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Intense Network Competition

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Abstract

First, we demonstrate how unregulated price setting in mobile telecommunications may lead to monopolization, even when networks are highly substitutable. Second, we demonstrate that a menu of structural rules, including (*i*) mandatory interconnection, (*ii*) reciprocal access prices and (*iii*) a ban on price discrimination of calls to other networks may restore competition. This regulation requires neither demand data nor information about call costs.
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1 Introduction

The European mobile telecommunications sector is currently under dual regulation.[1] Various *structural regulations* increase competition between mobile networks, by reducing differentiation and consumer switching costs. Extensive coverage obligations make the networks overlap geographically. Mandatory interconnection make it possible to make calls between networks. Number-portability allow consumers who want to switch networks to retain their phone number. Remaining network differentiation results mainly from tariff constructions and the creation of consumer switching costs, which can also be reduced by regulatory intervention.

At the same time, the sector is still under price regulation. The fees charged for terminating calls from one network to another must typically be cost-based. The problems of price regulation are well-documented. A recent official report describes excessive bureaucratic complications (SOU, 2006). Companies and regulators often have divergent views of most issues, from the competitive situation to the level of cost, and the decisions are usually appealed. Legal proceedings are known to drag on for years. In addition, companies have great difficulties in predicting the eventual decisions by regulatory agencies and courts.

We analyze whether further reductions of network differentiation and switching costs would be enough to create a competitive environment allowing the price regulations to be removed. In case competition would still be imperfect, we like to know if there are additional structural remedies that may be added to restore competition.

[1]The European regulations are laid down in the Framework Directive and the Access and Interconnection Directive. The Access Directive provides a menu of possible remedies for the interconnection markets, including mandatory access, price control, non-discrimination and transparency.

Limits to Competition First, we demonstrate why and how unregulated price setting may lead to monopolization even when networks are highly substitutable. Entry is deterred by imposing three *margin squeezes*. First, the access price charged by the monopolist for termination is so high in relation to the price the monopolist charges for calls inside its own network (on-net) that it is impossible for an entrant to compete on termination of calls in the monopolist's network. Second, the monopolist sets a price for calls to any entrant (off-net) which is so low that the entrant cannot even profit from termination of calls in its own network. Third, the monopolist pays so little for call termination in the entrant's network that the entrant earns no access revenues. To successfully monopolize the market when networks are close substitutes, a monopolist must squeeze the entrants in all three markets.

The risk of monopolization hinges crucially, however, on asymmetric bargaining power between the networks, or a mechanism for the monopolist to share its profits with an entrant. A prudent regulator will probably ask for additional tools to reduce the danger of monopolization .

A Simple Regulation Solves the Problem Second, we demonstrate that a menu of *structural rules* is sufficient to restore competition when networks are perfect substitutes. The menu include (*i*) mandatory interconnection; (*ii*) reciprocal access prices and (*iii*) a ban on price discrimination between on-net calls and off-net calls. Policies (*i*) and (*iii*) intensify call price competition by eliminating network externalities. Policy (*ii*) ensures that one firm cannot push the competitor out of the market by underpricing access.

The policies are informationally undemanding: no information is required about demand or the costs of completing calls. They are thus easy to implement and should also be transparent to the industry, thereby reducing the often substantial costs of regulation.

Mandatory interconnection and reciprocal access prices are standard elements in the regulator's toolbox. A ban on call price discrimination is novel, but consistent with e.g. EU regulations. In Estonia, for instance, the incumbent is disallowed to price discriminate between on-net and off-net prices of fixed calls (ERG, 2007).

Finally, it is well-known that an equilibrium in pure strategies, i.e. stable call prices, may fail to exist in telecom markets. We consider the problem of non-existence as real, not as an artefact of the model. We assume that regulators prefer stable prices. We will demonstrate that the policy sketched above guarantees the existence of a pure strategy equilibrium. Other policies (discussed below) do not.

Other Common Policies may Fail Finally, we also analyze the properties of a number of common regulatory policies. A simple *prohibition of margin squeezes inside one's own network* may be counter-productive. Implementing such a policy may lead to a softening of competition by establishing a credible floor on on-net prices instead of the intended ceiling on access prices.

A ban on *margin squeezes on termination of calls in the entrant's network*, would break the incumbent´s monopoly power, but would require information about the cost of calls in addition to being inefficient: calls would be priced above marginal cost.

Requiring a *reciprocal access price above the marginal termination cost* will induce competitive prices even without the additional call price regulation. However, the efficiency of this policy may be limited by arbitrage.

If the access price happens to be too high in relation to the competitive price, each network could open a subscription in the competitor's network and profit from making an unbounded number of off-net calls to one's own network. Competition would be non-viable. In the presence of arbitrage, the informational requirements of this policy are substantial.

Contributions We extend the analysis of network competition to the case of strong network externalities by assuming a low (zero) degree of network differentiation. Most papers in the literature have followed in the footstep of Armstrong (1998) and Laffont, Rey and Tirole (1998a and b), henceforth *LRT*, by assuming a very high degree of network differentiation. Exceptions are Doganoglu and Tauman (2002), Jeon and Hurkens (2008) and Stennek and Tangerås (2008), all of whom start from the premise that networks are interconnected, access prices are reciprocal and call price discrimination is illegal. The present paper makes no such restrictions, but presents a rationale for why these three policies might form the basis of a sound regulatory policy and, therefore, a rational for the modelling strategy.

Most of the literature concerned with market performance considers the problem of collusive access prices. Instead, Calzada and Valletti (2008) study how reciprocal access prices can be used to deter entry when networks are poor substitutes. The present paper complements Calzada and Valletti (2008) by showing that entry deterrence can be achieved even when networks are close substitutes, provided the access prices are asymmetric.

We show that networks may have an incentive not to interconnect because subsequent cut-throat competition will eradicate all profits. Carter and Wright (1999) also study the incentives to connect, but conclude that the problem is anti-competitive agreements, rather than reaching an agreement. Their conclusion which is based on a model of differentiated networks. This shows that the degree of network differentiation is likely to be important for the networks' incentives to connect.

Discussion of the Assumptions The market for telecommunications clearly is a lot more complicated than the model we consider. We have ignored fixed telephony as we expect mobile telephony to become the dominating medium for voice telephony. Mobile penetration exceeds 100% in many developed countries, the UK and Sweden being two examples. Mobile telephony is superior to fixed telephony in many dimensions, not only regarding mobility, but also in terms of available services, such as text messaging and multimedia access. With mobile prices converging to fixed prices, more and more subscribers can be expected to abandon fixed telephony altogether.

Twenty-six out of thirty OECD countries have three or more national operators. Moreover, switching between networks is costly due to lead times. Our assumptions of duopoly and perfect network substitutability seem at odds with reality, but are not critical. We show in the Appendix that monopoly call prices can be sustained in duopoly even in a standard Hotelling model of network differentiation, provided networks are not too differentiated. Obviously, a deregulated market can be monopolized independently of the number of networks, provided all challengers accept the same unfavorable asymmetric call termination conditions.

In a companion paper (Stennek and Tangerås, 2008), we extend the analysis to cover the case of many networks and imperfect network substitutability. It turns out that the market can be monopolized *even* if the networks are required to charge reciprocal access prices and are forbidden to create network externalities by means of differentiating call prices or making the networks incompatible. The problem now is collusion, rather than entry deterrence. Market concentration has two countervailing effects on network competition. A standard competitive effect working through the elasticity of demand pulls in favour of low prices. But a cost effect goes in the other direction. A larger fraction of each network's calls are terminated off-net when there are more networks. If off-net calls are subjected to an access price mark-up, the effective marginal call cost goes up when there are more networks. If the access price is high enough, the cost effect is so strong as to fully neutralize the competitive effect, leaving equilibrium prices unchanged at the monopoly level. To restore competition, we add a cost-independent cap on access prices to the regulatory menu (i)-(iii). For a sufficiently tight cap, call prices fall as more networks enter the market and converge to marginal cost as network differentiation goes to zero.

Typically, telecom operators have large fixed costs. Fixed costs would only serve to exacerbate the problem of monopolization under a laissez-faire policy. However, the networks are unable to recover their fixed costs under competitive prices. Therefore, some lump-sum transfers might be required to secure budget balance. Information about fixed costs may be more readily available than information about marginal call costs. Moreover, it is not socially optimal to strive for "perfect competition", for example by removing all network differentiation. We leave the issue of optimal network differentiation for future research.

We make the assumption that networks use linear prices, but show elsewhere (available upon request) that our main results do not hinge upon this assumption. The industry may be monopolized also under two-part call tariffs, and any equilibrium by necessity is competitive under the menu of regulatory policies (i)-(iii). We believe that an analysis with linear prices is interesting in its own right. Standard models of nonlinear network competition seem unable to capture interesting economic trade-offs and important inefficiencies. In the standard models there are no consumer losses due to distortions. All calls are priced at marginal cost. Marginal costs are too low since the networks tend to agree on access prices below the marginal cost of termination (Gans and King, 2001; Calzada and Valletti, 2008). In addition to producing counter-factual predictions - off-net prices are below on-net prices - call prices are *too low* from a welfare viewpoint. Most regulators are concerned with call prices being *too high*. In reality, operators offer menus of two-part tariffs, presumably to price discriminate between heterogeneous consumers. Under non-linear prices, operators would distort call prices to extract rent from subscribers, precisely as in a model without two-part tariffs. Thus, a model with homogeneous consumers which abstracts from two-part tariffs captures the same inefficiencies and trade-offs one would expect to materialize in a more elaborate (and complicated) model with two-part tariffs and heterogeneous consumers (see Dessein, 2003 for an early exploration).

2 Model

Consider a mobile telecom market with 2 operators/networks. The interaction is described as a game with four stages. First, the operators negotiate access prices. Subsequently, the operators unilaterally and simultaneously set the prices for calls inside their network, so-called on-net calls, and calls outside the network, so-called off-net calls. Third, the consumers choose a subscription based on the call prices, and finally decide how many calls to make. There is a continuum of consumers of unit measure, each of whom buys one subscription, i.e. the market is fully covered.

Call Demand In stage four, all consumers have taken a subscription in one of the networks, and they all make calls on the basis of the call prices. Every subscriber to network i makes q_{ii} calls to every subscriber in network i at the on-net price p_{ii} per call and q_{ij} calls to every subscriber in the other network j, at the off-net price p_{ij} per call. No subscriber attaches any value to incoming calls and therefore receives utility $s_iU(q_{ii})+(1-s_i)U(q_{ij})+y$, where $s_i \in [0,1]$ is the customer base of operator i, $U(\cdot)$ is a twice differentiable, strictly increasing and strictly concave utility function, and y is a numeraire good. Utility maximization subject to the budget constraint $s_iq_{ii}p_{ii} + (1 - s_i)q_{ij}p_{ij} + y \leq I$ yields demand $q_{ii} = D(p_{ii})$ and $q_{ij} = D(p_{ij})$, where D is the inverse of U', and I denotes income. We assume that $D(0)$ is finite and that $\lim_{p\to\infty} D(p) = 0$.

Network Subscriptions In stage three, consumers choose a network on the basis of the operators' announced end-user prices. Let $u(p) = U(D(p)) - D(p)p$ be the indirect call utility and assume that $\lim_{p\to\infty} u(p) = 0$. The indirect utility (net of income) of belonging to network i, then, is

$$V(p_i, s_i) = s_iu(p_{ii}) + (1 - s_i)u(p_{ij}), \tag{1}$$

where $p_i = (p_{ii}, p_{ij})$ is the call price profile of operator i. Let $\mathbf{p} = (p_1, p_2)$ be the vector of call prices.

Since all subscribers choose network simultaneously, customer bases are not observable when the choice of network is made. Instead, the network is chosen on the basis of the expected utility of belonging to different networks, i.e. on the basis of call prices and expected customer bases. In rational expectations equilibrium, however, expected and real customer bases are the same.

By assumption, networks are perfect substitutes. Consequently, network

i has a positive customer base only if $V(p_i, s_i) \geq V(p_j, s_j)$. If consumers are indifferent between both networks independently of customer bases, we assume the operators to share customers equally.[2]

With perfect network substitutability there are multiple equilibria in customer bases for some combinations of call prices. In particular, if $u(p_{ii}) > u(p_{ji})$ and $u(p_{ij}) < u(p_{jj})$, there are two extreme equilibria with either firm monopolizing the market and one interior equilibrium with positive market shares for both networks. The interior equilibrium is unstable, however. If just a small fraction of customers happened to switched network, it would be better for the rest of the customers to follow, as a result of positive network externalities. The process would not stop until one of the asymmetric equilibria is reached (cf. Tirole, 1988). The following Lemma describes the stable equilibria in customer bases.

Lemma 1 *The stable customer base of operator i is given by*

$$S_i(\mathbf{p}) = \begin{cases} 1 & \text{if } u(p_{ii}) \geq [>] u(p_{ji}) \quad \text{and } u(p_{ij}) > [\geq] u(p_{jj}) \\ 1/2 & \text{if } u(p_{ii}) = u(p_{ji}) \quad \text{and } u(p_{ij}) = u(p_{jj}) \\ 0 & \text{if } u(p_{ii}) \leq [<] u(p_{ji}) \quad \text{and } u(p_{ij}) < [\leq] u(p_{jj}) \\ 1 \text{ and } 0 & \text{if } u(p_{ii}) > u(p_{ji}) \quad \text{and } u(p_{ij}) < u(p_{jj}) \\ \frac{u(p_{ij}) - u(p_{jj})}{u(p_{ij}) + u(p_{ji}) - u(p_{ii}) - u(p_{jj})} & \text{if } u(p_{ii}) < u(p_{ji}) \quad \text{and } u(p_{ij}) > u(p_{jj}) \end{cases}$$

The proof of the above and all subsequent lemmas and propositions are contained in the Appendix.

In the three first cases, the prices are such that there are no network effects: one customer's preferred network does not depend on the choices of

[2] In reality, all differentiation could probably not be removed, nor would it be socially optimal to do so. In Section 4 we check the implications of variations in network differentiation on our main results.

other customers. All customers have the same preferences; thus, they all end up with the same operator. An exception is when the two operators offer the same benefits, in which case the operators share the market equally. In the fourth case, the prices induce positive network externalities, and the networks become strategic complements: all customers prefer to belong to the network with the largest customer base. In stable equilibrium, all customers belong to the same network, but which one is undetermined. In the final case, the prices induce negative network externalities whereby the networks become strategic substitutes. The larger is the customer base, the less attractive is the network. In this case, the equilibrium customer bases balance the prices to make the two networks equally attractive for customers.

Network Profit In stage two, network i sets (non-negative) call prices p_i to maximize its profit

$$\pi_i(p_i) = S_i \left[S_i \left(p_{ii} - c\right) D\left(p_{ii}\right) + S_j \left(p_{ij} - a_i - c_o\right) D\left(p_{ij}\right) + S_j \left(a_j - c_t\right) D\left(p_{ji}\right)\right], \tag{2}$$

where c_t is the marginal cost of call termination, c_o is the marginal cost of call origination, $c = c_t + c_o$ is the total marginal cost of a completed call, a_i is access price paid by network i for every call terminated in the competitor's network and a_j is the compensation i receives for every outside call it terminates in the own network. The term in brackets is the profit per subscriber. The first term is the profit from on-net calls, the second term is the profit from outgoing off-net calls, and the third term is the profit on incoming off-net calls.

To guarantee that the monopoly profit function $D\left(p\right)\left(p - c\right)$ has a unique interior maximum, $p^m > c$, we assume that $(p - c)\eta\left(p\right)/p$ is increasing in $p \geq c$ and that $\lim_{p\to\infty}(p - c)\eta\left(p\right)/p > 1$, where $\eta\left(p\right) = -pD'(p)/D(p)$

is the price elasticity of call demand. Common demand functions such as constant elasticity an linear demand fulfill these assumptions. Denote by $\pi^m = D(p^m)(p^m - c)$ the monopoly profit.

We consider Nash equilibria in pure strategy call prices, henceforth simply referred to as equilibria. Equilibria are indicated by an asterisk, so that $p_i^* = (p_{ii}^*, p_{ij}^*)$ is an equilibrium call price profile by operator i, $\mathbf{p}^* = (p_1^*, p_2^*)$ is a vector of equilibrium call prices, $S_i^* = S_i(\mathbf{p}^*)$ is a corresponding equilibrium customer base, and $\pi_i^* = \pi_i(p_i^*)$ denotes the equilibrium profit of network i.

In the first stage, the networks negotiate the pair (a_1, a_2) of access prices. As is standard in the literature, the access prices are set to maximize industry profit $\pi_1^* + \pi_2^*$. To guarantee that a network will not have an incentive to make phony calls to the other network, the marginal cost of off-net calls must be non-negative, i.e. $a_i \geq -c_o$.

Equilibrium with positive network externalities Certain call prices give rise to positive network externalities in the customers' choice of networks. One network attracting all customers is then a stable equilibrium outcome, but so is the other network attracting all customers. This happens if $u(p_{11}) > u(p_{21})$ and $u(p_{22}) > u(p_{12})$.

Positive network externalities are unlikely to arise in equilibrium, however. In such a case, one operator could always change its call price to get the same profit with certainty as it would *at most* obtain under positive network externalities.

Lemma 2 *Assume that for every price configuration which gives rise to positive network externalities, at least one operator assigns a positive subjective probability to both stable customer base configurations $S_i(\mathbf{p}) \in \{0, 1\}$. Then,* *there exists no equilibrium with positive network externalities.*

On the basis of Lemma 2 we henceforth disregard equilibria with positive network externalities.

Welfare Welfare is the sum of indirect consumer utility and firm profit:

$$\begin{aligned} W(\mathbf{p}) &= S_1 V(p_1, S_1) + S_2 V(p_2, S_2) + \pi_1 + \pi_2 \\ &= S_1^2(U(D(p_{11})) - cD(p_{11})) + S_2^2(U(D(p_{22})) - cD(p_{22})) \\ &\quad + S_1 S_2(U(D(p_{12})) - cD(p_{12}) + U(D(p_{21})) - cD(p_{21})). \end{aligned}$$

The social optimum is achieved by setting all prices equal to marginal cost in this model. The access prices have only an indirect bearing on welfare via the effect on call prices and firm profitability. The scope of regulation is to induce a competitive equilibrium, that is, an equilibrium in which all calls are made at marginal cost c.

3 Call Prices, Access Prices, Interconnection

3.1 Laissez-faire

Assume that operators are free to negotiate any access price between themselves, and that call prices are unregulated. A laissez-faire policy may lead to monopoly pricing:

Proposition 1 *If $a_i = c_t$ and a_j is sufficiently high ($a_j \geq \overline{a} = c_t + 2(p^m - c) + cD(0)/D(p^m)$), there exists an equilibrium. In any equilibrium, network i corners the market ($S_i^* = 1$), charges the monopoly price on on-net calls*

($p_{ii}^{*} = p^{m}$) and sets a price on off-net calls below the marginal cost of on-net calls ($p_{ij}^{*} \leq c$).

The proposition states that full monopolization is the unique equilibrium outcome. Access prices block competition even when networks are perfect substitutes and there are no fixed costs.

Since network i monopolizes the market, i will sometimes be referred to as the incumbent. Network j, then, is the entrant. The incumbent's pricing policy takes the form of a three so-called *margin squeezes*. First, the access price for termination in the incumbent's network is so high in relation to the incumbent's on-net price ($a_j > p_{ii}^{*} - c_o$) that it is impossible for an entrant to profitably compete on termination of calls in the incumbent's network. Second, the incumbent sets an off-net call price that is so low that the entrant cannot even profit from termination of calls in its own network ($p_{ij}^{*} \leq c$). Third, the incumbent's compensation to the competitor for call termination is so low that the entrant earns no access revenues ($a_i = c_t$). To successfully monopolize the market when the two networks are close substitutes, a firm must squeeze the competitor in all three markets. For example, a failure to squeeze the competitor in the competitor's own network ($p_{ij}^{*} > c$), would allow the competitor ample leeway to profitably monopolize the market for itself ($p_{jj} \in (c, p_{ij}^{*})$, $p_{ji} < p^{m} = p_{ii}^{*}$). Importantly, this suggests an extension of the margin squeeze criterion to include more markets than just termination in the squeezer's own network, the common policy today.

Full monopolization necessarily involves one of the operators cornering the market. It is impossible for the two operators to share the market and reach industry monopoly profits when the degree of network substitutability is very high.[3] We see three plausible reasons why such an asymmetric equilibrium may be a reasonable prediction. First, one of the networks may have substantially more bargaining power, e.g. resulting from an incumency advantage, starting out with all customers in the own network (cf. de Bijl and Peitz, 2002). Second, if both operators are international carriers with a home and a foreign market, they could agree on a geographical market segmentation allowing each operator to monopolize its home market. Third, profits can be distributed by means of joint ownership whereby each operator holds a 50% stake in the competitor.

We conclude there is a scope for a prudent regulator to intervene. Given the importance of margin squeezes, we consider first regulation of call prices.

3.2 Regulation of Call Prices

Prohibition of Margin Squeezes in the Own Network Assume that margin squeezes on the termination of calls inside one's own network are illegal, i.e. network i (the incumbent) is required to set a combination of on-net prices and access prices satisfying $p_{ii}^* \geq a_j + c_o$. This policy requires information about the cost of originating calls. But what is worse, a ban on margin squeezes might actually lead to *higher* call prices. The problem is that a ban on margin squeezes not only places a price ceiling on the access prices, but also a price floor on on-net calls. This price floor might serve to weaken competition. To illustrate this point we take advantage of the following result:

[3] Monopoly profits would require monopoly prices in all market segments, i.e. on both on-net and off-net calls. Such a pricing policy would lead to a division of the market and division of the monopoly profit between the two operators. However, each operator could easily corner the market by lowering its off-net price slightly below the monopoly level, thereby capturing the entire monopoly profit for itself.

Proposition 2 *Assume that* $a_i = c_t$ *and* $a_j \in [p^m - c_o, \overline{a})$*. Then, all equilibria have* i *cornering the market* ($S_i^* = 1$) *charging at most the monopoly price on on-net calls* ($p_{ii}^* \leq p^m$)*. There exists a* $\overline{p} \in (c, p^m)$*, such that all on-net prices* $p_{ii}^* \in (\overline{p}, p^m]$ *can be sustained in equilibrium.*

If, in laissez-faire, the monopolist is unable to raise the access price sufficiently (i.e. up to $\overline{a}$), the threat of entry might push down the equilibrium on-net price below the monopoly level, to say $p_{ii}^* < p^m = a_j + c_o$. With a ban on margin squeezes, the price would never fall below the floor $a_j + c_o = p^m$. In other words, a ban on margin squeezes may act as a commitment device, ensuring a high on-net price.

Prohibition of Predatory Prices in the Other Network Assume now that i is required to set $p_{ij}^* > c$. This policy may be described as a prohibition of predatory pricing. Such a policy would be as demanding as the previous one in terms on information about marginal cost, but would have the impact that no operator could establish monopoly power; see the argument above.

Three aspects are noteworthy. First, the threshold is the competitor's marginal cost $c = c_o + c_t$ and not $c_o + a_i$, which is operator i's marginal cost of sending calls to j. Second, the policy might lead to disturbances of call prices in the sense that a pure strategy equilibrium in call prices may then fail to exist. Moreover, the policy cannot possibly lead to the first-best outcome as it requires that off-net calls be priced above marginal cost c. Hence, better policies than this could exist.

Prohibition of Call Price Discrimination Both policies above are burdened with informational problems. A softer type of regulation than direct price regulation would be to prohibit price-discrimination between on- and off-net calls, i.e. requiring $p_{ii} = p_{ij} = p_i$. This policy requires no information at all about cost data. Moreover:

Proposition 3 *If call price discrimination is prohibited ($p_{ii} = p_{ij} = p_i$), the competitive outcome ($p_1^* = p_2^* = c$) is the only possible equilibrium.*

Uniform pricing forces firms into Bertrand competition with one another for customers, which necessarily forces prices down to marginal cost. It would appear that uniform pricing solves the monopoly problem. However, in the proposed equilibrium, operator i's profit is given by

$$\pi_i^* = \frac{1}{4}(a_j - a_i)D(c).$$

Unless operators charge reciprocal access prices, one operator will suffer a loss at marginal cost pricing. There is no reason to presume that the operators would actually agree on reciprocal access charges, as all profits would subsequently be competed away. If interconnection is mandatory, efficient bargaining suggests that the operators prefer to induce exit, although there is the issue of splitting the surplus from such an agreement. In case a pure strategy equilibrium fails to exist, prices are by definition stochastic, and the outcome cannot be efficient.

As we have seen, regulation of call prices is subject to a number problems including informational burdens, allocative inefficiency and a potentially destabilizing effect on call prices. Perhaps for this reason, mobile call prices are mostly left unregulated across Europe. Therefore, it seems fruitful to consider alternative policies for dealing with failures in the markets for electronic communications. Next, we consider regulation of access prices.

3.3 Regulation of Access Prices

Cost-based Regulation of Access Prices Since bans on margin squeezes may induce a floor on call prices rather than a ceiling on access prices, one may wish to consider policies that serve to reduce access prices. The standard regulatory solution is direct regulation:

Proposition 4 *If the operators are required to set access charges above termination cost ($a_i \geq c_t$) but below the "monopoly squeeze level," ($a_j < p^m - c_o$) no equilibrium exists which sustains monopoly profits.*

Regulation of access charges at least partly mitigates the monopolization problem. However, this policy is even more informationally demanding than call price regulation. Not only does the regulator need to have an assessment of marginal transmission costs, she also needs demand information to be able to infer the monopoly price. Second, even if this information were available, there is no guarantee that an equilibrium in call prices actually exists. Hence, access price regulation may destabilize the call price market when networks are highly substitutable.

Reciprocal Access Prices Above Marginal Termination Cost A policy which is less informationally demanding than the previous one is:

Proposition 5 *If the operators are required to set reciprocal access charges above termination cost ($a_1 = a_2 = a \geq c_t$) there exists an equilibrium, and every equilibrium is competitive.*

Proposition 5 states that call prices will be competed all the way down to marginal cost, provided the reciprocal access price is not too low.[4] At

[4] The exact distribution of customer bases is not determined. There are three possibilities, namely $S_1^* = 1/2$ and $p_{11}^* = p_{21}^* = p_{12}^* = p_{22}^* = c$, or $S_i^* = 1$ and $p_{ii}^* = c$, $i = 1, 2$; see the Appendix.

face value, the informational requirements of the policy appear to be weak. Reciprocity does not require demand data. Also, one can ensure that the lower bound is met by setting a generous access price. In reality, the feasible access price may be bounded above by an arbitrage condition. Suppose that off-net calls are priced at c, but access is priced at $a - c_t > c$. Then, each operator could open a subscription in the competitor's network and make off-net calls to one's own network. The gain from such a call would be $a - c_t$, but the cost would merely be c. Operators might be expected to take advantage of this arbitrage opportunity, which renders $c + c_t$ the upper bound to the feasible access price. Thus, information about the marginal cost of calls remains critical.

Since both networks have zero profit for any access price above, but may earn strictly positive profit from an access price below the marginal cost of termination (Gans and King, 2001), the networks have no incentive to reveal their costs. However, it is easy to determine whether the two operators price access below cost simply by looking at the call prices. With an access price below the marginal cost of termination, the marginal cost of an off-net call is lower than the marginal cost of an on-net call, which in turn implies that off-net calls are cheaper than on-net calls in equilibrium:

Proposition 6 *Assume that the two operators set a reciprocal access price below the marginal cost of termination ($a_1 = a_2 = a < c_t$). Then, every operator with a positive market share has an equilibrium off-net price which is lower than the on-net price ($p^*_{ij} < p^*_{ii}$ if $S^*_i > 0$).*

To eliminate any incentive to soften competition by underpricing access, the proposition suggests that access price regulation be combined with call price regulation. We now turn to an analysis of a combined regulation.

3.4 Combined Call Price and Access Price Regulation

As we have demonstrated above, call price or access price regulation alone seem incapable of achieving the competitive outcome. However, a combination of call price and access charge regulation is more promising.

Proposition 7 *Assume that call price discrimination is prohibited ($p_{ii} = p_{ij} = p_i$) and that operators are forced to charge reciprocal access prices ($a_1 = a_2 = a$). For every access price $a \geq -c_o$, the competitive outcome ($p_1^* = p_2^* = c$) is the unique equilibrium.*

A special case of the above is the much debated Bill-and-Keep regime ($a_1 = a_2 = 0$), which in combination with a ban on call-price discrimination would induce the competitive outcome with interconnected networks.

This policy requires neither demand nor cost data, when the networks are perfect substitutes as assumed here.

However, there is an additional problem. The result is derived under the implicit assumption that operators are interconnected. Without an interconnection agreement, there would be two incompatible networks in the market. Under perfect network substitutability there would be two stable equilibria, each leading to monopolization by one of the networks. The network externalities are so strong that both equilibria coexist for any price configuration satisfying $p_{11}^* < u^{-1}(0)$ and $p_{22}^* > u^{-1}(0)$.[5] In particular, monopoly call prices can be sustained in equilibrium. If the alternative is to sign an agreement and earn zero profits, it may be in both operators best interest not to sign any interconnection agreement. Thus, regulatory intervention into the retail and access markets may have to be assisted by mandatory interconnection.

[5] $S_i(\mathbf{p}^*) = 1$ is a stable customer base for any price configuration $\mathbf{p}^*$ where $p_{ii}^* < u^{-1}(0)$ and the two networks are disconnected. If a subscriber to network i expects a fraction s_j to choose network j. Then, the benefit $s_j u(p_{jj}^*)$ of subscribing to network j is lower than the benefit $(1 - s_j)u(p_{ii}^*)$ of subscribing to network i for s_j sufficiently low, but positive.

4 Concluding Remarks

The Monopolization Problem A laissez-faire approach to network competition may lead to monopoly call prices, even if all differentiation between networks would be removed. A network can sustain its monopoly power by using its access price to raise the rival's costs for terminating calls inside the own network while simultaneously setting predatory prices on calls terminated in the rival's network. Thus, there is a scope for regulatory intervention even in a market where network differentiation is low.

The crucial condition for monopolization is the possibility of setting asymmetric access prices. In other respects the result is robust.

Absence of differentiation is not crucial for the result. Monopoly call prices can be sustained in equilibrium even in a standard Hotelling model of network differentiation such as the one considered by *LRT*, provided networks are not too differentiated. This is shown in the proof of Proposition 1. In *LRT*, all equilibria break down when networks are sufficiently close substitutes, but this is because the networks are assumed to charge reciprocal access prices. There is no reason to presume that two networks would agree on symmetric access prices if the market could be monopolized under asymmetric access prices.

Absence of fixed costs is not crucial for the result. On the contrary. The higher are the investment costs, the less would one network have to squeeze the competitor(s) to preserve monopoly power.

Duopoly is not crucial for the result. Any number of competitors are blocked from gaining market power if they all accept same unfavorable call termination conditions.

Absence of two-part tariffs is not crucial either. With two-part tariffs there are no welfare losses due to distortions since the monopolist prices calls at marginal cost. Instead, the subscription fee is used to transfer all surplus from consumers to the monopolist. Still, intervention is warranted if the social planner values consumer surplus higher than network profit. The calculations are available on request.

A Regulatory Solution A combination of informationally undemanding policies are shown *sufficient* to restore competition under perfect network substitutability: (*i*) interconnection is mandatory; (*ii*) access prices are reciprocal; (*iii*) price discrimination between calls inside one's network (on-net calls) and calls to competing networks (off-net calls) is illegal.

Moreover, all three ingredients are *necessary* to generate a competitive environment. Unless interconnection is mandatory, the firms can create strong network externalities and thereby sustain monopoly on-net prices by making networks incompatible. With disconnected networks, access prices and off-net call prices do not matter for competition. If networks are allowed to differentiate their access prices, the network with the lowest access price is driven out of the market when networks are interconnected and call price discrimination prohibited. Thus, competition is non-viable. Finally, if the networks are interconnected and access prices are reciprocal, but call price discrimination is legal, the networks can soften competition by creating tariff-mediated network externalities through a manipulation of the joint access price (see, e.g. Gans and King, 2001; Armstrong and Wright, 2008; Gabrielsen and Vagstad, 2008).

Also the regulatory solution is robust in many dimensions. In a companion paper (Stennek and Tangerås, 2008) we analyze how the above menu of policies fare when networks are imperfect substitutes and there are more than two networks. If the regulator places an additional cap on all access prices -

we call this package STR (structural) regulation - call prices will converge to marginal cost as network differentiation goes to zero. Also, we find that a cap on access prices is necessary even when there are many networks and access prices are set in bilateral negotiations. Market concentration (the number of networks) has two countervailing effects on competition. The price elasticity of subscription demand increases when there are more networks. This competition effect pulls in favour of lower prices. However, a larger fraction of each network's calls are terminated off-net when there are more networks. If off-net calls are subjected to an access price mark-up, the effective marginal call cost goes up. In a fully covered, symmetric market where the call pattern is balanced, the cost effect fully neutralizes the competition effect, leaving equilibrium prices unchanged. Hence, reduced market concentration is no guarantee of low prices.

In the presence of fixed costs, perfect competition is not necessarily the social optimum. Instead, the networks must be allowed enough market power to be able to recover their costs. Jeon and Hurkens (2008) show that an access price *rule* given by $a_i - c_o = 2(p_i - c)$ would implement Ramsey prices when the networks have a fixed cost for each subscription. However, under this rule the problem still remains how the networks should cover costs that are independent of the amount of calls and of the number of subscribers. The STR (structural) regulation allows networks an operating profit which is declining as networks become closer substitutes. Consequently, there is an optimal degree of network substitutability under STR (structural) regulation at which the networks just break even. Optimal network differentiation is an interesting issue for future research.

Another possibility would be an auction, awarding licenses to the networks asking for the smallest subsidy for taking the costs of providing call services under the *STR* regulation.

The competitive result obtained for the menu of regulatory policies (i)-(iii) under perfect network substitutability extends to the case with two-part tariffs (available upon request). Any equilibrium would entail call prices equal to marginal cost and a subscription fee equal to the fixed subscription cost. However, the policy is demanding because the equilibrium exists if and only if the access price is equal to the marginal cost of termination. The networks themselves may prefer an access price different from the marginal termination cost thereby inducing stochastic call prices.

References

Armstrong, Mark (1998): Network Interconnection in Telecommunications, *Economic Journal* 108: 545-564.

——————— and Julian Wright (2008): Mobile Call Termination, mimeo, UCL.

Calzada, Joan and Tommaso M. Valletti (2008): Network Competition and Entry Deterrence, *Economic Journal* 118: 1223-1244.

Carter, Michael and Julian Wright (1999): Interconnection in Network Industries, *Review of Industrial Organization* 14: 1-25.

De Bijl, Paul and Martin Peitz (2002): *Regulation and Entry into Telecommunications Markets*. New York: Cambridge University Press.

Dessein, Wouter (2003): Network Competition in Nonlinear Pricing, *RAND Journal of Economics* 34: 593-611.

Doganoglu, Toker and Yair Tauman (2002): Network Competition and Access Charge Rules, *Manchester School* 70: 16-35.

Gabrielsen, Tommy S. and Steinar Vagstad (2008): Why is On-net Traffic Cheaper than Off-net Traffic? Access Markup as a Collusive Device, *European Economic Review* 52: 99-115.

Gans, Joshua S. and Stephen P. King (2001): Using 'Bill and Keep' Interconnect Arrangements to Soften Network Competition, *Economics Letters* 71: 413-420.

Jeon, Doh-Shin and Sjaak Hurkens (2008): A Retail Benchmarking Approach to Efficient Two-way Access Pricing: No Termination-based Price Discrimination, *RAND Journal of Economics*, forthcoming.

ERG (2007): Common Position on Symmetry of Fixed Call Termination Rates and Symmetry of Mobile Call Termination Rates, ERG (07) 83.

Laffont, Jean-Jacques, Patrick Rey and Jean Tirole (1998a): Network Competition: Overview and Nondiscriminatory Pricing, *RAND Journal of Economics* 29: 1-37.

——————, Patrick Rey and Jean Tirole (1998b): Network Competition: Price Discrimination, *RAND Journal of Economics* 29: 38-56.

——————, Scott Marcus, Patrick Rey and Jean Tirole (2003): Internet Interconnection and the Off-Net-Cost Pricing Principle, *RAND Journal of Economics* 34, 370-390.

SOU (2006): *Effektivare LEK*, Statens offentliga utredningar, Stockholm, 2006:88.

Stennek, Johan and Thomas P. Tangerås (2008): Competition vs. Regulation in Mobile Telecommunications: II - The Role of Market Concentration, manuscript, IFN and Gothenburg University.

Tirole, Jean (1988): *The Theory of Industrial Organization*, MIT Press, Cambridge, Massachusetts.

Appendix

Proof of Lemma 1

We do the proof case by case. First, assume that $u(p_{jj}) \leq u(p_{ij})$ and that $u(p_{ii}) \geq u(p_{ji})$, with at least one strict inequality. Note that

$$V(p_i, s_i) - V(p_j, s_j) = s_i(u(p_{ii}) - u(p_{ji})) + s_j(u(p_{ij}) - u(p_{jj}))$$

is strictly positive for all $s_i \in (0,1)$. Thus, $S_i \in \{0,1\}$. $S_i = 1$ is an equilibrium since $V(p_i, 1) - V(p_j, 0) = u(p_{ii}) - u(p_{ji}) \geq 0$. It is stable since $V(p_i, s_i) > V(p_j, s_j)$ for all $s_i < 1$. $S_i = 0$ is an equilibrium if and only if $u(p_{ij}) = u(p_{jj})$ since $V(p_i, 0) - V(p_j, 1) = u(p_{ij}) - u(p_{jj}) \geq 0$. The $S_i = 0$ equilibrium is unstable since $V(p_i, s_i) > V(p_j, s_j)$ for all $s_i > 0$.

Second, assume that $u(p_{jj}) = u(p_{ij})$ and $u(p_{ii}) = u(p_{ji})$. Then $V(p_i, s_i) = V(p_j, s_j)$ independently of the customer base s_i. In this case the operators by assumption divide the market, i.e. $S_i = 1/2$.

Third, assume that $u(p_{ji}) \geq u(p_{ii})$ and $u(p_{jj}) \geq u(p_{ij})$, with at least one strict inequality. $V(p_j, s_j) > V(p_i, s_i)$ for all $s_i \in (0,1)$ implies that $S_i \in \{0,1\}$. $S_i = 0$ is a stable equilibrium since $V(p_i, 0) - V(p_j, 1) = u(p_{ij}) - u(p_{jj}) \leq 0$, and $V(p_j, s_j) > V(p_i, s_i)$ for all $s_i > 0$. $S_i = 1$ is an equilibrium if and only if $u(p_{ii}) = u(p_{ji})$ since $V(p_i, 1) - V(p_j, 0) = u(p_{ii}) - u(p_{ji}) \leq 0$. The $S_i = 1$ equilibrium is unstable since $V(p_j, s_j) > V(p_i, s_i)$ for all $s_i < 1$.

Fourth, consider the case with positive network externalities, $u(p_{ii}) > u(p_{ji})$ and $u(p_{jj}) > u(p_{ij})$. $S_i = 1$ is an equilibrium because $V(p_i, 1) - V(p_j, 0) = u(p_{ii}) - u(p_{ji}) > 0$, $S_i = 0$ is an equilibrium because $V(p_i, 0) - V(p_j, 1) = u(p_{ij}) - u(p_{jj}) < 0$. The $S_i = 1$ equilibrium and $S_i = 0$ equilibria are both stable because $V(p_i, s_i) > V(p_j, s_j)$ even for positive, but small s_j and $V(p_i, s_i) < V(p_j, s_j)$ even for positive, but small s_i. There exists a unique interior equilibrium given by $\widehat{S}_i = \frac{u(p_{jj}) - u(p_{ij})}{u(p_{11}) + u(p_{22}) - u(p_{12}) - u(p_{21})}$, but this equilibrium is unstable since

$$V(p_i, \widehat{S}_i + \varepsilon) - V(p_j, \widehat{S}_j - \varepsilon) = \varepsilon(u(p_{11}) + u(p_{22}) - u(p_{12}) - u(p_{21}))$$

is strictly positive (negative) for all $\varepsilon > (<)\ 0$ and the equilibrium thus converges to one of the corner equilibria $S_i \in \{0, 1\}$ for every tremble $\varepsilon \neq 0$.

Finally, consider the case with negative network externalities. There can be no corner equilibrium since $V(p_i, 1) - V(p_j, 0) = u(p_{ii}) - u(p_{ji}) < 0$ and $V(p_i, 0) - V(p_j, 1) = u(p_{ij}) - u(p_{jj}) > 0$. The only possibility left is the interior equilibrium given by the solution to $V(p_i, S_i) = V(p_j, S_j)$:

$$S_i = \frac{u(p_{ij}) - u(p_{jj})}{u(p_{ij}) + u(p_{ji}) - u(p_{ii}) - u(p_{jj})}. \quad \blacksquare$$

Proof of Lemma 2

Assume throughout the proof that there are positive network externalities in equilibrium; $u(p^*_{11}) > u(p^*_{21})$ and $u(p^*_{22}) > u(p^*_{12})$. Let $\phi_i \in (0, 1)$ be the subjective probability operator i assigns to cornering the market. First, it cannot be the case that $p^*_{ii} < c$. In this case it would be strictly profitable for i to deviate to $p_{ii} = p^*_{ji}$ and $p_{ij} > p^*_{jj}$. The deviation would render i a unique market share $S_i = 0$ and, consequently, $\pi_i = 0 > \phi_i(p^*_{ii} - c)D(p^*_{ii}) = \pi^*_i$. Second, it cannot be the case that $p^*_{ii} \geq c$ and $p^*_{ii} \neq p^m$. It would then be strictly profitable for i to deviate to $p_{ii} \in (p^*_{ii}, \min\{p^m; p^*_{ji}\})$ and $p_{ij} = p^*_{jj}$ in case $p^*_{ii} \in [c, p^m)$ and to $p_{ii} \in (p^m, p^*_{ii})$ and $p_{ij} = p^*_{jj}$ in case $p^*_{ii} > p^m$. In both cases would i corner the market with certainty and earn $\pi_i = (p_{ii} - c)D(p_{ii}) > (p^*_{ii} - c)D(p^*_{ii}) \geq \pi^*_i$. Finally, it cannot be the case that $p^*_{ii} = p^m$ and $p_{ij} > p^*_{jj}$.

It would be strictly profitable for i to deviate to $p_{ij} = p^*_{jj}$, corner the market with certainty and earn $\pi_i = (p^m - c)D(p^m) > \phi_i(p^m - c)D(p^m) = \pi^*_i$. ■

Proof of Proposition 1

We first prove existence, and then move on to showing that all possible equilibria imply monopolization by operator i.

Existence Proposition 1, stating that laissez-faire leads to monopolization for very asymmetric access prices is a key result and the rationale for intervention in the market. To demonstrate that the result does not hinge crucially upon the assumption of perfect network substitutability, we shall prove the monopolization result in a standard Hotelling model of imperfect network competition based on the seminal work by Laffont, Rey and Tirole (1998a and b).

Assume that a continuum of consumers are uniformly distributed on the unit interval, and two networks are located at either end of the interval, operator 1 at 0 and operator 2 at 1. The (virtual) transportation cost $t \geq 0$ is a measure of horizontal differentiation, with perfect substitutability at $t = 0$. The utility of subscribing to network i for a consumer located at a distance k_i from operator i is

$$\alpha + V(p_i, s_i) - k_i t + I = \alpha + s_i u(p_{ii}) + (1 - s_i) u(p_{ij}) - k_i t + I,$$

where α is the direct utility of subscribing to a network and sufficiently large to guarantee that the market is fully covered. In interior equilibrium, $V(p_i, S_i) - S_i t = V(p_j, S_j) - S_j t$, which yields at most one interior customer base, given by

$$S_j(\mathbf{p}, t) = \frac{t + u(p_{ji}) - u(p_{ii})}{2t + u(p_{ij}) + u(p_{ji}) - u(p_{ii}) - u(p_{jj})}. \tag{3}$$

The interior equilibrium is stable if and only if $u(p_{12}) + u(p_{21}) \geq u(p_{11}) + u(p_{22})$ because

$$V(p_i, S_i + \varepsilon) - S_i t - V(p_j, S_j - \varepsilon) + S_j t = \varepsilon(u(p_{11}) + u(p_{22}) - u(p_{12}) - u(p_{21})),$$

and any tremble $\varepsilon > (<)\ 0$ pulls all subscribers towards operator i (j) if $u(p_{12}) + u(p_{21}) < u(p_{11}) + u(p_{22})$. We restrict attention to the set of stable customer bases.

The proof that monopolization is an equilibrium is a special case ($t = 0$) of the following more general result:

Proposition 8 *Let the customer bases be formed on the basis of the Hotelling model above. Assume that the networks are not too differentiated, i.e.* $t < u(p^m)$. *Let* $a_i = c_t$ *and* $a_j \geq u^{-1}(u(p^m) - t) - c_o$. *Then, the following price configuration constitutes an equilibrium:* $p_{ii}^* = p^m$, $p_{ij}^* = 0$, $p_{jj}^* > u^{-1}(u(0) - t)$ *and* $p_{ji}^* = a_j + c_o$. *The customer base is unique and stable and given by* $S_i^* = 1$.

Proof. We first show that it is a strictly dominating strategy for almost all subscribers to choose network i given $\mathbf{p}^*$. The net benefit

$$\begin{aligned}
& V(p_i^*, s_i) - k_i t - V(p_j^*, s_j) + (1 - k_i)t \\
> \ & V(p_i^*, s_i) - V(p_j^*, s_j) - t \\
= \ & s_i(u(p^m) - u(a_j + c_o) - t) + s_j(u(0) - u(p_{jj}^*) - t)
\end{aligned}$$

of choosing operator i over j for a consumer located at $k_i \in [0, 1)$ is strictly positive no matter the expectation s_i about i's market share. The strict inequality follows from $a_j + c_o \geq u^{-1}(u(p^m) - t)$ and $p_{jj}^* > u^{-1}(u(0) - t)$.

Next, we show that operator i has no incentive for deviating. $S_i^* = 1$ and $p_{ii}^* = p^m$ imply $\pi_i^* = \pi^m$. Substitute $a_j = p_{ji}^* - c_o$ and $a_i = c_t$ into the profit function $\pi_i(p_i)$ to get

$$\pi_i(p_i) = S_i[S_i(p_{ii} - c)D(p_{ii}) + S_j(p_{ij} - c)D(p_{ij}) + S_j(p_{ji}^* - c)D(p_{ji}^*)].$$

All deviations are unprofitable since for all p_i:

$$\begin{aligned}\pi_i^* - \pi_i(p_i) = \; & S_i^2(\pi^m - (p_{ii} - c)\, D\,(p_{ii})) + S_i S_j(\pi^m - (p_{ij} - c)\, D\,(p_{ij})) \\ + \; & S_i S_j(\pi^m - (p_{ji}^* - c) D(p_{ji}^*)) + S_j^2 \pi^m \geq 0,\end{aligned}$$

since p^m is the maximand of $(p - c)\, D\,(p)$.

We finally show that j cannot profit from gaining a positive market share, $S_j > 0$. First, consider the possibility of deviating to a stable interior equilibrium, $S_j(p_j, p_i^*, t) \in (0, 1)$ given by (3). By necessity, $p_{ji} < u^{-1}(u\,(p^m) - t) \leq a_j + c_o$, because $p_{ji} \geq u^{-1}(u\,(p^m) - t)$ implies $u(p_{ji}) \leq u\,(p^m) - t$ and thus $S_j(p_j, p_i^*, t) = 0$ since the denominator of (3) is positive in any stable equilibrium. Since $p_{ji} < a_j + c_o$, by necessity $(p_{jj} - c)D(p_{jj}) > 0$ because otherwise operator j's profit

$$\pi_j(p_j) = S_j\,[S_j\,(p_{jj} - c)\, D\,(p_{jj}) + S_i\,(p_{ji} - a_j - c_o)\, D(p_{ji})]$$

would be non-positive. $S_j(p_j, p_i^*, t)$ implicitly defines $p_{ji}(p_{jj})$ as a function of p_{jj}. Any movements of p_{ji} and p_{jj} along the *iso-market share curve*

$$u\,(p_{ji}) = \frac{S_j - S_i}{S_i} t + u\,(p_{ii}^*) + \frac{S_j}{S_i}(u(p_{ij}^*) - u\,(p_{jj})), \tag{4}$$

keep the market share of operator j constant. The slope of the iso-market share curve is

$$\frac{dp_{ji}}{dp_{jj}} = -\frac{S_j}{S_i}\frac{D(p_{jj})}{D(p_{ji})} \tag{5}$$

since $u' = -D$. Define profit $\widetilde{\pi}_j(p_{jj}) = \pi_j(p_{jj}, p_{ji}(p_{jj}))$. Along the iso-market curve, profits are affected as follows:

$$\begin{aligned}\partial\widetilde{\pi}_j/\partial p_{jj} = \; & S_j^2\,(D\,(p_{jj}) + (p_{jj} - c)\, D'\,(p_{jj})) \\ + \; & S_i S_j\,(D(p_{ji}) + (p_{ji} - a_j - c_o)\, D'(p_{ji}))\, \tfrac{dp_{ji}}{dp_{jj}} \\ = \; & S_j^2 D\,(p_{jj}) \left(1 - \tfrac{(p_{jj} - c)}{p_{jj}} \eta(p_{jj})\right) \\ - \; & S_i S_j D(p_{ji}) \left(1 - \tfrac{(p_{ji} - a_j - c_o)}{p_{ji}} \eta(p_{ji})\right) \tfrac{S_j D(p_{jj})}{S_i D(p_{ji})} \\ = \; & S_j^2 D\,(p_{jj}) \left[\tfrac{(p_{ji} - a_j - c_o)}{p_{ji}} \eta(p_{ji}) - \tfrac{(p_{jj} - c)}{p_{jj}} \eta(p_{jj})\right].\end{aligned} \tag{6}$$

Marginal profit is strictly negative for all $p_{jj} > c$ and $p_{ji} < a_j + c_o$. Thus

$$\widetilde{\pi}_j(p_{jj}) < \lim_{p_{jj}\to c} \widetilde{\pi}_j(p_{jj}) = S_i S_j \left(p_{ji}(c) - a_j - c_o\right) D\left(p_{ji}(c)\right) < 0,$$

and so a deviation by j to a stable equilibrium with $S_j(p_j, p_i^*, t) \in (0,1)$ is unprofitable since $\pi_j^* = 0$. The final possibility is a deviation by j to monopoly. Note that $S_j(p_j, p_i^*, t) = 1$ only if $V(p_j, 1) - t \geq V(p_i^*, 0)$, which is equivalent to $u(p_{jj}) \geq u(0) + t$. This is possible only if $p_{jj} = 0$. But $S_j = 1$ and $p_{jj} = 0$ imply $\pi_j = -cD(0) < 0 = \pi_j^*$, and a deviation to monopoly is not profitable, either. ■

Monopolization The proof that all possible equilibria imply monopolization by operator i, given the chosen access prices and given perfect network substitutability is collected in a series of seven claims. Claims 1 to 4 establish that $S_i^* = 1$ in every equilibrium. Claim 5 shows that $p_{ij}^* \leq c$ in every equilibrium with $S_i^* = 1$. Claims 6 and 7 demonstrate that $p_{ii}^* = p^m$ in every equilibrium with $S_i^* = 1$.

Claim 1 *If $p_{ji}^* > p^m$, then $S_i^* = 1$.*

Proof. For $p_{ji}^* > p^m$, i can ensure itself monopoly status and profit π^m by setting $p_{ii} = p^m < p_{ji}^*$ and $p_{ij} \leq p_{jj}^*$. Thus, $\pi_i^* \geq \pi^m$. Moreover, $\pi_i^* \leq \pi^m$ since $\pi_j^* \geq 0$, and $\pi_i^* + \pi_j^* \leq \pi^m$. Now, $\pi_i^* = \pi^m$ only if $S_i^* > 0$. To see that $p_{ji}^* > p^m$ implies $S_i^* = 1$, observe that industry profit

$$\begin{aligned}\pi_1^* + \pi_2^* &= (S_1^*)^2 \left(p_{11}^* - c\right) D(p_{11}^*) + (S_2^*)^2 \left(p_{22}^* - c\right) D(p_{22}^*) \\ &\quad + S_1^* S_2^* ((p_{12}^* - c)\, D\left(p_{12}^*\right) + (p_{21}^* - c)\, D\left(p_{21}^*\right)) < \pi^m\end{aligned}$$

for all $S_i^* \in (0,1)$ and $p_{ji}^* > p^m$. This is inconsistent with $\pi_i^* = \pi^m$ and $\pi_j^* = 0$. ■

Claim 2 *Assume that* $a_j \geq p^m - c_o$ *and* $a_i \leq c_t$. *Then,* $p^*_{ji} \leq p^m$ *and* $(p^*_{jj} - c)D(p^*_{jj}) > 0$ *imply* $S^*_i \in \{0, 1\}$.

Proof. There cannot exist any equilibrium with $(p^*_{jj} - c)D(p^*_{jj}) > 0$ and $S^*_j \in (0, 1)$. For $S^*_j \in (0, 1)$ to be an equilibrium, it must necessarily be the case that, $p^*_{21} < p^*_{11}$ and $p^*_{22} > p^*_{12}$, see Lemma 1. $\partial\widetilde{\pi}_j(p^*_{jj})/\partial p_{jj} < 0$ for every $(p^*_{jj} - c)D(p^*_{jj}) > 0$ and $p^*_{ji} \leq p^m \leq a_j + c_o$ imply that the proposed equilibrium configuration does not maximize profit. Consequently, $S^*_i \in \{0, 1\}$. ■

Claim 3 *Assume that* $a_j \geq p^m - c_o$ *and* $a_i \leq c_t$. *Then,* $p^*_{ji} \leq p^m$ *and* $(p^*_{jj} - c)D(p^*_{jj}) \leq 0$ *imply* $S^*_i \in \{0, 1\}$.

Proof. Neither $(p^*_{jj} - c)D(p^*_{jj}) \leq 0$, $p^*_{ji} \leq p^m \leq a_j + c_o$ and $S^*_j > 0$ nor $(p^*_{jj} - c)D(p^*_{jj}) = 0$, $p^*_{ji} < p^m \leq a_j + c_o$ and $S^*_j \in (0, 1)$ nor $(p^*_{jj} - c)D(p^*_{jj}) = 0$, $p^*_{ji} \leq p^m < a_j + c_o$ and $S^*_j \in (0, 1)$ can be equilibria because all cases would imply $\pi^*_j < 0$:

$$\pi^*_j \leq S^*_j[S^*_j(p^*_{jj} - c)D(p^*_{jj}) + S^*_i(p^*_{ji} - a_j - c_o)D(p^*_{ji})] < 0. \tag{7}$$

To complete the proof, we need to rule out the possibility that $(p^*_{jj} - c)D(p^*_{jj}) = 0$, $p^*_{ji} = p^m = a_j + c_o$ and $S^*_j \in (0, 1)$. In this case i can set $p_{ii} = p^m$, $p_{ij} < c$ corner the market and earn π^m. It cannot earn as much by letting j into the market since $p_{jj} \neq p^m$, hence $\pi^*_i + \pi^*_i < \pi^m$ and consequently $\pi^*_i < \pi^m$. ■

Claim 4 *Assume that* $a_j \geq p^m - c_o$ *and* $a_i \leq c_t$. *Then,* $p^*_{ji} \leq p^m$ *implies* $S^*_i > 0$.

Proof. Obviously, $S^*_j = 1$ can be an equilibrium only if $p^*_{jj} \geq c$, otherwise j would run a deficit in equilibrium. Moreover, $p^*_{ji} \leq c$ in any equilibrium $S^*_j = 1$. Otherwise, i can set $p_{ii} \in (c, p^*_{ji})$ and $p_{ij} < p^*_{jj}$, become the monopolist and earn profit $(p_{ii} - c)D(p_{ii}) > 0$, which is strictly preferred to having zero market share. Finally, $S_i^* = 1$ implies $p_{ii}^* \leq p^m$. If $p_{ii}^* > p^m$, i could lower p_{ii} to p^m, retain monopoly power and obtain the higher monopoly rent π^m. Hence, $p_{ji}^* \leq c \leq p_{jj}^* \leq p^m$ in any equilibrium with $S_j^* = 1$. Consider i's policy $p_{ii} > c \geq p_{ji}^*$ and $p_{ij} < p_{jj}^*$. In this case $S_i \in (0,1)$. Moreover,

$$\frac{\partial \pi_i}{\partial p_{ij}}|_{p_{ii}>c, p_{ij}=p_{jj}^*} = \frac{\partial S_i}{\partial p_{ij}} \left[(p_{jj}^* - a_i - c_o) D(p_{jj}^*) + (a_j - c_t)\, D(p_{ji}^*) \right] < 0,$$

since $u(p_{ii}) < u(p_{ji}^*)$ and $u(p_{ij}) = u(p_{jj}^*)$ imply $S_i = 0$, and $\partial S_i / \partial p_{ij}|_{p_{ii}>c, p_{ij}=p_{jj}^*}$ $= -D(p_{jj}^*)/(u(p_{ji}^*) - u(p_{ii})) < 0$. Thus, it is strictly profitable for i to lower p_{ij} slightly below p_{jj}^* and gain a positive market share. ■

Claim 5 $S_i^* = 1$ *implies* $p_{ij}^* \leq c$.

Proof. First, of all $S_i^* = 1$ implies $p_{ii}^* \geq c$. Otherwise i would run a strict loss. Any combination $p_{ii}^* \geq c$ and $p_{ij}^* > c$ allows j to charge a price $p_{jj} \in (c, p_{ij}^*)$ and $p_{ji} < p_{ii}^*$, become the monopolist and earn profit $\pi_j = (p_{jj} - c) D(p_{jj}) > 0$. This, of course, is better than having no market share at all. ■

Claim 6 $S_i^* = 1$ *implies* $p_{ii}^* \leq p^m$.

Proof. If $p_{ii}^* > p^m$, i could lower p_{ii} to p^m, retain monopoly power and obtain the higher monopoly rent π^m. ■

Claim 7 *Assume that* $a_j \geq \overline{a} = c_t + 2\,(p^m - c) + cD\,(0)\,/D\,(p^m)$ *and* $a_i \leq c_t$. *Then,* $S_i^* = 1$ *implies* $p_{ii}^* \geq p^m$.

Proof. The proof amounts to showing that $a_i \leq c_t$, $a_j \geq \overline{a}$ and $S_i^* = 1$ jointly imply $p_{ji}^* \geq p^m$. Any equilibrium with $S_i^* = 1$, $p_{ji}^* \geq p^m$ and $p_{ii}^* < p^m$ is impossible, because i then could increase profits to the monopoly rent by increasing p_{ii} slightly, but keeping it below p^m, while maintaining $p_{ij}^* \leq p_{jj}^*$, and thus monopoly status.

Next, $a_i \leq c_t$, $a_j \geq \overline{a}$ and $S_i^* = 1$ jointly imply $p_{ji}^* \geq p^m$: Any equilibrium with $S_i^* = 1$ and $p_{ji}^* < p^m$ necessarily has $p_{ii}^* \geq p_{ji}^*$. To see this, note that i cannot gain anything by setting $p_{ii} < p_{ji}^* < p^m$ because any price increase towards p_{ji}^* would still render i the monopoly power and thus increase profits per outgoing call. Note also that i would cease to be a certain monopolist should $p_{ii} > p_{ji}^*$, see Lemma 1. So $S_i^* = 1$ and $p_{ji}^* < p^m$ imply $\pi_i^* = (p_{ji}^* - c)D(p_{ji}^*)$. Consider i's incentive for setting a price $p_{ii} \in (p_{ji}^*, p^m)$, while holding $u(p_{ij}^*) > u(p_{jj}^*)$ which would imply a market share $S_i \in (0,1)$:

$$\begin{aligned}\lim_{p_{ii}\downarrow p_{ji}^*} \tfrac{\partial \pi_i}{\partial p_{ii}}\big|_{p_{ii}>p_{ji}^*, p_{ij}^*<p_{jj}^*} &= D(p_{ji}^*)(1 - \tfrac{(p_{ji}^*-c)}{p_{ji}^*}\eta(p_{ji}^*)) \\ &+ \tfrac{D(p_{ji}^*)}{u(p_{ij}^*)-u(p_{jj}^*)}\left[(p_{ij}^* - a_i - c_o)D(p_{ij}^*) + (a_j - c_t - 2(p_{ji}^* - c))D(p_{ji}^*)\right]\end{aligned}$$

The first term in square brackets is strictly positive for all $p_{ji}^* < p^m$. The second term in square brackets is positive for all $a_j \geq \overline{a}$ and $p_{ji}^* < p^m$. Hence, i has an incentive to increase its price and sacrifice its monopoly market share given $a_i \leq c_t$, $a_j \geq \overline{a}$, and $p_{ji}^* < p^m$. ■

Proof of Proposition 2

We know from Claims 1 to 4 in the proof of Proposition 1 that for $a_i = c_t$ and $a_j \geq p^m - c_o$, all pure strategy equilibria have $S_i^* = 1$. Moreover, $p_{ii}^* \leq p^m$; see Claim 6. Consider the following set of call prices, $c \leq p_{ii}^* = p_{ji}^* \leq p^m$, $p_{ij}^* = 0$ and $p_{jj}^* = \epsilon \in (0, p_{ji}^*)$. In this case $S_i^* = 1$. We further know, see the proof of Proposition 8, that j cannot strictly profit by gaining positive market share as long as $a_i \leq c_t$, $a_j \geq p^m - c_o$, $p_{ij}^* \leq c$ and $p_{ii}^* \leq p^m$. Considering next i's incentive to deviate, the first thing to observe is that i has no incentive to lower p_{ii} below p_{ji}^* as there is nothing to gain in terms of market share, but the price is lowered even further below p^m. If i increases p_{ii} above p_{ji}^*, it is necessary to keep $p_{ij} < p_{jj}^*$, otherwise i loses its entire market share to j. At $p_{ii} > p_{ji}^* \geq c$ and $p_{ij} < p_{jj}^* < p_{ji}^* = p_{ii}^*$, $S_i \in (0,1)$. Fix the market share

S_i and consider a move along the iso-market line, see (6). Along this line, profits are affected as follows

$$\widetilde{\pi}_i'(p_{ii}) = S_i^2 D\left(p_{ii}\right)\left[\frac{(p_{ij}-c)}{p_{ij}}\eta(p_{ij}) - \frac{(p_{ii}-c)}{p_{ii}}\eta(p_{ii})\right] \leq 0.$$

The inequality follows from the fact that $p_{ii} > c$, $p_{ij} < p_{ii}$, and $(p-c)\,\eta(p)/p$ is increasing in p. Hence, for any (p_{ii}, p_{ij}) such that $S_i \in (0,1)$, we have

$$\pi_i \leq S_i^2(p_{ji}^* - c)D(p_{ji}^*) + S_i S_j(p_{jj}^* - c)D(p_{jj}^*) + S_i S_j(a_j - c_t)D(p_{ji}^*)$$

since $p_{ij} \rightarrow p_{jj}^*$ as $p_{ii} \rightarrow p_{ji}^*$ along the iso-market line. At the proposed monopoly $\pi_i^* = (p_{ji}^* - c)D(p_{ji}^*)$, hence

$$\pi_i^* - \pi_i \geq S_j\left[(1+S_i)(p_{ji}^* - c)D(p_{ji}^*) - S_i(p_{jj}^* - c)D(p_{jj}^*) - S_i(a_j - c_t)D(p_{ji}^*)\right].$$

The term in square brackets is decreasing in a_j, and since $a_j < \overline{a}$,

$$\begin{aligned}\pi_i^* - \pi_i \;\; &\geq \;\; S_j\left((1+S_i)(p_{ji}^* - c)D(p_{ji}^*) - S_i(p_{jj}^* - c)D(p_{jj}^*)\right) \\ &\quad -S_i S_j(2(p^m - c) + c\frac{D(0)}{D(p^m)})D(p_{ji}^*)\end{aligned}$$

If $p_{ji}^* \rightarrow p^m$ and $p_{jj}^* \rightarrow 0$, the term on the right hand side of the inequality converges to $S_j^2(p^m - c)D(p^m) > 0$. Hence, there exists a p_{ji}^* sufficiently close to and below p^m and a p_{jj}^* sufficiently close to, but above zero such that i prefers to hold on to his monopoly power at $p_{ii}^* = p_{ji}^* < p^m$. ■

Proof of Proposition 3

We first demonstrate that $p_1^* = p_2^* = p^*$. There can be no equilibrium in which $p_i^* > p_j^* > c$ [$p_i^* > p_j^* = c$]. In equilibrium, $S_i^* = 0$ and $\pi_i^* = 0$ [$\pi_j^* = 0$], hence i [j] could strictly increase profits by cutting [raising] prices to $p_i \in (c, p_j^*)$ [$p_j \in (c, p_i^*)$]. There is no equilibrium in which $p_i^* > p_j^*$ and $p_j^* < c$ since then $S_j^* = 1$ and $\pi_j^* < 0$.

We next demonstrate that $p^* = c$. There can be no equilibrium with $p^* < c$ since this would imply negative industry profits, hence at least one firm running at a loss. Finally, there can be no equilibrium with $p^* > c$, either. Industry profits would then be equal to $(p^* - c)D(p^*)$, hence at least one firm would have profits strictly below $(p^* - c)D(p^*)$. This firm could deviate and steal the entire market by lowering prices below p^*. The only equilibrium candidate left is $p_1^* = p_2^* = c$. ■

Proof of Proposition 4

For the industry to reach the monopoly profit, either one operator has the monopoly ($S_i^* = 1$) and charges the monopoly price on on-net calls ($p_{ii} = p^m$), or both operators have a positive market share ($S_i^* \in (0, 1)$) and charge the monopoly price on all products ($p_{11}^* = p_{12}^* = p_{21}^* = p_{22}^* = p^m$). In the last case, the operators divide the market and obtain half the monopoly profit each ($\pi_1 = \pi_2 = \pi^m/2$). This cannot be an equilibrium. By lowering the off-net price below the monopoly price ($p_{ij} \in (0, p^m)$), any single operator can corner the market and reap the entire monopoly rent. Assume therefore that $p_{ii}^* = p^m$ and $S_i^* = 1$. Consider j's alternative policy $p_{ji} \in (a_j + c_o, p^m)$ and $p_{jj} > c \geq p_{ij}^*$. In this case $S_j \in (0, 1)$ and

$$\pi_j = S_j^2(p_{jj} - c)D(p_{jj}) + S_i S_j[(p_{ji} - a_j - c_o)D(p_{ji}) + (a_i - c_t)D(p_{ij})] > 0.$$

Thus $p_{ii}^* = p^m$ and $S_i^* = 1$ cannot be an equilibrium, either. ■

Proof of Proposition 5

The first claim proves the existence of a competitive equilibrium under the assumption that $a_1 = a_2 \geq c_t$.

Claim 8 *Assume that $a_1 = a_2 \geq c_t$. Then, $p_{11}^* = p_{21}^* = p_{12}^* = p_{22}^* = c$ is an equilibrium.*

Proof. At the proposed equilibrium, $\pi_i^* = 0$. Neither a deviation to $p_{ii} < c$ nor to $p_{ii} = c$ and $p_{ij} \neq c$ can be profitable. In both cases $S_i \in \{0, 1\}$. As $\pi_i = 0$ when $S_i = 0$ and $\pi_i = (p_{ii} - c)D(p_{ii}) \leq 0$ when $S_i = 1$, the deviation cannot be strictly profitable. Any profitable deviation must therefore be to $p_{ii} > c$. In this case it cannot be strictly profitable to also set $p_{ij} \geq c$, because then $S_i = 0$ and $\pi_i = 0$. We finally need to check the profitability of the deviation $p_{ii} > c > p_{ij}$. In this case $S_i \in (0, 1)$ due to the negative network externalities. Consider the marginal effect on profit of a movement among the iso-market-share curve $p_{ij}(p_{ii})$, see (4):

$$\widetilde{\pi}_i'(p_{ii}) = S_i^2 D(p_{ii}) \left[\frac{p_{ij} - c - (a_i - c_t)}{p_{ij}} \eta(p_{ij}) - \frac{(p_{ii} - c)}{p_{ii}} \eta(p_{ii}) \right] \leq 0$$

because $p_{ij} < c$, $p_{ii} > c$ and $a_i \geq c_t$. Note that $\lim_{p_{ii} \to c} p_{ij}(p_{ii}) = c$, and so

$$\pi_i(p_{ii}) < \lim \pi_i(p_{ii})_{p_{ii} \to c} = S_i S_j (a_j - a_i) D(c) = 0$$

by reciprocity of the access price. Any deviation $p_{ii} > c > p_{ij}$ is unprofitable, which completes the proof. ■

The final sequence of claims is used to prove that all possible equilibria are efficient. We invoke Lemma 2 and rule out equilibria with positive network externalities. Claim 9 demonstrates that under reciprocal access prices $a_1 = a_2 = a \geq c_t$, all equilibria in which one operator corners the market necessarily are efficient. Claim 10 proves that there is at most one equilibrium in which the two operators divide the market equally under reciprocal access prices $a_1 = a_2 = a \geq c_t$. This equilibrium is efficient. Finally, we show that there is no equilibrium with negative network externalities and reciprocal access prices $a_1 = a_2 = a \geq c_t$.

Claim 9 *Let $a_1 = a_2 = a \geq c_t$. Then, $S_i^* = 1$ implies $p_{ii}^* = c \geq p_{ij}^*$.*

Proof. Note first that $S_i^* = 1$ implies $p_{ii}^* \geq c \geq p_{ij}^*$. Obviously, $p_{ii}^* \geq c$ since $p_{ii}^* < c$ and $S_i^* = 1$ imply $\pi_i^* < 0$. $S_i^* = 1$ and $p_{ij}^* > c$ cannot simultaneously hold because j could then enter as a monopolist and earn a strictly positive profit by setting $p_{jj} \in (c, p_{ij}^*)$ and $p_{ji} \leq p_{ii}^*$. Suppose therefore that $S_i^* = 1$, $p_{ii}^* > c$ and $p_{ij}^* \leq c$. In this case operator j could enter at $(p_{jj} - c)D(p_{jj}) > 0$ and $p_{ji} \in (c, p_{ii}^*)$, induce negative network externalities, get a market share $S_j \in (0, 1)$ and profit

$$\pi_j = S_j \left[S_j \left(p_{jj} - c \right) D \left(p_{jj} \right) + S_i (p_{ji} - c) D \left(p_{ji} \right) + S_i \left(a - c_t \right) \left(D(p_{ij}^*) - D \left(p_{ji} \right) \right) \right],$$

which is strictly positive since $p_{jj} > c$, $p_{ji} > c$, $a \geq c_t$ and $D(p_{ij}^*) \geq D(c) > D\left(p_{ji}\right)$. Thus, $S_i^* = 1$ implies $p_{ii}^* = c \geq p_{ij}^*$ if $a_1 = a_2 = a \geq c_t$. ∎

Claim 10 *If* $u(p_{11}^*) = u(p_{21}^*) = u(p_1^*)$ *and* $u(p_{12}^*) = u(p_{22}^*) = u(p_2^*)$, *then* $p_1^* = p_2^* = c$.

Proof. We first show that $p_1^* = p_2^* = p^*$. Industry profit is

$$\pi_1^* + \pi_2^* = \tfrac{1}{2}((p_1^* - c) D \left(p_1^* \right) + (p_2^* - c) D(p_2^*)).$$

It cannot be the case that $(p_1^* - c) D \left(p_1^* \right) \leq 0$ and $(p_2^* - c) D \left(p_2^* \right) \leq 0$ with one inequality strict, because industry profit would then be negative. Either $(p_1^* - c) D \left(p_1^* \right) = (p_2^* - c) D \left(p_2^* \right) p_2^* = 0$, $(p_1^* - c) D \left(p_1^* \right) > 0$ or $(p_2^* - c) D \left(p_2^* \right) > 0$. Suppose first that $(p_i^* - c) D \left(p_i^* \right) > (p_j^* - c) D(p_j^*)$, and $(p_i^* - c) D \left(p_i^* \right) > 0$. In this case operator i could corner the market by lowering its on-net price slightly from p_i^* and earn a profit $\pi_i \approx (p_i^* - c) D \left(p_i^* \right) > \pi_1^* + \pi_2^* \geq \pi_i^*$. Suppose next that $(p_1^* - c) D \left(p_1^* \right) = (p_2^* - c) D(p_2^*)$. Either $p_1^* = p_2^*$, and we are done, or $p_i^* > p^m > p_j^*$, in which case i could lower its on-net price towards the monopoly price and earn $\pi_i > (p_i^* - c) D \left(p_i^* \right) = \pi_1^* + \pi_2^* \geq \pi_i^*$. Hence, $p_1^* = p_2^* = p^*$.

When both operators charge p^*, $\pi_i^* = \frac{1}{4}\left[2(p^* - c) + a_j - a_i\right] D\left(p^*\right)$, with $p^* \geq c$. If $p^* > c$, at least the operator with an unfavourable access price $a_i \geq a_j$ has an incentive to corner the market by reducing its price on off-net calls below p^*, thus earning $\pi_i = D\left(p^*\right)\left(p^* - c\right) > \pi_i^*$. ■

Finally, there can be no equilibrium with negative network externalities.

Claim 11 *Assume that $a_2 \geq c_t$ and $a_1 \geq c_t$. Any equilibrium with negative network externalities then satisfies $p_{11}^* > c$, $p_{22}^* > c$, $p_{12}^* > a_1 + c_o$, $p_{21}^* > a_2 + c_o$.*

Proof. Note first that it cannot be the case that both $p_{11}^* \leq c$ and $p_{22}^* \leq c$. Since $p_{21}^* < p_{11}^*$ and $p_{12}^* < p_{22}^*$ by the assumption of negative network externalities, industry profit

$$\begin{aligned}\pi_1^* + \pi_2^* &= \left(S_1^*\right)^2 \left(p_{11}^* - c\right) D(p_{11}^*) + \left(S_2^*\right)^2 \left(p_{22}^* - c\right) D(p_{22}^*) \\ &\quad + S_1^* S_2^* ((p_{12}^* - c) D\left(p_{12}^*\right) + (p_{21}^* - c) D\left(p_{21}^*\right))\end{aligned}$$

then would be negative, and so at least one operator would run at a strict loss. Assume wlog that $p_{ii}^* > c$. Define implicitly $p_{ij}(p_{ii})$ by means of the iso-market share relation (4) where $S_i = S_i^*$, and let $\widetilde{\pi}_i(p_{ij}) = \pi_i(p_{ii}(p_{ij}), p_{ij})$. Marginal profit

$$\widetilde{\pi}_i'(p_{ij}^*) = S_i^* S_j^* D(p_{ij}^*)[\frac{p_{ii}^* - c}{p_{ii}^*}\eta(p_{ii}^*) - \frac{(p_{ij}^* - a_i - c_o)}{p_{ij}^*}\eta(p_{ij}^*)]$$

is strictly positive for all $p_{ij}^* \leq a_i + c_o$ since $p_{ii}^* > c$. Thus, $p_{ii}^* > c$ implies $p_{ij}^* > a_i + c_o$. But then even $p_{jj}^* > c$, since $p_{jj}^* > p_{ij}^* > a_i + c_o \geq c$. One can then apply the same reasoning used to prove that $p_{ii}^* > c$ implies $p_{ij}^* > a_i + c_o$, to show that $p_{jj}^* > c$ implies $p_{ji}^* > a_j + c_o$. ■

From the above claim we observe that equilibrium prices are strictly positive. Thus, the first-order condition $\widetilde{\pi}_i'(p_{ij}^*) = 0$ must be satisfied, which implies the Ramsey relation

$$\frac{p_{ii}^* - c}{p_{ii}^*}\eta(p_{ii}^*) = \frac{(p_{ij}^* - a_i - c_o)}{p_{ij}^*}\eta(p_{ij}^*).$$

Subtract $\frac{p_{ij}^*-c}{p_{ij}^*}\eta(p_{ij}^*)$ from both sides of the Ramsey relation and rearrange:

$$\frac{p_{ii}^* - c}{p_{ii}^*}\eta(p_{ii}^*) - \frac{p_{ij}^* - c}{p_{ij}^*}\eta(p_{ij}^*) = -\frac{(a_i - c_t)}{p_{ij}^*}\eta(p_{ij}^*).$$

The right-hand side is non-positive, and since $(p-c)\eta(p)/p$ is increasing in $p > c$, it must be the case that $p_{ij}^* \geq p_{ii}^*$ in every interior equilibrium with $a_i \geq c_t$. Thus $p_{12}^* \geq p_{11}^*$ and $p_{21}^* \geq p_{22}^*$. Combine this with $p_{21}^* < p_{11}^*$ and $p_{12}^* < p_{22}^*$ from the assumption of negative network externalities to get $p_{12}^* \geq p_{11}^* > p_{21}^* \geq p_{22}^* > p_{12}^*$, a contradiction. ■

Proof of Proposition 6

The proof is done by going through the possible equilibrium customer bases case by case, see Lemma 1 for a description of the possible configurations. Also, we invoke Lemma 2 and rule out equilibria with positive network externalities. The first Claim proves that monopoly implies an on-net price below the off-net price if $a_1 = a_2 = a < c_t$.

Claim 12 *If $a_1 = a_2 = a < c_t$, then $S_i^* = 1$ implies $p_{ii}^* > p_{ij}^*$.*

Proof. Note first that $S_i^* = 1$ implies $p_{ii}^* \geq c \geq p_{ij}^*$. Obviously, $p_{ii}^* \geq c$ since $p_{ii}^* < c$ and $S_i^* = 1$ imply $\pi_i^* < 0$. $S_i^* = 1$ and $p_{ij}^* > c$ cannot simultaneously hold because j could then corner the market and earn a strictly positive profit by setting $p_{jj} \in (c, p_{ij}^*)$ and $p_{ji} \leq p_{ii}^*$. We finally show that $S_i^* = 1$ and $p_{ii}^* = p_{ij}^* = c$ cannot possibly hold. Assume that operator j in that case enters at $u(p_{ji}) = 2u(c) - u(p_{jj})$, which implicitly defines $p_{ji}(p_{jj})$ from the iso-market share relation (4). Operator j gets half the customers and earns

$$4\widetilde{\pi}_j(p_{jj}) = (p_{jj} - c)\, D\,(p_{jj}) + (p_{ji}(p_{jj}) - a - c_o)D\,(p_{ji}(p_{jj})) + (a - c_t)D\,(c)$$

Now,

$$4\widetilde{\pi}_j'(p_{jj}) = D\left(p_{jj}\right)\left(\frac{(p_{ji}(p_{jj}) - a - c_o)}{p_{ji}(p_{jj})}\eta\left(p_{ji}(p_{jj})\right) - \frac{(p_{jj} - c)}{p_{jj}}\eta\left(p_{jj}\right)\right)$$

and so $4\widetilde{\pi}_j'(c) = D\left(c\right)\eta\left(c\right)\left(c_t - a\right)/c > 0$. Consequently, $\widetilde{\pi}_j(p_{jj}) > \widetilde{\pi}_j(c) = 0$ for some p_{jj} slightly above c and $p_{ji}(p_{jj})$ slightly below c. Network j can profitably enter at $p_{ii}^* = p_{ij}^* = c$ if $a < c_t$. Hence, $S_i^* = 1$ cannot hold. ■

Consider next the case in which both operators have a positive market share, either they split the market, $u(p_{11}^*) = u(p_{21}^*)$ and $u(p_{12}^*) = u\left(p_{22}^*\right)$, or there are negative network externalities, $u(p_{11}^*) < u(p_{21}^*)$ and $u(p_{12}^*) > u\left(p_{22}^*\right)$. We first show that $p_{ii}^* \geq c$. Suppose on the contrary that $p_{ii}^* < c$. By necessity, $p_{ij}^* > a + c_o$ since otherwise $\pi_i^* < 0$. Recall the iso-market share relation

$$u(p_{ij}) = u(p_{jj}^*) + \frac{S_i^*}{S_j^*}(u(p_{ji}^*) - u(p_{ii}))$$

which implicitly defines $p_{ij}(p_{ii})$, with $p_{ij}'(p_{ii}) = -\frac{S_i^*}{S_j^*}\frac{D(p_{ii})}{D(p_{ij}(p_{ii}))}$ and $p_{ij}(p_{ii}^*) = p_{ij}^*$. Let $\widetilde{\pi}_i(p_{ii}) = \widetilde{\pi}_i(p_{ii}, p_{ij}(p_{ii}))$. The marginal profit

$$\widetilde{\pi}_i'(p_{ii}^*) = (S_i^*)^2 D\left(p_{ii}^*\right)\left[\frac{(p_{ij}^* - a - c_o)}{p_{ij}^*}\eta(p_{ij}^*) - \frac{(p_{ii}^* - c)}{p_{ii}^*}\eta\left(p_{ii}^*\right)\right]$$

is strictly positive for all $p_{ii}^* < c$ and $p_{ij}^* > a + c_o$, and so for all $p_{ii}^* < c$ network i strictly profits from an increase in the on-net price above p_{ii}^*. Note that $p_{ij}^* < p_{ii}^*$ for all $p_{ij}^* < c$ since $p_{ii}^* \geq c$. Consider therefore the case with $p_{ii}^* \geq c$ and $p_{ij}^* \geq c > a + c_o$. In interior equilibrium, $\widetilde{\pi}_i'(p_{ii}^*) = 0$, which from above implies the equilibrium Ramsey relation

$$\frac{p_{ii}^* - c}{p_{ii}^*}\eta\left(p_{ii}^*\right) = \frac{p_{ij}^* - a - c_o}{p_{ij}^*}\eta(p_{ij}^*).$$

Subtract $\frac{p_{ij}^* - c}{p_{ij}^*}\eta(p_{ij}^*)$ from both sides and simplify to

$$\frac{p_{ii}^* - c}{p_{ii}^*}\eta\left(p_{ii}^*\right) - \frac{p_{ij}^* - c}{p_{ij}^*}\eta(p_{ij}^*) = \frac{c_t - a}{p_{ij}^*}\eta(p_{ij}^*) > 0.$$

The strict inequality follows from $a < c_t$ and $u(p_{ij}^*) > u(p_{jj}^*) \geq 0$ which in turn implies $D(p_{ij}^*) > 0$. By assumption, $\frac{p-c}{p}\eta(p)$ is non-decreasing in p for all $p \geq c$. Hence $\frac{p_{ii}^*-c}{p_{ii}^*}\eta(p_{ii}^*) \leq \frac{p_{ij}^*-c}{p_{p_{ij}^*}}\eta(p_{ij}^*)$ for all $p_{ij}^* \geq p_{ii}^* \geq c$, and it must necessarily be the case that $p_{ij}^* < p_{ii}^*$. ■

Proof of Proposition 7

From the Proof of Proposition 3 we know that the only possible equilibrium when each operator is forced to charge a uniform price on all its products, is $p_1^* = p_2^* = c$. Under reciprocal access prices, $\pi_1^* = \pi_2^* = 0$. No operator has an incentive to increase its price as this would only lead to a complete eradication of market share. No operator would lower its price as it would become a monopolist pricing below marginal cost. ■