A Cross-Sectional Investigation of the Conditional ICAPM*

Turan G. Bali[a] and Robert F. Engle[b]

ABSTRACT

This paper provides a cross-sectional investigation of the conditional and unconditional intertemporal capital asset pricing model (ICAPM). The results indicate that estimating the conditional ICAPM with a pooled panel of time series and cross-sectional data in a multivariate GARCH-in-mean framework is crucial in identifying the positive risk-return tradeoff. Different from the traditional literature, the paper decomposes the aggregate stock market portfolio into ten book-to-market portfolios and then estimates a cross-sectionally consistent slope coefficient on the conditional variance-covariance matrix. The risk-aversion coefficient, restricted to be the same across all portfolios, is estimated to be positive and highly significant. This is the first study testing the cross-sectional consistency of the intertemporal relation by estimating the multivariate GARCH-in-mean model with different slopes. The statistical results indicate the equality of slope coefficients across all portfolios, supporting the empirical validity and sufficiency of the conditional ICAPM. The paper also provides evidence that the time-varying conditional covariances can explain the value premium because the average risk-adjusted return difference between the value and growth portfolios is economically and statistically insignificant within the conditional ICAPM framework.

JEL classifications: G12; G13; C51.

Keywords: ICAPM; Risk-return tradeoff; Risk aversion; Multivariate GARCH-in-mean.

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[a] Turan G. Bali is the David Krell Chair Professor of Finance at the Department of Economics and Finance, Zicklin School of Business, Baruch College, One Bernard Baruch Way, Box 10-225, New York, NY 10010. Phone: (646) 312-3506, Fax: (646) 312-3451, Email: turan_bali@baruch.cuny.edu.

[b] Robert F. Engle is the Michael Armellino Professor of Finance at New York University Stern School of Business, 44 West Fourth Street, Suite 9-62, New York, NY 10012, Phone: (212) 998-0710, Fax : (212) 995-4220, Email: rengle@stern.nyu.edu.

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This paper provides a cross-sectional investigation of the conditional and unconditional intertemporal capital asset pricing model (ICAPM). The results indicate that estimating the conditional ICAPM with a pooled panel of time series and cross-sectional data in a multivariate GARCH-in-mean framework is crucial in identifying the positive risk-return tradeoff. Different from the traditional literature, the paper decomposes the aggregate stock market portfolio into ten book-to-market portfolios and then estimates a cross-sectionally consistent slope coefficient on the conditional variance-covariance matrix. The risk-aversion coefficient, restricted to be the same across all portfolios, is estimated to be positive and highly significant. This is the first study testing the cross-sectional consistency of the intertemporal relation by estimating the multivariate GARCH-in-mean model with different slopes. The statistical results indicate the equality of slope coefficients across all portfolios, supporting the empirical validity and sufficiency of the conditional ICAPM. The paper also provides evidence that the time-varying conditional covariances can explain the value premium because the average risk-adjusted return difference between the value and growth portfolios is economically and statistically insignificant within the conditional ICAPM framework.

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1. Introduction

Merton's (1973) intertemporal capital asset pricing model (ICAPM) provides a positive equilibrium relation between the conditional mean and variance of excess returns on the aggregate market portfolio. However, Abel (1988), Backus and Gregory (1993), and Gennotte and Marsh (1993) develop models in which a negative relation between expected return and volatility is consistent with equilibrium. Similarly, empirical studies are still not in agreement on the direction of a time-series relation between expected return and risk. The estimates for the relative risk aversion coefficient are mostly insignificant or even negative. Due to the fact that the conditional mean and volatility of stock market returns are not observable, different approaches and specifications used by previous studies in estimating the two conditional moments are largely responsible for the conflicting empirical evidence.

Following the traditional literature, we estimate the risk-return tradeoff using a single series of the U.S. equity market index. Three different approaches are utilized when investigating the ICAPM based on the market portfolio: GARCH-in-mean, realized volatility, and range volatility models. Consistent with earlier studies, the risk aversion estimates provide no evidence for a significant link between the conditional mean and volatility of excess returns on the value-weighted NYSE/AMEX/NASDAQ index.

These results indicate that despite its fundamental importance, the intertemporal risk-return relation is inherently difficult to estimate. Most studies find low R-squares on the risk-return regressions, showing the existence of large noise in the realized market returns. Without a proper identification scheme, these large noises can completely disguise the fundamental relation between the expected excess return and the conditional variance. In this paper, we move away from the narrow focus on the single series of a market portfolio return. Instead, by using a cross-section of equity portfolios, we effectively enlarge our sample of observations and gain statistical power in identifying the risk-return tradeoff.

Merton's (1973) ICAPM predicts that an asset's expected return depends on its covariance with the market portfolio and with state variables that proxy for investment opportunities. In other words, Merton's prediction that expected returns should be related to conditional risk applies not only to the market portfolio, but also to individual stocks and stock portfolios. Indeed, for the model to be cross-sectionally consistent, the intertemporal relation between the expected excess return and its covariance with the market portfolio should universally be the same across all stock portfolios. In this paper, we exploit this cross-sectional universality of the risk-return relation and find a positive and significant intertemporal relation for equity portfolios.

Different from the traditional literature, we divide the aggregate stock market portfolio into ten book-to-market portfolios over the sample period of July 1926 to December 2007. We then estimate different forms of a multivariate GARCH-in-mean model with the constant conditional correlation (CCC) model of Bollerslev (1990) and the mean-reverting dynamic conditional correlation (DCC) model of Engle (2002). Following the original theoretical work of Merton (1973), we restrict the relative risk aversion coefficient to be the same across all portfolios and the common slope estimate turns out to be positive, highly significant, and robust to variations in the conditional covariance specifications and including a wide variety of state variables that proxy for the intertemporal hedging demand.

We investigate whether the power of our methodology is coming from (1) the GARCH-based time-varying conditional covariances, or (2) pooling the time series and cross section together, or (3) both. Consistent with the existing literature, we find that the GARCH-in-mean model cannot generate a positive and significant relation between risk and return on the aggregate market portfolio. That is, the GARCH-based methodology itself cannot resolve the issue if one narrowly focuses on a single series of the market portfolio return. When we pool the time-series and cross-section together, we find that the unconditional measures of market risk cannot predict expected future returns on equity portfolios, implying insufficiency of the unconditional ICAPM. Put differently, pooling the time series and cross section together without the time-varying conditional covariances cannot help identify a significant risk-return tradeoff either. Our results indicate that estimating the conditional ICAPM with a pooled panel of time series and cross sectional data in a multivariate GARCH-in-mean framework is essential in identifying the positive risk-return tradeoff.

This paper tests for the first time the cross-sectional consistency of the intertemporal relation. We estimate the multivariate GARCH-in-mean model with a different slope coefficient for each equity portfolio. The maximum likelihood estimates of the slope coefficients are found to be positive, similar in magnitude, and statistically significant for all the ten portfolios of book-to-market ratio. The statistical results indicate the equality of slope coefficients on the conditional variance-covariance matrix, supporting the empirical validity and sufficiency of the conditional ICAPM with a common slope.

While estimating the multivariate GARCH-in-mean models, we allow the intercepts to be different across portfolios. The intercepts capture the monthly abnormal returns on each portfolio that cannot be explained by the conditional measures of market risk. One implication of the ICAPM is that the intercepts should not be jointly different from zero assuming that the covariances of risky assets with the market portfolio have enough predictive power for expected future returns. To examine the empirical validity of the conditional ICAPM, we test the joint hypothesis that all intercepts equal zero. The Wald statistics fail to reject the null hypothesis, implying that the conditional measures of market risk have significant predictive power for the time-series and cross-sectional variations in expected returns on the book-to-market portfolios.

The paper also tests whether the return differences between the value and growth portfolios (value premium) can be explained by the conditional ICAPM. The results clearly indicate that the DCC- and CCC-based time-varying conditional covariances can explain the value premium because the average risk-adjusted return difference between the value and growth portfolios is found to be economically and statistically insignificant within the conditional ICAPM framework.

When investigating the intertemporal hedging demands and the associated risk premiums induced by the conditional covariation of portfolio returns with a set of macroeconomic variables, we find that the common slope coefficients on the conditional covariances with the unexpected news in the inflation rate and the aggregate dividend yield are statistically significant, implying that the inflation-related and dividend-related shocks contain systematic risks rewarded in the stock market and they can be viewed as a proxy for investment opportunities. However, the innovations in default spread, term spread, short-term interest rate, and the growth rate of industrial production do not play a significant role in intertemporal hedging demand. Incorporating the conditional covariation with any of these state variables does not change the positive risk premium induced by the conditional covariation of portfolio returns with the market portfolio.

The paper is organized as follows. Section 2 briefly discusses the intertemporal relation between risk and return. Section 3 describes the data. Section 4 presents results from estimating the risk-return tradeoff on the market portfolio. Section 5 provides a cross-sectional investigation of the conditional ICAPM. Section 6 estimates the intertemporal relation with the unconditional measures of market risk. Section 7 examines the risk-return tradeoff by accounting for the intertemporal hedging demand. Section 8 concludes the paper.

2. The intertemporal relation between expected return and risk

Merton's (1973) ICAPM implies the following equilibrium relation between expected return and risk for any risky asset *i*:

$$\mu_i - r = \beta \cdot \sigma_{im} + \lambda \cdot \sigma_{ix}, \tag{1}$$

where r is the risk-free interest rate, $\mu_i - r$ denotes the expected excess return on the risky asset i, σ_{im} denotes the covariance between the excess returns on the risky asset i and the market portfolio m, and σ_{ix} denotes a $(1 \times k)$ row of covariances between the excess returns on the risky asset i and the k state variables x. β is the relative risk aversion coefficient and λ measures the market's aggregate reaction to shifts in a k-dimensional state vector that governs the stochastic investment opportunity set. Equation (1) states that in equilibrium, investors are compensated in terms of expected return for bearing market (systematic) risk and for bearing the risk of unfavorable shifts in the investment opportunity set.

Many empirical studies focus on the time-series implication of the equilibrium relation in eq. (1) and apply it narrowly to the market portfolio. Without the hedging demand component ($\sigma_{ix} = 0$), this focus leads to the following risk-return relation:

$$\mu_m - r = \beta \cdot \sigma_m^2 . \tag{2}$$

When considering stochastic investment opportunity, the literature often implicitly or explicitly projects the covariance vector σ_{ix} linearly to the state variables x to obtain the following relation:

$$\mu_m - r = \beta \cdot \sigma_m^2 + \lambda \cdot x . \tag{3}$$

Our work in this article differs from the above literature in two major ways. First, we estimate the intertemporal relation eq. (1) not on the single series of the market portfolio, but simultaneously on equity portfolios, and constrain all these portfolios to have the same cross-sectionally consistent proportionality coefficients β and λ. Second, we directly estimate the conditional covariances σ_{im} and σ_{ix} using the constant conditional correlation model of Bollerslev (1990) and the dynamic conditional correlation model of Engle (2002). We do not make any linear projection assumptions on the state variables.

The second term in eq. (1) reflects the investors' demand for the asset as a vehicle to hedge against unfavorable shifts in the investment opportunity set. An "unfavorable" shift in the investment opportunity set is defined as a change in x such that future consumption c will fall for a given level of future wealth. That is, an unfavorable shift is an increase in x if $\partial c/\partial x < 0$ and a decrease in x if $\partial c/\partial x > 0$.

Merton (1973) shows that all risk-averse utility maximizers will attempt to hedge against such shifts in the sense that if $\partial c/\partial x < 0$ ($\partial c/\partial x > 0$), then, ceteris paribus, they will demand more of an asset, the more positively (negatively) correlated the asset's return is with changes in x. Thus, if the ex post opportunity set is less favorable than was anticipated, investors will expect to be compensated by a higher level of wealth through the positive correlation of the returns. Similarly, if the ex post returns are lower, they will expect a more favorable investment environment.

In this paper, we focus on the sign and statistical significance of the common slope coefficient (β) on σ_{im} in the following risk-return relation:

$$\mu_i - r = \alpha_i + \beta \cdot \sigma_{im} . \tag{4}$$

According to the original ICAPM of Merton (1973), the relative risk aversion coefficient β is restricted to be the same across all risky assets and it should be positive and statistically significant, implying a positive risk-return tradeoff.

We test whether the slopes on σ_{im} are different across risky assets. Earlier studies assume a common slope coefficient (β) following the original theoretical work of Merton (1973) and do not question the validity of this assumption. In this paper, we examine the sign and statistical significance of different slope coefficients (β_i) on σ_{im} in the following risk-return relation:

$$\mu_i - r = \alpha_i + \beta_i \cdot \sigma_{im} . \tag{5}$$

To determine whether there is a common slope coefficient (β) on σ_{im} corresponding to the average relative risk aversion, we examine the cross-sectional consistency of the intertemporal relation by testing the joint hypothesis that H_0: $\beta_1 = \beta_2 = ... = \beta_n$ assuming that we have n risky assets.

Another implication of the ICAPM is that the intercepts (α_i) in eq. (4) should not be jointly different from zero assuming that the covariances of risky assets with the market portfolio have enough predictive power for the time-series and cross-sectional variations in expected returns. To determine if σ_{im} has significant explanatory power, we test the joint hypothesis that H_0: $\alpha_1 = \alpha_2 = ... = \alpha_n = 0$.

We think that macroeconomic variables such as the default spread, term spread, relative T-bill rate, aggregate dividend yield, inflation rate, and economic growth can be viewed as potential state variables that may affect the stochastic investment opportunity set. Hence, we examine whether the innovations in these state variables are priced in the conditional ICAPM framework. In other words, we test if the changes in these macroeconomic variables are risks rewarded in the stock market. Specifically, we first test the statistical significance of the common slope coefficient (λ) on σ_{ix} in equation (6),

$$\mu_i - r = \alpha_i + \beta \cdot \sigma_{im} + \lambda \cdot \sigma_{ix} , \tag{6}$$

and then examine whether the common slope (β) on σ_{im} remains positive and significant after including σ_{ix} to the risk-return relation.

3. Data

We use the monthly excess returns on the value-weighted book-to-market portfolios and the monthly excess returns on the value-weighted stock market portfolio. The aggregate market portfolio is proxied by the value-weighted NYSE/AMEX/NASDAQ index, which is also known as the value-weighted index of the Center for Research in Security Prices (CRSP). The CRSP index contains all stocks trading at NYSE, AMEX, and NASDAQ and hence it can be viewed as the broadest possible index for the U.S. equity market. Excess returns on portfolios are obtained by subtracting the returns on the one-month Treasury bill from the raw returns on equity portfolios. The data are obtained from Kenneth French's online data library: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. We use the longest sample period available, from July 1926 to December 2007, yielding a total of 978 monthly observations.

3.1. Value-Weighted Book-to-Market Portfolios

As described in Fama and French (1993), book-to-market portfolios are formed on the ratio of book value equity (BE) to market value of equity (ME) at the end of each June using NYSE breakpoints. They use all NYSE, AMEX, and NASDAQ stocks for which they have market equity for December of year T-1 and June of year T, and book equity for year T-1. BE used in June of year T is the book equity for the last fiscal year end in T-1. ME is price times shares outstanding at the end of December of year T-1. In June of each year, Fama and French (1993) rank all NYSE stocks in CRSP based on the ratio of BE/ME to determine the NYSE decile breakpoints according to the ranked values of BE/ME for NYSE stocks. Then, they break all NYSE, AMEX, and NASDAQ stocks into 10 book-to-market groups based on the NYSE breakpoints.

Appendix A presents summary statistics for the monthly excess returns on the 10 value-weighted book-to-market (BM) portfolios. "Growth" (Decile 1) is the portfolio of growth stocks with the lowest book-to-market ratios and "Value" (Decile 10) is the portfolio of value stocks with the highest book-to-market ratios. Mean, median, maximum, minimum, and standard deviation are reported for each portfolio in Panel A.

For the sample period from July 1926 to December 2007, the value-weighted average excess return increases from 0.57% per month (growth portfolio) to 1.09% per month (value portfolio). The average return difference between the value and growth portfolios is 0.52% per month with the OLS t-statistic of 2.46 and the Newey-West (1987) adjusted t-statistic of 2.39, implying that value stocks on average generate higher returns than growth stocks. This indicates economically and statistically significant value premium (52 basis points per month) over the sample period of 1926-2007. As shown in Panel A, the value-weighted median excess return increases from 0.75% per month (growth portfolio) to 0.96% per month (value portfolio), indicating value premium based on the median return differences.

The last two columns of Panel A suggest that there can be a risk-based explanation of the return differences between growth and value stocks. Specifically, the standard deviation of excess returns increases from 5.75% to 9.34% per month as we move from the growth to value portfolios. That is, value stocks are expected to generate higher returns than growth stocks because value stocks are riskier. Furthermore, there is a significant difference between the market betas of the growth and value portfolios. As shown in the last column of Panel A, the market beta of the growth portfolio is 1.0063 whereas the market beta of the value portfolio is 1.4466. These results indicate that value stocks have higher systematic risk than growth stocks over the sample period of 1926-2007.

3.2. Value-Weighted Market Portfolio

Panel B of Appendix A presents summary statistics for the monthly excess returns on the value-weighted NYSE/AMEX/NASDAQ index. The average excess return on the market portfolio is 0.65% per month corresponding to the expected market risk premium of 7.8% per annum. The maximum excess return on the market portfolio is 38.27% per month observed in April 1933, whereas the minimum excess return on the market portfolio is –29.04% per month observed in September 1931. The standard deviation of excess returns on the market portfolio is 5.41% per month. The average Sharpe ratio (or the expected excess return per unit risk) is about 0.12 (0.0065/0.0541) for the aggregate market over the sample period of 1926-2007.

3.3. Macroeconomic Variables

Some studies find that macroeconomic variables associated with business cycle fluctuations can predict the stock market. The commonly chosen variables include Treasury bill rates, default spread, term spread, and dividend-price ratios. We study how variations in the default spread, term spread, de-trended short-term interest rate, and the aggregate dividend yield predict variations in the investment opportunity set and how incorporating the conditional covariances of portfolio returns with the innovations in these variables affects the intertemporal risk-return relation.

We obtain the monthly yields on the 3-month Treasury bill, 10-year Treasury bond, BAA-rated and AAA-rated corporate bonds from the H.15 database of the Federal Reserve Board. The default spread (*DEF*) is computed as the difference between the yields on the BAA-rated and AAA-rated corporate bonds. The term spread (*TERM*) is calculated as the difference between the yields on the 10-year Treasury bond and the 3-month Treasury bill. The relative T-bill rate (*RREL*) is the de-trended short-term interest rate, defined as the difference between the 3-month T-bill rate and its 12-month backward moving average. We download the monthly aggregate dividend yield from Robert Shiller's website: http://aida.econ.yale.edu/~shiller/. The aggregate dividend yield (*DIV*) is defined as the ratio of the monthly dividends of the S&P 500 index to the current level of the S&P 500 index.[1]

In addition to the aforementioned macroeconomic variables, we use the monthly inflation rate and the monthly growth rate of industrial production proxying for economic growth. The inflation rate (*INF*) is the monthly growth rate of the Consumer Price Index available at Robert Shiller's website. The economic or output growth (*OUT*) is defined as the monthly growth rate of the Industrial Production Index obtained from the G.17 database of the Federal Reserve Board. The sample period for all these macroeconomic variables is from July 1926 to December 2007.

4. Risk-return tradeoff on the market portfolio

Merton's (1973) ICAPM indicates that the conditional expected excess return on a risky market portfolio is a linear function of its conditional variance plus a hedging component that captures the investor's motive to hedge for future investment opportunities. Ignoring the hedging demand component, the equilibrium relation between risk and return is defined as:

$$E_t(R_{m,t+1}) = \beta \cdot E_t(\sigma^2_{m,t+1}), \tag{7}$$

where $E_t(R_{m,t+1})$ and $E_t(\sigma^2_{m,t+1})$ are, respectively, the conditional mean and variance of excess returns on the market portfolio, and $\beta > 0$ is the average risk aversion of market investors. Equation (7) establishes the dynamic relation that investors require a larger risk premium at times when the market is riskier.

A large number of studies fail to identify a robust and significant intertemporal relation between risk and return on the aggregate stock market portfolio. French, Schwert, and Stambaugh (1987) find that the coefficient estimate (β) in eq. (7) is not significantly different from zero when they use past daily returns to estimate the monthly conditional variance. Follow-up studies by Baillie and DeGennaro (1990), Campbell

[1] At an earlier stage of the study, we also used the aggregate dividend yield from the CRSP value-weighted index with and without dividends. The results from the CRSP data are very similar to our findings reported in the paper.

and Hentchel (1992), Chan, Karolyi, and Stulz (1992), Glosten, Jagannathan, and Runkle (1993), Harrison and Zhang (1999), Goyal and Santa-Clara (2003), and Bollerslev and Zhou (2006) rely on the GARCH-in-mean and realized volatility models that provide no evidence for a robust, significant link between expected return and risk on the aggregate market portfolio.

Some studies find that the intertemporal relation between risk and return is negative (e.g., Campbell (1987), Breen, Glosten, and Jagannathan (1989), Turner, Startz, and Nelson (1989), Nelson (1991), Glosten, Jagannathan, and Runkle (1993), Whitelaw (1994, 2000), Harvey (2001), and Brandt and Kang (2004)). Some studies do provide evidence supporting a positive and significant relation between expected return and risk on individual stocks (e.g., Bali and Engle (2008)) and equity portfolios (e.g., Bollerslev, Engle, and Wooldridge (1988), Scruggs (1998), Ghysels, Santa-Clara, and Valkanov (2005), Guo and Whitelaw (2006), Lundblad (2007), and Bali (2008)).

For comparison, we follow the traditional literature and estimate the risk-return tradeoff using a single series of the value-weighted NYSE/AMEX/NASDAQ index. We use three different approaches when testing ICAPM based on the market portfolio: Generalized Autoregressive Conditional Heteroskedasticity (GARCH), Realized Volatility, and Range Volatility models.

4.1. GARCH-in-Mean Model

The following GARCH-in-mean model is used to estimate the intertemporal relation between the expected excess return and risk on the market portfolio:[2]

$$R_{m,t+1} \equiv \alpha + \beta \cdot \sigma^2_{m,t+1|t} + \varepsilon_{m,t+1}\,, \tag{8}$$

$$E(\varepsilon^2_{m,t+1} \,|\, \Omega_t) = \sigma^2_{m,t+1|t} = \gamma_0 + \gamma_1 \varepsilon^2_{m,t} + \gamma_2 \sigma^2_{m,t}\,, \tag{9}$$

where $R_{m,t+1}$ is the excess return on the market portfolio for month t+1, $E(R_{m,t+1} \,|\, \Omega_t) \equiv \alpha + \beta \cdot \sigma^2_{m,t+1|t}$ is the conditional mean of excess market returns for month t+1 based on the information set up to time t denoted by Ω_t, $\varepsilon_{m,t+1} = z_{t+1} \cdot \sigma_{m,t+1|t}$ is the error term with $E(\varepsilon_{m,t+1}) = 0$, $\sigma_{m,t+1|t}$ is the conditional standard deviation of monthly excess returns on the market portfolio, and $z_{t+1} \sim N(0,1)$ is a random variable drawn from the standard normal density and can be viewed as information shocks in the equity market. $\sigma^2_{m,t+1|t}$ is the conditional variance of monthly excess returns based on the information set up to time t. The conditional variance, $\sigma^2_{m,t+1|t}$, in equation (9) follows a GARCH(1,1) process as defined by Bollerslev (1986) to be a function of the last period's unexpected news (or information shocks), z_t, and the last period's variance, $\sigma^2_{m,t}$.

[2] The GARCH-in-mean models are originally introduced by Engle, Lilien, and Robins (1987) and then used by a large number of studies including Engle, Ng, and Rothschild (1990), Baillie and DeGennaro (1990), Nelson (1991), and many recent papers on risk-return tradeoff.

Our focus is to examine the magnitude and statistical significance of the risk aversion parameter β in equation (8).

Table 1 presents the maximum likelihood parameter estimates and the t-statistics in parentheses for the GARCH-in-mean model. The risk aversion parameter (β) is estimated to be positive, 1.1442, but statistically insignificant based on the normal and the Bollerslev-Wooldridge (1992) robust t-statistics. For a robustness check of this finding, we consider the conditional standard deviation and the conditional log-variance in the conditional mean equation:

$$R_{m,t+1} \equiv \alpha + \beta \cdot \sigma_{m,t+1|t} + \varepsilon_{m,t+1}, \tag{10}$$

$$R_{m,t+1} \equiv \alpha + \beta \cdot \ln \sigma^2_{m,t+1|t} + \varepsilon_{m,t+1}, \tag{11}$$

As presented in Table 1, the slope coefficient on $\sigma_{m,t+1}$ is positive, 0.1156, but statistically insignificant with the Bollerslev-Wooldridge t-statistic of 1.04. Similarly, the slope coefficient on $\ln \sigma^2_{m,t+1|t}$ is positive, 0.0023, but insignificant with t-stat.= 0.88. The results from alternative specifications of the GARCH-in-mean model provide no evidence for a significant link between expected return and risk on the market portfolio.

Another notable point in Table 1 is the significance of volatility clustering. For all specifications of the volatility process, the conditional variance parameters (γ_1, γ_2) are positive, between zero and one, and highly significant. The results indicate the presence of rather extreme conditionally heteroskedastic volatility effects in the stock return process because the GARCH parameters, γ_1 and γ_2, are not only highly significant, but also the sum ($\gamma_1 + \gamma_2$) is close to one. This implies the existence of substantial volatility persistence in the stock market.

4.2. Realized Volatility Model

Earlier studies that investigate the monthly risk-return tradeoff generally rely on the GARCH-in-mean or Realized Volatility models. Following earlier studies, we calculate the monthly realized variance using the within-month daily return data:

$$\sigma^2_{m,t} = \sum_{d=1}^{D_t} R^2_{m,d} + 2\sum_{d=2}^{D_t} R_{m,d} \cdot R_{m,d-1}, \tag{12}$$

where D_t is the number of trading days in month t and $R_{m,d}$ is the daily return on the value-weighted market portfolio on day d. The aggregate market portfolio is measured by the value-weighted CRSP index. The second term on the right hand side adjusts for the autocorrelation in daily returns using the approach of French, Schwert, and Stambaugh (1987).[3]

[3] As discussed in French, Schwert, and Stambaugh (1987), equation (12) is not strictly speaking a variance measure because it does not demean returns before taking the expectation. However, for short holding periods, the impact of subtracting the means is trivial. Using daily data, French, Schwert, and Stambaugh (1987) and Schwert (1989) find the

We first generate the monthly realized variance based on equation (12) and then estimate the following risk-return regressions:

$$\begin{aligned} R_{m,t+1} &= \alpha + \beta \cdot \sigma_{m,t}^2 + \varepsilon_{m,t+1} \\ R_{m,t+1} &= \alpha + \beta \cdot \sigma_{m,t} + \varepsilon_{m,t+1} \\ R_{m,t+1} &= \alpha + \beta \cdot \ln \sigma_{m,t}^2 + \varepsilon_{m,t+1} \end{aligned} \tag{13}$$

where $R_{m,t+1}$ is the one-month ahead excess return on the value-weighted NYSE/AMEX/NASDAQ index and the expected conditional variance of the market portfolio, $E_t(\sigma_{m,t+1}^2)$, is proxied by the lagged realized variance measure, i.e., $E_t(\sigma_{m,t+1}^2) = \sigma_{m,t}^2$. As shown in equation (13), we consider the lagged realized standard deviation ($\sigma_{m,t}$) and the lagged log realized variance ($\ln \sigma_{m,t}^2$) in the risk-return regressions as well.

Table 2 presents the parameter estimates and their OLS and Newey-West (1987) adjusted *t*-statistics from the risk-return regressions with monthly realized volatility. The relative risk aversion parameter (β) on $\sigma_{m,t}^2$ is estimated to be positive, 0.1220, but statistically insignificant with the Newey-West t-statistic of 0.12. Similar results are obtained when the realized standard deviation and log-variance are used to test the significance of the intertemporal relation. Specifically, the slope coefficient on $\sigma_{m,t}$ is estimated to be 0.0397 with t-stat.= 0.28 and the slope on $\ln \sigma_{m,t}^2$ is 0.0009 with t-stat.= 0.32. These results indicate that the monthly realized volatility cannot predict future returns on the market portfolio, implying an insignificant intertemporal relation between expected return and risk.

4.3. Range Volatility Model

It is not common to use a range volatility estimator when testing the significance of a risk-return tradeoff. Following Alizadeh, Brandt, and Diebold (2002) and Brandt and Diebold (2006), monthly range volatility is defined as the difference between the logarithms of the highest and lowest index levels.

$$\sigma_{m,t}^{range} = \ln(P_{d,t}^{\max}) - \ln(P_{d,t}^{\min}), \tag{14}$$

where $P_{d,t}^{\max}$ and $P_{d,t}^{\min}$ are the maximum and minimum daily index levels of the NYSE/AMEX/NASDAQ on day *d* in month *t*.[4]

We first generate the monthly range variance, standard deviation, and log-variance based on equation (14) and then estimate the risk-return regressions given in equation (13). Table 3 presents the parameter estimates and their OLS and Newey-West corrected *t*-statistics from the risk-return regressions with monthly

squared mean term is irrelevant to variance calculations. At an earlier stage of the study, we used the demeaned daily returns in equation (12) and the qualititative results remained intact. We also eliminated the second term on the right of equation (12) and the qualitative results turned out to be very similar to those reported in our tables.

[4] Alizadeh et al. (2002) and Brandt and Diebold (2006) indicate that the range-based volatility estimator is highly efficient, approximately Gaussian and robust to certain types of microstructure noise such as bid-ask bounce.

range volatility. The relative risk aversion parameter (β) on $\sigma_{m,t}^2$ is estimated to be positive, 0.1529, but statistically insignificant with the Newey-West t-statistic of 0.46. Similar results are obtained when the range standard deviation and log-variance are used to test the significance of the intertemporal relation. Specifically, the slope coefficient on $\sigma_{m,t}$ is estimated to be 0.0414 with t-stat. = 0.92 and the slope on $\ln \sigma_{m,t}^2$ is 0.0010 with t-stat.= 0.42. These results indicate that the monthly range volatility cannot predict the time-series variation in excess market returns, implying an insignificant link between risk and return.

To make sure that our results from estimating equation (7) are not due to model misspecification, we added to the regressions a set of control variables that have been used in the literature to capture changes in the investment opportunity set. As discussed in Appendix B, including a wide variety of macroeconomic variables does not affect the insignificant relation between risk and return on the aggregate market portfolio. Among the control variables, we find a significantly negative (positive) relation between the excess market return and the inflation rate (the aggregate dividend yield), whereas the default spread, term spread, relative T-bill rate, and output growth cannot predict future returns on the market portfolio.

5. A cross-sectional investigation of the conditional ICAPM

Consistent with the existing literature, when we use a single series of the market portfolio return, we find no evidence for a significant link between risk and return in the aggregate stock market. Different from the traditional literature, in this section, we first divide the aggregate market portfolio into ten book-to-market portfolios and then estimate the cross-sectionally consistent slope coefficient on the conditional variance-covariance matrix. Second, we investigate the cross-sectional consistency of the intertemporal relation by testing the equality of slope coefficients in the multivariate GARCH-in-mean model. Finally, we test whether the value premium can be explained within the conditional ICAPM framework.[5]

5.1. Investigating ICAPM with Constant Conditional Correlations

We estimate the risk aversion coefficient (β) based on the following multivariate GARCH-in-mean model with constant conditional correlations (CCC):

$$R_{i,t+1} = \alpha_i + \beta \cdot \sigma_{im,t+1} + \varepsilon_{i,t+1} \tag{15}$$

$$R_{m,t+1} = \alpha_m + \beta \cdot \sigma_{m,t+1}^2 + \varepsilon_{m,t+1} \tag{16}$$

$$E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \gamma_0^i + \gamma_1^i \varepsilon_{i,t}^2 + \gamma_2^i \sigma_{i,t}^2 \tag{17}$$

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[5] In addition to the book-to-market portfolios, at an earlier stage of the study we used the ten value-weighted momentum and industry portfolios which are available at Kenneth French's online data library. We also used the ten equal-weighted size portfolios from the CRSP database. The qualitative results from the size, momentum and industry portfolios turn out to be similar to our findings reported in the paper. They are available upon request.

$$E_t\left[\varepsilon_{m,t+1}^2\right] \equiv \sigma_{m,t+1}^2 = \gamma_0^m + \gamma_1^m \varepsilon_{m,t}^2 + \gamma_2^m \sigma_{m,t}^2 \quad (18)$$

$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1} \quad (19)$$

where $R_{i,t+1}$ and $R_{m,t+1}$ denote the time (t+1) excess return on portfolio i and the market portfolio m over a risk-free rate, respectively, and $E_t[.]$ denotes the expectation operator conditional on time t information. $\sigma_{i,t+1}^2$ is the time-t expected conditional variance of $R_{i,t+1}$, $\sigma_{m,t+1}^2$ is the time-t expected conditional variance of $R_{m,t+1}$, and $\sigma_{im,t+1}$ is the time-t expected conditional covariance between $R_{i,t+1}$ and $R_{m,t+1}$. ρ_{im} is the constant conditional correlation between between $R_{i,t+1}$ and $R_{m,t+1}$. In the conditional mean given by equations (15)-(16), we have a common slope coefficient (β) but different intercepts for each book-to-market portfolio (α_i) and the stock market portfolio (α_m). The conditional variance parameters are assumed to be different for each stock $(\gamma_0^i, \gamma_1^i, \gamma_2^i)$ and the market portfolio $(\gamma_0^m, \gamma_1^m, \gamma_2^m)$.

We use the maximum likelihood methodology with the multivariate normal density to estimate equations (15)-(19) in one step. Using ε_{t+1} and Σ_{t+1} to denote, respectively, the multivariate residual vector and the conditional variance-covariance matrix,

$$\varepsilon_{t+1} = \begin{bmatrix} R_{i,t+1} - \alpha_i - \beta \cdot \sigma_{im,t+1} \\ R_{m,t+1} - \alpha_m - \beta \cdot \sigma_{m,t+1}^2 \end{bmatrix}, \qquad \Sigma_{t+1} = \begin{bmatrix} \sigma_{i,t+1}^2 & \sigma_{im,t+1} \\ \sigma_{im,t+1} & \sigma_{m,t+1}^2 \end{bmatrix},$$

we can write the conditional log-likelihood function as

$$L(\Theta) = -\frac{1}{2}\sum_{t=1}^{N}\left[\ln(2\pi) + \ln\left|\Sigma_{t+1}\right| + \varepsilon_{t+1}^T \Sigma_{t+1}^{-1} \varepsilon_{t+1}\right],$$

where Θ denotes the vector of parameters in the specifications in (15) to (19), and N denotes the number of monthly observations for each series. Since we have a total of 11 portfolios (including the market portfolio), ε_{t+1} is 11×1 and Σ_{t+1} is 11×11 matrix.

The parameters are estimated using the 10 value-weighted book-to-market portfolios and the value-weighted NYSE/AMEX/NASDAQ index as a market portfolio. Since we have a total of 11 portfolios, we estimate a total of 100 parameters simultaneously. Specifically, we have 11 conditional means with a total of 12 parameters (11 intercepts and a common slope), we have 11 conditional variances with a total of 33 parameters, and we have a total of 55 constant conditional correlations.

Table 4 reports the common slope estimate (β), the abnormal returns (α_i, α_m) for each portfolio, the t-statistics of the parameter estimates, and the maximized log-likelihood value. Estimation is based on the monthly excess returns over the sample period of July 1926 to December 2007. The risk aversion coefficient is estimated to be positive ($\beta = 4.1733$), and highly significant with the *t*-statistic of 4.69. This implies a positive and significant intertemporal relation between expected return and risk on equity portfolios.

When estimating the multivariate GARCH-in-mean model in equations (15)-(19), we allow the intercepts (α_i, α_m) to be different across portfolios. These intercepts capture the monthly abnormal returns on each portfolio that cannot be explained by the conditional covariances with the market portfolio. The first column of Table 4 shows that the abnormal return on the growth portfolio (with the lowest book-to-market ratio) is $\alpha_1 = -0.15\%$ per month with the t-statistic of –0.72, whereas the abnormal return on the value portfolio (with the highest book-to-market ratio) is $\alpha_{10} = 0.18\%$ per month with the t-statistic of 0.81.

One implication of the ICAPM is that the intercepts (α_i, α_m) should not be jointly different from zero assuming that the covariances of risky assets with the market portfolio have enough predictive power for expected future returns. To examine the empirical validity of ICAPM, we test the joint hypothesis that $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_{10} = \alpha_m = 0$. As presented in Table 4, the Wald statistic is 15.84 with a *p*-value of 14.71%, which fails to reject the null hypothesis that all intercepts equal zero. This result indicates that the conditional covariances of the book-to-market portfolios with the aggregate market portfolio have significant predictive power for the time-series and cross-sectional variations in expected returns.

According to the original ICAPM of Merton (1973), the relative risk aversion coefficient β is restricted to be the same across all risky assets and it should be positive and statistically significant. The common slope estimate in Table 4 provides empirical support for the positive risk-return tradeoff.

We now test whether the slopes on (σ_{im}, σ_m^2) are different across risky assets. Earlier studies assume a common slope coefficient (β) following the original theoretical work of Merton (1973) and do not question the validity of this assumption. In this paper, we examine the sign and statistical significance of different slope coefficients (β_i, β_m) on (σ_{im}, σ_m^2) in the following multivariate GARCH-in-mean model with constant conditional correlations:

$$R_{i,t+1} = \alpha_i + \beta_i \cdot \sigma_{im,t+1} + \varepsilon_{i,t+1}, \tag{20}$$

$$R_{m,t+1} = \alpha_m + \beta_m \cdot \sigma^2_{m,t+1} + \varepsilon_{m,t+1}, \tag{21}$$

$$E_t\left[\varepsilon^2_{i,t+1}\right] \equiv \sigma^2_{i,t+1} = \gamma_0^i + \gamma_1^i \varepsilon^2_{i,t} + \gamma_2^i \sigma^2_{i,t}, \tag{22}$$

$$E_t\left[\varepsilon^2_{m,t+1}\right] \equiv \sigma^2_{m,t+1} = \gamma_0^m + \gamma_1^m \varepsilon^2_{m,t} + \gamma_2^m \sigma^2_{m,t}, \tag{23}$$

$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1}. \tag{24}$$

To determine whether there is a common slope coefficient (β) on σ_{im} corresponding to the average relative risk aversion, we first estimate the portfolio-specific slope coefficients (β_i, β_m) and then test the joint hypothesis that $H_0 : \beta_1 = \beta_2 = ... = \beta_{10} = \beta_m$.

Table 5 presents the maximum likelihood parameter estimates based on the 10 value-weighted book-to-market portfolios and the value-weighted NYSE/AMEX/NASDAQ index. Compared to equations (15)-(19), we have an additional 10 slope coefficients to estimate in equations (20)-(24), yielding a total of 110 parameters. As shown in Table 5, all of the slope coefficients (β_i, β_m) are estimated to be positive and highly significant without any exception. Specifically, the minimum slope is about 3.57 with t-stat. = 2.81, and the maximum slope is about 4.72 with t-stat. = 3.97. We should note that the average of these 11 slope coefficients is about 4.13, which is close to the common slope estimate of 4.17 reported in Table 4. These results indicate a positive and significant intertemporal relation between risk and return on equity portfolios.

We examine the cross-sectional consistency of the intertemporal relation by testing the equality of slope coefficients based on the likelihood ratio (LR) test statistic. As shown in Tables 4 and 5, the maximized log-likelihood value of the restricted model (with a common slope coefficient) is 27,798.18, whereas the maximized log-likelihood value of the unrestricted model (with different slope coefficients) is 27,803.62. These maximized log-likelihood values yield the LR test statistic of 10.88 with a *p*-value of 45.34%, which cannot reject the joint hypothesis that $H_0 : \beta_1 = \beta_2 = ... = \beta_{10} = \beta_m$.[6]

When estimating the multivariate GARCH-in-mean model in equations (20)-(24), we allow the intercepts (α_i, α_m) to be different across portfolios. The first column of Table 5 shows that the abnormal return on the growth portfolio is $\alpha_1 = -0.02\%$ per month with the t-statistic of –0.07, whereas the abnormal return on the value portfolio is $\alpha_{10} = 0.15\%$ per month with the t-statistic of 0.81. We test whether there is a significant average risk-adjusted return difference between the value and growth portfolios. Specifically, we test the null hypothesis that $H_0 : \alpha_1 = \alpha_{10}$. The Wald statistic reported in Table 5 is about 0.20 with a *p*-value of 65.54%, indicating insignificant value premium over the sample period 1926-2007.

As shown in Appendix A, the average excess return is 0.57% per month for the growth portfolio and 1.09% per month for the value portfolio. The average *raw return difference* between the value and growth portfolios is 0.52% per month with the Newey-West t-statistic of 2.39, implying that value stocks on average generate higher raw returns than growth stocks. However, after controlling for the conditional covariance risk, the average *risk-adjusted return difference* between the growth and value portfolios reduces to 0.17%

[6] LR statistic is calculated as *LR* = –2(LogL* – LogL), where LogL* is the value of the log likelihood under the null hypothesis, and LogL is the log likelihood under the alternative: *LR* = –2(27,798.18 – 27,803.62) = 10.88. The critical Chi-squared values with 11 degrees of freedom are $\chi^2_{(11,0.10)} = 17.28$, $\chi^2_{(11,0.05)} = 19.68$, and $\chi^2_{(11,0.01)} = 24.72$ at the 10%, 5% and 1% level of significance, respectively.

per month and it is statistically insignificant. In other words, the conditional ICAPM explains the value premium for the sample period of July 1926–December 2007.[7]

To further examine the empirical validity and sufficiency of the conditional ICAPM, we test whether the abnormal returns on equity portfolios are jointly equal to zero. As shown in Table 5, the Wald statistic is 7.22 with a p-value of 78.14%, which fails to reject the joint hypothesis that $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_{10} = \alpha_m = 0$. This result implies that the conditional covariances of the book-to-market portfolios with the aggregate market portfolio can capture the time-series and cross-sectional variations in expected returns.

5.2. Investigating ICAPM with Dynamic Conditional Correlations

In this section, we estimate the risk aversion coefficient (β) based on the multivariate GARCH-in-mean model with the mean-reverting dynamic conditional correlations (DCC) of Engle (2002):

$$R_{i,t+1} = \alpha_i + \beta \cdot \sigma_{im,t+1} + \sigma_{i,t+1} u_{i,t+1} \tag{25}$$

$$R_{m,t+1} = \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \sigma_{m,t+1} u_{m,t+1} \tag{26}$$

$$E_t\left[\varepsilon^2_{i,t+1}\right] \equiv \sigma^2_{i,t+1} = \gamma^i_0 + \gamma^i_1 \sigma^2_{i,t} u^2_{i,t} + \gamma^i_2 \sigma^2_{i,t} \tag{27}$$

$$E_t\left[\varepsilon^2_{m,t+1}\right] \equiv \sigma^2_{m,t+1} = \gamma^m_0 + \gamma^m_1 \sigma^2_{m,t} u^2_{m,t} + \gamma^m_2 \sigma^2_{m,t} \tag{28}$$

$$E_t\left[\varepsilon_{i,t+1} \varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1} \tag{29}$$

$$\rho_{im,t+1} = \frac{q_{im,t+1}}{\sqrt{q_{ii,t+1} \cdot q_{mm,t+1}}}, \quad q_{im,t+1} = \overline{\rho}_{im} + a_1 \cdot \left(u_{i,t} \cdot u_{m,t} - \overline{\rho}_{im}\right) + a_2 \cdot \left(q_{im,t} - \overline{\rho}_{im}\right) \tag{30}$$

where $\overline{\rho}_{im}$ is the unconditional correlation between $u_{i,t}$ and $u_{m,t}$. In the conditional mean given by eqs. (25)-(26), we have a common slope coefficient (β) but different intercepts for book-to-market portfolios (α_i) and the stock market portfolio (α_m). The conditional variance parameters are assumed to be different for each stock $(\gamma^i_0, \gamma^i_1, \gamma^i_2)$ and the market portfolio $(\gamma^m_0, \gamma^m_1, \gamma^m_2)$. Following Engle (2002), we assume that the correlation parameters (a_1, a_2) in the mean-reverting DCC model are constant across portfolios.

To ease the parameter convergence, we use correlation targeting assuming that the time-varying correlations mean revert to the sample correlations $\overline{\rho}_{im}$. To reduce the overall time of maximizing the conditional log-likelihood, following Engle (2009), we first estimate all pairs of the bivariate GARCH-in-mean model and then use the median values of β, a_1 and a_2 as starting values along with the bivariate GARCH-in-mean estimates of the variance parameters (γ_0, γ_1, γ_2). Even after going through these steps to

[7] This result is consistent with Ang and Chen (2007) who estimated a conditional CAPM specification with stochastic market beta.

increase the speed of parameter convergence, it takes long time to obtain the full set of parameters in the multivariate GARCH-in-mean model with 11 portfolios and 978 time-series observations. The common slope and the intercepts are estimated using the monthly excess returns on the 10 value-weighted book-to-market portfolios for the sample period from July 1926 to December 2007. The aggregate market portfolio is proxied by the value-weighted NYSE/AMEX/NASDAQ index.

Table 6 shows that the risk-return coefficient (β) on the conditional variance-covariance matrix is estimated to be about 5.12 with the *t*-statistic of 6.47. The magnitude and statistical significance of the common slope turns out to be similar to our earlier findings from the constant conditional correlations. We test the joint hypothesis that all intercepts equal zero and the Wald statistic is found to be 16.19 with a *p*-value of 13.43%. Overall, the results in Table 6 provide similar evidence that there is a significantly positive relation between risk and return on equity portfolios and the abnormal returns are individually and jointly equal zero, suggesting validity of the conditional ICAPM. Put differently, the DCC-based conditional covariances can explain the time-series and cross-sectional variations in portfolio returns.

We now test whether the slopes on the conditional variance-covariance matrix are different across stock portfolios. Specifically, we examine the sign and statistical significance of different slope coefficients (β_i, β_m) on (σ_{im}, σ_m^2) in the multivariate GARCH-in-mean model with DCC:

$$R_{i,t+1} = \alpha_i + \beta_i \cdot \sigma_{im,t+1} + \sigma_{i,t+1} u_{i,t+1} \tag{31}$$

$$R_{m,t+1} = \alpha_m + \beta_m \cdot \sigma_{m,t+1}^2 + \sigma_{m,t+1} u_{m,t+1} \tag{32}$$

$$E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \gamma_0^i + \gamma_1^i \sigma_{i,t}^2 u_{i,t}^2 + \gamma_2^i \sigma_{i,t}^2 \tag{33}$$

$$E_t\left[\varepsilon_{m,t+1}^2\right] \equiv \sigma_{m,t+1}^2 = \gamma_0^m + \gamma_1^m \sigma_{m,t}^2 u_{m,t}^2 + \gamma_2^m \sigma_{m,t}^2 \tag{34}$$

$$E_t\left[\varepsilon_{i,t+1} \varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1} \tag{35}$$

$$\rho_{im,t+1} = \frac{q_{im,t+1}}{\sqrt{q_{ii,t+1} \cdot q_{mm,t+1}}}, \quad q_{im,t+1} = \bar{\rho}_{im} + a_1 \cdot \left(u_{i,t} \cdot u_{m,t} - \bar{\rho}_{im}\right) + a_2 \cdot \left(q_{im,t} - \bar{\rho}_{im}\right) \tag{36}$$

Table 7 shows that all of the slope coefficients (β_i, β_m) are estimated to be positive and highly significant without any exception. Specifically, the minimum slope is about 4.22 with t-stat.= 3.18, and the maximum slope is about 5.63 with t-stat.= 5.27. We should note that the average of these 11 slope coefficients is about 4.98, which is close to the common slope estimate of 5.12 reported in Table 6. These results show a positive and significant intertemporal relation between risk and return on equity portfolios.

We investigate the cross-sectional consistency of the intertemporal relation by testing the equality of slope coefficients based on the likelihood ratio (LR) test statistic. As shown in Tables 6 and 7, the maximized log-likelihood value of the restricted model (with a common slope coefficient) is 27,874.77, whereas the maximized log-likelihood value of the unrestricted model (with different slopes) is 27,881.38. These maximized log-likelihood values yield the LR statistic of 13.22 with p-value = 27.92%, which cannot reject the joint hypothesis $H_0 : \beta_1 = \beta_2 = ... = \beta_{10} = \beta_m$.

The first column of Table 7 shows that the abnormal return on the growth portfolio is $\alpha_1 = -0.05\%$ per month with t-stat.= –0.20, whereas the abnormal return on the value portfolio is $\alpha_{10} = 0.01\%$ per month with t-stat.= 0.04. We test whether there is a significant average risk-adjusted return difference between the value and growth portfolios. The Wald statistic from testing the null hypothesis $H_0 : \alpha_1 = \alpha_{10}$ is about 0.04 with p-value = 83.36%, indicating insignificant value premium over the sample period of 1926-2007. After controlling for the market risk with the DDC-based time-varying conditional covariances, the average risk-adjusted return difference between the growth and value portfolios becomes only 6 basis points per month and statistically insignificant. To further examine the empirical validity of the conditional ICAPM, we test whether the abnormal returns on equity portfolios are jointly equal to zero. As shown in Table 7, the Wald statistic is 9.46 with p-value = 57.98%, which fails to reject the joint hypothesis of zero intercepts. These results indicate that the conditional ICAPM with the dynamic conditional correlations has significant predictive power for the time-series and cross-sectional variations in expected returns and explains the value premium over the long sample period 1926- 2007.[8]

6. A cross-sectional investigation of the unconditional ICAPM

It is well documented in the literature that narrowly focusing on a single series of the market portfolio, it is difficult to find a significant intertemporal relation between risk and return in the aggregate stock market. Our results from the GARCH-in-mean, realized, and range volatility models are aligned with the existing literature. However, by pooling the time series and cross section together, we find that the DCC- and CCC-based time-varying conditional covariances generate a significantly positive intertemporal relation between expected return and risk on book-to-market portfolios. The significant, robust and sensible estimates highlight the added benefits of using the GARCH-based conditional measures of market risk and by estimating the conditional ICAPM with a pooled panel of time series and cross sectional data.[9]

We investigate whether the power of our methodology is coming from (1) the GARCH-based time-varying conditional covariances, or (2) pooling the time series and cross section together, or (3) both. In this section, we provide a cross-sectional investigation of the *unconditional* ICAPM. Specifically, we examine whether the *unconditional* measures of market risk can predict expected future returns on equity portfolios.

[8] Appendix C presents descriptive statistics of the DCC- and CCC-based conditional covariance estimates which are key to our cross-sectional investigation of the conditional ICAPM.

[9] Although some of our findings are not presented in the paper to save space, the results are robust across different equity portfolios (book-to-market, size, momentum, industry), different specifications of the conditional covariance process, and including a large set of control variables to the panel of multivariate GARCH-in-mean model.

We have already shown that the GARCH-in-mean model cannot generate a positive and significant relation between risk and return on the aggregate market portfolio. In other words, the GARCH-based methodology itself cannot resolve the issue if one narrowly focuses on a single series of the market portfolio return.

We now take a closer look at the effect of using a pooled panel of time series and cross sectional data on the identification of risk-return tradeoff. Earlier studies starting with Fama and French (1992) and French, Schwert, and Stambaugh (1987) show that the unconditional beta (or the unconditional covariance of risky assets with the market portfolio) cannot predict the cross-sectional variation in stock returns and that the intertemporal risk-return relation is not significantly positive in time-series regressions.[10] Based on the rolling regressions, Fama and French (1992) first estimate the unconditional market beta using the past 24 to 60 months of returns (as available) on individual stocks trading at the NYSE, AMEX, and NASDAQ. Then, they show that there is no significant relation between the unconditional measures of market risk and the cross-section of expected returns.

In this paper, following Fama and French (1992), we use the monthly rolling regressions and estimate the unconditional variance of excess returns on the market portfolio as well as the unconditional covariances between excess returns on the book-to-market portfolios and the market portfolio using the past 24, 36, 48, and 60 months of data. Given the unconditional covariances and the unconditional variance of the market portfolio for each month in our sample, we estimate the intertemporal relation from the following system of equations:

$$R_{i,t+1} = \alpha_i + \beta \cdot \sigma_{im,t} + \varepsilon_{i,t+1}, \tag{37}$$

$$R_{m,t+1} = \alpha_m + \beta \cdot \sigma_{m,t}^2 + \varepsilon_{m,t+1}, \tag{38}$$

where the expected conditional variance of the market portfolio, $E_t(\sigma_{m,t+1}^2)$, is proxied by the one-month lagged realized variance, i.e., $E_t(\sigma_{m,t+1}^2) = \sigma_{m,t}^2$, and similarly the expected conditional covariance of book-to-market portfolios with the market portfolio, $E_t(\sigma_{im,t+1})$, is proxied by the one-month lagged realized covariance, i.e., $E_t(\sigma_{im,t+1}) = \sigma_{im,t}$.

We estimate the system of equations (37)-(38) using a weighted least square method that allows us to place constraints on coefficients across equations. We constrain the slope coefficient (β) on the

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[10] Empirical studies of the ICAPM diverge into two perpendicular dimensions. Studies that focus on the intertemporal risk-return relation often choose to use merely one return series on the market portfolio, ignoring the model's implication that the same relation between excess returns and their conditional covariances with the market should hold across all stock portfolios to guarantee cross-sectional consistency. However, some of the earlier studies on conditional CAPM (e.g., Jagannathan and Wang (1996)) deal with an unconditional implication of the conditional relation by regressing excess returns on the unconditional beta and an unconditional covariance term that accounts for the covariation between the conditional beta and the conditional market risk premium. What this exercise ignores is that a more direct test of the conditional relation is to estimate the conditional relation itself, instead of dealing with the unconditional implication of the conditional relation. The narrow focus of both strands of the literature often leads to insignificant estimates on market beta or market variance.

unconditional variance-covariance matrix to be the same across all portfolios for cross-sectional consistency. We allow the intercepts (α_i, α_m) to differ across portfolios. Under the null hypothesis of the unconditional ICAPM, the intercepts should be jointly zero and the common slope coefficient (β) should be positive and statistically significant. We use insignificant estimates of β and the deviations of the intercept estimates from zero as a test against the validity and sufficiency of the unconditional ICAPM specification. We compute the *t*-statistics of the parameter estimates accounting for heteroskedasticity and autocorrelation as well as contemporaneous cross-correlations in the error terms. The estimation methodology for the system of equations (37)-(38) can be regarded as an extension of the seemingly unrelated regression (SUR) method.

Table 8 presents the SUR panel regression estimates of the portfolio-specific intercepts, common slope coefficients on the unconditional variance-covariance matrix, and their *t*-statistics (in parentheses). The parameters and their *t*-statistics are estimated using the excess returns on the market portfolio and the 10 book-to-market portfolios.[11] The last row presents the Wald statistics and their *p*-values in square brackets from testing the joint hypothesis that all intercepts equal zero: $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_{10} = \alpha_m = 0$. A notable point in Table 8 is that the common slope coefficient (β) is found to be positive, but statistically insignificant for all measures of unconditional market risk. Specifically, the risk aversion coefficient on the unconditional variance-covariance matrix is estimated to be in the range of 0.17 to 0.50 with the t-statistics ranging from 0.43 to 1.42. Another notable point in Table 8 is that all intercepts are positive and highly significant without any exception. The Wald statistics reject the joint hypothesis that all intercepts equal zero.

These results indicate that the unconditional measures of market risk cannot explain the time-series and cross-sectional variations in expected returns, implying insufficiency of the unconditional ICAPM. Put differently, pooling the time series and cross section together without the time-varying conditional covariances cannot help identify a significant risk-return tradeoff. Estimating the conditional ICAPM with a pooled panel of time series and cross sectional data in a multivariate GARCH-in-mean framework is essential in identifying the positive risk-return tradeoff.

7. Risk-return tradeoff with intertemporal hedging demand

In this section, we investigate the intertemporal relation between risk and return after taking into account the intertemporal hedging demand. Specifically, we test the significance of risk premia induced by the conditional covariation of book-to-market portfolios with the innovations in economic factors. We also

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[11] Although the original sample period is from July 1926 to December 2007, when $\sigma^2_{m,t}$ and $\sigma_{im,t}$ are estimated using the monthly rolling regressions with a fixed window of 24 months, the panel regression is run for the sample period of July 1928-December 2007 because the first 24 observations are lost for the estimation of realized variance-covariance matrix. Similarly, when $\sigma^2_{m,t}$ and $\sigma_{im,t}$ are estimated using a fixed rolling window of 60 months, the panel regression is run for the sample period of July 1931-December 2007.

examine the significance of market covariance risk after controlling for risk premia induced by the conditional covariation with the unexpected news in macroeconomic variables.

Financial economists often choose certain macroeconomic variables to control for stochastic shifts in the investment opportunity set. To investigate how these macroeconomic variables vary with investment opportunities and whether covariations of equity portfolios with them induce additional risk premia, we estimate the following GARCH-in-mean model with the mean-reverting DCC model and then analyze how the portfolios' excess returns respond to their conditional covariance with these economic factors:

$$\begin{aligned} R_{i,t+1} &= \alpha_i + \beta \cdot \sigma_{im,t+1} + \lambda \cdot \sigma_{ix,t+1} + \varepsilon_{i,t+1} \\ R_{m,t+1} &= \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \lambda \cdot \sigma_{mx,t+1} + \varepsilon_{m,t+1} \\ E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] &\equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1} \\ E_t\left[\varepsilon_{i,t+1}\varepsilon_{x,t+1}\right] &\equiv \sigma_{ix,t+1} = \rho_{ix,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{x,t+1} \\ E_t\left[\varepsilon_{m,t+1}\varepsilon_{x,t+1}\right] &\equiv \sigma_{mx,t+1} = \rho_{mx,t+1} \cdot \sigma_{m,t+1} \cdot \sigma_{x,t+1} \end{aligned} \tag{39}$$

where the parameters and their *t*-statistics are estimated using the excess returns on the market portfolio and the 10 value-weighted book-to-market portfolios for the sample period of July 1926–December 2007. $\sigma_{ix,t+1}$ measures the time-*t* expected conditional covariance between the excess returns on each portfolio $(R_{i,t+1})$ and the innovations in a macroeconomic variable proxied by the first difference $(\Delta X_{t+1} = X_{t+1} - X_t)$, and $\sigma_{mx,t+1}$ measures the time-*t* expected conditional covariance between the excess returns on the market portfolio $(R_{m,t+1})$ and the innovations in a macroeconomic variable (ΔX_{t+1}). Although not presented in eq. (39) to save space, the conditional variances are estimated based on the GARCH(1,1) specification, and the time-varying conditional correlations are estimated based on the mean-reverting DCC model of Engle (2002).

Table 9 reports the common slope estimates ($\beta, \lambda_1, \lambda_2, \lambda_3, \lambda_4$) and their *t*-statistics from the multivariate GARCH-in-mean model with the GARCH(1,1) conditional variances and the dynamic conditional correlations:

$$\begin{aligned} R_{i,t+1} &= \alpha_i + \beta \cdot \sigma_{im,t+1} + \lambda_1 \cdot \sigma_{i,DEF,t+1} + \lambda_2 \cdot \sigma_{i,TERM,t+1} + \lambda_3 \cdot \sigma_{i,RREL,t+1} + \lambda_4 \cdot \sigma_{i,DIV,t+1} + \varepsilon_{i,t+1} \\ R_{m,t+1} &= \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \lambda_1 \cdot \sigma_{i,DEF,t+1} + \lambda_2 \cdot \sigma_{i,TERM,t+1} + \lambda_3 \cdot \sigma_{i,RREL,t+1} + \lambda_4 \cdot \sigma_{i,DIV,t+1} + \varepsilon_{m,t+1} \end{aligned}$$

where $\sigma_{i,DEF,t+1}$, $\sigma_{i,TERM,t+1}$, $\sigma_{i,RREL,t+1}$, and $\sigma_{i,DIV,t+1}$ measure, respectively, the time-*t* expected conditional covariance between the excess returns on each portfolio *i* and the change in default spread (Δ*DEF*), the change in term spread (Δ*TERM*), the change in short-term interest rate (*RREL*), and the change in aggregate dividend yield (Δ*DIV*).

The parameter estimates in Panel A of Table 9 reveal several important results. The slope coefficients on $\sigma_{i,DEF,t+1}$, $\sigma_{i,TERM,t+1}$, and $\sigma_{i,RREL,t+1}$ $(\lambda_1, \lambda_2, \lambda_3)$ are all negative, but statistically insignificant, implying that the innovations in default spread, term spread, and short-term interest rate are not priced in the stock market. Incorporating the covariance of portfolio returns with any of these macroeconomic variables does not alter the magnitude and statistical significance of the risk aversion estimates. In all cases, the common slope coefficient (β) on $\sigma_{im,t+1}$ is positive, in the range of 5.32 and 6.07, and highly significant with the *t*-statistics between 4.90 and 5.39.

The other state variable considered in the paper is the aggregate dividend yield that moves positively with optimal consumption. As shown in Panel A of Table 9, the slope coefficient on $\sigma_{i,DIV,t+1}$ (λ_4) is positive and statistically significant with t-stat. = 2.23. The positive coefficient estimate, $\lambda_4 = 0.93$, on the covariance of portfolio returns with the dividend-related shocks indicates that an increase in a portfolio's covariance with the unexpected dividend yield predicts a higher excess return on the portfolio.

In addition to the commonly used macroeconomic variables (*DEF*, *TERM*, *RREL*, *DIV*), we investigate whether the innovations in fundamental economic factors (*INF*, *OUT*) are priced in the conditional ICAPM framework. Panel B of Table 9 presents the common slope estimates (β, λ_1, λ_2) and their *t*-statistics from the following GARCH-in-mean model with dynamic conditional correlations:

$$R_{i,t+1} = \alpha_i + \beta \cdot \sigma_{im,t+1} + \lambda_1 \cdot \sigma_{i,INF,t+1} + \lambda_2 \cdot \sigma_{i,OUT,t+1} + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \lambda_1 \cdot \sigma_{i,INF,t+1} + \lambda_2 \cdot \sigma_{i,OUT,t+1} + \varepsilon_{m,t+1}$$

where $\sigma_{i,INF,t+1}$ and $\sigma_{i,OUT,t+1}$ measure the time-*t* expected conditional covariance between the excess returns on each portfolio *i* and the change in the monthly inflation rate (ΔINF) and the change in the output growth (ΔOUT) proxied by the growth rate of industrial production.

As presented in Panel B of Table 9, the slope coefficient on the conditional covariances between the excess monthly returns and the inflation-related shocks are negative and statistically significant with t-stat. = –2.19. The slope estimate for the conditional covariances between the excess monthly returns and the output-related shocks is positive, but statistically insignificant. This result holds when the conditional covariances with both the inflation- and output-related shocks are used simultaneously in the multivariate GARCH-in-mean model. Similar to our earlier findings, incorporating the covariance of portfolio returns with ΔINF and ΔOUT does not change the magnitude and statistical significance of the risk aversion estimates. In all cases, the common slope coefficient (β) on $\sigma_{im,t+1}$ is positive, in the range of 5.53 and 6.40, and highly significant with the *t*-statistics between 3.12 and 5.30.

Our findings can be interpreted within the context of ICAPM. In Merton (1973)'s original setup, when the investment opportunity set is stochastic, investors adjust their investment to hedge against future shifts in the investment opportunity and achieve intertemporal consumption smoothing. If an asset return moves against the shifts in the investment opportunity, investors increase their investment in the asset for its positive role in intertemporal consumption smoothing. In equilibrium, investors are willing to accept a lower expected excess return on this asset for its intertemporal hedging function.

Note that inflation moves negatively with optimal consumption. Thus, the negative coefficient estimate on the conditional covariances of returns with inflation-related shocks indicates that an increase in a portfolio's covariance with unexpected inflation predicts a lower excess return on the portfolio. In the context of Merton (1973)'s ICAPM, this negative slope estimate suggests that an increase in unexpected inflation predicts a decrease in optimal consumption and hence an unfavorable shift in the investment opportunity set. This generates an increase in intertemporal hedging demand for the portfolio, which in equilibrium reduces the excess return on the portfolio, and hence a negative coefficient on the conditional covariance of returns with unexpected inflation.[12]

Note that the growth rate of industrial production moves positively with optimal consumption. Hence, the positive coefficient estimate on the covariance of returns with output-related shocks indicates that an increase in a portfolio's covariance with unexpected economic growth predicts a higher excess return on the portfolio. This positive slope estimate suggests that an increase in unexpected output growth predicts an increase in optimal consumption and hence a favorable shift in the investment opportunity set. However, as shown in Panel B of Table 9, the positive slope estimate for the conditional covariances of returns with the unexpected news in output growth is statistically insignificant with the t-statistics ranging from 1.43 to 1.54, implying that the growth rate of industrial production is not priced in the stock market.

Our results have so far indicated that the innovations in inflation rate and aggregate dividend yield are risks rewarded in the stock market, whereas the innovations in default spread, term spread, short-term interest rate, and output growth do not play a significant role in intertemporal hedging demand. To test whether the inflation and aggregate dividend yield can simultaneously play a significant role in optimal consumption smoothing, we consider their covariances jointly in the ICAPM specification. Specifically, we estimate the following GARCH-in-mean model with dynamic conditional correlations:

$$R_{i,t+1} = \alpha_i + \beta \cdot \sigma_{im,t+1} + \lambda_1 \cdot \sigma_{i,INF,t+1} + \lambda_2 \cdot \sigma_{i,DIV,t+1} + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \lambda_1 \cdot \sigma_{i,INF,t+1} + \lambda_2 \cdot \sigma_{i,DIV,t+1} + \varepsilon_{m,t+1}$$

[12] There are several channels by which inflation surprises may have effects on stock prices. A direct, negative effect could emerge if a positive surprise in announced inflation induces investors to raise their level of expected inflation. The explanation for this finding is that investors use inflation-swelled nominal interest rates to capitalize corporate earnings. Higher expected inflation leads to higher nominal interest rates. The anticipation of higher rates in the future causes investors to sell securities immediately, forcing interest rates upward. Higher interest rates then lead to lower stock prices, assuming investors view these assets as substitutes. A second channel by which inflation surprises may affect stock prices occur if investors believe that policymakers react to inflation news. Unexpectedly high inflation may lead to more restrictive policies, which in turn lead to reduced cash flows for firms and lower stock prices. Similarly, if a positive inflation surprise causes investors to revise upward their assessment of future money demand, higher interest rates and lower stock prices may result if investors further expect the Federal Reserve to maintain its previous monetary growth objectives. In any event, all of these potential links suggest that stock prices may be negatively related to surprises in announced measures of inflation.

Panel C of Table 9 demonstrates that the slope coefficient on the conditional covariances between the portfolio returns and the surprises in inflation is negative and significant with t-stat. = –1.98. However, the slope estimate for the conditional covariances between the portfolio returns and the innovations in aggregate dividend yield is positive but statistically insignificant with t-stat. = 1.28. Overall, we can conclude that the inflation-related shocks play a significant role in intertemporal hedging demand, whereas the intertemporal hedging function of the aggregate dividend yield is weaker. Panel C also shows that incorporating the covariance of portfolio returns with ΔINF and ΔDIV does not alter the magnitude and statistical significance of the risk-return coefficient. The common slope coefficient (β) on $\sigma_{im,t+1}$ remains positive, 6.02, and highly significant with t-stat.= 4.03.

8. Conclusion

A large number of studies examine the significance of an intertemporal relation between expected return and risk in the aggregate stock market. However, the existing literature has not yet reached a consensus on the presence of a positive risk-return tradeoff for stock market indices. For comparison, we follow the traditional literature and estimate the risk-return tradeoff using a single series of the value-weighted NYSE/AMEX/NASDAQ index. Consistent with earlier studies, the results from alternative specifications of the GARCH-in-mean, realized volatility, and range volatility models provide no evidence for a significant link between the conditional mean and volatility of excess returns on the market portfolio.

Different from the existing literature, we estimate the monthly intertemporal relation between risk and return using a cross section of book-to-market portfolios. By so doing, we not only guarantee the cross-sectional consistency of the estimated intertemporal relation, but also gain statistical power by pooling multiple time series together for a joint estimation with common slope coefficients. The average risk aversion of market investors is estimated to be positive, highly significant, and robust to variations in the conditional covariance process and including a large set of state variables proxying for the intertemporal hedging demand.

Following the original theoretical work of Merton (1973), we first restrict the relative risk aversion coefficient to be the same across all portfolios and the common slope estimate turns out to be positive and highly significant. Then, to test the cross-sectional consistency of the intertemporal relation, we estimate the multivariate GARCH-in-mean model with different slopes and the statistical results indicate the equality of slope coefficients on the conditional variance-covariance matrix. This result provides support for the empirical validity and sufficiency of the conditional ICAPM with a common slope coefficient.

One implication of the ICAPM is that the intercepts (or abnormal returns on each portfolio) should not be jointly different from zero assuming that the covariances of risky assets with the market portfolio have enough predictive power for expected future returns. To further examine the empirical sufficiency of the conditional ICAPM, we test the joint hypothesis that all intercepts equal zero. The Wald statistics fail to reject the null hypothesis, providing evidence that the time-varying conditional measures of market risk have significant predictive power for the time-series and cross-sectional variations in expected returns on book-to-market portfolios. The estimates on the abnormal returns also indicate that the conditional ICAPM can explain the return differences between the value and growth portfolios.

We estimate the risk-return tradeoff by accounting for the intertemporal hedging demand identified by the conditional covariation of portfolio returns with the innovations in a set of macroeconomic variables. The common slope coefficients on the conditional covariances with the unexpected news in the inflation rate and the aggregate dividend yield are found to be statistically significant, implying that the inflation-related and dividend-related shocks contain systematic risks rewarded in the stock market and they can be viewed as a proxy for investment opportunities. However, the innovations in default spread, term spread, short-term interest rate, and output growth do not play a significant role in intertemporal hedging demand. Incorporating the conditional covariation with any of these state variables does not change the positive risk premium induced by the conditional covariation of portfolio returns with the market portfolio.

The negative (positive) slope estimate suggests that an increase in inflation (dividend yield) predicts a decrease (increase) in optimal consumption and hence an unfavorable (favorable) shift in the investment opportunity set. Intertemporally, an increase in the covariance of returns with unexpected inflation (unexpected dividend growth) leads to an increase (decrease) in the hedging demand, which in equilibrium decreases (increases) the excess return on the portfolio, and hence a negative (positive) slope estimate for the conditional covariance of returns with the innovations in inflation (dividend yield). The importance of a negative (positive) and significant inflation-return (dividend-return) relations is that it may indicate hedging opportunities for investors and that the changes in inflation and dividend can be viewed as a priced factor.

Appendix A. Descriptive Statistics

Panel A. Monthly Excess Returns on the Value-Weighted Book-to-Market Portfolios

This table presents summary statistics for the monthly excess returns on the 10 value-weighted book-to-market portfolios. "Growth" is the portfolio of growth stocks with the lowest book-to-market ratios and "Value" is the portfolio of value stocks with the highest book-to-market ratios. Mean, median, maximum, minimum, standard deviation, and market beta are reported for each portfolio. The sample period is from July 1926 to December 2007 (978 monthly observations).

Portfolio	Mean	Median	Maximum	Minimum	Std. Dev.	Beta
Growth	0.0057	0.0075	0.3867	–0.2915	0.0575	1.0063
BM 2	0.0066	0.0097	0.3479	–0.2686	0.0551	0.9794
BM 3	0.0067	0.0080	0.3120	–0.2746	0.0537	0.9477
BM 4	0.0066	0.0097	0.5703	–0.2440	0.0605	1.0602
BM 5	0.0073	0.0091	0.4626	–0.2909	0.0564	0.9768
BM 6	0.0078	0.0098	0.5832	–0.3419	0.0617	1.0641
BM 7	0.0080	0.0095	0.6161	–0.3368	0.0670	1.1303
BM 8	0.0095	0.0088	0.7173	–0.3144	0.0696	1.1556
BM 9	0.0100	0.0107	0.6428	–0.3903	0.0762	1.2534
Value	0.0109	0.0096	1.0226	–0.4545	0.0934	1.4466

Panel B. Monthly Excess Returns on the Value-Weighted NYSE/AMEX/NASDAQ Index

This table presents summary statistics for the monthly excess returns on the value-weighted CRSP index. Mean, median, maximum, minimum, and standard deviation are reported for the value-weighted CRSP index. The sample period is from July 1926 to December 2007, yielding a total of 978 monthly observations.

Market Portfolio	Mean	Median	Maximum	Minimum	Std. Dev.
NYSE/AMEX/NASDAQ	0.0065	0.0097	0.3827	–0.2904	0.0541

Appendix B. Risk-Return Tradeoff after Controlling for Macroeconomic Variables

With the intertemporal hedging demand, Merton's ICAPM implies the following equilibrium relation between risk and return:

$$E_t(R_{m,t+1}) = \beta \cdot \sigma^2_{m,t+1|t} + \lambda \cdot \sigma_{mx,t+1|t}\,, \tag{B.1}$$

where $\sigma_{mx,t+1|t}$ is the time-*t* expected conditional covariance between the excess returns on the market portfolio and the innovations in a set of state variables *x* for time *t*+1. Earlier studies assume that $\sigma_{mx,t+1|t}$ is a linear function of state variables, i.e., $\sigma_{mx,t+1|t} = a + b \cdot X_t$, such that equation (B.1) can be rewritten as:

$$E_t(R_{m,t+1}) = \alpha + \beta \cdot \sigma^2_{m,t+1|t} + \lambda \cdot X_t\,, \tag{B.2}$$

where the state variables (X_t) are directly introduced to the risk-return regressions along with $\sigma^2_{m,t+1|t}$. The commonly chosen variables include the default spread (*DEF*), term spread (*TERM*), relative T-bill rate (*RREL*), and aggregate dividend yield (*DIV*). We study how variations in these macroeconomic variables affect the intertemporal risk-return relation. We test the significance of the risk aversion parameter, β, based on the GARCH-in-mean model after controlling for macroeconomic variables:

$$\begin{gathered}
R_{m,t+1} = \alpha + \beta \cdot \sigma^2_{m,t+1|t} + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1} \\
R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t+1|t} + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1} \\
R_{m,t+1} = \alpha + \beta \cdot \ln \sigma^2_{m,t+1|t} + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1} \\
E\left(\varepsilon^2_{m,t+1} \mid \Omega_t\right) = \sigma^2_{m,t+1|t} = \gamma_0 + \gamma_1 \varepsilon^2_{m,t} + \gamma_2 \sigma^2_{m,t}\,.
\end{gathered} \tag{B.3}$$

Panel A of Table B1 presents the maximum likelihood parameter estimates and the t-statistics in parentheses from estimating equation (B.3) based on the value-weighted CRSP index. For the variance, standard deviation, and log-variance specifications of the GARCH-in-mean model, the risk aversion parameter (β) is estimated to be very small both economically and statistically. The Bollerslev-Wooldridge robust t-statistics are 0.23 for the variance, –0.50 for the standard deviation, and –0.93 for the log-variance specification.

Another point worth mentioning in Table B1 is that the slope coefficients on the relative T-bill rate (*RREL*) are negative and highly significant. More specifically, the slope on *RREL* (λ_3) is estimated to –5.91 for the variance, –5.90 for the standard deviation, and –5.92 for the log-variance models. The normal t-statistics of these slope estimates are in the range of –3.10 and –3.12. The Bollerslev-Wooldridge robust t-statistics are somewhat lower, but they are still statistically significant, in the range of –2.41 and –2.43. The slope coefficients on the aggregate dividend yield (*DIV*) are positive and highly significant without any exception. The slope on *DIV* (λ_4) is estimated to be 0.22 for the variance and 0.23 for the standard deviation and log-variance specifications. The Bollerslev-Wooldridge robust t-statistics of these slope estimates are in the range of 2.42 and 2.51, implying strong statistical significance. As shown in Table B1, the default spread (*DEF*) and the term spread (*TERM*) have no predictive power for the one-month ahead return on the market portfolio since the slope coefficients (λ_1, λ_2) on *DEF* and *TERM* have very low t-statistics.

The empirical evidence on the intertemporal relation between macroeconomic factors and equity returns is inconclusive. Bodie (1976), Fama (1981), Geske and Roll (1983), Pearce and Roley (1983, 1985) document a negative impact of inflation and money growth on equity values. Chan, Chen, and Hsieh (1985), Chen, Roll, and Ross (1986), and Chen (1991) find that changes in aggregate production, inflation, and the short-term interest rates are important economic indicators in determining equilibrium expected returns on securities. In time-series analyses, Fama and Schwert (1977), Keim and Stambaugh (1986), Campbell (1987), Campbell and Shiller (1988), and Fama and French (1988, 1989) find that the short-term interest rates, expected inflation, and dividend yields can predict the expected returns of bonds and stocks. Cutler, Poterba, and Summers (1989) find that the growth rate of industrial production is significantly positively correlated with real stock returns over the period 1926-1986, but not in the 1946-1985 sub-period. Flannery and Protopapadakis (2002) identify six candidates for priced factors: three nominal variables (CPI, PPI, and a Monetary Aggregate) and three real variables (Balance of Trade, Employment Report, and Housing Starts). Popular measures of overall economic activity, such as Industrial Production or GNP are not represented.

The basic approach taken in previous empirical work has been to estimate time-series regression of the aggregate equity returns on a group of macro variables that proxy for inflation and/or real economic activity. In this paper, we use the monthly inflation rate and the monthly growth rate of industrial production proxying for economic growth. The inflation rate (*INF*) is the monthly growth rate of the Consumer Price Index available at Robert Shiller's website. The economic or output growth (*OUT*) is defined as the monthly growth rate of the Industrial Production Index obtained from the G.17 database of the Federal Reserve Board.

We examine the significance of the risk aversion parameter, β, based on the GARCH-in-mean model after controlling for inflation and output growth:

$$
\begin{aligned}
R_{m,t+1} &= \alpha + \beta \cdot \sigma^2_{m,t+1|t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1} \\
R_{m,t+1} &= \alpha + \beta \cdot \sigma_{m,t+1|t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1} \\
R_{m,t+1} &= \alpha + \beta \cdot \ln \sigma^2_{m,t+1|t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1} \\
E\left(\varepsilon^2_{m,t+1} \mid \Omega_t\right) &= \sigma^2_{m,t+1|t} = \gamma_0 + \gamma_1 \varepsilon^2_{m,t} + \gamma_2 \sigma^2_{m,t}
\end{aligned}
\tag{B.4}
$$

Panel B of Table B1 reports the maximum likelihood parameter estimates and the t-statistics in parentheses from estimating equation (B.4) based on the value-weighted NYSE/AMEX/NASDAQ index. For alternative specifications of the GARCH-in-mean model, the risk aversion parameter (β) is estimated to be positive, but statistically insignificant. The Bollerslev-Wooldridge robust t-statistics are 1.17 for the variance, 1.08 for the standard deviation, and 1.04 for the log-variance specification. Another notable point in Panel B of Table B1 is that the slope coefficients on the inflation rate (*INF*) are negative and highly significant. More specifically, the slope on *INF* (λ_1) is estimated to –1.10 for the variance, –1.12 for the standard deviation, and –1.13 for the log-variance models. The Bollerslev-Wooldridge robust t-statistics are statistically significant and in the range of –3.75 and –3.85. The slope coefficients on the growth rate of industrial production (*OUT*) are very small both economically and statistically. The Bollerslev-Wooldridge robust t-statistics of the slope coefficients on *OUT* (λ_2) are in the range of –0.31 and –0.60.

Table B2 presents the parameter estimates of the Realized Volatility model with control variables. As shown in Panel A, after incorporating *DEF*, *TERM*, *RREL*, and *DIV* to the risk-return regressions, the risk aversion parameter (β) is estimated to be negative, but statistically insignificant with very low t-statistics. The same qualitative results are obtained for the realized variance, standard deviation, and log-variance estimators. A notable point in Panel A of Table B2 is that the slope coefficients on the aggregate dividend yield (*DIV*) are positive and statistically significant. The slope on *DIV* (λ_4) is estimated to be 0.27 for the realized variance, standard deviation, and log-variance estimators. The Newey-West t-statistics of these slope estimates are in the range of 2.16 and 2.19. The default spread, term spread, and relative T-bill rate have no predictive power for the one-month ahead returns on the market portfolio since the slope coefficients (λ_1, λ_2, λ_3) on *DEF*, *TERM*, and *RREL* have very low t-statistics. Panel B of Table B2 reports the parameter estimates of the Realized Volatility model with the inflation rate and output growth. The risk aversion parameter (β) is estimated to be positive, but statistically insignificant with very low t-statistics. Similar to our earlier findings in Table B1, the slope coefficients on the inflation rate (*INF*) are negative and significant, whereas the slopes on the growth rate of industrial production (*OUT*) are positive but statistically insignificant.

Table B3 shows the parameter estimates of the Range Volatility model with *DEF*, *TERM*, *RREL*, and *DIV*. As reported in Panel A, the risk aversion parameter (β) is estimated to be small both economically and statistically. The slope coefficients on the aggregate dividend yield (*DIV*) are positive, in the range of 0.27 to 0.28, and statistically significant. The default spread, term spread, and relative T-bill rate cannot explain the time-series variation in monthly returns on the NYSE/AMEX/NASDAQ index. Panel B of Table B3 reports the parameter estimates of the Range Volatility model with *INF* and *OUT*. The risk aversion parameter is estimated to be positive, but statistically insignificant with very low t-statistics. The slope coefficients on the inflation rate are negative and significant although marginally in some cases, whereas the slopes on output growth are positive but statistically insignificant. Overall, the parameter estimates of the GARCH-in-mean, Realized, and Range Volatility models reported in Tables B1, B2, and B3 provide evidence that the inflation rate and aggregate dividend yield have significant predictive power for the one-month excess returns on the market portfolio.

Table B1. Testing ICAPM with GARCH-in-mean Models after Controlling for Macroeconomic Variables

This table presents the parameter estimates of the GARCH-in-mean models with conditional variance, conditional standard deviation (std dev), and conditional log-variance after controlling for macroeconomic variables. The detrended relative rate (RREL) is defined as the difference between the 3-month T-bill rate and its 12-month backward moving average. The term spread (TERM) is defined as the difference between the yields on the 10-year Treasury bond and the 3-month Treasury bill. The default spread (DEF) is defined as the difference between the yields on the BAA-rated and AAA-rated corporate bonds. The aggregate dividend-price ratio (DIV) is obtained from Robert Shiller's website: http://aida.econ.yale.edu/shiller/. The inflation rate (INF) is the monthly growth rate of the Consumer Price Index. The output growth (OUT) is defined as the growth rate of the Industrial Production Index. The parameters are estimated using the NYSE/AMEX/NASDAQ (CRSP) value-weighted index return for the sample period of January 1926 to December 2007, yielding a total of 984 monthly observations. The normal t-statistics are given in parentheses and Bollerslev-Wooldridge (1992) robust t-statistics are given in square brackets.

Panel A. Controlling for DEF, TERM, RREL, and DIV

GARCH-in-mean with Variance: $R_{m,t+1} = \alpha + \beta \cdot \sigma^2_{m,t+1|t} + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1}$

GARCH-in-mean with Std Dev: $R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t+1|t} + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1}$

GARCH-in-mean with Log-Variance: $R_{m,t+1} = \alpha + \beta \cdot \ln \sigma^2_{m,t+1|t} + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1}$

$$E\left(\varepsilon^2_{m,t+1} \mid \Omega_t\right) = \sigma^2_{m,t+1|t} = \gamma_0 + \gamma_1 \varepsilon^2_{m,t} + \gamma_2 \sigma^2_{m,t}$$

GARCH	α	β	λ_1	λ_2	λ_3	λ_4	γ_0	γ_1	γ_2
	–0.0003	0.2596	–0.7179	0.2564	–5.9066	0.2197	6.34×10^{-5}	0.1251	0.8556
Variance	(–0.07)	(0.22)	(–0.15)	(0.16)	(–3.12)	(2.38)	(3.11)	(6.23)	(46.04)
	[–0.06]	[0.23]	[–0.14]	[0.14]	[–2.41]	[2.42]	[2.22]	[4.88]	[28.64]
	0.0012	–0.0726	1.5863	0.2337	–5.9045	0.2255	6.18×10^{-5}	0.1243	0.8571
Std. Dev.	(0.21)	(–0.47)	(0.33)	(0.15)	(–3.10)	(2.47)	(3.15)	(6.32)	(46.95)
	[0.22]	[–0.50]	[0.30]	[0.13]	[–2.42]	[2.46]	[2.19]	[4.91]	[29.21]
	–0.0239	–0.0032	2.6856	0.2548	–5.9217	0.2324	6.15×10^{-5}	0.1234	0.8580
Log-Variance	(–0.93)	(–0.90)	(0.59)	(0.16)	(–3.10)	(2.59)	(3.17)	(6.34)	(47.35)
	[–0.92]	[–0.93]	[0.53]	[0.14]	[–2.43]	[2.51]	[2.17]	[4.94]	[29.61]

Table B1 (continued)

Panel B. Controlling for INF and OUT

GARCH-in-mean with Variance: $R_{m,t+1} = \alpha + \beta \cdot \sigma^2_{m,t+1|t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1}$

GARCH-in-mean with Std Dev: $R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t+1|t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1}$

GARCH-in-mean with Log-Variance: $R_{m,t+1} = \alpha + \beta \cdot \ln \sigma^2_{m,t+1|t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1}$

$$E\left(\varepsilon^2_{m,t+1} \mid \Omega_t\right) = \sigma^2_{m,t+1|t} = \gamma_0 + \gamma_1 \varepsilon^2_{m,t} + \gamma_2 \sigma^2_{m,t}$$

GARCH	α	β	λ_1	λ_2	γ_0	γ_1	γ_2
	0.0091	0.9888	–1.1021	–0.0654	6.37×10^{-5}	0.1329	0.8485
Variance	(4.04)	(1.21)	(–4.19)	(–0.80)	(3.29)	(6.62)	(47.16)
	[3.92]	[1.17]	[–3.75]	[–0.31]	[2.28]	[4.90]	[29.41]
	0.0063	0.1137	–1.1168	–0.0633	6.28×10^{-5}	0.1327	0.8492
Std. Dev.	(1.32)	(1.02)	(–4.26)	(–0.77)	(3.22)	(6.64)	(47.21)
	[1.44]	[1.08]	[–3.81]	[–0.60]	[2.27]	[5.10]	[28.52]
	0.0281	0.0026	–1.1282	–0.0622	6.20×10^{-5}	0.1327	0.8497
Log-Variance	(1.58)	(0.97)	(–4.30)	(–0.76)	(3.16)	(6.66)	(47.40)
	[1.69]	[1.04]	[–3.85]	[–0.59]	[2.26]	[5.11]	[28.77]

Table B2. Testing ICAPM with Realized Volatility after Controlling for Macroeconomic Variables

This table presents the parameter estimates of the risk-return regressions with lagged realized variance, lagged realized standard deviation (std dev), and lagged realized log-variance. Monthly realized variance is calculated as the sum of squared daily returns within a month. The detrended relative rate (RREL) is defined as the difference between the 3-month T-bill rate and its 12-month backward moving average. The term spread (TERM) is defined as the difference between the yields on the 10-year Treasury bond and the 3-month Treasury bill. The default spread (DEF) is defined as the difference between the yields on the BAA-rated and AAA-rated corporate bonds. The aggregate dividend yield (DIV) is obtained from Robert Shiller's website: http://aida.econ.yale.edu/shiller/. The inflation rate (INF) is the monthly growth rate of the Consumer Price Index. The output growth (OUT) is defined as the growth rate of the Industrial Production Index. The parameters are estimated using the NYSE/AMEX/NASDAQ (CRSP) value-weighted index return for the sample period of January 1926 to December 2007, yielding a total of 984 monthly observations. The OLS t-statistics are given in parentheses and the Newey-West (1987) adjusted t-statistics are given in square brackets.

Panel A. Controlling for DEF, TERM, RREL, and DIV

Realized Variance: $$R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t}^2 + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1}$$

Realized Std Dev: $$R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t} + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1}$$

Realized Log-Variance: $$R_{m,t+1} = \alpha + \beta \cdot \ln \sigma_{m,t}^2 + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1}$$

$$\sigma_{m,t}^2 = \sum_{d=1}^{D_t} R_{m,d}^2 + 2\sum_{d=2}^{D_t} R_{m,d} \cdot R_{m,d-1}$$

Realized	α	β	λ_1	λ_2	λ_3	λ_4
	–0.0057	–0.3279	1.6662	0.4706	–4.1246	0.2721
Variance	(–1.17)	(–0.73)	(0.39)	(0.21)	(–1.56)	(2.21)
	[–0.71]	[–0.36]	[0.24]	[0.21]	[–1.39]	[2.16]
	–0.0046	–0.0402	1.5086	0.4622	–4.1462	0.2690
Std. Dev.	(–0.89)	(–0.50)	(0.33)	(0.20)	(–1.56)	(2.17)
	[–0.55]	[–0.31]	[0.20]	[0.20]	[–1.38]	[2.19]
	–0.0124	–0.0010	1.3979	0.4595	–4.1603	0.2663
Log-Variance	(–0.76)	(–0.45)	(0.30)	(0.20)	(–1.57)	(2.13)
	[–0.60]	[–0.41]	[0.17]	[0.20]	[–1.40]	[2.18]

Table B2 (continued)

Panel B. Controlling for INF and OUT

Realized Variance: $R_{m,t+1} = \alpha + \beta \cdot \sigma^2_{m,t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1}$

Realized Std Dev: $R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1}$

Realized Log-Variance: $R_{m,t+1} = \alpha + \beta \cdot \ln \sigma^2_{m,t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1}$

$$\sigma^2_{m,t} = \sum_{d=1}^{D_t} R^2_{m,d} + 2\sum_{d=2}^{D_t} R_{m,d} \cdot R_{m,d-1}$$

Realized	α	β	λ_1	λ_2
	0.0078	0.0269	–0.6674	0.0858
Variance	(3.63)	(0.07)	(–2.07)	(0.91)
	[3.12]	[0.03]	[–1.99]	[0.65]
	0.0068	0.0256	–0.6531	0.0882
Std. Dev.	(2.06)	(0.39)	(–2.02)	(0.94)
	[1.40]	[0.19]	[–1.94]	[0.68]
	0.0124	0.0007	–0.6599	0.0885
Log-Variance	(1.07)	(0.39)	(–2.06)	(0.94)
	[0.66]	[0.26]	[–1.94]	[0.68]

Table B3. Testing ICAPM with Range Volatility after Controlling for Macroeconomic Variables

This table presents the parameter estimates of the risk-return regressions with lagged range variance, lagged range standard deviation (std dev), and lagged range log-variance. Monthly range volatility (std dev) is calculated as the difference between the natural logarithms of the highest daily index and the lowest daily index levels in a month. The detrended relative rate (RREL) is defined as the difference between the 3-month T-bill rate and its 12-month backward moving average. The term spread (TERM) is defined as the difference between the yields on the 10-year Treasury bond and the 3-month Treasury bill. The default spread (DEF) is defined as the difference between the yields on the BAA-rated and AAA-rated corporate bonds. The aggregate dividend-price ratio (DIV) is obtained from Robert Shiller's website: http://aida.econ.yale.edu/shiller/. The inflation rate (INF) is the monthly growth rate of the Consumer Price Index. The output growth (OUT) is defined as the growth rate of the Industrial Production Index. The parameters are estimated using the NYSE/AMEX/NASDAQ (CRSP) value-weighted index return for the sample period of January 1926 to December 2007, yielding a total of 984 monthly observations. The OLS t-statistics are given in parentheses and the Newey-West (1987) adjusted t-statistics are given in square brackets.

Panel A. Controlling for DEF, TERM, RREL, and DIV

Range Variance: $$R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t}^2 + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1}$$

Range Std Dev: $$R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t} + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1}$$

Range Log-Variance: $$R_{m,t+1} = \alpha + \beta \cdot \ln \sigma_{m,t}^2 + \lambda_1 \cdot DEF_t + \lambda_2 \cdot TERM_t + \lambda_3 \cdot RREL_t + \lambda_4 \cdot DIV_t + \varepsilon_{m,t+1}$$

$$\sigma_{m,t} = \ln(P_{d,t}^{\max}) - \ln(P_{d,t}^{\min})$$

Range	α	β	λ_1	λ_2	λ_3	λ_4
	–0.0053	0.0255	–0.0386	0.6096	–4.0960	0.2760
Variance	(–1.09)	(0.16)	(–0.01)	(0.27)	(–1.54)	(2.25)
	[–0.66]	[0.08]	[–0.01]	[0.26]	[–1.36]	[2.14]
	–0.0054	–0.0016	0.3114	0.5676	–4.1118	0.2760
Std. Dev.	(–1.07)	(–0.03)	(0.07)	(0.25)	(–1.55)	(2.25)
	[–0.65]	[–0.02]	[0.04]	[0.24]	[–1.35]	[2.17]
	–0.0099	–0.0007	1.0128	0.4445	–4.1914	0.2728
Log-Variance	(–0.76)	(–0.37)	(0.23)	(0.20)	(–1.58)	(2.21)
	[–0.58]	[–0.33]	[0.13]	[0.19]	[–1.39]	[2.16]

Table B3 (continued)

Panel B. Controlling for INF and OUT

Range Variance: $R_{m,t+1} = \alpha + \beta \cdot \sigma^2_{m,t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1}$

Range Std Dev: $R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1}$

Range Log-Variance: $R_{m,t+1} = \alpha + \beta \cdot \ln \sigma^2_{m,t} + \lambda_1 \cdot INF_t + \lambda_2 \cdot OUT_t + \varepsilon_{m,t+1}$

$$\sigma_{m,t} = \ln(P^{\max}_{d,t}) - \ln(P^{\min}_{d,t})$$

Range	α	β	λ_1	λ_2
	0.0071	0.1213	–0.6318	0.0891
Variance	(3.40)	(0.87)	(–1.96)	(0.95)
	[2.99]	[0.38]	[–1.79]	[0.68]
	0.0058	0.0339	–0.6342	0.0918
Std. Dev.	(1.84)	(0.86)	(–1.97)	(0.98)
	[1.30]	[0.42]	[–1.83]	[0.70]
	0.0131	0.0009	–0.6564	0.0905
Log-Variance	(1.38)	(0.56)	(–2.05)	(0.96)
	[0.86]	[0.38]	[–1.91]	[0.70]

Appendix C. DCC- and CCC-based Conditional Covariance Estimates

Figure 1 displays the DCC-based conditional covariance estimates between the monthly excess returns on the value-weighted book-to-market portfolios and the value-weighted market portfolio over the sample period of July 1926 to December 2007. A notable point in Figure 1 is that the conditional covariances exhibit significant time variation for all portfolios. The conditional covariances of the value portfolio (BM 10) with the market are greater than the conditional covariances of the growth portfolio (BM 1) throughout the sample period. A common observation in Figure 1 is that there is a big spike in September 1932 for all portfolios, and the covariance stays at an extremely high level for about 3 months including October and November 1932. A similar spike is observed in June 1933 and it also lasts for about 3 months including May and July 1933. The next big spike is observed in July 1938, and it remains to be high in August and September 1938. Because of the significant level differences in the conditional covariance estimates before and after the World War II, Figure 1 seems to suggest that there is not much variation after the 1940s. Hence, in Figure 2, we plot the same DCC-based conditional covariance estimates for the sample period of January 1946 to December 2007. Figure 2 shows that the fluctuations in the conditional covariances are so significant that one should take into account the dynamics of conditional covariances when investigating the significance of an intertemporal relation between risk and return.

Figures 3 and 4 demonstrate the CCC-based conditional covariance estimates for the sample period of July 1926–December 2007 and January 1946–December 2007, respectively. The time-series pattern (including the dates of the spikes) observed in the CCC-based conditional covariances is very similar to the pattern obtained from the DCC-based conditional covariances. The time-series plots in Figures 1 to 4 suggest that investigating ICAPM with the DCC- and CCC-based conditional covariances provides similar conclusions.

To test whether the mean-reverting DCC model of Engle (2002) and the constant conditional correlation model of Bollerslev (1990) generate reasonable conditional covariance estimates, we compute the value-weighted averages of the conditional covariances of the book-to-market portfolios with the aggregate market portfolio. Then, we compare the weighted average conditional covariances with the benchmark of the conditional market variance. In Figure 5, the dashed line denotes the conditional variance of monthly excess returns on the market portfolio. The solid line denotes the value-weighted average of the conditional covariances. Panel A (Panel B) compares the empirical performance of the DCC-based (CCC-based) conditional covariance estimates. In both panels, the value-weighted average covariances are in the same range as the conditional variance of the market portfolio. The two series in both panels move very closely together. In fact, it is almost impossible to visually distinguish the two series in Figure 5. Specifically, in Panel A the sample correlation between the value-weighted average DCC-based covariances and the market variance is 99.96% and in Panel B the sample correlation between the value-weighted average CCC-based covariances and the market variance is 99.92%. The affinity in magnitudes and time-series fluctuations between the weighted average covariances and market portfolio variance provides evidence for reasonable conditional variance and covariance estimates from the DCC and CCC models.

The table below compares the sample mean, median, maximum, minimum, and standard deviation measures for the value-weighted average DCC- and CCC-based conditional covariance estimates and the conditional variance of the market portfolio for the sample period of July 1926–December 2007. Although the descriptive statistics indicate superior performance of both models, the mean-reverting DCC model provides slightly more accurate estimates of the conditional measures of market risk.

	Value-Weighted Average Conditional Covariance		*Conditional Variance of the Market*
	CCC Model	DCC Model	GARCH Model
Mean	0.002954	0.002963	0.002991
Median	0.001857	0.001848	0.001849
Maximum	0.038903	0.040164	0.039648
Minimum	0.000646	0.000625	0.000540
Std. Dev.	0.003914	0.004029	0.004036

Figure 1. DCC-based conditional covariances: January 1926 – December 2007

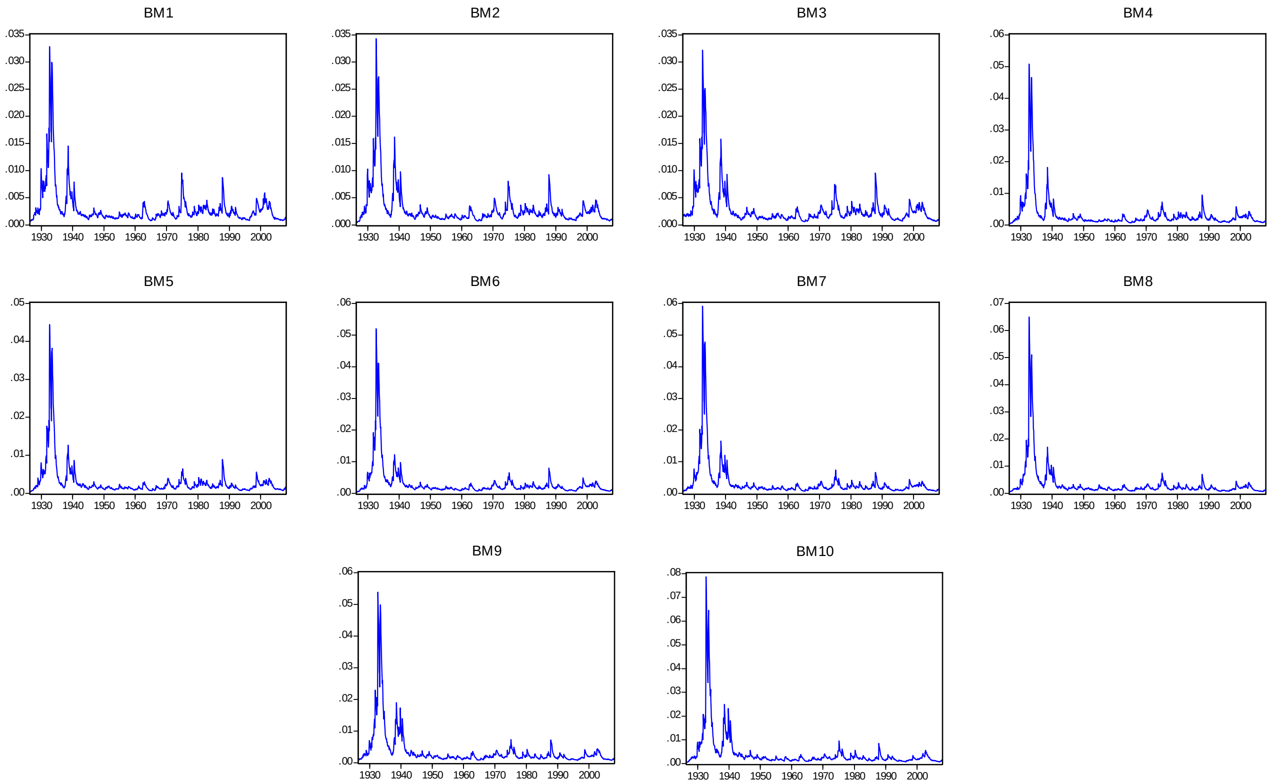


Figure 2. DCC-based conditional covariances: January 1946 – December 2007

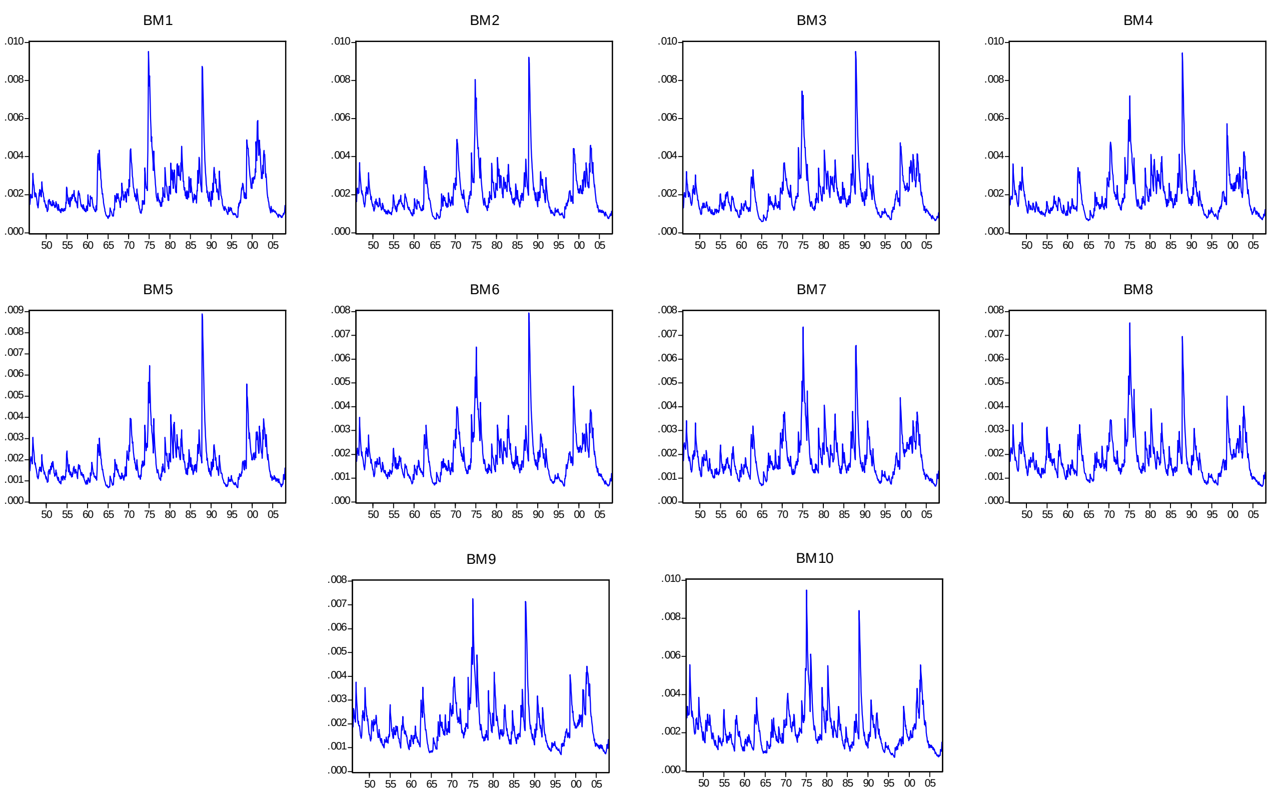


Figure 3. CCC-based conditional covariances: January 1926 – December 2007

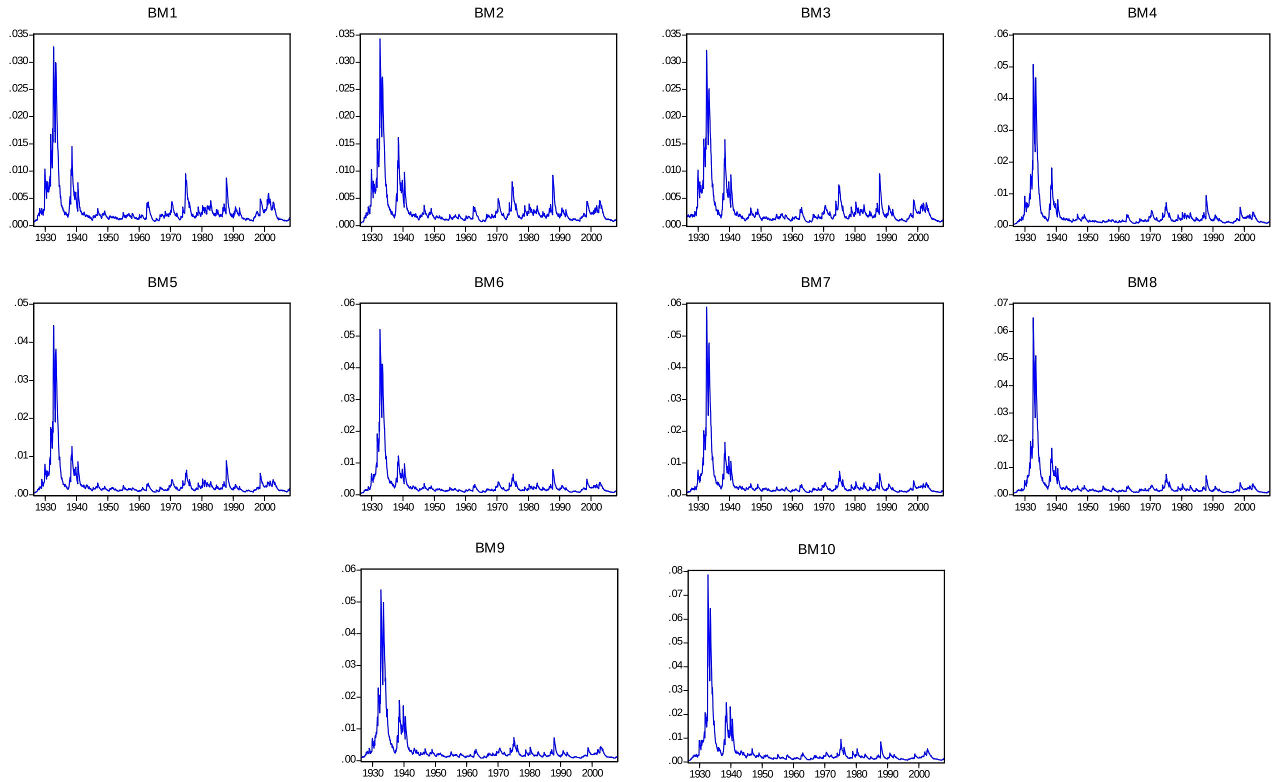


Figure 4. CCC-based conditional covariances: January 1946 – December 2007

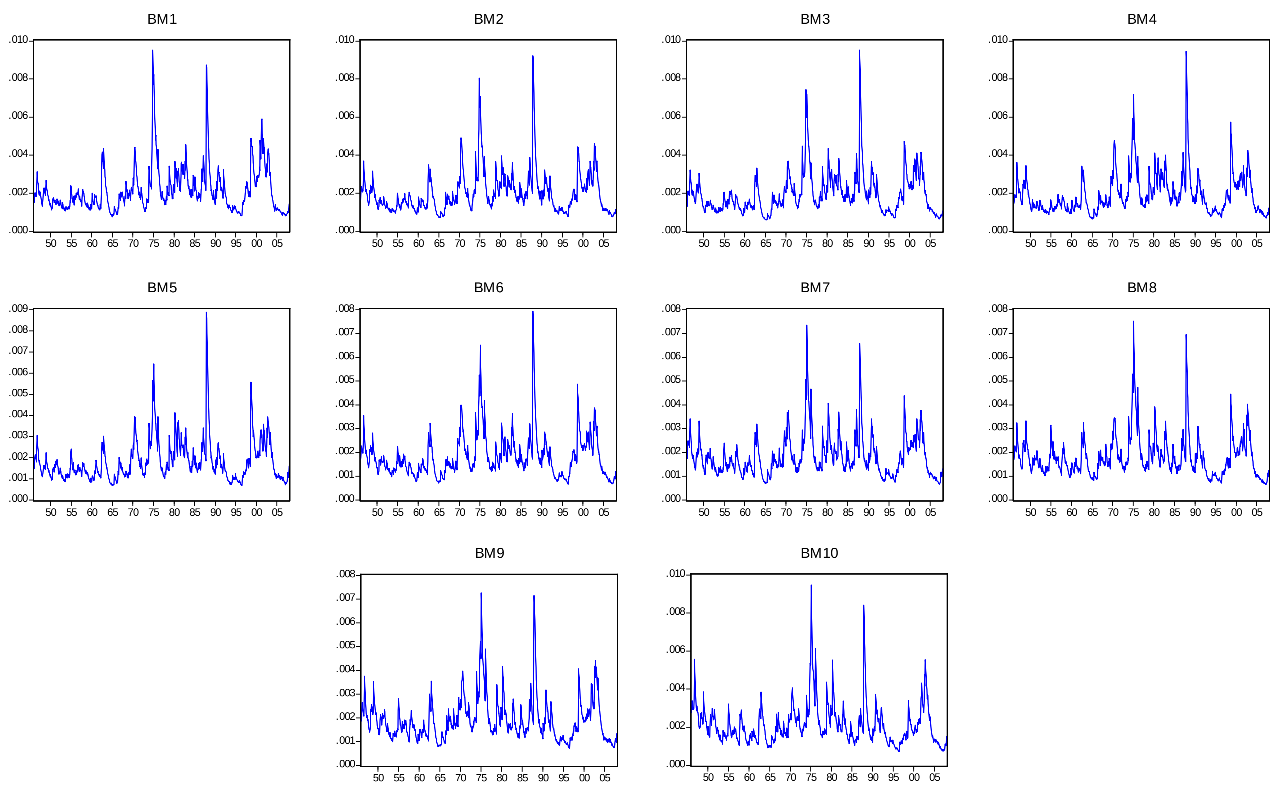
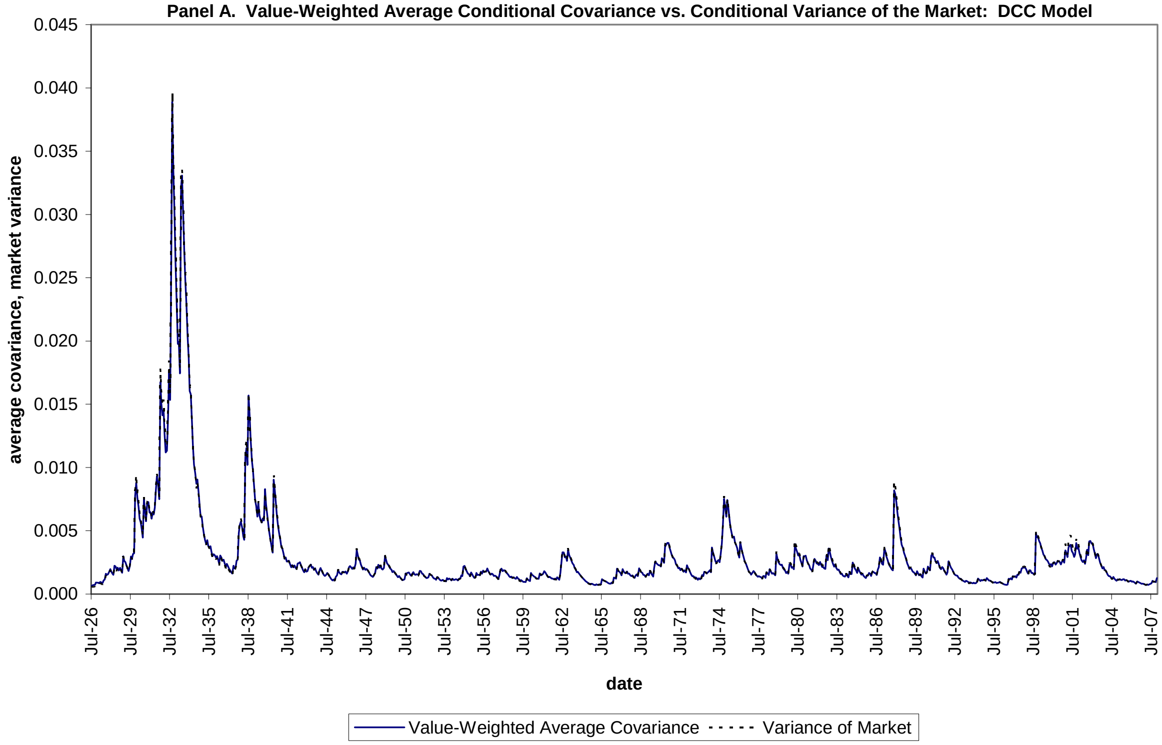


Figure 5. Weighted Average Conditional Covariance vs. Conditional Variance of the Market

Panel A presents the value-weighted average covariance and market variance estimates from the Dynamic Conditional Correlation (DCC) model. Panel B presents the value-weighted average covariance and market variance estimates from the Constant Conditional Correlation (CCC) model. The dashed line denotes the conditional variance of monthly excess returns on the value-weighted market portfolio. The solid line denotes the value-weighted average of the conditional covariances of monthly excess returns on the ten value-weighted book-to-market portfolios with the monthly excess returns on the value-weighted market portfolio. The market portfolio is measured by the value-weighted NYSE/AMEX/NASDAQ index.

Panel A. Value-Weighted Average Conditional Covariance vs. Conditional Variance of the Market: DCC Model



Panel B. Value-Weighted Average Conditional Covariance vs. Conditional Variance of the Market: CCC Model

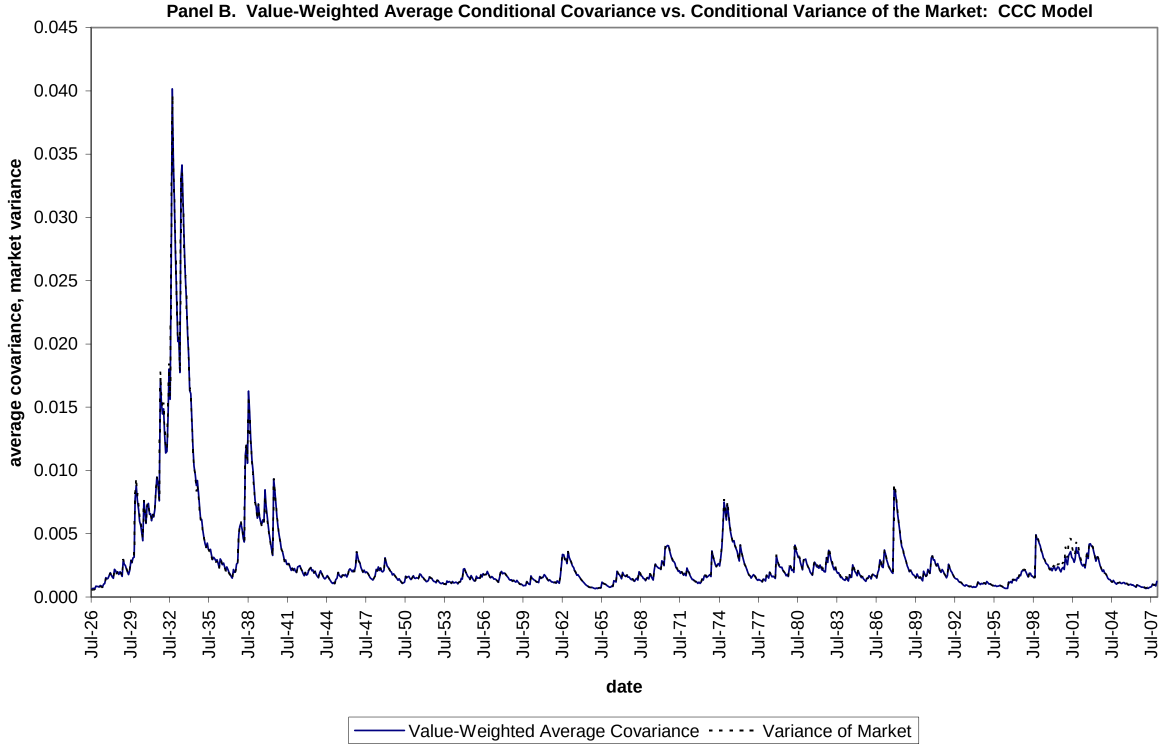


Table 1. Testing ICAPM with GARCH-in-mean Model

This table presents the parameter estimates of the GARCH-in-mean model with conditional variance, conditional standard deviation (std dev), and conditional log-variance. The parameters are estimated using the NYSE/AMEX/NASDAQ (CRSP) value-weighted index return for the sample period of January 1926 to December 2007, yielding a total of 984 monthly observations. The normal t-statistics are given in parentheses and Bollerslev-Wooldridge (1992) robust t-statistics are given in square brackets.

GARCH-in-mean with Variance: $R_{m,t+1} = \alpha + \beta \cdot \sigma^2_{m,t+1|t} + \varepsilon_{m,t+1}$

GARCH-in-mean with Std Dev: $R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t+1|t} + \varepsilon_{m,t+1}$

GARCH-in-mean with Log-Variance: $R_{m,t+1} = \alpha + \beta \cdot \ln \sigma^2_{m,t+1|t} + \varepsilon_{m,t+1}$

$$E\left(\varepsilon^2_{m,t+1} \mid \Omega_t\right) = \sigma^2_{m,t+1|t} = \gamma_0 + \gamma_1 \varepsilon^2_{m,t} + \gamma_2 \sigma^2_{m,t}$$

GARCH Volatility	α	β	γ_0	γ_1	γ_2
	0.0056	1.1442	6.39×10^{-5}	0.1269	0.8540
Variance	(2.80)	(1.41)	(3.21)	(6.38)	(46.71)
	[2.66]	[1.20]	[2.30]	[5.05]	[29.54]
	0.0029	0.1156	6.30×10^{-5}	0.1265	0.8549
Std. Dev.	(0.63)	(1.05)	(3.16)	(6.40)	(46.88)
	[0.63]	[1.04]	[2.29]	[5.04]	[29.74]
	0.0228	0.0023	6.23×10^{-5}	0.1265	0.8554
Log-Variance	(1.27)	(0.84)	(3.11)	(6.43)	(47.15)
	[1.31]	[0.88]	[2.28]	[5.03]	[29.88]

Table 2. Testing ICAPM with Realized Volatility

This table presents the parameter estimates of the risk-return regressions with lagged realized variance, lagged realized standard deviation (std dev), and lagged realized log-variance. Monthly realized variance is calculated as the sum of squared daily returns within a month. The parameters are estimated using the NYSE/AMEX/NASDAQ (CRSP) value-weighted index return for the sample period of January 1926 to December 2007, yielding a total of 984 monthly observations. The OLS t-statistics are given in parentheses and the Newey-West (1987) adjusted t-statistics are given in square brackets.

Realized Variance: $R_{m,t+1} = \alpha + \beta \cdot \sigma^2_{m,t} + \varepsilon_{m,t+1}$

Realized Std Dev: $R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t} + \varepsilon_{m,t+1}$

Realized Log-Variance: $R_{m,t+1} = \alpha + \beta \cdot \ln \sigma^2_{m,t} + \varepsilon_{m,t+1}$

$$\sigma^2_{m,t} = \sum_{d=1}^{D_t} R^2_{m,d} + 2\sum_{d=2}^{D_t} R_{m,d} \cdot R_{m,d-1}$$

Realized Volatility	α	β
	0.0062	0.1220
Variance	(3.19)	(0.32)
	[2.88]	[0.12]
	0.0049	0.0397
Std. Dev.	(1.58)	(0.62)
	[1.00]	[0.28]
	0.0123	0.0009
Log-Variance	(1.07)	(0.51)
	[0.63]	[0.32]

Table 3. Testing ICAPM with Range Volatility

This table presents the parameter estimates of the risk-return regressions with lagged range variance, lagged range standard deviation (std dev), and lagged range log-variance. Monthly range volatility (std dev) is calculated as the difference between the natural logarithms of the highest daily index and the lowest daily index levels in a month. The parameters are estimated using the NYSE/AMEX/NASDAQ (CRSP) value-weighted index return for the sample period of January 1926 to December 2007, yielding a total of 984 monthly observations. The OLS t-statistics are given in parentheses and the Newey-West (1987) adjusted t-statistics are given in square brackets.

Range Variance: $R_{m,t+1} = \alpha + \beta \cdot \sigma^2_{m,t} + \varepsilon_{m,t+1}$

Range Std Dev: $R_{m,t+1} = \alpha + \beta \cdot \sigma_{m,t} + \varepsilon_{m,t+1}$

Range Log-Variance: $R_{m,t+1} = \alpha + \beta \cdot \ln \sigma^2_{m,t} + \varepsilon_{m,t+1}$

$$\sigma_{m,t} = \ln(P^{\max}_{d,t}) - \ln(P^{\min}_{d,t})$$

Range Volatility	α	β
	0.0056	0.1529
Variance	(2.99)	(1.11)
	[2.90]	[0.46]
	0.0040	0.0414
Std. Dev.	(1.38)	(1.06)
	[0.50]	[0.92]
	0.0125	0.0010
Log-Variance	(1.33)	(0.65)
	[0.80]	[0.42]

Table 4. Multivariate GARCH-in-mean Model with Constant Conditional Correlations and a Common Slope Coefficient: 10 Value-Weighted Book-to-Market Portfolios

Entries report the maximum likelihood parameter estimates of the multivariate GARCH-in-mean model with constant conditional correlations and a common slope coefficient on the conditional variance-covariance matrix:

$$R_{i,t+1} = \alpha_i + \beta \cdot \sigma_{im,t+1} + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \varepsilon_{m,t+1}$$
$$E_t\left[\varepsilon^2_{i,t+1}\right] \equiv \sigma^2_{i,t+1} = \gamma^i_0 + \gamma^i_1 \varepsilon^2_{i,t} + \gamma^i_2 \sigma^2_{i,t}$$
$$E_t\left[\varepsilon^2_{m,t+1}\right] \equiv \sigma^2_{m,t+1} = \gamma^m_0 + \gamma^m_1 \varepsilon^2_{m,t} + \gamma^m_2 \sigma^2_{m,t}$$
$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1}$$

where ρ_{ij} is the constant conditional correlation between $R_{i,t+1}$ and $R_{j,t+1}$. The parameters and their *t*-statistics are estimated using the excess returns on the aggregate market portfolio and the 10 value-weighted book-to-market (value vs. growth) portfolios for the sample period from July 1926 to December 2007.

Portfolio	Intercepts	Coefficient	t-statistic
Growth	α_1	-0.0015	-0.72
BM 2	α_2	-0.0008	-0.41
BM 3	α_3	-0.0005	-0.29
BM 4	α_4	-0.0002	-0.10
BM 5	α_5	0.0009	0.54
BM 6	α_6	0.0010	0.57
BM 7	α_7	0.0010	0.56
BM 8	α_8	0.0019	1.09
BM 9	α_9	0.0020	1.04
Value	α_{10}	0.0018	0.81
Market	α_m	-0.0006	-0.34
Common Slope	β	4.1733	4.69
Maximized log-likelihood	LogL	27,798.18	

$H_0 : \alpha_1 = \alpha_2 = ... = \alpha_m = 0 \;\Rightarrow$ Wald statistic = 15.84 (*p*-value = 14.71%)

Table 5. Multivariate GARCH-in-mean Model with Constant Conditional Correlations and Different Slope Coefficients: 10 Value-Weighted Book-to-Market Portfolios

Entries report the maximum likelihood parameter estimates of the multivariate GARCH-in-mean model with constant conditional correlations and different slopes on the conditional variance-covariance matrix:

$$R_{i,t+1} = \alpha_i + \beta_i \cdot \sigma_{im,t+1} + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + \beta_m \cdot \sigma^2_{m,t+1} + \varepsilon_{m,t+1}$$
$$E_t\left[\varepsilon^2_{i,t+1}\right] \equiv \sigma^2_{i,t+1} = \gamma^i_0 + \gamma^i_1 \varepsilon^2_{i,t} + \gamma^i_2 \sigma^2_{i,t}$$
$$E_t\left[\varepsilon^2_{m,t+1}\right] \equiv \sigma^2_{m,t+1} = \gamma^m_0 + \gamma^m_1 \varepsilon^2_{m,t} + \gamma^m_2 \sigma^2_{m,t}$$
$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1}$$

where ρ_{ij} is the constant conditional correlation between $R_{i,t+1}$ and $R_{j,t+1}$. The parameters and their t-statistics are estimated using the excess returns on the aggregate market portfolio and the 10 value-weighted book-to-market (value vs. growth) portfolios for the sample period July 1926-December 2007.

Intercepts	Coefficient	t-statistic	Slopes	Coefficient	t-statistic
α_1	-0.0002	-0.07	β_1	3.5879	2.38
α_2	-0.0014	-0.55	β_2	4.5015	3.82
α_3	-0.0016	-0.65	β_3	4.7199	3.97
α_4	0.0004	0.19	β_4	3.9099	3.39
α_5	0.0020	0.89	β_5	3.6286	2.96
α_6	0.0022	0.94	β_6	3.5694	2.81
α_7	0.0008	0.36	β_7	4.3335	3.74
α_8	0.0011	0.56	β_8	4.6363	4.30
α_9	0.0016	0.73	β_9	4.4346	4.17
α_{10}	0.0015	0.56	β_{10}	4.3777	3.72
α_m	0.0003	0.14	β_m	3.7612	3.62
LogL	27,803.62				

$H_0 : \beta_1 = \beta_2 = \ldots = \beta_m$ $\Rightarrow$ LR statistic = 10.88 (p-value = 45.34%)

$H_0 : \alpha_1 = \alpha_2 = \ldots = \alpha_m = 0$ $\Rightarrow$ Wald statistic = 7.22 (p-value = 78.14%)

$H_0 : \alpha_1 = \alpha_{10}$ $\Rightarrow$ Wald statistic = 0.20 (p-value = 65.54%)

Table 6. Multivariate GARCH-in-mean Model with Dynamic Conditional Correlations and a Common Slope Coefficient: 10 Value-Weighted Book-to-Market Portfolios

Entries report the maximum likelihood parameter estimates of the multivariate GARCH-in-mean model with dynamic conditional correlations and a common slope coefficient on the conditional variance-covariance matrix:

$$R_{i,t+1} = \alpha_i + \beta \cdot \sigma_{im,t+1} + \sigma_{i,t+1} u_{i,t+1}$$

$$R_{m,t+1} = \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \sigma_{m,t+1} u_{m,t+1}$$

$$E_t\left[\varepsilon^2_{i,t+1}\right] \equiv \sigma^2_{i,t+1} = \gamma^i_0 + \gamma^i_1 \sigma^2_{i,t} u^2_{i,t} + \gamma^i_2 \sigma^2_{i,t}$$

$$E_t\left[\varepsilon^2_{m,t+1}\right] \equiv \sigma^2_{m,t+1} = \gamma^m_0 + \gamma^m_1 \sigma^2_{m,t} u^2_{m,t} + \gamma^m_2 \sigma^2_{m,t}$$

$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1}$$

$$\rho_{im,t+1} = \frac{q_{im,t+1}}{\sqrt{q_{ii,t+1} \cdot q_{mm,t+1}}}, \quad q_{im,t+1} = \overline{\rho}_{im} + a_1 \cdot \left(u_{i,t} \cdot u_{m,t} - \overline{\rho}_{im}\right) + a_2 \cdot \left(q_{im,t} - \overline{\rho}_{im}\right)$$

where $\overline{\rho}_{im}$ is the unconditional correlation between $u_{i,t}$ and $u_{m,t}$. The parameters and their *t*-statistics are estimated using the excess returns on the aggregate market portfolio and the 10 value-weighted book-to-market (value vs. growth) portfolios for the sample period from July 1926 to December 2007.

Portfolio	Intercepts	Coefficient	t-statistic
Growth	α_1	-0.0023	-1.30
BM 2	α_2	-0.0017	-1.05
BM 3	α_3	-0.0014	-0.91
BM 4	α_4	-0.0011	-0.72
BM 5	α_5	0.00010	0.07
BM 6	α_6	0.00012	0.09
BM 7	α_7	0.00010	0.07
BM 8	α_8	0.0009	0.63
BM 9	α_9	0.0007	0.45
Value	α_{10}	0.0008	0.46
Market	α_m	-0.0014	-0.91
Common Slope	β	5.1185	6.47
Maximized log-likelihood	LogL	27,874.77	

$H_0 : \alpha_1 = \alpha_2 = ... = \alpha_m = 0 \;\Rightarrow\;$ Wald statistic = 16.19 (*p*-value = 13.43%)

Table 7. Multivariate GARCH-in-mean Model with Dynamic Conditional Correlations and Different Slope Coefficients: 10 Value-Weighted Book-to-Market Portfolios

Entries report the maximum likelihood parameter estimates of the multivariate GARCH-in-mean model with dynamic conditional correlations and different slopes on the conditional variance-covariance matrix:

$$R_{i,t+1} = \alpha_i + \beta_i \cdot \sigma_{im,t+1} + \sigma_{i,t+1} u_{i,t+1}$$

$$R_{m,t+1} = \alpha_m + \beta_m \cdot \sigma^2_{m,t+1} + \sigma_{m,t+1} u_{m,t+1}$$

$$E_t\left[\varepsilon^2_{i,t+1}\right] \equiv \sigma^2_{i,t+1} = \gamma^i_0 + \gamma^i_1 \sigma^2_{i,t} u^2_{i,t} + \gamma^i_2 \sigma^2_{i,t}$$

$$E_t\left[\varepsilon^2_{m,t+1}\right] \equiv \sigma^2_{m,t+1} = \gamma^m_0 + \gamma^m_1 \sigma^2_{m,t} u^2_{m,t} + \gamma^m_2 \sigma^2_{m,t}$$

$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1}$$

$$\rho_{im,t+1} = \frac{q_{im,t+1}}{\sqrt{q_{ii,t+1} \cdot q_{mm,t+1}}}, \quad q_{im,t+1} = \overline{\rho}_{im} + a_1 \cdot \left(u_{i,t} \cdot u_{m,t} - \overline{\rho}_{im}\right) + a_2 \cdot \left(q_{im,t} - \overline{\rho}_{im}\right)$$

where $\overline{\rho}_{im}$ is the unconditional correlation between $u_{i,t}$ and $u_{m,t}$. The parameters and their *t*-statistics are estimated using the excess returns on the aggregate market portfolio and the 10 value-weighted book-to-market (value vs. growth) portfolios for the sample period from July 1926 to December 2007.

Intercepts	Coefficient	t-statistic	Slopes	Coefficient	t-statistic
α_1	-0.0005	-0.20	β_1	4.2193	3.18
α_2	-0.0021	-0.98	β_2	5.3753	4.97
α_3	-0.0023	-1.13	β_3	5.6301	5.27
α_4	-0.00012	-0.06	β_4	4.6176	4.37
α_5	0.0014	0.74	β_5	4.3834	4.13
α_6	0.0015	0.78	β_6	4.3359	3.80
α_7	0.00009	0.05	β_7	5.1603	4.94
α_8	0.00019	0.11	β_8	5.5864	5.65
α_9	0.00031	0.17	β_9	5.3793	5.43
α_{10}	0.00009	0.04	β_{10}	5.5420	5.36
α_m	-0.00036	-0.20	β_m	4.5907	4.91
LogL	27,881.38				

$H_0 : \beta_1 = \beta_2 = ... = \beta_m$ $\Rightarrow$ LR statistic = 13.22 (*p*-value = 27.92%)

$H_0 : \alpha_1 = \alpha_2 = ... = \alpha_m = 0$ $\Rightarrow$ Wald statistic = 9.46 (*p*-value = 57.98%)

$H_0 : \alpha_1 = \alpha_{10}$ $\Rightarrow$ Wald statistic = 0.0442 (*p*-value = 83.36%)

Table 8. Estimating Risk-Return Tradeoff with Unconditional Measures of Risk and a Common Slope Coefficient: 10 Value-Weighted Book-to-Market Portfolios

Entries report the SUR panel regression estimates of the portfolio-specific intercepts, common slope coefficients on the unconditional variance-covariance matrix, and their *t*-statistics (in parentheses) from the following system of equations:

$$R_{i,t+1} = \alpha_i + \beta \cdot \sigma_{im,t} + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + \beta \cdot \sigma_{m,t}^2 + \varepsilon_{m,t+1}$$

where the parameters and their *t*-statistics are estimated using the excess returns on the market portfolio and the 10 value-weighted book-to-market portfolios. The *t*-statistics adjust for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The one-month lagged unconditional variance of excess returns on the market portfolio ($\sigma_{m,t}^2$) and the one-month lagged unconditional covariances between excess returns on the book-to-market portfolios and excess returns on the market portfolio ($\sigma_{im,t}$) are obtained from the past 24, 36, 48, and 60 months of data. The last row presents the Wald statistics and their *p*-values in square brackets from testing the joint hypothesis that all intercepts equal zero $H_0 : \alpha_1 = \alpha_2 = ... = \alpha_{10} = \alpha_m = 0$.

Portfolio	Intercepts	24-month	36-month	48-month	60-month
Growth	α_1	0.0046	0.0045	0.0045	0.0042
		(2.32)	(2.21)	(2.19)	(2.00)
BM 2	α_2	0.0060	0.0057	0.0055	0.0051
		(3.11)	(2.87)	(2.76)	(2.52)
BM 3	α_3	0.0057	0.0056	0.0054	0.0052
		(3.05)	(2.91)	(2.79)	(2.62)
BM 4	α_4	0.0059	0.0058	0.0056	0.0053
		(2.81)	(2.66)	(2.52)	(2.35)
BM 5	α_5	0.0067	0.0065	0.0063	0.0062
		(3.43)	(3.23)	(3.07)	(2.96)
BM 6	α_6	0.0071	0.0071	0.0068	0.0066
		(3.30)	(3.21)	(3.00)	(2.88)
BM 7	α_7	0.0073	0.0073	0.0069	0.0068
		(3.14)	(3.03)	(2.83)	(2.75)
BM 8	α_8	0.0088	0.0089	0.0087	0.0084
		(3.65)	(3.58)	(3.40)	(3.25)
BM 9	α_9	0.0093	0.0092	0.0090	0.0087
		(3.52)	(3.39)	(3.24)	(3.12)
Value	α_{10}	0.0099	0.0101	0.0100	0.0096
		(3.09)	(3.06)	(2.98)	(2.81)
Market	α_m	0.0057	0.0056	0.0054	0.0052
		(2.98)	(2.84)	(2.73)	(2.55)
Common Slope	β	0.1729	0.1184	0.2864	0.5030
		(0.73)	(0.43)	(0.92)	(1.42)
H_0: Intercepts = 0	Wald	23.38	22.06	20.60	19.28
		[0.0156]	[0.0239]	[0.0377]	[0.0563]

Table 9. Risk Premiums Induced by Conditional Covariation with Macroeconomic Variables

Entries report the maximum likelihood parameter estimates of the ICAPM with intertemporal hedging demand based on the dynamic conditional correlation model with a common slope coefficient on the conditional variance-covariance matrix:

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + \beta \cdot \sigma_{im,t+1} + \lambda \cdot \sigma_{ix,t+1} + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \lambda \cdot \sigma_{mx,t+1} + \varepsilon_{m,t+1} \\
E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] &\equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1} \\
E_t\left[\varepsilon_{i,t+1}\varepsilon_{x,t+1}\right] &\equiv \sigma_{ix,t+1} = \rho_{ix,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{x,t+1} \\
E_t\left[\varepsilon_{m,t+1}\varepsilon_{x,t+1}\right] &\equiv \sigma_{mx,t+1} = \rho_{mx,t+1} \cdot \sigma_{m,t+1} \cdot \sigma_{x,t+1}
\end{aligned}
$$

where $\rho_{ij|t+1}$ is the dynamic conditional correlation between $R_{i,t+1}$ and $R_{j,t+1}$. The parameters and their *t*-statistics are estimated using the excess returns on the aggregate market portfolio and the 10 value-weighted book-to-market (value vs. growth) portfolios for the sample period from July 1926 to December 2007. $\sigma_{im,t+1}$ measures the time-*t* expected conditional covariance between the excess returns on each portfolio $(R_{i,t+1})$ and the market portfolio $(R_{m,t+1})$, $\sigma_{ix,t+1}$ measures the time-*t* expected conditional covariance between the excess returns on each portfolio $(R_{i,t+1})$ and the innovations in a macroeconomic variable proxied by the first difference $(\Delta X_{t+1} = X_{t+1} - X_t)$, and $\sigma_{mx,t+1}$ measures the time-*t* expected conditional covariance between the excess returns on the market portfolio $(R_{m,t+1})$ and the innovations in a macroeconomic variable (ΔX_{t+1}).

Panel A. Controlling for Conditional Covariation of Portfolio Returns with DEF, TERM, RREL, and DIV

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + \beta \cdot \sigma_{im,t+1} + \lambda_1 \cdot \sigma_{i,DEF,t+1} + \lambda_2 \cdot \sigma_{i,TERM,t+1} + \lambda_3 \cdot \sigma_{i,RREL,t+1} + \lambda_4 \cdot \sigma_{i,DIV,t+1} + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \lambda_1 \cdot \sigma_{i,DEF,t+1} + \lambda_2 \cdot \sigma_{i,TERM,t+1} + \lambda_3 \cdot \sigma_{i,RREL,t+1} + \lambda_4 \cdot \sigma_{i,DIV,t+1} + \varepsilon_{m,t+1}
\end{aligned}
$$

where $\sigma_{i,DEF,t+1}$, $\sigma_{i,TERM,t+1}$, $\sigma_{i,RREL,t+1}$, and $\sigma_{i,DIV,t+1}$ measure, respectively, the time-*t* expected conditional covariance between the excess returns on each portfolio *i* and the change in default spread (Δ*DEF*), the change in term spread (Δ*TERM*), the change in short-term interest rate (*RREL*), and the change in aggregate dividend yield (Δ*DIV*).

Cov(R_i, R_m)	Cov(R_i, Δ*DEF*)	Cov(R_i, Δ*TERM*)	Cov(R_i, *RREL*)	Cov(R_i, Δ*DIV*)
6.0727	–1.3573			
(5.39)	(–0.41)			
5.2349		–0.7462		
(4.76)		(–0.64)		
5.3244			–0.3459	
(4.90)			(–0.50)	
5.1246				0.9327
(4.65)				(2.23)

Table 9 (continued)

Panel B. Controlling for Conditional Covariation of Portfolio Returns with INF and OUT

$$R_{i,t+1} = \alpha_i + \beta \cdot \sigma_{im,t+1} + \lambda_1 \cdot \sigma_{i,INF,t+1} + \lambda_2 \cdot \sigma_{i,OUT,t+1} + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \lambda_1 \cdot \sigma_{i,INF,t+1} + \lambda_2 \cdot \sigma_{i,OUT,t+1} + \varepsilon_{m,t+1}$$

where $\sigma_{i,INF,t+1}$ measures the time-t expected conditional covariance between the excess returns on each portfolio i and the change in the monthly inflation rate (ΔINF) and $\sigma_{i,OUT,t+1}$ measures the time-t expected conditional covariance between the excess returns on each portfolio i and the change in the output growth (ΔOUT).

Cov(R_i, R_m)	Cov(R_i, ΔINF)	Cov(R_i, ΔOUT)
5.5301	–1.9004	
(5.30)	(–2.19)	
5.9496		0.8379
(3.12)		(1.54)
6.3966	–1.6447	0.7057
(3.34)	(–2.04)	(1.43)

Panel C. Controlling for Conditional Covariation of Portfolio Returns with INF and DIV

$$R_{i,t+1} = \alpha_i + \beta \cdot \sigma_{im,t+1} + \lambda_1 \cdot \sigma_{i,INF,t+1} + \lambda_2 \cdot \sigma_{i,DIV,t+1} + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + \beta \cdot \sigma^2_{m,t+1} + \lambda_1 \cdot \sigma_{i,INF,t+1} + \lambda_2 \cdot \sigma_{i,DIV,t+1} + \varepsilon_{m,t+1}$$

where $\sigma_{i,INF,t+1}$ measures the time-t expected conditional covariance between the excess returns on each portfolio i and the change in the monthly inflation rate (ΔINF) and $\sigma_{i,DIV,t+1}$ measures the time-t expected conditional covariance between the excess returns on each portfolio i and the change in the aggregate dividend yield (ΔDIV).

Cov(R_i, R_m)	Cov(R_i, ΔINF)	Cov(R_i, ΔDIV)
6.0231	–1.3154	0.2396
(4.03)	(–1.98)	(1.28)

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