Fire Sales, Foreign Entry and Bank Liquidity[1]

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J.E.L. Classification: G21, G28, G32, E58, D61

Keywords: Cash, Cash holdings, Crises, Systemic risk, Distress, Liquidation cost, Limited pledgeability

This Draft: 16 March, 2008

[1]We are grateful to Franklin Allen, Heitor Almeida, Giacinta Cestone (discussant), Douglas Gale, Alan Morrison (discussant), Leonard Nakamura, Raghuram Rajan and Ellis Tallman for their suggestions, to seminar participants at London Business School, Bank of England, University of Durham, International Conference on Asset Prices and Firm Policies in June 2006 in Verona, Workshop on Banking, Risk and Regulation hosted by BCBS and FDIC, and the UniCredit Group Conference on Banking in Finance in December 2007 in Naples for their comments, and to Yili Zhang for excellent research assistance. All errors remain our own. The views expressed here are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of New York or the Federal Reserve System.

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Abstract

Bank liquidity is a crucial determinant of the severity of banking crises. We consider the effect of fire sales and foreign entry during crises on banks' ex-ante choice of liquid asset holdings. In a setting with limited pledgeability of risky cash flows and differential expertise between banks and outsiders in employing banking assets, the market for assets clears only at fire-sale prices following the onset of a crisis – and outsiders may enter the market if prices fall sufficiently low. While fire sales make it attractive for banks to hold liquid assets, foreign entry reduces this incentive. We show that in this setting, bank liquidity is counter-cyclical whereas bank capital measured as bank profits is pro-cyclical. We derive conditions under which privately optimal levels of bank liquidity are higher or lower than benchmark levels that maximize total output of the banking sector. We present and discuss evidence on bank liquidity that is consistent with model predictions.

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1 Introduction

A central difficulty during banking crises is one of finding ready buyers of distressed assets: If a bank needs to restructure its balance sheet during a crisis, the potential buyers of its assets are other banks that may have also been severely affected and thus may not have enough equity capital or debt capacity to purchase assets. This theme is a familiar one from corporate finance (see Williamson, 1988, and Shleifer and Vishny, 1992), but leads to especially severe problems in a banking crisis given the relative opacity of bank balance-sheets and the high sensitivity of banking assets to macroeconomic shocks. Allen and Gale (1998) have shown in the context of the Diamond and Dybvig (1983) setup that asset prices fall below their fundamental value in some states of the world, giving rise to "cash-in-the-market" (or fire-sale) pricing. Surviving banks that do have enough liquidity during such states stand to make windfall profits from purchasing assets at fire-sale prices. Even if crises arrive infrequently, the potential gains from acquisitions at fire sales could be large. This gives banks incentives to hold liquid assets so that in the event that they survive the crisis, they will have resources to take advantage of fire sales.[1]

Our objective in this paper is to present a model of banks' choice of ex-ante liquidity that is driven by such strategic considerations. We examine the portfolio choice of banks maximizing their profits in the presence of fire sales that are endogenously derived in an equilibrium setup of the banking industry. While risky assets are attractive to banks given their limited liability, risky assets' cash flows are illiquid and have limited pledgeability compared to cash flows of safe assets (which we assume are fully pledgeable). This limited pledgeability of risky cash flows, coupled with the potential for future acquisitions at fire-sale prices, induces banks to hold liquid assets in their portfolios. In essence, bank portfolio choice acquires an inter-temporal dimension even in our otherwise myopic set-up.[2]

In this setting, we show that banks' equilibrium holding of liquid assets is decreasing

[1] Cleveland and Thomas in their book *Citibank* provide a memorable account of how National City Bank, that eventually became Citibank, grew from a small treasury unit into one of the biggest commercial banks under its president Stillman, who anticipated the 1893 and 1907 crises and built up liquidity and capital before the crises to benefit from the difficulties of its competitors. We describe this case in greater detail in Section 5.

[2] Indeed, we show that this dimension arises in our model purely due to benefit from holding liquid assets while purchasing failed banks, and not from the more standard precautionary desire for holding liquidity to avoid default.

in the pledgeability of risky cash flows. In turn, bank liquidity is counter-cyclical, that is, decreasing in the health of the economy: During economic upturns, expected profits from risky assets are high and so is their pledgeability. An important implication of this result is that adverse asset-side shocks that follow good times result in deeper fire-sale discounts since bank balance-sheets feature low liquidity in such times, whereby conditional on adverse shocks, there is lower aggregate liquidity to clear market for assets. In sharp contrast to this counter-cyclical behavior of bank liquidity, bank "capital" – measured as the expected value of bank profits or in other words market value of bank equity – is pro-cyclical. The intuition is simple: during upturns, opportunity cost of not investing in risky assets is high and anticipated pledgeability of future profits is high too. Put another way, during upturns, capital in the form of expected profits is a more efficient way for banks to create future liquidity, whereas in downturns, liquid assets play this role more efficiently.

We also compare the privately optimal levels of bank liquidity with benchmark levels that maximize the overall banking sector output. The pledgeability of risky cash flows turns out to be the critical determinant of whether banks hold too little or too high liquidity relative to the socially optimal level. When pledgeability is high, banks hold less liquidity than is socially optimal due to the preference for risk induced by limited liability; otherwise, banks may hold even more liquidity than is socially optimal in order to capitalize on fire sales. This latter result may seem surprising but is explained simply: Fire sales result in transfers of value amongst banks but do not lead to any aggregate welfare gains or costs, and thus, liquidity hoarded to capitalize on fire sales may in some cases be excessive from the standpoint of maximizing banking sector output. In particular, inefficiently high levels of bank liquidity and by implication inefficiently low levels of intermediation arise when pledgeability of risky cash flows is sufficiently low, for example, during crises or in banking sectors of emerging markets.

We present descriptive cross-country evidence on the asset liquidity of banks across countries. This evidence suggests that banks' choice of liquidity seems to vary along dimensions that would be correlated with difficulty in raising external finance and the severity of financial distress. We show that banks hold more liquid assets in those countries that have (i) less developed accounting standards; (ii) lower total market capitalization relative to GDP; and, (iii) lower liquidity in stock markets. We also discuss how our model's implication that bank liquidity would be counter-cyclical squares up with existing evidence and the recently documented facts concerning leverage targeting by banks.

We conclude by reverting to the model and analyzing the effect of entry by outsiders (outside of the banking sector) for acquisition of assets during crises. Since outsiders may lack expertise relative to surviving banks, they may enter only when fire sales are sufficiently deep. Nevertheless, once they enter, they have the effect of increasing the aggregate pool of liquidity and stabilizing prices. This reduces ex-ante returns to liquidity for banks and they hold lower levels of liquid assets in their portfolios. This implies that even when outsiders are second-best users of assets, their entry can potentially unlock liquid hoardings of banks in emerging markets and lead to greater intermediation by their banking sectors.

Section 2 presents the related literature. Sections 3 and 4 set up the benchmark model without outsiders and characterize the effect of fire sales on bank liquidity. Section 5 provides descriptive empirical evidence. Section 6 considers the effect of entry by liquidity-endowed outsiders on bank liquidity. Section 7 concludes. All proofs not in the main text are in the Appendix.

2 Related literature

The relationship between liquidity and asset prices has been used in the literature to examine a number of interesting issues such as financial market runs (Bernardo and Welch, 2004, and Morris and Shin, 2004), strategic lending and trading (Donaldson, 1992, Brunnermeier and Pedersen, 2005 and Acharya, Gromb and Yorulmazer, 2007), contagion through asset prices (Gorton and Huang, 2004, Schnabel and Shin, 2004, Allen and Gale, 2005, Cifuentes, Ferrucci and Shin, 2005, and Diamond and Rajan, 2005), and optimal resolution of bank failures (Acharya and Yorulmazer, 2005, 2007). While liquidity can affect asset prices, most of the literature cited above treats the level of liquidity (of banks) as exogenous, excepting a few papers that are discussed below.

Banks can hold liquidity for various reasons such as the "precautionary" motive of insuring against their depositors' uncertain liquidity needs and the "strategic" motive of being able to take advantage of profitable options when they arise. We abstract from the precautionary motive which finds a strong parallel in the corporate-finance literature on firms' propensity to save in the form of cash holdings (see, for example, Acharya, Almeida and Campello, 2007, and the large literature cited therein). Instead, we focus exclusively on the strategic one, the profitable option being the opportunity to acquire other banks' assets cheap. On this score, our paper is more in the spirit of recent papers by Allen and Gale (2004a,b) and Gorton and Huang (2004) who also investigate how liquidity is endogenously determined.[3,4]

Our paper differs from these recent papers along the following dimensions. First, in contrast to these papers, we do not model bank runs as an endogenous choice of depositors. Instead, we simply assume that deposits are hard contracts and thus bank failures occur whenever asset shocks are sufficiently adverse. Second, and more substantively, we derive interesting comparative static results as to how bank liquidity is affected by ease of external finance and the business cycle. These results are made possible by the fact that our model does not rule out external financing against risky cash flows altogether, but instead considers as a model parameter the limited pledgeability of risky cash flows arising from moral hazard considerations, as in Holmstrom and Tirole (1998). Under this limited pledgeability framework, we are also able to address the key welfare issues connected with entry of outside capital and whether the private level of bank liquidity is the efficient one.[5]

We acknowledge that since liquid assets usually have lower returns than illiquid assets, banks may rationally choose to rely on liquidity from an interbank market or a lender of last resort (LOLR). Bhattacharya and Gale (1987) build a model of the interbank market where individual banks that are subject to liquidity shocks coinsure each other against these shocks through a borrowing-lending mechanism. However, in their model, the composition of liquid and illiquid assets in each bank's portfolio and the liquidity shocks are private information.

[3] Allen and Gale (2004b), for example, build a model where runs by depositors result in fire-sale liquidation of banking assets. Banks endogenously choose the level of the liquid asset, which they use to purchase banking assets. Since on average the liquid asset has a lower return than the risky asset, banks have to be compensated for holding liquid assets, which is possible in equilibrium if they can purchase the risky asset at a discount in some states of the world, leading to cash-in-the-market pricing.

[4] Note that Diamond and Rajan (2005) also features fire-sale opportunities. However, in their model, date-0 depositors are assumed to care only about date-1 return and banks maximize their date-0 financing from these depositors. Hence, even though there is a high return to cash holdings in their model in some states of the world, banks never hold cash as cash yields benefits only in future (beyond date 1). While we do not endogenize debt capacity of the firm at date 0 as Diamond and Rajan do, the critical difference is that banks in our model maximize the sum of current and *future* profits.

[5] It should also be noted that since moral hazard arises only because insiders have expertise with regard to risky assets, our modeling assumptions are consistent with outside capital being second-best user of banking assets. This explains why outside capital does not enter during crises until discounts are steep so that a high shadow value of capital may co-exist with ready funds (waiting) outside the system at the onset of crises. A similar theme is explored in an asset pricing context by He and Krishnamurthy (2006) and the international finance context by Caballero and Krishnamurthy (2001).

Hence, banks have an incentive to under-invest in liquid assets and free-ride on the common pool of liquidity in the interbank market. Repullo (2005) shows that the existence of LOLR results in banks holding a lower level of the liquid asset as they rely on the LOLR for liquidity. While we do not consider inter-bank lending in this paper, we do discuss in Section 7 the implications in our setup of regulatory closure policies on bank liquidity.

Finally, while our model shares some similarities with Acharya and Yorulmazer (2005), especially in the way in which fire sales are derived and entry by outsiders is modeled, the focus of the two papers is completely different. In Acharya and Yorulmazer (2005), it is assumed that banks invest *all* their funds in risky assets whereas the focus of the current paper is the endogenous *interior* choice of liquidity in bank portfolios.

3 Benchmark model

Before presenting the model formally, we give an informal description of the building blocks and the key assumptions. We consider a setting with a large number of banks. Banks solve a portfolio choice problem: they maximize their profits, or in other words, equity values, by choosing how much to invest in risky assets, which are assumed to have diminishing returns to scale, and how much to park in the safe asset as liquid reserves. This portfolio choice problem acquires an inter-temporal dimension given the limited pledgeability of risky cash flows and the benefit from holding liquidity in states where banks can profit from asset purchases at fire-sale discounts. Specifically, while banks have a preference for the risky asset due to its "option" value in the traditional risk-shifting sense, there is a counteracting preference for the safe asset due to its greater liquidity relative to the risky asset.

Banks' choice of liquidity trades off the expected returns from the two kinds of assets taking account of the option value of the risky asset and the need for inter-temporal transfers of liquidity. The benchmark socially optimal level of liquid asset holdings in banks' portfolio maximizes the value of banking sector as a whole, that is, the sum of the values of banks (rather than just banks' equity values). For most of our analysis, we assume that when banks fail, the only potential purchasers are other banks. We also assume that deposits are insured by the regulator and that there is no cost of providing insurance to depositors, in which case the assumption of insured deposits does not play a key role in determination of liquidity choices of banks. We introduce entry by outsiders in Section 6 and discuss in Section 7 the implications of costly deposit insurance.

The time line of the benchmark model is outlined in Figure 1. We consider an economy where time is indexed by t, where $t \in \left\{0, \frac{1}{2}, 1, 2\right\}$. In the benchmark model, there are banks, bank owners, depositors and a regulator, who provides deposit insurance.

In particular, we assume that there is a continuum of banks with measure 1, where each bank has access to its own depositor base. The depositor base of a bank is itself a continuum of depositors of measure 1. Bank owners as well as depositors are risk-neutral, so that banks aim to maximize the equity value of the sum of expected profit over time.

Depositors receive a unit endowment at $t = 0$ and at $t = 1$. Depositors have access to a reservation investment opportunity that gives them a utility of 1 per unit of investment. At dates $t = 0$ and $t = 1$, depositors choose to invest their good in this reservation opportunity or in their bank. Deposits take the form of a simple debt contract with maturity of one period and the promised deposit rate is not contingent on bank's investment decisions or its realized returns.

Banks collect one unit of deposits from depositors and make investments to maximize the sum of expected profits at $t = 1$ and $t = 2$. There is no discounting. In particular, banks choose a portfolio by investing l units in a safe asset and the remaining $(1-l)$ units in a risky asset, which is to be thought of as a portfolio of loans to firms in the corporate sector.

The payoff of the bank from its loan is $\widetilde{R}_t$, where $\widetilde{R}_t$ is the random variable:

$$\widetilde{R}_t = \begin{cases} R_t & \text{with prob } \alpha_t \\ 0 & \text{with prob } 1 - \alpha_t \end{cases} . \tag{1}$$

R_t can be viewed as the notional value of the loan. The realization of $\widetilde{R}_t$ is independent across banks, so that by the law of large numbers, precisely α_t of the banks have positive payoff. Moreover, the returns are assumed to be independent over time.

However, there is aggregate uncertainty in that α_t is itself random. Hence, there is uncertainty over the proportion of banks that receive positive payoff. In what follows, we will denote by $E(\alpha_t)$ the expected realization of α_t.

We assume that the risky technology $\widetilde{R}_0$ has diminishing returns to scale, that is, the return R_0 is decreasing in $(1 - l)$. In order to get a closed form solution, we use a setup similar to Holmstrom and Tirole (2001) and let

$$R_0(l) = b - \frac{(1 - l)}{2}. \tag{2}$$

Hence, R_0 takes values between $\left(b - \frac{1}{2}\right)$ and b, and $\frac{dR_0}{dl} = \frac{1}{2} > 0$. For simplicity we assume that $\widetilde{R}_1$ is a constant returns to scale technology with $R_1 > \frac{1}{\alpha_1}$. This helps us concentrate on the effect of choice of liquid asset only in the first period and simplifies the analysis.

At the intermediate date $t = \frac{1}{2}$, the outcome of the first-period investments in the risky asset becomes public information, though banks can collect these returns fully only at $t = 1$.

The safe asset is completely liquid and pays one unit at any date for each unit invested. The risky asset is however not completely liquid due to a moral-hazard problem at the bank level. From date $t = 1/2$ to date $t = 1$, if the bank does not exert effort, then when the return is high, it cannot generate R_0 but only $\left(R_0 - \overline{\Delta}\right)$ and its owners enjoy a non-pecuniary benefit of $B \in (0, \overline{\Delta})$. For the bank owners to exert effort, appropriate incentives have to be provided by giving bank owners a minimum share of the bank's profits. We denote this share as θ. If r_0 is the cost of borrowing deposits, then the incentive-compatibility constraint is:

$$\alpha_0 \theta (R_0 - r_0) \geqslant \alpha_0 \left[\theta (R_0 - \overline{\Delta} - r_0) + B\right] \tag{IC}$$

Under this constraint, bank owners need a minimum share of $\overline{\theta} = B/\overline{\Delta}$ to monitor loans properly.[6] Therefore, the bank can raise at most a fraction $\tau = \left(1 - \overline{\theta}\right)$ of its income at $t = 1$ in the capital market at date $t = 1/2$ if it is required to exert effort to monitor loans.[7,8] We assume that at $t = 0$, the entire share of the bank profits belongs to the bank owners, and therefore, moral hazard is not a concern at the beginning, whereas it can become an issue at

[6]See Hart and Moore (1994) and Holmstrom and Tirole (1998) for models with similar incentive-compatibility constraints.

[7]In other words, bank-level moral hazard in our model can be addressed by greater ownership of banks by insiders. Caprio et al (2005) document that banks in general are not widely held (a widely-held bank is one that has no legal entity owning 10 percent or more of the voting rights), similar to the findings of La Porta et al (1999) for corporations in general. This observation is stronger for countries with weaker shareholder protection laws. They also find that greater inside ownership enhances bank valuation in such countries. Overall, these findings are consistent with the key assumptions of our model since weaker protection laws should imply a greater risk of cash-flow appropriation by insiders, and, in turn, lead to greater inside ownership of banks in equilibrium.

[8]Alternatively, we could have assumed that when the bank does not exert effort, the value of the high return is R_0, but the probability of having the high return is lower, say $\alpha_0^L < \alpha_0$, and its owners enjoy a non-pecuniary benefit of B, with $\left(\alpha_0 - \alpha_0^L\right) R_0 > B$. In that case, the incentive compatibility constraint can be written as $\alpha_0 \theta \left(R_0 - r_0\right) \geqslant \alpha_0^L \left[\theta \left(R_0 - r_0\right) + B\right]$. Hence, bank owners need a minimum share of $\overline{\theta} = \left(\frac{\alpha_0^L B}{\left(\alpha_0 - \alpha_0^L\right)\left(R_0 - r_0\right)}\right)$ to monitor these loans prudently. Therefore, the bank can raise at most a fraction $\tau = \left(1 - \overline{\theta}\right)$ of its future income in the capital market if it is required to exert effort. For simplicity, we model moral hazard using returns, rather than probabilities, and assume that the returns are not verifiable. While this does not change any of our results, it simplifies the expressions considerably.

$t = 1/2$ when the bank wants to pledge its future cash flow in the capital market.

We assume that deposits are fully insured in the first period. Note that the second period is the last period in our model and there is no further investment opportunity. As a result, our analysis is not affected by whether deposits are insured for the second investment or not. Finally, we make technical assumptions (A1)–(A4) which are contained in the Appendix. We refer to these at a few relevant points of our analysis.

If a bank's return from the first-period investment is high, then the bank operates one more period and makes the second-period investment. If the return is low, then the bank is in default and its assets are put up for sale at $t = \frac{1}{2}$.

When banks with the high return from the first period investment want to acquire failed banks' assets, they use the liquid asset in their portfolio and/or try to raise funds from the capital market against their future return. However, because of moral hazard, banks cannot fully pledge their future income, but only a fraction τ of it.

Depending on the first period returns, a proportion $k = (1 - \alpha_0)$ of banks fail. Since banks are identical at $t = 0$, we denote without loss of generality the possible states at $t = 1$ with k.

4 Analysis

We analyze the model proceeding backwards from the second period to the first period.

The surviving banks operate for another period at $t = 1$. The probability of having the high return is equal to α_1 for each bank. As this is the last period, there is no further investment opportunity and no asset sales take place in this period. Since the risky asset has a higher expected return than the safe asset and there is no asset purchase opportunity, banks invest all their funds in the risky asset at $t = 1$. The expected payoff to the bank from its second-period investment is thus $\alpha_1 (R_1 - r_1) = \overline{p}$.

Next, we investigate the sale of failed banks' assets and the resulting asset prices.

4.1 Asset sales and liquidation values

In examining the purchase of failed banks' assets, several interesting issues arise. First, surviving banks may compete with each other if there are enough resources with them to acquire all failed banks' assets. Second, unless the game for asset acquisition is specified with reasonable restrictions, an abundance of equilibria arises. To keep the analysis tractable and at the same time reasonable, we make the following assumptions:

(i) The regulator pools all failed banks' assets and auctions these assets to the surviving banks.

(ii) Denoting the surviving banks as $i \in [0, 1-k]$, each surviving bank submits a schedule $y_i(p)$ for the amount of assets they are willing to purchase as a function of the price p at which a unit of the banking asset (inclusive of associated deposits) is being auctioned.

(iii) The regulator determines the auction price p so as to maximize the output of the banking sector subject to the natural constraint that portions allocated to surviving banks add up at most to the number of failed banks, that is,

$$\int_0^{1-k} y_i(p)di \leq k. \tag{3}$$

(iv) We focus on the symmetric outcome where all surviving banks submit the same schedule, that is, $y_i(p) = y(p)$ for all $i \in [0, 1]$.

First, we derive the demand schedule for surviving banks. Note that a surviving bank can generate a maximum return of $\overline{p}$ from the risky asset in the second period. Hence, the maximum price a surviving bank is willing to pay for a failed bank's asset is $\overline{p}$. Also, a surviving bank can generate $(\tau [(1-l)R_0 - r_0])$ units from the capital market at $t = 1/2$.[9] Hence, the resources available with a surviving bank for purchasing failed banking assets is equal to

$$L = l + \tau [(1-l)R_0 - r_0], \tag{4}$$

[9] We assume that at $t = 1/2$ the bank can generate funds *only* against the return at $t = 1$, which has been publicly revealed. This is because at $t = 1/2$, the second investment has not yet arrived and the bank cannot generate any funds against such investments due to what is often called in corporate finance literature as the "bogus entrepreneur" problem. This refers to the severe adverse selection problem that firms face in raising funds against their growth options due to pooling with firms and entrepreneurs who may also approach capital markets for funding against "bogus" growth plans.

provided that the return from the risky asset is enough to pay old depositors, that is, when $(1-l)R_0 \geqslant r_0$. For $R_0 = \left(b - \frac{1-l}{2}\right)$, this condition is met when $l \leqslant l_{max} = \left[\sqrt{b^2-2} + (1-b)\right]$, and we show later on that under assumptions (A1) and (A2), banks never hold a level of liquidity l greater than l_{max} in equilibrium. Finally, Assumption (A2) also guarantees that the liquidity banks have for asset purchases increases as they hold more liquid asset in their portfolio, that is, $\frac{\partial L}{\partial l} > 0$.

Note that the expected profits of a surviving bank from the asset purchase can be calculated as: $y(p)[\overline{p} - p]$. The surviving bank wishes to maximize these profits subject to the resource constraint $y(p) \cdot p \leq L$. Hence, for $p < \overline{p}$, surviving banks are willing to purchase the maximum amount of assets using their resources. Thus, demand schedule for surviving banks is $y(p) = \frac{L}{p}$. For $p > \overline{p}$, the demand is $y(p) = 0$, and for $p = \overline{p}$, banks are indifferent between values of $y(p)$ over the range $\left[0, \frac{L}{\overline{p}}\right]$. In words, as long as purchasing assets is profitable, a surviving bank wishes to use up all its resources to purchase failed banks' assets.

Next, we analyze how the regulator allocates failed banks' assets and the resulting price function. The regulator cannot set $p > \overline{p}$ since in this case $y(p) = y_2(p) = 0$. If $p \leqslant \overline{p}$, and the proportion of failed banks is sufficiently small, then the surviving banks have enough funds to pay the full price for all failed banks' assets. Specifically, for $k \leq \underline{k}$, where

$$\underline{k} = \left(\frac{L}{L + \overline{p}}\right), \tag{5}$$

the regulator sets the price at $p^*(k) = \overline{p}$. At this price, surviving banks are indifferent between any quantity of assets purchased. Hence, we assume that the regulator allocates a share $y(p^*) = \frac{k}{1-k}$ to each surviving bank.

For values of $k > \underline{k}$, surviving banks cannot pay the full price for all failed banks' assets and the regulator sets the price at

$$p^*(k) = \frac{(1-k)L}{k}. \tag{6}$$

Note that, in this region, surviving banks use all available funds and the price falls as the number of failures increase. This effect is basically the cash-in-the-market pricing as in Allen and Gale (1994, 1998) and is also akin to the industry-equilibrium hypothesis of Shleifer and Vishny (1992). The resulting price function is formally stated in the following proposition and is illustrated in Figure 2.

Proposition 1 *The price of failed banks' assets as a function of the proportion of failed banks is as follows:*

$$p^*(k) = \begin{cases} \overline{p} & \text{for} \quad k \leqslant \underline{k} \\ \frac{(1-k)L}{k} & \text{for} \quad k > \underline{k} \end{cases} . \tag{7}$$

From equation (5), one can easily see that as banks hold less of the liquid asset, $\underline{k}$ decreases, that is, the region over which the price is equal to the fundamental price $\overline{p}$ shrinks. In turn, from Proposition 1 and Figure 2, one can easily see that for all values of k, when banks hold less of the liquid asset, prices deviate more from the fundamental price, that is, $(\overline{p} - p^*(k))$ (weakly) increases. This gives us the following Corollary.

Corollary 1 *For all* $k \in [0, 1]$, *as liquidity in banks' portfolio* l *decreases, price deviates more from the fundamental price* $\overline{p}$, *that is,* $(\overline{p} - p^*(k))$ *(weakly) increases.*

4.2 Banks' choice of liquidity

Consider a representative bank at the ex-ante stage. Formally, the objective of each bank at date 0 is to choose a portfolio of the safe and the risky asset, namely $(l, 1-l)$, that maximizes the sum of expected profits at $t = 1$ and $t = 2$, which consists of the expected profits from (i) their own investments taking account of the opportunity cost of holding liquid assets in their portfolio, and (ii) asset purchases when they survive.

Using the prices derived in Proposition 1, we can calculate profits for surviving banks from asset purchases. When only a small proportion of banks fail, $k \leq \underline{k}$, surviving banks pay the full price for the acquired assets and do not capture any surplus from the asset purchase. In these cases, from an ex-post stand point, banks carry excess liquidity in their portfolio and incur losses from forgone investment in the risky asset.

When the proportion of failed banks is high, $k > \underline{k}$, each surviving bank captures a surplus from asset purchase that equals

$$y(p^*) \cdot (\overline{p} - p^*) = \frac{k\overline{p}}{1-k} - L. \tag{8}$$

In all cases, bank owners of failed banks have no continuation payoffs.

Given this analysis, we can formalize each bank's portfolio choice that gives rise to a competitive equilibrium as follows. Banks' problem is to choose l that maximizes

$$E(\pi(l)) = E\left(\alpha_0\left[\underbrace{[l+(1-l)R_0(l)-r_0]}_{\text{profit 1}^{\text{st}}\text{ investment}} + \underbrace{L\left(\frac{\overline{p}-p^*(k)}{p^*(k)}\right)}_{\text{profit asset purchase}} + \underbrace{\overline{p}}_{\text{profit 2}^{\text{nd}}\text{ investment}}\right]\right), \quad (9)$$

where $p^*(k)$ is the market clearing price given in Proposition 1. The first order condition (FOC) for the maximization problem is given as:

$$E\left[\alpha_0\left(\left[1-R_0+(1-l)\frac{dR_0}{dl}\right]+\left[1-\tau R_0+\tau(1-l)\frac{dR_0}{dl}\right]\left[\frac{\overline{p}-p^*(k)}{p^*(k)}\right]\right)\right]=0 \quad (10)$$

We define

$$\phi = \alpha_0\left[\frac{\overline{p}-p^*(k)}{p^*(k)}\right], \quad (11)$$

as the expected benefit from asset purchase per unit of liquidity. See Figure 3 for an illustration of ϕ as a function of k. Note that ϕ is independent of l when viewed from a price-taking bank's perspective, but in equilibrium, $p^*(k)$ depends on the aggregate liquidity in state k. Hence, banks' equilibrium choice of liquid asset holdings is given by a fixed point that is formally stated below and illustrated in Figure 4.

Proposition 2 *Banks' choice of liquidity $\widehat{l}$ that satisfies the FOC in (10) is given by*

$$\widehat{l} = \min\left\{1, \max\left\{0, 1-b+\frac{E(\alpha_0)+E(\phi)}{E(\alpha_0)+\tau E(\phi)}\right\}\right\}. \quad (12)$$

The unique aggregate level of liquidity l^* is the fixed-point of

$$\widehat{l}(E(\alpha_0),\tau,E(\phi(\alpha_0,\tau,l^*))) = l^*. \quad (13)$$

Let $l_{max} = \left[\sqrt{b^2-2}+(1-b)\right]$. Then, we have $l^* \leq l_{max} < 1$. In particular, it is not optimal for banks to invest everything in the safe asset.

Note that $\widehat{l}$ is a (weakly) declining function of aggregate liquidity l. The intuition for this is that if aggregate liquidity is low, then the deviation of prices from the fundamental value is high, creating a motive to hold liquidity to acquire failed banks at lower prices. Conversely, if aggregate liquidity is high, then the expected gain from asset purchases is low and the incentives of a bank to carry liquid buffers is low as well. Note also that the strategic benefit of holding liquid assets for an individual bank, given by ϕ, depends on the liquidity in the whole market, since the market liquidity l^* affects the price $p^*(k)$. The endogenous determination of prices, and, in turn, of the strategic benefit to banks from acquiring other banks, is an important feature of our model.

Several aspects of banks' private choice of liquidity l^* deserve mention. First, under assumption A1 (that $b > 2$), banks never have an incentive to hold liquidity if pledgeability of future cash flows (τ) is sufficiently high. Specifically, if $\tau = 1$, then $l^* = 0$, the portfolio choice that trades off simply the expected returns to the bank owners from the risky asset and the safe asset. In particular, in this case both assets are fully liquid so that the optimal portfolio choice is not affected by inter-temporal liquidity considerations and consists of only risky investment.

Second, if the pledgeability of future cash flows is sufficiently low ($\tau < 1$), then liquidity cannot be generated against full expected value of uncertain cash flows. As a result, there is an inter-temporal motive to hold liquidity: Specifically, liquid holdings exceed those from the myopic portfolio choice problem as liquid assets dominate risky assets in future states where there is a strategic benefit from acquiring failed banks at cash-in-the-market prices ($k > \underline{k}$).

Finally, there is no precautionary motive for holding liquidity in our set-up. Since banks raise one unit of deposits at date 0, the only way banks can avoid default altogether is to store the entire unit in liquid holdings, but this leaves bankowners with no residual claim after paying back deposits. This is suboptimal since even with an infinitesimally small likelihood of success from the risky asset, a bank investing in at least some risky assets can survive when other banks are in default and purchase them all. Indeed, we show in the appendix that regardless of the likelihood of success of the risky asset, banks never invest everything in the safe asset. This lack of liquid holdings for precautionary reasons is specific to the model feature that default or lack thereof is tied in a binomial fashion to success and failure of the risky asset, rather than in a continuous fashion as mix between safe and risky assets changes. Nevertheless, it illustrates that the motivation for liquid asset holdings in our model stems purely from the strategic benefit they provide to banks in acquiring failed banks.

4.3 Comparative statics

In this section, we analyze how banks' choice of liquidity and their equity value, measured by expected profits, are affected by model parameters. We investigate the effects of the development of capital markets and the business cycle. These effects form the primary testable implications of our model.[10]

In developed economies, we would expect highly developed capital markets where banks can generate funds easily against future profits. Hence, one can interpret τ in our model as a proxy for the level of development in capital markets in the context of different countries. Also, we know that the cost of issuing capital rises, or in other words, the pledgeability of future returns, τ, decreases during economic downturns and crises, when viewed in the context of a single economy. We show below that for low values of τ, that is for less-developed economies and during economic downturns and crises, banks hoard more liquidity since they do not expect to have easy or cheap access to capital markets for raising funds in case acquisition opportunities arise.

Also, during boom periods it is more likely that risky investments will pay off well. To this end, we consider two different probability distributions, f and g, for α_0 to represent recessions and boom periods, respectively, by assuming that g first order stochastically dominates (FOSD) f. We show that in equilibrium, banks invest less in the liquid asset during boom periods.

Combining these two results, we get the following formal Proposition.

Proposition 3 *Let f and g be two probability distributions for α_0, where g FOSD f. Banks' choice of liquidity l^* and value of banks' equity $E(\pi)$ have the following features:*

(i) As the pledgeability of future returns, τ, increases, l^* decreases.

(ii) Let l_f^* and l_g^* be the liquid asset holdings of banks under probability distributions f and g, respectively, and let $E_f(\pi(l_f^*))$ and $E_g(\pi(l_g^*))$ be the corresponding values of banks' equity. Then, we have $l_f^* > l_g^*$, and $E_g(\pi(l_g^*)) > E_f(\pi(l_f^*))$.

[10] The comparative static with the parameter b is straightforward and not formally stated: Since $R_0 = b - \left(\frac{1-l}{2}\right)$, as b increases, the return from the risky asset increases, which also increases the liquidity banks can generate against their profits in the first period. Hence, as b increases, the liquid asset becomes less attractive and banks choose a lower level of the liquid asset l^*. This relation is apparent from equation (12).

Note that from expression (11), ϕ is (weakly) decreasing in α_0 (see Figure 3). Increased probability of the high return has two effects on banks' choice of liquidity that work in the same direction. First, the expected return from the risky asset increases, which makes the risky asset more attractive. Also, the proportion of failed banks decreases, which limits the opportunity for making profits from asset purchases at cash-in-the-market prices. This, in turn, makes the liquid asset less attractive. Similarly, as τ increases, banks can generate more funds from the capital market. Hence, banks do not have to heavily rely on their liquid asset holdings which yield lower return than risky assets.

It is striking that while bank liquidity is counter-cyclical in the sense that it is higher in downturns than in upturns, bank "capital" – measured as the expected value of bank profits or equity value – is pro-cyclical. The intuition is as follows. In our model, banks optimize on the portfolio mix between risky and safe assets to maximize direct return on investments as well as the strategic return from acquiring failed banks in future. During economic upturns, the opportunity cost of investing in liquid assets is high and anticipated pledgeability of future profits is high too. Put another way, during upturns, capital in the form of expected profits is a more efficient way for banks to create current profits as well as future liquidity, whereas in downturns, safe assets play the role of creating future liquidity more efficiently. In contrast, the effect of pledgeability parameter τ on bank profits is ambiguous: on the one hand, greater pledgeability implies greater ex-post liquidity with surviving banks and thus higher prices for acquiring assets, which reduces bank profits; on the other hand, greater pledgeability allows banks to hold lower liquid buffers which raises expected profits.

We can combine the two effects on bank liquidity in Proposition 3 by modeling the business cycle in a simple way by assuming that if g FOSD f then $\tau_g > \tau_f$. This assumption amplifies the effect of the business cycle on banks' choice of liquidity. Also, from Corollary 1, we know that as liquidity decreases, we observe bigger deviations in the price of banking assets from its fundamental value of $\overline{p}$. Hence, crises preceded by boom periods result in lower asset prices and higher price volatility, giving us the following result.

Corollary 2 *During economic upturns, banks' choice of liquidity l^* decreases. This, in turn, results in bigger deviations in the price of banking assets from their fundamental value of $\overline{p}$, that is, $(\overline{p} - p^*(k))$ increases.*

4.4 Socially optimal liquidity

In the following analysis, we derive as a benchmark the liquidity level of banks l^{**} that maximizes the expected total output generated by the banking sector, given as:

$$E(\Pi) = E\left[l + \alpha_0(1-l)R_0(l)\right] + \alpha_1 R_1. \tag{14}$$

We call this benchmark the "socially optimal" level of bank liquidity. The first-order condition for the socially optimal level of l is thus given as:

$$1 - E\left(\alpha_0\right)\left[R_0(l) - (1-l)\frac{dR_0}{dl}\right] = 0, \tag{15}$$

which gives us the following Proposition.

Proposition 4 *The socially optimal level of liquidity l^{**} satisfying the FOC in (15) is given as:*

$$l^{**} = \min\left\{1, \max\left\{0, 1 - b + \frac{1}{E(\alpha_0)}\right\}\right\}. \tag{16}$$

Furthermore, we have $l_f^{**} > l_g^{**}$, when g FOSD f.

Note that the socially optimal level of liquidity is determined by only the myopic portfolio choice. In contrast to the private choice of banks, asset sales do not play a role as they simply result in transfers across banks. When b increases, the return from the risky asset increases and the socially optimal level of liquidity l^{**} decreases, which can be seen from equation (16). Furthermore, l^{**} is independent of τ. Note that under Assumption (A1), privately optimal liquidity l^* is zero when τ equals one. But the socially optimal level of liquidity l^{**} may be positive. This is because while the private portfolio choice suffers from the risk-shifting problem, this is not the case for the social portfolio choice. Finally, socially optimal liquidity is higher during recessions (low $E(\alpha_0)$) as was the case with privately optimal bank liquidity.

4.5 Comparing privately and socially optimal levels of liquidity

In this section, we compare the privately and socially optimal levels of liquidity. We show that a crucial determinant of this comparison is the extent of pledgeability of risky cash flows: When pledgeability is high, banks hold less liquidity than is socially optimal due to risk-shifting incentives, whereas when pledgeability is sufficiently low, (somewhat counter-intuitively) banks may hold even more liquidity than is socially optimal. The intuition for this latter result in the context of our model is that privately banks stand to gain from acquiring failed banks in some states. However, from a social standpoint, these gains are only transfers within the banking system and there is no misallocation cost associated with asset sales within the banking sector.

Similarly, we also show that the privately optimal level of liquidity is inefficiently low during economic downturns and crises (even though in terms of absolute magnitude it is higher in downturns than in boom times). The result is stated in the following proposition and is illustrated in Figure 5.

Proposition 5 *Comparing the privately and socially optimal liquidity levels, we obtain that:*

(i) There exist critical values $\tau^*(E(\alpha_0))$, such that, the privately optimal level of liquidity is higher than the socially optimal level if and only if $\tau < \tau^*(E(\alpha_0))$.

(ii) There exist critical values $\alpha_0^*(\tau)$, such that, the privately optimal level of liquidity is higher than the socially optimal level if and only if $E(\alpha_0) > \alpha_0^*(\tau)$.

Furthermore, $\tau^*(E(\alpha_0)) < E(\alpha_0)$ and conversely $\alpha_0^*(\tau) > \tau$.

5 Some empirical evidence

So far, our focus has mainly been to present a theoretical model for analyzing the liquidity choice of banks in anticipation of financial crises. In this section, we provide some anecdotal and descriptive empirical evidence that is consistent with the model's implications for liquidity holdings of banks.

5.1 Hoarding of liquidity by banks for gains during crises

We focus below on one salient historical anecdote of a bank hoarding liquidity for strategic gains during crises – that of National City Bank from the United States banking system during the pre-Federal Reserve era.[11]

Cleveland and Thomas in their book *Citibank* provide a memorable account of how National City Bank, that eventually became Citibank, grew from a small treasury unit into one of the biggest commercial banks under its president Stillman, who anticipated the 1893 and 1907 crises and built up liquidity and capital before the crises to benefit from the difficulties of its competitors. In terms of actual levels of bank liquidity, the reserve ratio of National City Bank was 42.6% and 26.9% right before the 1893 and 1907 crises, respectively, while these ratios were lower at 25.2% and 24.9% for all other New York City banks. Also, for the 1907 crisis, the capital to net deposits stood at 35.2% for National City Bank, whereas it was 27.5% for all other New York City banks.

What was the impact of such positioning by National City Bank of the balance-sheet in terms of cash and capital? Cleveland and Thomas report that during the 1893 (1907) crises, while National City Bank increased its deposits by 12.4% (23.5%), deposits in all other New York City banks decreased by 14.5% (increased by only 9.2%). Furthermore, during the 1893 (1907) crises, while National City Bank increased its loans and discounts by 14.7% (10.2%), loans and discounts in all other New York City banks decreased by 9.1% (increased by only 3.7%). In other words, evidence shows that National City Bank expanded its business operations while other banks were simultaneously experiencing a shrinkage. We document below that *hoarding liquidity to acquire business that belonged to distressed institutions* (in case of 1907 crisis, the New York-based trusts) was indeed the strategy followed by the bank. Below is the paragraph about the 1907 crisis from Cleveland and Thomas' book (page 52) which illustrates this point succinctly:

National City Bank again emerged from the panic a larger and stronger institution. At the start, National City had higher reserve and capital ratios than its competitors, and during the panic it gained in deposits and loans relative to its competitors. Stillman (President) had anticipated and planned for this result. In response to Vanderlip's (Vice President) complaint in early 1907 that National City's low leverage and high reserve ratio was depressing profitability, Stillman replied: "I have felt for sometime that the next panic and low interest rates

[11] Casual empiricism suggests however that such cases are not uncommon. In fact, our private communications with bankers suggest that during the most recent sub-prime crisis of 2007 too, one of the perceived reasons for drying up of inter-bank lending markets has been the hoarding of liquidity by banks for acquisitions of troubled institutions at fire-sale prices, the other two reasons being precautionary motive from the risk of being distressed oneself and adverse selection about borrowing institutions.

following would straighten out good many things that have of late years crept into banking. What impresses me most important is to go into next Autumn (usually a time of financial stringency) ridiculously strong and liquid, and now is the time to begin and shape for it... If by able and judicious management we have money to help our dealers when trust companies have suspended, we will have all the business we want for many years."

5.2 Bank liquidity and ease of external finance

To provide more systematic evidence on bank liquidity, we appeal first to the robust implication of our analysis that the greater is the difficulty banks face in raising external finance, the more would banks hold liquid assets. We explore this hypothesis by examining the liquid asset holdings of banks in a cross-section of countries.

In a recent paper, Freedman and Click (2006) show that banks in developing countries choose to channel only a modest portion of their funds to private sector borrowers, while keeping a sizeable percentage of their deposits in liquid assets, such as cash, deposits with other banks, central bank debt, and short-term government securities. They construct a liquidity ratio for banks, defined as the ratio of Liquid Assets to Total Deposits, using from International Financial Statistics provided by the IMF.[12] They show that for developing countries the ratio ranges from 14% in South Africa to 126% in Argentina, with a mean value of 45%, with values of 2% for the UK, 6% for the US, 21% for Japan, 31% for France and 34% for Germany, with an average of 19% for developed countries.

They attribute this difference among developed and developing countries to banks' reluctance to lend in developing countries. Such reluctance, they argue, could be a response to inefficiencies in credit markets resulting from factors such as higher reserve requirements, greater macroeconomic risk and volatility, and significant deficiencies in the legal and regulatory environment which make it difficult to enforce contracts and foreclose on collateral. In this paper, we argue that an alternative channel may also be at work. Banks in poor legal and regulatory environments may find it difficult to raise liquidity against future profits and thus end up hoarding greater liquidity. Such cash hoardings may be inefficiently high and

[12] In particular, they calculate Liquid Assets as the sum of Reserves (line 20) and Claims on Central Government (usually line 22A), and Total Deposits as the sum of Demand Deposits (line 24), Time and Savings Deposits (line 25), Money Market Instruments (line 26A), and Central government Deposits (line 26D).

result in low levels of intermediation by the banking sector.

We expand on the data set of Freedman and Click (2006) to cover about 70 countries with data on liquidity ratios dating back to September 2003. First, we link bank liquidity to a number of institutional variables that capture country's financial development in terms of quality of disclosures, and the extent of stock and credit intermediation (relative to country's size). These proxies should thus all measure the ease of raising external finance. Specifically, we employ five measures based on Rajan and Zingales (1998, 2003), which are:

1. **Accounting standards** is an index developed by the Center for International Financial Analysis & Research ranking the amount of disclosure in annual company reports in each country. Though this index from Rajan and Zingales dates back to 1990, they report that it does not change much over time.

2. **Total capitalization to GDP** is the ratio of the sum of equity market capitalization (as reported by the IFC) and domestic credit (IFS line 32a-32f but not 32e) to GDP. Stock market capitalization is measured at the end of the earliest year in the 1980's for which it is available.

3. **Domestic credit to GDP** is the ratio of domestic credit to the private sector, which is from IFS line 32d, over GDP.

4. **Deposits to GDP** is the ratio of domestic deposits to the GDP, based on data for 1999.

5. **Stock market capitalization to GDP** is the ratio of the aggregate market value of equity of domestic companies divided by GDP, based on data for 1999.

We find that in the cross-section of countries, the correlation of country-level average for the banking system's ratio of Liquid Assets to Total Deposits with these five measures is uniformly and significantly negative, the values being -0.55, -0.38, -0.36, -0.33, and -0.50, respectively. We also plot the best regression fit of the Liquidity Ratio to Accounting standards (Figure 6) and to Total capitalization to GDP (Figure 7). The graphs illustrate that the negative relationship is quite robust to exclusion of outliers such as Argentina, whose Liquidity Ratio has been inflated due to the recent economic and political turmoil.

While this evidence is striking, it is potentially also consistent with the explanation of Freedman and Click (2006) that these measures of financial development (especially Domestic credit to GDP and to some extent Accounting standards) also proxy for frictions in the market for lending. That is, the negative relationship may be due to lower attractiveness of risky loans in these countries rather than due to greater attractiveness of safe assets. To help at least partially address this issue, we examine data on international stock market liquidity measured over the period 1989 to 2000 from Levine and Schmukler (2005). In particular, we consider for a subset of countries three measures of stock-market liquidity, namely **Turnover in Domestic Market**, and two inverse proxies, **Illiquidity Ratio** of Amihud (2002), and **Proportion of Zero Return Days** advocated by Bekaert, Harvey and Lundblad (2003).

While the first two measures show little correlation with the banking-system liquidity ratio, we find that the third measure of stock-market illiquidity, the proportion of zero return days, is significantly positively correlated. The correlation is 0.25 (Figure 8 shows the best regression fit of banks' Liquidity Ratio versus the Proportion of Zero Return Days). When the Brazil outlier is excluded, the correlation is around 0.35, the corresponding correlations with Accounting standards and Total Capitalization to GDP being -0.25 and -0.60, respectively (for the limited sample where stock market liquidity proxies are available).

This suggests that the relationship between financial development and bank liquidity may not entirely be due to credit-market frictions. A part of this relationship may also stem from the fact that financial development is associated with greater ease of external finance, which reduces the attractiveness of liquidity in banks' portfolio choice. Overall, this cross-country evidence is suggestive, even if not conclusive, that the hoarding of liquidity buffers for profitable investments such as acquisitions may be a potentially important determinant of equilibrium levels of bank liquidity.

5.3 Bank liquidity and the business cycle

In order to provide further evidence in support of our model's implications, we next appeal to the second robust implication that bank liquidity is counter-cyclical, that is, lower during economic upturns and higher as recessions approach (or are anticipated). On this implication, we rely on extant empirical evidence.

Aspachs et al. (2005) analyze the determinants of UK banks' liquidity holdings and find evidence supportive of this hypothesis. They use balance sheet and profit and loss data, for a panel of 57 UK-resident banks, on a quarterly basis, over the period 1985Q1 to 2003Q4. These data are obtained from the Bank of England Monetary and Financial statistics and relate to the banks' resident (UK) activity, excluding activities abroad. They measure liquidity as the sum of cash, reverse Repos, bills and commercial papers and comprise in addition all types of investments securities, such as equities and bonds. They use two alternative liquidity ratios. The first is the share of liquid assets in the bank's total assets. This measure captures the split between liquid and illiquid assets on the bank's balance sheet. And, to capture the liquidity mismatch inherent in the bank's balance sheet, they use a second measure, which is the ratio of liquid assets to total deposits. However, their results do not change materially whether they use ratio of liquidity over assets, or the ratio of liquidity over deposits.

In their regression analysis, they test among other effects the role of GDP growth in determining banks' liquid asset holdings. They find that banks in the UK appear to hold smaller (larger) amounts of liquidity, relative to both total assets and total deposits, in periods of stronger (weaker) economic growth. In particular, a 1% increase in GDP growth results in about a 2% decrease in liquidity, where the effect is significant at the 1% level. In other words, banks appear to build up their liquidity buffers during economic downturns and draw them down in economic upturns. Again, while business cycle fluctuations are certainly associated with fluctuations in demand for risky loans, their evidence, put together with the cross-country evidence, provides at least preliminary support for our model's business-cycle hypothesis. More research differentiating the alternative determinants of banks' liquid asset holdings and perhaps employing other empirical measures for the overall health of banking system is warranted.

Some recent literature (most notably, Adrian and Shin, 2006, Figures 1, 2, 7 and 10) has focused on targeting of leverage ratios by banks and its implications for the business cycle. In particular, this literature has argued that individual bank risk management leads to unwinding of assets in response to negative asset-side shocks, which depresses prices and leads to more unwinding, causing significant price drops. It has also been documented that there is a negative relationship between equity cushion maintained by banks and their total assets. We elaborate below that these facts are potentially consistent with risk management at banks being primarily achieved by management of their liquidity.

Leverage ratios would be targeted by banks in a "net" sense, that is, with leverage being net of cash reserves or liquid holdings of banks. Negative asset-side shocks increase the risk of a crisis giving banks incentives to build up their liquid buffers, for example, by liquidating risky assets and saving the proceeds. If such shocks are systematic, there may not be a sufficiently large pool of outsider buyers (such as pension funds, insurance companies, university endowments, hedge funds, etc., depending on the type of assets) to absorb liquidations by banks, resulting in fire-sale discounts in prices.[13] As asset liquidations increase, size of banking assets falls but due to liquidation proceeds and the anticipated gains on cash balances, the net equity cushions rise. These effects would be exaggerated if negative asset-side shocks are associated with a deterioration in market liquidity and cost of raising external finance (see, for example, Acharya and Pedersen, 2005, Figure 1) since this would strengthen banks' strategic (and precautionary) motives to increase liquid buffers.

While this cross-country and business-cycle support for our model's implications is arguably preliminary and only suggestive, we find it intriguing and promising for detailed investigation in future research.

6 Entry and inefficient liquidations

We revert to the analysis of liquidity choice of banks. In the benchmark model, only banks are present in the market for purchasing banking assets. In this section, we analyze the effect of entry by outsiders for purchase of distressed assets. One interpretation of these outsiders is as "foreign" banks.[14] Another interpretation of outsiders is as non-bank financial institutions such as hedge funds.[15]

[13] While we have not yet discussed the role of outside capital in our model, we do so in the next section.

[14] Krugman (1998) and Aguiar and Gopinath (2005) have recently documented evidence that the high foreign direct investment (FDI) flows into the crisis-stricken countries of the 1997 Asian financial crisis had many of the features of capitalizing on fire-sales: median offer price to book ratios were substantially lower for the purchase of cash-strapped firms, especially in 1998 when national players had low liquidity, resulting in a boost in mergers and acquisitions involving foreign players.

[15] The article entitled "Cashing in on the crash" in the *Economist* on August 23, 2007, provides a discussion of the incentives to hold liquidity for hedge funds to take advantage of fire sales during the recent financial turmoil. The article argues that vulture funds raised \$15.1 billion in the first seven months of 2007, more than the \$13.9 billion in all of 2006, to acquire assets at fire-sale discounts due to expected distress in financial markets. The same article points out that while some hedge funds suffered as the sub-prime crisis unfolded in the Summer of 2007, the others, such as Citadel, Ellington, and Marathon Asset Management had the ready cash to benefit from the crisis. For example, the article highlights the strategy of Citadel to keep more than a third of its assets in cash or liquid securities, allowing it to take advantage of fire sales when opportunities

Formally, we introduce outside investors who are risk-neutral and competitive and have funds w to purchase banking assets were these assets to be liquidated. These are investors outside the banking sector, and even though they have funds for asset purchases, they do not have the skills to generate the full value from banking assets. Hence, outsiders are inefficient users of banking assets relative to bank owners, provided that bank owners exert effort. Often such outsiders are short-term holders who re-package or securitize the assets for selling on to portfolio investors. Such outsiders may be unable to realize the full value of the assets for the familiar reason that bank assets (loans in particular) derive much of their value from the monitoring and collection efforts of loan officers who can influence the actions of the debtors. Hence, when distressed assets end up in the hands of outsiders, we may expect deadweight costs from inefficient allocation of assets. To capture this formally, we assume that outsiders cannot generate R_t in the high state but only $(R_t - \Delta)$. We also assume that $\overline{\Delta} > \Delta$ so that outsiders can generate more than what banks can generate from bad projects.

The notion that outsiders may not be able to use the banking assets as efficiently as the existing bank owners is akin to the notion of *asset-specificity*, first introduced in the corporate-finance literature by Williamson (1988) and Shleifer and Vishny (1992) and employed extensively since then in the banking literature as well.[16] In the evidence of such specificity for banks and financial institutions, James (1991) shows that the liquidation value of a bank is typically lower than its market value as an ongoing concern. In particular, his empirical analysis of the determinants of the losses from bank failures reveals a significant difference in the value of assets that are liquidated and similar assets that are assumed by acquiring banks.[17]

In terms of analysis, we first examine the sale of failed banks and the resulting prices in the presence of outsiders. The demand schedule for surviving banks does not change and we

arise.

[16] There is strong empirical support for the idea of asset specificity in the corporate-finance literature, as shown, for example, by Pulvino (1998) for the airline industry, and by Acharya, Bharath, and Srinivasan (2006) for the entire universe of defaulted firms in the US over the period 1981 to 1999 (see also Berger, Ofek, and Swary, 1996, and Stromberg, 2000).

[17] Acharya, Shin and Yorulmazer (2007) extend the evidence on fire-sale FDI of Krugman (1998) and Aguiar and Gopinath (2005) for the banking and financial sector. They document that banks and financial institutions acquired by foreign firms during the South East Asian crisis were more likely to be "flipped" back to domestic firms once the crisis abated. This evidence is consistent with FDI entering during the crisis to take advantage of low prices, but lacking in expertise relative to domestic firms which leads to the flipping once domestic firms' financing constraints are relaxed.

can derive the demand schedule for outsiders in a similar way. Let $\underline{p} \equiv \alpha_1 (R_1 - \Delta - r_1) = \overline{p} - \alpha_1 \Delta$, be the expected profit for outsiders from the risky asset in the second period. For $p < \underline{p}$, outsiders are willing to supply all their funds for the asset purchase. Thus, demand schedule is $y_2(p) = k$. For $p > \underline{p}$, the demand is $y_2(p) = 0$, and for $p = \underline{p}$, outsiders are indifferent between $y_2(p)$ over the range $[0, k]$. Thus, for $p > \underline{p}$, there is limited participation in the market for banking assets in that only insiders submit bids to purchase assets.

Next, we analyze how the regulator optimally allocates failed banks' assets. In the absence of financial constraints faced by surviving banks, it is optimal to sell all assets to surviving banks. However, surviving banks may not be able to pay the threshold price of $\underline{p}$ for all assets. If prices fall further, these assets become profitable for outsiders and they participate in the auction. Formally, as long as price is higher than $\underline{p}$, outsiders do not participate in the asset market. However, for $k > \overline{k}$, where

$$\overline{k} = \left(\frac{L}{L + \underline{p}} \right), \tag{17}$$

surviving banks cannot pay the threshold price of $\underline{p}$ for all assets, and outsiders are willing to supply all their funds for the asset purchase. For $w \geqslant \underline{p}$, with the injection of outsider funds, the price is sustained at $\underline{p}$ for all $k > \overline{k}$. However, when $w < \underline{p}$, if the crisis is very severe (sufficiently large k), the total liquidity available within the surviving banks and outsiders is not be enough to sustain the price at $\underline{p}$. Thus, we observe a second region where the price is downward sloping as a function of k. In other words, there is cash-in-the-market pricing in this region given the limited liquidity of the *entire* set of market players bidding for assets. In particular, for $k > \overline{\overline{k}}$, where

$$\overline{\overline{k}} = \left(\frac{L + w}{L + \underline{p}} \right), \tag{18}$$

the price is again strictly decreasing in k and is given by

$$p_w^*(k) = \left(\frac{(1 - k) L + w}{k} \right), \tag{19}$$

and $y(p_w^*) = \left(\frac{L}{p_w^*} \right)$ and $y_2(p_w^*) = \left(\frac{w}{p_w^*} \right)$. The resulting price function is illustrated in Figure 9 and is stated in the following proposition.

Proposition 6 *The price of assets as a function of the proportion k of failed banks and* *outsiders' wealth w is:*

$$p_w^*(k) = \begin{cases} \overline{p} & \textit{for} \quad k \leqslant \underline{k} \\ \frac{(1-k)L}{k} & \textit{for} \quad k \in (\underline{k}, \overline{k}] \\ \underline{p} & \textit{for} \quad w \geqslant \underline{p} \textit{ and } k > \overline{k}, \textit{ or } w < \underline{p} \textit{ and } k \in (\overline{k}, \overline{\overline{k}}] \\ \frac{[(1-k)L]+w}{k} & \textit{for} \quad w < \underline{p} \textit{ and } k > \overline{\overline{k}} \end{cases} . \tag{20}$$

Note that the introduction of outsiders (weakly) increases the price for failed banks. In particular, for $k > \overline{k}$, with the injection of outsider funds, the price stays at $\underline{p}$, at least for a range of values of k, and the price is higher than the price without outsiders, that is, $p_w^*(k) > p^*(k)$, given in equations (20) and (7), respectively.

Since the introduction of outsiders (weakly) increases the price for acquiring failed banks, the expected benefit to banks from holding the liquid asset to purchase assets declines. In this case, bank i's problem can be stated in the same way as in the benchmark case (equation (9)), except for the fact that instead of ϕ, we have

$$\phi_w = \alpha_0 \left[\frac{\overline{p} - p_w^*(k)}{p_w^*(k)} \right], \tag{21}$$

as the expected benefit from asset purchase per unit of liquidity. Note that for $k \leqslant \overline{k}$, $\phi_w = \phi$, whereas for $k > \overline{k}$, we have $\phi_w < \phi$.[18] Since $E(\phi_w) < E(\phi)$, the unique aggregate level of liquidity l_w^* with outsiders is lower than l^* given in Proposition 2. Furthermore, as outsider wealth w increases, the price p_w^* (weakly) increases for each k. This, in turn, decreases the private benefit ϕ_w and induces banks to hold less liquid asset: as w increases, l_w^* (weakly) decreases.[19]

[18] See Figure 10 for an illustration of ϕ_w. For $k \in [\overline{k}, \overline{\overline{k}}]$, we have $\phi_w = \alpha_0 (\alpha_1 \Delta)$. Note that ϕ_w is not monotone increasing in k. The reason for this is that, for $k \in [\overline{k}, \overline{\overline{k}}]$, with the participation of outsiders, price stays at $\underline{p}$ and the profit for a surviving bank from purchasing a unit of failed banks' asset is bounded by $(\alpha_1 \Delta)$, whereas a bank survives only with probability α_0. Hence, as α_0 decreases, the marginal gain from holding the liquid asset goes down for $k \in [\overline{k}, \overline{\overline{k}}]$. Since, ϕ_w is no longer monotone in α_0, the comparative statics result on $E(\alpha_0)$ in this case is not as clean as the result in the benchmark case. However, we provide interesting results on the effect of expertise $(\alpha_1 \Delta)$ and the wealth of outsiders (w) on banks' choice of liquidity.

[19] We observe a similar effect of $(\alpha_1 \Delta)$ on banks' choice of liquidity. As the wedge between the expertise of banks and outsiders widens, that is, as $(\alpha_1 \Delta)$ increases, the price for assets weakly decreases for all $k > \overline{k}$. Just like a decrease in outsider wealth w, this increases ϕ_w and banks hold more liquidity.

The socially optimal liquidity level l of each bank maximizes the objective function

$$E(\Pi) = E\left[l + \alpha_0(1-l)R_0(l)\right] + \alpha_1 R_1 - (\alpha_1\Delta) \int_{\bar{k}}^{1} f(k)\left[k - \frac{(1-k)L}{p_w^*(k)}\right] dk, \tag{22}$$

where $\left[k - \frac{(1-k)L}{p_w^*(k)}\right]$ represents the units of assets purchased by outsiders at the price $p_w^*(k)$, which, multiplied by $(\alpha_1\Delta)$, gives the misallocation cost arising from outsiders' lack of expertise relative to banks. On the one hand, as banks hold more liquid assets, the first expression decreases since in expected terms, risky asset has a higher return than the safe asset. On the other hand, as banks hold more liquid assets, they have more resources to acquire failed banking assets, which decreases the misallocation cost.

We thus obtain the following proposition.

Proposition 7 *The socially optimal level of liquidity that maximizes (22) is given as:*

$$\widehat{\widehat{l}}_w = \min\left\{1, \max\left\{0, 1 - b + \frac{1 + E(\gamma_w)}{E(\alpha_0) + \tau E(\gamma_w)}\right\}\right\}, \tag{23}$$

where $E(\gamma_w)$ is the marginal reduction in expected misallocation cost for an additional unit of liquidity within the set of surviving banks and is given in equation (43) in the Appendix. The unique level of socially optimal liquidity l_w^{**} is the fixed-point of $\widehat{\widehat{l}}_w(\alpha_0, \tau, E(\gamma(\alpha_0, \tau, l_w^{**}))) = l_w^{**}$. Furthermore, for $w > \left(\frac{L\underline{p}}{2L+\underline{p}}\right)$, the socially optimal level of liquidity l_w^{**} decreases as outsider wealth w increases.

As a function of equilibrium liquidity l, $\widehat{\widehat{l}}_w$ behaves similar to $\widehat{l}$ and $\widehat{l}_w$. When aggregate liquidity is high, misallocation costs are low and it becomes less desirable to carry additional liquidity. Similarly, if aggregate liquidity is low, the misallocation region is large and carrying additional liquidity is attractive from a social standpoint.

We show below that as in the benchmark case without outsider funds, the privately optimal level of liquidity may exceed the socially optimal level in the case when pledgeability of future profits is low.[20] The following proposition states the relationship between the privately optimal and the socially optimal levels of liquidity in relation to τ, the pledgeability of future returns.

[20] We present our results for the case $w \geqslant \underline{p}$. Similar results hold for the case $w < \underline{p}$.

Proposition 8 *With the possibility of outsider entry, there exist critical values $\tau^*(\Delta)$ and $\tau^{**}(\Delta)$, such that, for $\tau > \tau^*(\Delta)$, the socially optimal level of liquidity is higher than the privately optimal level, and for $\tau < \tau^{**}(\Delta)$, the privately optimal level of liquidity is higher than the socially optimal level, where $\tau^{**}(\Delta) \leqslant \tau^*(\Delta)$.*

The intuition for this result is overall similar to that for the case without outsider entry. If $\tau < 1$, bankowners have an inter-temporal motive to hold liquidity: surviving banks make profits from asset purchases when the proportion of failures is above $\underline{k}$, that is, $k > \underline{k}$, but since $\tau < 1$ they cannot pledge risky cash flows fully to capitalize on this benefit. Hence, there is a benefit from carrying liquidity into such states. In contrast, misallocation costs materialize only when the proportion of failures is above $\overline{k}$, that is, $k > \overline{k}$. For the intermediate region $[\underline{k}, \overline{k}]$, while banks gain by purchasing assets at cash-in-the-market prices, there is no social welfare loss. Thus, if τ is sufficiently small, then the private incentive to hold liquidity for inter-temporal transfers can prevail over the risk-shifting incentive, and, in turn, privately optimal level of liquidity can exceed the social one.

Conversely, if τ is sufficiently high or for a given value of τ if Δ is not very large so that the region $[\underline{k}, \overline{k}]$ is not too wide, then banks hold less than the socially optimal level of liquidity: The risk-shifting incentive dominates in this case. In other words, when the difference between the fundamental value of bank assets and the price outsiders are willing to pay for them is not very high, banks choose to hold less than socially optimal levels of liquidity.

To summarize, if sufficient liquidity cannot be raised against risky cash flows in a contingent fashion in future, then banks may carry excess liquidity (inefficiently bypassing profitable lending opportunities) in order to stand ready for acquiring failed banks at attractive prices. Note, however, that since outsider wealth (weakly) raises asset-sale prices, it has the effect of lowering bank liquidity in privately as well as socially optimal choices. The important implication of this is that when private level of bank liquidity is inefficiently high, entry by foreign banks can unlock the liquid hoardings and lead to greater intermediation by domestic banks. This effect of outside capital on ex-ante bank portfolio choice can be an important criterion in assessing welfare effects of foreign entry in banking sectors of developing countries and during crises, settings where cost of external finance is high.

7 Concluding remarks and policy implications

Our objective in this paper has been to develop a theoretical framework for bank portfolio choice between liquid (safe) and illiquid (risky) assets that is driven by strategic acquisition motives in the backdrop of potential crises and foreign entry. We have focused somewhat narrowly on the implications of this strategic motive, but it remains an important empirical question to differentiate it and measure its importance relative to the more traditional precautionary motive for holding liquidity. While our positive analysis of determinants of bank liquidity lends itself naturally to empirical work, of greater consequence for policy is that we have been able to conduct a normative analysis comparing the equilibrium liquidity choice of banks to a benchmark socially optimal level. The most striking result here is that when liquidity in market for external finance is low, for example, in business downturns or as crises become imminent, banks may hoard excessive liquidity to capitalize on private, strategic gains from acquiring distressed banks' assets.

In this context, it is important to recognize that entry of capital from outside of the banking sector to acquire distressed assets can raise asset prices, reduce hoarding incentives of banks, and free up bank balance-sheets for greater intermediation. This suggests that the role of foreign investors in bank restructuring presents important trade-offs for a country in the aftermath of a financial crisis. Foreign capital will be attracted by the very low prices of distressed assets, and fulfil the role of the "purchaser of last resort" when domestic capital is exhausted. However, the ultimate welfare effects of such foreign entry will depend on the complex interplay between the cushioning of price in the event of a crisis, the ex-ante portfolio choices in anticipation of such entry, and the ability of the foreign entrant to manage the assets they acquire.

Such policy implications seem pertinent and are worthy of further study, both theoretically, for example, by allowing for formal business-cycle dynamics, and empirically, for example, in the form of specific case studies linking the ex-ante choice of bank liquidity to the regulatory choice of closure and asset-sale policies as well as of liquidity injections. It also appears interesting to test jointly the implication of our model that while bank liquidity is counter-cyclical, bank capital is pro-cyclical.

We conclude by touching on one theme concerning the effect of regulatory closure policies on bank liquidity in our setup. In unreported results, we have incorporated in our setup costly provision of deposit insurance and crises resolution policies such as granting of liquidity to failed banks (government-sponsored bailouts) and granting of liquidity to surviving banks (government-assisted sales). It can be shown that (i) fiscal costs of deposit insurance make it more likely that banks will hold less liquidity than is socially optimal; (ii) bailouts result in lower bank liquidity only if they are excessive in the sense of covering more banks than is necessary to avoid costly liquidations to outsiders; (iii) in contrast, liquidity grants to surviving banks that are not contingent on banks' liquidity holdings always lower bank liquidity; (iv) however, if the amount of liquidity provided to surviving banks is increasing in their liquid holdings, then ex-ante incentives for banks to hold liquid assets are in fact strengthened. Such extensions suggest that our model holds promise for further normative analysis and related empirical work.

References

Acharya, Viral, Heitor Almeida and Murillo Campello (2007), Is Cash Negative Debt? - A Hedging Perspective on Corporate Financial Policies, *Journal of Financial Intermediation*, 16(4), 515-554.

Acharya, Viral, Sreedhar T. Bharath and Anand Srinivasan (2006), Does Industry-wide Distress Affect Defaulted Firms? - Evidence from Creditor Recoveries, *Journal of Financial Economics*, 85(3),787-821.

Acharya, Viral and Lasse H. Pedersen (2005), Asset Pricing with Liquidity Risk, *Journal of Financial Economics*, 77 (2), 375-410

Acharya, Viral and Tanju Yorulmazer (2005), Cash-in-the-Market Pricing and Optimal Resolution of Bank Failures, forthcoming, *Review of Financial Studies*.

Acharya, Viral and Tanju Yorulmazer (2007), Too Many to Fail - An Analysis of Time-inconsistency in Bank Closure Policies, *Journal of Financial Intermediation*, 16 (1), 1-31.

Acharya, Viral, Denis Gromb and Tanju Yorulmazer (2007), Imperfect Competition in the Inter-Bank Market for Liquidity as a Rationale for Central Banking, Working Paper, London Business School.

Acharya, Viral, Hyun Song Shin and Tanju Yorulmazer (2007), Fire-sale FDI, Working Paper, London Business School.

Adrian, Tobias and Hyun Song Shin (2006), Money, Liquidity and Financial Cycles, Working Paper, Princeton University.

Aguiar, Mark and Gita Gopinath (2005) Fire-Sale FDI and Liquidity Crises, *The Review of Economics and Statistics*, 87 (3), 439-542

Allen, Franklin and Douglas Gale (1994), Limited Market Participation and Volatility of Asset Prices, *American Economic Review*, 84, 933-955.

Allen, Franklin and Douglas Gale (1998), Optimal Financial Crises, *Journal of Finance*, 53, 1245-1284

Allen, Franklin and Douglas Gale (2004a), Financial Fragility, Liquidity and Asset Prices, *Journal of the European Economic Association*, 2 (6), 1015-1048.

Allen, Franklin and Douglas Gale (2004b), Financial Intermediaries and Markets, *Econometrica*, 72, 1023-1061.

Allen, Franklin and Douglas Gale (2005), From Cash-in-the-Market Pricing to Financial Fragility, *Journal of the European Economic Association*, 3 (2-3), 535-546.

Amihud, Yakov (2002), Illiquidity and Stock Returns: Cross-Section and Time- Series Effects, *Journal of Financial Markets*, 5, 31–56.

Aspachs, Oriol, Erlend Nier and Muriel Tiesset (2004), Liquidity, Banking Regulation and the Macroeconomy, mimeo., Bank of England.

Bekaert, G., C. R. Harvey, and C. Lundblad (2003), Liquidity and Expected Returns: Lessons from Emerging Markets, Working Paper, Columbia University.

Berger, Philip, Eli Ofek and Itzhak Swary (1996) Investor Valuation of the Abandonment Option, *Journal of Financial Economics,* 42, 257–287.

Bernardo, Antonio E. and Ivo Welch (2004), Liquidity and Financial Market Runs, *Quarterly Journal of Economics*, 199 (1), 135-158.

Bhattacharya, Sudipto and Douglas Gale (1987), Preference Shocks, Liquidity and Central Bank Policy, in William Barnett and Kenneth Singleton, eds.: *New Approaches to Monetary Economics*, Cambridge University Press, New York.

Brunnermeier, Markus and Lasse Heje Pedersen (2005), Predatory Trading, *Journal of Finance*, Vol. 60, No. 4, 1825-63.

Caballero, Ricardo and Arvind Krishnamurthy (2001), International and Domestic Collateral Constraints in a Model of Emerging Market Crises, *Journal of Monetary Economics*, 48(3), 513-548.

Caprio, Gerard, Luc Laeven, and Ross Levine (2005) Governance and Bank Valuation, Working Paper, Department of Economics, Brown University.

Cifuentes, Rodrigo, Gianluigi Ferucci and Hyun Song Shin (2005), Liquidity Risk and Contagion, *Journal of the European Economic Association*, 3 (2-3), 556-566.

Cleveland, van B. Harold and Thomas F. Huertas (1985) *Citibank, 1812-1970*, Harvard University Press, Cambridge, Massachusetts.

Diamond, Douglas and Philip Dybvig (1983) Bank Runs, Deposit Insurance, and Liquidity, *Journal of Political Economy*, 91, 401-419.

Diamond, Douglas and Raghuram Rajan (2005), Liquidity Shortages and Banking Crises, *Journal of Finance*, Vol. LX(2), 615–647.

Donaldson, R.G. (1992), Costly Liquidation, Interbank Trade, Bank Runs and Panics, *Journal of Financial Intermediation*, 2, 59-85.

Freedman, Paul and Reid Click (2006) Banks that Don't Lend? Unlocking Credit to Spur Growth in Developing Countries, *Development Policy Review*, 24 (3), 279-302.

Gonzalez-Eiras, Martin (2003), Banks' Liquidity Demand in the Presence of a Lender of Last Resort, mimeo, Universidad de San Andrés.

Gorton, Gary and Lixin Huang (2004) Liquidity, Efficiency, and Bank Bailouts, *American Economic Review*, 94(3), 455–483.

Hart, Oliver and John Moore (1994) A Theory of Debt Based on the Inalienability of Human Capital, *Quarterly Journal of Economics*, 109(4), 841–879.

He, Zhiguo and Arvind Krishnamurthy (2006) Intermediation, Capital Immobility and Asset Prices, working paper, Northwestern University, Kellogg School.

Holmstrom, Bengt and Jean Tirole (1998) Private and Public Supply of Liquidity, *Journal of Political Economy*, 106(1), 1–40.

Holmstrom, Bengt and Jean Tirole (2001) LAPM – A Liquidity-Based Asset Pricing Model, *Journal of Finance*, Vol 56 (5), 2001.

James, Christopher (1991) The Losses Realized in Bank Failures, *Journal of Finance*, 46(4), 1223–1242.

Krugman, Paul (1998) Fire-sale FDI, available at http://web.mit.edu/krugman/www/FIRESALE.htm

La Porta, Rafael, Florencio Lopez de Silanes and Andrei Shleifer (1999) Corporate Ownership Around the World, *Journal of Finance,* 54, 471-517.

Levine, Ross and Sergio Schmukler (2005) Internalization and Stock Market Liquidity, NBER Working Paper 11894.

Morris, Stephen and Hyun Shin (2004), Liquidity Black Holes, *Review of Finance*, 8, 1-18.

Pulvino, Todd C. (1998) Do Asset Fire Sales Exist: An Empirical Investigation of Commercial Aircraft Sale Transactions, *Journal of Finance*, 53, 939-978.

Rajan, Raghuram and Luigi Zingales (1998) Financial Dependence and Growth, *American Economic Review*, volume 88, 559-586.

Rajan, Raghuram and Luigi Zingales (2003) The Great Reversals: The Politics of Financial Development in the 20th Century, *Journal of Financial Economics*, 69 (1), 5-50.

Repullo, Rafael (2005) Liquidity, Risk Taking, and the Lender of Last Resort, *International Journal of Central Banking*, 2, 47–80.

Schnabel, Isabel and Hyun Shin (2004), Liquidity and Contagion: the Crisis of 1763, *Journal of the European Economic Association*, 2 (6), 929-968.

Shleifer, Andrei and Robert W. Vishny (1992), Liquidation Values and Debt Capacity: A Market Equilibrium Approach, *Journal of Finance,* 47, 1343-1366.

Stromberg, Per (2000) Conflicts of Interest and Market Illiquidity in Bankruptcy Auctions: Theory and Tests, *Journal of Finance*, 55, 2641–2692.

Williamson, Oliver E. (1988) Corporate Finance and Corporate Governance, *Journal of Finance*, 43, 567–592.

Appendix

Technical model assumptions: We make the following parametric assumptions to analyze the model.

(A1) $b > 2$: In this case, the return from the bank's portfolio, $[l + (1-l)R_0]$, without the profits from the asset purchase, is decreasing as the liquid asset l in bank's portfolio increases. This creates the trade off between the liquid and the illiquid asset *only once benefits from fire-sales are introduced*. In other words, the pure portfolio choice problem would lead to liquidity choice of $l = 0$. Furthermore, this condition also guarantees that $R_0 > r_0 = 1$.

(A2) $\tau < 1/b$: This guarantees that the liquidity banks have for asset purchases increases as they hold more liquid asset l in their portfolio, that is, $\frac{\partial L}{\partial l} > 0$.

(A3) $\overline{\Delta} < \left(b - \frac{3}{2}\right)$: Note that the maximum value $\overline{\Delta}$ can take, denoted by $\overline{\Delta}_{max}$, is equal to $(R_0 - r_0)$. This condition guarantees that $\overline{\Delta} < \overline{\Delta}_{max}$.

(A4) $B \leqslant \left(\frac{\overline{\Delta}^2}{b-1}\right)$: This condition guarantees that banks cannot generate a higher proportion of their future profits in the capital market when they invest in the bad project. In particular, when bank owners are left with a share of profits less than $\overline{\theta}$, they shirk, which results in a lower return from these investments. However, in that case, they can generate a higher proportion of their future profits in the capital market, that is, they can generate up to $\left(R_0 - \overline{\Delta} - r_0\right)$. Banks can generate higher funds from the capital market when they choose the good project if

$$\left(1 - \overline{\theta}\right)(R_0 - r_0) \geqslant R_0 - \overline{\Delta} - r_0, \tag{24}$$

which gives us $\overline{\theta} \leqslant \frac{\overline{\Delta}}{(R_0 - r_0)}$. Thus, we have $\overline{\theta} = \frac{B}{\overline{\Delta}} \leqslant \frac{\overline{\Delta}}{(R_0 - r_0)}$. In that case, it is optimal to leave a minimum share of $\overline{\theta}$ of future profits to bank managers, both for higher output as well as better liquidity generation through the capital market. This condition simplifies to $B \leqslant \left(\frac{\overline{\Delta}^2}{b-1}\right)$.

Proof of Proposition 2: We have $R_0(l) = \left[b - \frac{(1-l)}{2}\right]$ and $\frac{dR_0}{dl} = \frac{1}{2}$. Plugging these expressions into the FOC in (10), we get:

$$E(\alpha_0)\ \ [1 - b + (1 - l)] + E(\phi)\ \ [1 + \tau\,[-b + (1 - l)]] = 0, \tag{25}$$

where ϕ is the expected benefit per unit of failed banks' assets from asset purchase. From here, we can find banks' choice of liquidity $\widehat{l}$ that satisfies the FOC as:

$$\widehat{l} = 1 - b + \frac{E\left(\alpha_0\right) + E(\phi)}{E(\alpha_0) + \tau E(\phi)}, \tag{26}$$

which is given in Proposition 2.

We have the following:

$$\phi = \begin{cases} 0 & for \quad k \leqslant \underline{k} \\ \alpha_0 \left(\frac{(1-\alpha_0)\overline{p}}{\alpha_0 L} - 1\right) & for \quad k > \underline{k} \end{cases}. \tag{27}$$

Note that $\underline{k}$ is continuos in l. Thus, $E(\phi)$ is continuos in l. Hence, $\widehat{l}$ is continuos in l. Since, $\widehat{l}$ is a continuos function from the compact, convex set $[0, 1]$ into itself, by Brouwer's fixed point theorem, a fixed point of the mapping in equation (26) exists. Next, we show that the fixed point is unique.

Note that as l increases, the aggregate liquidity increases, and the region over which the price of the failed banks' assets fall below their fundamental value shrink, that is, $\frac{\partial \underline{k}}{\partial l} > 0$. Hence, we have $\frac{\partial E(\phi)}{\partial l} < 0$. Note that, we have $sign\left(\frac{\partial \widehat{l}}{\partial E(\phi)}\right) = sign\left[(1-\tau)E(\alpha_0)\right]$, where $(1-\tau)E(\alpha_0) > 0$. Hence, as the expected private benefit from holding the liquid asset decreases, banks hold less liquid asset in their portfolio. Thus, we have $\frac{\partial \widehat{l}}{\partial l} < 0$. As a result, the fixed point is unique.

Next, we show that $l^* \leqslant l_{\max}$. Note that by investing everything in the safe asset at $t = 0$, banks can avoid failure. First, we look at the case where banks take risks, that is, they choose $l < 1$. Then, we show that banks in equilibrium choose $l < 1$.

Consider first the case when $(1 - l)R_0 \geqslant r_0$. In this case, a proportion τ of the remaining return from the risky asset, that is, $\tau\left[(1 - l)R_0 - r_0\right]$ can be pledged in the capital market.

Thus, from equation (4), we have

$$\frac{\partial L}{\partial l} = 1 - \tau R_0 + \tau(1-l)\left(\frac{\partial R_0}{\partial l}\right) = 1 - \tau b + \tau\left(1-l\right). \tag{28}$$

Hence, for $\tau < \left(\frac{1}{b-1+l}\right)$, we have $\frac{\partial L}{\partial l} \geqslant 0$ and liquidity available for asset purchase increases as banks hold more of the liquid asset in their portfolio. A sufficient condition for this to hold is $\tau < 1/b$, which holds by our assumption (A2).

For the other case, $l > l_{max}$ and the return from the risky asset is not enough to pay old depositors. Hence, some of the liquid asset l has to be used to pay old depositors, and

$$L = l + (1-l)R_0 - r_0 < l. \tag{29}$$

Thus, for $b > 2$, which holds by (A1), $\frac{\partial L}{\partial l} < 0$ and the liquid asset available for asset purchase decreases as banks hold more of the liquid asset. Furthermore, without the asset purchase, the expected return on bank's portfolio $E\left[\alpha_0\left(l + (1-l)\left(b - \frac{1-l}{2}\right)\right)\right]$, is decreasing in l for $b > 2$. Hence, for $b > 2$, banks never hold a level of liquidity l greater than l_{max} in equilibrium.

Finally, we show that banks never invest everything in the safe asset so that our assumption $l < 1$ is verified.

The bank is safe only when it invests everything in the safe asset, that is, $l = 1$. Note that, even if the bank does not fail, it has no funds for the asset purchase as it has to pay back 1 unit to its depositors and the bank's profit is $E(\pi(1)) = \overline{p}$.

Alternatively, the bank can choose to take some risk and choose $l < 1$. In that case, the bank's profit is

$$\pi(l) = \alpha_0\left[l + (1-l)R_0(l) - r_0\right] + L\phi + \alpha_0\overline{p}. \tag{30}$$

We can show that $\pi(\alpha_0, l)$ is increasing in α_0 for all l. We do this in two steps:

- For $k \leqslant \underline{k}$ (that is, $\alpha_0 \geqslant \overline{\alpha} = 1 - \underline{k}$), we have $p^*(k) = \overline{p}$, hence
$$\frac{\partial \pi}{\partial \alpha_0} = l + (1-l)R_0(l) - r_0 + \overline{p} > 0. \tag{31}$$
- For $k > \underline{k}$ (that is, $0 < \alpha_0 < \overline{\alpha} = 1 - \underline{k}$), using the price function $p^*(k)$ in equation (7), we get:

$$\begin{aligned} \pi &= \alpha_0\left[l + (1-l)R_0(l) - r_0\right] + \alpha_0 L\left(\frac{(1-\alpha_0)\overline{p} - \alpha_0 L}{\alpha_0 L}\right) + \alpha_0\overline{p} \\ &= \alpha_0\left[l + (1-l)R_0(l) - r_0\right] + \overline{p} - \alpha_0 L > 0. \end{aligned} \tag{32}$$

Using $L = l + \tau\left[(1-l)R_0(l) - r_0\right]$, we get:

$$\frac{\partial \pi}{\partial \alpha_0} = (1-\tau)\left[(1-l)R_0(l) - r_0\right] > 0. \tag{33}$$

Note that $\underset{\alpha_0 \to 0}{lim}\ \pi = \overline{p}$. Hence, $\pi(l) > \overline{p}$ for all $\alpha_0 \in (0,1]$ and $l < 1$. Thus, we have $E(\pi(l)) > \overline{p} = E(\pi(1))$ for any continuos probability distribution f over $\alpha_0 \in [0,1]$, and banks do not invest everything in the safe asset. $\diamondsuit$

Proof of Proposition 3: First, we prove part (i). Note that if $\widehat{l}$ given in equation (12) increases, the privately optimal level of liquidity l^* increases. We have

$$\begin{aligned} sign\left(\frac{\partial \widehat{l}}{\partial \tau}\right) &= sign\left[\left(\frac{\partial E(\phi)}{\partial \tau}\right)\left[E(\alpha_0) + \tau E(\phi)\right] - \left[E(\alpha_0) + E(\phi)\right]\left(E(\phi) + \tau\left(\frac{\partial E(\phi)}{\partial \tau}\right)\right)\right] \\ &= sign\left[\left(\frac{\partial E(\phi)}{\partial \tau}\right)\left[(1-\tau)E(\alpha_0)\right] - E(\phi)\left[E(\alpha_0) + E(\phi)\right]\right]. \end{aligned}$$

We have $\frac{\partial E(\phi)}{\partial \tau} < 0$, since $\frac{\partial E(\phi)}{\partial p^*} < 0$ and $\frac{\partial p^*}{\partial \tau} > 0$. Hence, we have $\frac{\partial \widehat{l}}{\partial \tau} < 0$, that is, the privately optimal level of liquidity l^* decreases as τ increases.

Next, we prove part (ii). Note that ϕ is (weakly) increasing in k, therefore, is (weakly) decreasing in α_0. Hence, if g FOSD f, we have $E_g(\phi) < E_f(\phi)$. We have $\frac{\partial \widehat{l}}{\partial E(\phi)} > 0$. Hence, if g FOSD f, then we have $l^*_g < l^*_f$.

Finally, we prove part (iii). We have $E_g(\pi(l^*_g)) \geqslant E_g(\pi(l^*_f))$. Hence, a sufficient condition for (iii) to hold is $E_g(\pi(l^*_f)) > E_f(\pi(l^*_f))$. Since, g FOSD f, showing that $\pi(\alpha_0, l)$ is increasing in α_0 for all l is sufficient. We already showed in the proof of Proposition 2 that $\pi(\alpha_0, l)$ is increasing in α_0 for all l. $\diamondsuit$

Proof of Proposition 5: From the expressions for these two values of liquidity, we have

$$\widehat{l} - \widehat{\widehat{l}} = \frac{E(\alpha_0) + E(\phi)}{E(\alpha_0) + \tau E(\phi)} - \frac{1}{E(\alpha_0)} = \frac{E(\phi)\left[E(\alpha_0) - \tau\right] - E(\alpha_0)\left[1 - E(\alpha_0)\right]}{E(\alpha_0)\left[E(\alpha_0) + \tau E(\phi)\right]}. \tag{34}$$

Note that a sufficient condition for the socially optimal level of liquidity to be higher that the privately optimal level of liquidity is $E(\alpha_0) < \tau$. Hence, we analyze the case where $E(\alpha_0) \geqslant \tau$. As $E(\alpha_0)$ converges to 1, we have the privately optimal level of liquidity to be higher than the socially optimal level. Next, note that $\widehat{l} = \widehat{\widehat{l}}$ when

$$E(\phi)\left[E(\alpha_0) - \tau\right] = E(\alpha_0)\left[1 - E(\alpha_0)\right]. \tag{35}$$

Since the left hand side is decreasing in τ, but the right hand side is not affected by τ, this equation implicitly defines a unique critical $\tau^*(E(\alpha_0))$ such that $\widehat{l} < \widehat{\widehat{l}}$ if and only if $\tau > \tau^*(E(\alpha_0))$.

Using the implicit function theorem, we get:

$$E(\phi)\left[\frac{d\tau^*}{dE(\alpha_0)}\right] = E(\phi) + [E(\alpha_0) - \tau]\left[\frac{dE(\phi)}{dE(\alpha_0)}\right] - [1 - 2E(\alpha_0)]\,. \tag{36}$$

Note that $\left(\frac{dE(\phi)}{dE(\alpha_0)}\right) < 0$ so that we obtain

$$\frac{d\tau^*}{dE(\alpha_0)} < 0 \text{ for } E(\alpha_0) < \left(\frac{1 - E(\phi)}{2}\right). \tag{37}$$

See Figure 5 for an illustration. $\diamondsuit$

Proof of Proposition 7: We provide the proof for the case $w \geqslant \underline{p}$.[21] We have the FOC as:

$$1 + E(\alpha_0)\ \left[-b + (1 - l)\right] + T_2, \tag{38}$$

where, using Leibniz's rule, we get:

$$\begin{aligned} T_2 \;\; &= \;\; -(\alpha_1\Delta)\ \left[\int_{\overline{k}}^{1} f(k)\left[-\frac{(1-k)}{\underline{p}}\right]\left[1 - \tau R_0(l) + \tau(1-l)\left(\frac{dR_0}{dl}\right)\right]\ dk\right] \\ &\quad + (\alpha_1\Delta)\ \underbrace{\left[\overline{k} - \frac{(1-\overline{k})L}{\underline{p}}\right]}_{=0}\left(\frac{d\overline{k}}{dl}\right). \end{aligned} \tag{39}$$

Note that at $k = \overline{k}$, all failed banks' assets are purchased by surviving banks and the second term in equation (39) is equal to 0. Using $R_0(l) = \left[b - \frac{(1-l)}{2}\right]$ and $\frac{dR_0}{dl} = \frac{1}{2}$, we get:

$$T_2 = \left[1 + \tau\left(-b + 1 - l\right)\right]\ \left[\left(\frac{\alpha_1\Delta}{\underline{p}}\right)\int_{\overline{k}}^{1} f(k)(1-k)\ dk\right]. \tag{40}$$

Thus, the FOC can be written as

$$1 + E(\alpha_0)\ \left(-b + 1 - l\right) + E(\gamma_w)\ \left[1 + \tau\left(-b + 1 - l\right)\right] = 0, \tag{41}$$

[21] The proof for the case $w < \underline{p}$ is available upon request.

where $E(\gamma_w) = \left(\frac{\alpha_1 \Delta}{\underline{p}}\right) \left(\int_{\overline{k}}^{1} f(k)(1-k)\, dk\right)$. Equation (41) looks very much like the FOC for banks' choice of liquidity with the slight difference that in the first expression, we have 1 instead of α_0, since banks can benefit from their liquid assets only when they survive, which happens with a probability of α_0, whereas the regulator always benefits from banks' liquid assets.

From here, we can find the socially optimal level of liquidity $\widehat{\widehat{l}}_w$ that satisfies the FOC as:

$$\widehat{\widehat{l}}_w = 1 - b + \frac{1 + E(\gamma_w)}{E(\alpha_0) + \tau E(\gamma_w)}, \text{ where} \tag{42}$$

$$\gamma_w = \begin{cases} 0 & for & k \leqslant \overline{k} \\ \alpha_0 \left(\frac{\alpha_1 \Delta}{\underline{p}}\right) & for & w \geqslant \underline{p} \text{ and } k < \overline{k} \text{ or } w < \underline{p} \text{ and } k \in (\overline{k}, \overline{\overline{k}}] \\ \alpha_0 \left(\frac{\alpha_1 \Delta}{p_w^*(k)}\right) \left(\frac{w}{(1-k)L+w}\right) & for & w < \underline{p} \text{ and } k > \overline{\overline{k}} \end{cases}, \tag{43}$$

in its general form. We can show that a fixed point of the mapping in equation (42) exists and is unique.

Note that $\overline{k}$ is continuos in l. Thus, $E(\gamma_w)$ is continuos in l. Hence, $\widehat{\widehat{l}}_w$ is continuos in l. Since, $\widehat{\widehat{l}}_w$ is a continuos function from the compact, convex set $[0, 1]$ into itself, by Brouwer's fixed point theorem, a fixed point of the mapping in equation (42) exists. Next, we show that the fixed point is unique.

Note that as l increases, the aggregate level of liquidity increases, the region over which sales to outsiders take place shrinks, that is, $\frac{\partial \overline{k}}{\partial l} > 0$. Hence, we have $\frac{\partial E(\gamma_w)}{\partial l} < 0$. We have

$$\frac{\partial \widehat{\widehat{l}}_w}{\partial E(\gamma_w)} = \frac{E(\alpha_0) + \tau E(\gamma_w) - \tau\left[1 + E(\gamma_w)\right]}{\left[E(\alpha_0) + \tau E(\gamma_w)\right]^2} = \frac{E(\alpha_0) - \tau}{\left[E(\alpha_0) + \tau E(\gamma_w)\right]^2}.$$

Thus, for $E(\alpha_0) > \tau$, we have $\frac{\partial \widehat{\widehat{l}}_w}{\partial E(\gamma_w)} > 0$, which gives us $\frac{\partial \widehat{\widehat{l}}_w}{\partial l} < 0$. Hence, the fixed point is unique.

Next we analyze how the socially optimal level of liquidity l_w changes with outsider wealth w. Now, let

$$h = \frac{kw\underline{p}}{\left[(1-k)L + w\right]^2}. \tag{44}$$

Note that as the function h increases, $E(\gamma_w)$ and the socially optimal level of liquidity l_w increases. We have

$$\frac{\partial h}{\partial w} = \frac{k\underline{p}\left[(1-k)L+w\right]^2 - 2kw\underline{p}\left[(1-k)L+w\right]}{\left[(1-k)L+w\right]^4} = \frac{k\underline{p}\left[(1-k)L-w\right]}{\left[(1-k)L+w\right]^3}. \tag{45}$$

If $\frac{\partial h}{\partial w} < 0$ for $k \in [\overline{\overline{k}}, 1]$, then the socially optimal level of liquidity l_w decreases as outsider wealth w increases.

We have $\frac{\partial h}{\partial w} < 0$ when $w > (1-k)L$. For $k = 1$, this trivially holds.

For $k = \overline{\overline{k}}$, we have

$$w > \left(\frac{\underline{p}-w}{L+\underline{p}}\right) L \Longleftrightarrow w\left(L+\underline{p}\right) > \left(\underline{p}-w\right) L \Longleftrightarrow w > \left(\frac{L\underline{p}}{2L+\underline{p}}\right).$$

Hence, for $w > \left(\frac{L\underline{p}}{2L+\underline{p}}\right)$, we have $\frac{\partial h}{\partial w} < 0$ for $k \in [\overline{\overline{k}}, 1]$ and the socially optimal level of liquidity l_w decreases as outsider wealth w increases. $\diamondsuit$

Proof of Proposition 8: We investigate how the difference between the privately and socially optimal levels of liquidity behaves as a function of τ and Δ. See Figure 11 for an illustration.

Note that for $w \geqslant \underline{p}$, the actual value of w does not have an effect on the price. Hence, to simplify the notation, we suppress the subscript w in the expressions.

We have

$$\widehat{l} = 1 - b + \frac{E(\alpha_0) + E(\phi)}{E(\alpha_0) + \tau E(\phi)} \quad \text{and} \quad \widehat{\widehat{l}} = 1 - b + \frac{1 + E(\gamma)}{E(\alpha_0) + \tau E(\gamma)}. \tag{46}$$

Note that for regions where $k \in [0, \underline{k}]$ and $k \in [\overline{k}, 1]$, ϕ and γ are identical. However, in the interim range of failures, $k \in [\underline{k}, \overline{k}]$, surviving banks gain from asset purchases through cash-in-the-market prices while there is no social welfare loss since all banking assets are operated by the most efficient users. Thus, in this region, we have $\gamma = 0$ and $\phi > 0$. This implies that $E(\phi) > E(\gamma)$ for a given level of aggregate liquidity. Given these facts, we first prove part (i).

In the extreme case where $\tau = 1$, $E(\phi) = 0$ and $E(\gamma) = 0$, so that for all Δ,

$$\widehat{l} = 2 - b \leqslant \widehat{\widehat{l}} = 1 - b + \frac{1}{E(\alpha_0)}. \tag{47}$$

Since $\left(\widehat{\widehat{l}} - \widehat{l}\right)$ is continuos in τ, there exists a critical level $\tau^*(\Delta) \leqslant 1$, such that, for all $\tau > \tau^*(\Delta)$, socially optimal level of liquidity is higher than the privately optimal level of liquidity.

Next, for $\tau = 0$, we obtain that $\widehat{l} > \widehat{\widehat{l}}$ if and only if

$$h(\Delta) = E(\alpha_0) + E(\phi) - [1 + E(\gamma)] > 0. \tag{48}$$

For $\Delta = 0$, we know that $E(\phi) = E(\gamma) = 0$, so that $h(0) = E(\alpha_0) - 1 < 0$. Next, we have

$$\frac{\partial h}{\partial \Delta} = \frac{\partial}{\partial \Delta}\left[E(\phi) - E(\gamma)\right], \tag{49}$$

which is greater than 0 as shown below.

We know that except for the region $k \in [\underline{k}, \overline{k}]$, ϕ and γ are identical. Thus we have:

$$E(\phi) - E(\gamma) = \int_{1-\overline{k}}^{1-\underline{k}} \phi f(\alpha_0) d\alpha_0. \tag{50}$$

In this region, we have

$$\phi = \alpha_0 \left(\frac{(1-\alpha_0)\overline{p}}{\alpha_0 L} - 1\right) = \frac{\overline{p} - \alpha_0(\overline{p} + L)}{L} = \frac{\overline{p}}{L} - \frac{\alpha_0}{\underline{k}}. \tag{51}$$

Note that as Δ increases, $\underline{p}(= \overline{p} - (\alpha_1 \Delta))$ decreases. Thus, $\overline{k}$ increases whereas $\underline{k}$ does not change. Hence, the interval $[\underline{k}, \overline{k}]$ widens and $(E(\phi) - E(\gamma))$ increases. Formally, using Leibniz's rule, we get

$$\frac{\partial\left(E(\phi) - E(\gamma)\right)}{\partial \Delta} = \phi(1-\underline{k})\left[\frac{\partial\left(1-\underline{k}\right)}{\partial \Delta}\right] - \phi(1-\overline{k})\left[\frac{\partial\left(1-\overline{k}\right)}{\partial \Delta}\right] + \int_{1-\overline{k}}^{1-\underline{k}} \frac{\partial\left(\phi\right)}{\partial \Delta} f(\alpha_0) d\alpha_0. \tag{52}$$

Note that $\phi(1 - \underline{k}) = 0$. And since $\underline{k}$ does not change with $(\alpha_1 \Delta)$, from equation (51), we have $\frac{\partial(\phi)}{\partial \Delta} = 0$. Thus, we have

$$\frac{\partial\left(E(\phi) - E(\gamma)\right)}{\partial \Delta} = -\phi(1-\overline{k})\left[\frac{\partial\left(1-\overline{k}\right)}{\partial \Delta}\right]. \tag{53}$$

Note that $\overline{k}$ increases with Δ so that $\left[\frac{\partial(E(\phi)-E(\gamma))}{\partial \Delta}\right] > 0$. In other words, there exists a critical Δ^* such that $h(\Delta^*) = 0$ and $h(\Delta) > 0$ for all $\Delta > \Delta^*$, and $h(\Delta) < 0$ otherwise.

Since $\left(\widehat{\widehat{l}} - \widehat{l}\right)$ is continuos in τ, there exists a critical level $\tau^{**}(\Delta) \leqslant 1$, such that, for all $\tau \leqslant \tau^{**}(\Delta)$, the privately optimal level of liquidity is higher than the socially optimal level of liquidity. $\diamondsuit$

***t* = 0**	***t* = 1/2**	**States**	
• Banks borrow deposits.	• Returns from the risky investments are realized.		
• Banks choose their portfolio: o l units in the liquid asset. o 1 - l units in the risky asset.	• A proportion of k banks fail. • Failed banks' assets are auctioned to surviving banks.	$k \leq \underline{k}$	• Price is the full price, $\overline{p}$. • Surviving banks do not make profits from asset purchases.
		$k \leq \underline{k}$	• Price is decreasing as a function of k and is below $\overline{p}$. • Cash-in-the-market price. • Surviving banks make profits from asset purchases.

Figure 1: Timeline of the benchmark model.

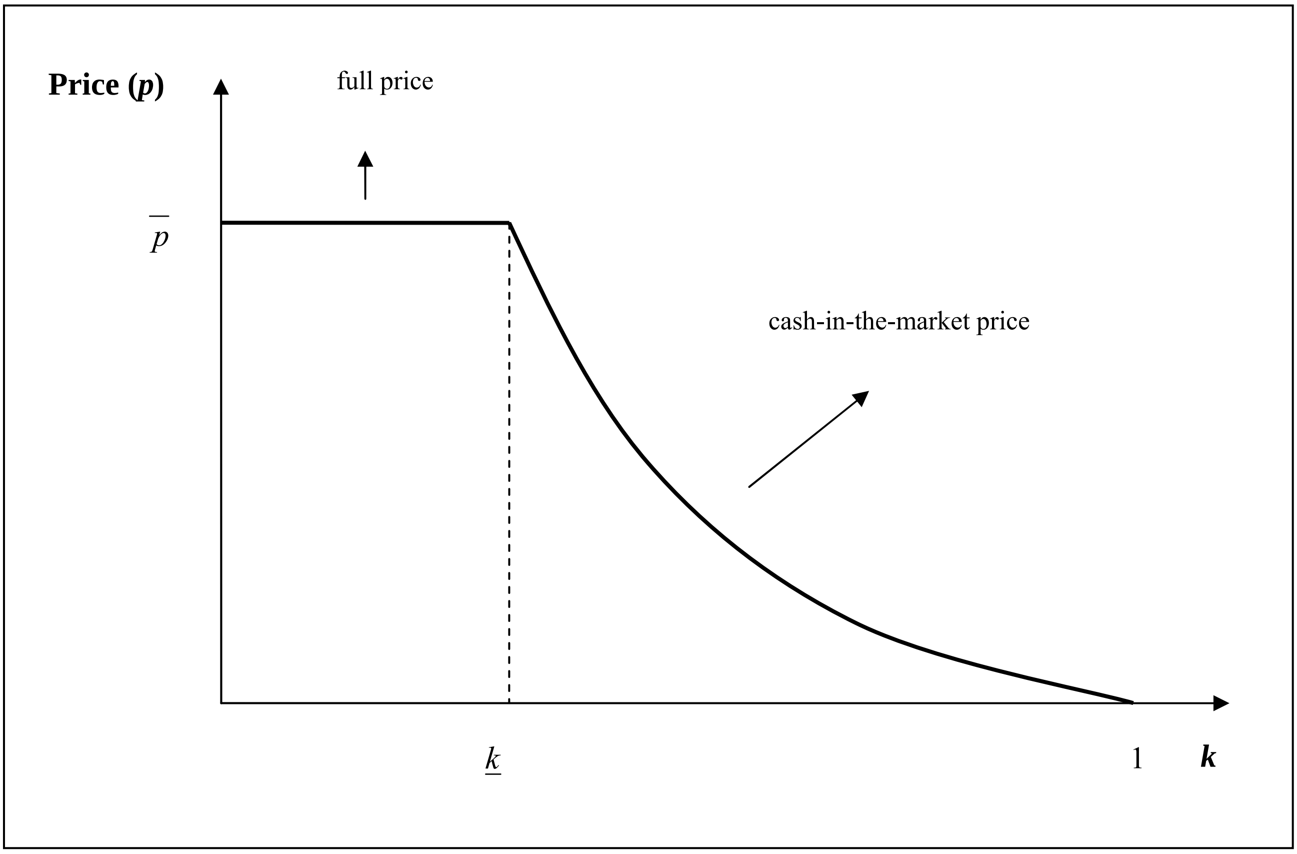


Figure 2: Price in Proposition 1.

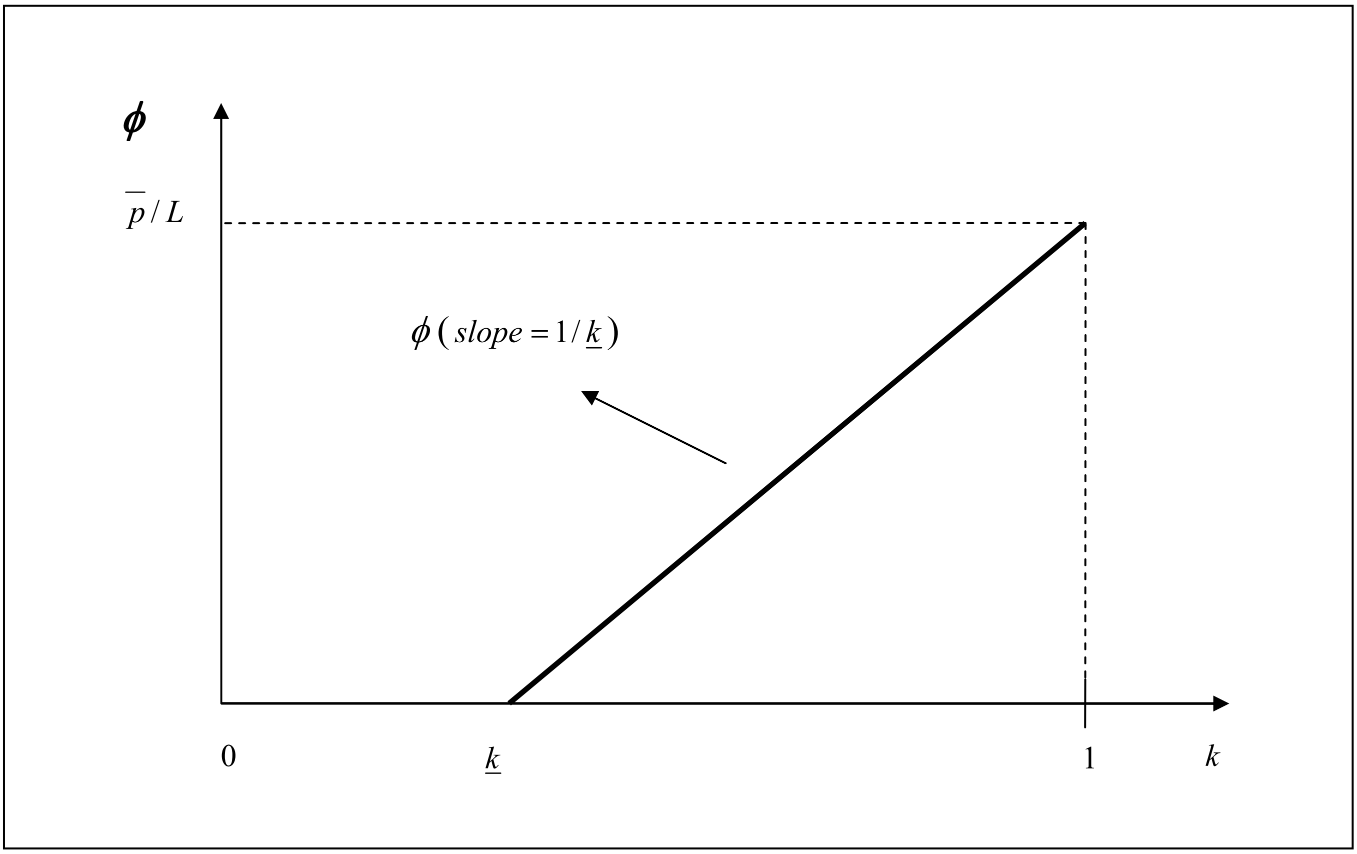


Figure 3: Marginal private (ϕ) benefit from the liquid asset (no outsiders).

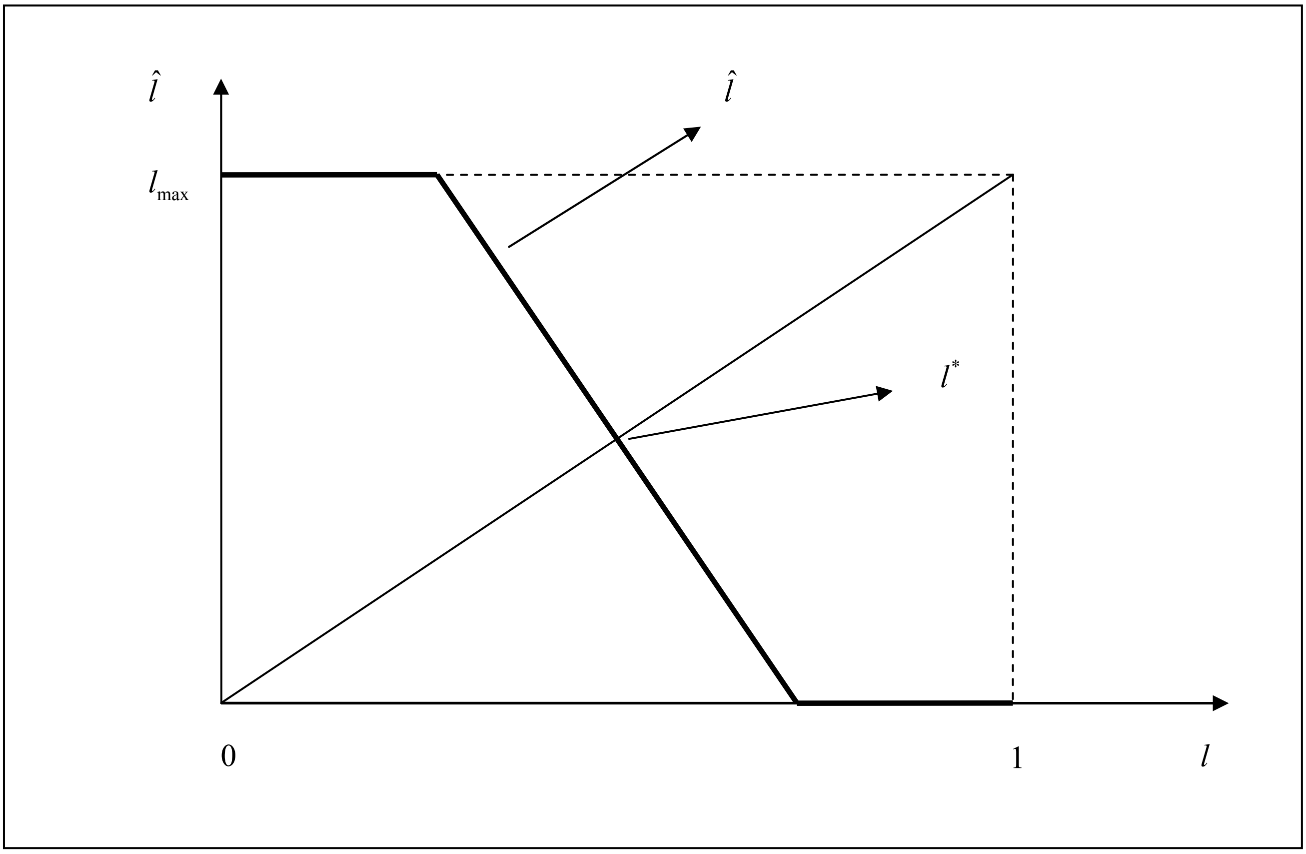


Figure 4: Privately optimal choice of liquidity and the equilibrium (Proposition 2).

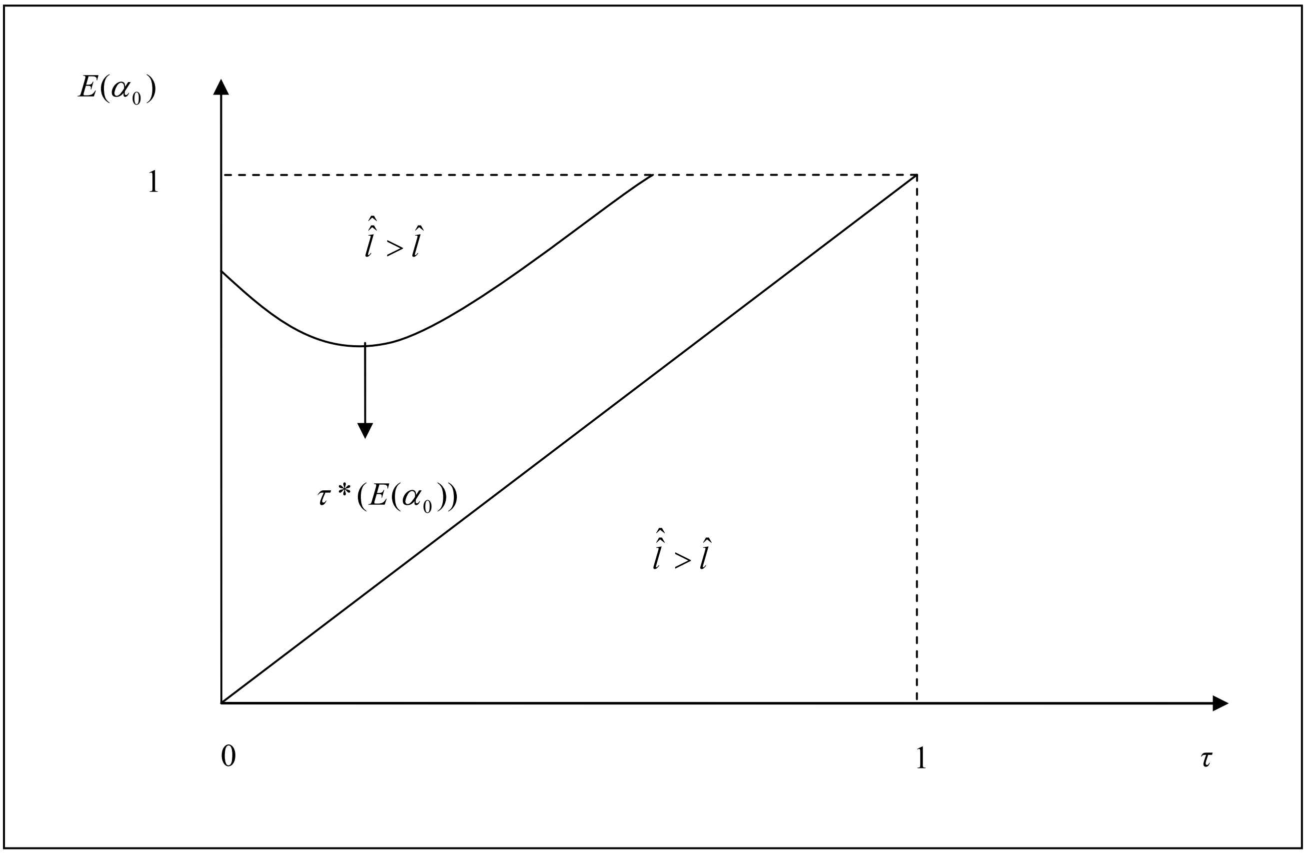


Figure 5: Comparison of privately and socially optimal levels of liquidity.

Figure 6: Liquidity ratio and its Fitted value vs Accounting standards

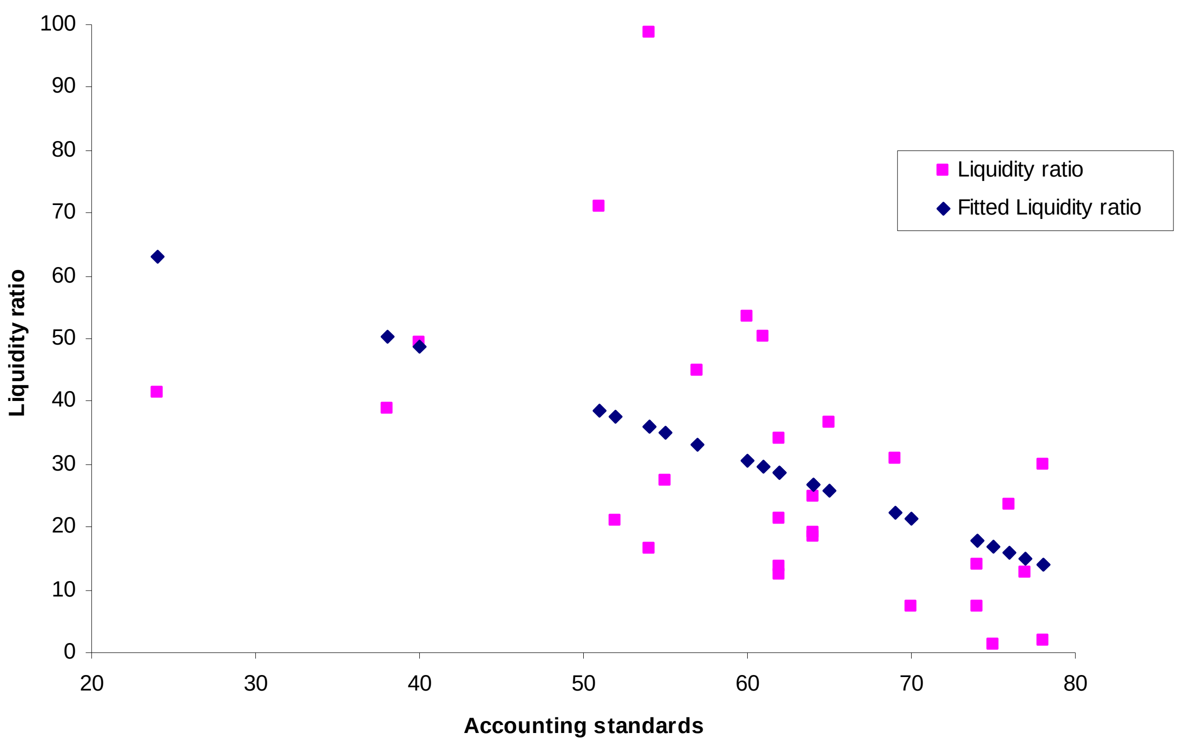


Figure 7: Liquidity ratio and its Fitted value vs Total Cap to GDP ratio

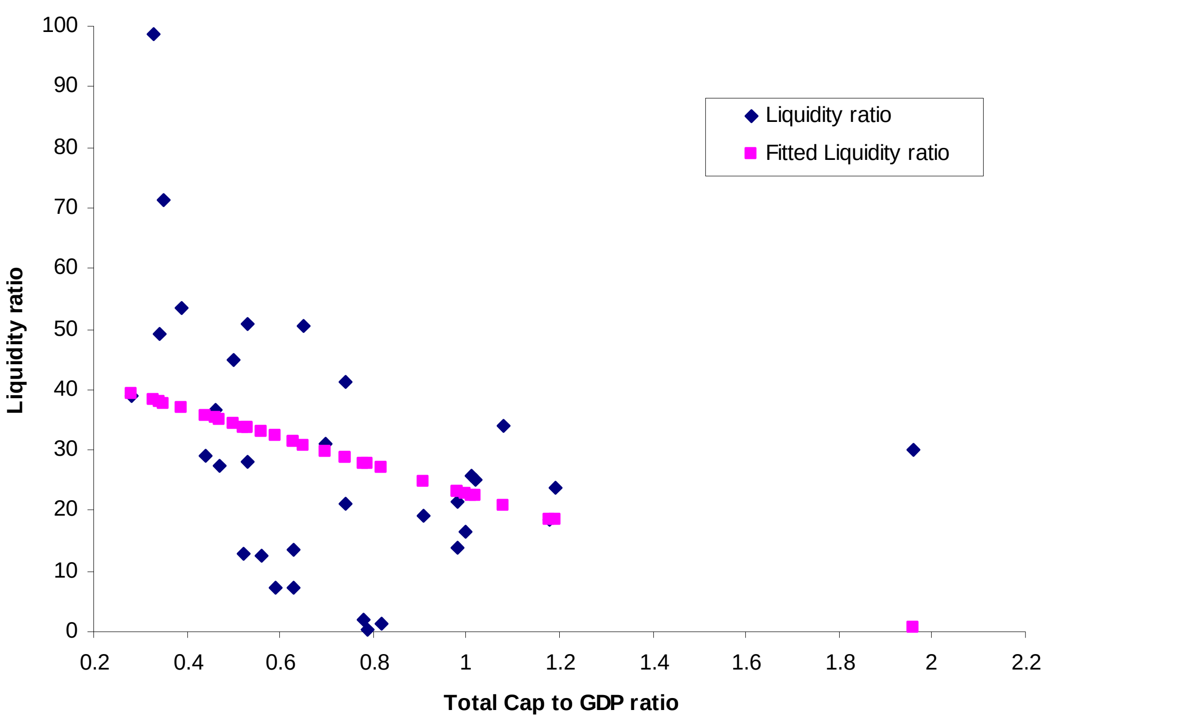
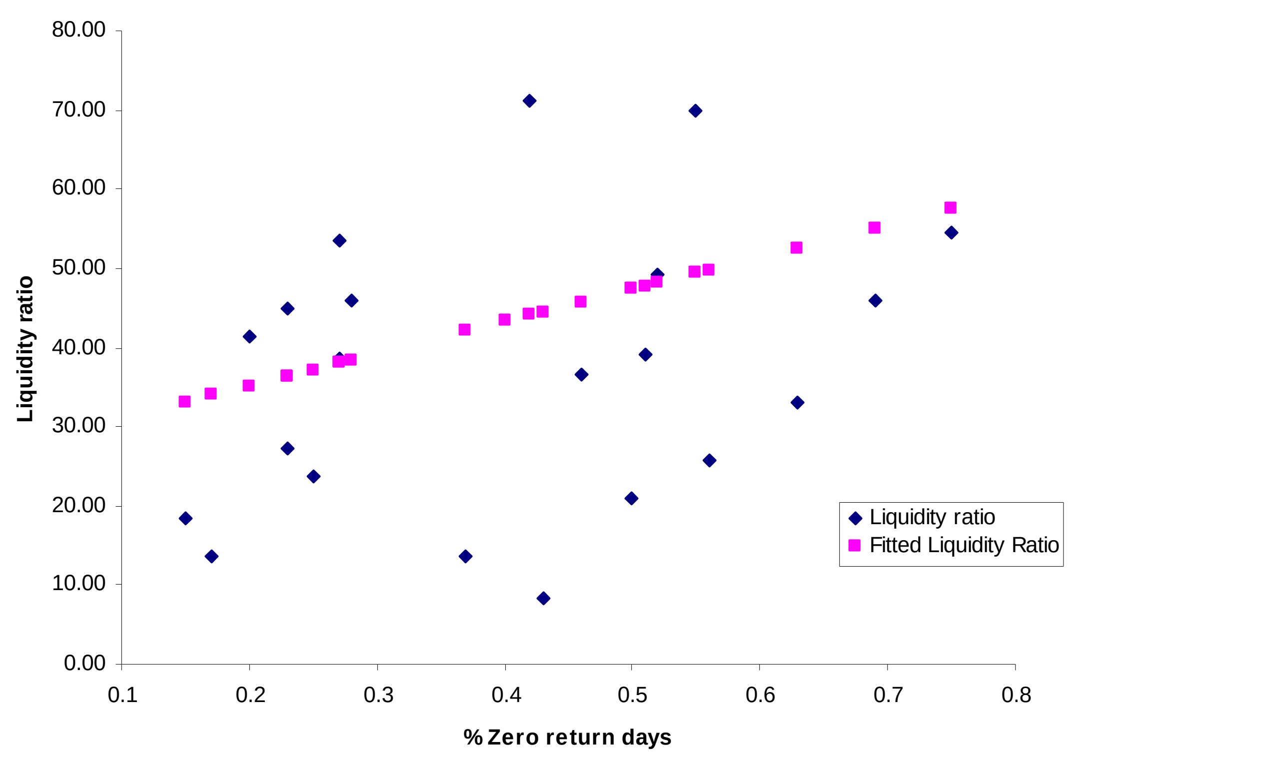


Figure 8: Liquidity ratio vs Stock market illiquidity (% Zero return days)



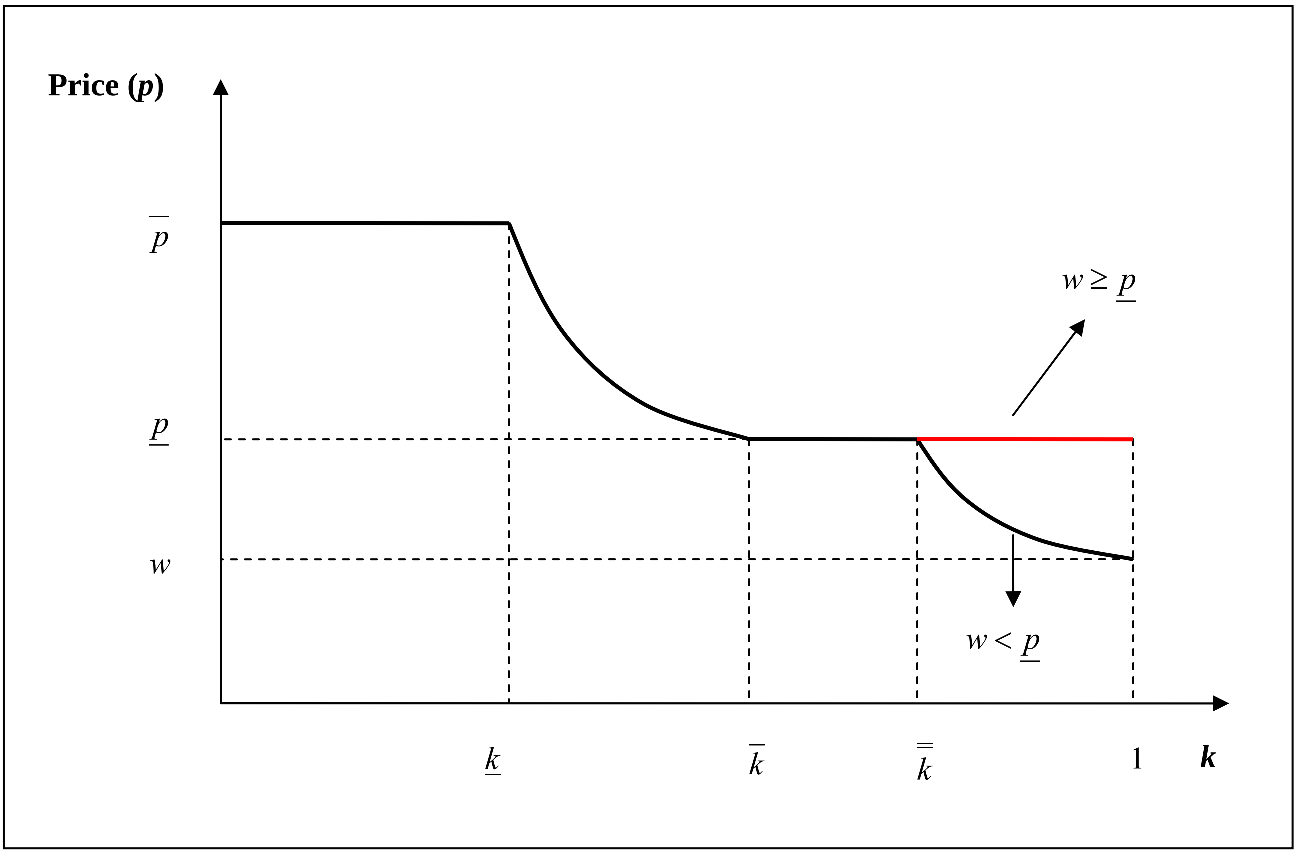


Figure 9: Price in Proposition 6.

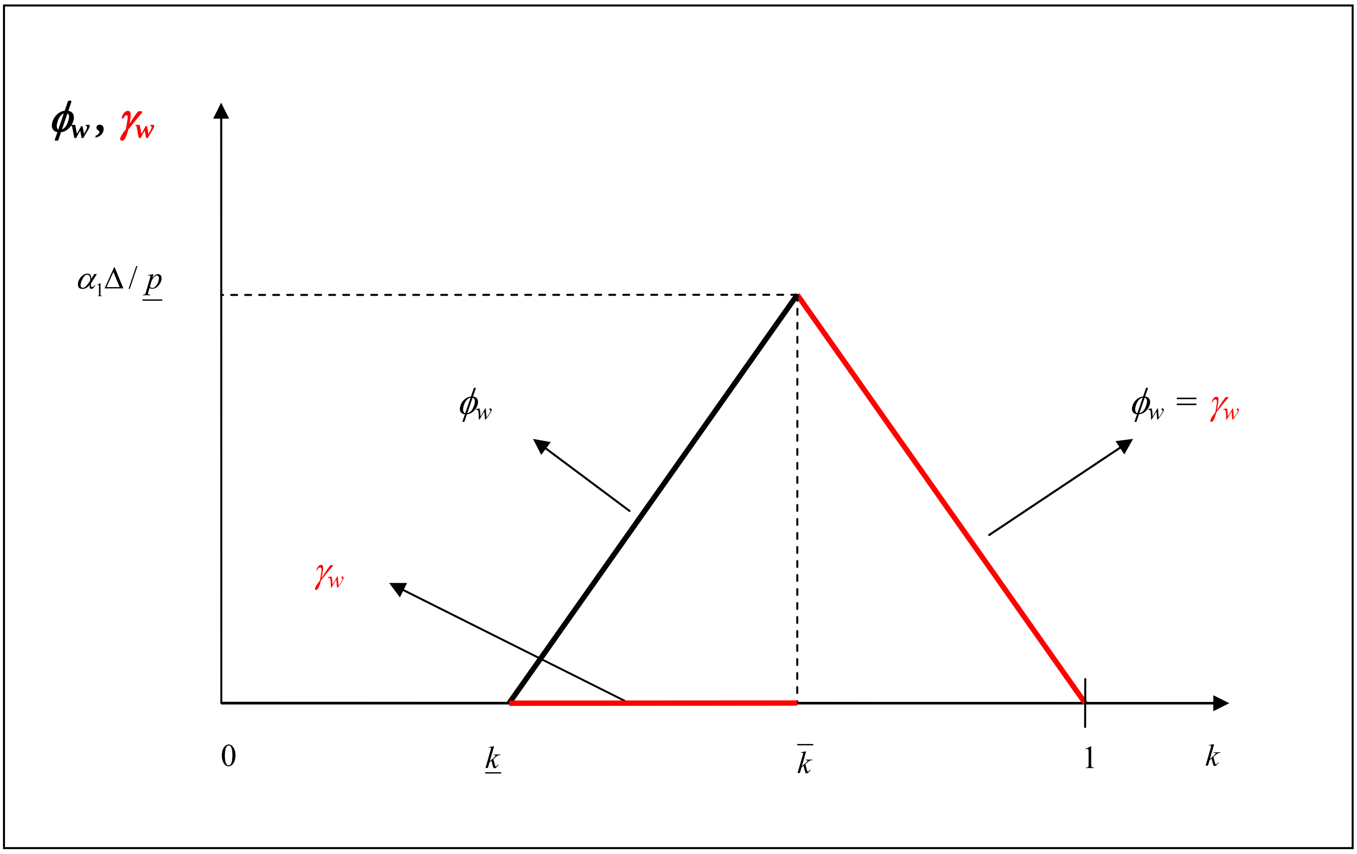


Figure 10: Marginal private (ϕ_w) and social (γ_w) benefit from the liquid asset for $w \geq \underline{p}$.

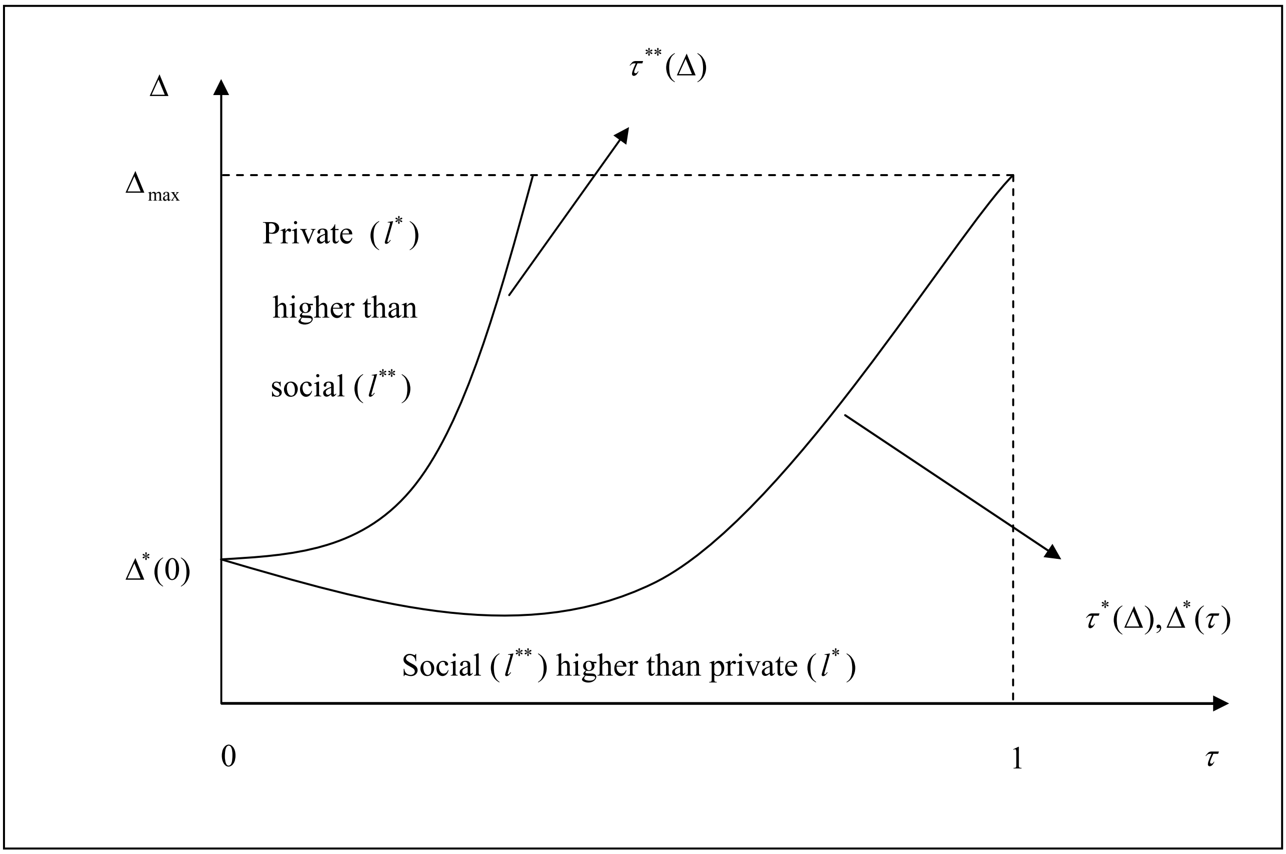


Figure 11: Comparing socially and privately optimal levels of liquidity (Proposition 8).