Wooldridge (2003) and Petersen (2008).

[5]The data were obtained as part of a research grant written by the authors and funded by the Society of Actuaries. In addition, we thank Terrence Adamson at AON Consulting who also provided data for this study.

sector (SIC 2) experienced significantly different revenue growth rates, with a mean growth rate of 8.5%, than the computer and measurement sector of manufacturing (SIC 3) with a mean revenue growth rate of 22.1%. On the whole, the computer high-tech corporations dominate the SIC 3 sector firms and these are smaller high growth companies over the study period. The services (SIC 7 and 8), Retail (SIC 5), and Finance, Insurance and Real Estate, (SIC 6) firms are on average somewhat smaller corporations and the average growth rates for both the retail and services (SIC 7) firms were in the mid to high twenties over the sample period. The educational, engineering, and accounting services sector (SIC 8) and the FIRE sector (SIC 6) both experienced slower growth rates over the period.

Our unit of analysis is an employee-grant-day. For each option grant we merge the appropriate path of daily split-adjusted stock prices and dividends, starting at the initial grant date, to the path of outstanding option vesting and exercise events for all grants and employees. These daily paths are constructed using detailed information on the contractual option vesting structure, the exercise events, and the cancellation events recorded for each grant. We track the employee-grant-days and a series of time-varying covariates until the options in the grant are full exercised, the options are cancelled, or we reach the end of the sample period of December 30, 2005.

There is significant heterogeneity in the contractual structure of the option grants both across and within firms in our sample. In Table 2, we report the summary statistics for the number of grants per employee over the sample period. To preserve the anonymity of the firms, we report the summary statistics by two-digit SIC groupings using the SIC designation for each firm that is reported in CRSP. As shown in the Table 2, there is considerable variability in the numbers of grants per employee across firms. The mean number of grants per employee ranges from 1.61 grants for firms in the Educational Services sector (SIC 82) to 5.64 grants per employee in the retail sector. There is also considerable variability within sectors where the number of grants per employee ranges from one to as many as sixty four in the chemicals and allied products sector (SIC 28). For the firms, where we have the employee ranking of the employee, the largest grant recipients are typically the CEO or senior managers.

Another feature of the option granting structure is the number of options that are associated with each grant. As shown in Table 3, there is again considerable variability both across industry sectors and within firms. The number of options have been multiplied by the ratio of the firms stock price to the global average stock price so that summary statistics control for the effect of relatively valuable options where the grants are small and large grants when the stock price is relatively low compared to the sample average. As shown, the mean standardized grant sizes are largest in the chemicals and allied products sector

(SIC 28), communications (SIC 48), and industrial machinery and computers sector (SIC 35). The range of the standardized option grant sizes is also large. For example, in the retail sector (SIC 59) the number of options per grant ranges from 0.5 to more than seventeen million standardized and split-adjusted options in a single grant. The combined effects of the large number of grants per employee and the size of these grants implies that individual employees are likely to hold large inventories of options with different strikes and vesting structures. This feature of the data introduces significant correlation across the exercise decisions of individual employees. As discussed above, the exercise decision for one option in a given grant held by an individual is highly correlated with the exercise of another option in the same grant held by the same individual. In addition, there would be correlation in the exercise decisions across grants that are held by the same individual. A particular strength of our fractional logistic estimator is that is does not require assumptions of independence across exercise events. We also pool by employee and correct our standard errors to account for our pooled structure.

Table 4 provides summary statistics for the maximum number of contractual vesting dates required before the options in a given grant fully vest. The means range between 1 for the petroleum, refining and related sector (SIC 29) and 11.75 for firms in the electronic and other electrical sector (SIC 36). Here again, the range of vesting structures across firms within a sector are very important. An example of a vesting structure that would lead to a large maximum would be a grant with a 25% vest at the end of the first year and then 2.08% monthly vests over the next 36 months. The minima are generated by what are termed “cliff vests” where all the options in a given grant vest on the same day. Another feature of the grants that exhibits important heterogeneity across firms is the percentage of options that vest on the first vesting dates. As shown in Table 5, the range of the mean percentage of options that vest on the first vesting date is between 20% to 100% across the SIC defined sectors. The within sector variance is also important and the percentage of the grants that vest on the first vesting date ranges between 1% to 100%. An 100% vest is also associated with the common “cliff vest” structure and these vests can occur within one month of issuance.

The summary statistics for the overall time that it takes to fully vest the grants are reported in Table 6. As shown, the mean number of months required for the grants to fully vest is between 7.11 months in the communications sector (SIC 48) and 59.99 months in the educational services sector (SIC 82). The within sector variation is between zero, in which case the grant fully vests on the first grant date, to a maximum of 121 months in the communications sector (SIC 48). The only homogeneous contractual feature of employee stock option grants across firms is the maturity in months from the issuance date to the date of expiry. As shown in Table 7, the term of executive stock options is quite uniformly ten years although there are some twenty year and one year maturity options granted on the part of some firms. At the employee-level, the employees in our sample are in some cases managing as many as ten different contractual option vesting structures in their inventory of options.

The vesting structure also affects the observed patterns of exercise events. Table 8 presents the summary statistics for the remaining term on the option at the time the option is exercised. Since on average the options in our sample have terms of ten years, these high mean values imply that on average the options in our sample are exercised when there is nearly two-thirds of the option term remaining. These patterns are consistent with those documented by Huddart and Lang (1996). The only sector that does not exhibit this persistent early exercise behavior on the part of employees is fabricated metal products (SIC 34). The summary statistics reported in Table 9 suggest that on average the options are exercised close to the vesting dates, controlling for the number of days in the interval that the option was in-the-money. The chemicals and allied products firms (SIC 28) and communications (SIC 48) sectors, however, appear to be exceptions to this pattern with mean times between the prior vesting date and the exercise date of 1,205.4 and 1830 days respectively.

Table 10 reports the summary statistics for the ratio of the stock price to the exercise price on the option. Not at all surprisingly, these ratios are all greater than one, but again there is very important variation in the ratios both across and within sectors. The very high ratios reflect the run-up in the stock market during our sample period. Overall, both the vesting structure and the option exercise patterns lead to the persistence of early exercise behavior in some industrial sectors but with considerable heterogeneity both within and across the sectors.

Another characteristic that we document in these data is the importance of fractional exercise behavior. As shown in Table 11, the mean percentage of vested options in a grant that are exercised is significantly less than one across all the industry sectors. The means range from about 30% in the miscellaneous retail sector (SIC 59) to a high of about 92% in the electric, gas, and sanitary services sector (SIC 49). There is also consider heterogeneity within firms in the industry sectors. Thus, on average employees typically exercise less than 100% of the outstanding vested options of a grant even though all the vested options in the grant would also be in-the-money. It is this central feature of option exercise behavior that motivates the development of our fractional logistic estimation strategy.

In summary, there are three features of the stock option exercise patterns observed in our sample of 870,624 option grants and 429,371 option exercises. First, many employees hold more than one option grant and make exercise decisions over more than one vested option at any given time. For this reason, estimation strategies must account for the correlated decision structure of employee option exercise. Second, both the contractual vesting structure and the exogenous price paths appear to have strong effects on option exercise patterns, thus careful controls for both of these feature on a daily basis must be included in a successful estimation strategy. Finally, most option positions are exercised fractionally, that is the proportion of the outstanding options that are exercised at exercise events is, on average, substantially less than one. For this reason, a successful econometric methodology must account for path dependent fractional exercise behavior or risk introducing significant misspecification bias and inaccurate forecasts of exercise timing.

3.1 The covariates

As discussed at length in our companion paper, Carpenter, Stanton, and Wallace (2008), since employee stock options are non-transferable, the optimal exercise policy for these options can look quite different from that for standard American call options. The intuition that the need for diversification can lead an employee to sacrifice some option value by exercising early is well understood in the theoretical literature. However, an explicit theory of the optimal exercise of executive stock options that could be taken directly to data is still developing. Formal theory of the optimal exercise of multiple executive stock option grants is even less developed, although intuition suggests that the greater the option holder's total forced exposure to the stock risk, the greater the exercise rate. Despite the potential limitations of prior theoretical research, we consider two types of path-dependent covariates in our empirical specification of fractional exercise timing. We use both fundamental state variables that have appeared in prior theoretical analyzes of optimal stock option exercise policies as well as "behavioral" variables that have been identified in recent empirical options exercise studies (See Huddart and Lang (1996); Heath et al. (1999); Huddart and Lang (2003); Malmendier and Tate (2005); and Armstrong et al. (2006)).

Even the most parsimonious theoretical models define the optimal exercise policies of executive stock options in terms of the time series dynamics of the underlying stock price in conjunction with the contractual timing of options' vesting structures and the timing of expirations. Successful development of empirical models of exercise timing require richer theoretical treatments of the exercise function in terms of variables such as the wealth of the option holder, the holdings of other options and restricted stock, the volatility of the underlying stock price, the portion of risk that is hedgeable using other assets, and the dividend payout structure of the firm. Unfortunately, the theoretical literature that focuses on these richer models is not well developed. The broader theoretical framework of Carpenter et al. (2008) allows for a number of empirically testable predictions concerning the average lives of employee stock options, and we draw upon a number of these predictions to motivate the our specification choices for our fractional logistic estimator. A first strong result from the model is that the higher the firm's dividend rate the more likely that the employee will exercise, since loss of dividend through option ownership is a cost of the option that could be avoided through exercise. A second prediction is that the higher correlation of the stock's price with the market (the higher beta), the less likely that exercise will occur. In addition we find that, unlike the standard intuition for the exercise of American calls, volatility does not lead to a clean prediction and may lead both to earlier or later exercise in the case of non-transferable stock options.[6] For this reason, we consider the effect of stock price volatility on the optimal exercise policy as an open empirical question. Standard intuition holds that risk averse employees are likely to exercise later and the cost of their options is higher. Similarly, employees with decreasing absolute risk aversion will exercise later, implying greater option cost, if they have more non-option wealth and this intuition is again supported in Carpenter et al. (2008). This final prediction not only requires information on the stock price paths but also requires proxies for the risk preferences of the employees who hold the options.

In Table 12 we present the summary statistics for the path dependent covariates. The first four rows of the table present the common state variables in theoretical analyzes of optimal employee option exercise policies. The *price to strike ratio* is the employee-grant-day observation of the ratio of the split-adjusted price of the stock to the split-adjusted strike price on the option (the zero valued minimum is due to our reporting precision). The *stock return volatility* is the standard deviation of the daily stock returns, as reported in CRSP, over the prior thirty trading days from each employee-grant-day. The *dividend payment* is equal to the next split-adjusted dividend divided by the current share price during the 14 days up to each dividend payment, and is equal to zero on all other days. The *firm beta* was estimated for each firm using market and firm data from CRSP and interest rate data from the Federal Reserve Board over a five year interval (in some cases the interval is shorter) and is a constant for each firm. The estimated mean beta is about 1.00 and the standard deviation is about .31 over the sample period.

Recent empirical studies of employee stock option exercise report links between behavioral indicators, or "rules of thumb", that employees appear to rely upon in making their option exercise decisions. Armstrong et al. (2006) find a statistically significant association between the timing of vesting events and option exercise. They argue that recent exercise events both mechanically affect an employees' ability to exercise their options and may also serve as a

[6]Option values increase in volatility in both standard option pricing theory and in the case of executive stock option models with an exogenously specified exercise boundary Cvitanić et al. (2004).

periodic reminder to employees to evaluate the value of their option positions. Heath et al. (1999) and Armstrong et al. (2006) also find a statistically significant positive association between option exercise and the occurrence of the current stock price exceeding the 90th percentile of the past year's price distribution. They argue that this association is driven by cognitive benchmarks that employees use in their decision rules. Given the importance of these variables in prior studies, we also include them as controls in all of our specifications. Our measure of vesting structure is the variable *vesting period within two weeks*. It is an indicator variable that is one if the employee-grant-day is within two weeks of a vesting event and zero otherwise. Our cognitive benchmark proxy is the variable *price relative to the 90th percentile*. It is an indicator variable that is equal to one if the observed stock price is greater than or equal to 90th percentile of the stock price distribution over the prior year of trading and zero otherwise. We report the summary statistics for these variables in the lower panel of Table 12. As shown in the table, about 2% of employee-grant-days are within two weeks of a vesting date, on average, and about 59% of the employee-grant-days are at prices that exceed the 90th percentile of the past year's price distribution, reflecting the large stock price run-ups in the sample period.

A prior empirical literature has found evidence that older individuals are more risk averse in financial decision making than younger individuals and that females appear to be more risk averse than males in their financial decisions (See Bajtelsmit and Bernasek (2001); Bellante and Green (2004); and Armstrong et al. (2006)). As previously discussed, economic intuition suggests that less risk averse employees are likely to exercise later and consequently the cost of their options is greater. For a subset of firms in our sample, we have information on the age, sex, and salary of the employees. The summary statistics for these variables are reported in Table 13. As shown, the subset of the data that includes salary is quite small. The mean salary is about $160,000 (the median about $75,000) and there is significant range between about $9,800 and $36 million dollars in annual salary. We have information for a subset of about sixty two thousand employees and as shown in the table the mean age is about forty three years old. Surprisingly, the youngest option recipient is only seventeen years old and the oldest is eighty five years old. In a somewhat smaller subsample, we have information about the gender of the employee and their employment status. Sixty eight percent of the subsample is male. About 8% of the employees are executives and about 3% are board members. The manager category (about 40% of the subsample) includes all employees who have manager as their employment designation and the other category includes all administrative personnel, logistics personnel, and engineers among many other categories. Our expectation is that, everything else equal, women and older employees will exercise later and that the larger non-diversifiable wealth positions of executives will lead them to exercise earlier.

In Table 14, we report the frequencies of option cancellation events. These covariates are important in the valuation simulations since they add three additional reasons for option terminations and they have potentially important effects on the probability of option exercise. The employee stock options in our sample can be cancelled for four reasons: repricing, death, retirement, and termination from the firm for either voluntary or involuntary reasons. Since we only observed about 4,000 repricings in the sample we ignored the effects of these events. Cancellations as the result, of death, retirement and job termination would be expected to force early option exercise since employees are required to relinquish their option positions within 60 days of any of the three events. For this reason, we drop all exercises within a six month window of these events, since presumably these exercises are motivated only in part by the current draw on the stock price path. As shown in Table 14, there are a large number of cancellation events where we were not provided with the reason for the cancellation. Of the 222,168 employees for whom we have cancellation information, the most common reason was job termination. We also had three hundred retirements and one thousand eight hundred and thirty deaths.

In summary, the covariates used in the fractional logistic specification include the salient state variables related to stock price paths, volatility, and market risk that have been the focus of the recent theoretical literature on employee stock option valuation and cost. In addition, we proxy for factors such as risk aversion and possible cognitive benchmarks using the covariates gender, age, salary, and employment status. We use this rich set of covariates to explore a set of theoretically motivated null hypotheses that have appeared in the recent literature. These nulls include: 1) higher dividend rates should make option exercise more likely; 2) more risk aversion should make early option exercise more likely; 3) higher volatility is an empirical question, since theoretically it could lead to either earlier or later exercise in a utility maximizing framework; 4) higher correlation with the market makes earlier exercise less likely; and 5) an employee's optimal exercise policy may involve exercising when the stock price exceeds a critical boundary. We report the results of these tests in the next section of the paper.

4 Estimation Results

Table 15 shows the results of applying our procedure to the sample of voluntary option exercises described above. We report our results for four alternative specifications. Specifications 1 and 2 are estimated on the full sample but excludes age and sex. Specification 1 excludes firm fixed effects and specification 2 includes them. Specifications 3 and 4 are estimated on a smaller sub-sample of firms that included information on the age and sex of the employee. Specification 3 excludes firm fixed effects and specification 4 includes them. The standard errors are reported in parentheses below the coefficient estimates and the estimator clusters at the level of the individual employee.

As shown in the table, the rate of voluntary option exercise is consistently positively related to the level of the stock price as expected. As previously discussed, Carpenter et al. (2008) argue that unlike the standard intuition for the exercise of American calls, volatility does not lead to a clearly signed prediction on an employee's optimal exercise policy. The results reported in Table 15 indicate that once we introduce controls for firm fixed effects, increased levels of stock return volatility are associated with larger fractions of options exercised. These results suggest that the employee's seek to minimize the costs associated with holding an undiversified option portfolio so they exercise their options as risk increases.

The effect of the timing of dividend payments on the fraction of options exercised appears to be very sensitive to controls for possible differential risk aversion associated sex and age. The theoretical prediction is that the higher the firm's dividend rate, the more likely it is that the employee will exercise, since the loss of dividend is a cost to option ownership. We find a statistically significant and positive effect only in specification 4 where we control for both firm fixed effects, sex, and age.[7] Specification 4 is our preferred specification. However, these results suffer from the important caveat that the subsample used for estimation is considerably smaller than the full sample. Another potential limitation with our dividend measure is that many firms did not pay dividends during out sample period, which includes the technology driven stock price boom.

Our prediction that the more highly the stock's price is correlated with the market, the less likely that exercise will occur, is not consistently borne out across the four specifications. As shown in Table 15, the coefficient estimate on firm beta is negative and statistically significant in specification 1 with no fixed effects and the coefficient is negative but not statistically significant in the sub-sample results that are reported as specification 4.[8] We interpret this result as only weak support of our *ex ante* expectations concerning the effect of beta on executive stock option exercise.

As reported in Table 15, with the exception of the results in specification 2, when the stock price is greater than or equal to the 90^{th} percentile of the trading price distribution in the prior three hundred and sixty trading days, the fraction of options that are exercised is

[7]We do not yet have usable standard errors for specification 2 with firm fixed effects.

[8]Part of the problem with the firm beta covariate is that we are currently measuring a fixed beta for each firm. In later versions of this paper, we plan to use annual estimates of firms' betas using CRSP aggregate files

also statistically significant and positive. We also find a statistically significant and positive association between the percentage of options that are exercised and recent vesting events, measured by an indicator of whether the exercise event was within a two week window of the last vesting date. As discussed previously, these results are consistent with the earlier empirical studies of Heath et al. (1999) and Armstrong et al. (2006), who argue that employees may tie their exercise decisions to cognitive benchmarks as a means of reducing monitoring costs. The exercise of vested options in a grant is also statistically associated with the remaining time on the option. Without controls for age and sex, our results indicate that the more days until maturity the more likely that employees exercise large fractions of vested outstanding options in a grant. However, after controlling for age and sex, this coefficient changes sign, suggesting that our proxies for risk aversion are confounded with the effects of the remaining term.

Since we only have a sub-set of firms that reported the age and sex of their employees, our preferred specification 4 is based on a substantially smaller number of employee grant days than the full sample. In specifications 3 and 4, as seen in Table 15, we find that age has a statistically significant and negative effect on the percentage of vested options that are exercised. Controlling for firm fixed effects in specification 5, however, we find that men are more likely to exercise large fractions of options. These effects suggest that the options costs associated with older and female employees would be higher since the life of their options is longer.

In summary, the covariates related to the stock price paths, the dividend payouts, and market risk have the expected sign with appropriate controls for firm fixed effects, age, and sex. We also sign the effects of stock return volatility and find that it has a positive effect on early exercise which is at variance with standard intuition on the optimal exercise policies associated with American call options. In addition, the contractual vesting structure of executive stock option contracts was found to have an important effect on option exercise behavior, since we find that options that have just vested are more likely to exercise. Finally, there appear to be important differences in the exercise behavior of employees by age and sex. Men are more likely to exercise larger fractions of options early and older employees appear to hold their options. These differential effects on life of the options would also be expected to affect option cost which we address with simulations in the next section.

5 Option Cost to the Firm

For an individual option, the exercise function describes the expected proportion of each outstanding option grant to be voluntarily exercised at a given time and state, conditional on having survived to that point. If the event that the option is actually exercised is sufficiently independent across option holders with identical exercise functions, conditional on the given time and state, then in a large enough pool of such option holders, the fraction of options exercised voluntarily will exactly equal the exercise function. Similarly, the termination function describes the fraction of options stopped through termination in a diversified pool. We assume that such diversification is possible, or, more generally, that the conditional variance in the number of options actually exercised around the expected value is not a priced risk in the market, so that option valuation proceeds as if perfect diversification were possible.

Given the estimated exercise rate per period, G, the value of the option is given by its expected risk-neutral discounted payoff,

$$\begin{aligned} O_t &= E_t^* \int_t^T e^{-r(\tau-t)} \left(\hat{S}_\tau - K\right)^+ \hat{G}_\tau e^{-\int_t^\tau \hat{G}_s \, ds} \, d\tau, \qquad \text{where} \\ d\hat{S}/\hat{S} &= (r-\delta)\, dt + \sigma \, dZ. \end{aligned}$$

To understand the intuition for this expression, note that G measures the expected fraction of a grant exercising, measured as a fraction of the options still unexercised one period earlier. To calculate the expected fraction of *today's* options that exercise at date t, we therefore need to multiply by the proportion of the grant outstanding today that has not exercised prior to t, given by

$$e^{-\int_t^\tau \hat{G}_s \, ds} \, d\tau.$$

We estimate this value with Monte Carlo simulation, using antithetic variates and importance sampling to increase precision. Table 16 reports option values, labeled ESO Value, for a variety of parameterizations, assuming the option holder voluntarily exercises according to the exercise rate function estimated in specification (4) of Table 15, and in addition terminates employment at a constant rate. For the base case, we set the employee age, termination rate, vesting period, stock return volatility, and dividend rate equal to their sample average values and we set the firm beta to one and option expiration date to ten years. Throughout the table, option values for the male employee are slightly lower than for the female employee because males have a higher exercise rate.

For comparison, the column labeled FASB Approx gives option value approximated as the probability of vesting times the Black-Scholes value adjusted for dividends, with contractual expiration date replaced by the option's expected life, conditional on vesting. While new methodologies are developing, the FASB accepts this approximation for accounting valuation, and it is used by the vast majority of firms. Like ESO Value, we compute the option's expected life using Monte Carlo simulation assuming the option holder follows the estimated exercise rate function and terminates employment at a constant rate. This expectation is with respect to the true probability measure, so it depends on the true expected return on the stock. We assume the mean stock return is determined by the CAPM, with a 6% excess expected return on the market.

In theory, the FASB approximation can either understate or overstate the true option value, depending on the exercise policy. To understand why, consider two special cases, and for simplicity assume immediate vesting. First, if the option holder follows the value-maximizing exercise policy in the presence of dividends, as in standard theory, then the true option value will be greater than the Black-Scholes value to any deterministic expiration date, so it will exceed the FASB approximation. Alternatively, suppose the option is stopped, either through exercise or cancellation, at a purely exogenous rate, independent of the stock price, without regard to whether it is in or out of the money. Then the true option value is the average Black-Scholes value over possible stopping dates, while the FASB approximation is Black-Scholes value to the average stopping date, and since the Black-Scholes value tends to be concave in the option expiration date, the FASB approximation will overstate the true value. The exercise policies followed in practice contain elements of both of these examples.

The first panel of Table 16 shows how option value varies with the termination rate. Higher termination rate increases the chance of pre-vesting forfeiture, the chance of post-vesting cancellation, and the rate of suboptimal early exercise, so it reduces option value. It also reduces option life, so the FASB approximation also declines. The increased noise in the exercise policy should in principle increase the FASB overstatement, but here that effect is only slight.

The second panel of Table 16 illustrates the effect of increasing the vesting period. A longer vesting period increases the risk of pre-vesting forfeiture, which reduces option value. Conditional on vesting, the option stopping time has less room to vary, so the difference between the option value and the FASB approximation shrinks.

The third panel of Table 16 presents volatility effects. Because higher volatility increases the exercise rate, option value increases much more slowly than standard theory would predict. It even increases more slowly than the FASB value, which accounts for the reduced expected option life, so that the overstatement increases significantly with volatility. The fourth panel shows that option value declines with the dividend rate. The FASB approximation declines even faster so the error declines in algebraic value.

The fifth panel of Table 16 shows the effect of increasing firm beta. This reduces the exercise rate, perhaps because of increased hedging opportunities, which increases option value. Beta has conflicting effects on option expected life. On one hand, its direct effect on the estimated exercise function is to reduce the exercise rate, which can increase expected life. On the other hand, under the true measure, it increases the mean stock return, which means the exercise function is more likely to be evaluated at higher stock prices, which increases the exercise rate and reduces expected option life. For the male employee, the first effect dominates, and the FASB approximation error is relatively constant. However, for the female employee, who has the lower overall rate of exercise, the second effect dominates, so that the FASB approximation actually moves in the opposite direction as the true value, and the error declines monotonically in algebraic value.

6 Conclusions

This paper is the first to perform a complete empirical estimation of employee stock option exercise behavior and option cost to firms. We develop a methodology for estimating option exercise and cancellation rates as a function of the stock price path, time to expiration, and firm and option holder characteristics. Our estimation is based on a fractional logistic approach, and accounts for correlation between exercises by the same executive. Valuation proceeds by using the estimated exercise rate function to describe the option's expected payoff along each stock price path, and then computing the present value of the payoff. The estimation of empirical exercise rates also allows us to test the predictions of theoretical models of option exercise behavior.

We apply our estimation technique to the largest dataset yet analyzed in the literature, consisting of a comprehensive sample of option exercise grant and exercise data for all employees at 47 publicly traded firms from 1980 to 2005. Our results indicate that using standard pricing approximations, such as the adjusted Black-Scholes method suggested by FASB, can lead to significant errors. The proprietary data used in this study were provided by corporate participants in a sponsored research project that was funded by the Society of Actuaries, who hope that the results of our study will eventually be used as the standard set of exercise assumptions to be used in calculating ESO values on firms' income statements.

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Table 1: Financial Performance Summary - Sample Period Annual Means

This table provides financial performance summary statistics for the forty seven firms in our sample. The firm data are organized into one digit SIC codes to preserve the anonymity of the firms. The start date is the earliest option grant date in our sample and the end date is the last day that we track all grants and exercise events in the sample for all of the firms. Employees is the sample period mean of the annual number of employees for the firms as reported in Compustat as *"Employees: Thousands"* (DATA29). The Market Cap is the sample period mean of *"Common Shares Outstanding: Millions"* in Compustat (DATA25) multiplied by *"Price minus Fiscal Year Close"* in Compustat (DATA199). The Market-to-Book is the sample period mean of the the Market Cap scaled by common equity as reported in Compustat (DATA60). Revenue is the sample period mean of *"Sales/Turnover"* in Compustat (DATA12). Revenue growth is the sample period mean of ($Revenue_t/Revenue_{t-1} - 1$). Net income is the sample period mean of *"Net Income"* in Compustat (DATA172).

	Manufacturing		Transportation, Communications & Utilities	Retail	Finance, Insurance, & Real Estate	Services	
SIC (one digit)	2	3	4	5	6	7	8
Start Sample	9/30/1981	3/28/1980	1/15/1990	9/20/1989	9/27/1991	9/10/1990	2/11/1994
End Sample	12/30/2005	12/30/2005	12/30/2005	12/30/2005	12/30/2005	12/30/2005	12/30/2005
Employees (000's)	19.460	3.969	13.878	21.728	3.271	2.938	4.308
Market Cap ($ Millions)	4,884.141	1,311.949	7,806.397	2,184.146	771.925	797.783	1,041.137
Market-to-Book	4.808	16.051	6.059	4.417	1.839	4.773	5.415
Revenue ($ Millions)	4,696.855	529.830	2,560.016	1,766.914	832.930	325.524	751.081
Revenue Growth ($ Millions)	0.085	0.221	0.107	0.292	0.129	0.241	0.089
Net Income $ Millions)	267.777	33.871	164.508	62.637	62.415	25.911	59.745

Table 2: Number of Grants Per Employee

This table provides the summary statistics over the sample period for the number of grants that were received by each employee in the forty seven firms. We organize the summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Industry	Number of Employees	Mean	Standard Deviation	Minimum	Maximum
Chemicals and allied products	180681	2.23	1.52	1.00	64.00
Petroleum refining and related	166	3.11	3.06	1.00	16.00
Rubber and plastics products	156	2.29	2.79	1.00	18.00
Primary metal industries	5498	3.60	3.50	1.00	34.00
Fabricated metal products	127	2.00	1.87	1.00	18.00
Industrial machinery and computers	30535	2.13	1.32	1.00	33.00
Electronic and other electrical	8075	3.85	3.18	1.00	39.00
Transportation equipment	254	3.69	2.31	1.00	10.00
Measuring, analyzing, and controlling instruments	5124	2.45	2.32	1.00	22.00
Miscellaneous manufacturing	5600	3.13	2.53	1.00	18.00
Communications	120119	1.79	1.40	1.00	34.00
Electric, gas, and sanitary services	375	4.64	3.32	1.00	12.00
Wholesale trade - Durable goods	455	2.06	1.61	1.00	11.00
Wholesale trade - Non-durable goods	4275	4.18	3.08	1.00	16.00
General merchandise stores	1929	4.29	3.39	1.00	17.00
Eating and drinking places	3091	2.60	1.91	1.00	20.00
Miscellaneous retail	10231	5.64	4.98	1.00	30.00
Brokers, dealers,and exchanges	1823	3.62	3.24	1.00	27.00
Business services	3144	3.17	2.72	1.00	48.00
Amusement and recreation services	504	2.76	2.04	1.00	14.00
Educational services	2614	1.61	1.65	1.00	26.00
Engineering, accounting, and management services	855	4.78	3.14	1.00	14.00

Table 3: Number of Options Per Grant

This table reports the number of options per grant at issuance. We multiply the total number of shares granted times the ratio of the price of the stock at the date of issuance to the sample average stock price at the date of issuance ($S_i/\bar{S}$), where ($\bar{S}$) is the global average stock price over all the firms in the sample. The summary statistics are computed over all grants in the sample period. We organize the summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Industry	Number of Grants	Mean	Standard Deviation	Minimum	Maximum
Chemicals and allied products	403811	1,015.2	9,928.6	0.2	3,308,325.8
Petroleum refining and related	516	3,408.1	3,163.9	145.4	24,915.8
Rubber and plastics products	358	24,775.5	64,514.3	227.8	918,803.0
Primary metal industries	19814	5,704.1	27,240.9	1.1	1,454,373.6
Fabricated metal products	254	4,947.5	7,204.4	84.6	70,435.6
Industrial machinery and computers	64900	6,506.3	63,081.0	0.5	6,136,619.3
Electronic and other electrical	31117	3,547.7	23,709.4	1.4	1,712,959.0
Transportation equipment	937	6,525.9	12,405.2	391.0	148,884.6
Measuring, analyzing, and controlling instruments	12554	3,077.7	12,370.8	14.6	641,626.9
Miscellaneous manufacturing	17549	5,714.7	39,731.4	33.2	3,036,514.4
Communications	214621	785.0	8,680.1	0.4	1,442,160.2
Electric, gas, and sanitary services	1740	3,646.6	4,360.0	272.0	85,366.1
Wholesale trade - Durable goods	936	2,733.1	11,404.7	33.9	233,971.4
Wholesale trade - Non-durable goods	17871	3,480.4	18,145.7	9.4	1,650,241.0
General merchandise stores	8276	14,364.3	60,144.7	66.0	2,004,211.9
Eating and drinking places	8041	3,855.9	15,458.4	1.2	411,897.9
Miscellaneous retail	57652	4,242.3	91,337.2	0.5	17,560,983.5
Brokers, dealers,and exchanges	6602	1,837.9	4,453.7	123.2	198,281.6
Business services	9975	6,071.6	41,742.7	2.6	1,214,947.0
Amusement and recreation services	1393	8,667.4	25,531.3	329.3	286,108.5
Educational services	4221	761.7	3,065.0	5.1	67,650.7
Engineering, accounting, and management services	4086	3,196.4	13,331.4	52.0	347,417.4

Table 4: Maximum Number of Vesting Periods per Grant

This table reports the maximum number of vesting periods required to fully vest a given option grant. The summary statistics are computed over all grants in the sample period. We organize the summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Industry	Number of Grants	Mean	Standard Deviation	Minimum	Maximum
Chemicals and allied products	403811	1.19	2.26	1.00	43.00
Petroleum refining and related	516	1.00	0.00	1.00	1.00
Rubber and plastics products	358	3.05	0.21	3.00	4.00
Primary metal industries	19814	3.58	1.04	1.00	4.00
Fabricated metal products	254	2.19	1.89	1.00	6.00
Industrial machinery and computers	64900	3.97	0.65	2.00	6.00
Electronic and other electrical	31117	11.75	13.81	1.00	49.00
Transportation equipment	937	2.91	0.42	1.00	3.00
Measuring, analyzing, and controlling instruments	12554	9.15	14.01	1.00	48.00
Miscellaneous manufacturing	17549	3.41	0.49	3.00	4.00
Communications	214621	1.07	0.45	1.00	4.00
Electric, gas, and sanitary services	1740	3.00	0.00	3.00	3.00
Wholesale trade - Durable goods	936	3.00	0.00	3.00	3.00
Wholesale trade - Non-durable goods	17871	2.44	1.22	1.00	5.00
General merchandise stores	8276	4.03	0.21	3.00	7.00
Eating and drinking places	8041	3.51	1.88	1.00	5.00
Miscellaneous retail	57652	3.04	1.11	1.00	34.00
Brokers, dealers,and exchanges	6602	3.00	0.12	1.00	8.00
Business services	9975	5.09	4.98	1.00	28.00
Amusement and recreation services	1393	3.86	0.63	1.00	4.00
Educational services	4221	5.00	0.00	5.00	5.00
Engineering, accounting, and management services	4086	3.00	0.00	3.00	3.00

Table 5: Percentage of the Options that Vest on the First Vesting Date

This table reports the percentage of the option grants that vest on the first vesting date. The summary statistics are computed over all grants in the sample period. We organize the summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Industry	Number of Grants	Mean %	Standard Deviation %	Minimum %	Maximum %
Chemicals and allied products	403811	97.14	13.30	12.50	100.00
Petroleum refining and related	516	100.00	0.00	100.00	100.00
Rubber and plastics products	358	32.94	1.77	25.00	33.33
Primary metal industries	19814	35.54	26.06	25.00	100.00
Fabricated metal products	254	74.80	35.92	16.67	100.00
Industrial machinery and computers	64900	26.15	6.27	16.67	50.00
Electronic and other electrical	31117	28.56	15.87	20.00	100.00
Transportation equipment	937	36.39	13.96	33.33	100.00
Measuring, analyzing, and controlling instruments	12554	26.38	21.02	2.08	100.00
Miscellaneous manufacturing	17549	29.95	4.09	25.00	33.33
Communications	214621	98.28	10.06	1.15	100.00
Electric, gas, and sanitary services	1740	33.30	0.00	33.30	33.30
Wholesale trade - Durable goods	936	33.33	0.00	33.33	33.33
Wholesale trade - Non-durable goods	17871	44.83	33.37	20.00	100.00
General merchandise stores	8276	24.83	1.06	14.29	33.33
Eating and drinking places	8041	48.85	38.09	20.00	100.00
Miscellaneous retail	57652	33.43	1.95	25.00	100.00
Brokers, dealers,and exchanges	6602	59.81	2.44	25.00	100.00
Business services	9975	27.06	11.02	20.00	100.00
Amusement and recreation services	1393	28.50	15.82	25.00	100.00
Educational services	4221	20.00	0.00	20.00	20.00
Engineering, accounting, and management services	4086	33.33	0.00	33.33	33.33

Table 6: Vesting Time (in Months) of the Options

This table reports the total time required to fully vest an option grant. Time is measured in months from the date of issuance to the month at which the grant is fully vested. The summary statistics are computed over all grants in the sample period. We organize the summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Industry	Number of Grants	Mean	Standard Deviation	Minimum	Maximum
Chemicals and allied products	403811	17.21	10.76	0.00	79.00
Petroleum refining and related	516	12.00	0.00	12.00	12.00
Rubber and plastics products	358	36.38	2.92	25.00	48.00
Primary metal industries	19814	38.48	14.39	0.00	48.00
Fabricated metal products	254	54.59	15.98	0.00	72.00
Industrial machinery and computers	64900	47.63	8.26	12.00	72.00
Electronic and other electrical	31117	34.00	18.21	0.00	60.00
Transportation equipment	937	34.79	5.52	0.00	36.00
Measuring, analyzing, and controlling instruments	12554	48.61	16.05	1.00	60.00
Miscellaneous manufacturing	17549	30.18	14.73	12.00	48.00
Communications	214621	7.11	13.28	0.00	121.00
Electric, gas, and sanitary services	1740	35.92	0.82	25.00	36.00
Wholesale trade - Durable goods	936	36.00	0.00	36.00	36.00
Wholesale trade - Non-durable goods	17871	15.02	15.37	0.00	48.00
General merchandise stores	8276	49.12	7.09	36.00	108.00
Eating and drinking places	8041	48.79	14.94	0.00	60.00
Miscellaneous retail	57652	35.85	1.59	0.00	36.00
Brokers, dealers,and exchanges	6602	35.98	0.96	1.00	48.00
Business services	6681	47.72	12.49	1.00	60.00
Amusement and recreation services	1393	45.76	9.99	0.00	48.00
Educational services	4221	59.99	0.27	50.00	60.00
Engineering, accounting, and management services	4086	36.00	0.16	26.00	36.00

Table 7: Maturity of the Options (in months) at their Issuance Date

This table reports the maturity on option grants at the issuance date. The maturity is measured as the number of months between the issuance date and the expiry date on the grant. The summary statistics are computed over all grants in the sample period. We organize the summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Industry	Number of Grants	Mean	Standard Deviation	Minimum	Maximum
Chemicals and allied products	403811	102.95	32.30	1.00	240.00
Petroleum refining and related	516	120.00	0.00	120.00	120.00
Rubber and plastics products	358	120.00	0.00	120.00	120.00
Primary metal industries	19814	120.00	0.00	120.00	120.00
Fabricated metal products	254	120.00	0.00	120.00	120.00
Industrial machinery and computers	64900	93.62	16.56	72.00	120.00
Electronic and other electrical	31117	120.00	0.00	120.00	120.00
Transportation equipment	937	120.00	0.00	120.00	120.00
Measuring, analyzing, and controlling instruments	12554	120.00	0.00	120.00	120.00
Miscellaneous manufacturing	17549	120.00	0.00	120.00	120.00
Communications	214621	92.81	39.83	3.00	240.00
Electric, gas, and sanitary services	1740	120.00	0.00	120.00	120.00
Wholesale trade - Durable goods	936	120.00	0.00	120.00	120.00
Wholesale trade - Non-durable goods	17871	120.00	0.00	120.00	120.00
General merchandise stores	8276	165.66	25.59	120.00	180.00
Eating and drinking places	8041	120.01	0.49	120.00	151.00
Miscellaneous retail	57652	120.00	0.00	120.00	120.00
Brokers, dealers,and exchanges	6602	120.00	0.00	120.00	120.00
Business services	6681	111.88	29.33	6.00	120.00
Amusement and recreation services	1393	120.00	0.00	120.00	120.00
Educational services	4221	120.00	0.00	120.00	120.00
Engineering, accounting, and management services	4086	120.00	0.00	120.00	120.00

Table 8: Remaining Term on the Option at the Time of Exercise

This table provides the summary statistics over the sample period for the remaining term (in days) on the option at the date the option is exercised. The remaining term is measured as the difference between the date of expiry and the exercise date. We organize the summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Industry	Number of Exercise Events	Mean	Standard Deviation	Minimum	Maximum
Chemicals and allied products	262372	1555.9	1209.7	0.0	3613.0
Petroleum refining and related	441	2215.9	894.3	0.0	3283.0
Rubber and plastics products	385	1989.9	966.1	1.0	3525.0
Primary metal industries	13503	2553.5	634.0	0.0	3646.0
Fabricated metal products	32	254.7	341.8	0.0	1081.0
Industrial machinery and computers	31814	1466.2	912.9	0.0	3616.0
Electronic and other electrical	9549	3010.7	464.5	203.0	3636.0
Transportation equipment	1000	2760.5	457.9	408.0	3483.0
Measuring, analyzing, and controlling instruments	10870	2428.1	669.0	0.0	3643.0
Miscellaneous manufacturing	7942	2673.8	678.3	0.0	3469.0
Communications	6523	1173.0	1165.4	0.0	3407.0
Electric, gas, and sanitary services	1644	2509.3	580.9	25.0	3469.0
Wholesale trade - Durable goods	65	2891.5	237.9	2316.0	3280.0
Wholesale trade - Non-durable goods	6160	2495.3	630.2	307.0	3515.0
General merchandise stores	6059	2907.8	1441.8	0.0	5292.0
Eating and drinking places	4891	1465.2	1186.7	0.0	3429.0
Miscellaneous retail	50948	2522.1	509.8	0.0	3643.0
Brokers, dealers,and exchanges	3900	1897.8	341.3	480.0	3638.0
Business services	5363	2335.9	955.2	0.0	3624.0
Amusement and recreation services	1842	2645.3	474.7	420.0	3385.0
Educational services	933	2265.5	597.7	872.0	3367.0
Engineering, accounting, and management services.	3135	1930.2	1018.8	0.0	3368.0

Table 9: Number of Days that the Option was In-the-Money from the Vesting Day to the Exercise Day

This table reports the summary statistics for the number of days that the option was in-the-money from the vesting day on the options to the date of exercise. We sum all days between the vesting date and the exercise date where the ratio of stock price to the strike price is greater than or equal to one. We organize the summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Industry	Number of Exercise Events	Mean	Standard Deviation	Minimum	Maximum
Chemicals and allied products	262372	1205.4	1045.3	0.0	3663.0
Petroleum refining and related	441	818.4	661.3	0.0	3309.0
Rubber and plastics products	385	688.8	788.5	0.0	2561.0
Primary metal industries	13503	217.2	330.0	0.0	2210.0
Fabricated metal products	32	99.8	171.5	0.0	608.0
Industrial machinery and computers	31814	294.8	354.1	0.0	2216.0
Electronic and other electrical	9549	70.2	134.0	0.0	1366.0
Transportation equipment	1000	86.5	201.7	0.0	2162.0
Measuring, analyzing, and controlling instruments	10870	242.2	419.6	0.0	2927.0
Miscellaneous manufacturing	7942	184.6	390.0	0.0	2205.0
Communications	6523	1830.7	1343.1	0.0	3988.0
Electric, gas, and sanitary services	1644	24.5	138.8	0.0	2002.0
Wholesale trade - Durable goods	65	47.8	101.1	0.0	357.0
Wholesale trade - Non-durable goods	6160	689.0	763.7	0.0	3206.0
General merchandise stores	6059	444.0	551.2	0.0	2217.0
Eating and drinking places	4891	105.1	199.6	0.0	2363.0
Miscellaneous retail	50948	113.2	219.9	0.0	2575.0
Brokers, dealers,and exchanges	3900	602.0	330.6	0.0	1699.0
Business services	5363	205.3	462.5	0.0	3612.0
Amusement and recreation services	1842	146.3	248.2	0.0	2541.0
Educational services	933	156.9	144.3	0.0	969.0
Engineering, accounting, and management services	3135	335.9	660.7	0.0	2565.0

Table 10: Ratio of the Stock Price to the Strike Price at the Time of Exercise

This table reports summary statistics over the sample period for the ratio of the stock price to the strike price (S/K) on the date the option is exercised. Both the price and the strike are adjusted for splits. We organize the summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Industry	Number of Exercise Events	Mean	Standard Deviation	Minimum	Maximum
Chemicals and allied products	262372	3.48	1.18	1.00	13.92
Petroleum refining and related	441	1.95	0.89	1.01	9.22
Rubber and plastics products	385	1.64	0.41	1.00	3.59
Primary metal industries	13503	4.03	2.75	1.00	23.19
Fabricated metal products	32	3.19	2.41	1.03	8.76
Industrial machinery and computers	31814	3.46	2.18	1.00	29.66
Electronic and other electrical	9549	3.28	2.80	1.00	28.91
Transportation equipment	1000	4.08	2.14	1.53	21.62
Measuring, analyzing, and controlling instruments	10870	3.49	2.18	1.00	21.51
Miscellaneous manufacturing	7942	2.07	0.82	1.00	9.82
Communications	6523	3.97	2.18	1.02	21.76
Electric, gas, and sanitary services	1644	2.06	0.55	1.06	7.77
Wholesale trade - Durable goods	65	1.79	0.93	1.05	5.39
Wholesale trade - Non-durable goods	6160	4.99	3.03	1.00	18.27
General merchandise stores	6059	3.69	2.89	1.00	21.43
Eating and drinking places	4891	1.69	0.77	1.00	10.17
Miscellaneous retail	50948	1.90	1.23	1.00	26.75
Brokers, dealers,and exchanges	3900	3.22	1.23	1.16	8.73
Business services	5363	3.91	3.75	1.01	24.53
Amusement and recreation services	1842	7.18	2.38	2.61	14.14
Educational services	933	2.04	1.17	1.01	6.60
Engineering, accounting, and management ser.	3135	3.92	3.14	1.01	17.31

Table 11: Percentage of the Grant's Vested Options That Are Exercised

This table reports the summary statistics for the percentage of each grant's vested options that are exercised on the date of exercise. The percentage is computed as the ratio of the number of options exercised to the total number of number of vested options on the grant that are still unexercised. We then compute the summary statistics using the total vested options outstanding to control for the size of the employee's option position. We organize the summary statistics by the two-digit firm-level SIC categories as reported in CRSP.

Industry	Number of Exercise Events	Mean %	Standard Deviation %	Minimum %	Maximum %
Chemicals and allied products	262372	73.29	31.47	0.10	100.00
Petroleum refining and related	441	90.63	31.66	6.70	100.00
Rubber and plastics products	385	78.75	29.41	1.70	100.00
Primary metal industries	13503	66.22	26.52	0.10	100.00
Fabricated metal products	32	64.56	29.20	6.90	100.00
Industrial machinery and computers	31814	76.95	30.39	0.10	100.00
Electronic and other electrical	9549	51.87	22.81	0.20	100.00
Transportation equipment	1000	79.92	28.15	0.60	100.00
Measuring, analyzing, and controlling instruments	10870	64.61	23.92	0.60	100.00
Miscellaneous manufacturing	7942	56.50	25.21	0.10	100.00
Communications	6523	34.46	17.41	0.20	100.00
Electric, gas, and sanitary services	1644	91.56	32.84	10.00	100.00
Wholesale trade - Durable goods	65	52.97	15.31	9.00	100.00
Wholesale trade - Non-durable goods	6160	87.23	31.87	1.60	100.00
General merchandise stores	6059	59.09	27.03	0.20	100.00
Eating and drinking places	4891	72.57	33.10	1.30	100.00
Miscellaneous retail	50948	40.63	17.62	0.10	100.00
Brokers, dealers,and exchanges	3900	79.97	28.43	3.30	100.00
Business services	5363	73.49	30.67	0.10	100.00
Amusement and recreation services	1842	75.37	28.84	7.70	100.00
Educational services	933	88.96	30.43	6.10	100.00
Engineering, accounting, and management services	3135	31.37	15.64	0.10	100.00

Table 12: Summary Statistics for the Covariate Paths

This table reports the summary statistics for the employee-grant-day paths for option grants in the sample. The *price to strike ratio* is the daily observation of the ratio of the split-adjusted price of the stock to the split-adjusted strike price on the option (the zero valued minimum is due to our reporting precision). The *dividend payment* is equal to the split-adjusted dividend payment on each day. This value is zero on all but dividend payment days. The *firm beta* was estimated for each firm using market and firm data from CRSP and interest rate data from the Federal Reserve Board over a five year interval (in some cases the interval is shorter) and is a constant for each firm. The *vesting period within two weeks* is an indicator variable that is one if the day is within two weeks of a vesting event and zero otherwise. The *stock return volatility* is the standard deviation of the daily stock returns, as reported in CRSP, over the past thirty trading days. The *price relative to the 90th percentile* is an indicator variable that is equal to one if the observed stock price is greater than or equal to 90th percentile of the stock price distribution over the prior year of trading. It is equal to zero otherwise.

Variable	Mean	Standard Deviation	Minimum	Maximum
Price to Strike Ratio	1.7124	1.5972	0	31.226
Stock return volatility past thirty days	0.019922	0.011672	0.0016	0.1644
Dividend payment	0.0011888	0.024931	0	1
Firm Beta	1.0482	0.30754	0.21289	2.3403
Vesting event within two weeks (Indicator variable)	0.019675	0.13888	0	1
Price relative to 90th percentile (indicator variable)	0.59843	0.49022	0	1
Remaining time (days)	2,444	754.83	181	7319
n = 313,767,112				

Table 13: Employee Demographics

This table presents summary statistics for the demographic information that is reported by a subset of the firms in the sample. We summarize the information by employees over the sample period.

	Number of Employees	Mean	Standard Deviation	Minimum	Maximum
Salary	8096	167026.38	917499.24	9760.00	3683520.00
Sex	34470	0.68	0.47	0.00	1.00
Age	62259	43.59	9.26	17.00	85.00
Executive	18683	0.08	0.27	0.00	1.00
Other employee type	18683	0.40	0.49	0.00	1.00
Managers	18683	0.49		0.00	1.00
Board member	18683	0.03	0.18	0.00	1.00

Table 14: Cancellation Counts and Reasons for Cancellations

This table presents the total number of cancellation events for all firms over the sample period. Only some of the firms reported the reason for cancellations. Of those firms that report the reason for the termination, the category Terminate Position includes all cancellations arising from employment terminations that are either voluntary or involuntary. The other categories that are reported include deaths and retirements. The majority of cancellations do not have further information concerning the reason for the cancellation.

year	Terminate Position	Retirement	Death	Reason Unknown	Total
1983	0	0	0	0	4
1984	18	0	0	4	23
1985	2	1	0	2	8
1986	0	0	0	3	10
1987	5	0	0	5	14
1988	12	0	1	5	19
1989	19	0	0	2	27
1990	60	1	1	19	148
1991	149	2	47	774	996
1992	178	4	10	2040	11670
1993	142	1	19	873	8915
1994	153	2	98	459	8175
1995	358	3	101	1282	5768
1996	557	6	68	1036	17944
1997	932	5	58	25391	48235
1998	2121	10	308	15829	28661
1999	2452	22	52	6539	14931
2000	3189	34	138	4699	10315
2001	6145	37	133	14442	25760
2002	4754	37	141	4307	10260
2003	5797	44	156	8247	16707
2004	3932	52	179	16069	24664
2005	4518	39	322	62830	153771
Total	35493	300	1832	164857	387025

Table 15: Estimation Results

This table presents the results for alternative specifications of the fractional logistic estimator. Specifications 1 and 2 are estimated on the full sample. In specification 1, we exclude both the sex and age covariates and the firm fixed effects. Specification 2 includes firm fixed effects and again excludes the sex and age covariates. Specifications 3 and 4 are estimated with a smaller subsample of firms that reported information on sex and age. Specification 3 includes sex and age and excludes firm fixed effects. Specification 4 includes sex, age and firm fixed effects. The standard errors are reported in parentheses below the coefficient estimates. The estimator clusters at the level of the individual employee.

	Alternative Specifications			
Covariates	1	2	3	4
Constant	-6.147	-10.594	-5.577	-6.359
	(0.011)		(0.053)	(0.341)
Price to strike ratio	0.109	0.153	0.060	0.108
	(0.001)		(0.002)	(0.002)
Stock return volatility in prior 30 trading days	-0.865	6.697	-4.156	2.792
	(0.201)		(0.586)	(0.403)
Dividend payment within 14 trading days (Indicator Variable)	-0.302	-7.375	7.586	24.807
	(0.074)		(9.670)	(4.368)
Firm beta	-0.128	2.211	0.181	-0.176
	(0.008)		(0.020)	(0.437)
Vesting event in prior 14 calendar days (Indicator Variable)	2.266	2.357	3.586	3.725
	(0.008)		(0.014)	(0.012)
Price $\geq 90^{th}$ percentile of the prior 360 trading days (Indicator Variable)	0.062	-0.005	0.258	0.283
	(0.005)		(0.015)	(0.008)
Remaining calendar days until expiration at the time of exercise	0.0005	0.0004	-0.001	-0.001
	(0.000)		(0.000)	(0.000)
Age			-0.012	-0.010
			(0.001)	(0.000)
Sex			-0.003	0.150
			(0.017)	(0.010)
Firm Fixed Effects	No	Yes	No	Yes
Number of observations	313.8M	313.8M	67.6M	67.6M

Table 16: Valuation Results.

The table shows option values for a 43-year old employee based on the estimated exercise rate function. The initial stock price and strike price are $100. The option expires in ten years. The base case assumes 11.52% annual termination rate, two-year vesting period, 30% volatility, 1% dividend rate, and beta of one. The riskless rate is 5% and the expected market return is 11%. FASB Approx is the probability that the option vests times the Black-Scholes value of the option assuming expiration at the option's average life given vesting.

	Male Employee					Female Employee				
Changing Parameter	ESO Value	Prob Vest	Av Vested Life	FASB Approx	Percent Error	ESO Value	Prob Vest	Av Vested Life	FASB Approx	Percent Error
Termination Rate Effects										
8.5%	27.86	0.84	5.53	28.46	2.15%	28.34	0.84	5.71	28.92	2.04%
10.0%	26.56	0.81	5.38	27.14	2.19%	27.00	0.81	5.55	27.56	2.10%
11.5%	25.30	0.78	5.23	25.86	2.20%	25.70	0.78	5.38	26.25	2.13%
13.0%	24.13	0.76	5.09	24.67	2.20%	24.50	0.76	5.24	25.03	2.15%
14.5%	23.00	0.73	4.95	23.50	2.19%	23.33	0.73	5.09	23.84	2.15%
Vesting Period Effects										
1	26.28	0.88	4.63	27.48	4.56%	26.77	0.88	4.79	27.96	4.44%
2	25.30	0.78	5.23	25.86	2.20%	25.70	0.78	5.38	26.25	2.13%
3	23.91	0.69	5.82	24.13	0.92%	24.24	0.69	5.97	24.45	0.86%
4	22.37	0.61	6.40	22.38	0.03%	22.63	0.61	6.55	22.62	-0.03%
5	20.77	0.54	6.99	20.65	-0.57%	20.96	0.54	7.12	20.83	-0.62%
Volatility Effects										
20%	20.62	0.78	5.22	20.39	-1.09%	20.96	0.78	5.39	20.77	-0.89%
30%	25.30	0.78	5.23	25.86	2.20%	25.70	0.78	5.38	26.25	2.13%
40%	29.63	0.78	5.20	31.24	5.43%	30.09	0.78	5.35	31.63	5.14%
50%	33.47	0.78	5.19	36.42	8.81%	33.98	0.78	5.32	36.81	8.31%
60%	36.95	0.78	5.18	41.32	11.84%	37.52	0.78	5.30	41.69	11.13%
Dividend Rate Effects										
0%	27.92	0.78	5.21	28.81	3.20%	28.43	0.78	5.37	29.31	3.08%
1%	25.30	0.78	5.23	25.86	2.20%	25.70	0.78	5.38	26.25	2.13%
3%	20.69	0.78	5.23	20.59	-0.47%	20.90	0.78	5.39	20.81	-0.44%
5%	16.87	0.78	5.20	16.16	-4.20%	16.95	0.78	5.36	16.26	-4.07%
7%	13.80	0.78	5.13	12.54	-9.16%	13.80	0.78	5.29	12.56	-8.98%
Beta Effects										
0.0	24.80	0.78	5.03	25.38	2.33%	25.23	0.78	5.55	26.64	5.59%
0.5	25.05	0.78	5.13	25.62	2.26%	25.47	0.78	5.46	26.44	3.80%
1.0	25.30	0.78	5.23	25.86	2.20%	25.70	0.78	5.38	26.25	2.13%
1.5	25.54	0.78	5.32	26.09	2.16%	25.92	0.78	5.31	26.08	0.60%
2.0	25.77	0.78	5.41	26.31	2.12%	26.14	0.78	5.25	25.93	-0.78%